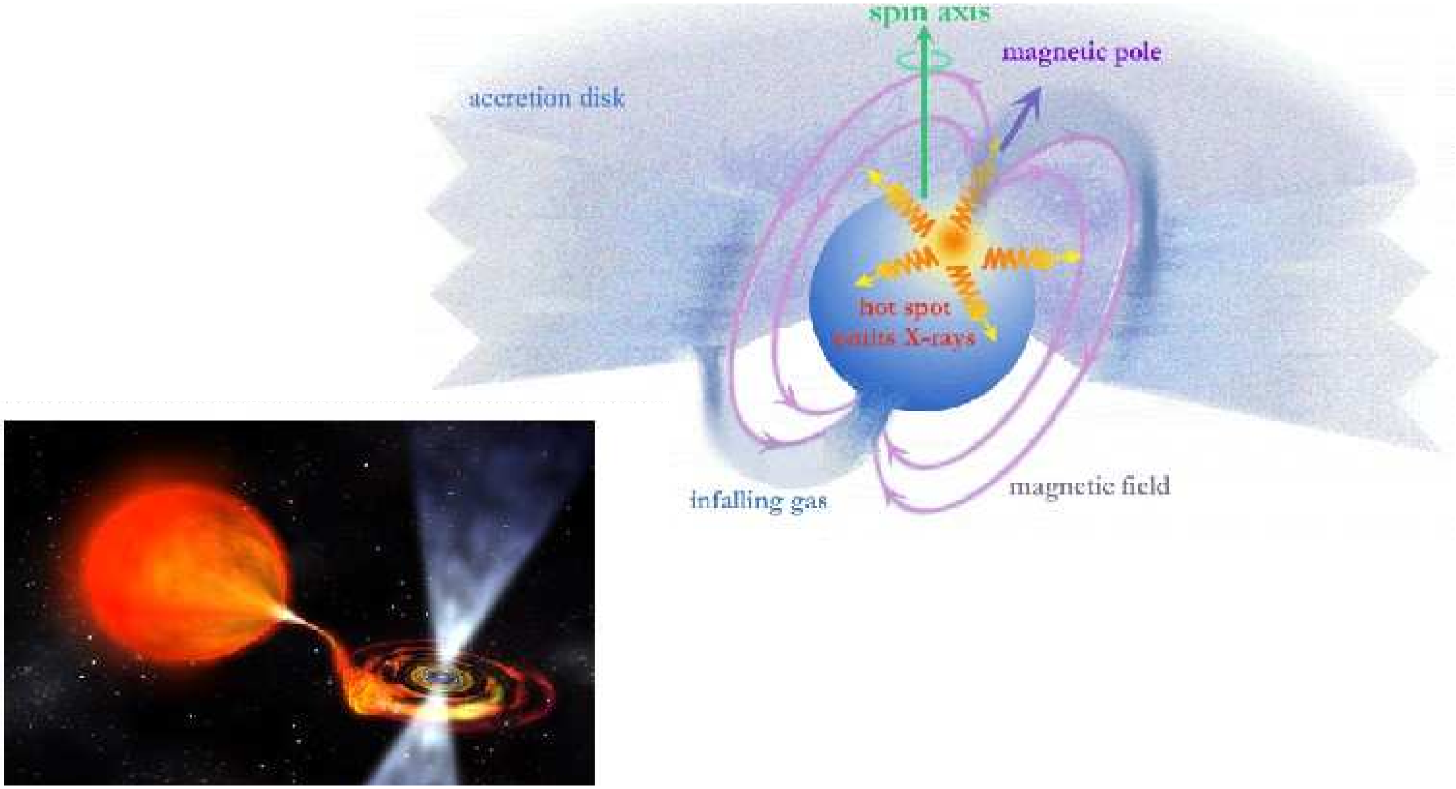
WHAT SETS THE MAXIMUM SPIN RATE OF NEUTRON STARS?

COLLABORATION:

P. Arras (UVa), R. Bondarescu (PSU),
D. Madison (Cornell), J. Brink (Caltech)
E. E. Flanagan (Cornell), K. Schenk (UCSF),
S. Morsink (Alberta), S. Teukolsky (Cornell), IW

Presented by: Ira Wasserman

ACCRETION & SPINUP



ACCRETION & SPIN-UP

- *FASTEST neutron star* is *716 Hz*
- *BREAKUP* is *f* ≈ **2000 Hz**
- ***ACCRETION IMPLIES SPINUP:***

$$\frac{\Delta f}{f_{max}} \approx \frac{\Delta M\, R^{3/2}\, R_{inner}^{1/2}}{I}$$

$$\frac{\Delta M}{M} \approx (0.3 - 0.4)\frac{\Delta f}{f_{max}}\frac{R^{1/2}}{R_{inner}^{1/2}}$$

WHY NOT FASTER?

GR MAXIMUM SPIN

(from Cook, Shapiro & Teukolsky 1994, ApJ, 424, 823 Table 5,7 & 8)

EOS	M	f	M	f	f(1.4)
	(absolute)		(`normal')		
Reid SC	1.95	2170	1.71	1590	1370
BJI	2.17	1700	1.92	1250	940
BJV	1.95	1750	1.70	1330	1160
MF(P&S)	3.27	1320	2.81	1030	720
RMF	3.22	1470	2.76	1150	840
AV14+UVII	2.55	2170	2.25	1690	1130

SPIN EQUILIBRIUM

- Magnetic spindown = accretion spinup
- Equilibrium spin and spin-up line

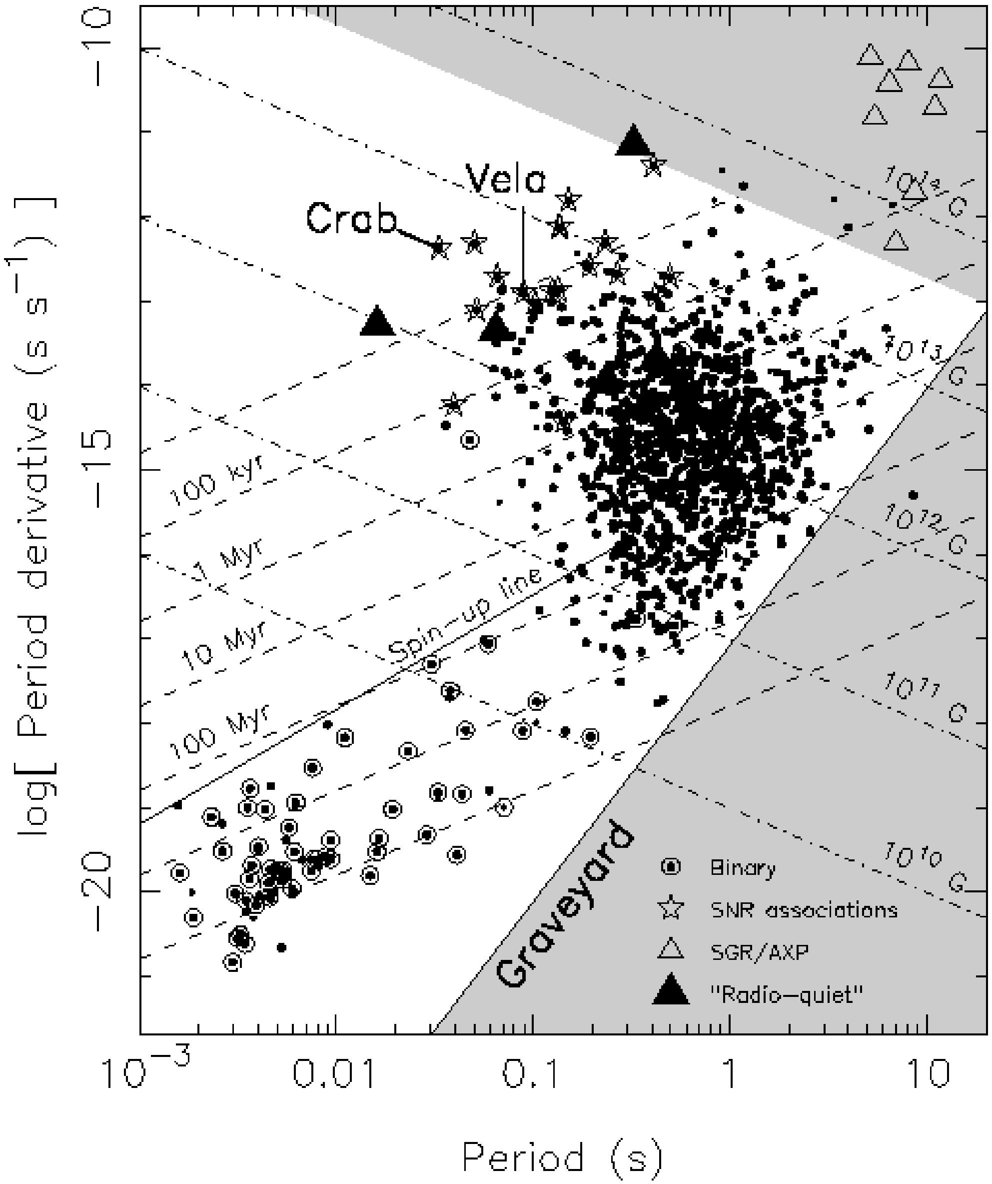
$$P_{eq} \approx \zeta^{3/2} \mu_{30}^{6/7} (\kappa_{0.4} / \dot{m} R_{10})^{3/7} M_{1.4}^{-5/7} \, seconds$$

$$\dot{P} = \alpha P^{4/3}$$

$$\alpha \approx 10^{-15} \eta \zeta^{-7/2} (\dot{m} R_{10} / \kappa_{0.4}) M_{1.4}^{5/3} I_{45}^{-1}$$

PULSAR AGES , DIPOLE B FIELDS

(*M. Kramer*)



NEUTRON STARS > 500 Hz

RADIO:

J1748-2446ad	716	(B,GC)
B1937+21	642	(I)
B1957+20	622	(B)
J1748-2446O	596	(B,GC)
J1748-2446P	578	(B,GC)
J0034-0534	532	(B)

X-RAY BURSTS:

4U 1608-52	619
SAX J1750.8-2900	601
X1743-29	589
4U 1636-53	581
X1658-298	567
Aql X-1	549
KS1731-260	524
(XTE J1739-285	1122?!)

GR ACCRETION & SPINUP

(from Cook, Shapiro & Teukolsky 1994, ApJL, 423, L117 Table I)

EOS	M(final)	ΔM_b	f(max)	M(max;0)
Reid SC	*1.77*	*0.428*	*1660*	*1.66*
BJI	1.74	0.389	1120	1.86
BJV	*1.76*	*0.405*	*1370*	*1.65*
MF(P&S)	1.80	0.443	800	2.70
RMF	1.84	0.484	926	2.64
AV14+UVII	1.79	0.446	1430	2.13

(From M=1.4 M⊙ to mass shedding or collapse)

GRAVITATIONAL RADIATION

(Wagoner 1984, Bildsten 1998)

- ***Spinup*** via ***accretion***

$$\dot{J}_{acc} \approx +\dot{M}(GMR)^{1/2}$$

- ***Spindown*** via ***gravitational radiation***

$$\dot{J}_{GR} \sim -G\,\Omega^{5}(\epsilon I)^{2} c^{-5}$$

- ***BALANCE***

$$\epsilon \approx 10^{-7}[\dot{M}/10^{-9} M_{Sun}\, y^{-1}]^{1/2}[300 \;\; Hz/\nu]^{5/2}$$

QUESTION: SOURCE OF Q ?

RADIATION BY CURRENTS

$$\dot{J}_{GR} \sim -G\,\Omega^5 I^2 c^{-5} (\Omega R/c)^2 (\delta v/\Omega R)^2$$

BALANCE when

$$\frac{\delta v}{\Omega R} \sim 10^{-6}[\dot{M}/10^{-9} M_{Sun}\, y^{-1}]^{1/2}[300\, Hz/\nu]^{7/2}$$

SOURCE OF CURRENTS ?

★ **CFS INSTABILITY OF R-MODES** ★

CFS INSTABILITY

(Chandrasekhar 1970; Friedman & Schutz 1978)

- **NORMAL MODE** $\delta v \propto e^{-i(\omega t - m\phi)}$
- **INERTIAL FRAME FREQUENCY ω >0**
- **ROTATING FRAME FREQUENCY**

$$\sigma = \omega - m\Omega \qquad (\sigma > 0 \vee \sigma < 0)$$

- **ROTATING FRAME *ENERGY*** $E_{ROT} = E - \Omega J_z$
- ***RADIATION*** $\dot{J}_z = m\dot{E}/\omega \;\rightarrow\; \dot{E}_{ROT} = \sigma \dot{E}/\omega$
- ***RETROGRADE*** ⇨ ***INSTABILITY***

R-MODES

(Andersson 1998; Bildsten 1998;
Friedman & Morsink 1998)

Restoring force = CORIOLIS

$$\frac{\omega}{\Omega} = \frac{(L-1)(L+2)}{L+1} \qquad \frac{\sigma}{\Omega} = -\frac{2}{L+1}$$

★ SUBSET OF INERTIAL MODES ★

$$-2\Omega \le \sigma \le +2\Omega$$

ROTATING SLAB

Magnetic fields, solid crust, buoyancy

(Hide QJRAS, 12, 380 [1971]+shear)

Modes $\boldsymbol{\xi} \propto \exp[i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)]$

$$\omega^2 = \frac{1}{2}\left(\omega_R^2 + |\boldsymbol{N}\times\hat{\boldsymbol{k}}|^2\right) + k^2 s^2 \pm \sqrt{\frac{1}{4}\left(\omega_R^2 + |\boldsymbol{N}\times\hat{\boldsymbol{k}}|^2\right)^2 + \omega_R^2 k^2 s^2}$$

$$\omega_R = 2\boldsymbol{\Omega}\cdot\hat{\boldsymbol{k}} \quad k^2 s^2 = k^2 c_t^2 + (\boldsymbol{k}\cdot\boldsymbol{v}_A)^2$$

$$\omega \approx \omega_R + \frac{|\boldsymbol{N}\times\hat{\boldsymbol{k}}|^2}{2\omega_R} + \frac{k^2 s^2}{\omega_R}$$

BASICS I: GROWTH & DAMPING

- Growth rate for R mode:

$$\gamma_{GR} = \frac{8\mathrm{GM}\langle r^4 \rangle}{225\mathrm{c}^7}\left(\frac{4\,\Omega}{3}\right)^6 \approx \frac{M_{1.4}\,R_6^4\,\nu_{kHz}^6}{46\ \ sec} \qquad [N = 1\ \ \ polytrope]$$

- Weak dependence on structure

$$\langle r^4 \rangle = (0.06 - 0.09)\left(\frac{M}{\rho_{1/2}}\right)^{4/3} \qquad [N \leq 3.0\ \ \ polytropes]$$

BASICS I: GROWTH & DAMPING

- Shear boundary layer (low T) [m=2, N=1]

$$\gamma_{BL}=\frac{\sqrt{4/3}\,\pi\,\mathbf{S}^{2}\rho_{b}r_{b}^{6}\sqrt{\Omega\,\boldsymbol{\eta}_{b,visc}}}{M\langle r^{4}\rangle}\approx\frac{\mathbf{S}^{2}\rho_{b,14}R_{6}^{2}\sqrt{\nu_{kHz}[\mathbf{T}_{8}^{2}\boldsymbol{\eta}_{b,visc}]_{4}}}{200\,\mathbf{T}_{8}M_{1.4}\quad sec}$$

Crust-core boundary; shear modulus

- Other modes:

$$\gamma_{BL}\sim\frac{\rho_{b}r_{b}^{2}\mathbf{S}_{mode}^{2}\mathbf{v}_{mode}^{2}(\mathbf{r}_{b})\sqrt{\Omega\,\eta_{b,visc}}}{\mathbf{E}_{mode}}$$

BASICS I: GROWTH & DAMPING

- Bulk viscosity: (High T)

$$\gamma_{BULK} = \frac{1}{2E_{mode}} \int dV\, \Re\, \Lambda \frac{P}{n} |\nabla \cdot \xi|^2$$

$$\Re = \frac{xn\Gamma\omega^2}{\omega^2 + \Gamma^2} \leq \frac{xn\omega}{2} \qquad \Lambda = -\frac{\partial \ln P}{\partial x}\frac{\partial \ln x}{\partial \ln n} \sim 1$$

- Suppressed for R mode(s) $\left(|\nabla \cdot \xi| \sim \Omega^2 / G\rho\right)$
- Depends on composition, EOS

$$\gamma_{BULK} = \frac{\nu_{kHz}^2 R_{10}^6}{1100 \mathrm{M}_{1.4}^2\, sec}\left(\frac{\langle \Re\, \Lambda \rangle}{n_{nuc}\, sec^{-1}}\right) < \frac{20 \langle \Lambda x \rangle \gamma_{GR}}{\nu_{kHz}^3 R_{10} M_{1.4}^2} \qquad [R\, mode]$$

SPINUP PROBLEM

- Accretion ⇒ $T \approx 3\times 10^{8} K$
- Low T ⇒ BL shear vs. GR ⇒

$$\nu_{S,kHz} \approx 0.3 \left(\frac{\rho_{b,14}}{(T_{S,8}/3)} \right)^{2/11} \left[\frac{10 S_{R} (R_{b}/R)}{M_{1.4} R_{10}} \right]^{4/11} \left(T_{8}^{2} \eta_{b,visc}\right)_{4}^{1/11}$$

$$\frac{\gamma_{BULK}}{\gamma_{GR}} < \frac{20 \langle \Lambda x \rangle}{\nu_{kHz}^{3} R_{10} M_{1.4}^{2}}$$

BULKING UP?

$$\gamma_{BULK} = \gamma_{GR} \to \langle \Re \Lambda \rangle \approx 24\, M_{1.4}^{3} R_{10}^{-2} \nu_{kHz}^{4} \quad n_{nuc}\, s^{-1}$$

$$\gamma_{BULK} = \gamma_{BL} \to \langle \Re \Lambda \rangle \approx 5.6\, S_{R}^{2} M_{1.4} R_{b,10}^{2} \left[T_{8}^{2} \eta_{b,visc} \right]_{4}^{1/2} T_{8}^{-1} R_{10}^{-6} \nu_{kHz}^{-3/2}$$

$$\bar{n}\,\omega \approx 10^{4}\, M_{1.4} R_{10}^{-3} \nu_{kHz}$$

PROBLEMS : ***Slow rates , superfluid suppression***

$\to \langle \Re \Lambda \rangle \ll 1$ ***typically*** $[\,Hyperons\,?\,]$

BASICS II: DYNAMICS

- **RESONANCES :** $\delta\omega = |\omega_R - \omega_2 - \omega_3| \ll \Omega$
- ***SELECTION RULES & COUPLING***
- **Parametric instability threshold *PIT*** (3 mode)
- ***Steady state amplitudes*** (3 mode)
- ***LOWEST PIT*** (3 or many mode)

$$|c_R|^2_{PIT} = \frac{\gamma_2 \gamma_3}{4\kappa^2 \omega_2 \omega_3}\left[1 + \frac{(\delta\omega)^2}{(\gamma_2+\gamma_3)^2}\right] \sim \frac{\gamma_2\gamma_3}{\kappa^2\omega_2\omega_3} \sim \frac{(\delta\omega)^2}{\kappa^2\omega_2\omega_3}$$

BASICS III: HEATING & COOLING

- Heating by modes: (expect $\Delta \sim 1$)

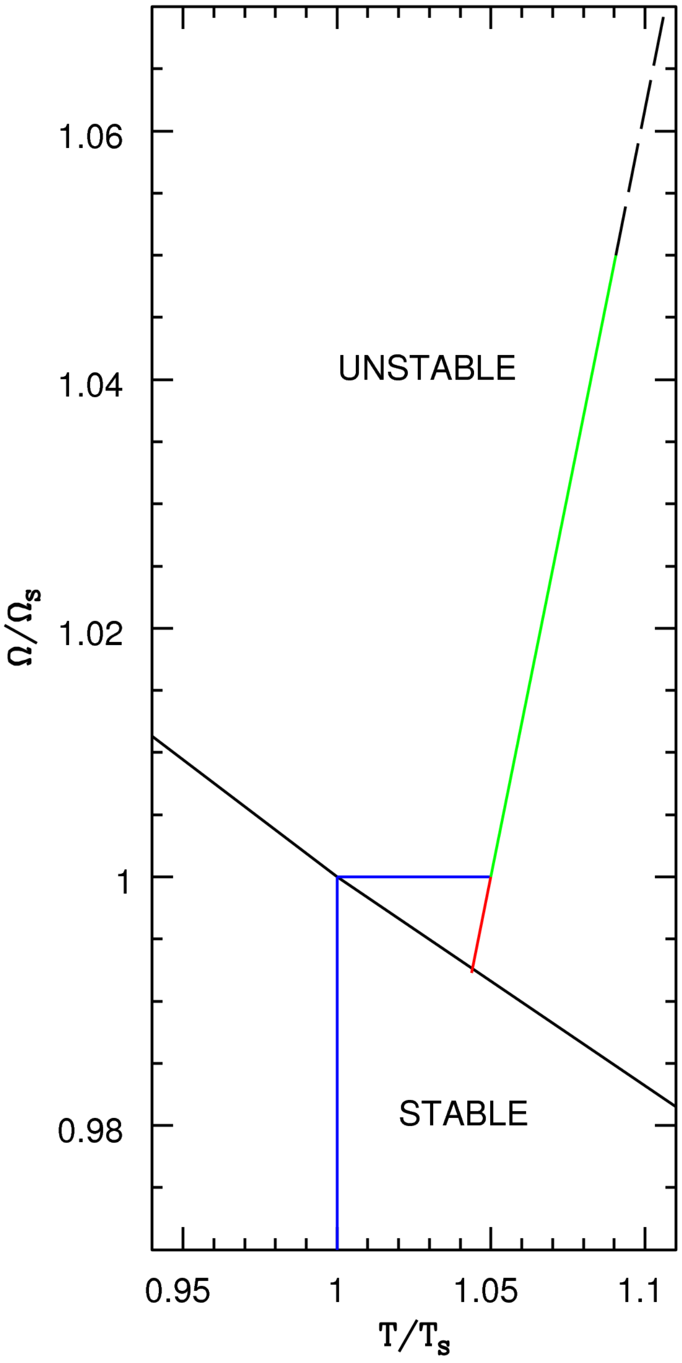
$$H_{modes} \approx 2\mathrm{MR}^2 \Omega^2 \gamma_{GR} |c_R|^2 = \frac{MR^2 \Omega^2 \gamma_{GR} \gamma_2 \gamma_3}{2 \kappa^2 \omega_2 \omega_3} (1+\Delta^2)$$

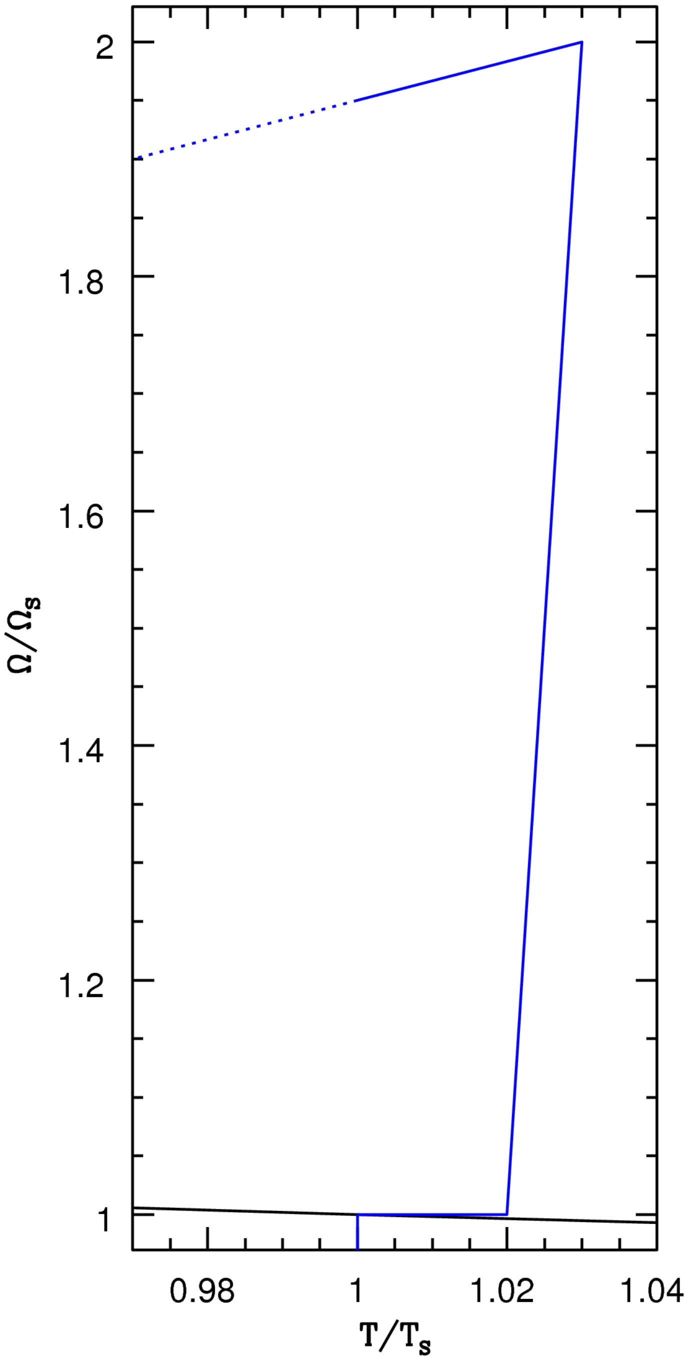
- Neutrino cooling: $L_\nu(T)$ $[dL_\nu(T)/dT > 0]$

- Heating by accretion: $H_{accretion} = \epsilon \dot{M} c^2$

- Evolve toward ***BALANCE***

$$L_\nu(T) = H_{modes} + H_{accretion} \rightarrow \Omega(T)$$


UNSTABLE
STABLE
1.06
1.04
1.02
1
0.98
Ω/Ω_s
0.95
1
1.05
1.1
T/T_s


2
1.8
1.6
1.4
1.2
1
Ω/Ω_s
0.98
1
1.02
1.04
T/T_s

SUMMARY

- R mode instability sets in below breakup
- Nonlinear evolution frustrates spinup unless:
  - -- enhanced dissipation, stable spinup
  - -- much less dissipation, GR spindown not significant.