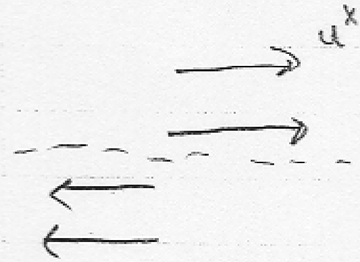
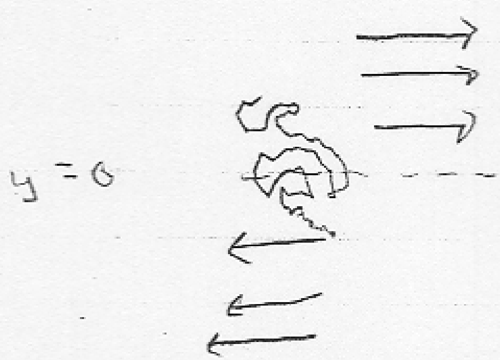
1

Shear Viscosity - Basics:

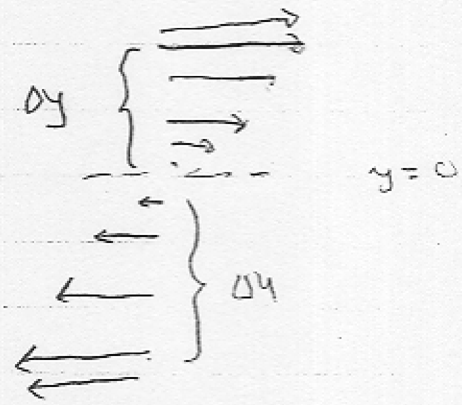


$$\frac{F^x}{A} = -\eta \frac{\partial u^x}{\partial y}$$

Now momentum of lower steam diffuses



so after some time



Find:

Ideal Fluid ↓

$$\Delta p^x = \int_{y>0} \Delta T^{0x} = \int_0^{\Delta y} dy\, (e+p)\, \Delta u^x$$

$$= -\int_0^{\Delta y} dy\, (e+p) \frac{\partial u^x}{\partial y}\, y$$

$$\Delta p^x = -(e+p) \frac{\partial u^x}{\partial y} \frac{(\Delta y)^2}{2}$$

(2)

Kinetics

$$\ell_{mfp} \sim \frac{1}{n\sigma}$$

$$\tau_c \sim \frac{1}{n\sigma v_{th}}$$

So

$$(\Delta y)^2 \sim \frac{\Delta t}{\tau_c} (\ell_{mfp})^2$$

Define the momentum diffusion coefficient

$$(\Delta y)^2 \equiv 2 D_\eta \Delta t \qquad D_\eta \sim \frac{\ell_{mfp}^2}{\tau_c}$$

So

$$\frac{\Delta p}{\Delta t} = - \overbrace{(e+p) D_\eta}^{\equiv \eta} \frac{\partial u^x}{\partial y}$$

i.e.

$$D_\eta \equiv \frac{\eta}{e+p} \qquad \eta \sim \frac{(e+p) v_{th}}{n\sigma}$$

So $(e+p)/n \cdot v_{th} \sim p_{typ}$ and find

$$\boxed{\eta \sim \frac{p_{typ}}{\sigma}}$$

High temperature QCD (Above Deconfinement)

$$p_{typ} \sim T$$

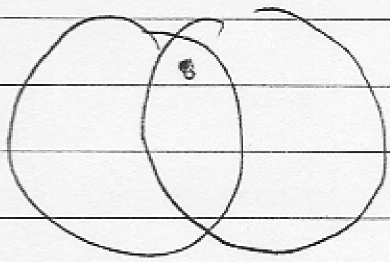
And

$$\sigma \sim \frac{\alpha_s^2}{T^2}$$

So

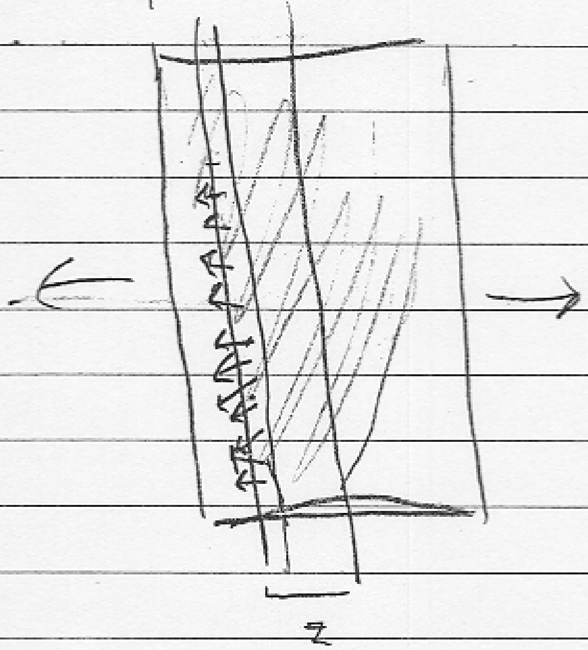
$$\eta \sim \frac{T^3}{\alpha_s^2}$$

For Hydro need:



$$\frac{l_{mfp}}{L} \ll 1$$

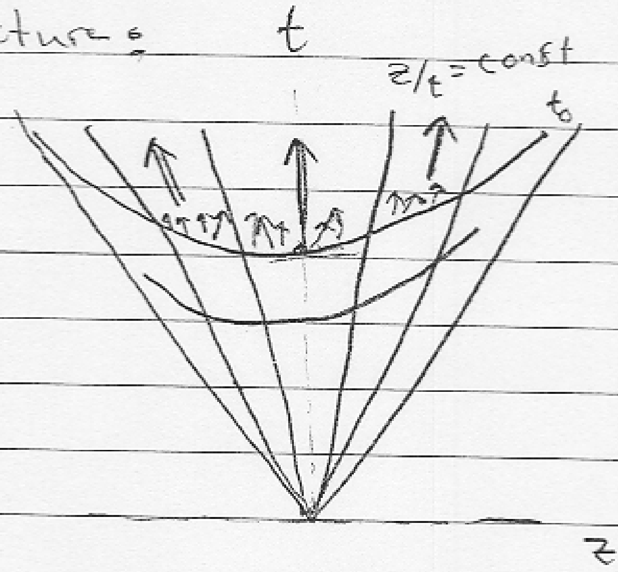
More importantly the system is expanding like mad



After a time t_0: $u^\mu = (u^0, \vec{u}) = (\gamma, \gamma$

$$\frac{z}{t_0} \simeq \frac{p_z}{E} \simeq v_f^z \leftarrow \text{Fluid Velocity} = \frac{u^z}{u^0}$$

Picture:



Expansion rate:

$$\frac{1}{V}\frac{dV}{dt} = \partial_\mu u^\mu \Big|_{z=0} = \partial_z u^z = \frac{1}{t_0}$$

So we have:

$$\frac{\tau_{coll}}{t_0} \ll 1 \qquad \text{or} \qquad \frac{1}{c_s^2}\frac{\eta}{(e+p)}\frac{1}{t_0} \ll 1$$

using $(e+p) = sT$ $\quad v_{th}^2 \sim c_s^2$

$$\frac{1}{c_s^2}\frac{\eta}{s}\frac{1}{\tau_0 T} \ll 1$$

$T \sim 300$ MeV

$c_s^2 \sim \sqrt{\frac{1}{3}}$

$\tau_0 \sim 1$ fm

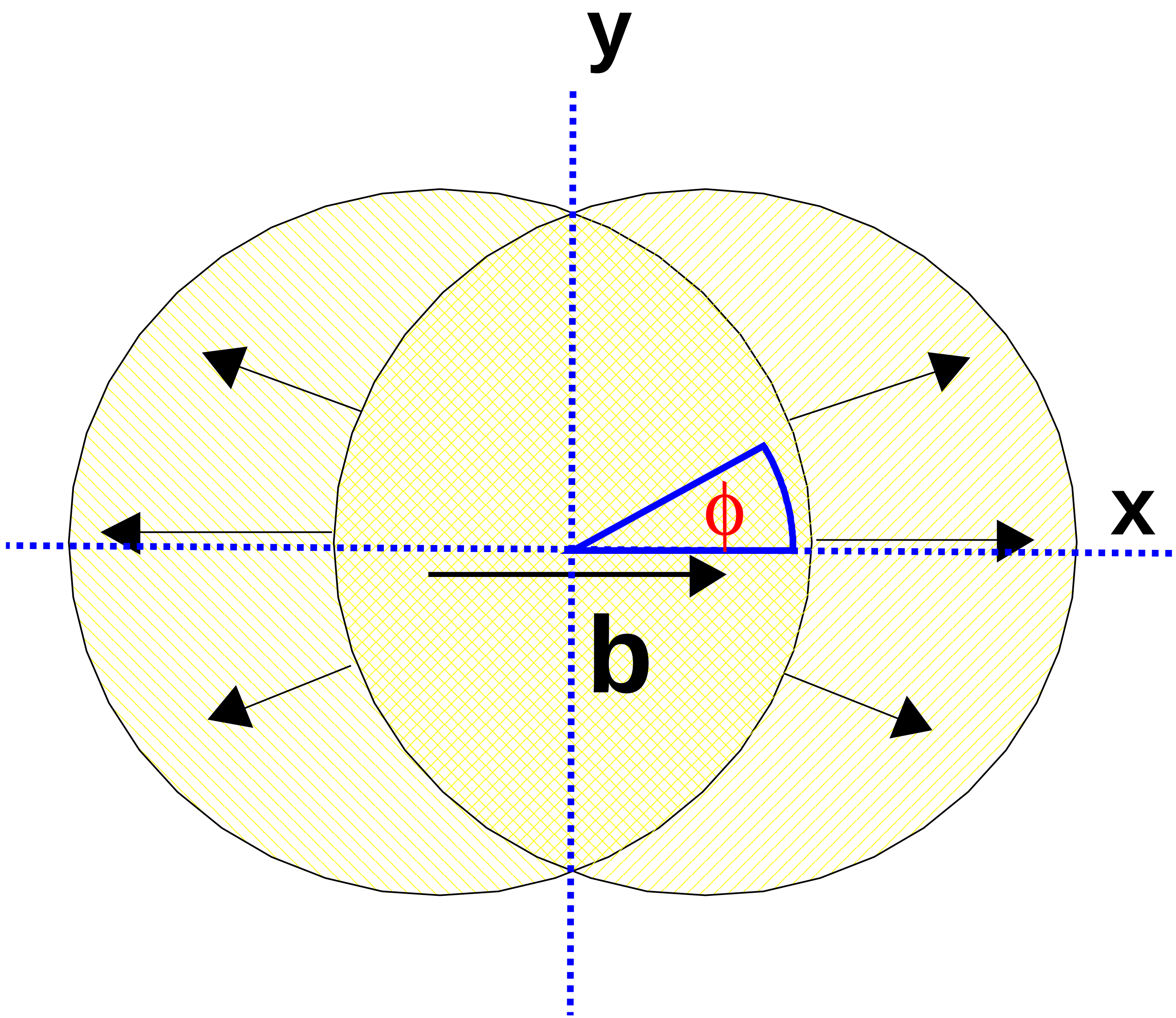
Now

$$0.36 \left(\frac{\eta/s}{0.3}\right)\left(\frac{1\,fm}{\tau_0}\right)\left(\frac{300\,MeV}{T_0}\right) \ll 1$$

There is good! evidence that there is hydro at RHIC

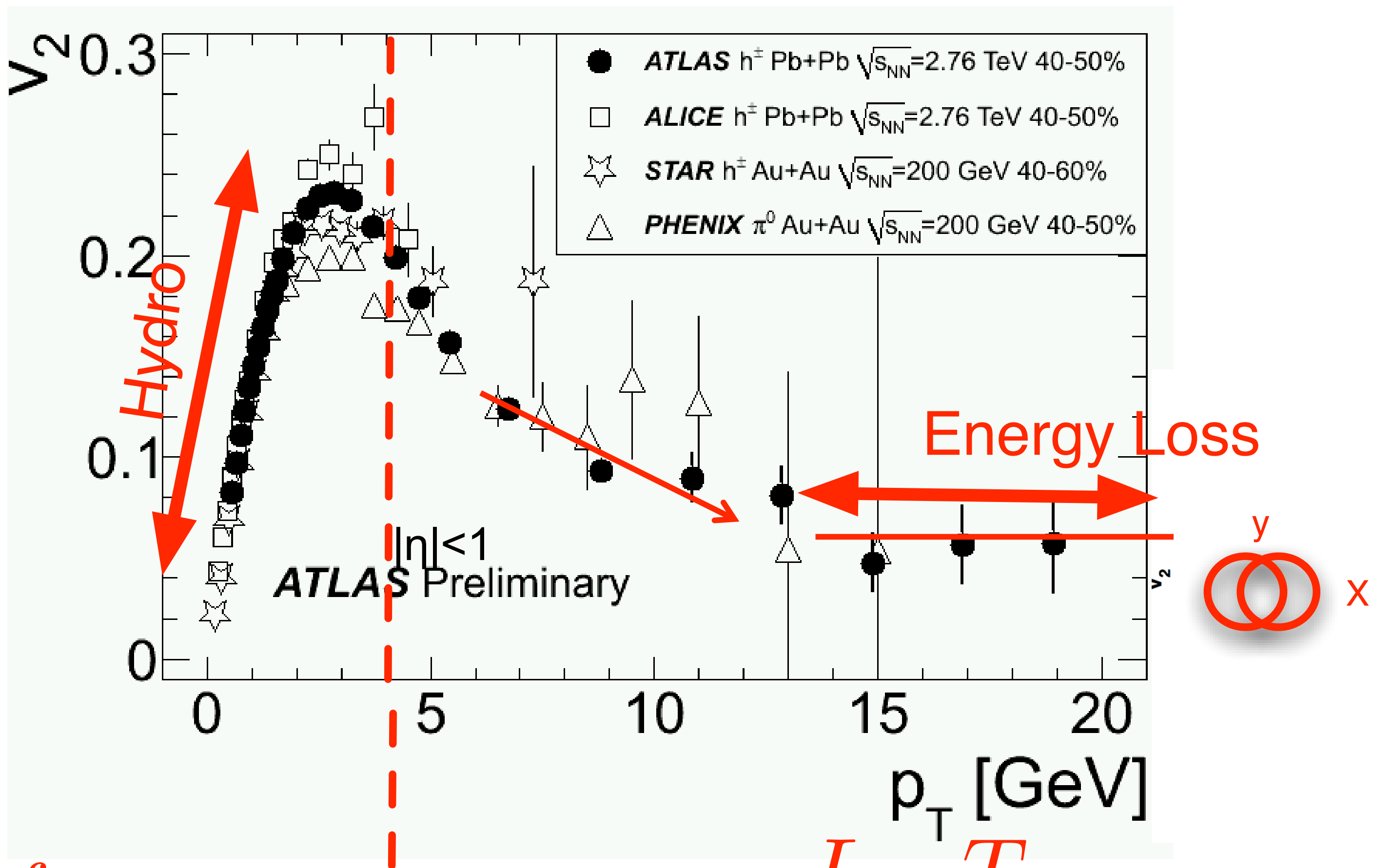
See Next Slides

From Krishna's Talk



$$\frac{dN}{dp_T d\phi} = \frac{dN}{dp_T}\left(1 + 2v_2(p_T)\cos 2\phi_{\mathbf{p}} + \ldots\right)$$

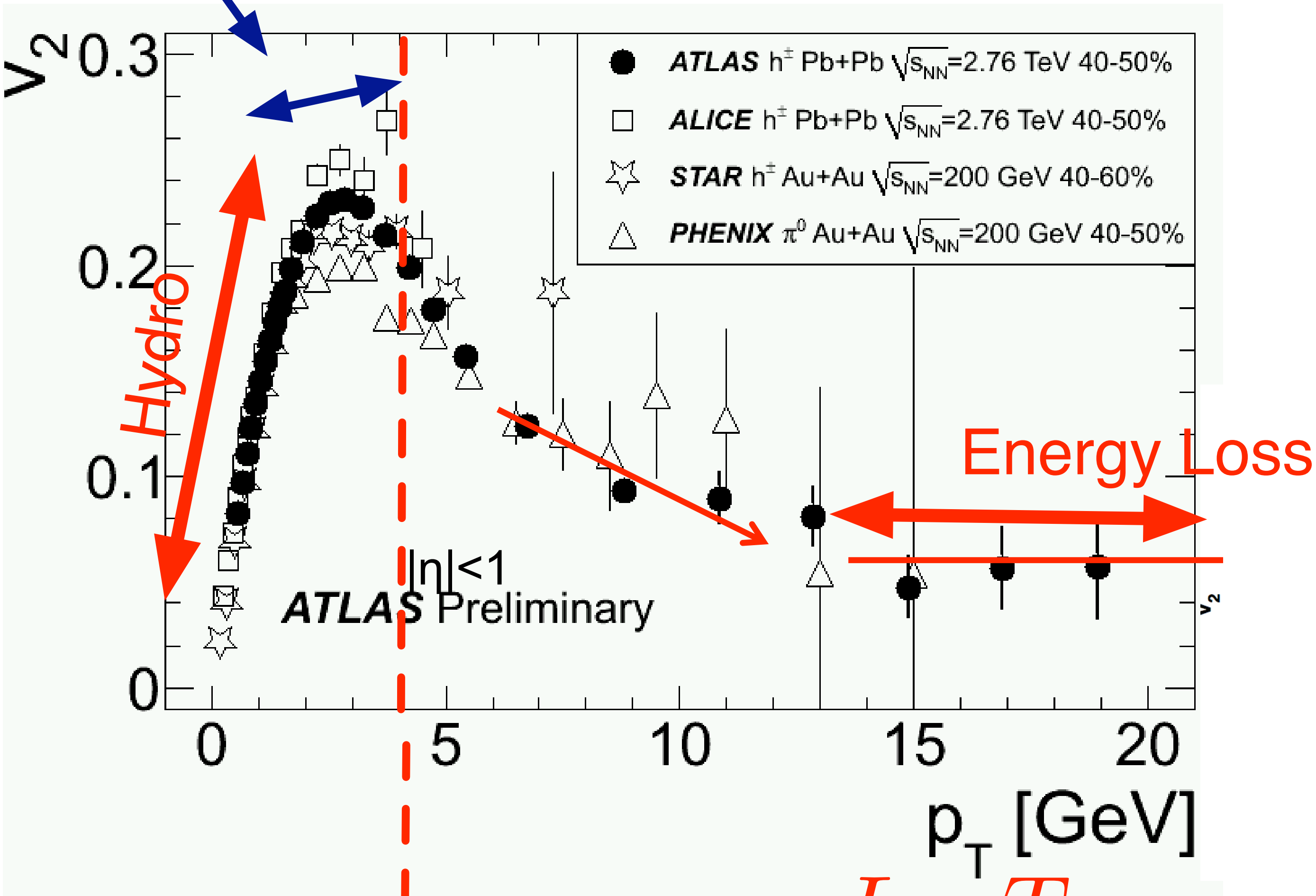
Hydro and Energy loss:



$$\frac{\ell_{\mathrm{mfp}}}{L} \to 0 \qquad \frac{L}{\ell_{\mathrm{mfp}}}\frac{T}{p_T} \to 0$$

Higher pt but still hydro

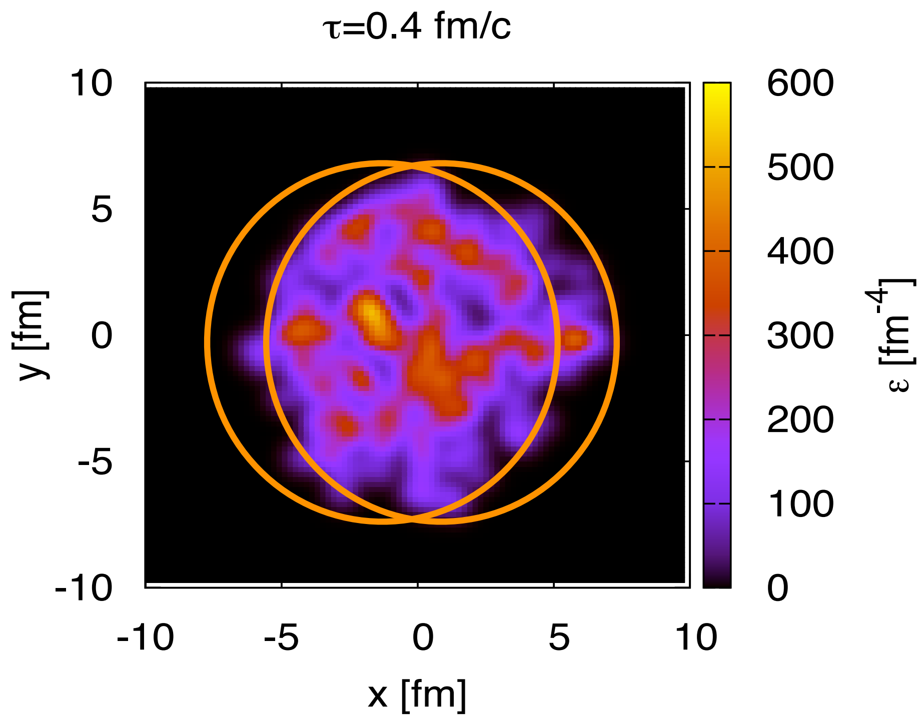
$$(\ell_{\mathrm{mfp}}/L)\frac{p_T}{T} < 1$$



$$\frac{\ell_{\mathrm{mfp}}}{L} \to 0$$

$$\frac{L}{\ell_{\mathrm{mfp}}}\frac{T}{p_T} \to 0$$

Determining the Shear Viscosity of QGP with Correlations:



1. Characterize energy density with ellipse
   - Elliptic Shape gives elliptic flow

$$v_2 = \langle \cos 2\phi_{\mathbf{p}} \rangle$$

2. Around almond shape are *fluctuations*
   - Triangular Shape gives v_3 (Alver)

$$v_3 = \langle \cos 3(\phi_{\mathbf{p}} - \Psi_3) \rangle$$

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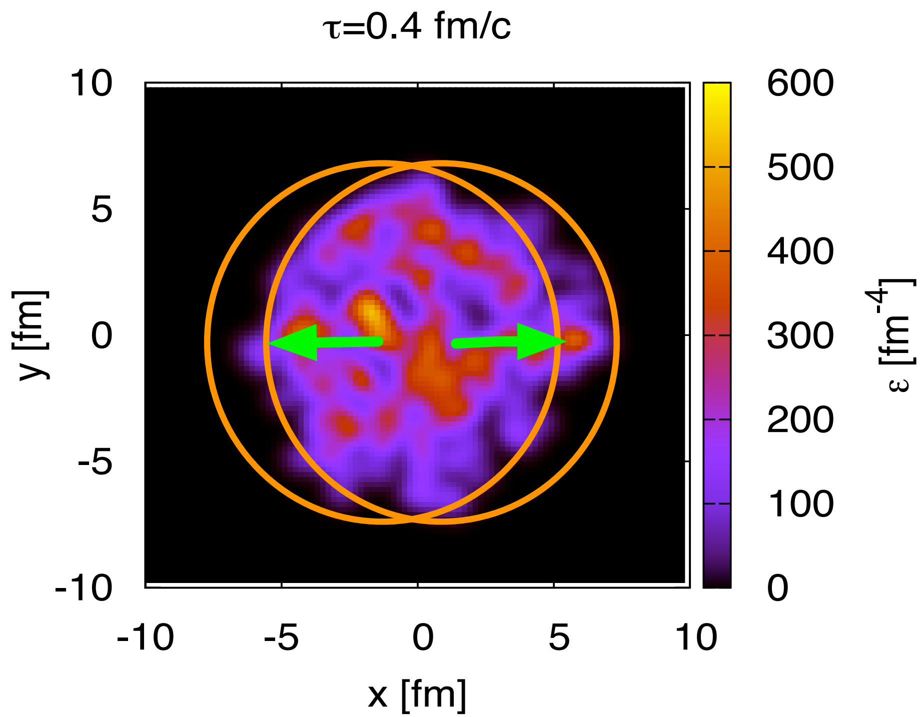

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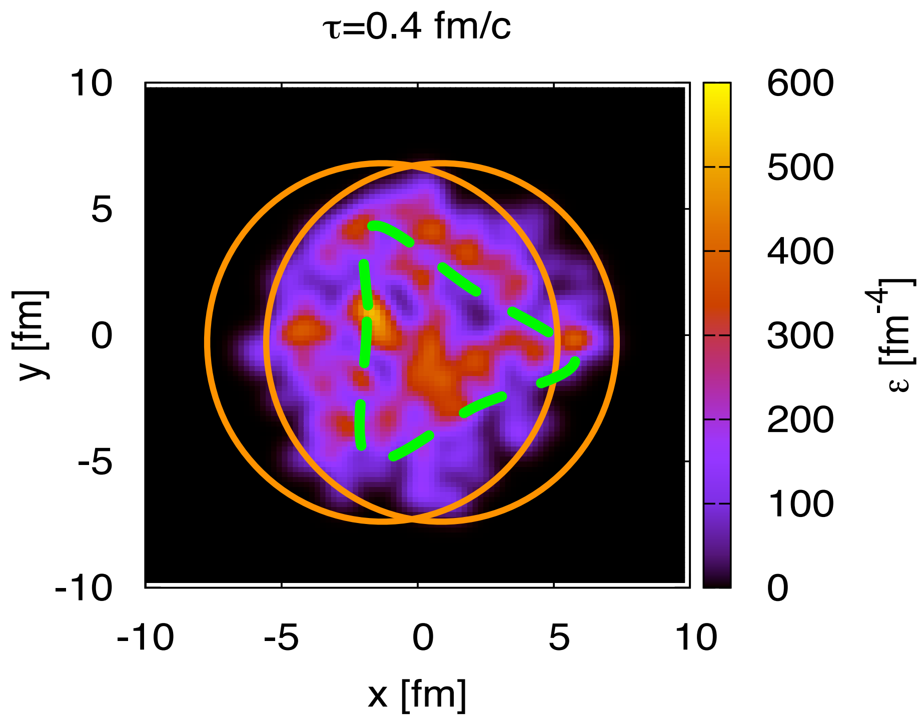

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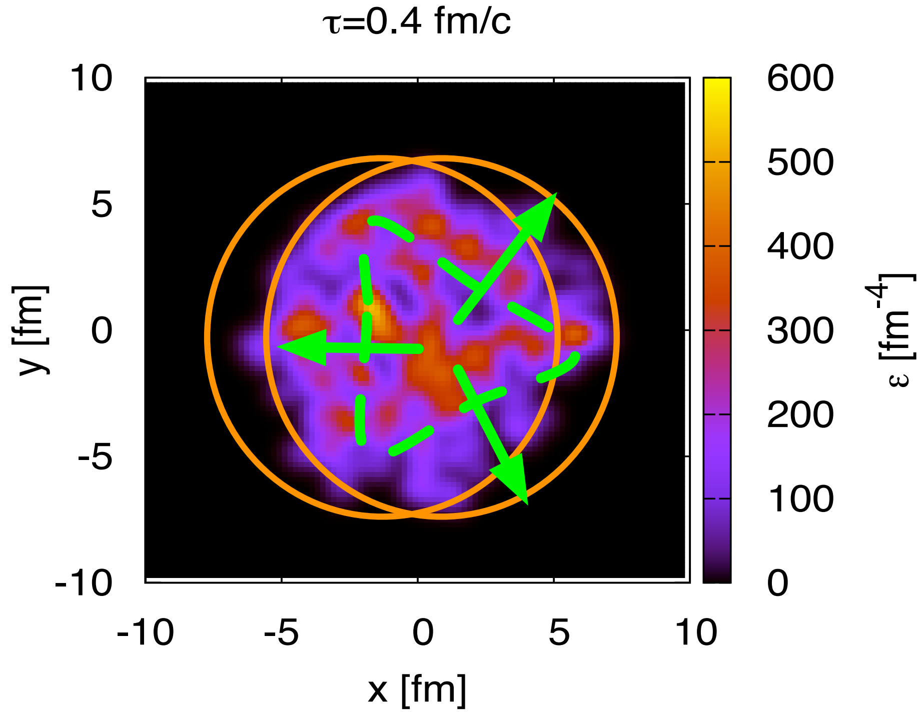
Determining the Shear Viscosity of QGP with Flow:



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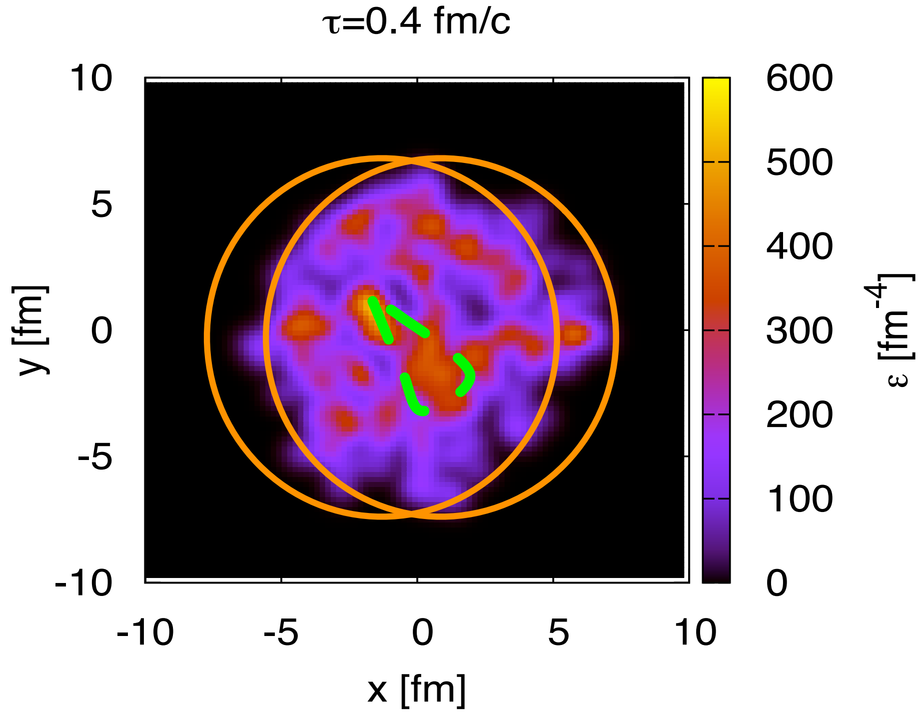

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$$v_2 = \langle \cos 2\phi_{\mathbf{p}} \rangle$$

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   - Triangular Shape gives v_3 (Alver)

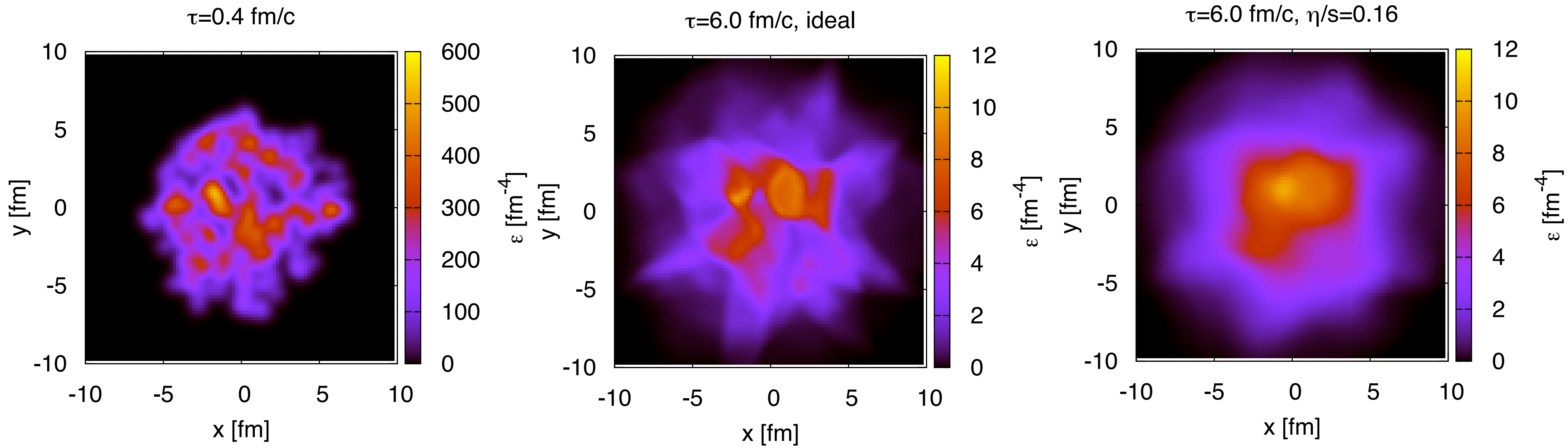
$$v_3 = \langle \cos 3(\phi_{\mathbf{p}} - \Psi_3) \rangle$$

3. Hot-spots give *correlated* higher harmonics

$$v_n = \langle \cos n(\phi_{\mathbf{p}} - \Psi_n) \rangle$$

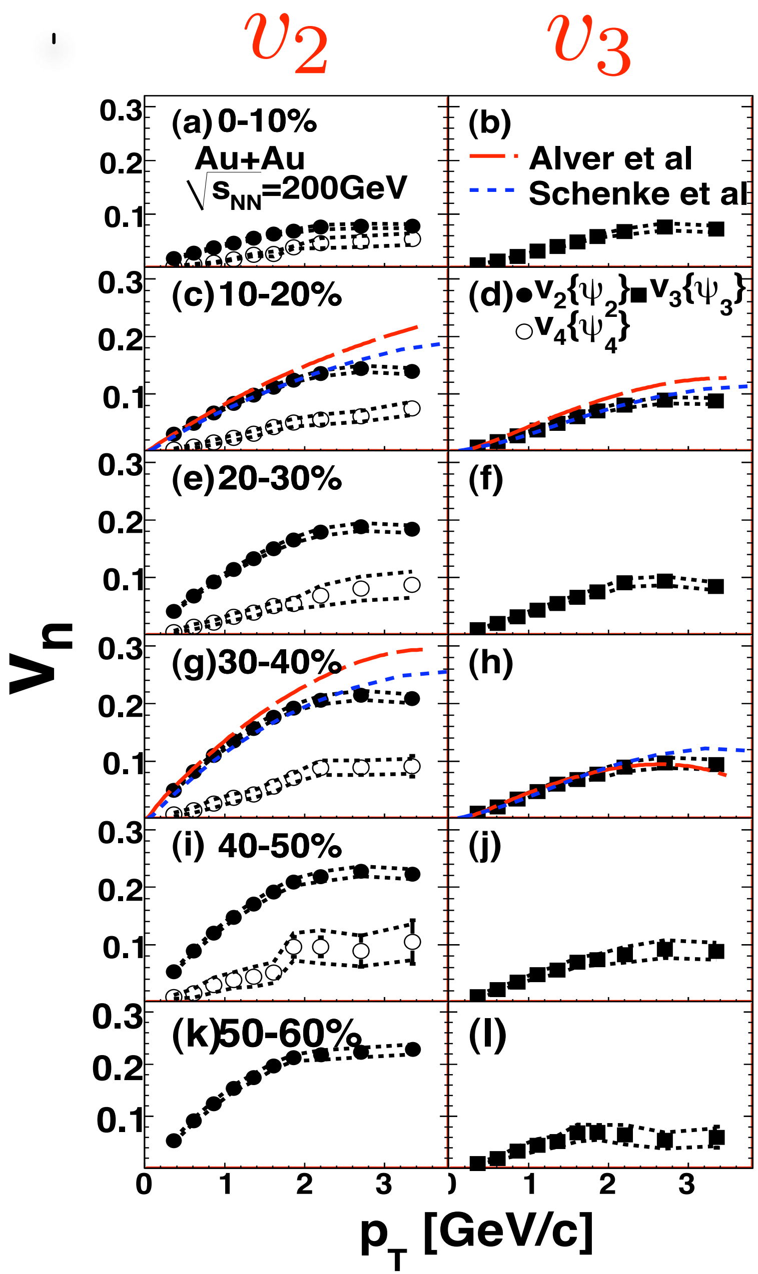
3+1 E by E viscous hydro simulations by Schenke *et al*

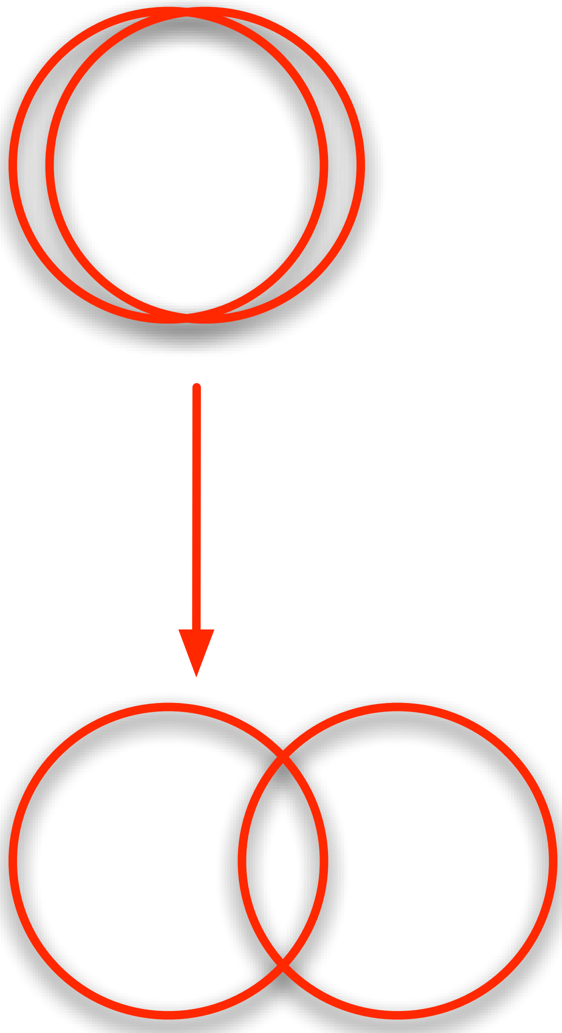
Initial   Final Ideal   Final Visc.



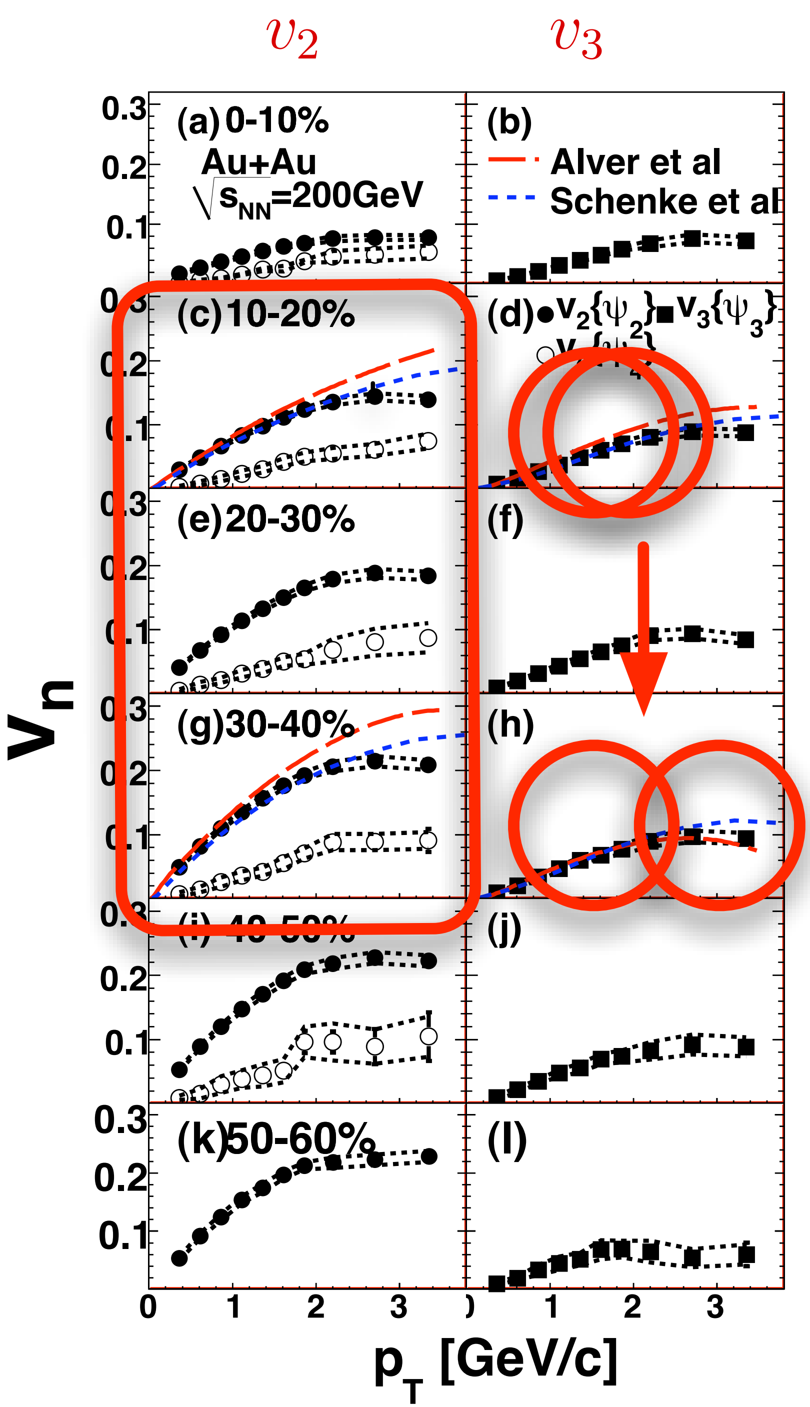
Higher harmonics are damped most by viscosity

Hydro Working





Phenix flow data



Hydro Working:

(schenke, luzum)

1. Centrality dependence of v_2 and v_3

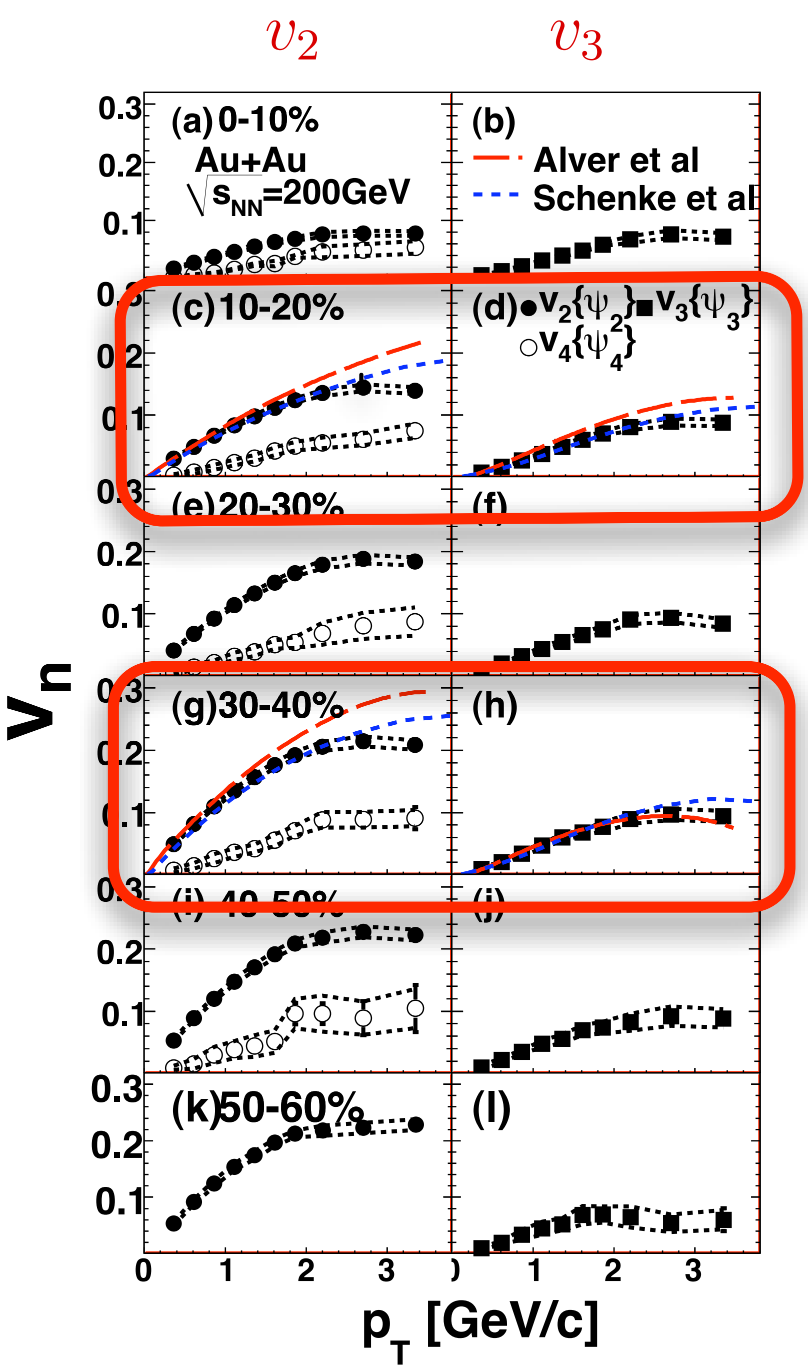
$$\sim (\ell_{\rm mfp}/L)$$

2. Relative strength of v_2 and v_3

3. p_T dependence of viscous corrections

$$\sim (\ell_{\rm mfp}/L)\frac{p_T}{T}$$

Phenix flow data



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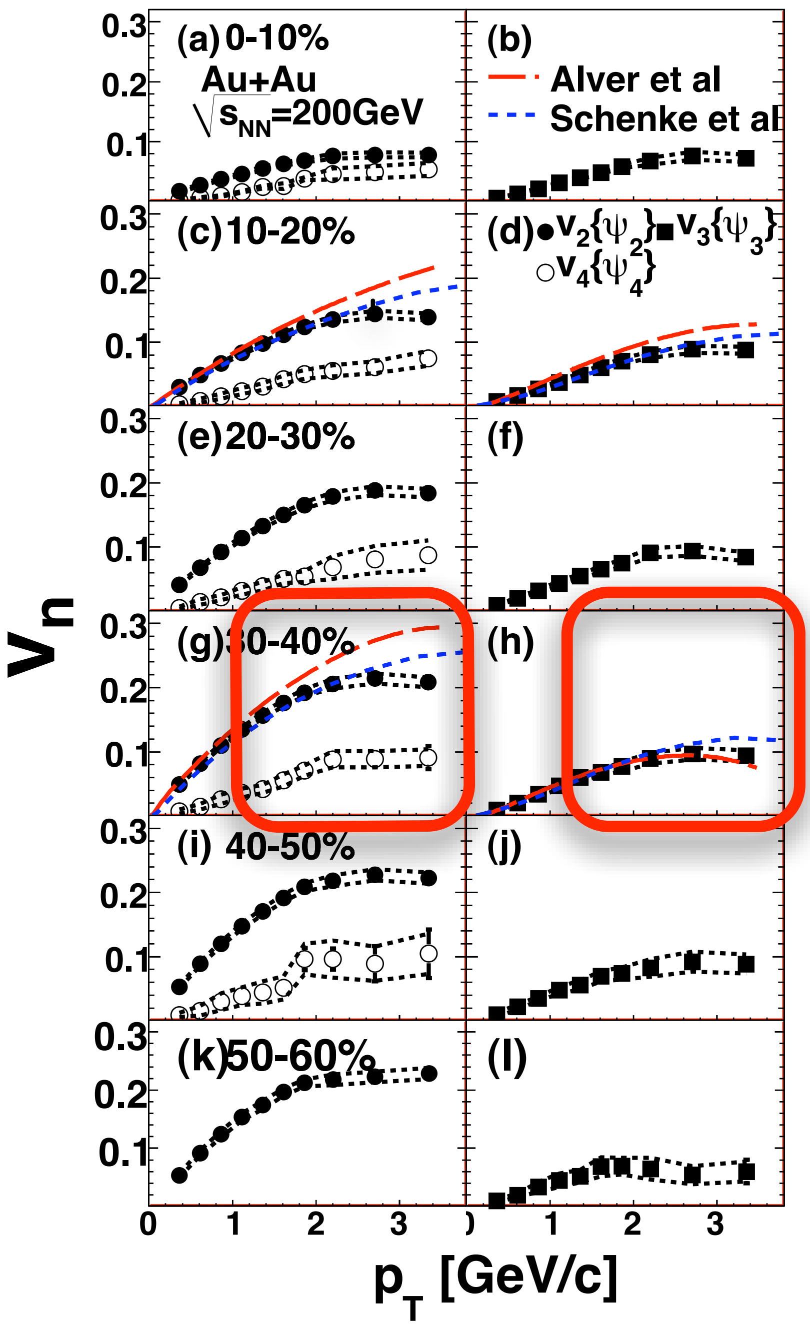
Phenix flow data

v_2 v_3



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3. p_T dependence of viscous corrections

$$\sim (\ell_{\rm mfp}/L)\frac{p_T}{T}$$

Hydro Working

Why I believe that there's hydro at RHIC (and why you should too):

- ✓ Ideal hydro works kind-of (not for today)
- ✓ Viscous corrections systematically capture deviations of data from ideal hydro

Makes the bounds $1/4\pi < \eta/s < 4/4\pi$ kind of convincing

Calculating Shear Viscosity @ Kinetics:

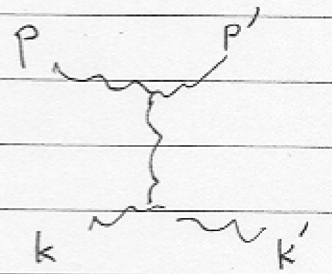
$$\left(\partial_t + v_p \frac{\partial}{\partial x}\right) f = -C[f]$$

Then

$$C[f] = \int_{kp'k'} \Gamma_{pk \to p'k'} \left[f_p f_k (1+f_{p'})(1+f_{k'}) - f_{p'} f_{k'} (1+f_k)(1+f_p) \right]$$

① $= \int_p \frac{d^3p}{(2\pi)}$

② $\Gamma_{pk \to p'k'} = \frac{|M|^2}{2p\,2k\,2p'\,2k'}$ e.g. $|M|^2 = 8 g^4 C_A^2 \frac{s^2}{t^2}$



Near Equilibrium: temperature variation and flow

$\vec{u}^i(x)$, $T(x) = T_0 + \delta T(x)$ — $u(x)$ and $T(x)$ obey hydro

(1) $f_p = f_e(\vec{p}) + \delta f$

$$f_e = \frac{1}{e^{(E_p - \vec{p}\cdot\vec{u}(x))/T(x)} - 1} \simeq n_p + n_p(1+n_p)\left[\frac{\vec{p}\cdot\vec{u}}{T_0} + E_p \frac{\delta T(x)}{T_0^2}\right.$$

(2) We want to know δf - first viscous correct

- Proportional Strains $\langle \partial^i u^j \rangle$, others (ignore for no

$$\delta f(p) = n_p(1+n_p)\, \chi_{\vec{p}}$$

$$\delta f(p) = n_p(1+n_p)\, \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle X(p)$$

Now

$$T^{ij} = p\,\delta^{ij} - \eta \langle \partial^i u^j \rangle = \int_p f_e + \delta f$$

$$\eta = \frac{2}{15} \int_p n_p(1+n_p)\, X(p)$$

Now $f_p = f_e + \delta f$

$$\left[\partial_t + v_p \frac{\partial}{\partial x}\right] f_e = C[f_e + \delta f] \qquad C[f_e] = 0$$

Work:

① Use (Hydro) EOM to write time derivs as spatial derivs

② Thermodynamic identities

③ Detailed Balance $\quad n_p n_k (1+n_{p'})(1+n_{k'}) \Gamma_{pk\to p'k'}$

$$= n_{p'} n_{k'} (1+n_p)(1+n_k) \Gamma_{pk\to p'k'}$$

Find an equation for χ

$$\underbrace{(\ldots)\partial_i u^i}_{} + \frac{p^i p^j}{2TE}\langle \partial_i u_j \rangle = -\int_{kp'k'} \Gamma_{pk\to p'k'} n_p n_k (1+n_{p'})(1+n_{k'}) \left[\chi_{\vec{p}} + \chi_{\vec{k}} - \chi_{\vec{p}'} - \chi_{\vec{k}'}\right]$$

Responsible for Bulk viscosity

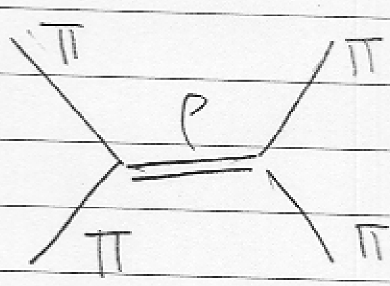
General Structure: A matrix eqn for $\chi_{\bar{p}}$

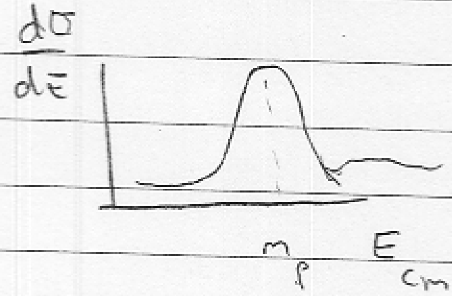
$$\frac{p^i p^j}{2TE}\langle \partial_i u_j \rangle = C_{p\bar{p}}\, \chi_{\bar{p}}$$

Numerically invert matrix eqn determine $\chi_{\vec{p}}$

QCD Scattering

Hadronic Phase:

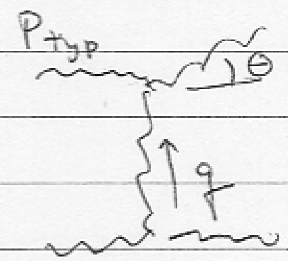




Mostly known experimentally - pretty robust

$\frac{\eta}{s}$ computed by Prakash et al ~ 1992

High temperature :



$$\frac{d\sigma}{d\Omega} \propto \frac{g^4 C_A^2}{\theta^4} \leftarrow \text{coulomb scattering}$$

$$\sigma_{tr} \propto \int \frac{d\sigma}{d\Omega} (1-\cos\theta) \; d\Omega$$

$$\sigma_{tr} \propto g^4 C_A^2 \log \theta_{min}$$

$$\propto g^4 C_A^2 \log \frac{T}{m_D}$$

So shear viscosity has the form (more next time)

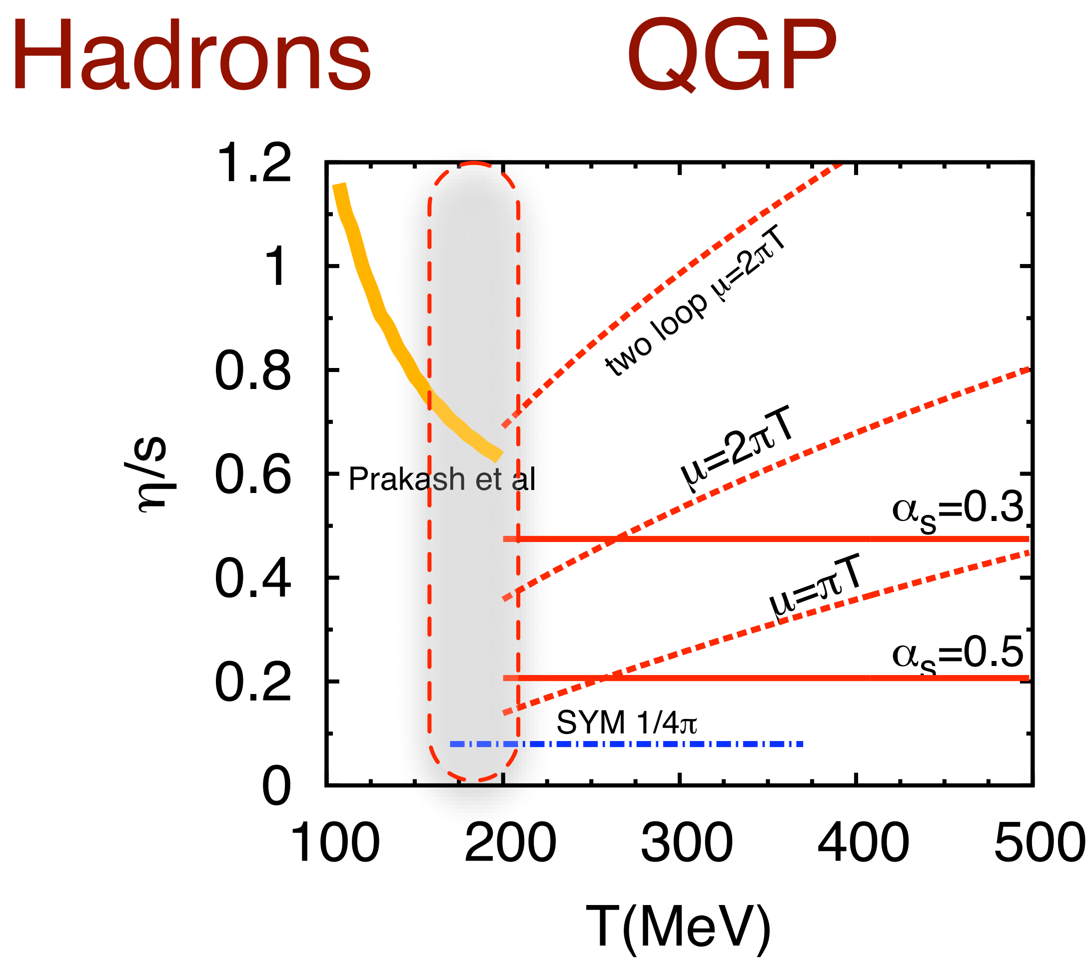
$$\frac{\eta}{s} = \frac{1}{\alpha_s^2} F(m_D/T)$$ Computed by AMY (Arnold moore Yaf...

$\nwarrow$ depends on how regulate -- uncert... coulomb log

$\nearrow$ depends on how to set the scale uncertain

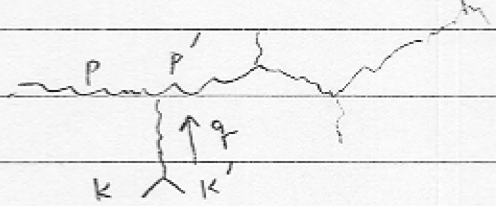
$$\frac{\eta}{s} = 0.22 \left(\frac{0.5}{\alpha_s}\right)^2$$

compliation from D. A. Teaney, “Viscous Hydrodynamics and the Quark Gluon Plasma,” arXiv:0905.2433 [nucl-th].



$$0.36\left(\frac{\eta/s}{0.3}\right)\left(\frac{1\,\mathrm{fm}}{\tau_o}\right)\left(\frac{300\,\mathrm{MeV}}{T_o}\right) \ll 1$$

Now interpret soft coulomb scattering:



→ gives rise to a random walk, diffusion in mom space

$$\partial_t + v_p \frac{\partial}{\partial x} \delta f = \int_{pk'p'} \cdots \left[\chi_{\vec{p}} + \chi_{\vec{k}} - \chi_{\vec{p}'} - \chi_{\vec{k}'} \right]$$

Approx

$$\chi_{p'} = \chi_p + \vec{q} \frac{\partial \chi}{\partial \vec{p}} + \frac{1}{2} \vec{q}^i \vec{q}^j \frac{\partial \chi}{\partial p^i \partial p^j} + \ldots$$

momentum diffusion eqn

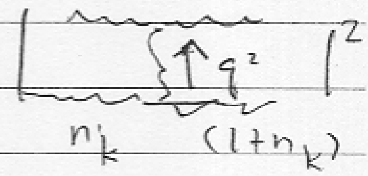
$$\partial_t + v_p \frac{\partial}{\partial x} \delta f = \frac{\hat{q}_{soft}}{2} \frac{\partial}{\partial \vec{p}} n_p (1+n_p) \frac{\partial}{\partial \vec{p}} \chi$$

↑ transverse momentum diffusion coefficient

Here $\hat{q}$

① $\hat{q}_{soft} =$ mean $\langle q_T^2 \rangle$ per length

$$= \int_{\vec{q}\,\vec{k}} |M|^2 \, n_k(1+n_k) \times q^2$$



$$\hat{q}_{soft} = \frac{g^4 C_A^2 T^3}{12\pi} \log\left(\frac{T}{m_D}\right)$$

②

$$\frac{\eta}{s} = 0.92 \frac{T^3}{\hat{q}_{soft}}$$

Comments

- So if coupling is too strong such that the Debye sector is non perturbative -- at least we can parametrize @ one number
- Get $\hat{q}$ from experiment?
- AdS?