6 Baryons

Baryons as wrapped D4-branes

[Witten, Gross-Ooguri 1998]

- Baryons in AdS/CFT are constructed by wrapped D-branes
  In our case,

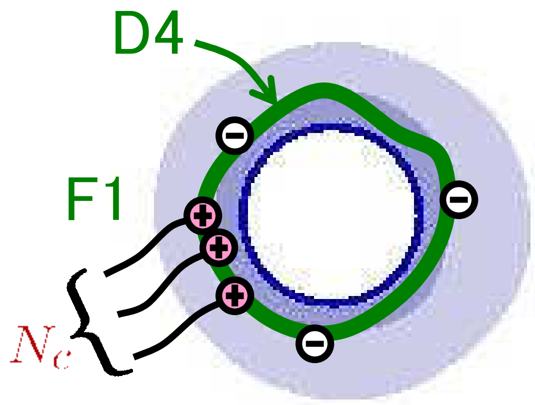
Baryon $\simeq$ D4-brane wrapped on the S^4

- RR flux $\frac{1}{2\pi}\int_{S^4} dC_3 = N_c$ forces N_c F-strings to be attached on it.

$$S_{\rm CS}^{\rm D4} = \int_{\mathbf{R}\times S^4} C \wedge e^{F^{\rm D4}/2\pi} \sim -N_c \int_{\mathbf{R}} A^{\rm D4}$$

source of $-N_c$ electric charge on D4

➡ F-string should be attached.
➡ Bound state of N_c quarks
➡ Baryon



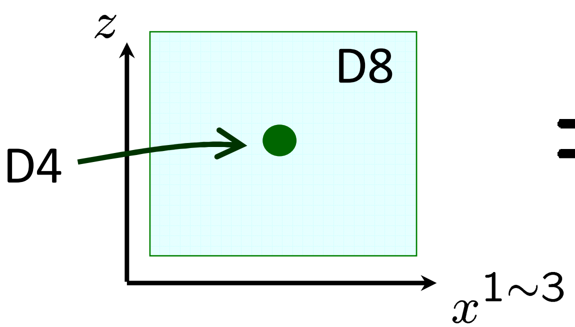
Baryon mass (∝ vol. of S^4) is generated by the geometry!

Baryons as “instantons”

D4-brane D8-brane Topology of the background

$$\underset{x^0}{\mathbf{R}} \times S^4 \subset \underset{x^\mu}{\mathbf{R}^{1,3}} \times \underset{z}{\mathbf{R}} \times S^4 \subset \mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$$

baryon



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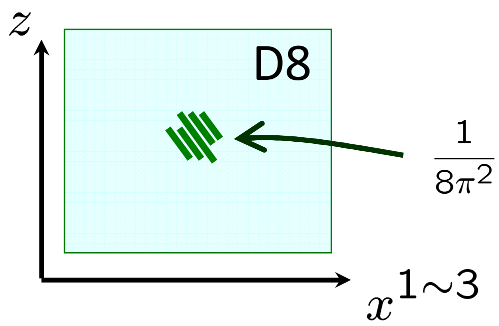


[Douglas 1995]

D4 within D8

$M = 1, 2, 3, z$

Instanton in $x^M = (\vec{x}, z) \in \mathbf{R}^4$

$$\text{Baryon number} = \text{number of D4} = \frac{1}{8\pi^2}\int_{\text{4dim}} \mathrm{tr} F \wedge F$$

(x^1, x^2, x^3, z)

Classical solution (We consentrate on the $N_f = 2$ case.)

- The instanton solution for the Yang-Mills action

$$S_{\rm YM} = \kappa \int d^4x dz \, {\rm Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right)$$

shrinks to zero size !

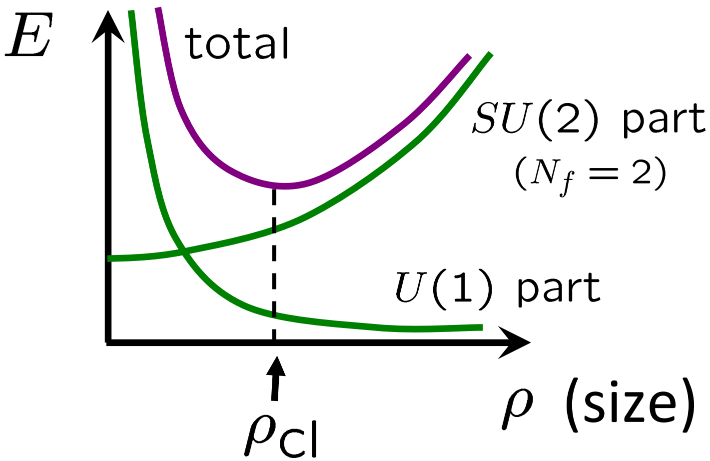
- The Chern-Simons term makes it larger

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A) = \frac{N_c}{16\pi^2} \int d^4x dz \, A_0^{U(1)} \underbrace{\epsilon^{ijk} {\rm Tr} F_{ij} F_{kz}} + \cdots$$

$U(1)$ part

Non-zero for instanton

➡ source of the U(1) charge



➡ Stabilized at $\rho_{\rm cl}^2 = \frac{N_c}{8\pi^2\kappa}\sqrt{\frac{6}{5}}$

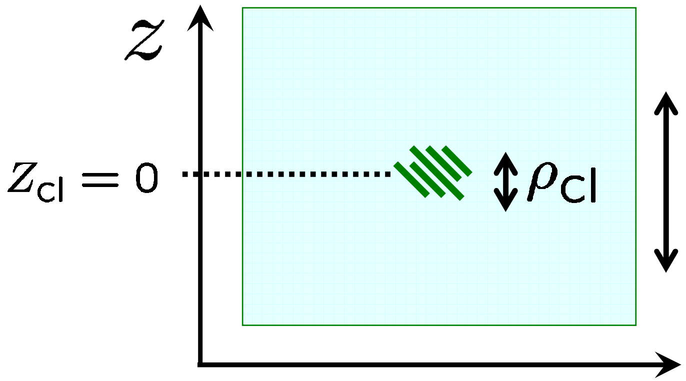
[Hong-Rho-Yee-Yi 2007]
[Hata-Sakai-S.S.-Yamato 2007]

- Note that $\rho_{\text{cl}} \sim \mathcal{O}(\lambda^{-1/2})$

λ : 't Hooft coupling (assumed to be large)

If λ is large enough, the 5 dim space-time can be approximated by the flat space-time.

(The effect of the non-trivial z-dependence is taken into account perturbatively.)



$k(z) = 1 + z^2, \; h(z) = (1 + z^2)^{-1/3}$

$k(z) \simeq h(z) \simeq 1 \quad (\text{for } |z| \ll 1)$

➡ The leading order classical solution is the BPST instanton with $\rho = \rho_{\text{cl}}$ and $Z = Z_{\text{cl}} = 0$

$$A_M^{\text{cl}} = -i \frac{\xi^2}{\xi^2 + \rho^2} g \partial_M g^{-1} \qquad g = \frac{(z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi}$$

$$\xi = \sqrt{(\vec{x} - \vec{X})^2 + (z - Z)^2}$$

$M = 1, 2, 3, z$

ρ : size $\quad (\vec{X}, Z)$: position of the instanton

Quantization

[Hata-Sakai-S.S.-Yamato 2007]

- Consider a slowly moving (rotating) baryon configuration.
  moduli space approximation method :

  Instanton moduli $\mathcal{M} \ni (X^\alpha) \rightarrow (X^\alpha(t))$ $(\alpha = 1, 2, \cdots, \dim \mathcal{M})$

  time

  $A_M(t,x) \sim A_M^{\text{cl}}(x; X^\alpha(t))$

  $S_{5\text{dim}} \rightarrow$ **Quantum Mechanics for** $X^\alpha(t)$

- For SU(2) one instanton,

$$\mathcal{M} \simeq \{(\vec{X}, Z, \rho)\} \times SU(2)/\mathbf{Z}_2 \qquad \mathbf{Z}_2 : \mathbf{a} \to -\mathbf{a}$$

position, size, Ψ $\mathbf{a}$ ← SU(2) orientation

$$\rightarrow \quad L_{\text{QM}} = \frac{G_{\alpha\beta}}{2}\dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \qquad U(X^\alpha) = 8\pi^2\kappa\left(1 + \left(\frac{\rho^2}{6} + \frac{3^6\pi^2}{5\,\lambda^2\rho^2} + \frac{Z^2}{3}\right) + \cdots\right)$$

<u>Note</u> $(\vec{X}, \mathbf{a})$: genuine moduli (the same as in the Skyrme model)

(ρ, Z) : new degrees of freedom, added since they are light compared with the other massive modes.

- Solving the Schrodinger equation for this Quantum mechanics, we obtain the baryon states

→ Generalization of Adkins-Nappi-Witten including vector mesons and ρ , Z modes

We can construct baryon states for

$$n, p, \Delta(1232), N(1440), N(1530), \cdots$$

<u>Example</u> Nucleon wave function:

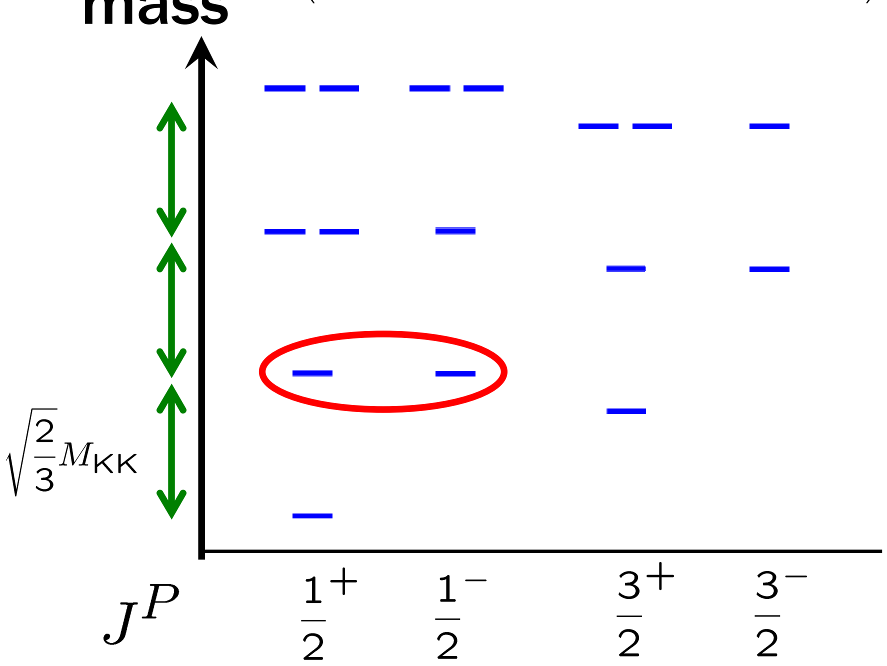
$$\psi(\vec{X}, \mathbf{a}, \rho, Z) \propto e^{i\vec{p}\cdot\vec{X}} R(\rho)\psi_Z(Z)T(\mathbf{a})$$

$$\left(\begin{array}{ll} R(\rho) = \rho^{\tilde{l}} e^{-A\rho^2} & \tilde{l} = -1 + 2\sqrt{1 + N_c^2/5} \\ \psi_Z(Z) = e^{-AZ^2} & A = \frac{8\pi^2\kappa}{\sqrt{6}} \\ T(\mathbf{a}) = a_1 + ia_2 \ \text{ for } \ |p\uparrow\rangle \quad \text{etc.} & \end{array} \right)$$

Baryon spectrum

Theory

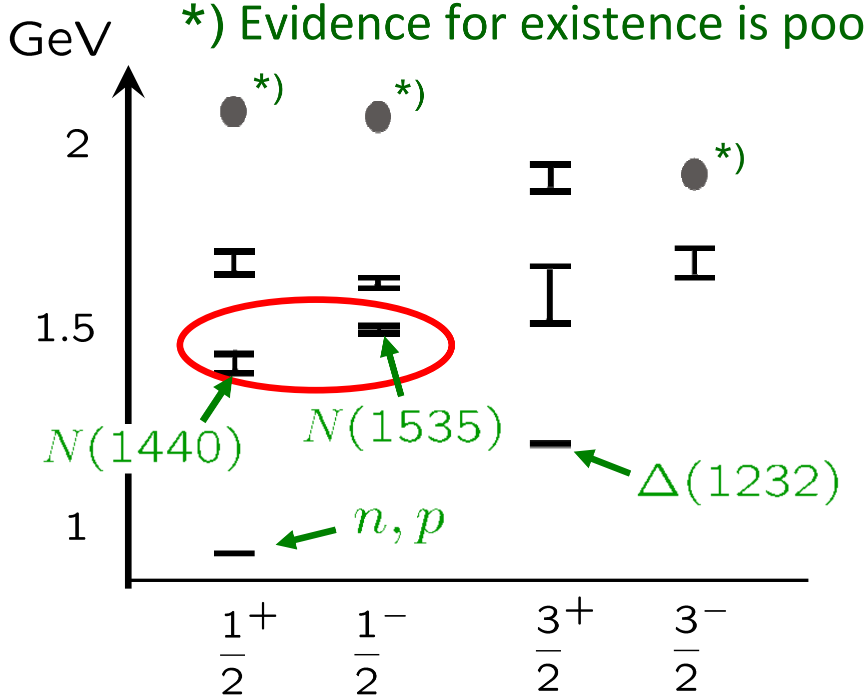
$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15}N_c^2} + \sqrt{\frac{2}{3}}(n_\rho + n_z)\right) M_{KK}$$



Experiment

($I = J$ states from PDG)

*) Evidence for existence is poor



<u>Note:</u>

- We only consider the mass difference, since $\mathcal{O}(N_c^0)$ term in M_0 is not known.
- $M_{KK} \simeq 949$ MeV (fixed by ρ -meson mass) is a bit too large. It looks better if M_{KK} were around 500 MeV .

Currents

- Chiral symmetry

$$U(N_f)_L \times U(N_f)_R \xrightarrow{\text{gauge}} (A_{L\mu}(x), A_{R\mu}(x))$$

- Interpreted as

$$A_{L\mu}(x) = \lim_{z \to +\infty} A_\mu(x,z) \qquad A_{R\mu}(x) = \lim_{z \to -\infty} A_\mu(x,z)$$

$$\Rightarrow \quad S_{5\,\mathrm{dim}}\Big|_{\mathcal{O}(A_L,A_R)} = -\int d^4x \left(A^a_{L\mu} J^{a\mu}_L + A^a_{R\mu} J^{a\mu}_R \right)$$

with

$$J_{L\mu} = -\kappa \left(k(z) F_{\mu z} \right)\Big|_{z=+\infty} \qquad J_{R\mu} = +\kappa \left(k(z) F_{\mu z} \right)\Big|_{z=-\infty}$$

- vector and axial vector currents

$$J^\mu_V \equiv J^\mu_L + J^\mu_R = -\kappa \Big[k(z) F^{\mu z} \Big]^{z=+\infty}_{z=-\infty}$$

$$J^\mu_A \equiv J^\mu_L - J^\mu_R = -\kappa \Big[\psi_0(z) k(z) F^{\mu z} \Big]^{z=+\infty}_{z=-\infty} \qquad (\psi_0(\pm\infty) = \pm 1)$$

- Using these currents, we can calculate

$$\langle r^2 \rangle = \int d^3x \, r^2 \, \langle J^0 \rangle$$: charge radius

g-factor

$$\mu^i = \frac{1}{2}\epsilon^{ijk} \int d^3x \, x^j \langle J^k \rangle = \frac{g}{2M_N} S^i$$: magnetic moment

spin

Summary of the results

	our result	exp.
$\langle r^2\rangle_{I=0}^{1/2}$	0.742 fm	0.806 fm
$\langle r^2\rangle_{I=1}^{1/2}$	0.742 fm	0.939 fm
$\langle r^2\rangle_{A}^{1/2}$	0.537 fm	0.674 fm
$g_{I=0}$	1.68	1.76
$g_{I=1}$	7.03	9.41
g_A	0.734	1.27

[Hashimoto-Sakai-S.S. 2008]

[See also,
Hong-Rho-Yee-Yi 2007,
Hata-Murata-Yamato 2008,
Kim-Zahed 2008]

- We can also evaluate these for excited baryons such as $\Delta(1232), N(1440), N(1535), \cdots$