Electronic screening and damping in magnetars

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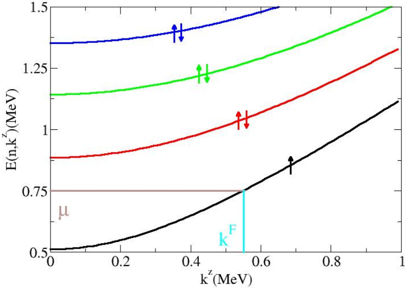
Outline

- Electronic response in magnetars is anisotropic, and features long range oscillations in the static screening potential parallel to the magnetic field
- Electronic scattering off lattice phonons is anisotropic and leads to anisotropic heat transport contribution from the lattice phonons

Electrons in large magnetic fields

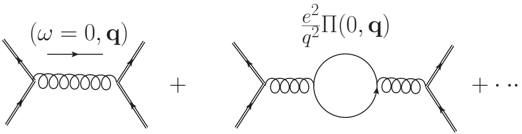
- Magnetars are neutron stars with magnetic fields as large as 2×10^{15}G
- Large magnetic fields can change the structure and properties of matter
- For example, atoms on the surface of the star become elongated along B *(Ruderman 1971; Medin, Lai)*
- What happens in the crust where the electrons are "deconfined"?

Landau levels



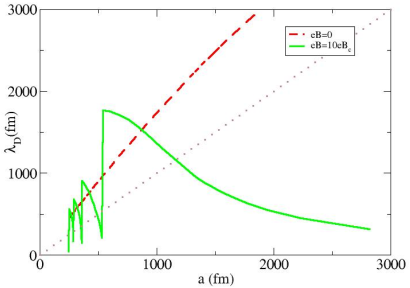
- The single electron energy levels are $E_n = \sqrt{k_z^2 + 2neB + m_e^2}$
- We consider the case where only the lowest Landau level is occupied
- $n_e = \frac{eB}{2\pi^2}\sqrt{\mu^2 - m_e^2}$

Screening by electrons



- $\Re e(\Pi(0,\mathbf{q}))$ appears in the calculation of the screened potential
- $V(\mathbf{q}) = \frac{Z_1 Z_2 e^2}{q^2 - e^2\Pi(\mathbf{q})}$, $V(\mathbf{r}) = \int đ^3\mathbf{q} e^{-i\mathbf{q}\cdot\mathbf{r}} V(\mathbf{q})$
- Taking out the Coulomb part, $V(\mathbf{r}) = \frac{Z_1 Z_2 e^2}{r} g(\mathbf{r})$.
- $\Pi(\omega, |\mathbf{q}|) = -i \int d^4x e^{-iq\cdot x} \langle T\left\{\psi^\dagger(x)\psi(x)\psi^\dagger(0)\psi(0)\right\}\rangle$ $\sim \sum_{E_m<\mu,\, E_n>\mu} |\langle E_n|\psi^\dagger(0)\psi(0)|E_m\rangle|^2 \frac{1}{\omega + E_n - E_m + i\epsilon}$

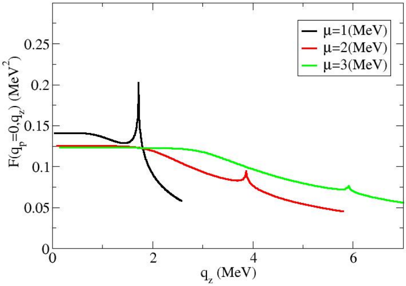
Debye Screening



- Usually for large r, taking $-e^2 \lim_{q\to 0} \Pi(\mathbf{q}) = \lambda_D^{-2} = e^2 \frac{dn}{d\mu}$ gives Debye screening
- $g(r) = \exp(-r/\lambda_D)$. For the lowest Landau level, $\lambda_D^{-2} = e^2 \frac{eB}{2\pi^2} \frac{\mu}{\sqrt{\mu^2 - m_e^2}}$ $(m_D = \lambda_D^{-1})$

Non-analyticities

- $\Pi(\mathbf{q})$ has non-analyticities due to a sharp Fermi surface
- For $B = 100B_c$, we have

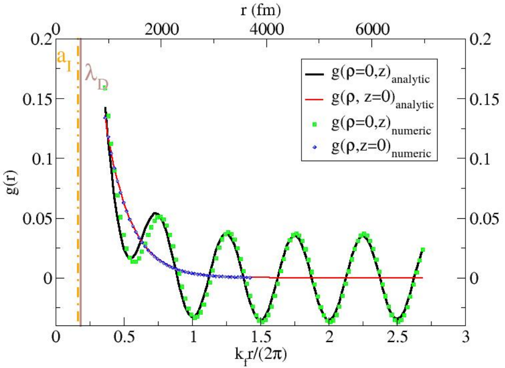


- Friedel oscillations more prominent in the non-relativistic regime *Kapusta, Toimela (1988)*

Friedel oscillations

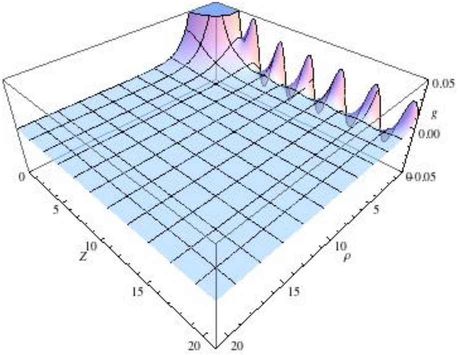
- Simplified expression for $k_z^F = \sqrt{\mu^2 - m_e^2} \ll \mu$
- $\Pi(q_z, q_\perp) \sim \frac{-m_e}{2q_z\pi^2} eB \exp(-\frac{q_\perp^2}{2eB}) \log(\frac{|q_z+2k_z^F|}{|q_z-2k_z^F|})$
- Branch cuts along $q_z = \pm 2k_z^F + i\eta$ give a long range structure to the position space potential, $V(\rho, z)$, in the z direction
- $g(\rho, 0) \simeq \exp(-\rho m_D)$
- $g(0, z) \simeq e^{-m_D z} - \frac{1}{16}\frac{m_D^2}{(k_f)^2+m_D^2 \ln(4k_f z)/8} \cos(2k_z^F z)$
- See also *Horing (1969)*
- At higher densities, Friedel oscillations become unimportant

Friedel oscillations



- $B = 100 B_c$, $\mu_e = 0.7$MeV, $\lambda_F = 1296$fm, $\lambda_D = 468$fm
- For $Z = 26$, $d_{bcc} = 743$fm

Friedel oscillations



Conclusions

- Overscreening of charge by electrons leads to long range oscillations parallel to the magnetic field
- Superposition of the screened potential can give rise to an anisotropic potential
- May affect the structure and the properties of the lattice formed by ions. May form an anisotropic crystal structure

Electron-lattice phonon interaction

- The electron ion interaction is simply screened Coulomb,

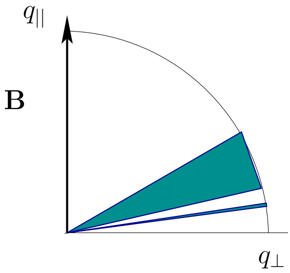
$$\mathcal{L}_{el} = \frac{Ze^2}{m_D^2} \int d^4x \Psi_I^\dagger \Psi_I \psi_e^\dagger \psi_e \tag{1}$$

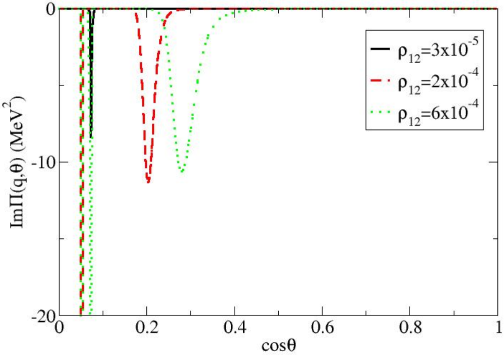
- This gives the electron-lattice phonon interaction strength $\frac{1}{f_{el}} = \frac{Ze^2 n_I}{m_D^2 \sqrt{m_I n_I}}$ *(eg. Kittel)*
- Therefore one can calculate the mean free path of lattice phonons due to scattering off electrons

$\Im m \Pi(\omega, \mathbf{q})$

- $\Im m \Pi(\omega, \mathbf{q})$ is related to the rate of absorption of lattice phonons by electrons and therefore to lattice phonon transport properties
- The heat conductivity $\kappa = \frac{1}{3} C_V \lambda_\xi c_\xi = -\frac{c_\xi 2\pi^2 T^2 f_{el}^2}{45 \Im m \Pi(\omega, \mathbf{q})}$
- The conductivity depends on the angle between the magnetic field and **q**, the direction of propagation of the lattice phonon
- We calculate the decay rate at $\omega = 3T$, and $q = \omega / c_\xi$

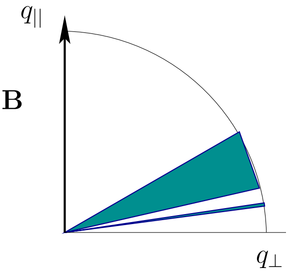
Anisotropic scattering

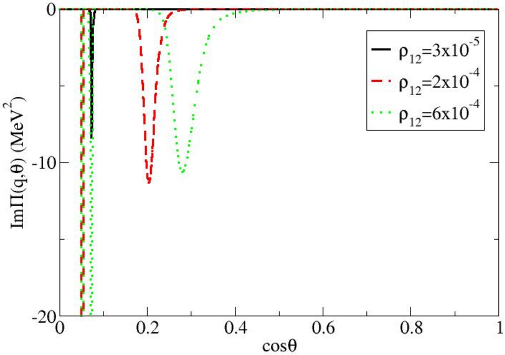




- $B = 10B_c$, $T = 5\text{keV}$, $c_\xi = 0.05$

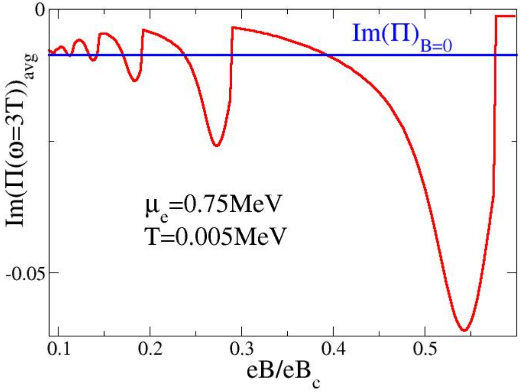
Anisotropic scattering



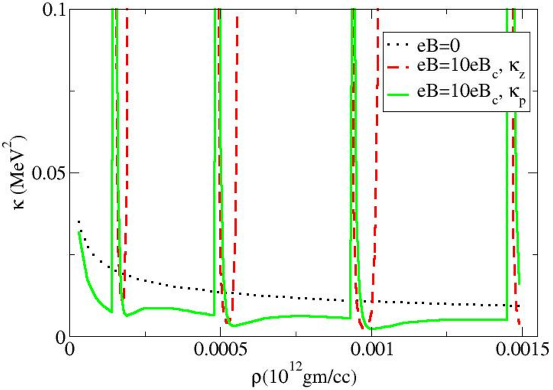


- Intuition: Momentum and energy conservation require $\sqrt{(k^z + q^z)^2 + m_e^2} - \sqrt{(k^z)^2 + m_e^2} = c_s q$

Angle averaged scattering



Anisotropic thermal conductivity

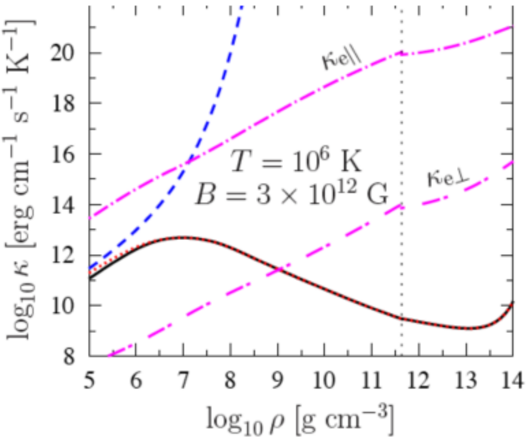


- For comparison, the electronic conductivity at $\rho_{12} = 0.0005$ is $\kappa_z^e \sim 1\text{MeV}^2$, $\kappa_\perp^e \sim 10^{-5}\text{MeV}^2$

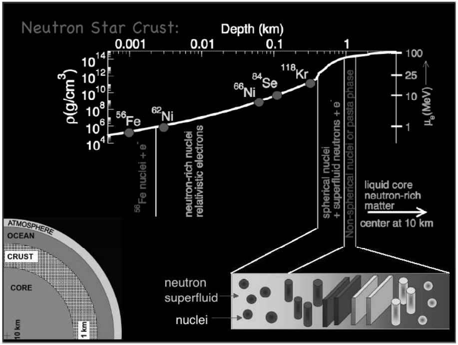
Conclusion

- Even for modest magnetic fields, the lattice phonon transport is highly anisotropic
- The anisotropy in the heat transport of electrons in large magnetic fields is well known *eg. (Chugunov, Haensel, Perez, Azorin, Yakovlev...)*
- Anisotropic lattice transport has not been taken into account and may be important for large magnetic fields

Conductivities

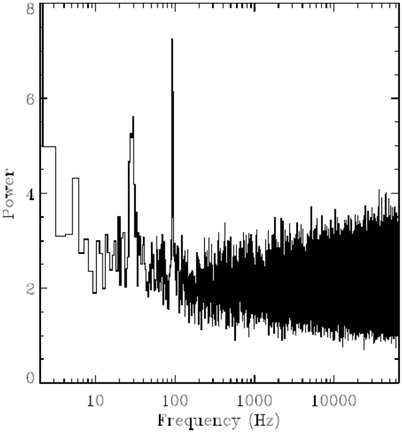


Neutron star profile



- *(Baym, Pethick, Sutherland; Reddy)*

Magnetar oscill



- *(Watts, Strohmayer)*
- 18Hz, 26Hz, 92Hz, 625Hz, ...