

# Quantum matter and gauge-gravity duality

Institute for Nuclear Theory, Seattle  
Summer School on Applications of String Theory  
July 18-20

Subir Sachdev



## Outline

1. Conformal quantum matter
2. Compressible quantum matter

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*The boson Hubbard model*

*and the superfluid-insulator transition*

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*The fermion Hubbard model*

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*The  $AdS_4$  - Reissner-Nordström black-brane*

*and  $AdS_2 \times R^2$*

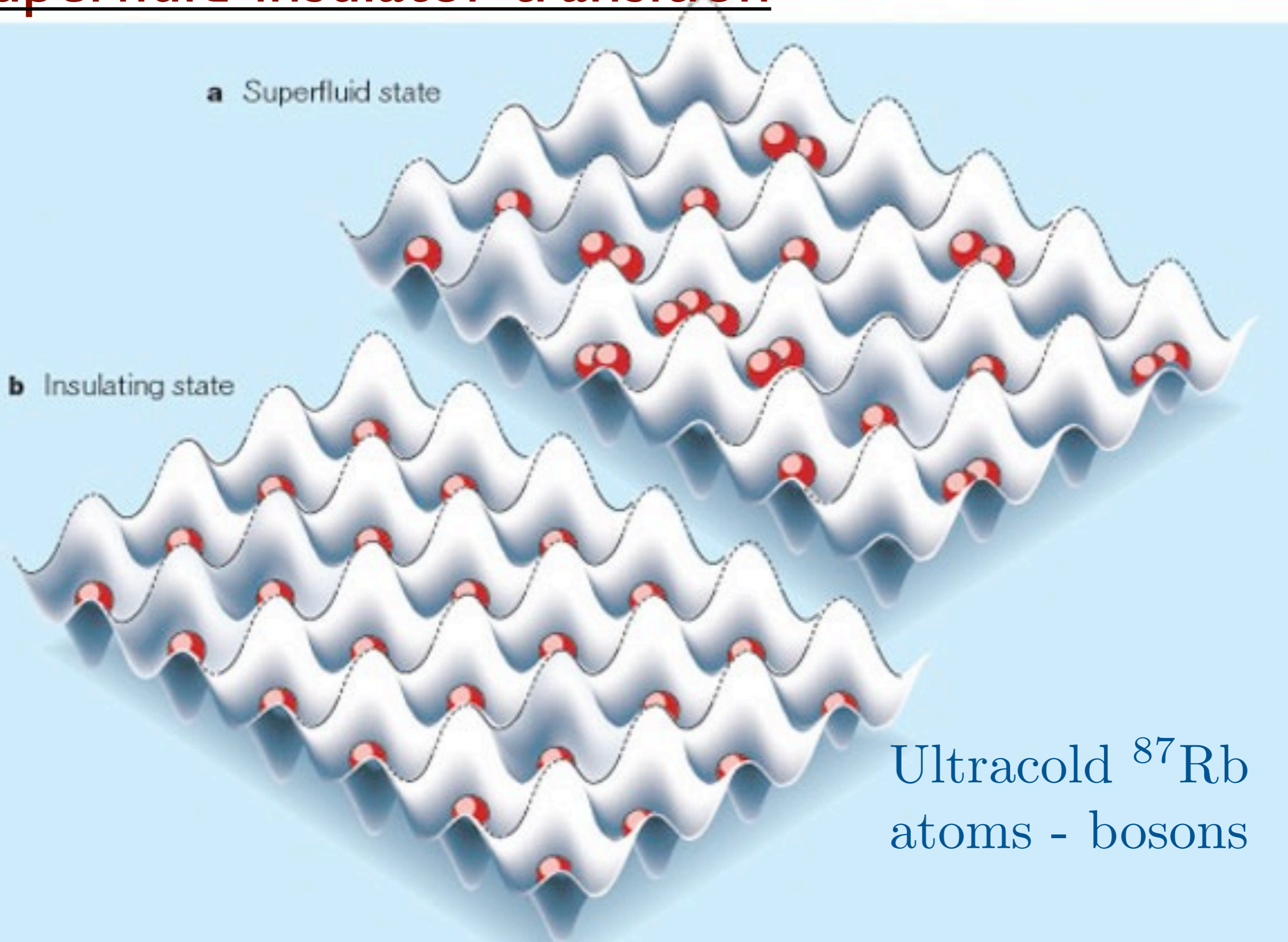
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# Superfluid-insulator transition



Ultracold  $^{87}\text{Rb}$   
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



# The Superfluid-Insulator transition

## Boson Hubbard model

Degrees of freedom: Bosons,  $b_j^\dagger$ , hopping between the sites,  $j$ , of a lattice, with short-range repulsive interactions.

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \dots$$

$$n_j \equiv b_j^\dagger b_j$$

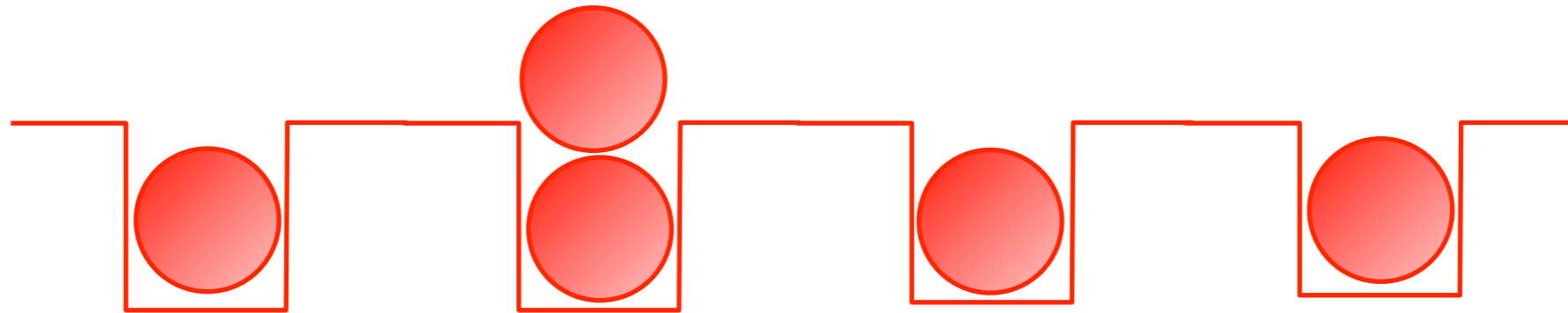
$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).



Insulator (the vacuum)  
at large repulsion between bosons

# Excitations of the insulator:



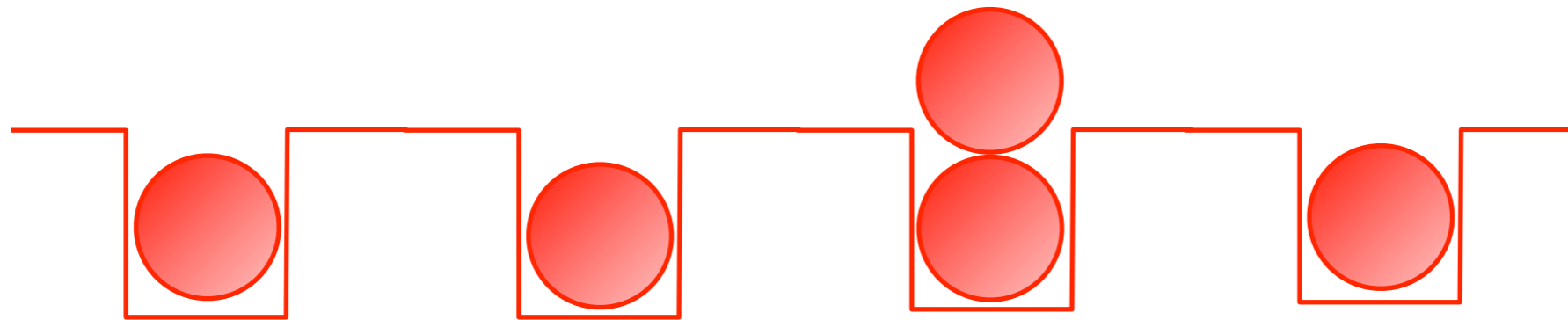
Particles  $\sim \psi^\dagger$

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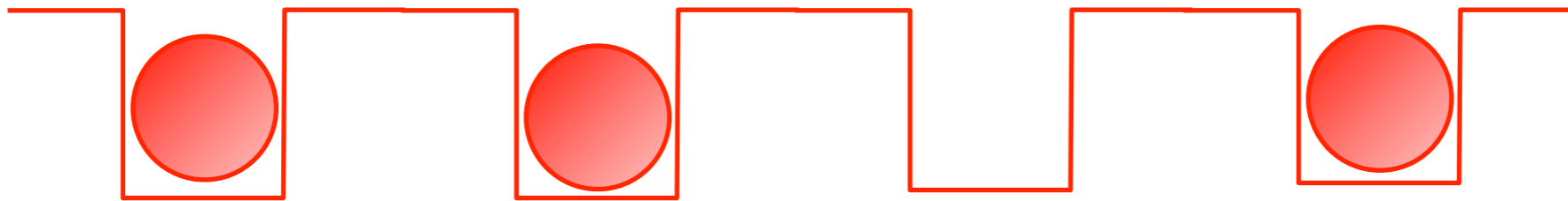


Holes  $\sim \psi$

# Excitations of the insulator:



Particles  $\sim \psi^\dagger$



Holes  $\sim \psi$

Density of particles = density of holes  $\Rightarrow$

“relativistic” field theory for  $\psi$ :

$$\mathcal{S} = \int d^2r d\tau \left[ |\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c) |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

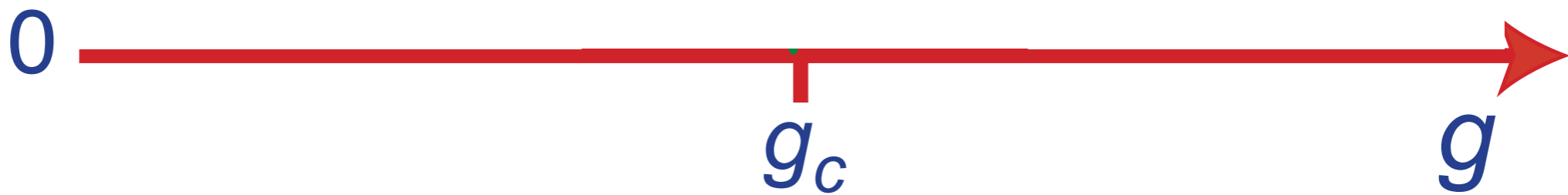
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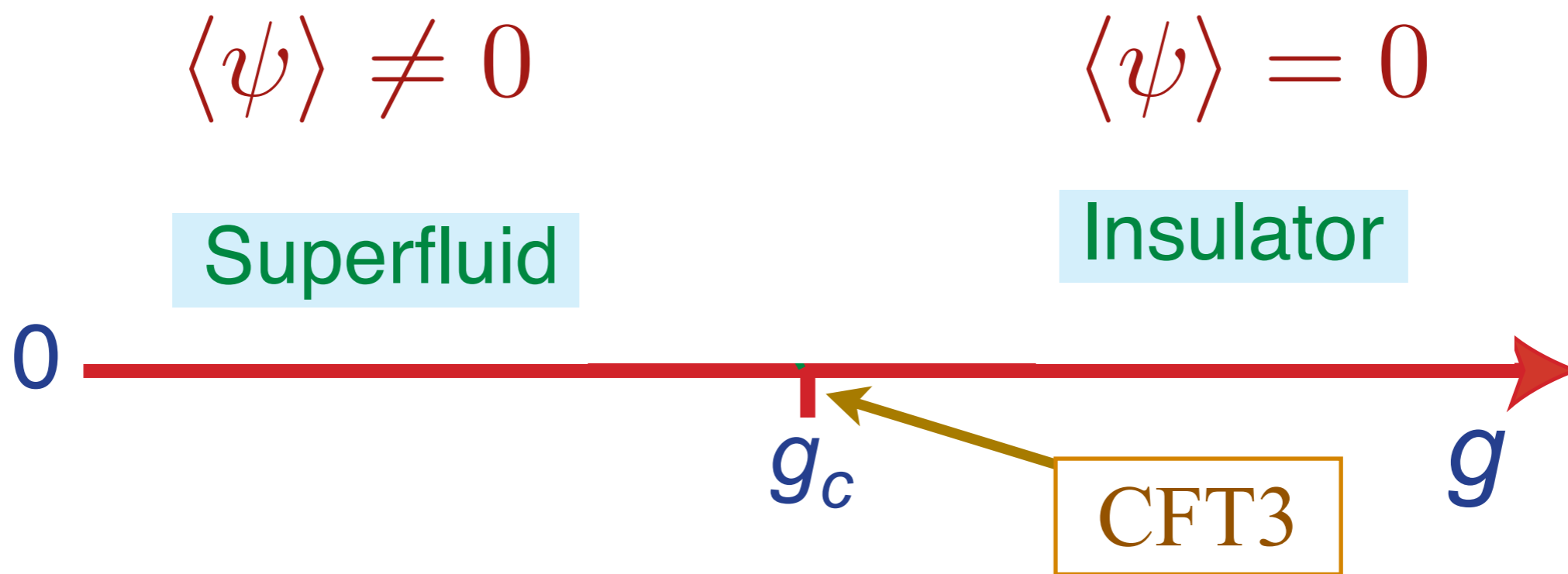
$$\langle \psi \rangle \neq 0$$

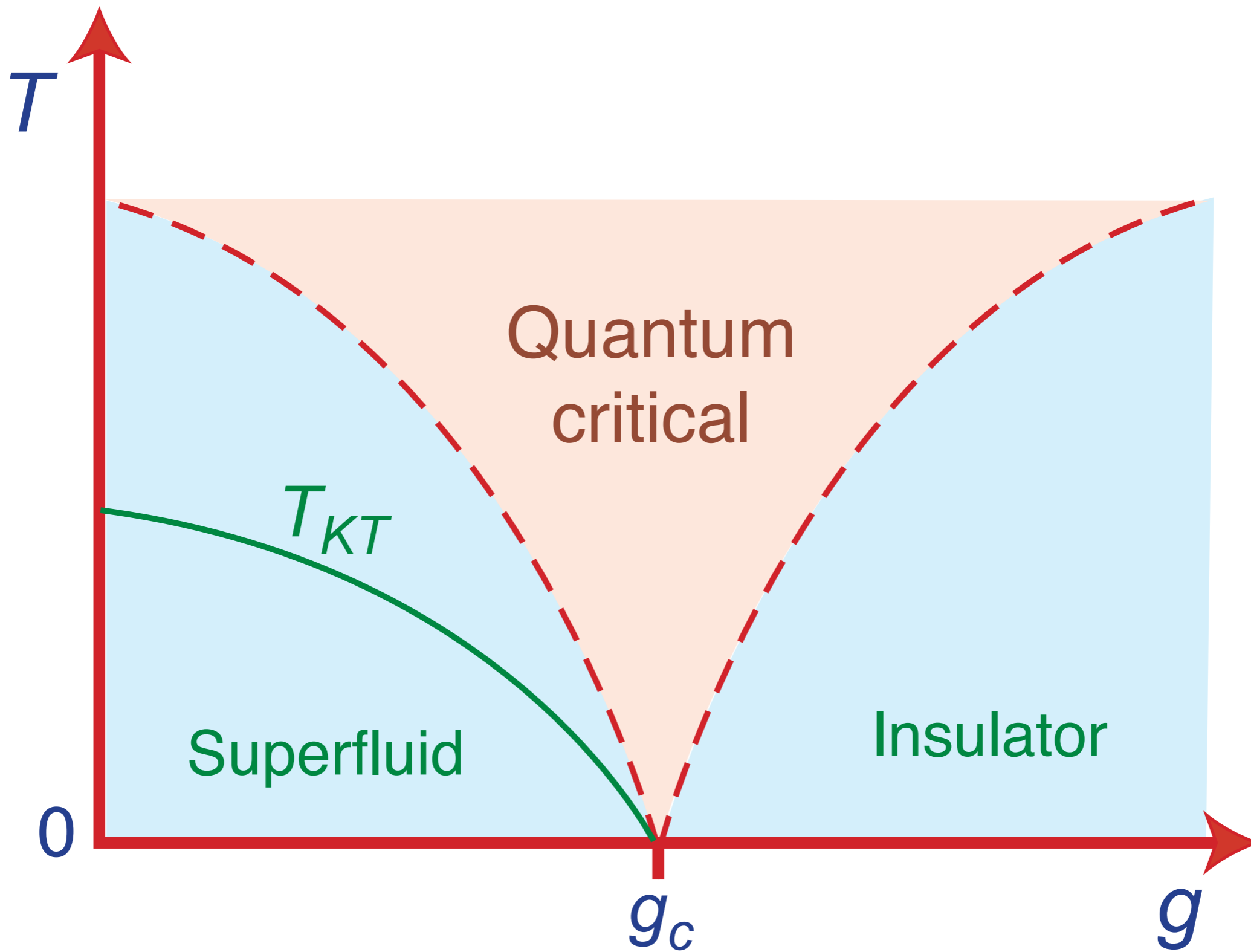
Superfluid

$$\langle \psi \rangle = 0$$

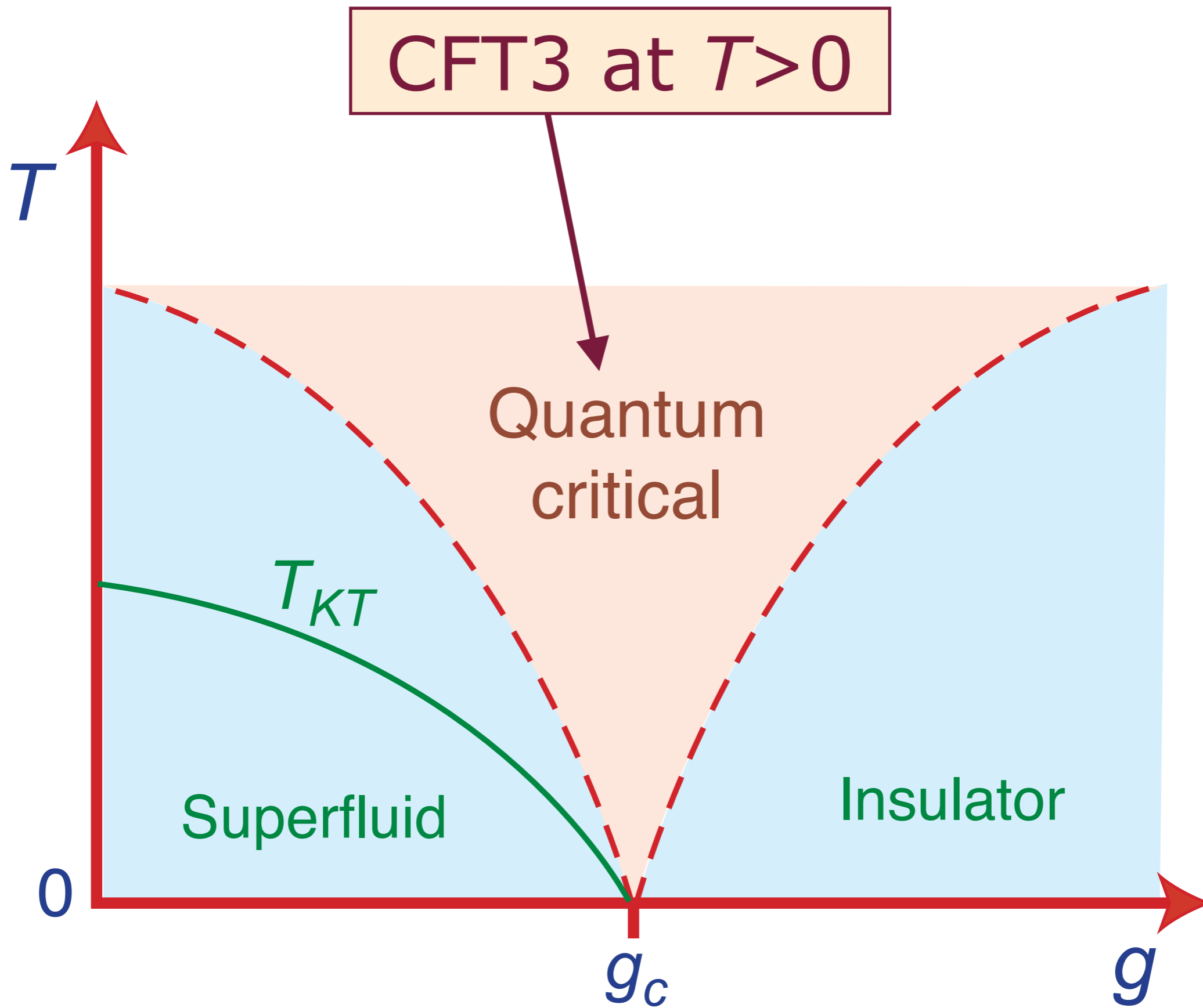
Insulator











# Quantum critical transport

Quantum “*nearly perfect fluid*”  
with shortest possible  
equilibration time,  $\tau_{\text{eq}}$

$$\tau_{\text{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where  $\mathcal{C}$  is a *universal* constant

# Quantum critical transport

Transport co-efficients not determined  
by collision rate, but by  
universal constants of nature

## Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1) ]$$

( $Q$  is the “charge” of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

# Quantum critical transport

Transport co-efficients not determined  
by collision rate, but by  
universal constants of nature

## Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94**, 11601 (2005)

# Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency ( $\omega$ ) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau_c}$$

where  $\tau_c \sim \hbar/(k_B T)$  is the time between boson collisions.

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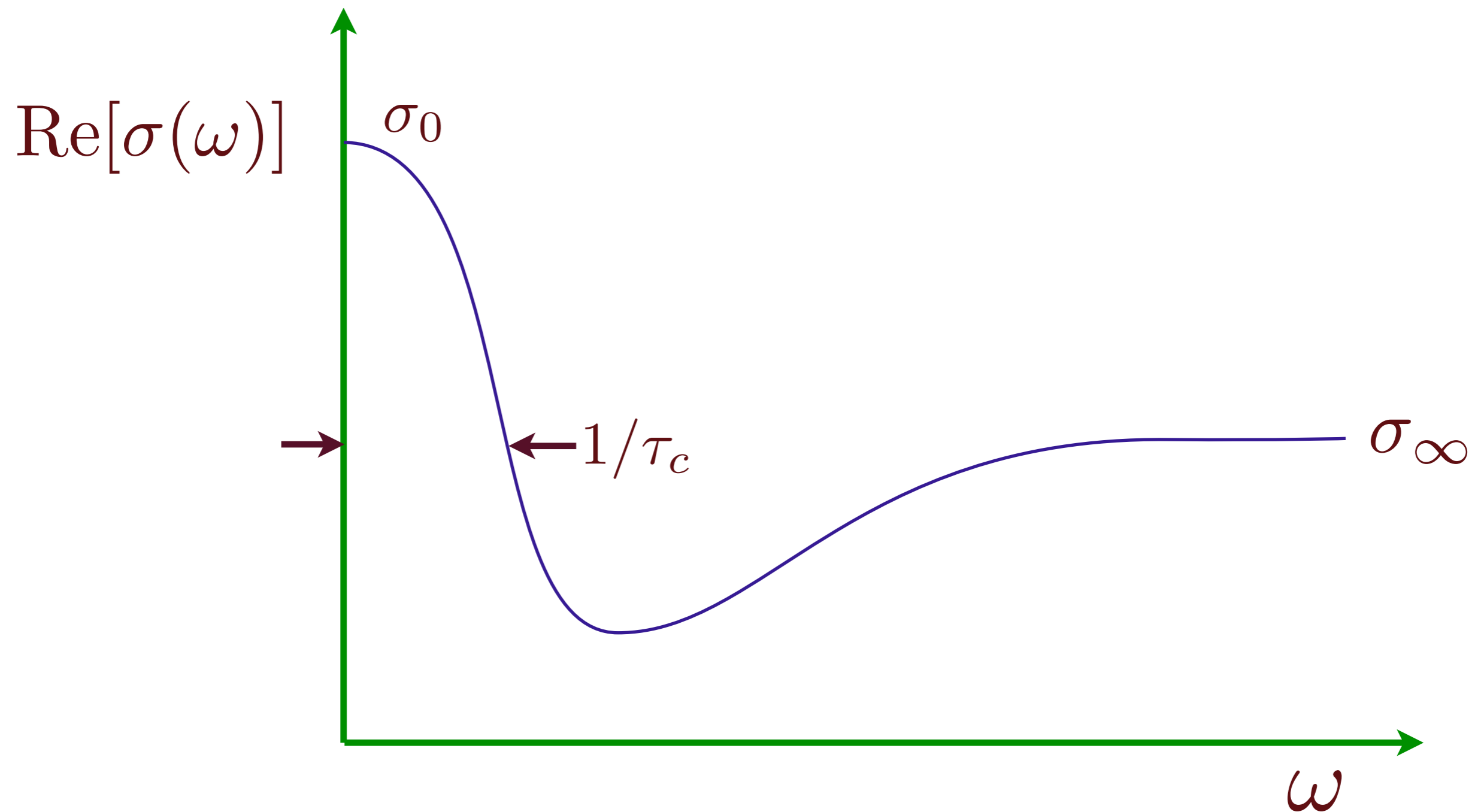
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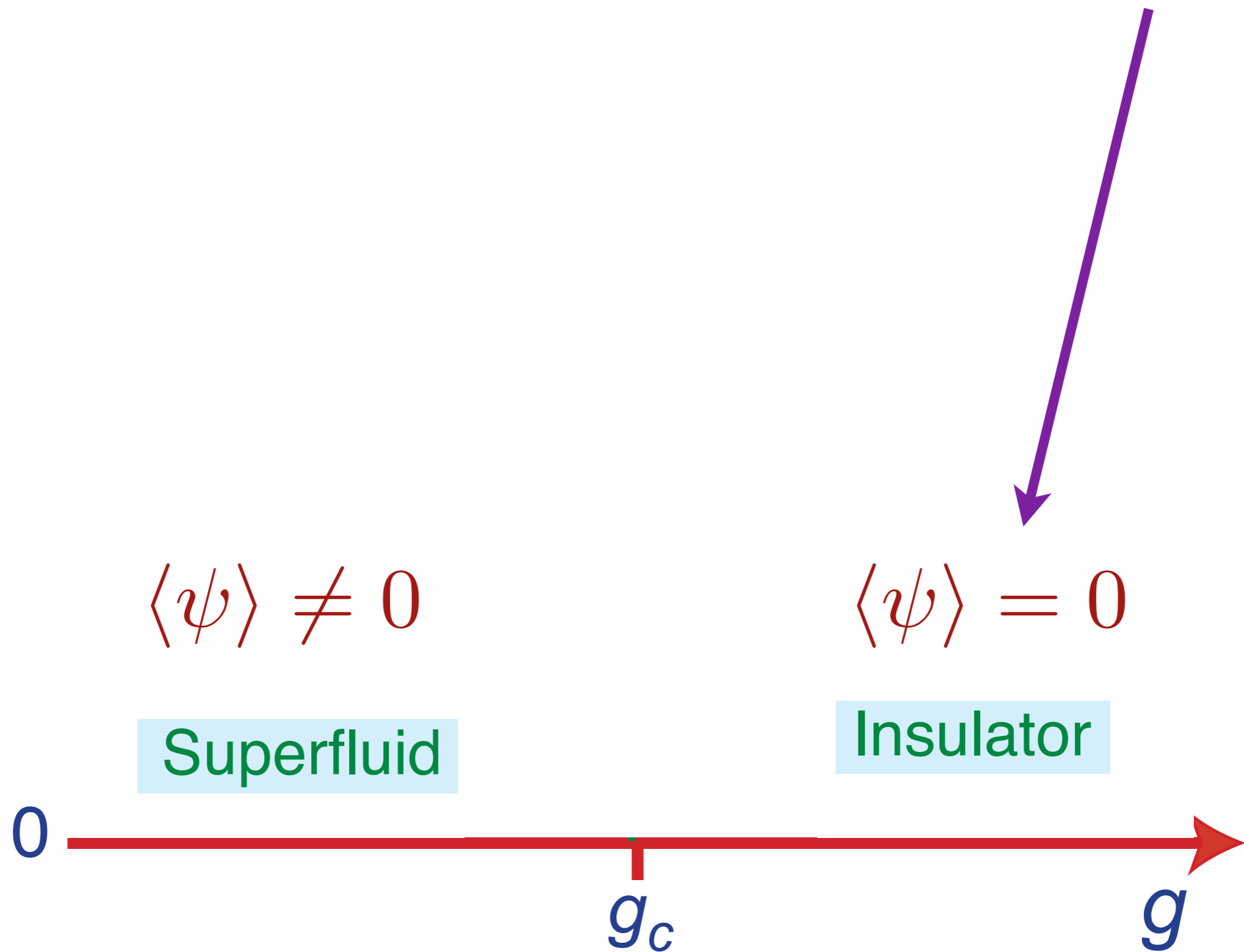
where  $\tau_c \sim \hbar/(k_B T)$  is the time between boson collisions.

Also, we have  $\sigma(\omega \rightarrow \infty) = \sigma_\infty$ , associated with the density of states for particle-hole creation (the “optical conductivity”) in the CFT3.

# Boltzmann theory of bosons

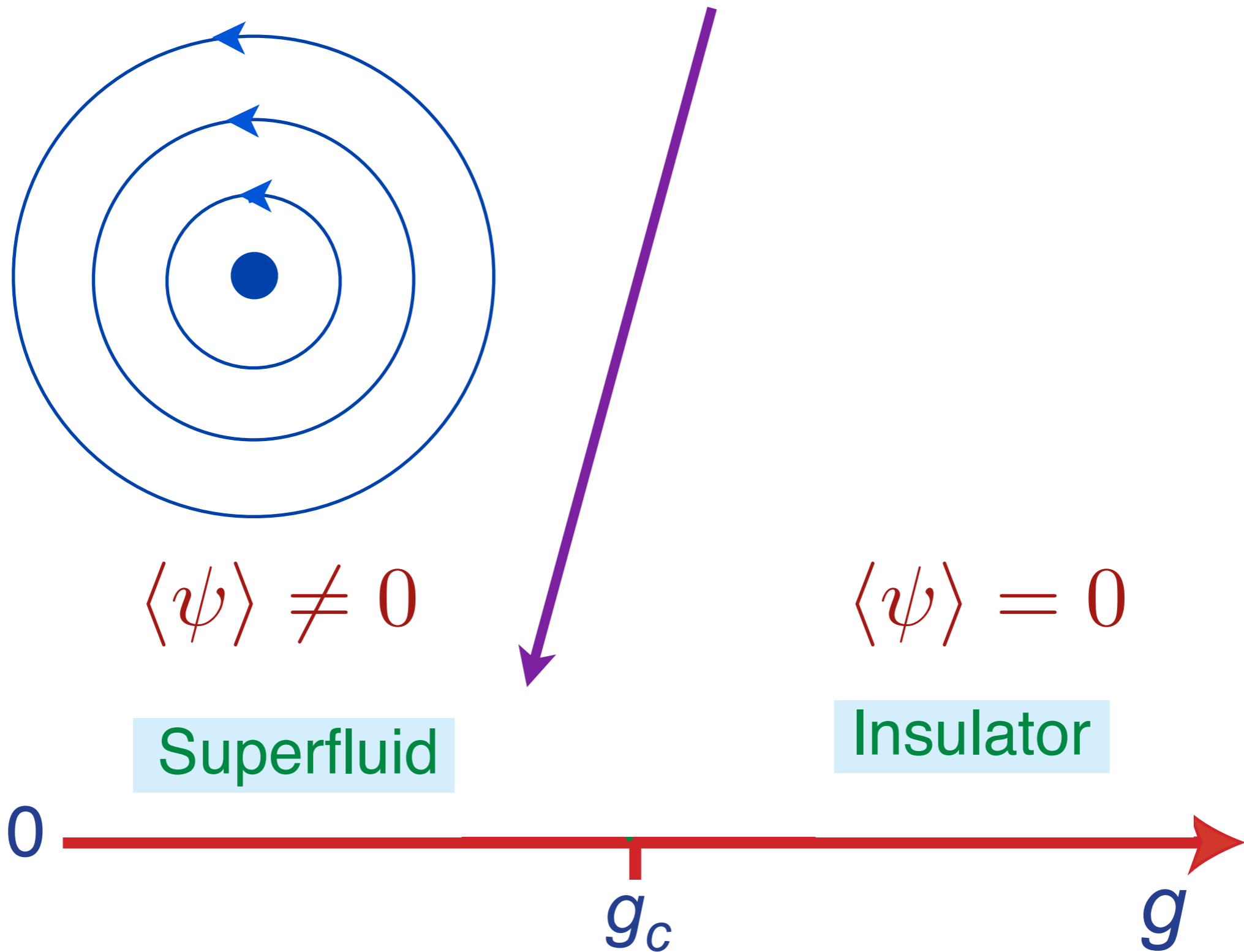


So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.





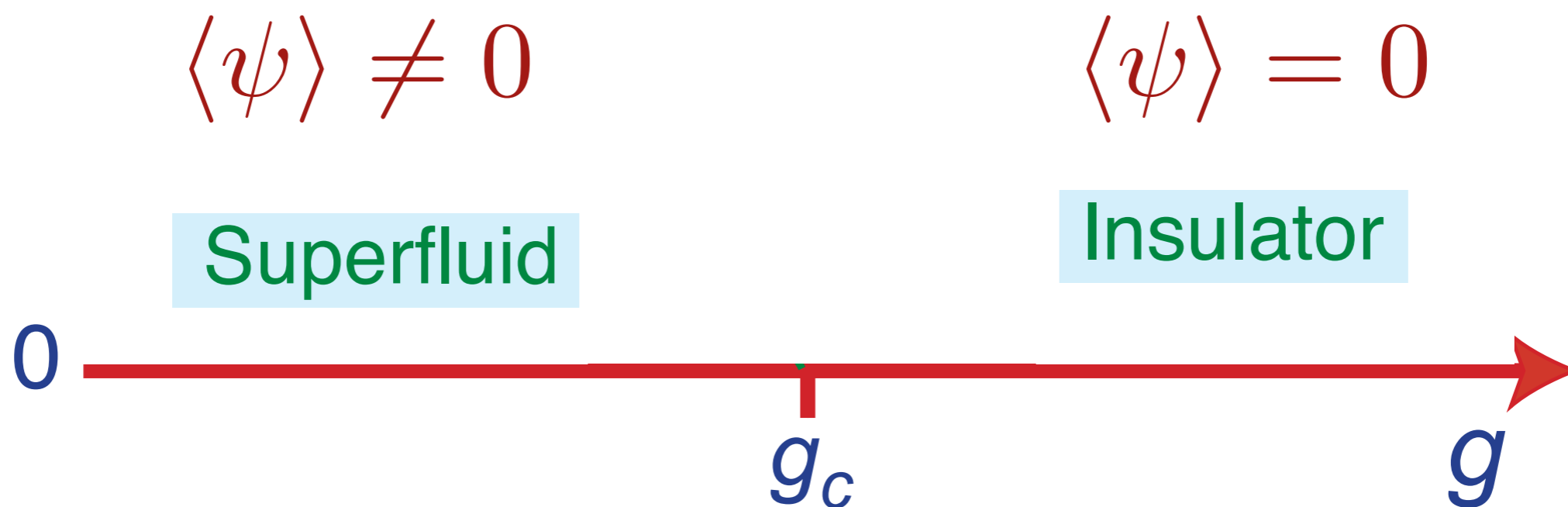
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



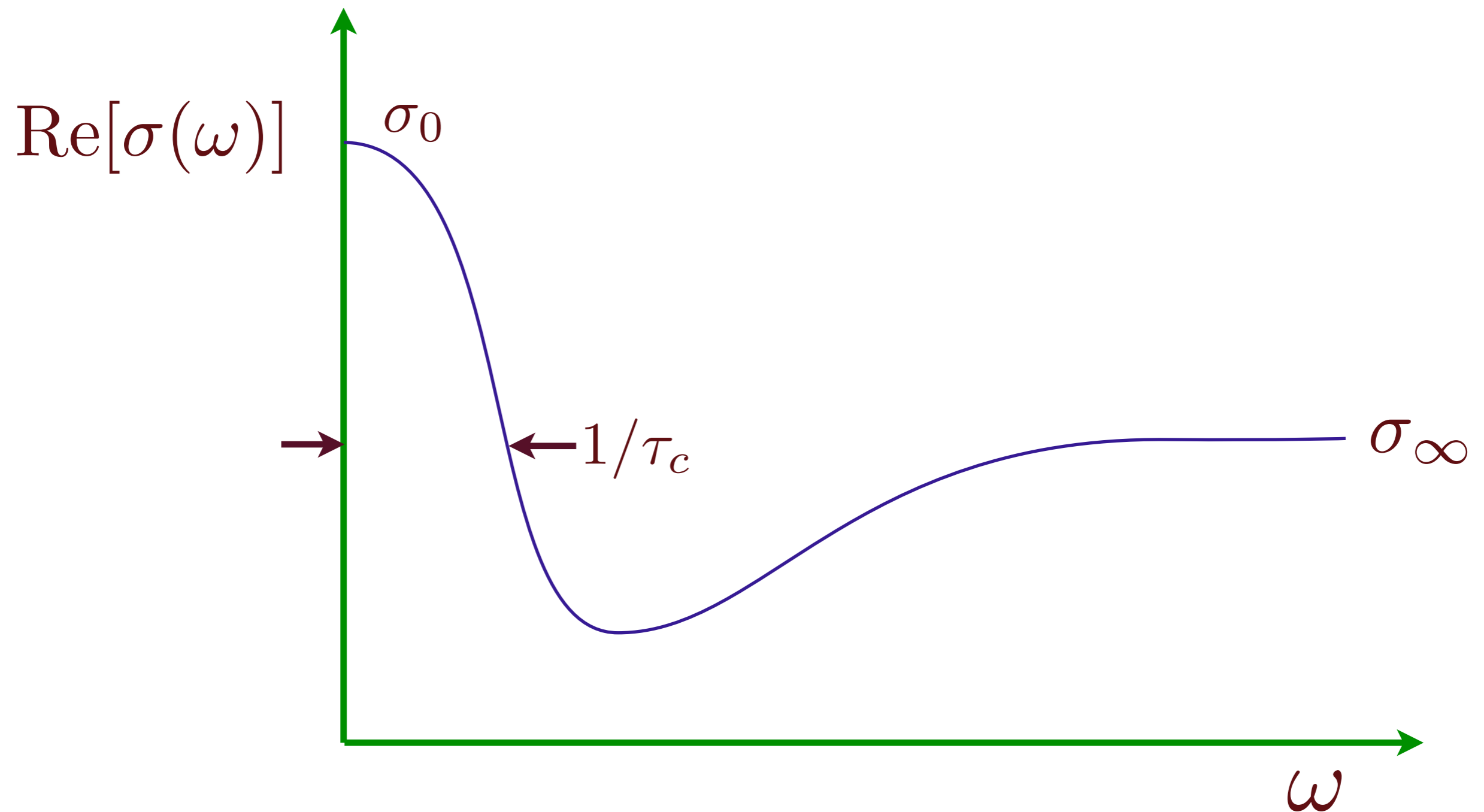
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

These are quantum particles (in 2+1 dimensions) which are described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their  $T > 0$  dynamics can also be described by a Boltzmann equation:

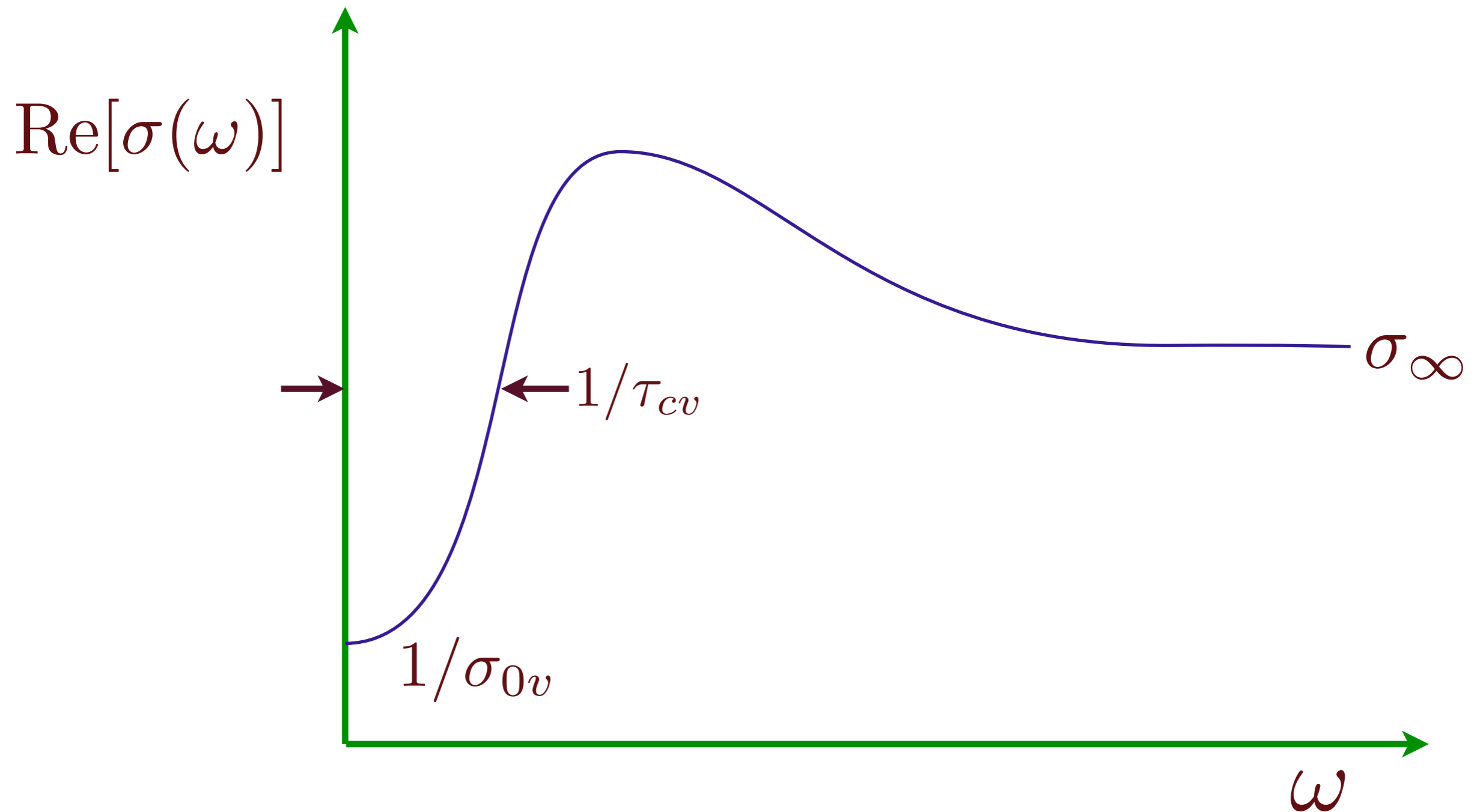
Conductivity = Resistivity of vortices



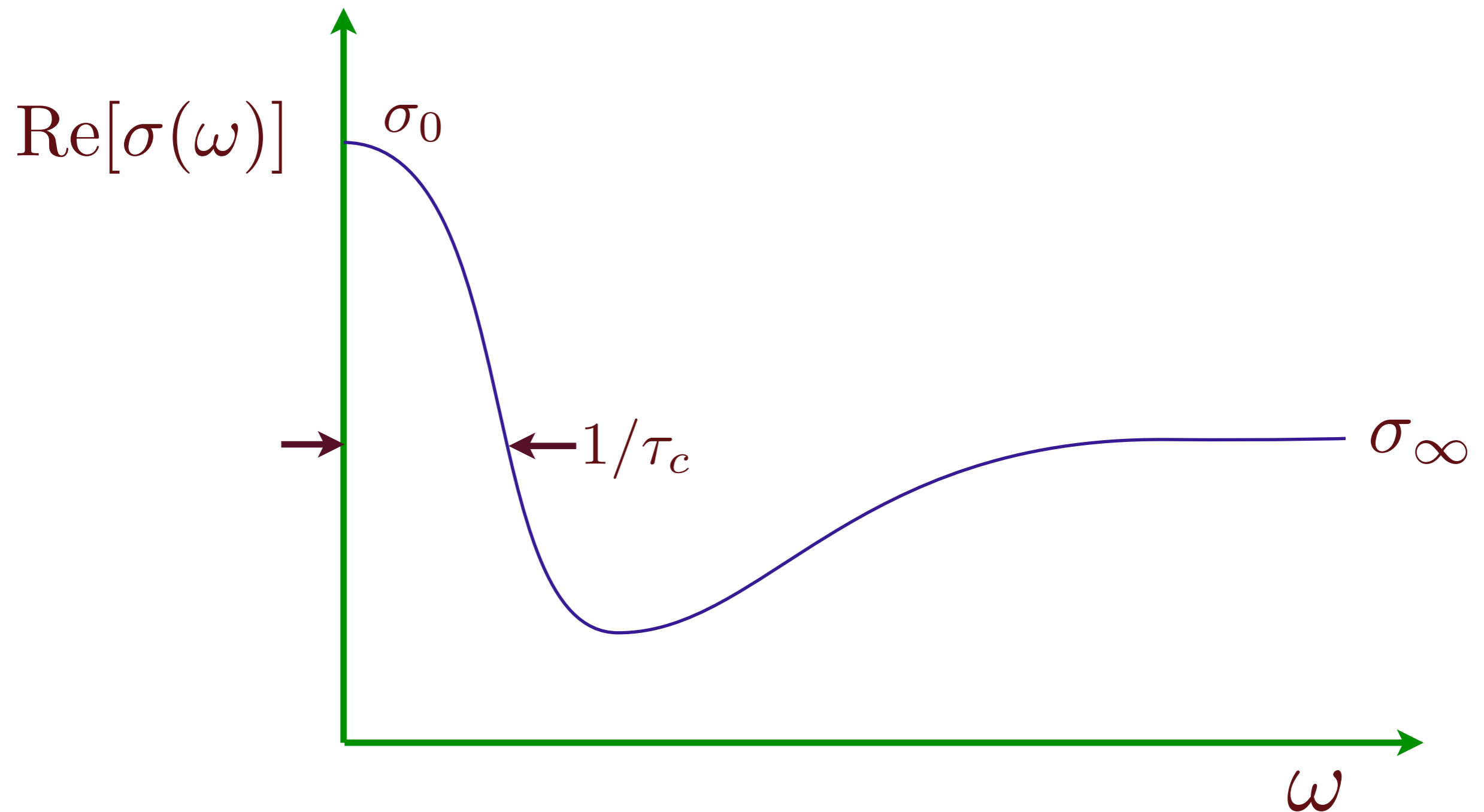
# Boltzmann theory of bosons



# Boltzmann theory of vortices

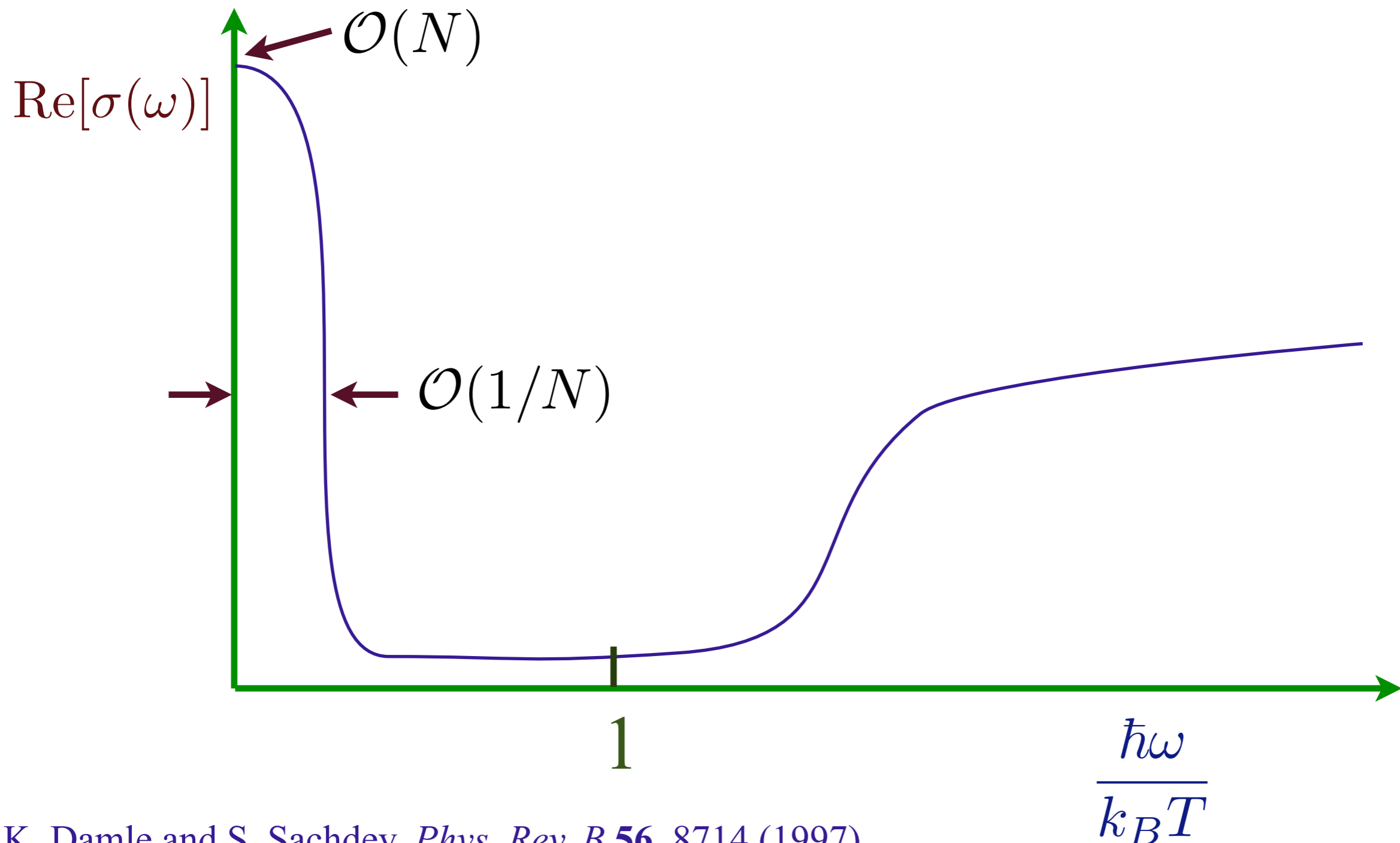


# Boltzmann theory of bosons



# Vector large $N$ expansion for CFT3

$$\sigma = \frac{Q^2}{h} \Sigma \left( \frac{\hbar\omega}{k_B T} \right); \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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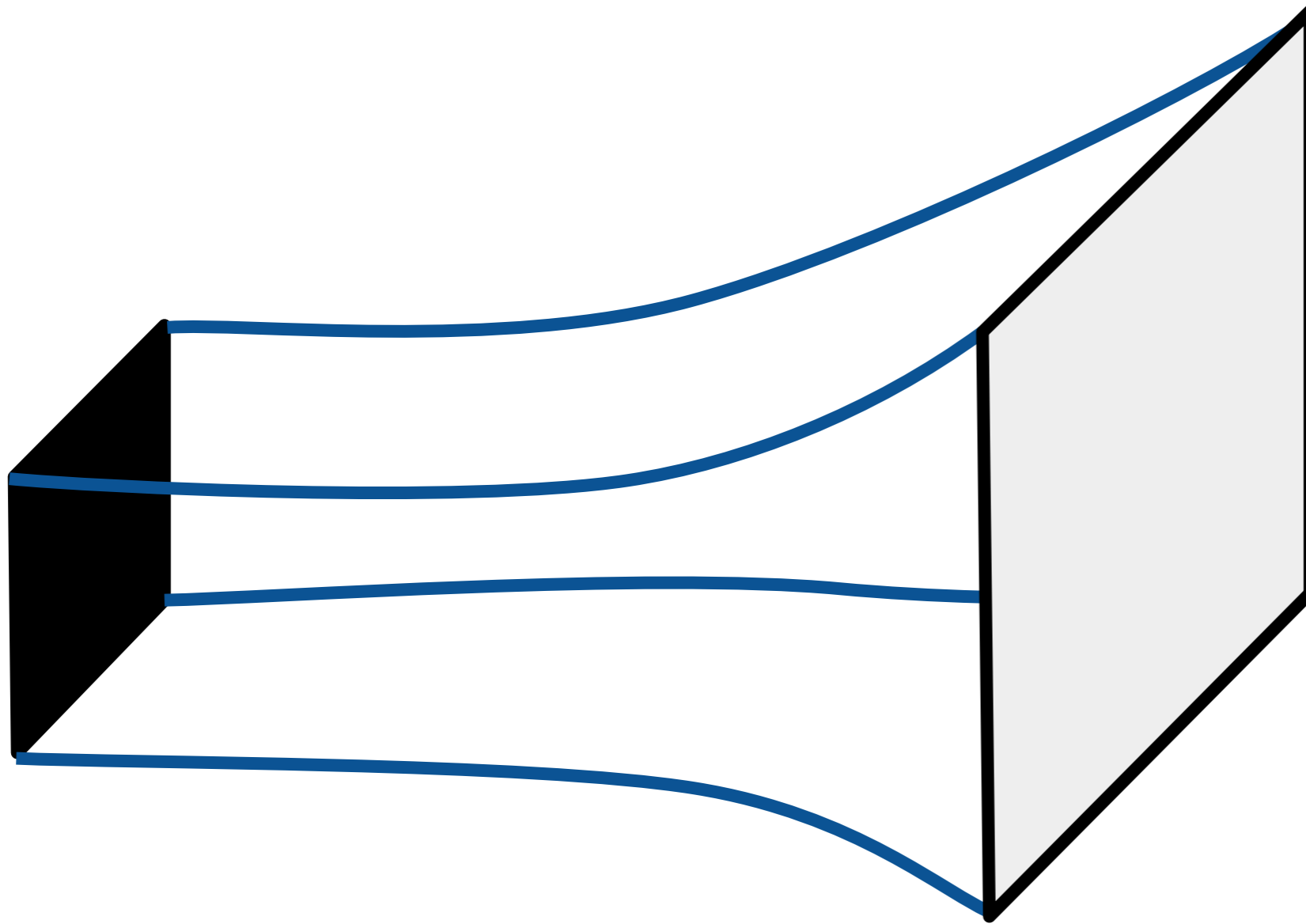
*The  $AdS_4$  - Schwarzschild black brane*

# 2. Compressible quantum matter



# AdS/CFT correspondence

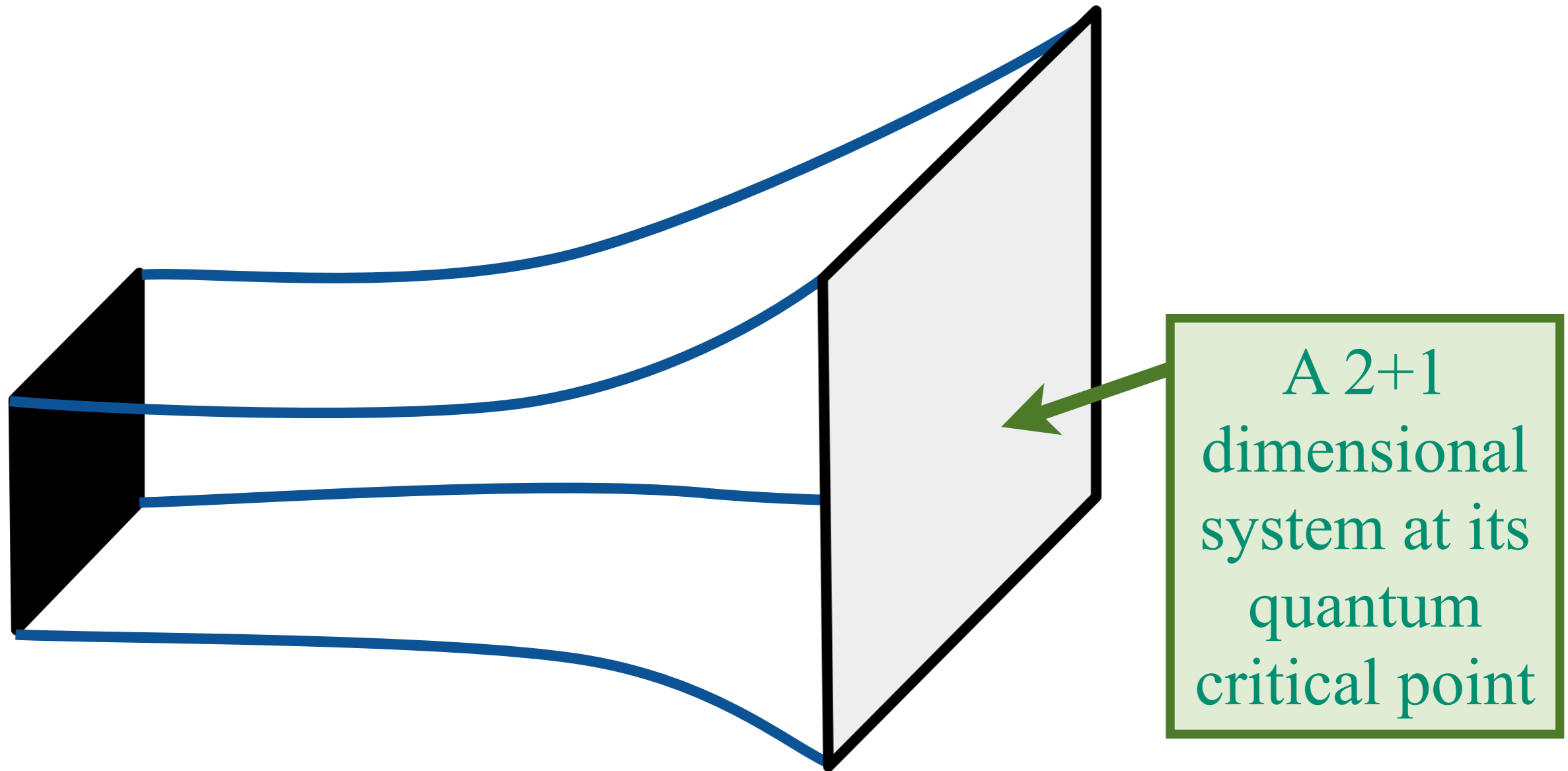
## AdS<sub>4</sub>-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$

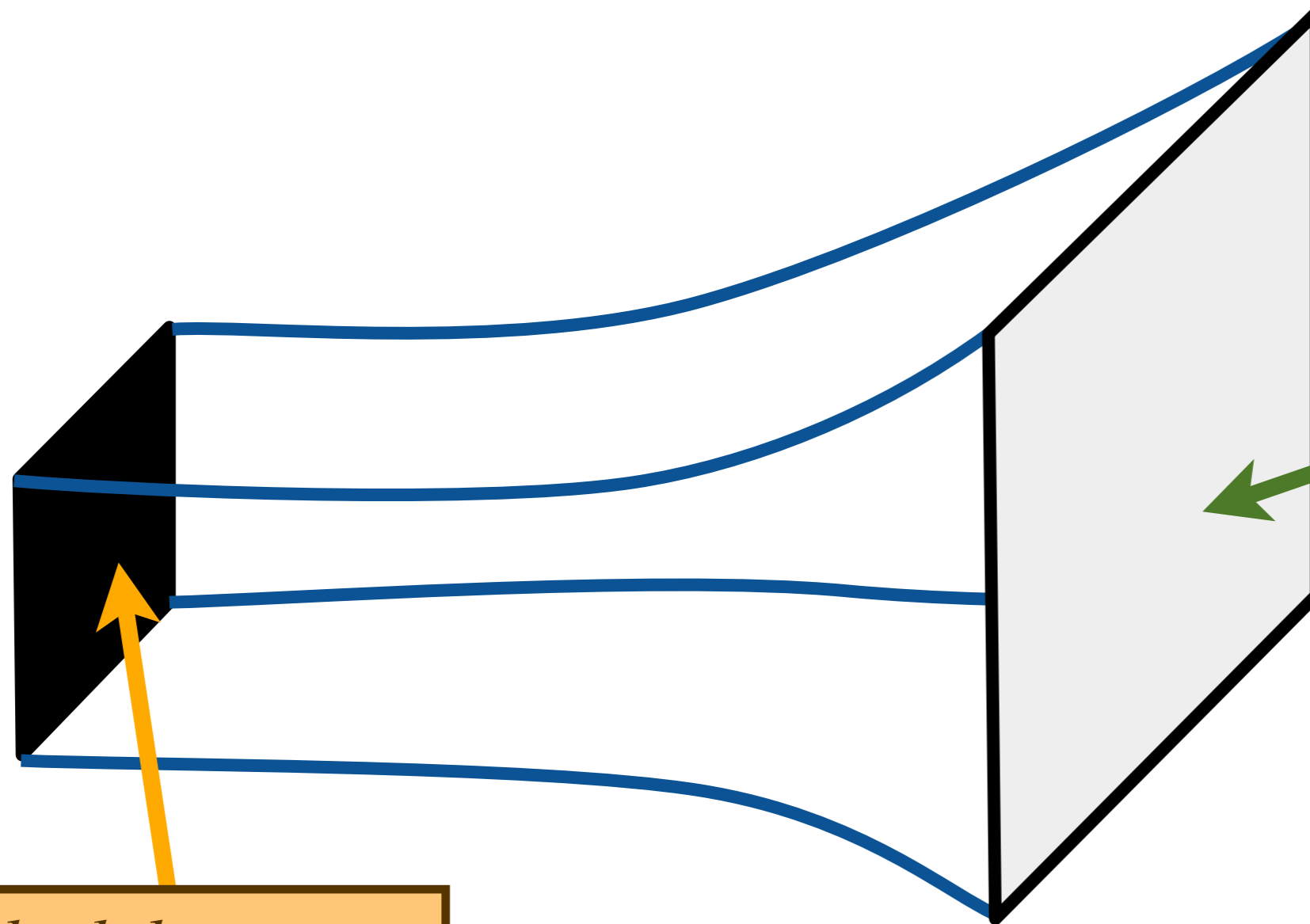
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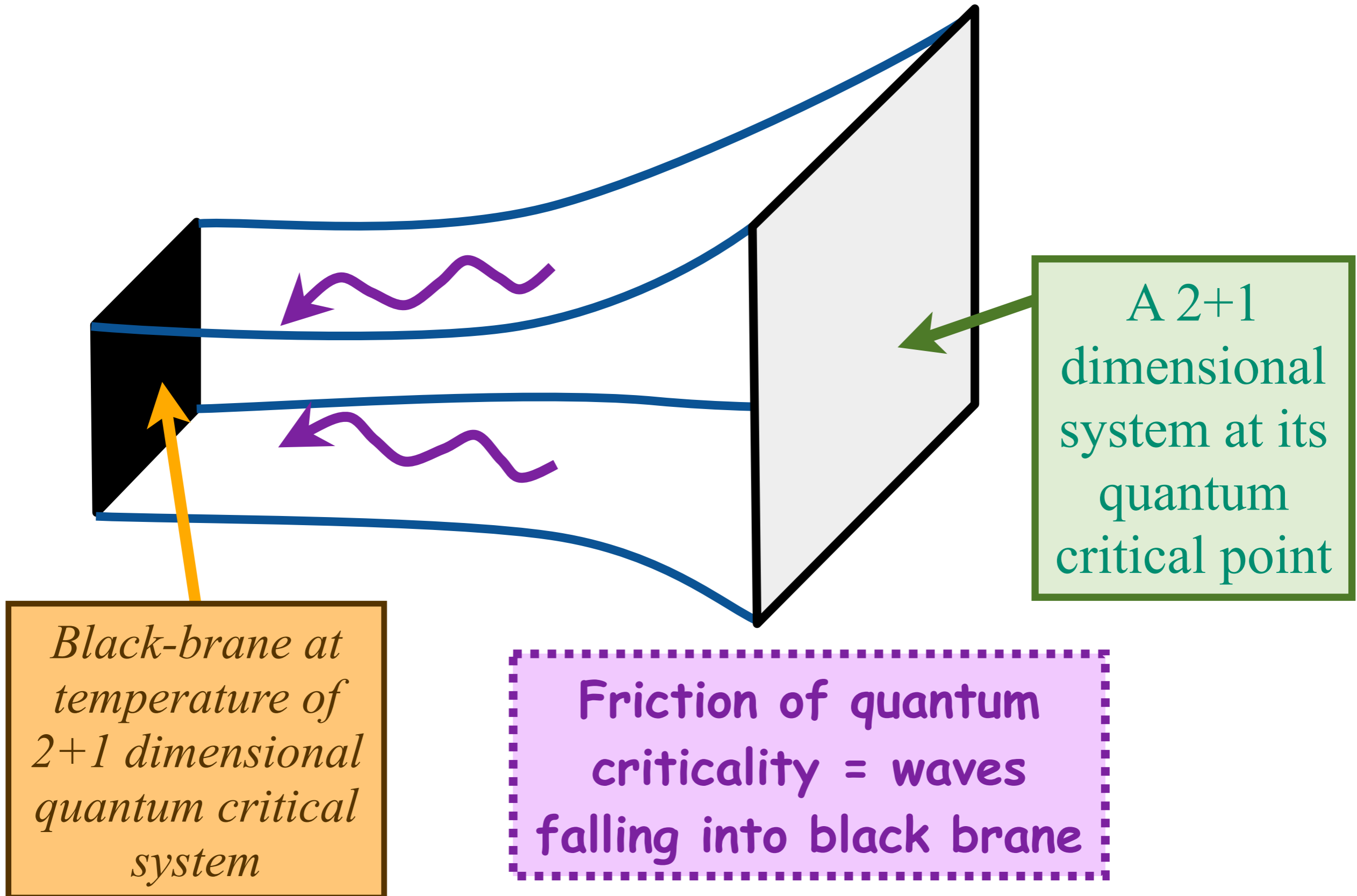
*Black-brane at temperature of 2+1 dimensional quantum critical system*

A 2+1 dimensional system at its quantum critical point

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$

# AdS/CFT correspondence

## AdS<sub>4</sub>-Schwarzschild black-brane



# AdS<sub>4</sub> theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS<sub>4</sub>-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[ -\frac{1}{4e^2} F_{ab} F^{ab} \right].$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,  
*Phys. Rev. D* **75**, 085020 (2007).

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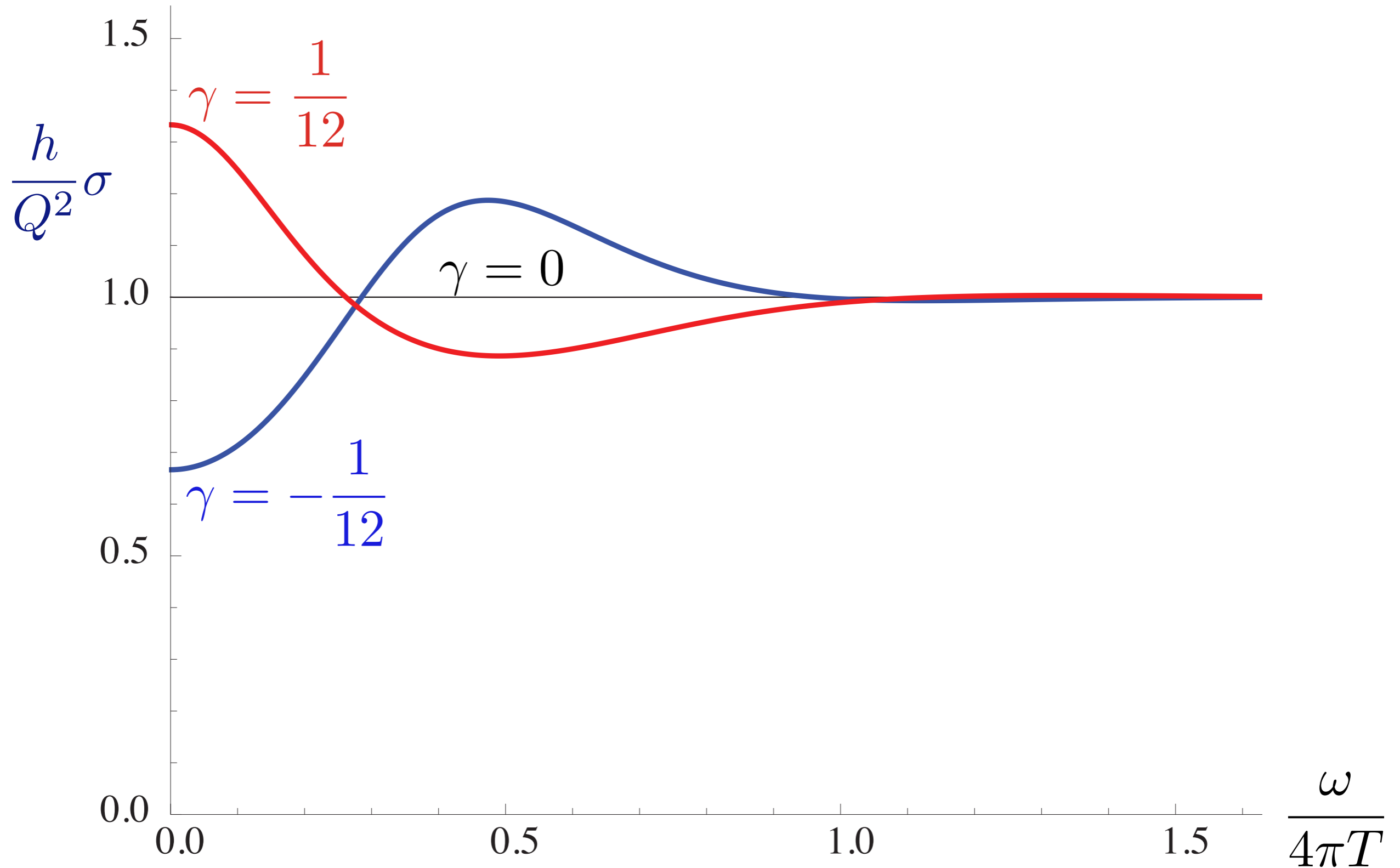
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant  $\gamma$  ( $L$  is the radius of AdS<sub>4</sub>):

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[ -\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right],$$

where  $C_{abcd}$  is the Weyl curvature tensor.

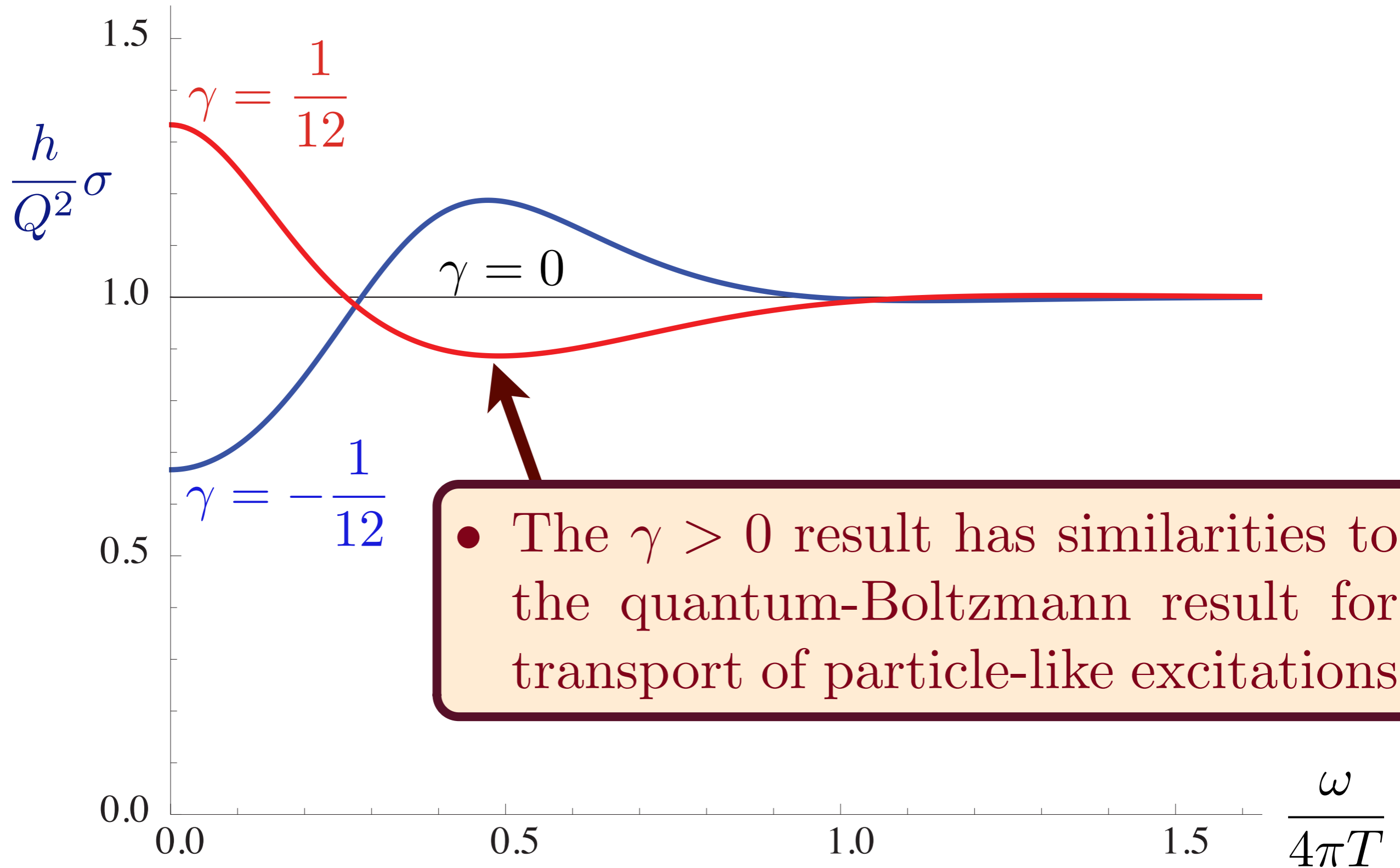
*Stability and causality constraints restrict  $|\gamma| < 1/12$ .*

# AdS<sub>4</sub> theory of strongly interacting “perfect fluids”



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

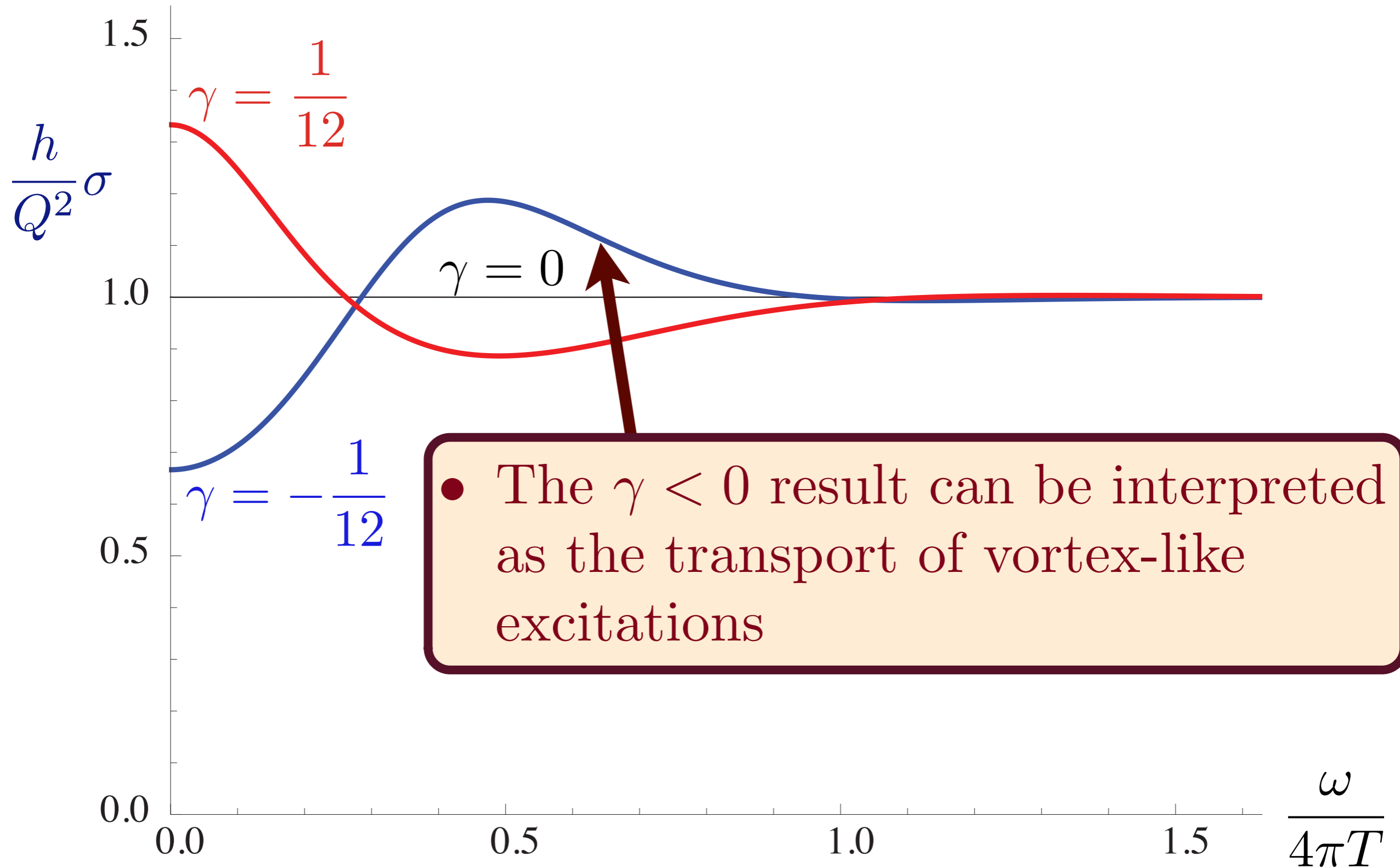
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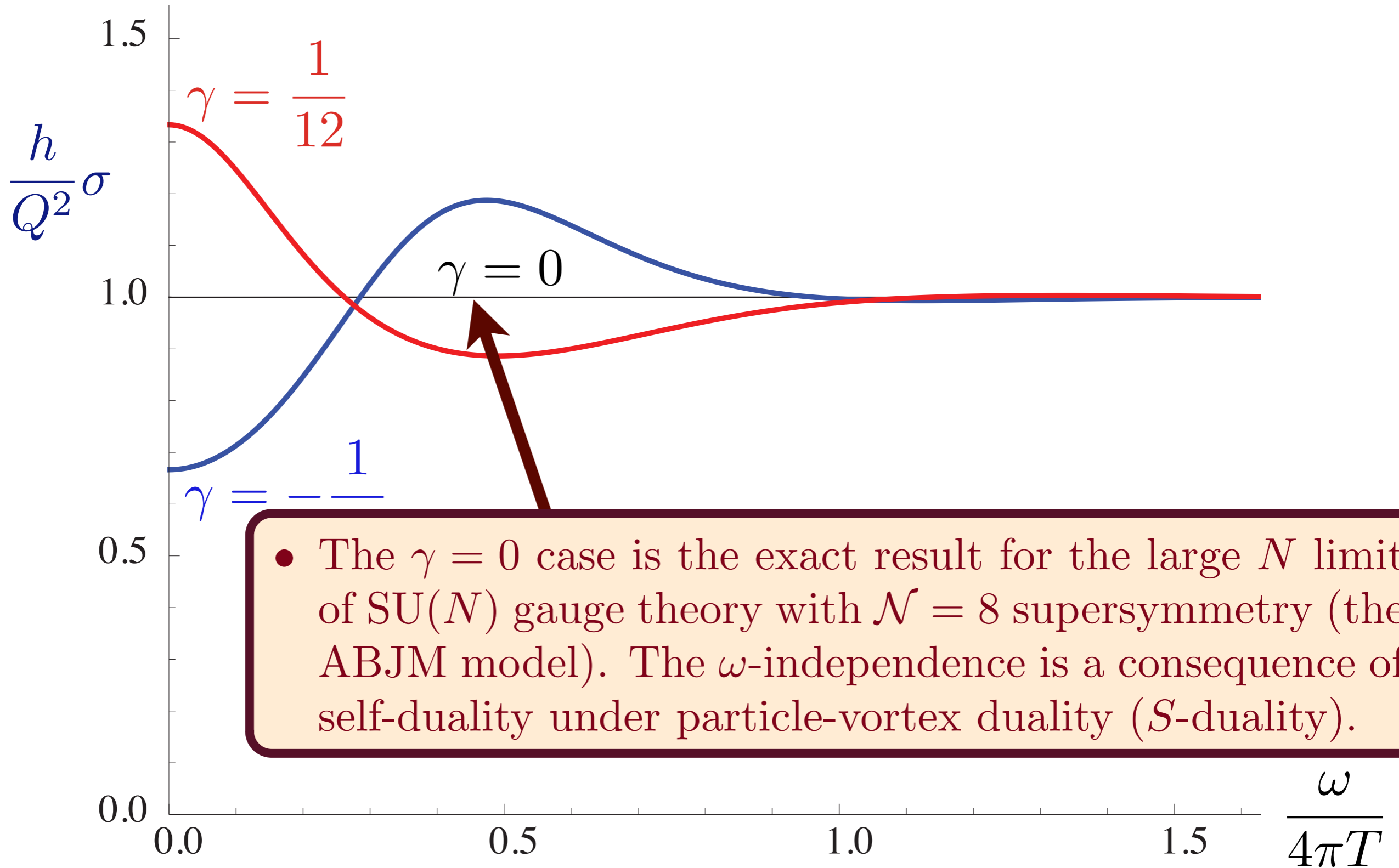


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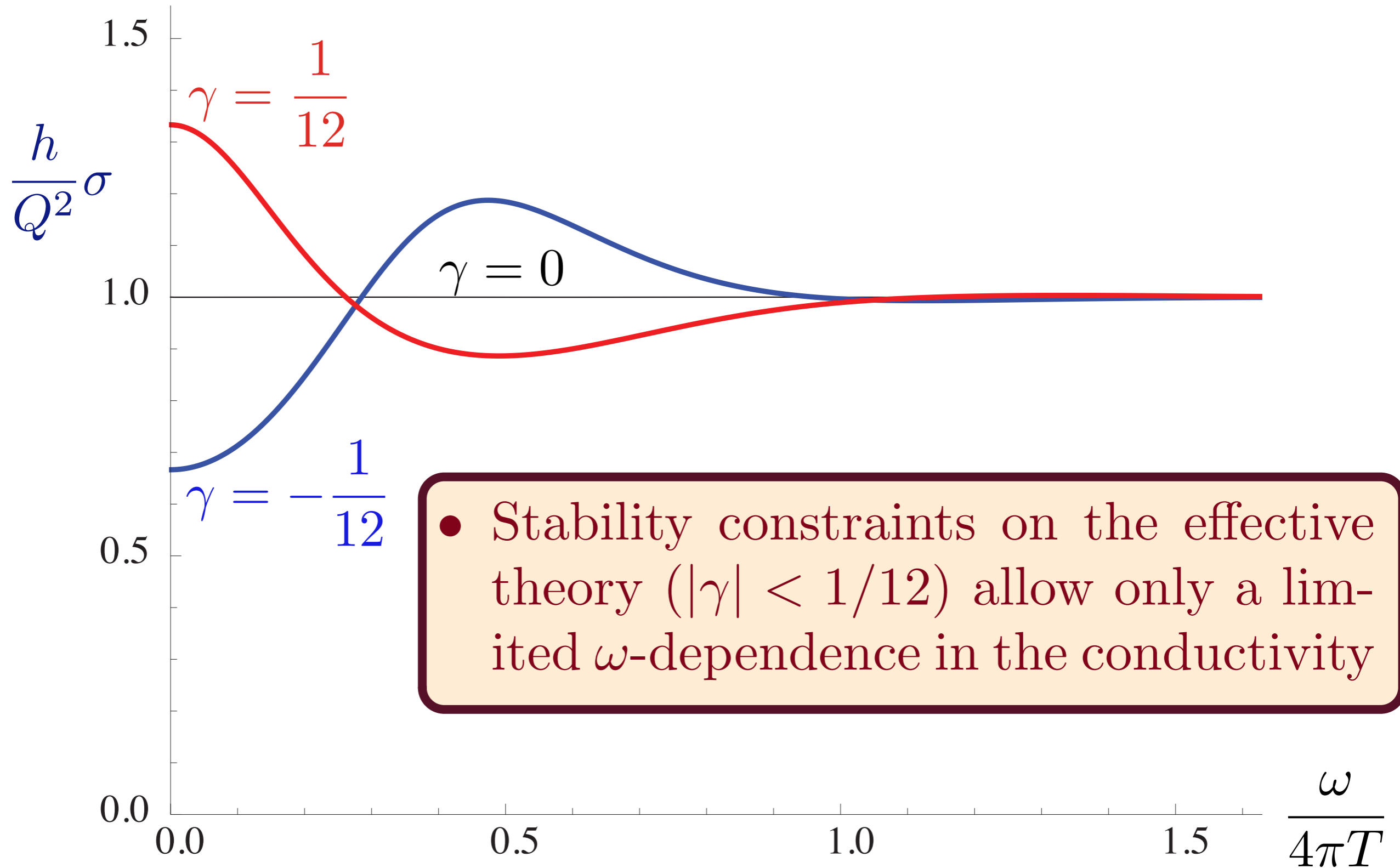
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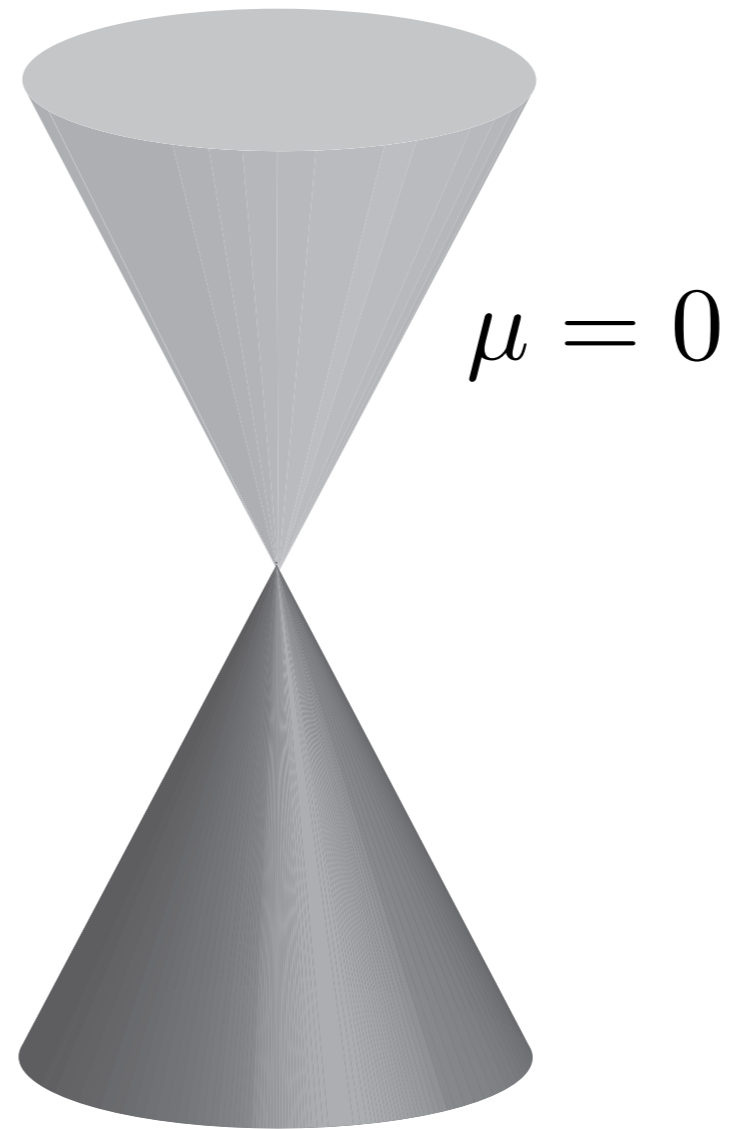
# Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved  $U(1)$  charge  $Q$  (the “electron density”) in spatial dimension  $d > 1$ .

# Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge  $Q$  (the “electron density”) in spatial dimension  $d > 1$ .
- Describe zero temperature phases where  $\langle Q \rangle$  varies smoothly as a function of  $\mu$  (the “chemical potential”) which changes the Hamiltonian,  $H$ , to  $H - \mu Q$ .

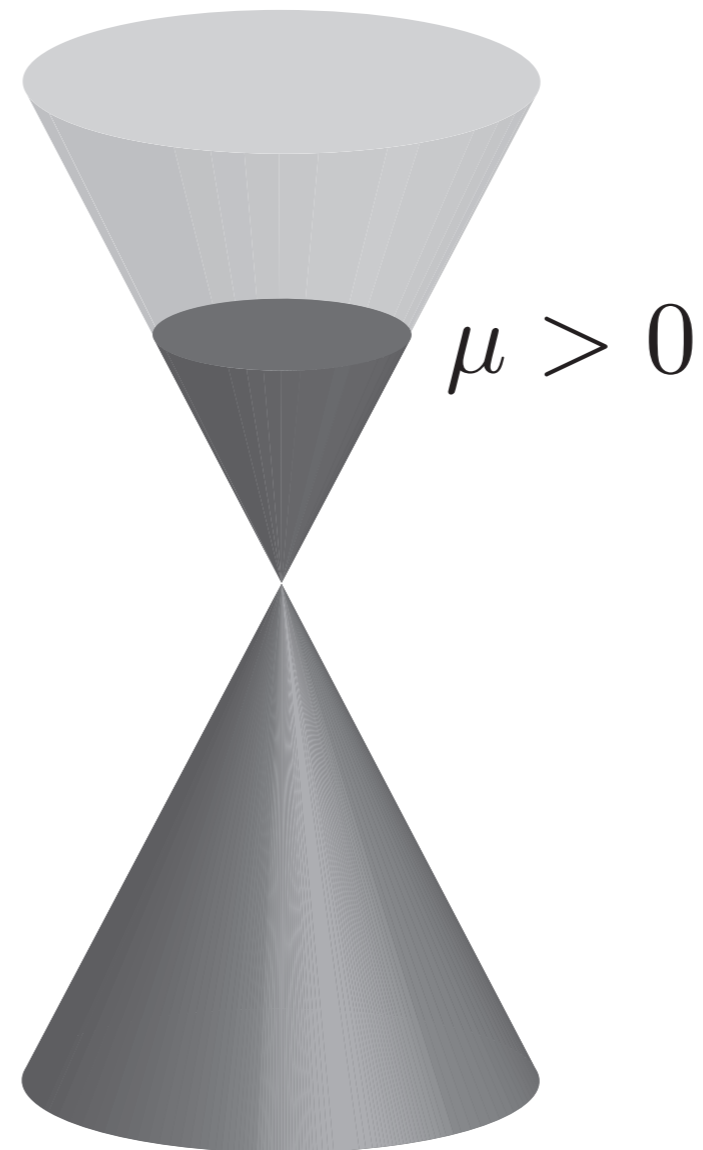
# Turning on a chemical potential on a CFT



Massless Dirac fermions  
(e.g. graphene)



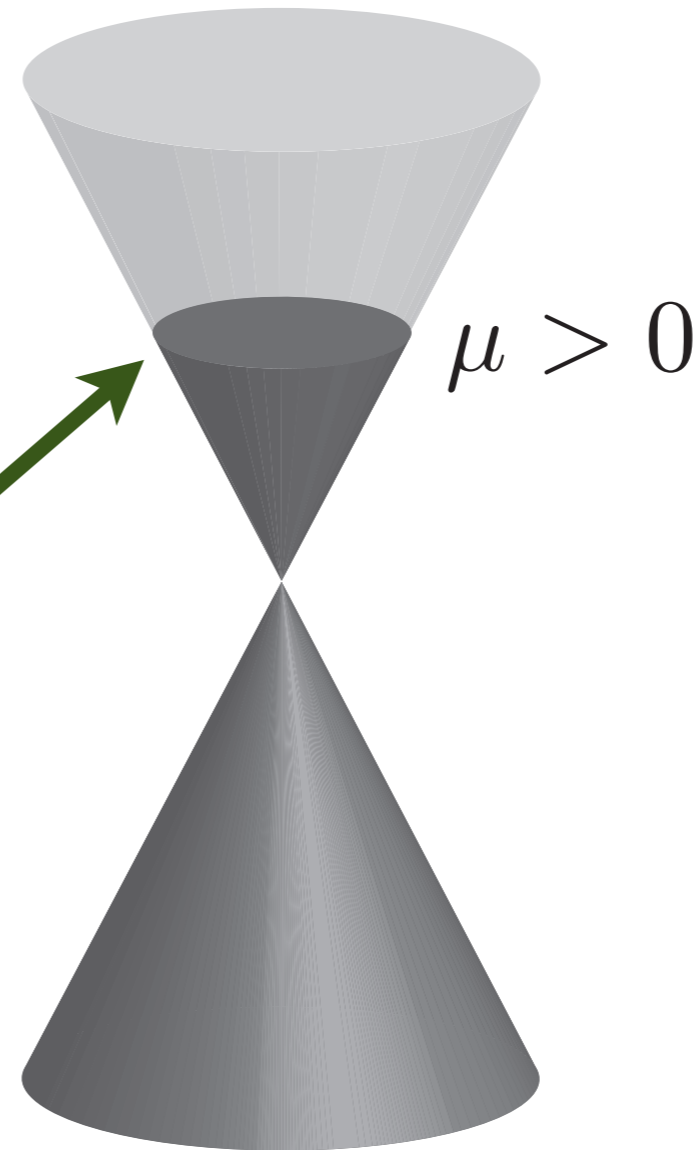
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# Turning on a chemical potential on a CFT

Compressible  
phase is a  
**Fermi Liquid**  
with a  
**Fermi surface**



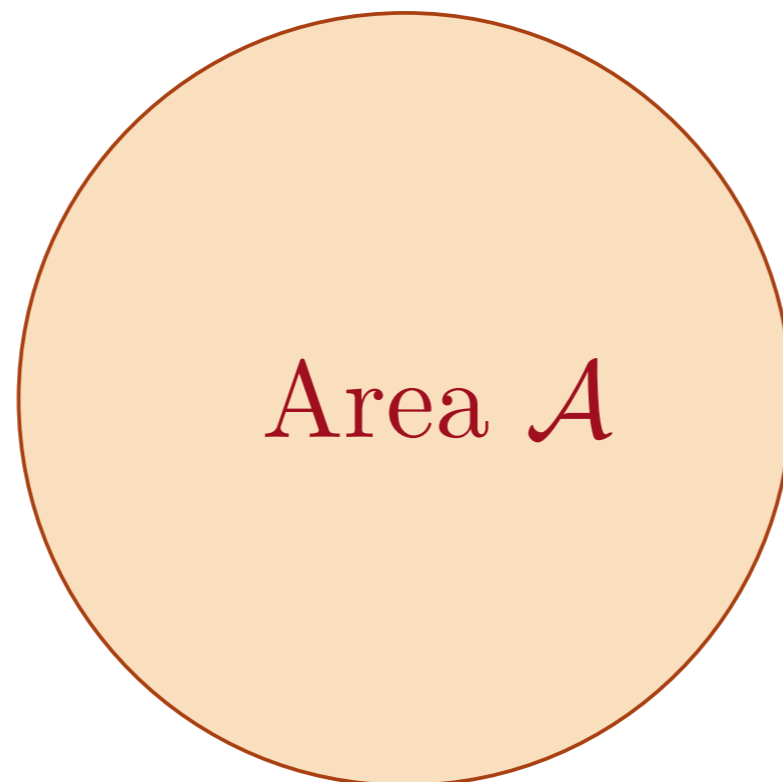
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# The Fermi surface

This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge  $Q$ .

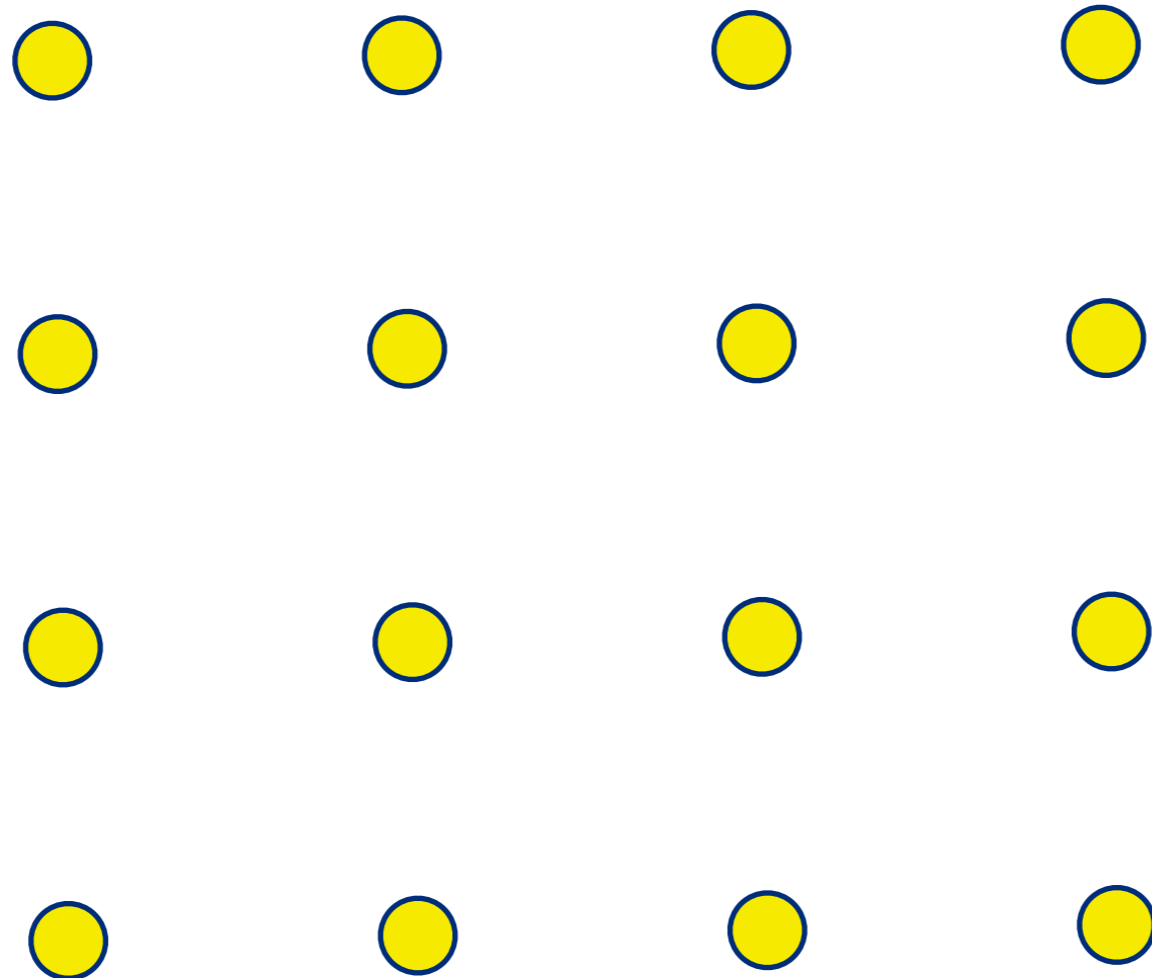
$$G_{\text{fermion}}^{-1}(k = k_F, \omega = 0) = 0.$$

**Luttinger relation:** The total “volume (area)”  $\mathcal{A}$  enclosed by the Fermi surface is equal to  $\langle Q \rangle$ . This is a *key* constraint which allows extrapolation from weak to strong coupling.



# Compressible quantum matter

Another compressible state is the **solid**  
(or “Wigner crystal” or “stripe”).  
This state breaks translational symmetry.



# Compressible quantum matter

The only other familiar compressible state is the **superfluid**.

This state breaks the global  $U(1)$  symmetry associated with  $Q$



Condensate of  
fermion pairs

# Compressible quantum matter

Conjecture: All compressible states which preserve translational and global  $U(1)$  symmetries must have FERM SURFACES, but they are not necessarily Fermi liquids.

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- Such states obey the Luttinger relation

$$\sum_{\ell} q_{\ell} \mathcal{A}_{\ell} = \langle Q \rangle,$$

where the  $\ell$ 'th Fermi surface has fermionic quasiparticles with global  $U(1)$  charge  $q_{\ell}$  and encloses area  $\mathcal{A}_{\ell}$ .

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- Non-Fermi liquids have quasiparticles coupled to deconfined gauge fields (or gapless bosonic modes at quantum critical points).



# The Hubbard Model

$$H = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$  “hopping”.  $U \rightarrow$  local repulsion,  $\mu \rightarrow$  chemical potential

Spin index  $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}$$

$$c_{i\alpha}^\dagger c_{j\beta} + c_{j\beta} c_{i\alpha}^\dagger = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

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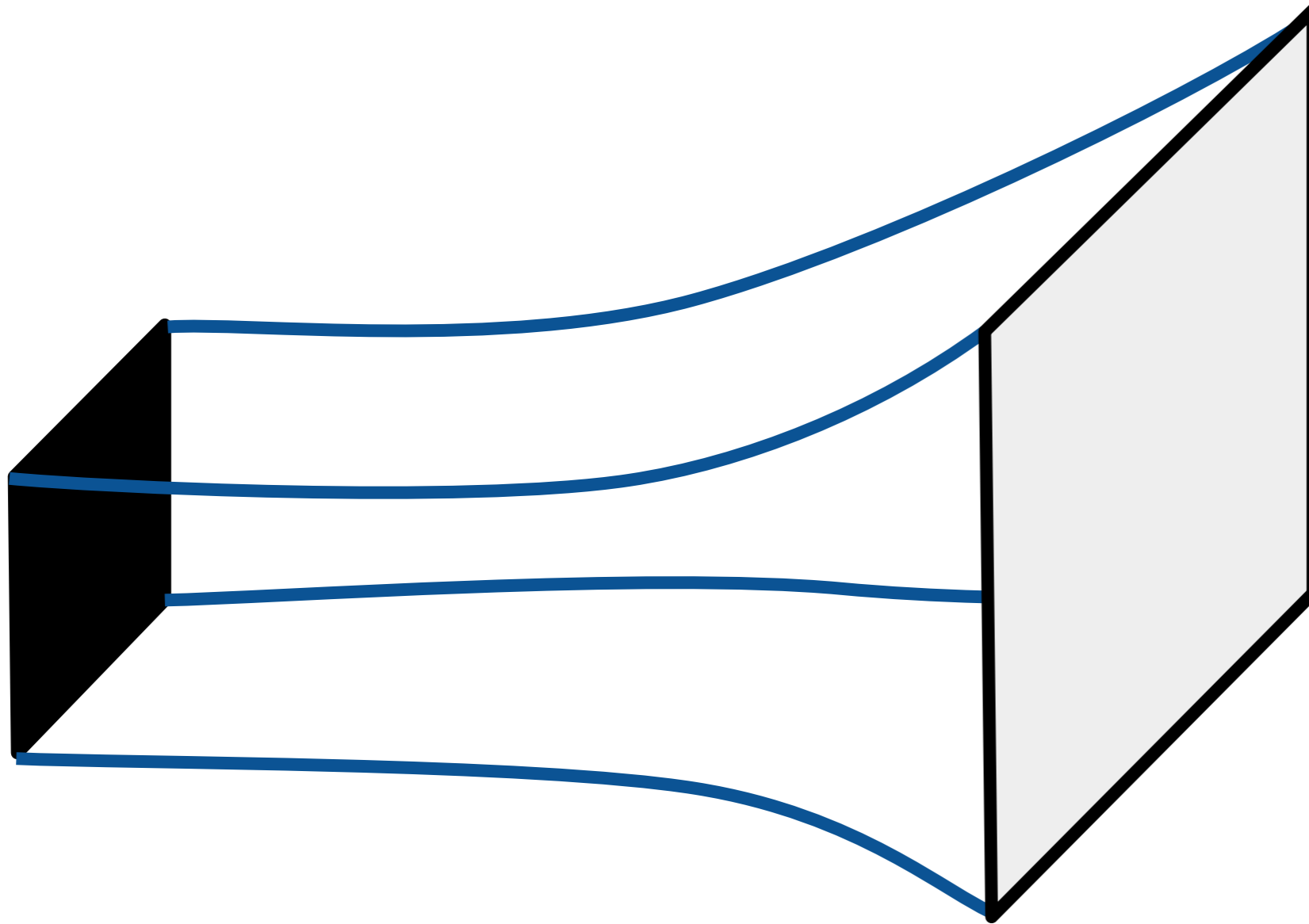
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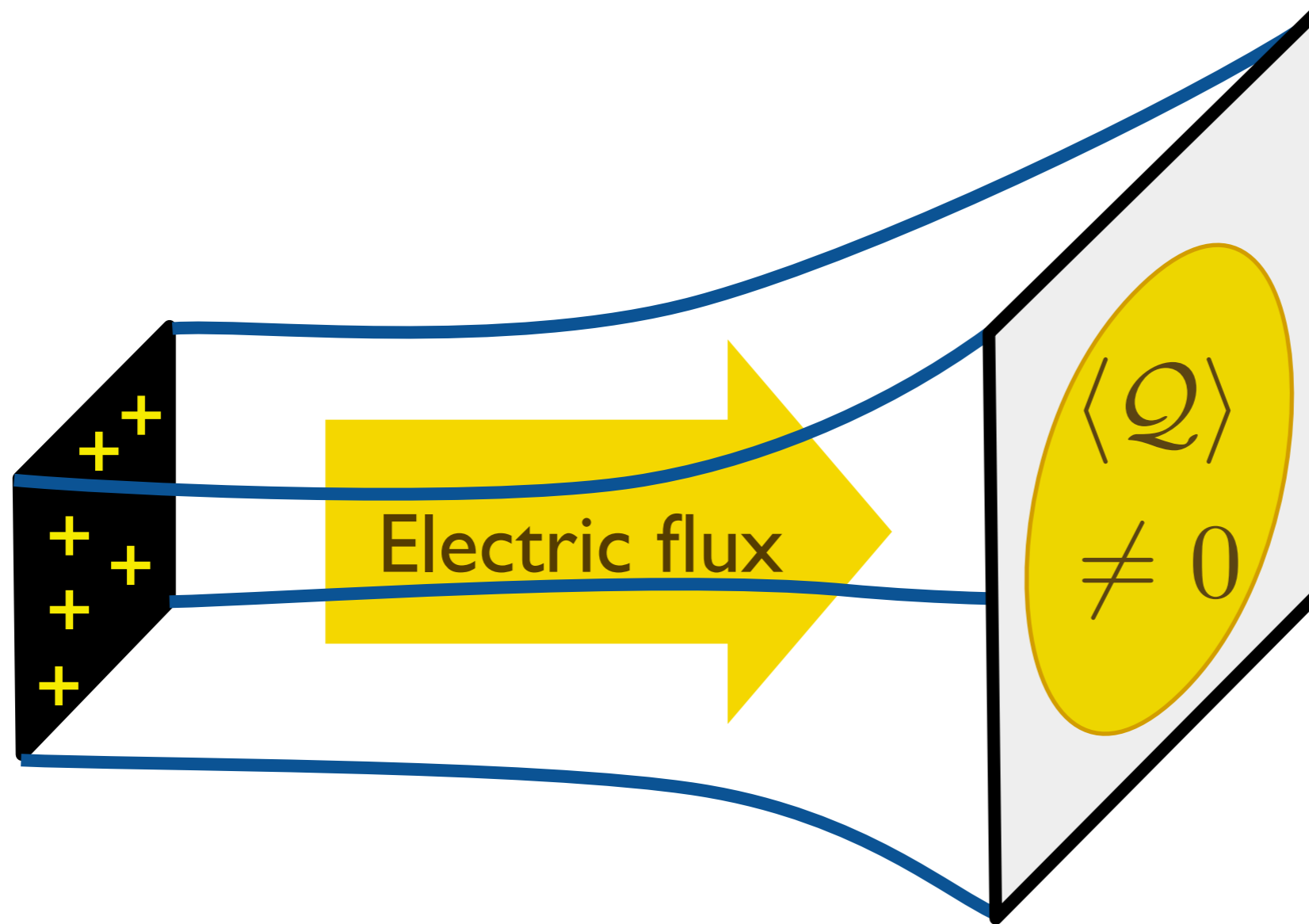
*and  $AdS_2 \times R^2$*

# AdS<sub>4</sub>-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$

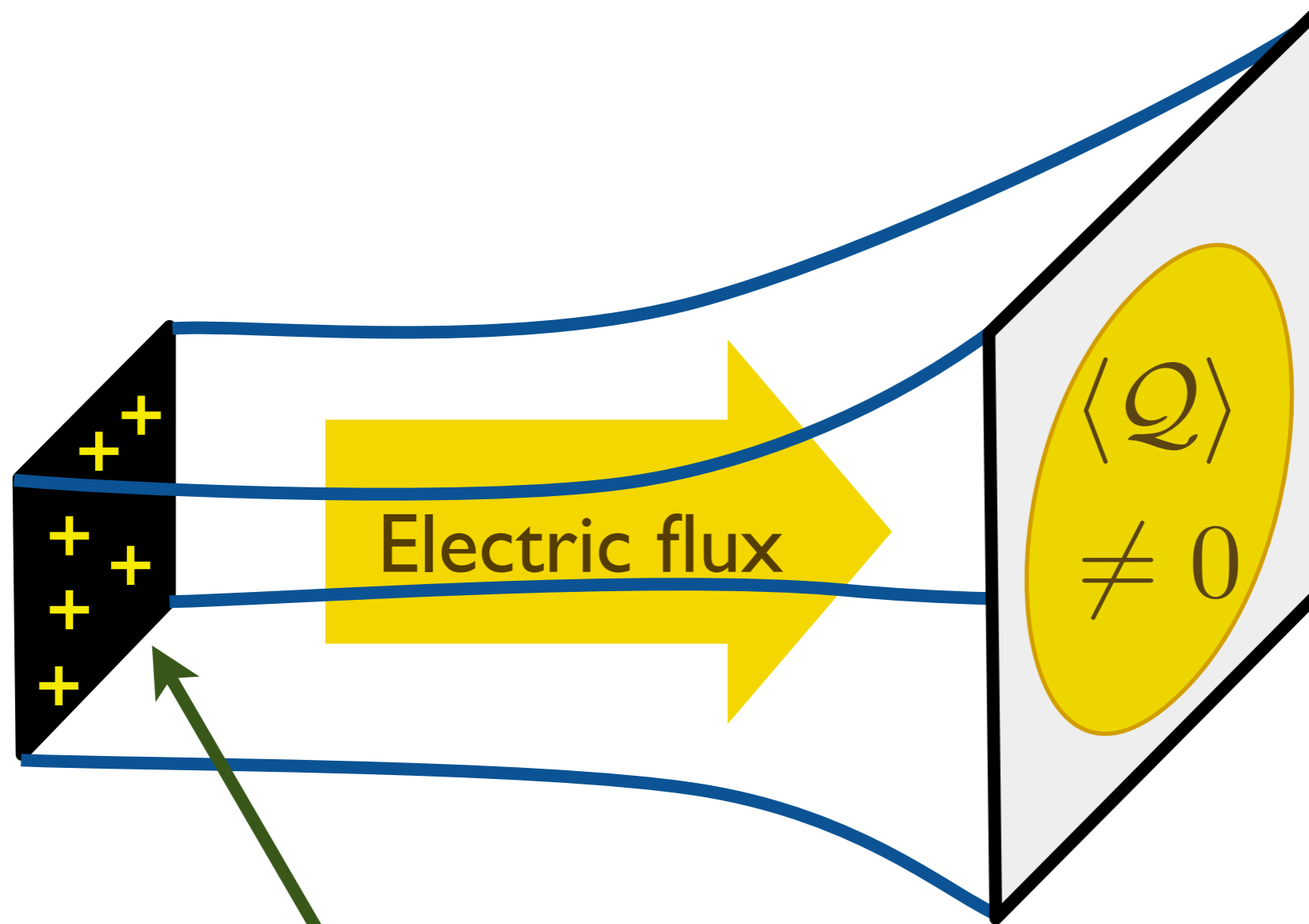
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S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

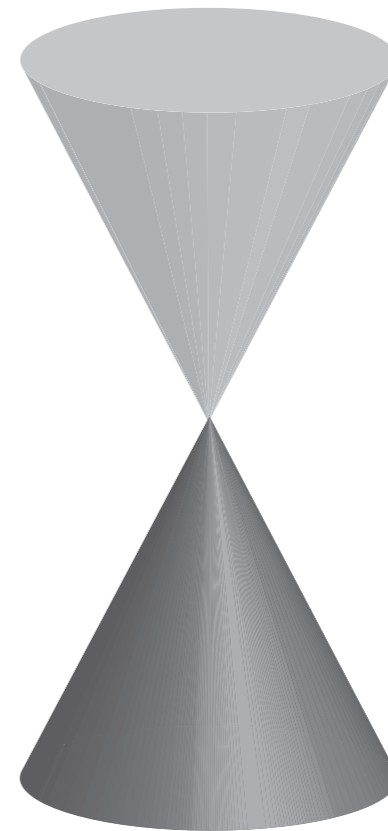
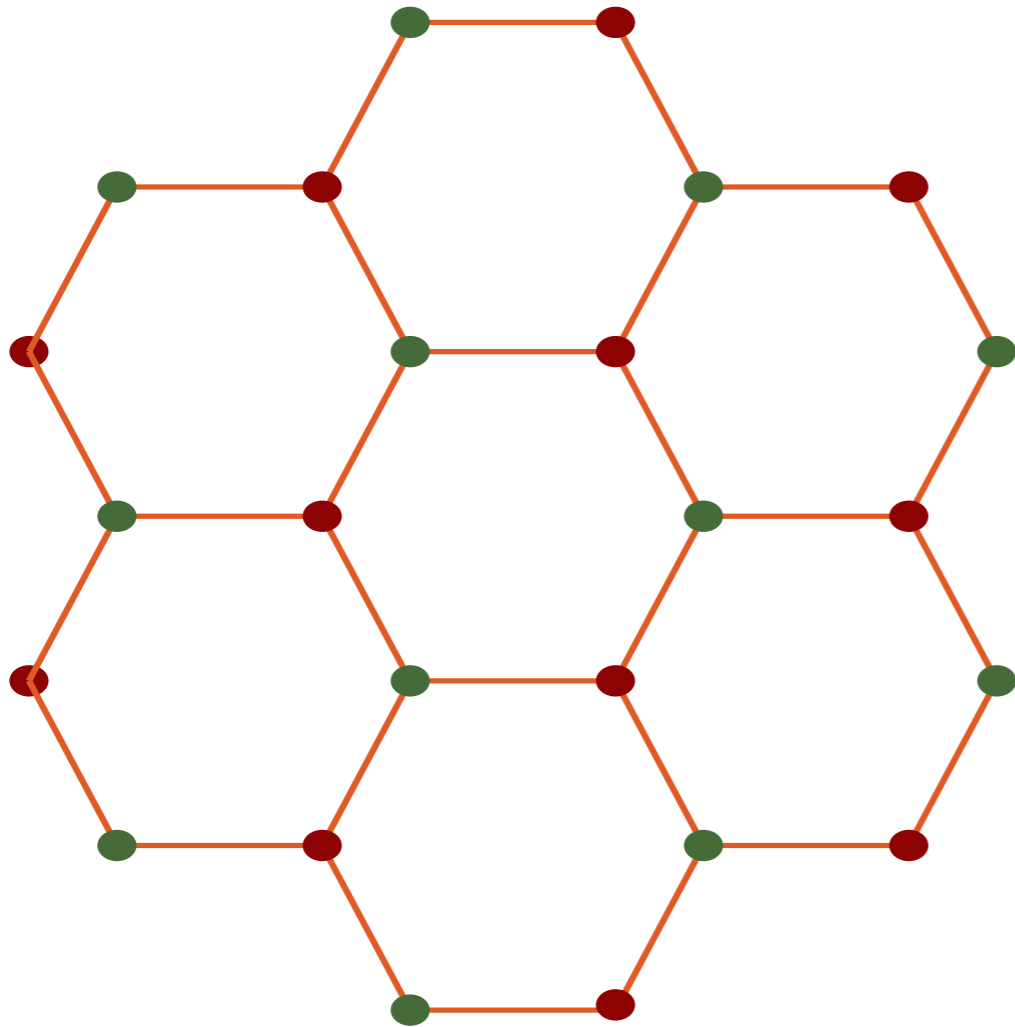
# AdS<sub>4</sub>-Reissner-Nordström black-brane



At  $T = 0$ , we obtain an extremal black-brane, with a near-horizon (IR) metric of  $\text{AdS}_2 \times R^2$

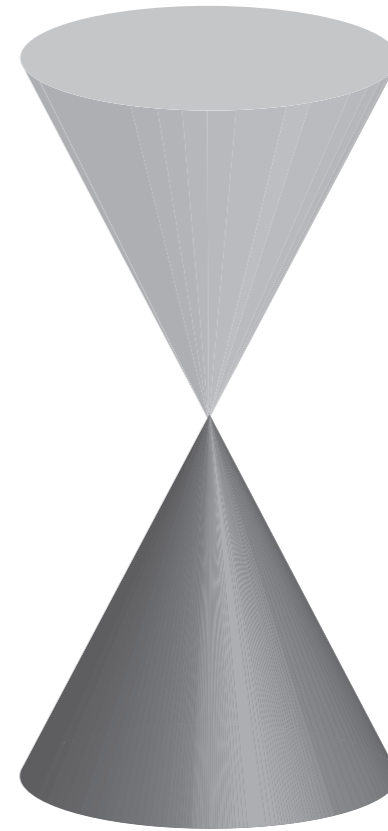
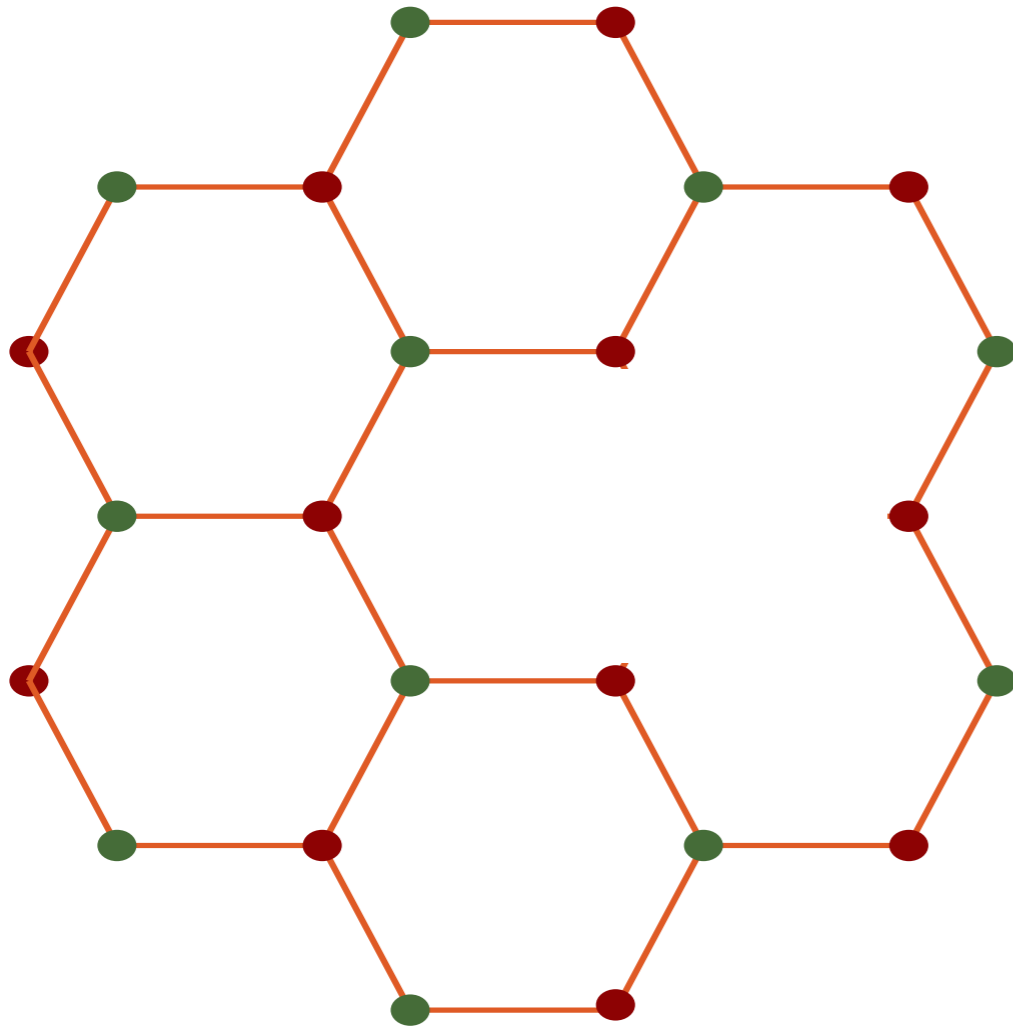
$$ds^2 = \frac{L^2}{6} \left( \frac{-dt^2 + dr^2}{r^2} \right) + dx^2 + dy^2$$

# Interpretation of $AdS_2$



CFT on graphene

# Interpretation of AdS<sub>2</sub>

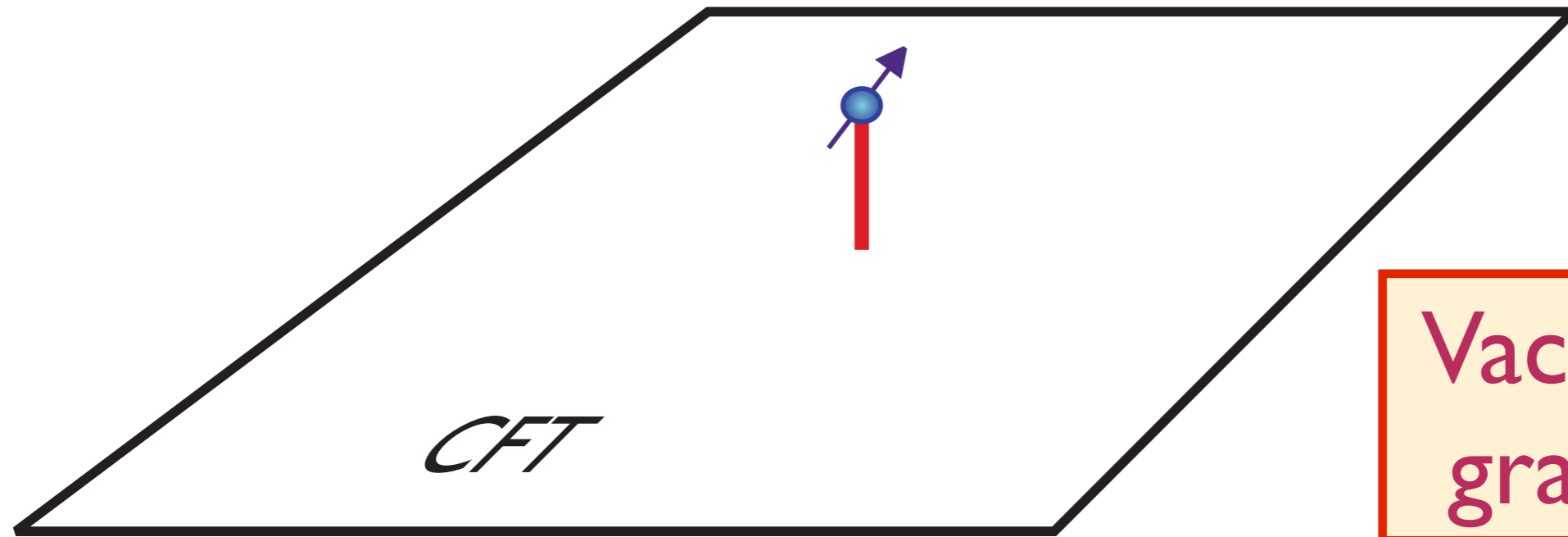


Add “matter” one-at-a-time: honeycomb lattice with a vacancy.

There is a zero energy quasi-bound state with  $|\psi(r)| \sim 1/r$ .  
We represent this by a localized fermion field  $\chi_\alpha(\tau)$ .



# Interpretation of AdS<sub>2</sub>



Vacancy in  
graphene

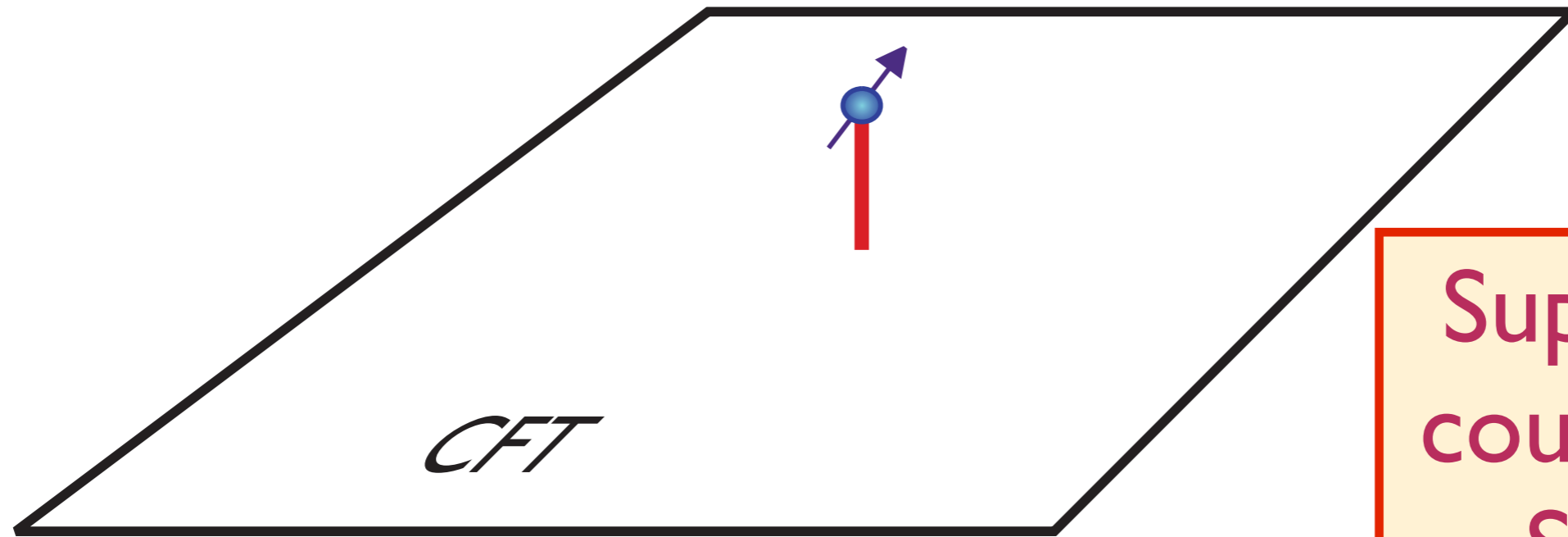
$$\mathcal{S} = \int d^3x \mathcal{L}_{\text{CFT}} - \int d\tau \mathcal{L}_{\text{imp}}$$

$$\mathcal{L}_{\text{imp}} = \chi_{\alpha}^{\dagger} \frac{\partial \chi_{\alpha}}{\partial \tau} - \kappa \chi_{\alpha}^{\dagger} \sigma_{\alpha\beta}^a \chi_{\beta} \varphi^a(\mathbf{r} = 0, \tau)$$

AdS<sub>2</sub>: “Boundary” conformal field theory obtained when  $\kappa$  flows to a fixed point  $\kappa \rightarrow \kappa^*$ .

S. Sachdev, C. Buragohain, and M. Vojta, *Science* **286**, 2479 (1999)

# Interpretation of AdS<sub>2</sub>



Superspin  
coupled to  
SYM4

$$\mathcal{S} = \int d^4x \mathcal{L}_{\text{SYM}} + \int d\tau \mathcal{L}_{\text{imp}}$$

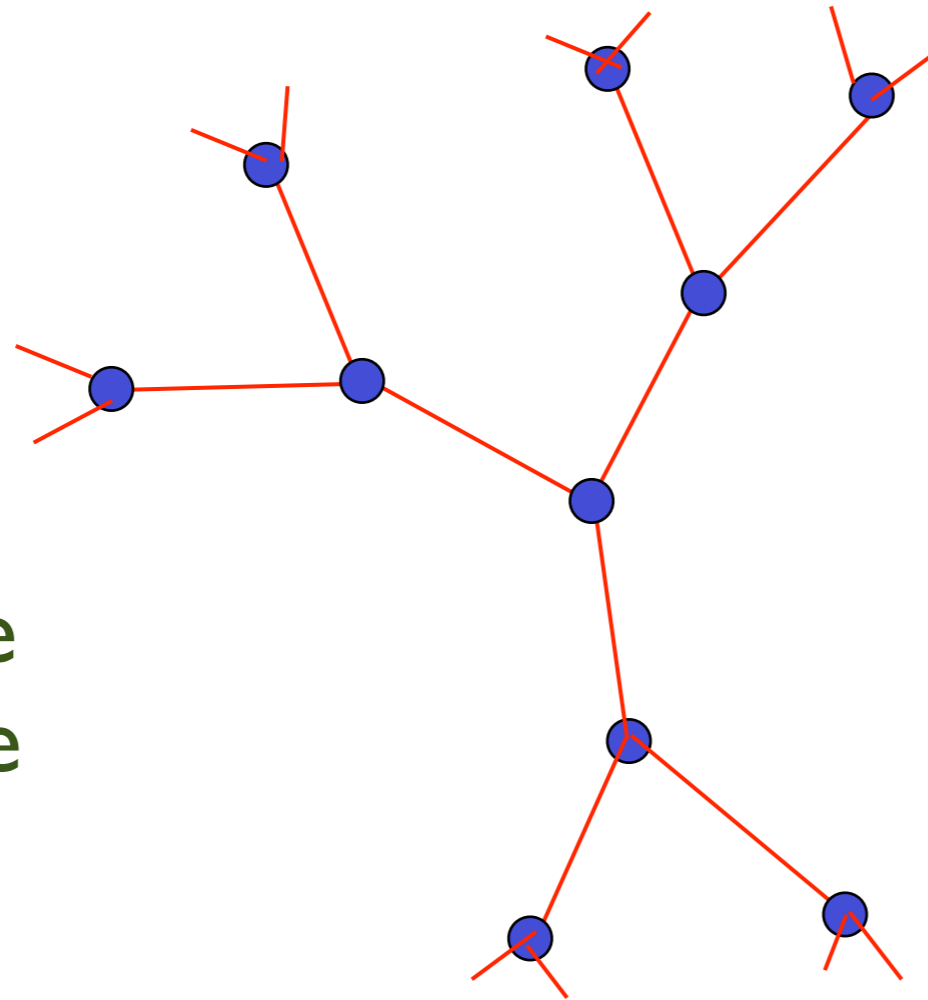
$$\mathcal{L}_{\text{imp}} = \chi_b^\dagger \frac{\partial \chi^b}{\partial \tau} + i\chi_b^\dagger \left[ (A_\tau(0, \tau))^b_c + v^I (\phi_I(0, \tau))^b_c \right] \chi^c$$

Solution in large  $N$  limit shows low energy theory of impurity is described by AdS<sub>2</sub>

S. Kachru, A. Karch, and S. Yaida, Phys. Rev. D **81**, 026007 (2010)

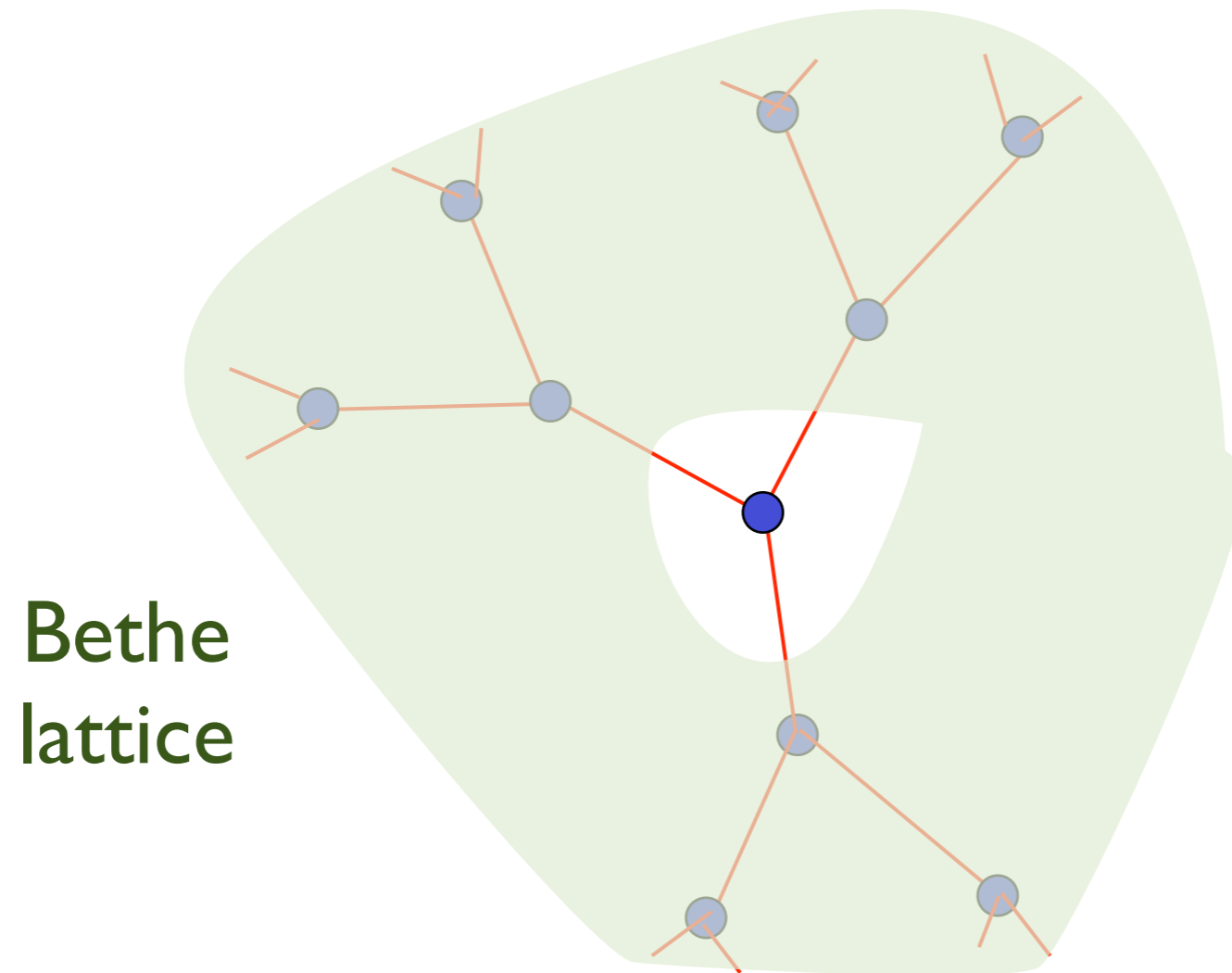
# Interpretation of $AdS_2 \times R^2$

Bethe  
lattice



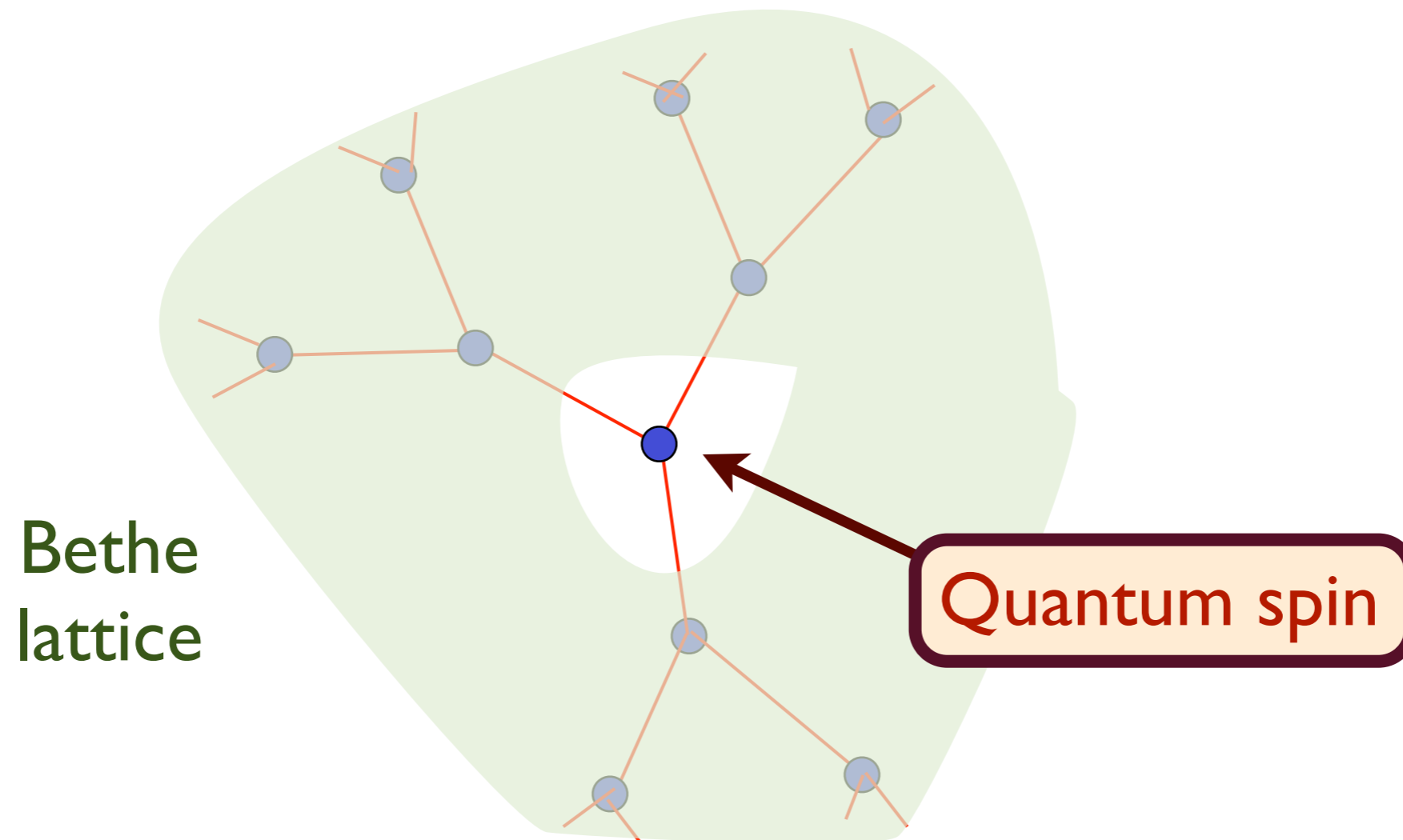
Solve electronic models in the limit of large  
number of nearest-neighbors

# Interpretation of $AdS_2 \times R^2$



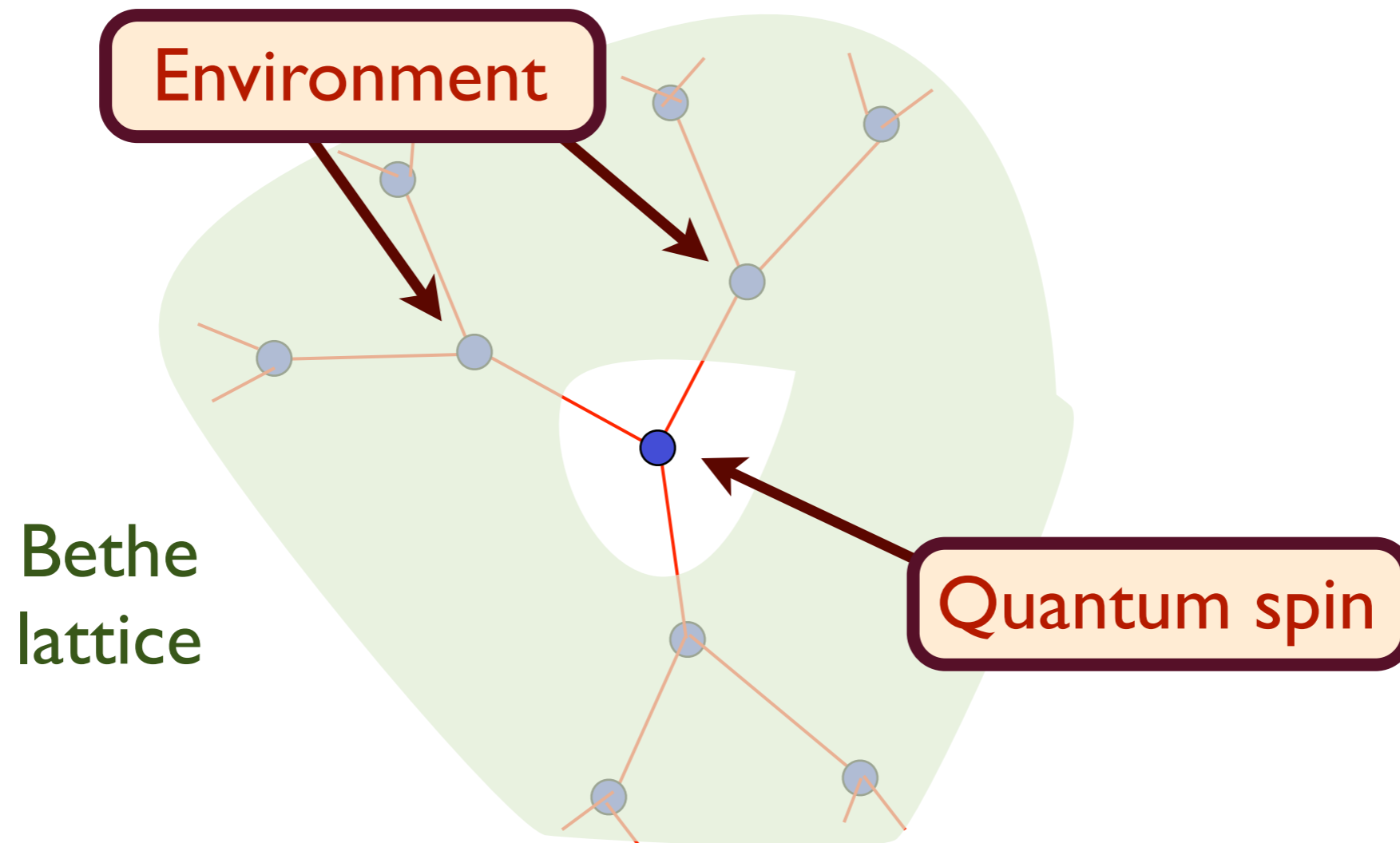
Theory is expressed as a “quantum spin” coupled  
to an “environment”:  
solution is often a boundary CFT in  $0+1$  dimension

# Interpretation of $AdS_2 \times R^2$



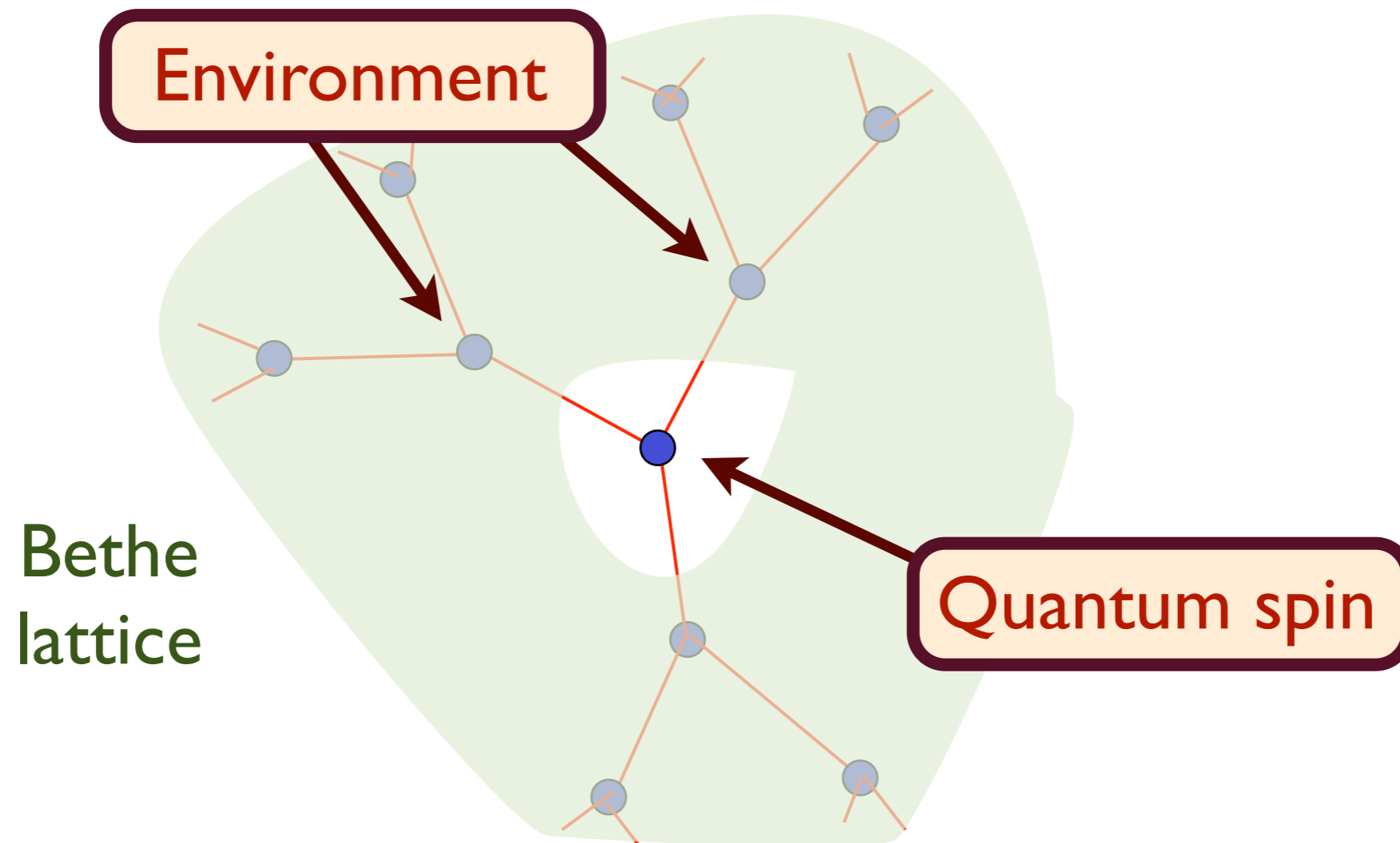
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# Interpretation of $AdS_2 \times R^2$



Theory is expressed as a “quantum spin” coupled to an “environment”:  
solution is often a boundary CFT in  $0+1$  dimension

# Interpretation of $AdS_2 \times R^2$



Exponents are determined by self-consistency condition between “spin” and “environment”.

# Artifacts of $\text{AdS}_2 \times R^2$

- The large-neighbor-limit solution matches with those of the  $\text{AdS}_2 \times R^2$  holographic solutions:
  - A non-zero ground state entropy.
  - Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
  - A marginal Fermi liquid spectrum for fermionic quasi-particles (for the holographic solution, this requires tuning a free parameter).
  - The low energy sector has conformally invariant correlations.

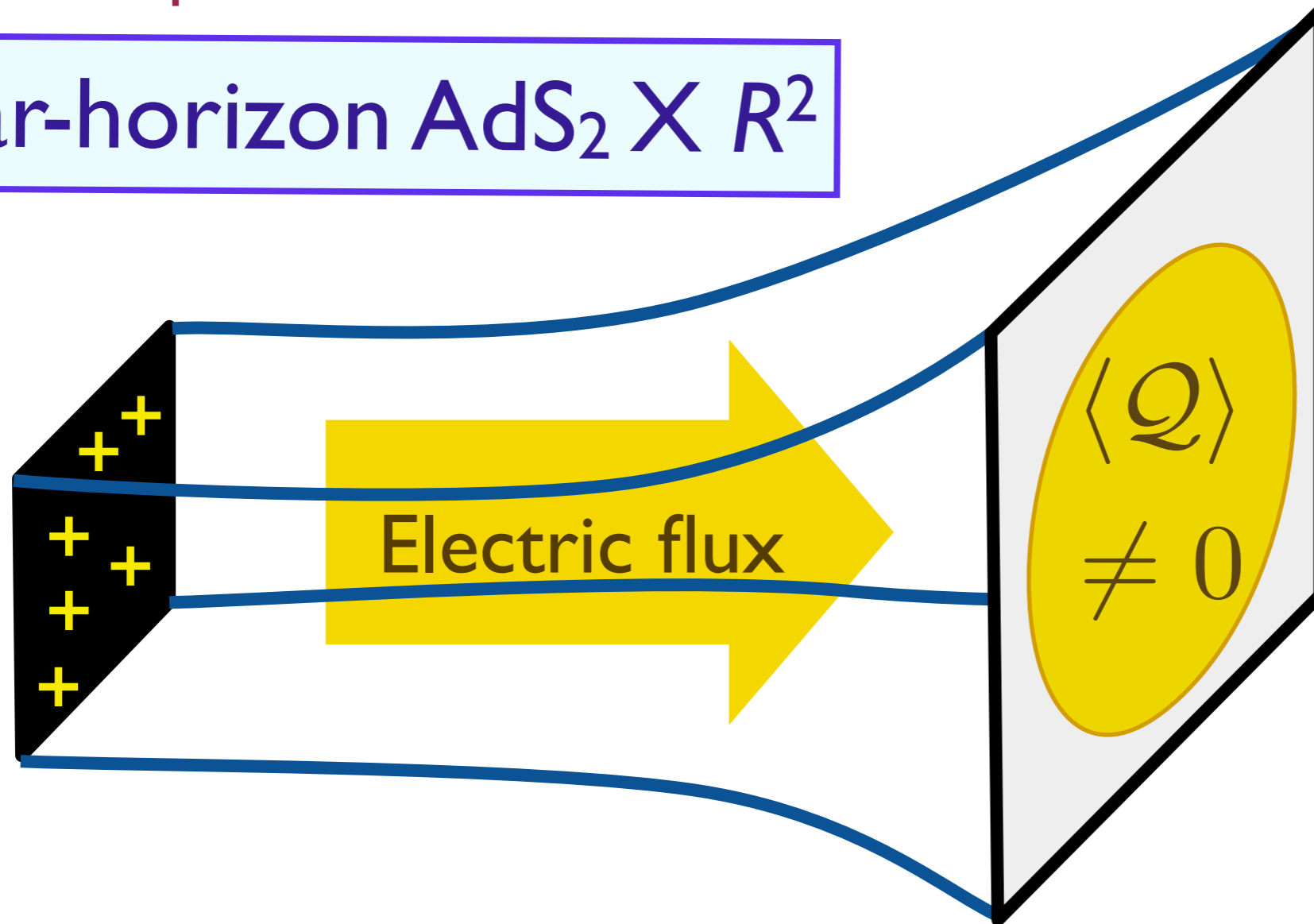
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).



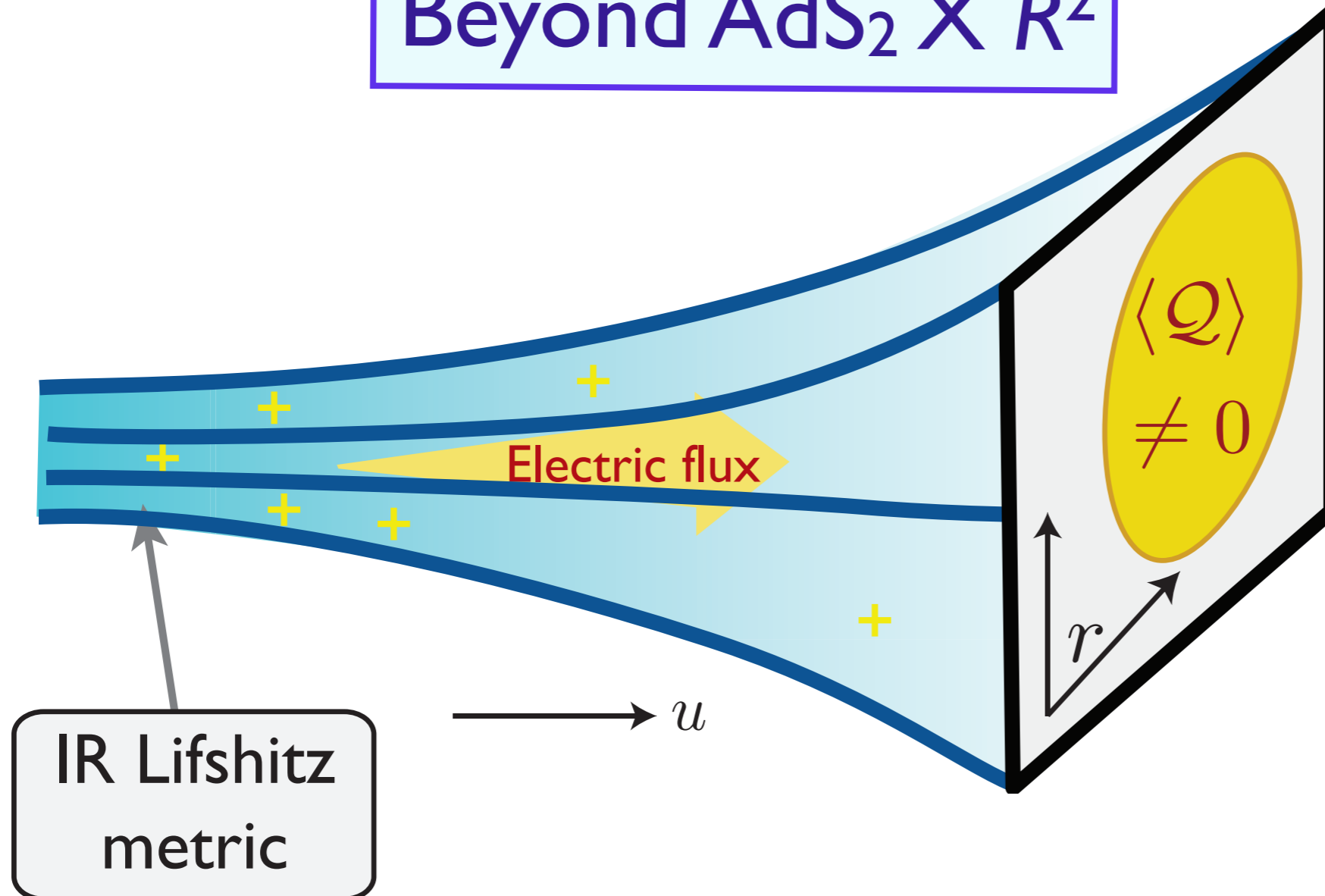
# AdS<sub>4</sub>-Reissner-Nordström black-brane

Near-horizon AdS<sub>2</sub> × R<sup>2</sup>



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

# Beyond $\text{AdS}_2 \times R^2$



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} + \mathcal{L}_{\text{matter}} \right]$$

Sufficiently light matter undergoes Schwinger pair-creation, back-reacts on the metric, the horizon may disappear, and the charge density is delocalized in the bulk spacetime

# Beyond $\text{AdS}_2 \times R^2$

- The metric often has a “Lifshitz” form in the IR:

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dr^2 + dx^2 + dy^2}{r^2}$$

with dynamic scaling exponent  $z$ . This possibly indicates Landau-damped transverse gauge modes. The  $\text{AdS}_2 \times R^2$  case corresponds to  $z \rightarrow \infty$ .

Kachru, Liu, Mulligan; Horowitz, Roberts; Gubser, Nellore; Hartnoll, Polchinski, Silverstein, Tong; Hartnoll, Tavanfar; Charmousis, Gouteraux, Kim, Kiritsis, Meyer; Goldstein, Iizuka, Kachru, Prakash, Trivedi, Westphal; Herzog, Klebanov, Pufu, Tesileanu

# Beyond $\text{AdS}_2 \times R^2$

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with dynamic scaling exponent  $z$ . This possibly indicates Landau-damped transverse gauge modes. The  $\text{AdS}_2 \times R^2$  case corresponds to  $z \rightarrow \infty$ .

- For bosons, back-reaction on metric appears when bosons condense, leading to a holographic description of superfluids. The Lifshitz metric is mysterious, indicating the presence of additional low energy modes not found in traditional superfluids.

Gubser; Hartnoll, Herzog, Horowitz; Nishioka, Ryu, Takayanagi;  
Gauntlett, Sonner, Wiseman; Gubser, Pufu, Rocha; Denef, Hartnoll;  
Gubser, Herzog, Pufu, Teitelbaum;  
Faulkner, Horowitz, McGreevy, Roberts, Vegh;  
Erdmenger, Grass, Kerner, Ngo; Ammon, Erdmenger, Kaminski, O’Bannon

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- For fermions, multiple Fermi surfaces are obtained, whose total enclosed area is consistent with the Luttinger count. This appears to be a Fermi liquid, but the Lifshitz metric is still mysterious.

Arsiwalla, de Boer, Papadodimas, Verlinde;  
Hartnoll, Hofman, Vegh; Iqbal, Liu, Mezei;  
Cubrovic, Schalm, Sun, Zaanen

## Conclusions

# Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

# Conclusions

## Compressible quantum matter

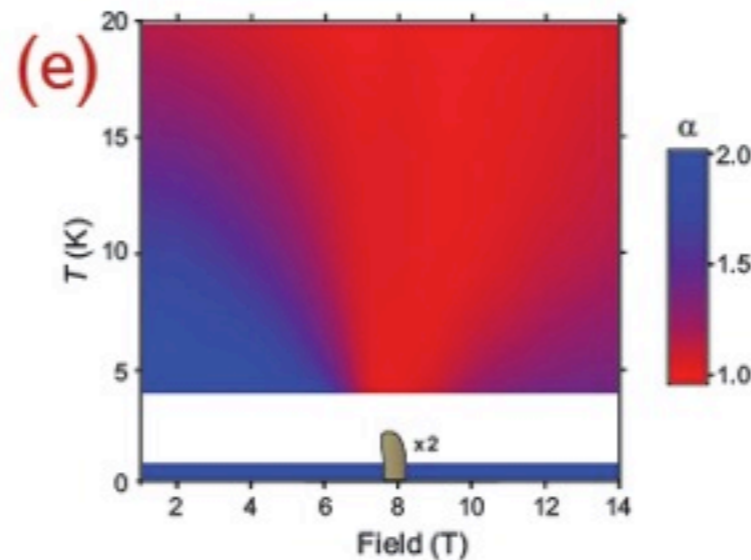
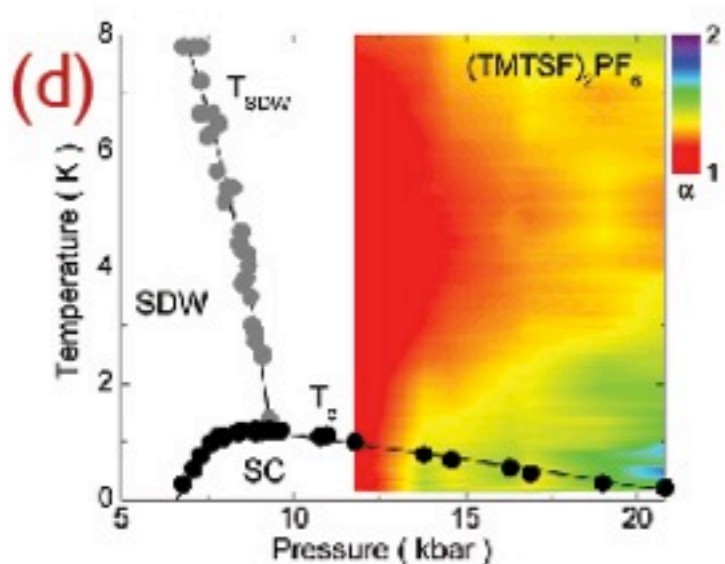
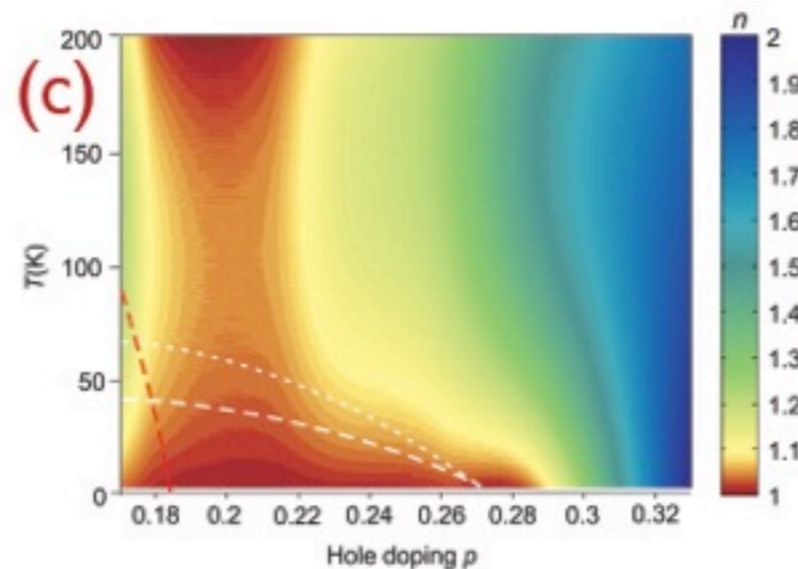
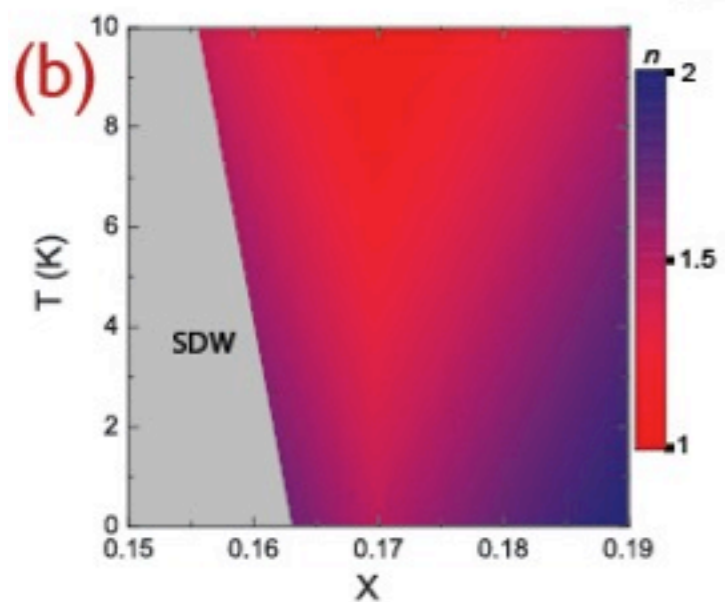
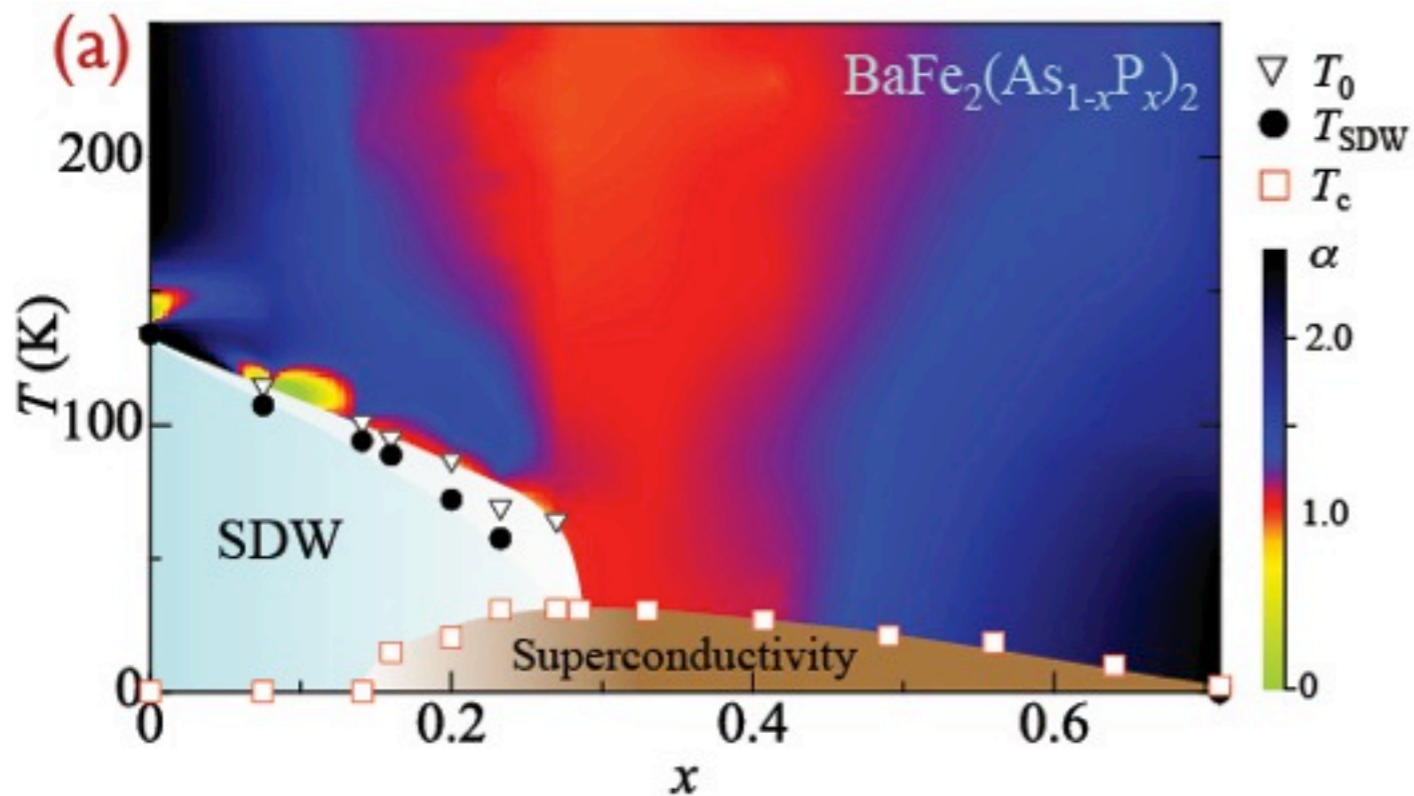
- The Reissner-Nordström solution provides the simplest holographic theory of a compressible state
- The RN solutions has many problems: finite ground-state entropy density, violation of Luttinger relation.
- Condensation of a scalar leads to the holographic theory of a superfluid. The IR metric has a Lifshitz form, indicating the presence of neutral gapless excitations not found in a superfluid.

# Conclusions

## Compressible quantum matter

- Fermion back-reaction leads to a Fermi liquid with many Fermi surfaces which do obey the Luttinger relation. However, the IR Lifshitz metric, and the very small Fermi wavevectors appear to be unwanted artifacts.
- Needed: a complete holographic theory of non-Fermi liquids and “fractionalized” Fermi liquids, obeying the Luttinger relations, to describe experiments on “strange metals”.





Plots of the resistivity exponent  $\frac{d \ln(\rho)}{d \ln T}$

- (a) Pnictide
- (b) e-doped cuprate
- (c) h-doped cuprate
- (d) organic superconductor
- (e)  $\text{Sr}_2\text{Ru}_3\text{O}_7$

Umklapp scattering likely crucial

S. Sachdev and B. Keimer, Physics Today, February 2011, page 29