Color superconducting compact stars: Structure, Cooling and Gravitational waves

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Astrophysical Transients: Multi-messenger Probes of Nuclear Physics July 11 - August 5, 2011, INT, Seattle

Multi-messengers for studying quark superconductivity in compact stars

Structure: CSC phases, stability, NS masses

Cooling: processes, tuning, fitting the data

Vortices: nucleation, interactions, dynamics

Gravity waves: generation by crystalline color superconducting phase, upper limits

Luca Bonanno, AS, 2011, submitted to arxiv;
Daniel Hess, AS, 2011, arXiv:1104.1706;
Mark Alford, AS, 2010, J. Phys. G, 37, 075202
Bettina Knippel, AS, 2009, Phys.Rev.D79:083007

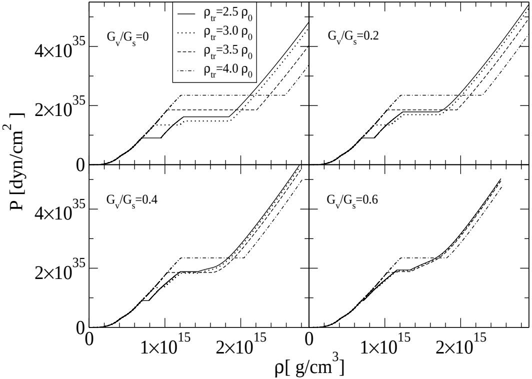
The set-up

The low density nuclear equation of state: Relativistic Lagrangian at Hartree-Fock + Crust model (BPS)

Can be and should be matched to a relativistic-BHF model

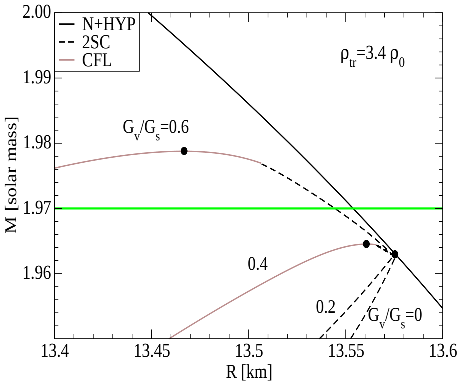
The high-density quark equation of state: NJL+ vector interactions + t'Hooft + Bag from gluon dynamics (not MIT Bag)

EOS



- Phase equilibrium is constructed via Maxwell prescription
- Sequential phase transition $NM \to 2SC \to CFL$.

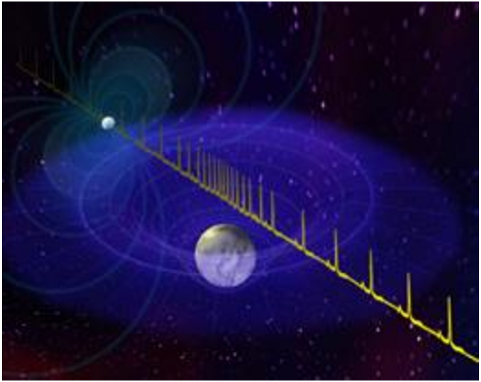
Maximal Masses



- Dashed: 2SC, grey-full: CFL.
- Stability is achieved for $G_V > 0.2$ and transition densities few ρ_0

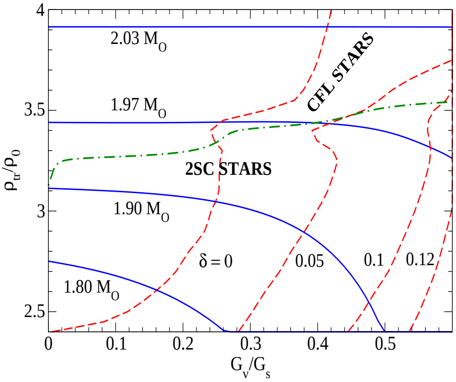
Nature, online 27 October 2010

The largest pulsating star yet observed casts doubts on exotic matter theories (?)



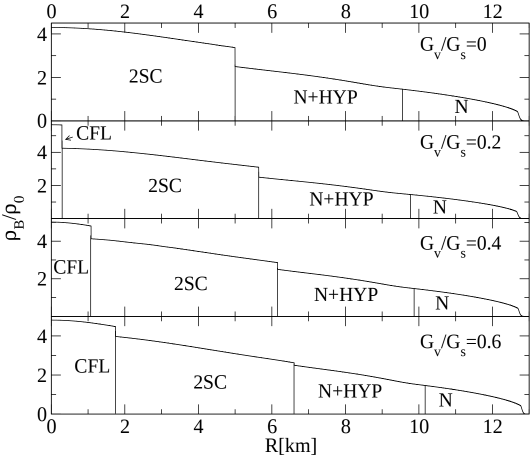
Radio pulses from a neutron star suggest exotic particles are absent from its core

Parameter space



- Below dashed-dotted: 2SC stars are stable
- To the right of dashed curves CFL are stable few ρ_0

Composition



- Fix transition density $2.5 \times \rho_0$.
- Increasing G_V stabilizes the stars + “exotic matter”

Conclusions on the EOS

Need a stiff NM equation state above saturation. NL3 parameter set accommodates hyperons, GM3 - not.

Need vector interactions to stabilize color superconducting quark stars (more generally a mechanism that will make the quark EOS stiffer).

A $2M_{\odot}$ mass star does not exclude exotic matter in the cores of NS.

Cooling processes

Quark cores of NS emit neutrons via: $d \to u + e + \bar{\nu}_e \quad u + e \to d + \nu_e$. The rate of the process is

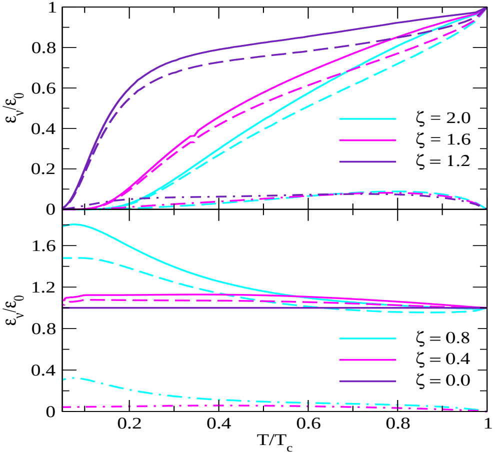
$$\begin{aligned}\epsilon_{\nu\bar{\nu}} &= -2\left(\frac{G}{2\sqrt{2}}\right)^2 \sum_f \int \frac{d^3q_2}{(2\pi)^3 2\omega_\nu(\vec{q}_2)} \int \frac{d^3q_1}{(2\pi)^3 2\omega_\nu(\vec{q}_1)} \int \frac{d^4q}{(2\pi)^4} \\ &\quad (2\pi)^4 \delta^3(\vec{q}_1 + \vec{q}_2 - \vec{q})\delta(\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2) - q_0)\ [\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2)] \\ &\quad g_B(q_0)\,[1 - f_\nu(\omega_\nu(\vec{q}_1))]\,[1 - f_{\bar{\nu}}(\omega_\nu(\vec{q}_2))]\,\Lambda^{\mu\lambda}(q_1, q_2)\mathrm{Im}\,\Pi^R_{\mu\lambda}(q).\end{aligned}$$

via the response function

$$\Pi_{\mu\lambda}(q) = -i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[(\Gamma_-)_\mu S(p)(\Gamma_+)_\lambda S(p+q)\right], \quad \Gamma_\pm(q) = \gamma_\mu(1-\gamma_5) \otimes \tau_\pm$$

with propagators

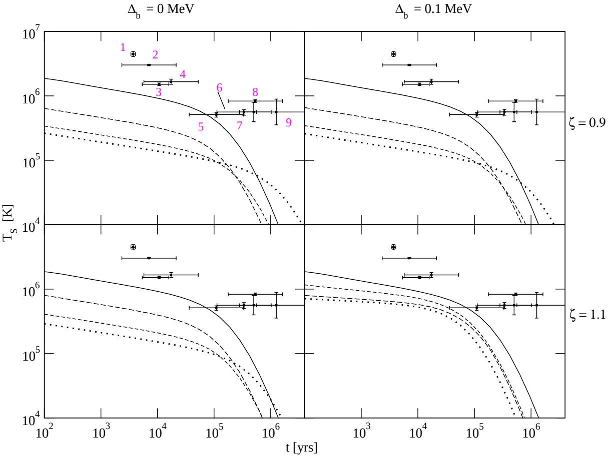
$$S_{f=u,d} = i\delta_{ab}\frac{\Lambda^+(p)}{p_0^2 - \epsilon_p^2}(\not{p} - \mu_f\gamma_0), \quad F(p) = -i\epsilon_{ab3}\epsilon_{fg}\Delta\frac{\Lambda^+(p)}{p_0^2 - \epsilon_p^2}\gamma_5 C$$



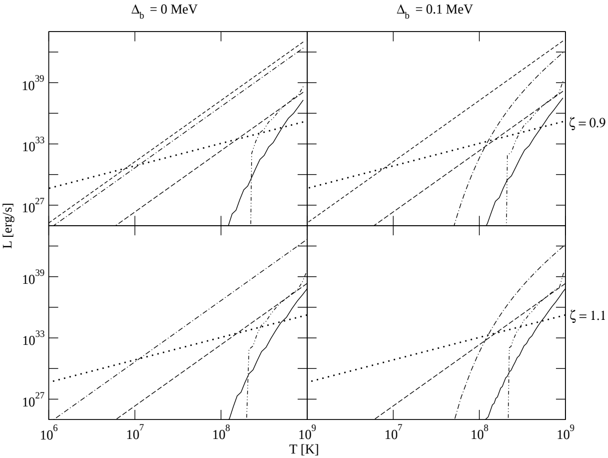
Gapless vs gapped emissivities (Jaikumar, Roberts, AS, 2006)

Here $\zeta = \Delta/\delta\mu$, where $\delta\mu = \mu_d - \mu_u = \mu_e$.

Temperature evolution



Luminosities



Conclusions on cooling

Red-green quarks in the 2SC phase may or may not be fast cooling agents depending on the gaplessness parameter.

The blue quarks act as a BCS superconductor and can contribute to fast cooling if their gaps are small, inversely, can be ineffective in cooling if their gaps are lagre (unlikely above keV).

We have new tools to control the cooling of 2CS-phase of quarks!

GL functional

Ginzburg-Landau functional for 2SC superconductor

$$\mathcal{F} = \mathcal{F}_n + \alpha\psi^*\psi + \frac{1}{2}\beta(\psi^*\psi)^2 + \gamma(\boldsymbol{\nabla}\psi^* - 2ie\boldsymbol{A}\psi^*)(\boldsymbol{\nabla}\psi + 2ie\boldsymbol{A}\psi) + \frac{1}{2\mu_0}(\boldsymbol{B} - \mu_0\boldsymbol{H})^2. \quad (1)$$

The boundary between the type-I and type-II superconductors is set by the GL parameter

$$\kappa_{2\mathrm{SC}} \approx 11\frac{\Delta}{\mu}\ . \quad (2)$$

Need to have fields larger than the lower critical field

$$H_{c1} = \frac{\Phi_X}{4\pi\lambda^2}\ln\kappa \simeq 6.5\times 10^{17}\left(\frac{\mu}{400\ \mathrm{MeV}}\right)^2\left(\frac{g^2}{4\pi}\right)\left(1-\frac{T}{T_c}\right)\quad \mathrm{G}, \quad (3)$$

The density of color-magnetic flux tubes is of the order of those in the core

$$n_v = \frac{B_X}{\Phi_X} = \frac{2\sqrt{3}e}{g}\frac{B_X}{\Phi_0} \simeq 0.3\,\frac{B_X}{\Phi_0}, \quad (4)$$

Aharonov-Bohm cross-section

A single $U(1)$ gauge group (electromagnetism) the cross-section (per unit length)

$$\frac{d\sigma}{d\vartheta} = \frac{\sin^2(\pi\beta)}{2\pi k \sin^2(\vartheta/2)}, \quad \beta = \frac{q_p}{q_c}. \tag{5}$$

q_p is the charge of the scattering particle.

- The cross-section vanishes if β is an integer, but is otherwise non-zero.
- The cross section is *independent of the thickness of the flux tube*: the scattering is not suppressed in the limit where the symmetry breaking energy scale goes to infinity, and the flux tube thickness goes to zero.
- The cross section diverges both at low energy and for forward scattering.

Effect of rotated electromagnetism (Alford, Berges, Rajagopal) "Rotated" gauge fields

$$\begin{aligned} A^{\tilde{Q}} &= \cos\alpha_0 A^Q - \sin\alpha_0 A^T, \\ A^X &= \sin\alpha_0 A^Q + \cos\alpha_0 A^T, \end{aligned}$$

There is a massless $A^{\tilde{Q}}$ field and massive (Higgsed) A^X-field.

AB-cross-section in quark matter

In the basis $\psi = (ru, gd, rd, gu, bu, bd, e^-)$

$$\beta^\psi = \mathrm{diag}\Big(\frac{1}{2} + \frac{e^2}{2g^2}, \frac{1}{2} - \frac{e^2}{2g^2}, \frac{1}{2} - \frac{e^2}{2g^2}, \frac{1}{2} + \frac{e^2}{2g^2}, -1 + \frac{e^2}{g^2}, -1, -\frac{e^2}{g^2}\Big) \,. \tag{6}$$

We conclude

- The gapped quarks have $\tilde{\beta}$ close to $1/2$, which means that they have near-maximal Aharonov-Bohm interactions with an X-flux tube.
- The light fermions the $\tilde{Q}$-neutral bd has zero Aharonov-Bohm interaction with the flux tube.
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- The bu and electron have a $\tilde{\beta}$ that differs from an integer by $e^2/g^2 = \alpha/\alpha_s \sim 1/100$, near-maximal Aharonov-Bohm interactions with an X-flux tube.

Flux dynamics

Aharonov-Bohm scattering contribution from relaxation time

$$\boxed{\tau = \frac{c n_v}{p_{Fi}} \sin^2(\pi \tilde{\beta}_i),} \qquad \boldsymbol{f}_{\rm mf} + \boldsymbol{f}_{\rm ML} + f_b + f_{\rm Iord.} = 0. \tag{7}$$

- Boundary force

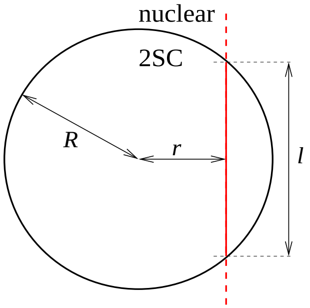
$$f_b \approx \frac{r}{R^2 - r^2} \frac{\mu_q^2}{3\pi} \ln \kappa_X$$

- Mutual friction force

$$f_{\rm mf} = \eta V_L = \frac{p_{Fi} n_i \tau_{if}^{-1}}{n_v} = n_i \sin^2(\pi \tilde{\beta}_i) v_L .$$

- Magnus-Lorentz force

$$\boldsymbol{f}_{\rm ML} = -(\boldsymbol{j}_X \times \hat{n} \Phi_X)$$



The expulsion time for a flux tubes is of the order of 10^{10} years.

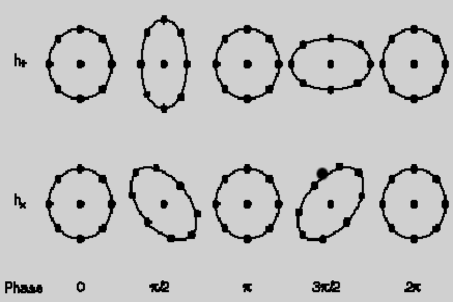
Conclusions on the flux tubes

Dynamics of flux tubes is two slow to expel the magnetic field from the 2SC core.

Flux-tube-fermion scattering can contribute to the transport in some color superconducting phases

Gravitation radiation

Two independent polarization of GW; perturbations of metric $h_{ij} = h_+ e^+_{ij} + h_X e^X_{ij}$.

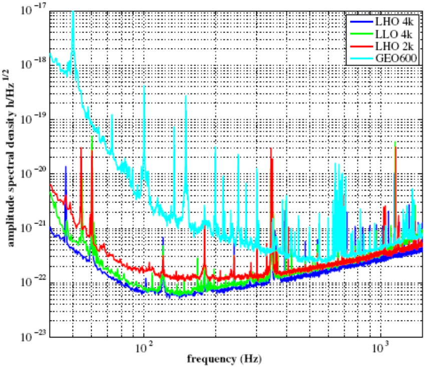


Weak field limit, linearized GR equations, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $h_{\mu\nu}$ perturbation

$$\Box \bar{h}_{\mu\nu} = 0, \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

Gravitation radiation

LIGO is sensitive to GW emitting by rotating NS, which is at 2Ω, e.g. Crab pulsar $\Omega \simeq 30$ Hz.



Gravitation radiation

Given a deformation the characteristic strain amplitude:

$$\boxed{h_0 = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_{zz}\nu^2}{r},}$$

$\epsilon = (I_{xx} - I_{yy})/I_{zz}$ is the equatorial ellipticity. Strain amplitude can be expressed in terms of the $m = 2$ mass quadrupole moment as

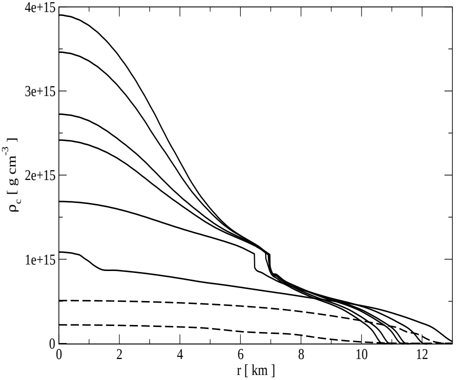
$$h_0 = \frac{16\pi^2 G}{c^4}\left(\frac{32\pi}{15}\right)^{1/2}\frac{Q_{22}\nu^2}{r},$$

Quadrupole moment

$$Q_{22} = \int_0^{R_{\rm core}} \frac{drr^3}{g(r)}\left[\frac{3}{2}(4-U)t_{rr} + \frac{1}{3}(6-U)t_\Lambda + \sqrt{\frac{3}{2}}\left(8 - 3U + \frac{1}{3}U^2 - \frac{r}{3}\frac{dU}{dr}\right)t_{r\perp}\right],$$

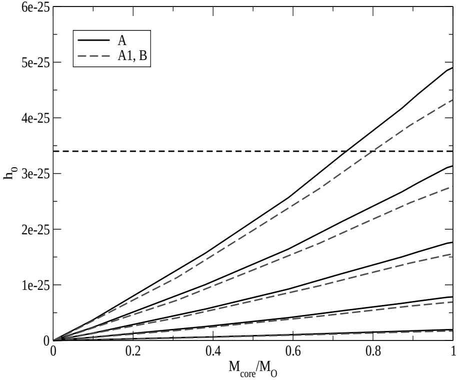
where $U = 2 + d\ln g(r)/d\ln r$ and t_{rr}, t_Λ and $t_{r\perp}$ are the coefficients of the expansion of the shear stress tensor in spherical harmonics.

Internal structure



- Idea: L.-M. Lin, Phys. Rev. D **76**, 081502(R) (2007), *incompressible models without nuclear crusts*
- B. Knippel, A. Sedrakian, Phys. Rev. D **79**, 083007 (2009), *microscopic equations of state*

Strain amplitudes



GW strain amplitudes for breaking strain 10^{-4}, Gaps from 10 to 50 MeV.
Dashed line Crab pulsars' upper limit from S5 run

h_0 can pin down the product $\sigma\Delta^2$, currently $\bar{\sigma}_{\max}\Delta^2 \sim 0.25$ MeV^2 (under the assumptions of the present model).

Conclusions on the gravity waves

Future detection of gravity wave from an isolated NS can place bounds on the properties of solid phases in the NS

Crystalline quark matter can produce gravitational waves that are strong enough to be detected by the LIGO or advanced LIGO experiments

A $2M_{\odot}$ mass star does not exclude exotic matter in the cores of NS.