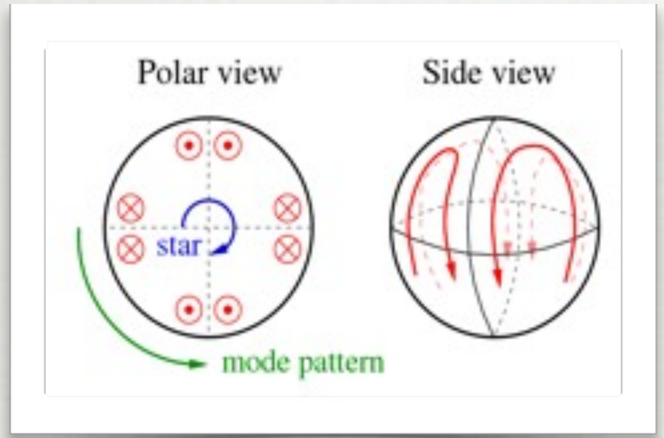
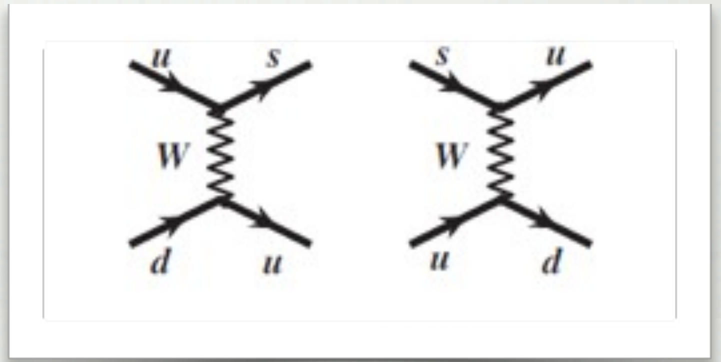
THE (IN-)DEPENDENCE OF R-MODE DAMPING ON THE MICROPHYSICS





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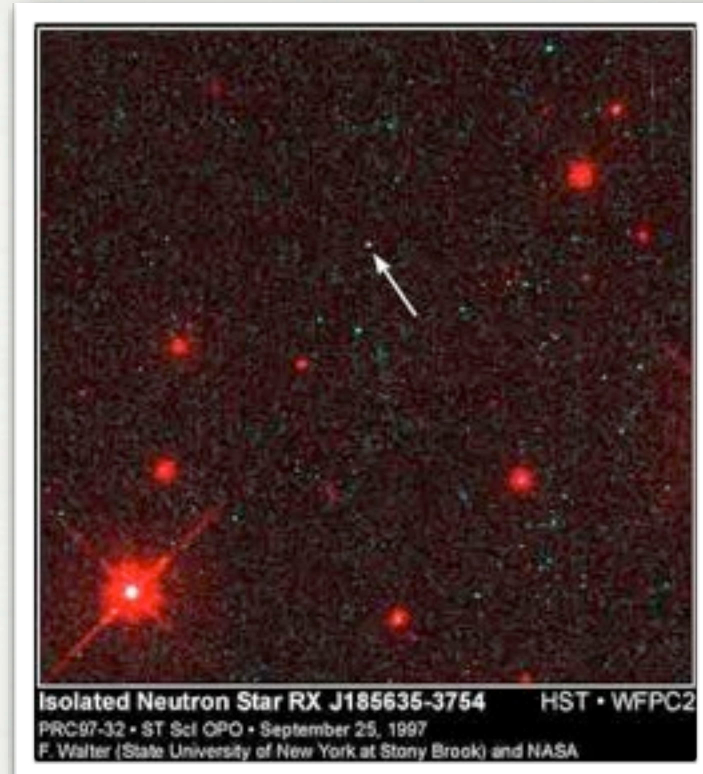
INT SEATTLE, JULY 25. 2010

IN COLLABORATION WITH MARK ALFORD AND SIMIN MAHMOODIFAR

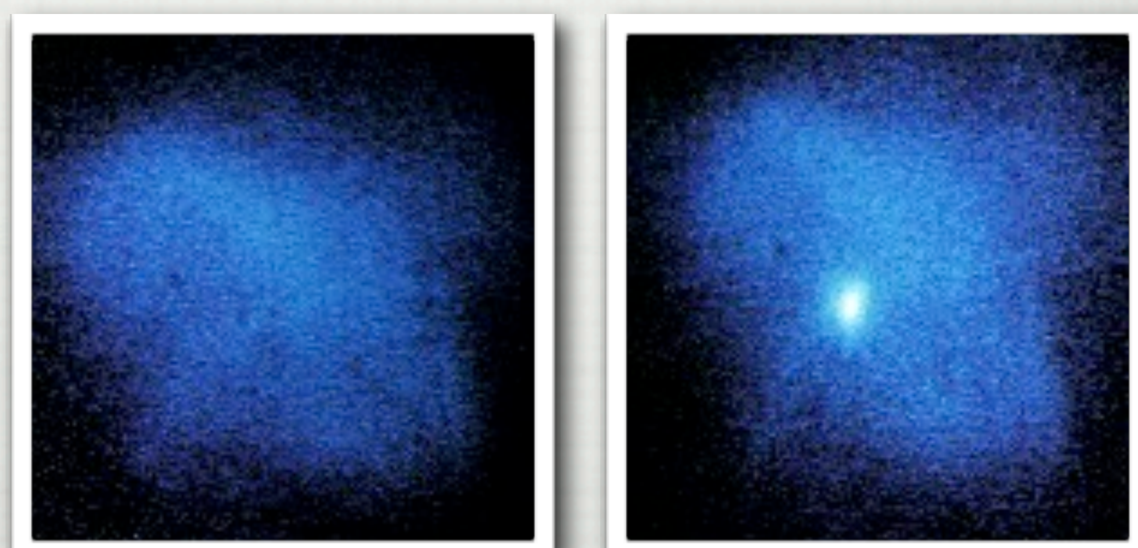
ARXIV:1012.4883

COMPACT STARS

- COMPACT STARS ARE THE DENSEST KNOWN OBJECTS ($\sim M_\odot$ @ $\sim 10\text{km}$ RADIUS)
- ... AND COULD CONTAIN EXOTIC FORMS OF MATTER
- REQUIRES TO CONNECT OBSERVATIONAL ASPECTS TO MICROSCOPIC PROPERTIES
- MANY MICROSCOPIC ASPECTS OF STRONG INTERACTION ARE PURELY UNDERSTOOD
- MANY OBSERVABLES ARE RATHER INDIRECT AND INVOLVE MODEL ASSUMPTIONS



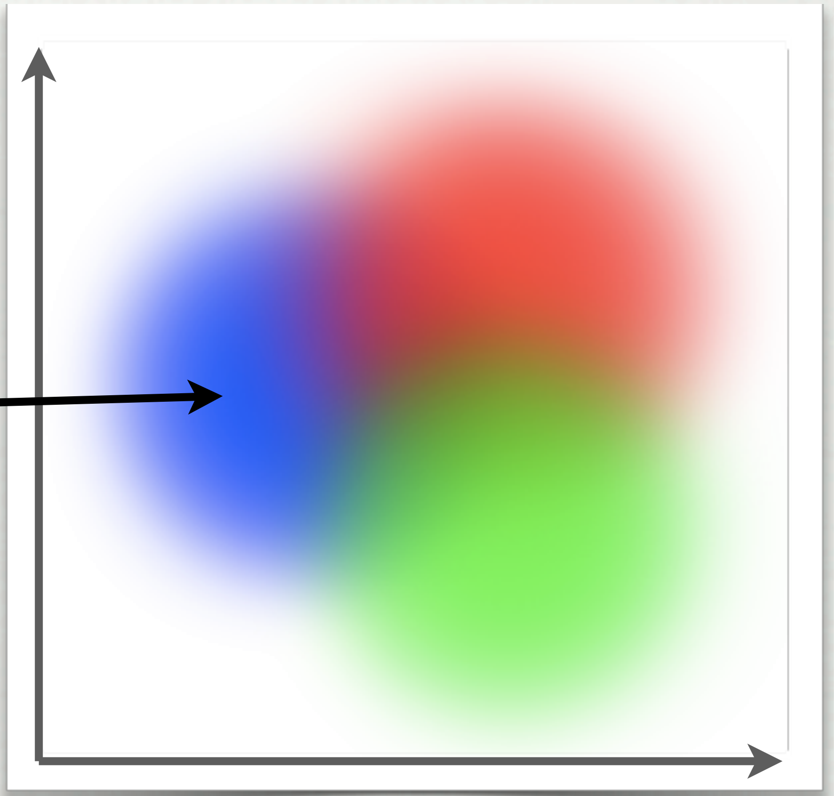
ISOLATED COMPACT STAR



X-RAY EMISSION FROM CRAB PULSAR(*1054 A.D)

DISCRIMINATION

- DETAILED PROPERTIES OF A GIVEN COMPACT STAR CAN NEITHER BE OBSERVED NOR THEORETICALLY PREDICTED
- YET IT WOULD BE GREAT IF ONE COULD ANSWER THE QUESTION IF A STAR IS WITHIN A GIVEN CLASS (E.G.: "QUARK MATTER")



- FOR STATIC PROPERTIES (E.G. MASS/RADIUS RELATION) THIS IS HARD ...

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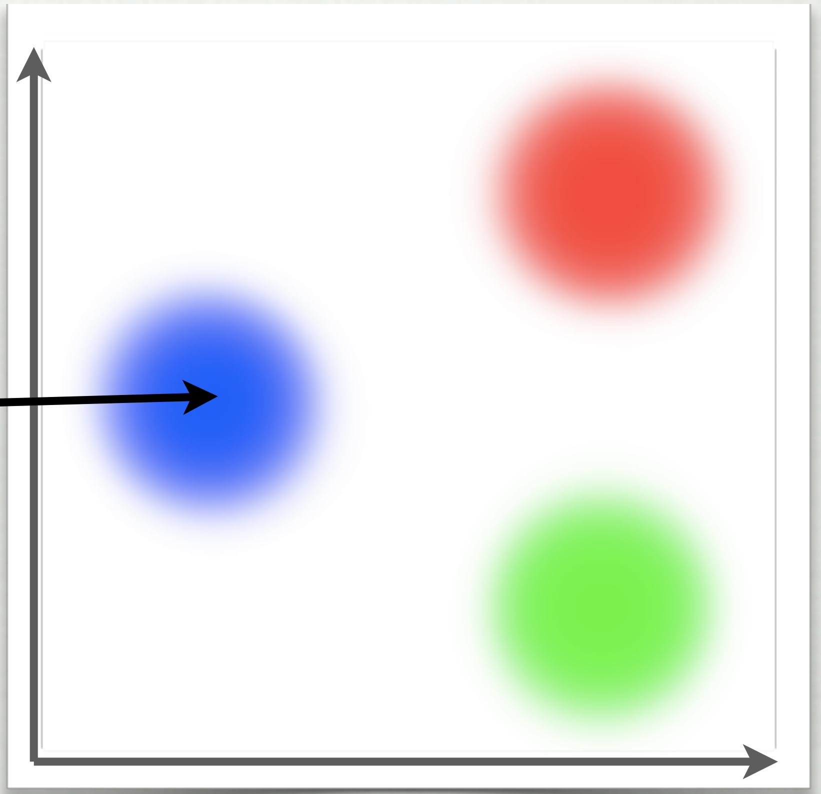
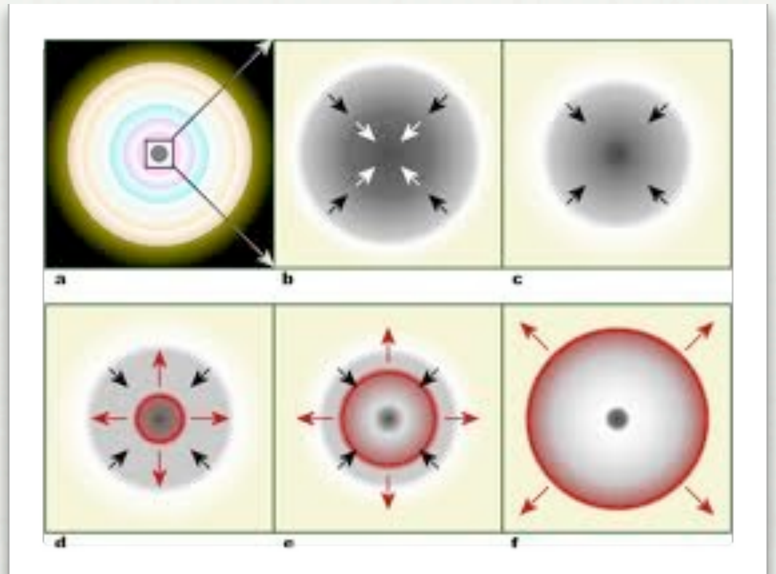
- ... BUT FOR DYNAMIC PROPERTIES CERTAIN CLASSES OF STARS CAN BEHAVE VERY DIFFERENTLY DUE TO PARAMETRICALLY DIFFERENT MICROPHYSICAL MECHANISMS



PULSAR FREQUENCIES

- YOUNG STARS CREATED IN CORE-COLLAPSE SUPERNOVA



  - REMNANT SHRINKS STRONGLY
  - ANGULAR MOMENTUM $J = I\Omega = \tilde{I}MR^2\Omega$ IS CONSERVED => SPIN UP

- OLD STARS IN BINARIES CAN BE GRADUALLY SPUN UP BY ACCRETION

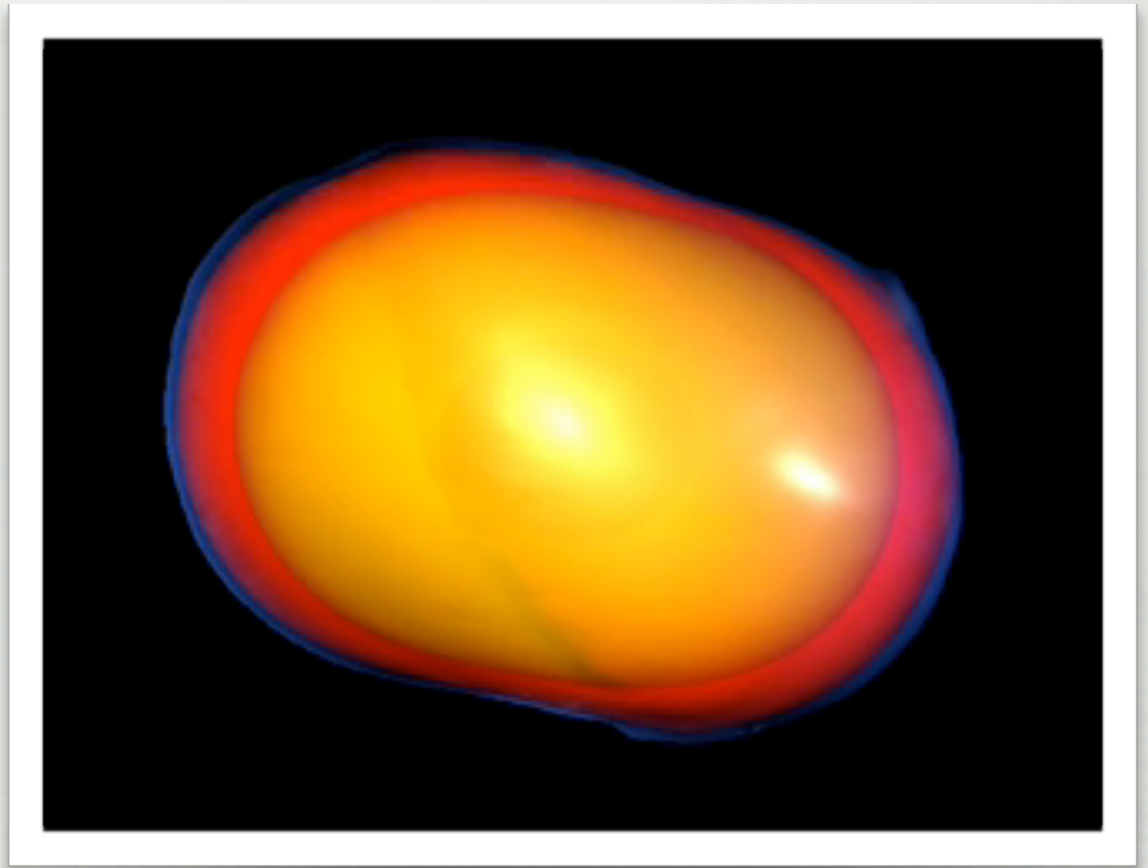
AGE DEPENDENCE

- AGES ARE MOSTLY UNKNOWN, BUT THE SPIN-DOWN AGE COULD GIVE A ROUGH ORDER OF MAGNITUDE ESTIMATE
- PULSAR FREQUENCIES AND SPIN-DOWN RATES PRESENT IMPORTANT DIRECT OBSERVABLES AND SHOW A STRIKING AGE-DEPENDENCE
- OSCILLATIONS COULD SPIN DOWN STARS DUE TO GRAVITATIONAL RADIATION ...

MANCHESTER, ET. AL. ASTRO-PH/0412641

R-MODE OSCILLATIONS

- R-MODE: EIGENMODE OF A ROTATING STAR WHICH IS UNSTABLE AGAINST GRAV. WAVE EMISSION
  N. ANDERSSON, ASTROPHYS. J. 502 (1998) 708
- LARGE AMPLITUDE R-MODE OSCILLATIONS COULD QUICKLY SPIN DOWN A STAR
  B. J. OWEN, ET. AL., PHYS. REV. D 58 (1998) 084020
- BUT R-MODE GROWTH HAS TO BE STOPPED BY SOME NON-LINEAR DAMPING MECHANISM
  --> MARK ALFORDS TALK
- SIMPLE POSSIBILITY: NON-LINEAR VISCOUS DAMPING
- OTHERWISE, LARGE AMPLITUDE R-MODES COULD BE DESTROYED BY DECAY INTO OTHER MODES
  L.M. LIN AND W.M. SUEN, MON. NOT. ROY. ASTRON. SOC. 370 (2006) 1295



SIMULATION BY L. LINDBLOM

VELOCITY OSCILLATION:

$$\delta\vec{v} = \alpha R \Omega \left(\frac{r}{R}\right)^l \vec{Y}^B_{ll} \mathrm{e}^{i\omega t}$$

DENSITY OSCILLATION (NLO IN Ω):

$$\left|\frac{\Delta n}{\bar{n}}\right| \approx \sqrt{\frac{16m}{(m+1)^5(2m+3)}} \frac{\alpha A R^2 \Omega^2}{\kappa(\Omega)} \cdot \left(\left(\left(\frac{r}{R}\right)^{m+1} + \delta\Phi_0\right) \left|Y^m_{m+1}(\theta,\phi)\right| + \cdots\right)$$

VISCOUS DAMPING

- VISCOSITIES DESCRIBE THE DISSIPATION DUE TO MICROSCOPIC INTERACTIONS:

$$\left.\frac{d\epsilon}{dt}\right|_{visc} = -\underbrace{\eta\left(\nabla_a v_b + \nabla_b v_a - \frac{2}{3}\delta_{ab}\nabla_c v_c\right)^2}_{\text{SHEAR}} - \underbrace{\zeta\left(\vec{\nabla}\cdot\vec{v}\right)^2}_{\text{BULK}}$$

- SHEAR VISCOSITY DUE TO PARTICLE SCATTERING HAS USUALLY A POWER LAW TEMPERATURE DEPENDENCE $\eta = \tilde{\eta} T^{\sigma}$

- BULK VISCOSITY DUE TO LOCAL DENSITY OSCILLATION ...

$$n\left(\vec{r}, t\right) = \bar{n} + \Delta n\left(\vec{r}\right)\sin\left(\frac{2\pi t}{\tau}\right)$$

... WHICH INDUCES A CORRESPONDING CHEMICAL POTENTIAL OSCILLATION

$$\mu_\Delta = C\frac{\delta n}{\bar{n}} + B\bar{n}\delta x$$

(E.G. $\mu_\Delta = \mu_n - \mu_p - \mu_e$)

WITH SUSCEPTIBILITIES THAT CHARACTERIZE THE MATTER $C \equiv \bar{n}\frac{\partial \mu_\Delta}{\partial n}$ AND $B \equiv \frac{1}{\bar{n}}\frac{\partial \mu_\Delta}{\partial x}$

BULK VISCOSITY

- STRONG PROCESSES ARE VERY FAST AND LEAD TO AN IMMEDIATE THERMAL EQUILIBRATION WHEREAS WEAK PROCESSES CAN BE SLOW SO THAT CHEMICAL EQUILIBRIUM IS NOT ESTABLISHED

- GENERAL FORM ARISING FROM MICROSCOPIC COMPUTATIONS:

$$\Gamma^{(\leftrightarrow)} = -\tilde{\Gamma} T^{\delta} \mu_{\Delta} \left(1 + \sum_{j=1}^{N} \chi_j \left(\frac{\mu_{\Delta}^2}{T^2} \right)^j \right) \qquad \text{(WHERE } \Gamma^{(\leftrightarrow)} \equiv \Gamma - \Gamma^{(inv)} \text{)}$$

- NON-LINEAR TERMS IMPORTANT AT LARGE AMPLITUDE

--> MARK ALFORDS TALK

- GENERAL ANALYTIC EXPRESSION IN THE SUBTHERMAL LIMIT

$$\zeta^{<} = \frac{C^2 \tilde{\Gamma} T^{\delta}}{\omega^2 + \left(B \tilde{\Gamma} T^{\delta} \right)^2}$$

- TWO ASYMPTOTIC LIMITS $\omega \lessgtr B\tilde{\Gamma} T^{\delta}$

DAMPING PROCESSES

- HADRONIC MATTER: "APR" EOS A. AKMAL, ET. AL., PRC 58 (1998) 1804
  - SHEAR VISCOSITY FROM LEPTONIC SCATTERING
    P.S. SHTERNIN, D.G. YAKOVLEV, PRD 78 (2008) 063006
  - BULK VISCOSITY FROM WEAK URCA PROCESSES:
    - STANDARD MODIFIED $n+n \rightarrow n+p+e+\bar{\nu}_e\ ,\ n+p+e \rightarrow n+n+\nu_e$
      R.F. SAWYER, PLB 233 (1989) 412
    - OR DIRECT AT HIGH DENSITY $n \rightarrow p+e+\bar{\nu}_e\ , p+e \rightarrow n+\nu_e$
      P. HAENSEL AND R.SCHAEFFER, PRD 45 (1992) 4708
- STRANGE QUARK MATTER: $p_{par} = \frac{1-c}{4\pi^2}\left(\mu_d^4+\mu_u^4+\mu_s^4\right) - \frac{3m_s^2\mu_s^2}{4\pi^2} - \mathcal{B} + \frac{\mu_e^4}{12\pi^2}$
  M. ALFORD, ET. AL., APJ 629 (2005) 969
  - SHEAR VISCOSITY FROM QUARK SCATTERING
    H. HEISELBERG, C.J. PETHICK, PRD 48 (1993) 2916
  - BULK VISCOISTY FROM NON-LEPTONIC WEAK PROCESSES $s+u \leftrightarrow d+u$ J. MADSEN, PRD 46 (1992) 3290; M. ALFORD, S. MAHMOODIFAR, K. S., J. PHYS. G 37 (2010) 125202

PARAMETERS

- STRONG INTERACTION PARAMETERS:

$$A \equiv \frac{d\rho}{dp}$$

ENTERS IN R-MODE PROFILE

	A	B	C
hadronic matter	$m_N \left(\frac{\partial p}{\partial n}\right)^{-1}$	$\frac{8S}{n} + \frac{\pi^2}{(4(1-2x)S)^2}$	$4(1-2x)\left(n\frac{\partial S}{\partial n} - \frac{S}{3}\right)$
(hadronic gas)	$\frac{3m_N^2}{(3\pi^2 n)^{\frac{2}{3}}}$	$\frac{4m_N^2}{3(3\pi^2)^{\frac{1}{3}} n^{\frac{4}{3}}}$	$\frac{(3\pi^2 n)^{\frac{2}{3}}}{6m}$
quark matter (gas: $c=0$)	$3+\frac{m_s^2}{(1-c)\mu_q^2}$	$\frac{2\pi^2}{3(1-c)\mu_q^2}\left(1+\frac{m_s^2}{12(1-c)\mu_q^2}\right)$	$-\frac{m_s^2}{3(1-c)\mu_q}$

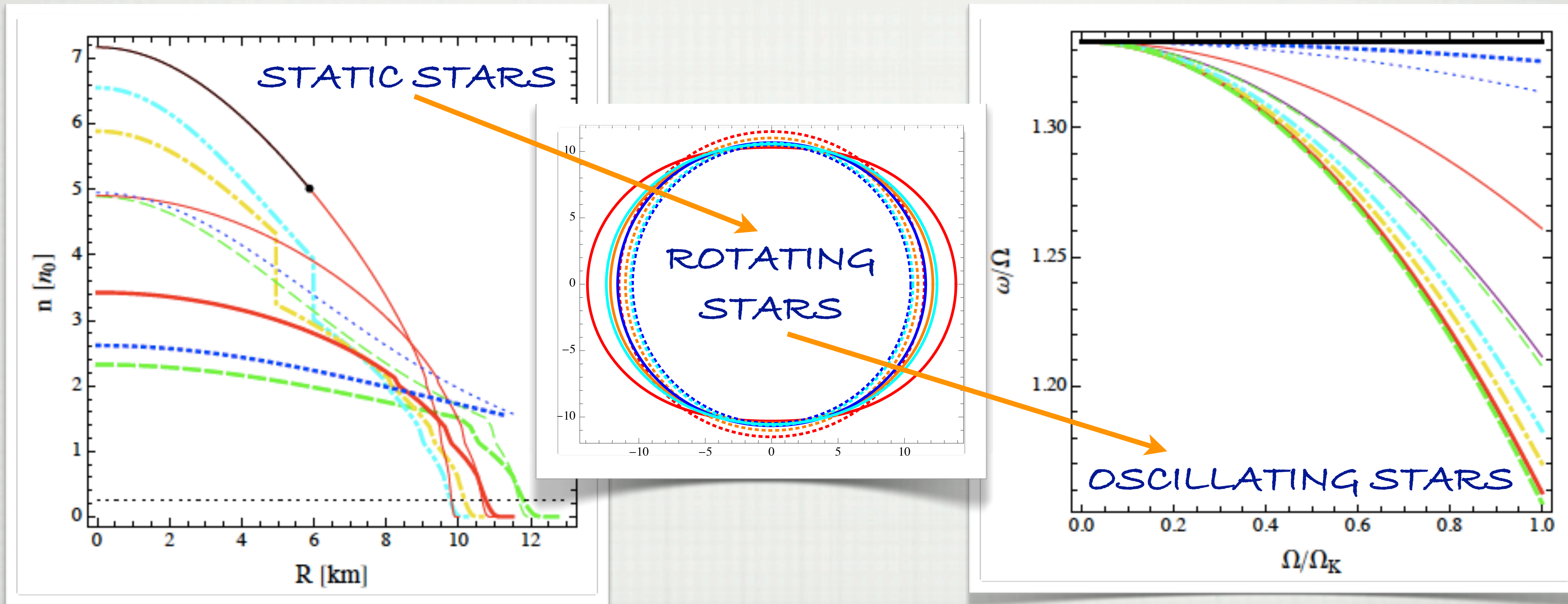
- BULK VISCOSITY PARAMETERS:

Weak process	$\tilde{\Gamma}\,[\mathrm{MeV}^{(3-\delta)}]$	δ	χ_1	χ_2	χ_3
quark non-leptonic	$6.59\times10^{-12}\left(\frac{\mu_q}{300\,\mathrm{MeV}}\right)^5$	2	$\frac{1}{4\pi^2}$	0	0
hadronic direct Urca	$5.24\times10^{-15}\left(\frac{x\,n}{n_0}\right)^{\frac{1}{3}}$	4	$\frac{10}{17\pi^2}$	$\frac{1}{17\pi^4}$	0
hadronic modified Urca	$4.68\times10^{-19}\left(\frac{x\,n}{n_0}\right)^{\frac{1}{3}}$	6	$\frac{189}{367\pi^2}$	$\frac{21}{367\pi^4}$	$\frac{3}{1835\pi^6}$

- SHEAR VISCOSITY PARAMETERS: (NON-FERMI LIQUID ENHANCED SCATTERING)

Strong/EM process	$\tilde{\eta}\,[\mathrm{MeV}^{(3+\sigma)}]$	σ
quark scattering	$1.98\times10^{9}\alpha_s^{-\frac{5}{3}}\left(\frac{\mu_q}{300\,\mathrm{MeV}}\right)^{\frac{14}{3}}$	$\frac{5}{3}$
leptonic scattering	$1.40\times10^{12}\left(\frac{x\,n}{n_0}\right)^{\frac{14}{9}}$	$\frac{5}{3}$
nn-scattering	$5.46\times10^{9}\left(\frac{\rho}{m_N n_0}\right)^{\frac{9}{4}}$	2

STAR MODELS



- SOLUTIONS OF STATIC TOLMAN-OPPENHEIMER-VOLKOV EQS.
  - BOTH $1.4 M_\odot$ AND HEAVY $2 M_\odot$ STAR MODELS — P. B. DEMOREST, ET. AL., NATURE 467 (2010) 1081
- "SLOW ROTATION" EXPANSION IN Ω AND LINEAR MODE ANALYSIS — ,L. LINDBLOM, ET. AL., PRL 80 (1998) 4843; PRD 60 (1999) 064006

GENERAL DAMPING TIME EXPRESSIONS

- THE DAMPING TIMES FOR GENERAL FORMS OF DENSE MATTER CAN BE WRITTEN IN A SEMI-ANALYTIC FORM WHERE ALL DEPENDENCE ON THE MICROPHYSICS (EQUATION OF STATE, TRANSPORT PROPERTIES) AND THE STAR MODEL (DENSITY PROFILES) IS CONTAINED IN A FEW DIMENSIONLESS CONSTANTS:

$$\frac{1}{\tau_G} = -\frac{32\pi\,(m-1)^{2m}}{((2m+1)!!)^2}\left(\frac{m+2}{m+1}\right)^{2m+2}\tilde{J}_m G M R^{2m}\Omega^{2m+2}$$

$$\frac{1}{\tau_S} = \frac{(m-1)\,(2m+1)\,\tilde{S}_m \Lambda_{QCD}^{3+\sigma} R}{\tilde{J}_m M T^{\sigma}}$$

$$\frac{1}{\tau_B^{<}} \xrightarrow[f\ll 1]{} \frac{16m}{(2m+3)\,(m+1)^5\,(\kappa-m)^2}\frac{\Lambda_{QCD}^{9-\delta}\tilde{V}_m R^5 \Omega^2 T^{\delta}}{\Lambda_{EW}^4 \tilde{J}_m M}$$

$$\frac{1}{\tau_B^{<}} \xrightarrow[f\gg 1]{} \frac{16m}{(2m+3)\,(m+1)^5}\frac{\Lambda_{EW}^4 \Lambda_{QCD}^{\delta-1}\tilde{W}_m R^5 \Omega^4}{\tilde{J}_m M T^{\delta}}$$

WITH

$$\tilde{J}_m \equiv \frac{1}{M R^{2m}}\int_0^R \rho(r)\, r^{2m+2} dr$$

$$\tilde{S}_m \equiv \frac{1}{R^{2m+1}\Lambda_{QCD}^{3+\sigma}}\int_{R_i}^{R_o} \tilde{\eta} r^{2m} dr$$

$$\tilde{V}_m \equiv \frac{\Lambda_{EW}^4}{R^3 \Lambda_{QCD}^{9-\delta}}\int_{R_i}^{R_o} dr\, r^2 A^2 C^2 \tilde{\Gamma}\,\left(\delta\Sigma(r)\right)^2$$

$$\tilde{W}_m \equiv \frac{1}{R^3 \Lambda_{EW}^4 \Lambda_{QCD}^{\delta-1}}\int_{R_i}^{R_o} dr\, r^2 \frac{A^2 C^2}{\tilde{\Gamma} B^2}\left(\delta\Sigma(r)\right)^2$$

R-MODE DENSITY FLUCTUATION

- DEPENDENCE ON MACROSCOPIC PARAMETERS EXPLICIT

INSTABILITY REGIONS

- THE BOUNDARY OF THE INSTABILITY REGION IS GIVEN BY:

$$\frac{1}{\tau_G} + \underbrace{\sum_s}_{\text{SHELL}} \left(\frac{1}{\tau_S^{(s)}} + \frac{1}{\tau_B^{(s)}} \right) = 0$$

  - IN GENERAL COMPLICATED NON-ANALYTIC EQUATION IN T AND Ω
  - YET THE DEPENDENCE OF THE INDIVIDUAL TERMS IS VERY PRONOUNCED SO THAT NEARLY ALWAYS ONE DAMPING TERM CLEARLY DOMINATES
  - APPROXIMATE ANALYTIC SOLUTION IN THIS REGIME
- CONDITION FOR BOUNDARY SEGMENT: $\tau_V^{(i)} = |\tau_G|$
- CONDITION FOR EXTREMA: $\tau_V^{(i)} = \tau_V^{(j)} = |\tau_G|/2$

SEMI-ANALYTIC RESULTS

- GENERAL RESULT FOR MINIMUM OF THE INSTABILITY REGION:

$$\Omega_{min} \approx \left(\left(\frac{m\,(m+1)^{2m-1}\,((2m+1)!!)^2}{4\pi\,(2m+3)\,(m+2)^{2m+2}\,(m-1)^{2m}} \right)^{\sigma} \left(\frac{((2m+1)!!)^2\,(2m+1)\,(m+1)^{2m+2}}{16\pi\,(m-1)^{2m-1}\,(m+2)^{2m+2}} \right)^{\delta} \frac{\tilde{V}_m^{\sigma}\tilde{S}_m^{\delta}\Lambda_{QCD}^{3\delta+9\sigma}}{\tilde{J}_m^{2(\delta+\sigma)}\Lambda_{EW}^{4\sigma}G^{\delta+\sigma}R^{2m(\delta+\sigma)-\delta-5\sigma}M^{2(\delta+\sigma)}} \right)^{\frac{1}{2m(\delta+\sigma)+2\delta}}$$

  - IMPORTANT BECAUSE IT DETERMINES THE LIMITING FREQUENCY TO WHICH R-MODES CAN SPIN DOWN A STAR

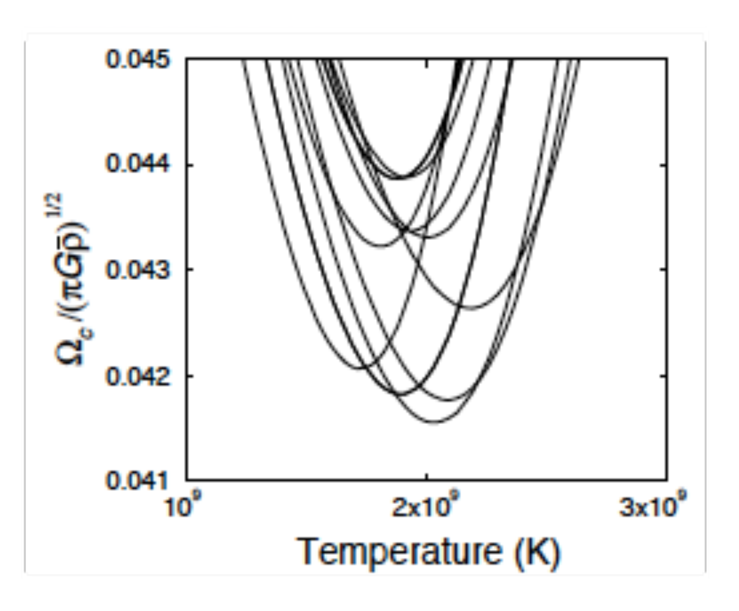
- LOWEST R-MODE OF A NEUTRON STAR WITH MODIFIED URCA:

$$\Omega_{min}^{(NS)} \approx 1.06\,\frac{\Lambda_{QCD}^{\frac{99}{128}}\tilde{S}^{\frac{9}{64}}\tilde{V}^{\frac{5}{128}}}{R^{\frac{49}{128}}\tilde{J}^{\frac{23}{64}}G^{\frac{23}{128}}M^{\frac{23}{64}}\Lambda_{EW}^{\frac{5}{32}}}\,,\quad T_{min}^{(NS)} \approx 1.89\,\frac{\tilde{S}^{\frac{3}{32}}\Lambda_{QCD}^{\frac{1}{64}}\Lambda_{EW}^{\frac{9}{16}}G^{\frac{3}{64}}\tilde{J}^{\frac{3}{32}}M^{\frac{3}{32}}}{\tilde{V}^{\frac{9}{64}}R^{\frac{27}{64}}}$$

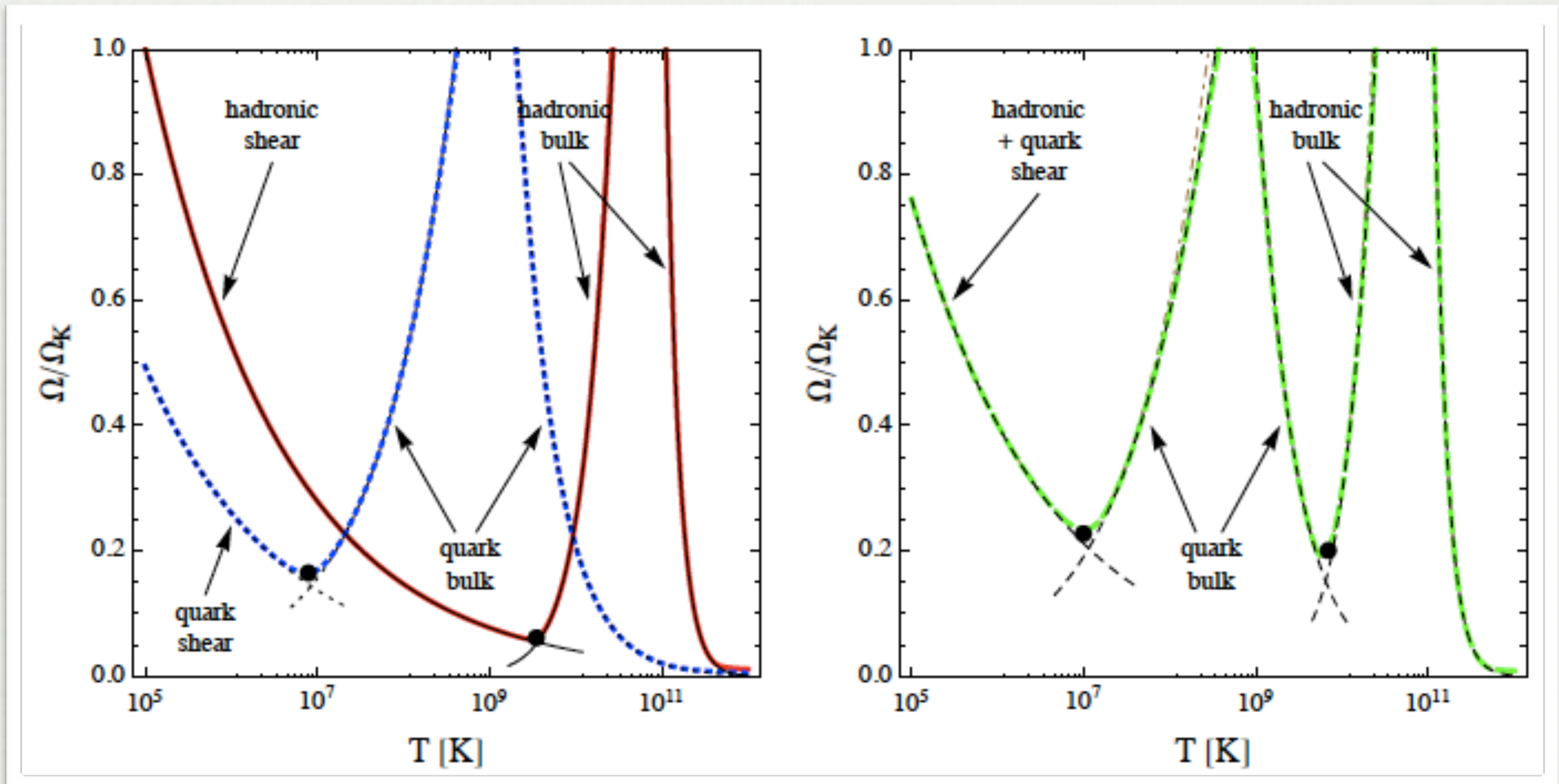
  - EXTREMELY LOW POWERS OF $\tilde{S}$ & $\tilde{V}$!
  - INSENSITIVE TO DETAILS WITHIN A CLASS ...
  - ... BUT NOT TO DIFFERENT CLASSES (δ & σ)

- GENERALIZATION OF A PREVIOUS RESULT

L. LINDBLOM, ET. AL., PRL 80 (1998) 4843

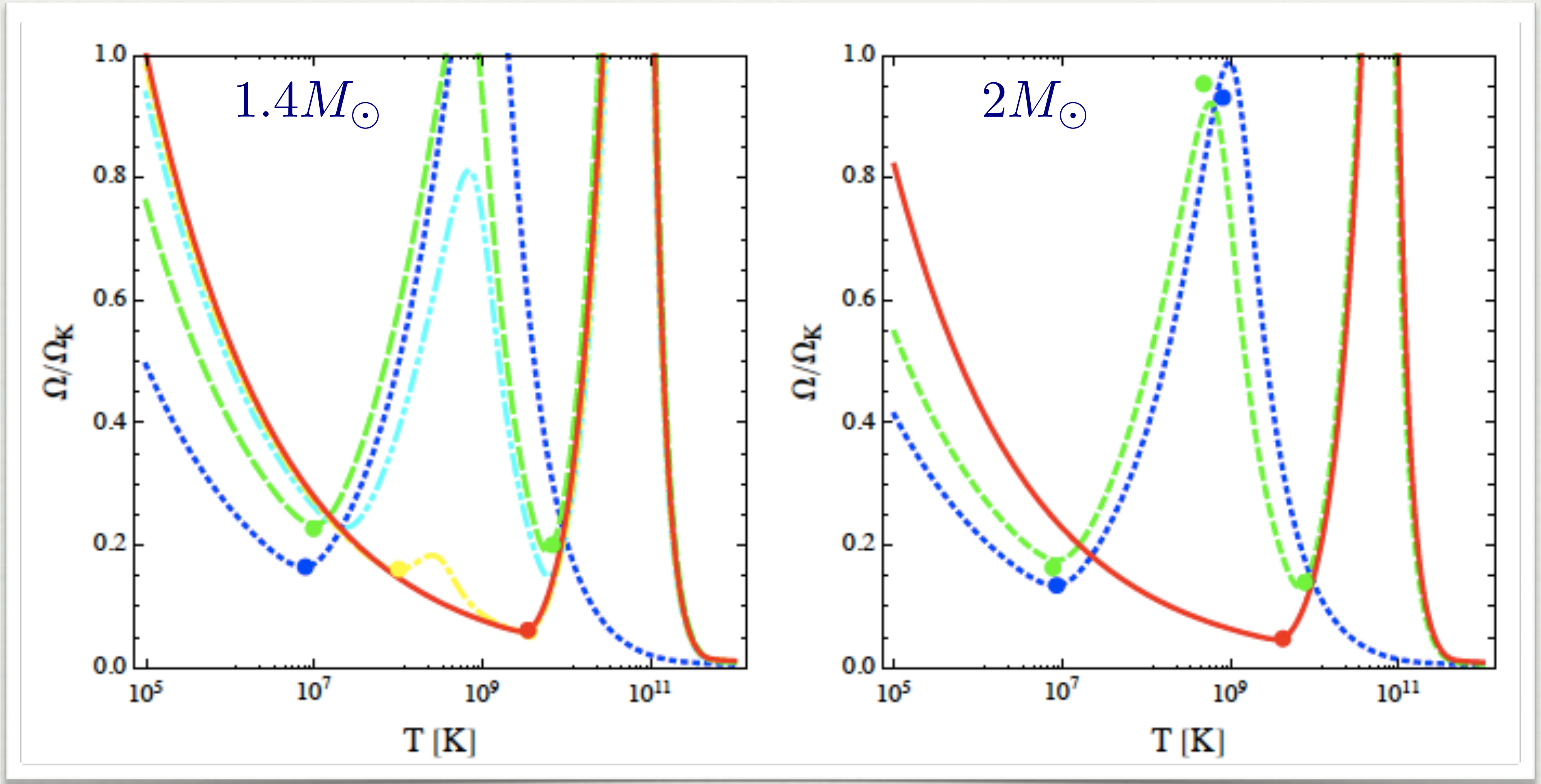


ANALYTIC VS. NUMERIC



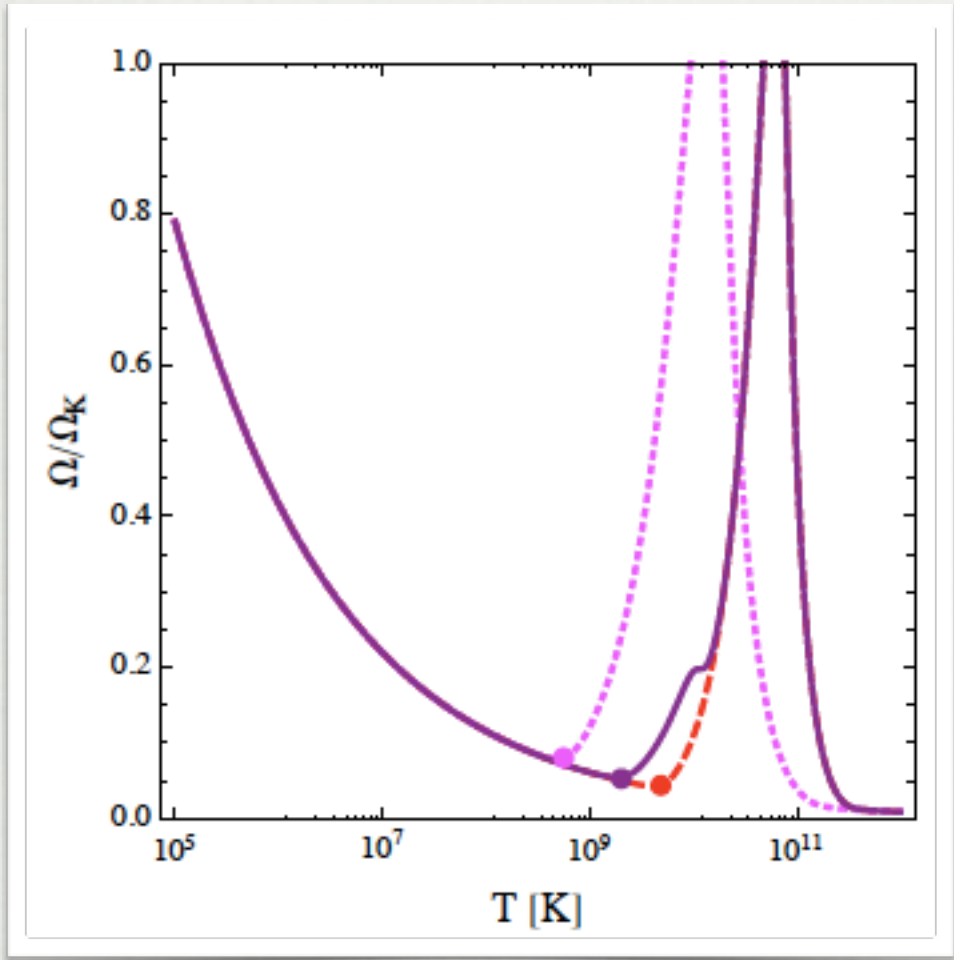
- VERY GOOD AGREEMENT BETWEEN THE SEMI-ANALYTIC AND NUMERIC RESULTS
- ANALYTIC EXPRESSIONS COVER THE BASICALLY ENTIRE INSTABILITY BOUNDARY

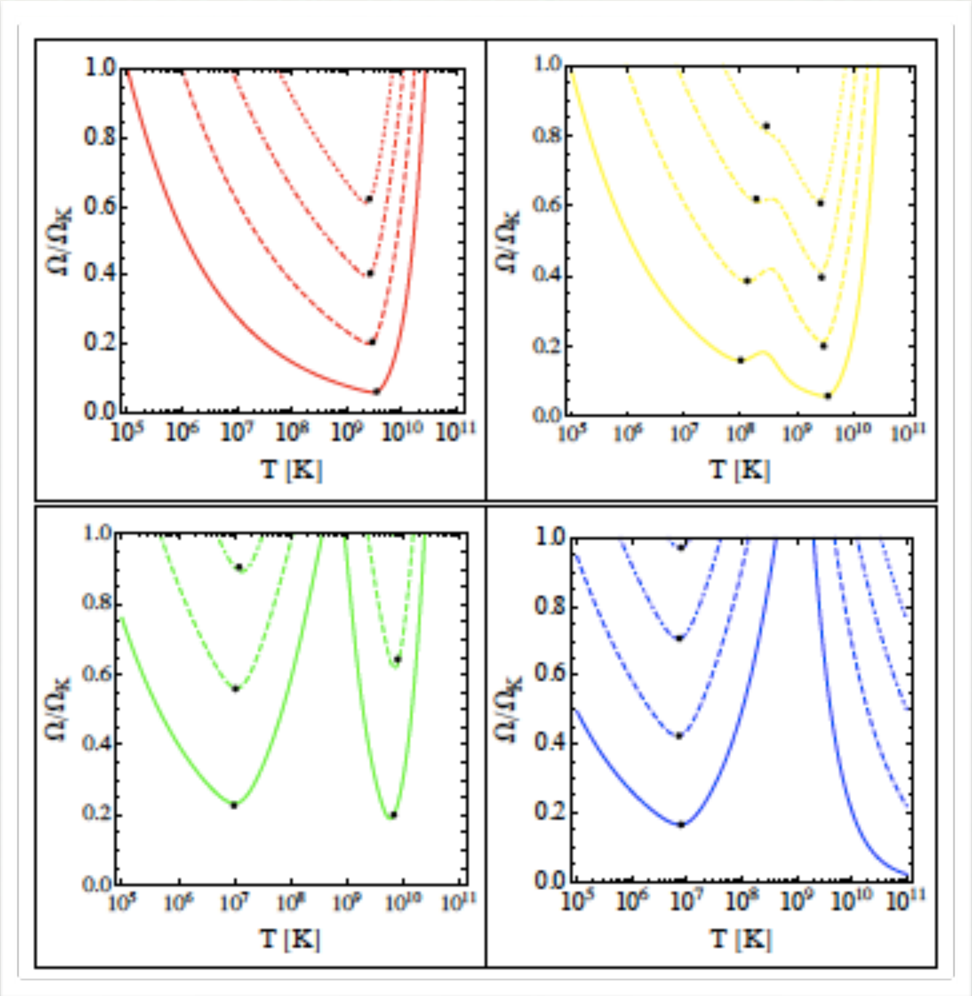
MASS DEPENDENCE



- LARGE MASS STARS ARE SLIGHTLY MORE UNSTABLE
- HYBRID STARS WITH A SMALL QUARK CORE ARE INDISTINGUISHABLE FROM ORDINARY NEUTRON STARS

OTHER ASPECTS

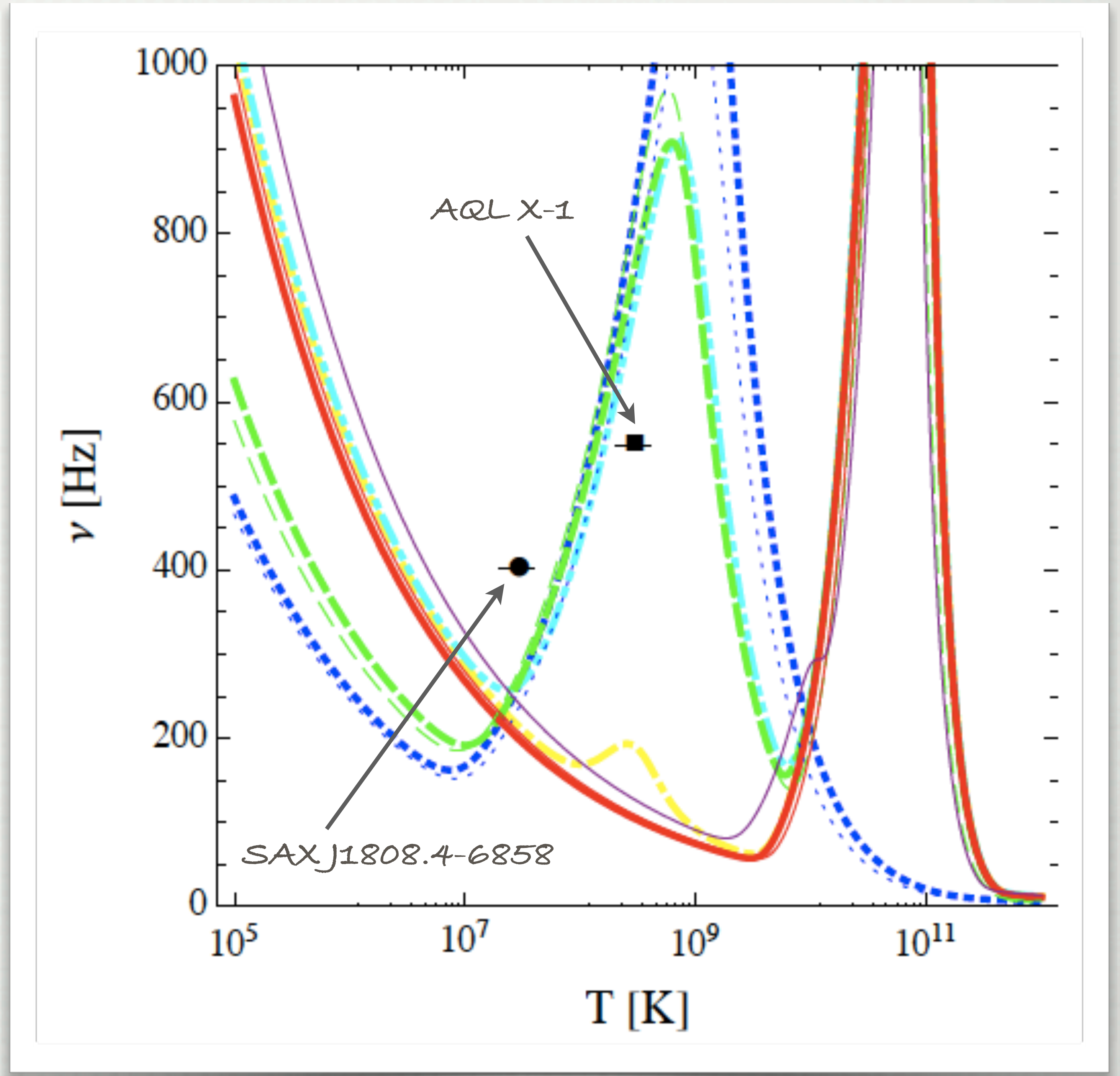




- DIRECT URCA HAS A SMALL EFFECT (NOTCH AT THE R.H.S)
- HIGHER R-MODES MORE STABLE

CONFRONTING DATA

- LMXB'S FAR INSIDE THE NEUTRON STAR INSTABILITY REGION
- ANALYTIC ANALYSIS: ROBUST STATEMENT!
- POSSIBLE EXPLANATION: EXOTIC MATTER WHICH PROVIDES ENHANCED DAMPING
  - STABILITY WINDOW
- OTHER OPTIONS:
  - CRUST EFFECTS
  - SUPERFLUIDITY?



CONCLUSION

- SEMI-ANALYTIC EXPRESSIONS FOR THE BOUNDARY OF THE INSTABILITY REGION
- SURPRISINGLY INSENSITIVE TO THE DETAILED MICROPHYSICS FOR A GIVEN FORM OF DENSE MATTER
- ... BUT COULD DISTINGUISH CLEARLY BETWEEN DIFFERENT CLASSES OF DENSE MATTER
- DEFINITE CONCLUSIONS REQUIRE A BETTER UNDERSTANDING OF THE DYNAMIC R-MODE ASPECTS (SATURATION, EVOLUTION, ...)