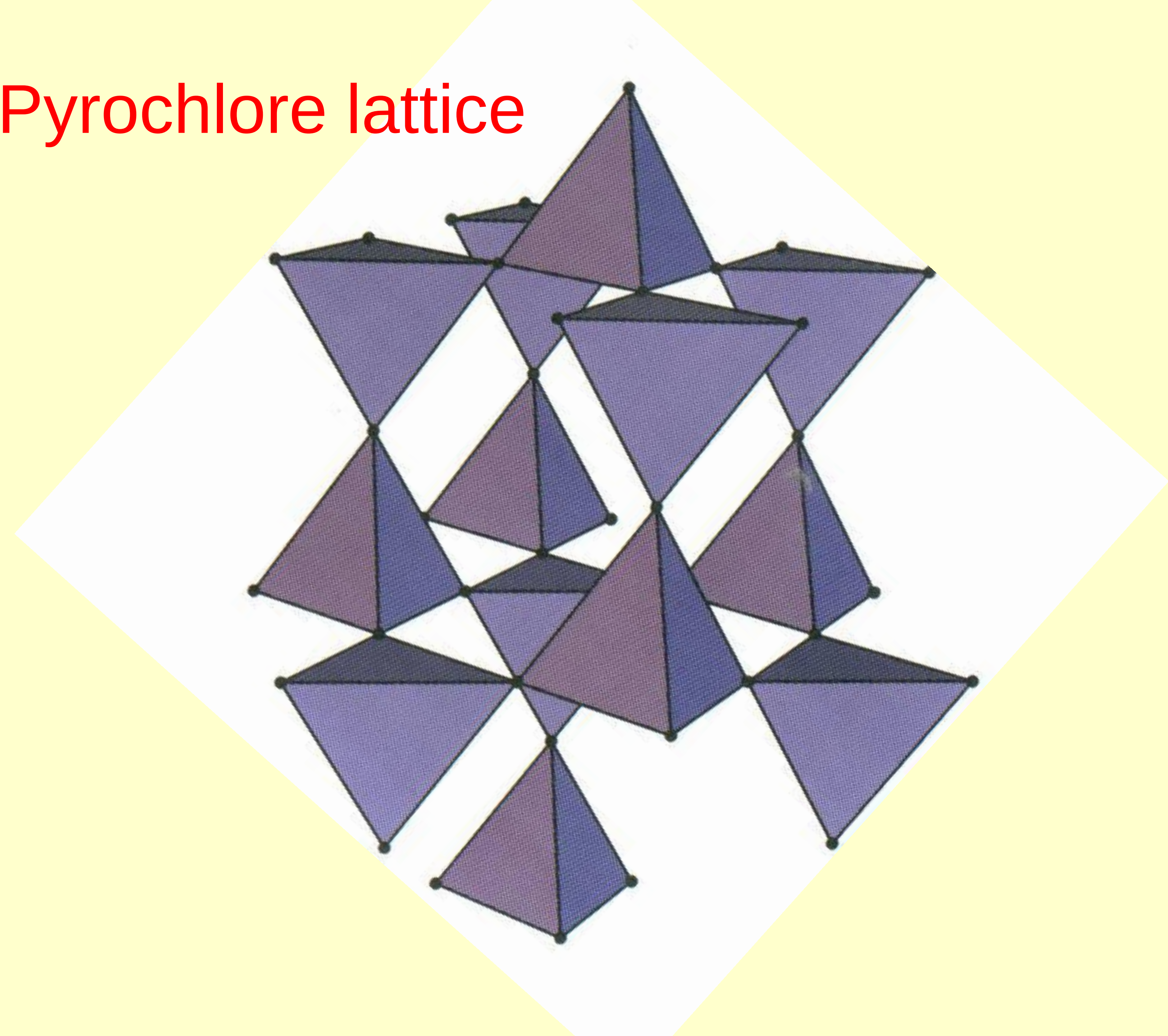
The Spin Ices

- The Spin Ices are the insulating compounds $(Ho,Dy)_2(Ti,Sn)_2O_7$ where the magnetic ions (Ho,Dy) sit on a pyrochlore lattice
- Low energy degrees of freedom are spins with a strong easy axis anisotropy that points in the local [111] direction (locally Ising – 4 different orientations).
- The spins are classical and interact dominantly via their dipolar interaction

$$\mathcal{H} = \mathcal{H}_{nn} + \frac{\mu_0}{4\pi} \sum_{ij} \frac{\vec{\mu}_i \cdot \vec{\mu}_j - 3(\vec{\mu}_i \cdot \hat{r}_{ij})(\vec{\mu}_i \cdot \hat{r}_{ij})}{r_{ij}^3}$$

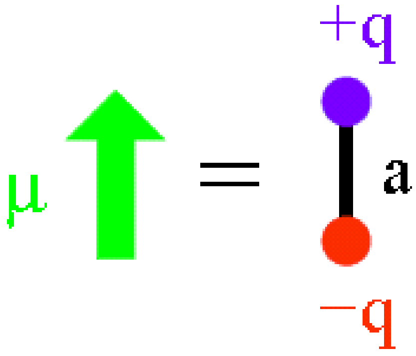
Pyrochlore lattice



The 'dumbell' model

Dipole ≈ pair of opposite charges ($\mu = qa$):

- Sum over dipoles ≈ sum over charges:



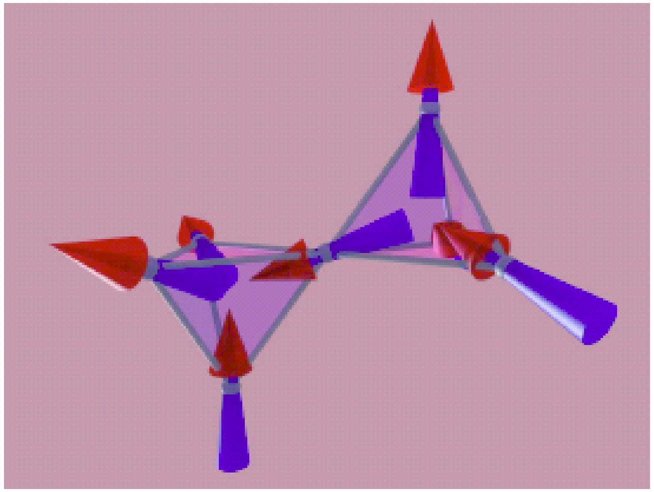
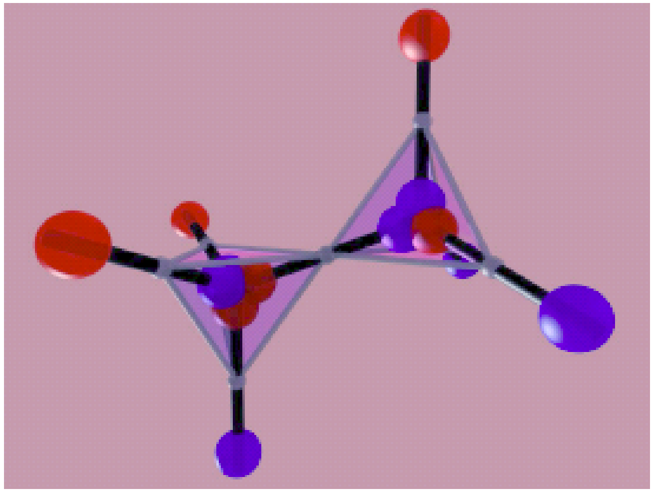
$$\mathcal{H}_{ij} = \sum_{m,n=1}^{2} v(r_{ij}^{mn})$$

- $v \propto q^2/r$ is the usual Coulomb interaction (regularised):

$$v(r_{ij}^{mn}) = \begin{cases} \mu_0\, q_i^m q_j^n/(4\pi r_{ij}^{mn}) & i \neq j \\ v_o(\frac{\mu}{a})^2 = \frac{J}{3} + 4\frac{D}{3}(1+\sqrt{\frac{2}{3}}) & i = j, \end{cases}$$

Origin of the ice rules

Choose $a = a_d$, separation between centres of tetrahedra

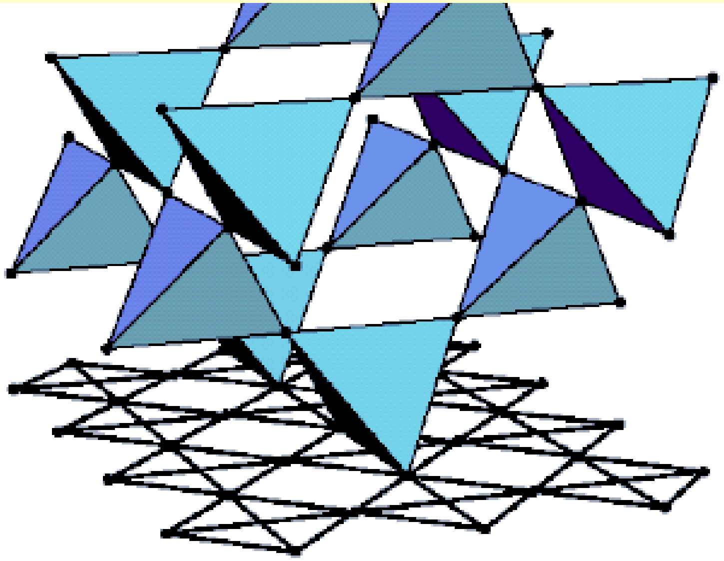
Resum tetrahedral charges $Q_\alpha = \sum_{r_i^m \in \alpha} q_i^m$:

$$\mathcal{H} \approx \sum_{ij}^{mn} v(r_{ij,mn}) \longrightarrow \sum_{\alpha\beta} V(r_{\alpha\beta}) = \begin{cases} \frac{\mu_0}{4\pi} \frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} & \alpha \neq \beta \\ \frac{1}{2} v_o Q_\alpha^2 & \alpha = \beta \end{cases}$$

- Ice configurations ($Q_\alpha = 0$) degenerate⇒ Pauling entropy!

- Pauling estimate of ground state entropy $S_0 = \ln N_{gs}$:

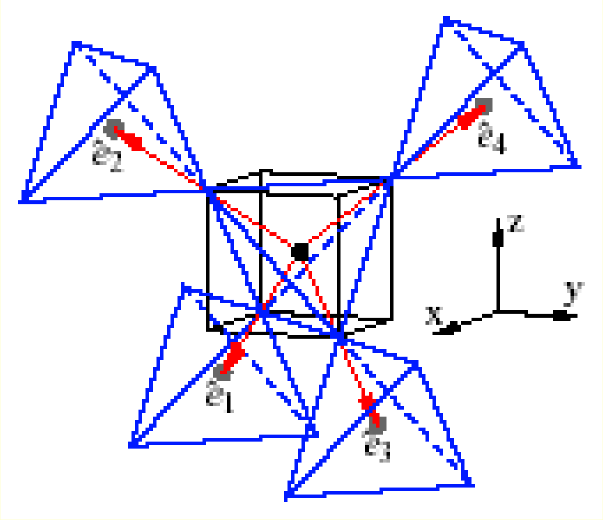
$$N_{gs} = 2^N \left(\frac{6}{16}\right)^{N/2} \Rightarrow S_0 = \frac{1}{2}\ln\frac{3}{2}$$



- microstates vs. constraints; N spins, $N/2$ tetrahedra

Conservation law

Orient bonds on the dual diamond lattice from one sublattice to the other



Define a vector field on bonds

$$\vec{B}^a(\mathbf{x}) = S^a(\mathbf{x})\hat{e}(\mathbf{x})$$

$$\sum S^a(\mathbf{x}) = 0$$

on each tetrahedron in grounds states, implies

$$\nabla \cdot \vec{B}^a = 0$$

at each dual site

Second ingredient: rotation of closed loops of **B** connects ground states
Which implies large density of states near $\mathbf{B}_{av} = 0$

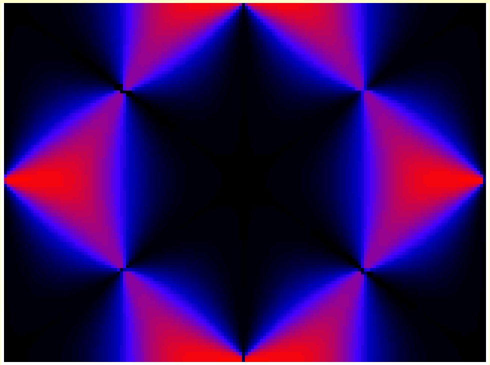
Using these "magnetic" fields we can construct a coarse grained partition function

$$\sum_{\text{spin configs}} \text{"1"} \rightarrow \sum_{\vec{B}^a(\mathbf{x})} \delta(\nabla \cdot \vec{B}^a)\; e^{-\frac{K}{2}\int d^3x \sum_a (\vec{\mathbf{B}}^a)^2}$$

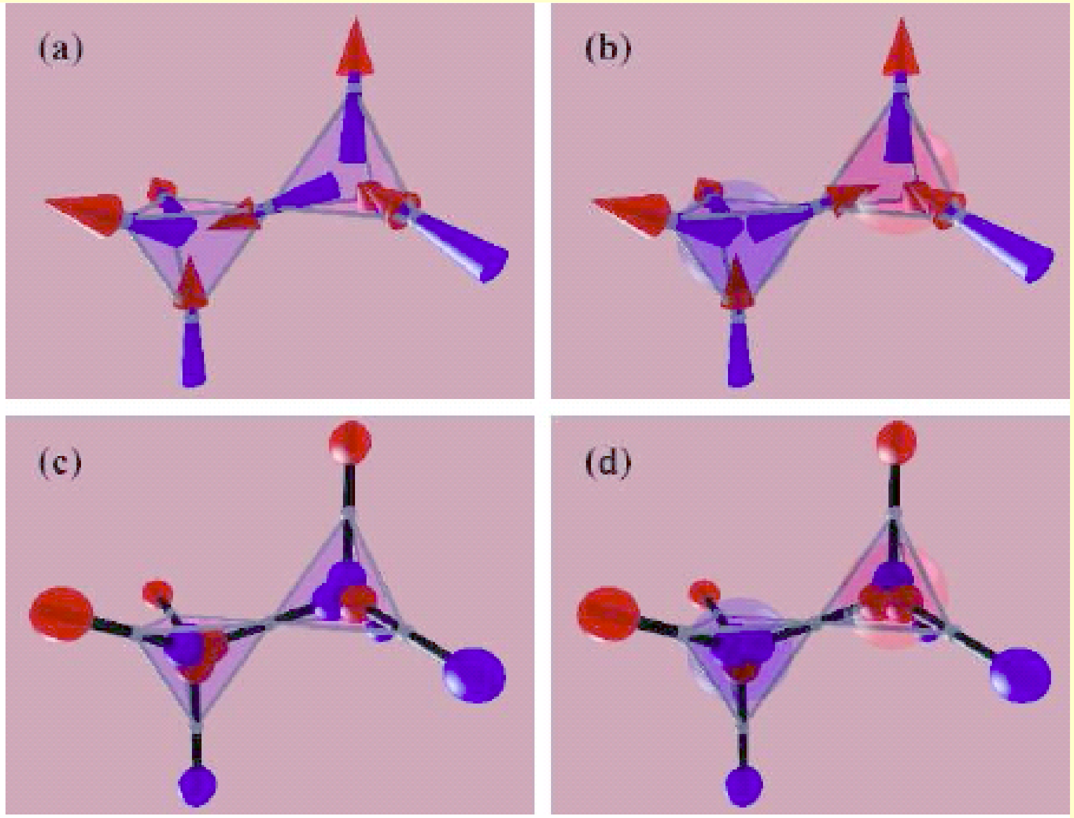
Solve constraint $\vec{B} = \nabla \times \vec{A}$ to get Maxwell theory for gauge field

$$\sum_{\vec{A}^a(\mathbf{x})} e^{-\frac{K}{2}\int d^3x \sum_a (\nabla \times \vec{\mathbf{A}}^a)^2}$$

Leads to dipolar spin correlations.



$$S_{[hhk]} = \frac{32\left(\cos(\frac{q_x}{4}) - \cos(\frac{q_z}{4})\right)^2 \sin(\frac{q_x}{4})^2}{5 - \cos(q_x) - 4\cos(\frac{q_x}{2})\cos(\frac{q_z}{2})}.$$



Flipping spin creates two ice rule violating tetrahedra which can be separated to infinite distance as finite free energy cost – fractionalized monopoles. These are monopoles in two senses.

First, they are charged under the emergent gauge field.

$$\nabla \bullet \vec{B} = \pm 2$$

The entropy of the background spins depends upon their separation and leads to an entropic Coulomb interaction between monopole-antimonopole pair:

$$F(\vec{r}) = -\#\frac{T}{r}$$

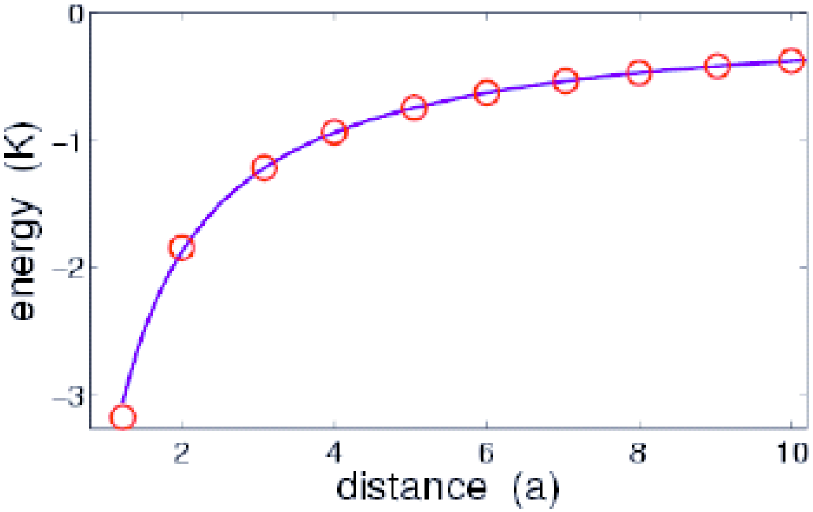
Second, the are “charged” under the magnetostatic gauge field. More precisely we see that in the dumbbell model they have an energetic Coulomb interaction with charge q introduced earlier

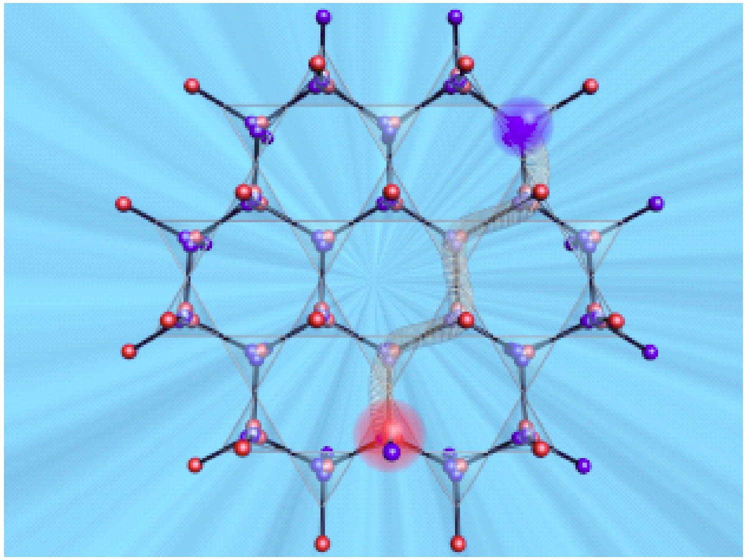
Deconfined magnetic monopoles

Dumbell Hamiltonian gives

$$E(r) = -\frac{\mu_0}{4\pi}\frac{q_m^2}{r}$$

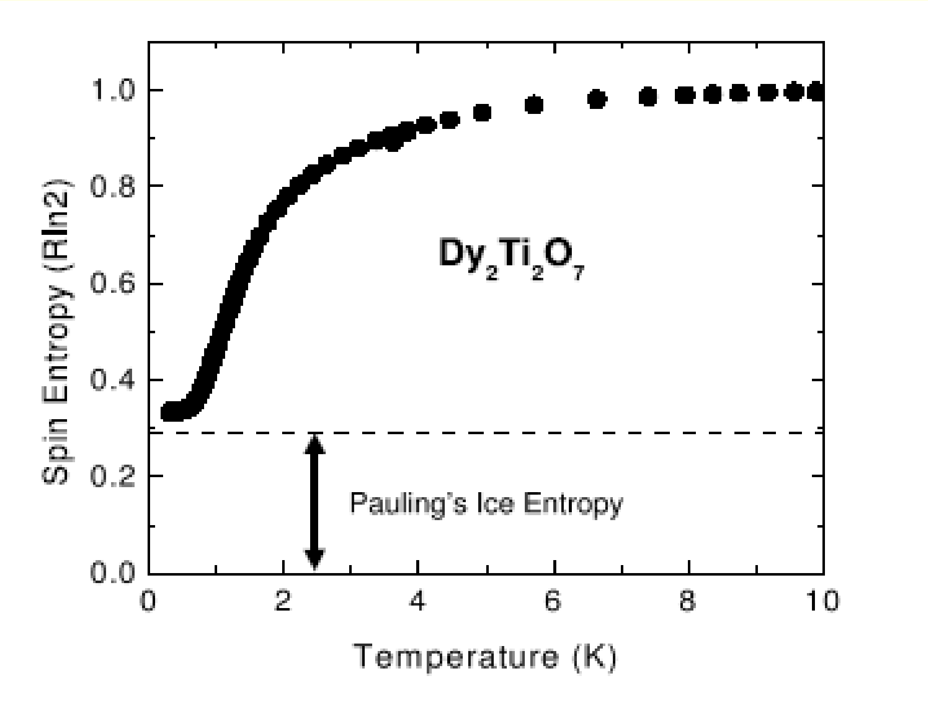
- magnetic Coulomb interaction
- deconfined monopoles
  - charge $q_m = 2\mu/a = (2\mu/\mu_b)(\alpha\lambda_C/2\pi a_d)q_D \approx q_D/8000$
  - monopoles in H, not B



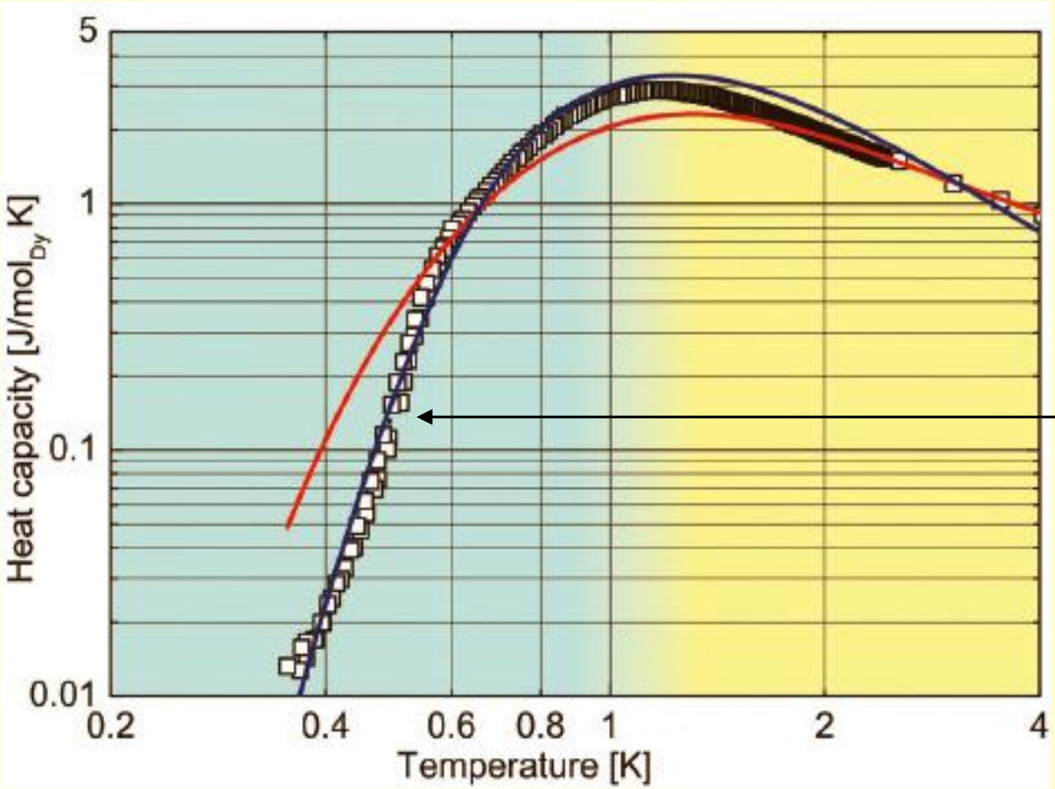


Experiments: I

Macroscopic low temperature entropy (*Ramirez et al, 1999*)



II: Interacting Coulomb Liquid



Debye-Huckel theory

Dirac Strings and Magnetic Monopoles in Spin Ice $Dy_2Ti_2O_7$

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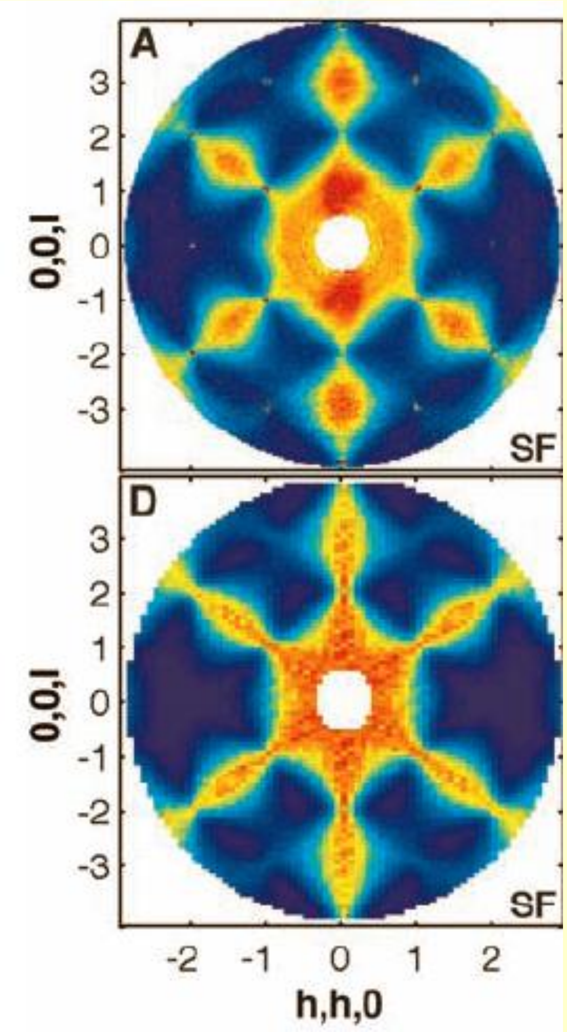
III: Measuring the gauge fluctuations

Experiment

Theory



Magnetic Coulomb Phase in the Spin Ice $Ho_2Ti_2O_7$

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