MAXIMALLY MODEL INDEPENDENT EQUATION OF STATE FOR NEUTRON STAR MATTER

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Content

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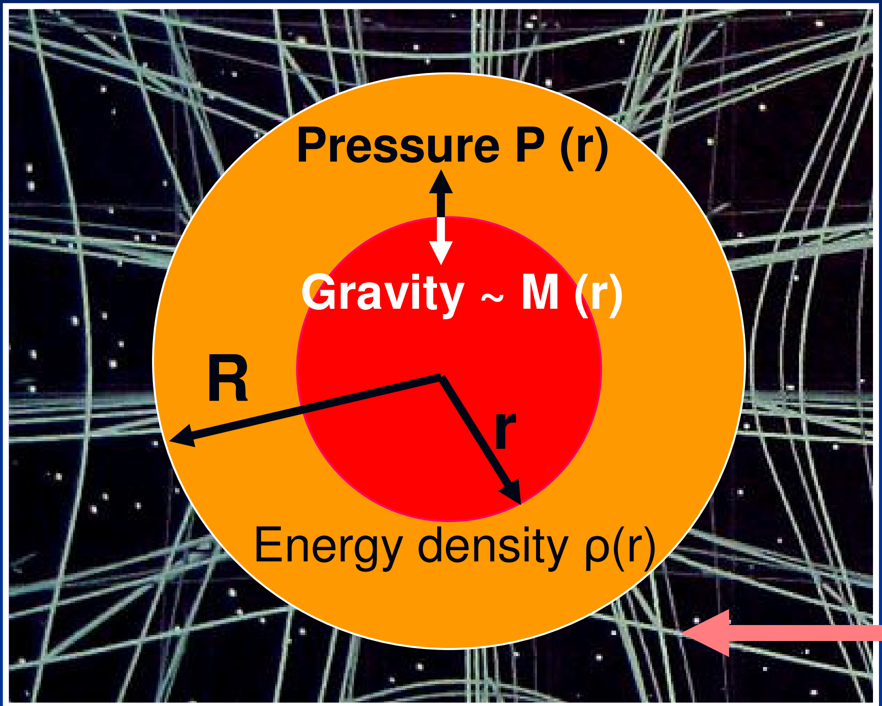
- The Tolman-Oppenheimer-Volkov (TOV) equations.
- Potentially observable parameters of a NS.
- Observational data with errors and their effects on the inferred Equation of State (EOS).
- Argument for smoothness of EOS.
- Schemes to generate wide class of EOS.
- Simplest inference from 5 stars using X-ray bursts data.
- Sequential Bayesian analysis.
- The TOOL for astronomers.
- **Recent 2 Solar mass star**
- **Taking steps further.**
- **Summary.**

Objectives

1. Use observations of masses and radii (binding energy, moment of inertia, Love number, period and etc.) of several individual NS stars to determine the dense nuclear matter EOS.

2. To provide a benchmark maximally model-independent dense matter EOS for ongoing microscopic studies.

Neutron Star in hydrostatic equilibrium
Tolman-Oppenheimer-Volkov (TOV) equations



1) Equations of stellar structure connects the matter pressure P(r) and the enclosed gravitational mass M(r) at the macrophysical level

2) Equation of State (EOS): $P \equiv P(\rho)$ connects pressure and energy density through microphysics of dense matter

3) Due to compactness, gravitational force is large and General Relativity (GR) must be considered

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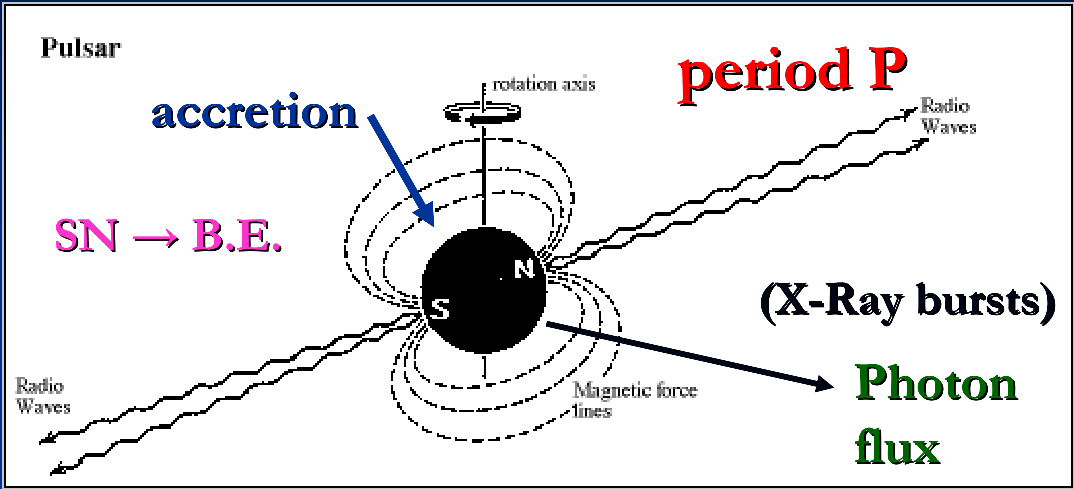
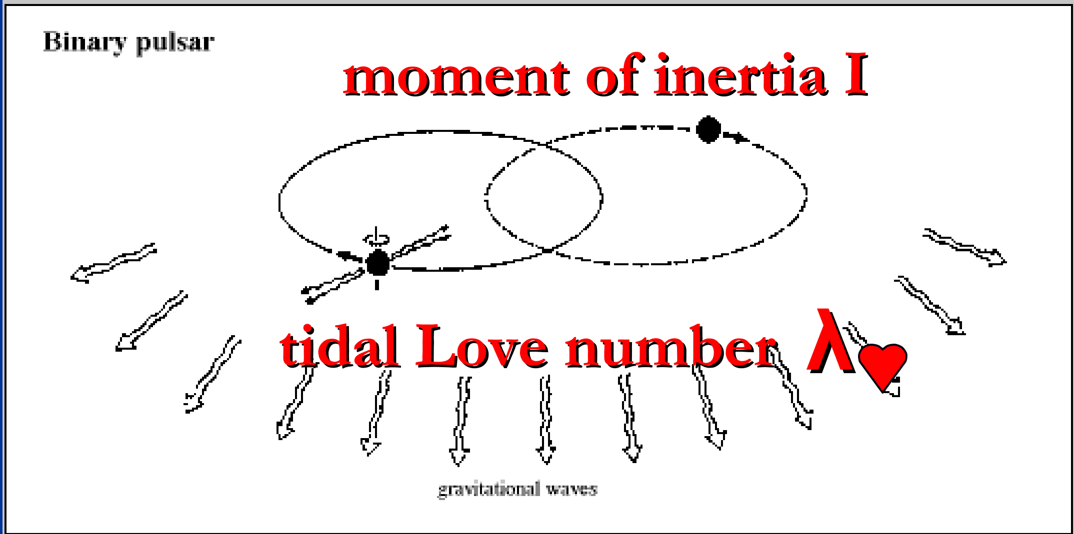
Center: $\begin{cases} \rho(0) = \rho_c \\ P(0) = P_c = P(\rho_c) \end{cases}$ ← TOV / EOS → Surface: $\begin{cases} M(R) \text{ total mass} \\ P(R) = 0 \end{cases}$

microphysics — the bridge — macrophysics

Potentially observed properties of a NS





(http://nobelprize.org/nobel_prizes/physics/laureates/1993/press.html)

$$\beta \equiv GM/Rc^2$$

Radiation radius R_∞ and effective temperature T_∞ is measured from photon flux spectra.

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}\frac{1}{f_c^2 T_\infty^2}}$$

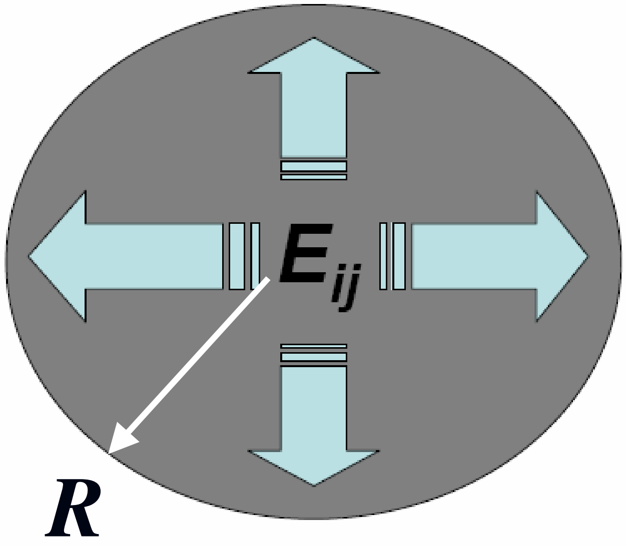
Redshift z can be measured from lines or peak curvature of spectrum.

$$z = (1-2\beta)^{-1/2} - 1$$

And the other ways and parameters…

Love number of a star

An external tidal field $\mathcal{E}_{ij}$ induces a quadrupole moment Q_{ij} in the star, disturbing it from its spherical configuration.



$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$k_2 = \frac{3}{2} G \lambda R^{-5}$$

λ is the Love number.

k_2 is its dimensionless form.

A good analogy is the electric dipole moment of an atom in an external electric field.
λ carries the info about the structure of the compact object.

First order DE and surface parameter *y*

$$-\frac{dy}{dh}\nu'/2 + y^2/r(h) - y/r(h) + y\left(2/r(h) + e^{\lambda}\left[2m(h)/r^2(h) + 4\pi r(h)(p-\rho)\right]\right) + \\ + \left(-6e^{\lambda}/r(h) + 4\pi r(h)e^{\lambda}(5\rho + 9p + (p+\rho)/c_s^2) - \nu'^2\right) = 0.$$

h defined through $dh = \frac{dp}{\rho + p}$, $y(r) = r\frac{H'(r)}{H(r)}$,

with initial condition at the star's center

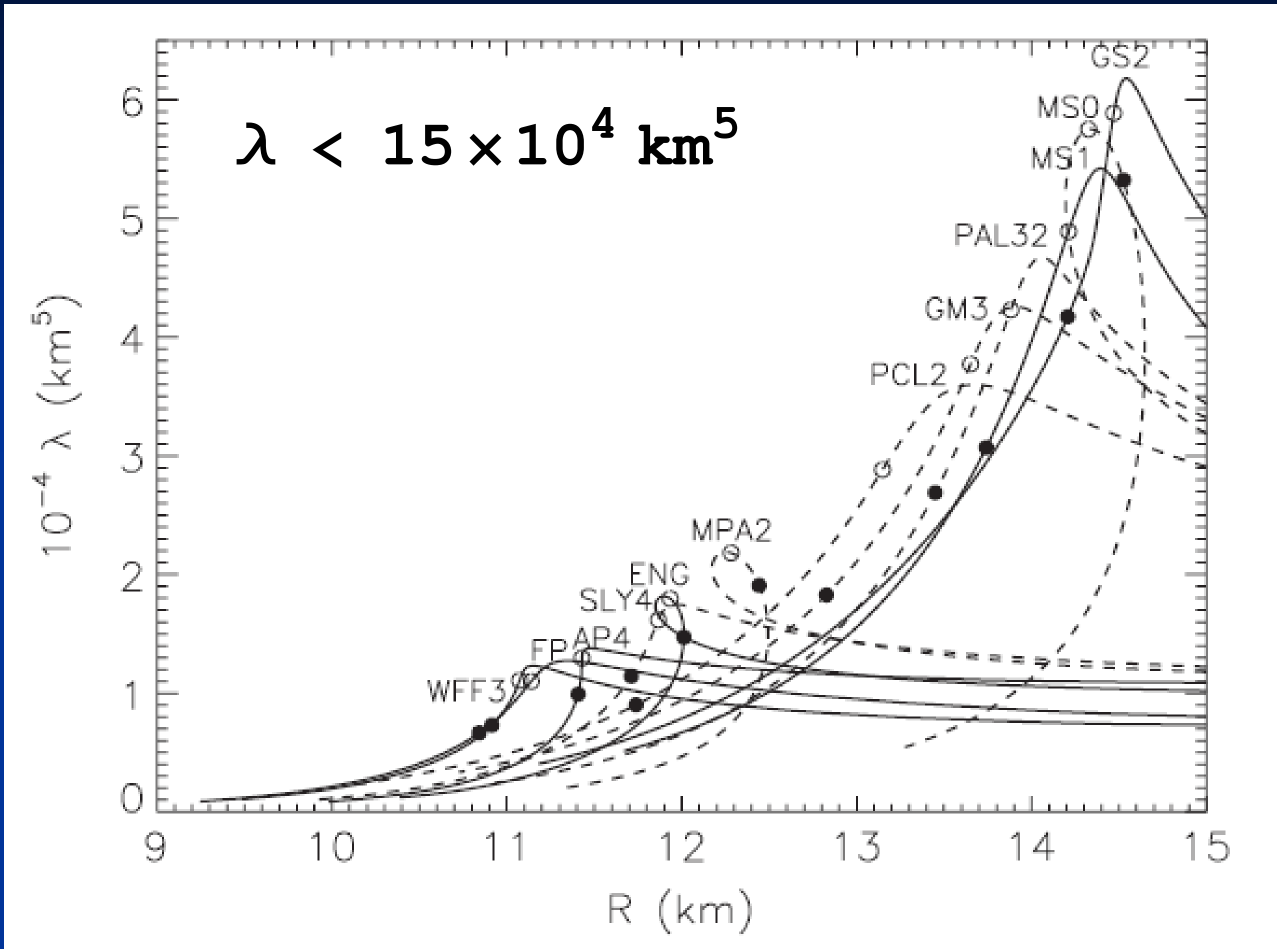
$$y = 2 - \frac{9}{7}\frac{5\rho_c + 9p_c + (p_c + \rho_c)/c_{sc}^2}{3p_c + \rho_c}(h_c - h) + O\left((h_c - h)^2\right).$$

$$y = y(h = 0).$$

$$y(r_d + \epsilon) = y(r_d - \epsilon) - \frac{\rho(r_d + \epsilon) - \rho(r_d - \epsilon)}{m(r_d)/(4\pi r_d^3)}$$

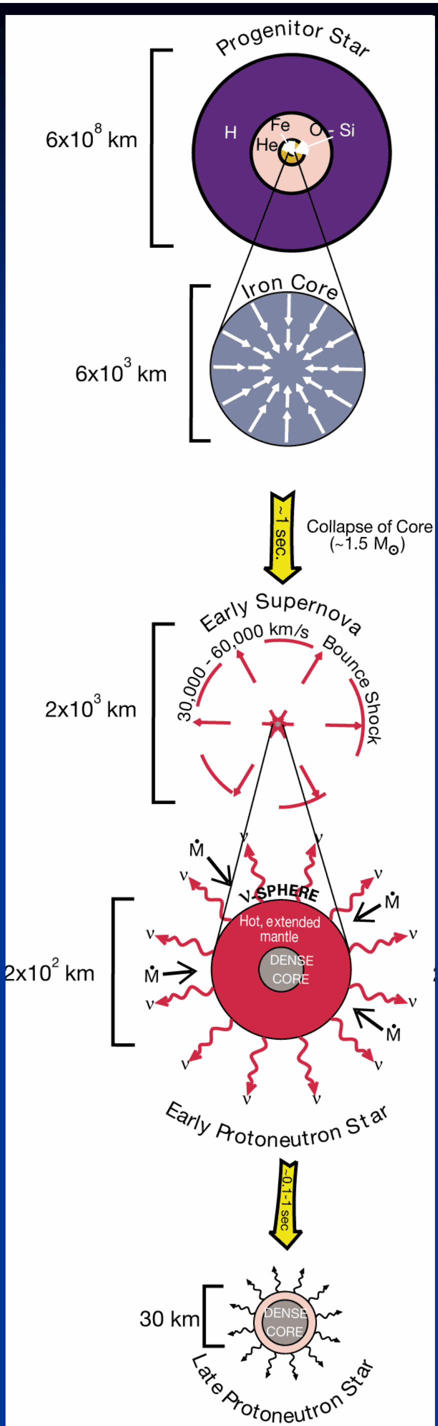
$$k_2(C) = \frac{8}{5}C^5(1-2C)^2(2 + 2C(y-1) - y) \times \\ \times \left[2C(6 - 3y + 3C(5y - 8)) + 4C^3(13 - 11y + C(3y - 2) + 2C^2(1 + y)) + \\ + 3(1-2C)^2(2 - y + 2C(y-1)\log(1-2C))\right]^{-1}$$

Love Numbers for NS



Supernovae and Binding Energy

(Adam Burrows, Nature 403, 727-733)



$$M = 4\pi \int_0^R \epsilon r^2 dr. \qquad (3)$$

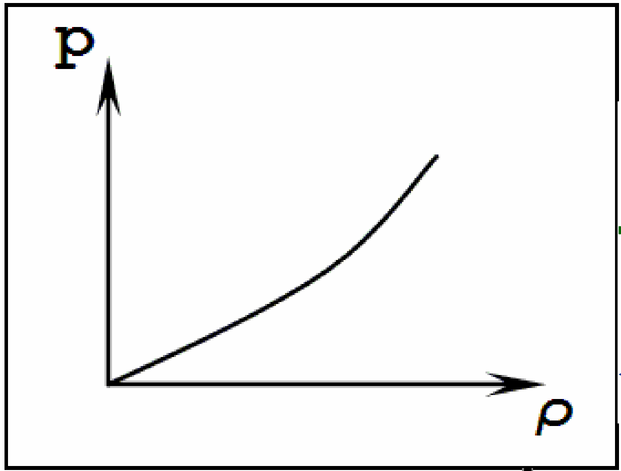
This is the mass which is measured by Kepler's law when a satellite orbits the star. For this reason, M is often called the "gravitational mass". The baryon mass, M_0, of the star is given by the volume integral

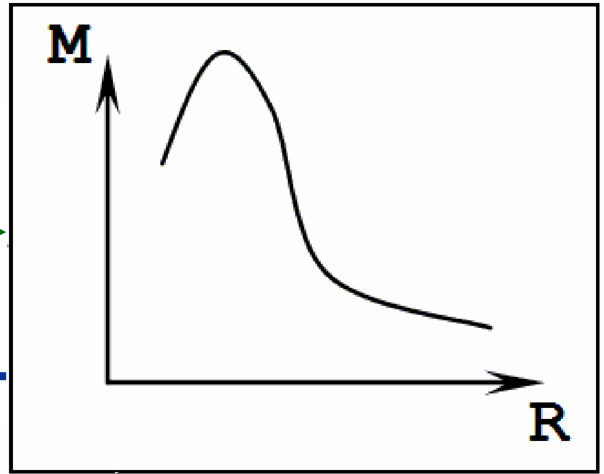
$$M_0 = m_B N = 4\pi m_B \int_0^R n\left(1 - 2m(r)/r\right)^{-1/2} r^2 dr.$$

B.E. = M_0 - M

"Neutrino observations of supernovae, validated by the serendipitous observations of SN 1987A, which yielded about 20 neutrinos, should detect thousands of neutrinos from a galactic supernova. This could yield neutron star binding energies to a few percent accuracy and provide estimates of their masses, radii, and interior compositions, as well as details of neutrino opacities in dense matter." (Lattimer and Prakash, Nature **304** (2004): 536-542)

Mass and radius of a NS



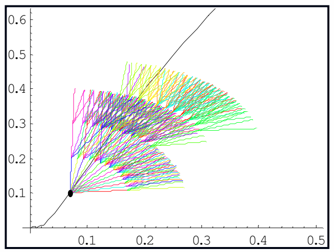


Lindblom (ApJ 1992)

Harada (PRC 2001)

TOV

EOS:



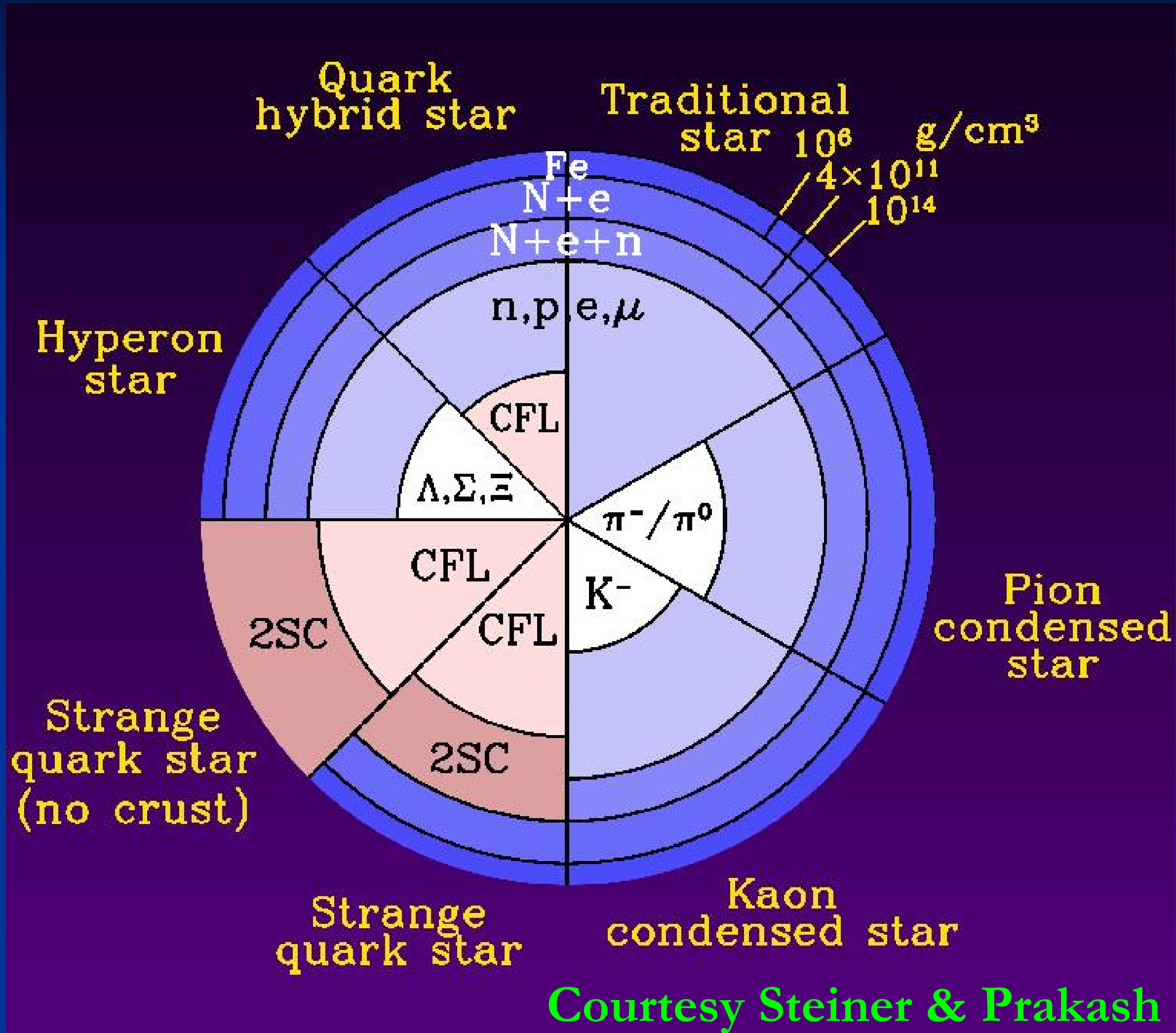
$$\frac{dm}{dr} = 4\pi r^2 \rho \,, \qquad f(R)$$

$$\frac{dp}{dr} = -(\rho + p)\,\frac{m + 4\pi r^3 p}{r\,(r - 2m)}$$

$(G = c = 1)$ (+ other relevant eqns.)

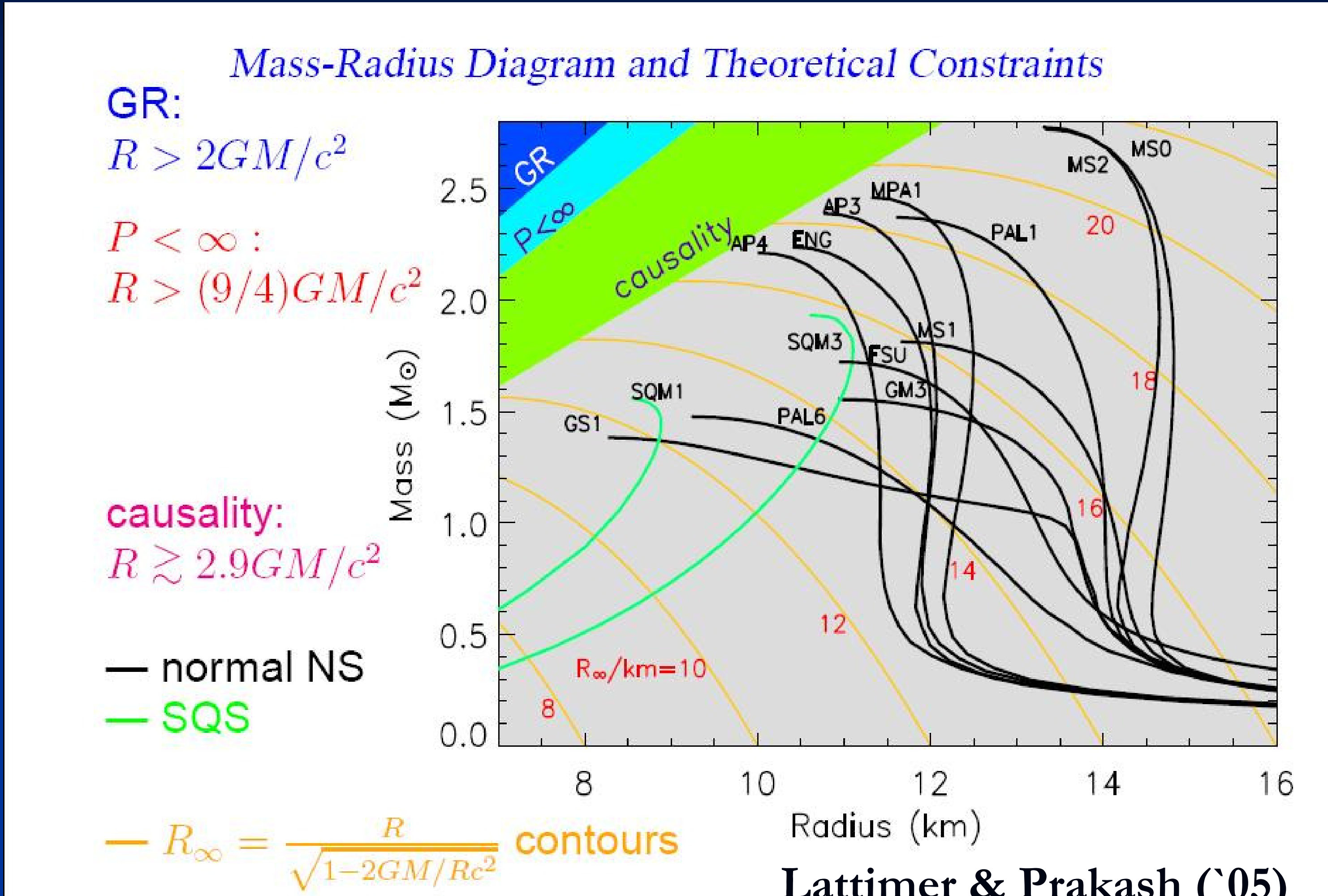
- I
- Z
- B.E.
- ⋮

"The American Pie"



Many exotic scenarios for interior composition, but none confirmed!

Direct approach: from EOS to M vs R



Lattimer & Prakash (`05)

Note: Diverse predictions for masses and radii; not satisfactory!

Neutron Star Matter Pressure and the Radius

$p \simeq K\epsilon^{1+1/n}$ **- polytrope**

$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$

$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$

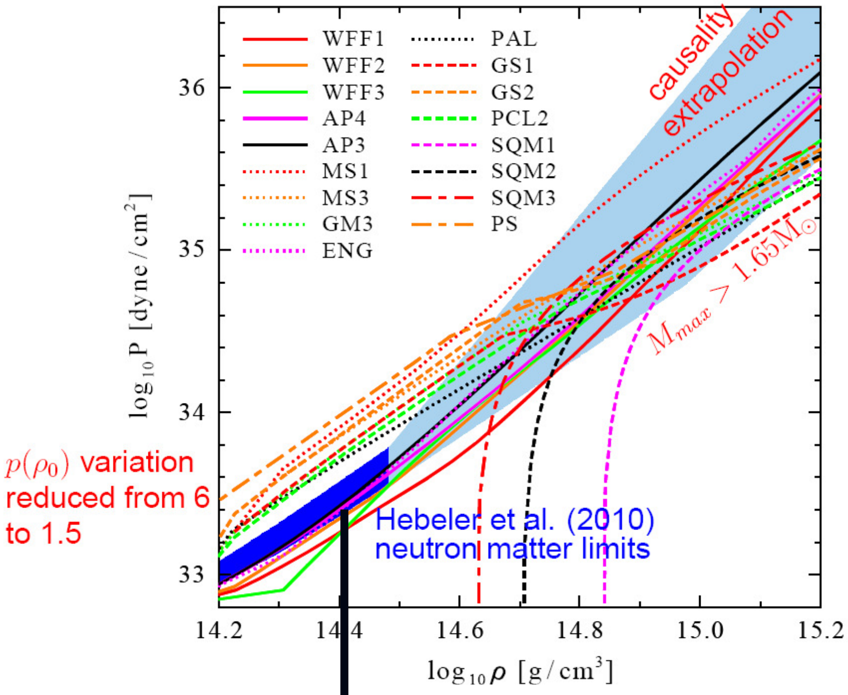
$R \propto p_*^{1/2} \epsilon_*^{-1} M^0$

$(1 < \epsilon_*/\epsilon_0 < 2)$

Wide variation:

$1.2 < \frac{p(\epsilon_0)}{\mathrm{MeV\ fm^{-3}}} < 7$

(Lattimer & Prakash 2001)

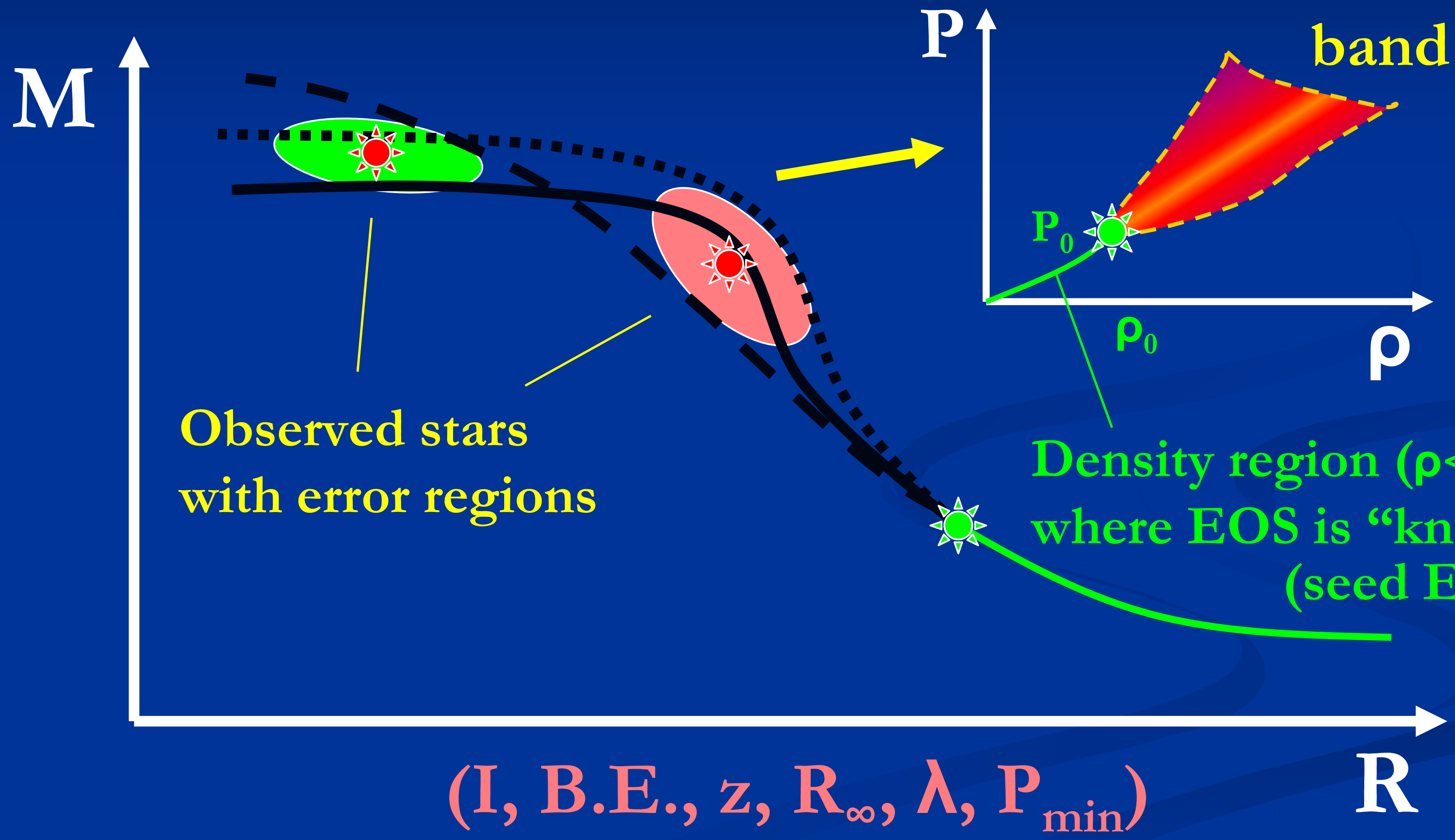


Note: Even at nuclear density, predicted pressures vary up to a factor of 5 - 6 !

Experiment with Nuclei

Nuclear Symmetry Energy

How several observed individual stars determine the EOS.



Smoothness of the EOS in NS matter

Phase transitions with more than one conserved charge

Glendenning Phys Rev D 46 (1992) 1274

Gibbs' rules

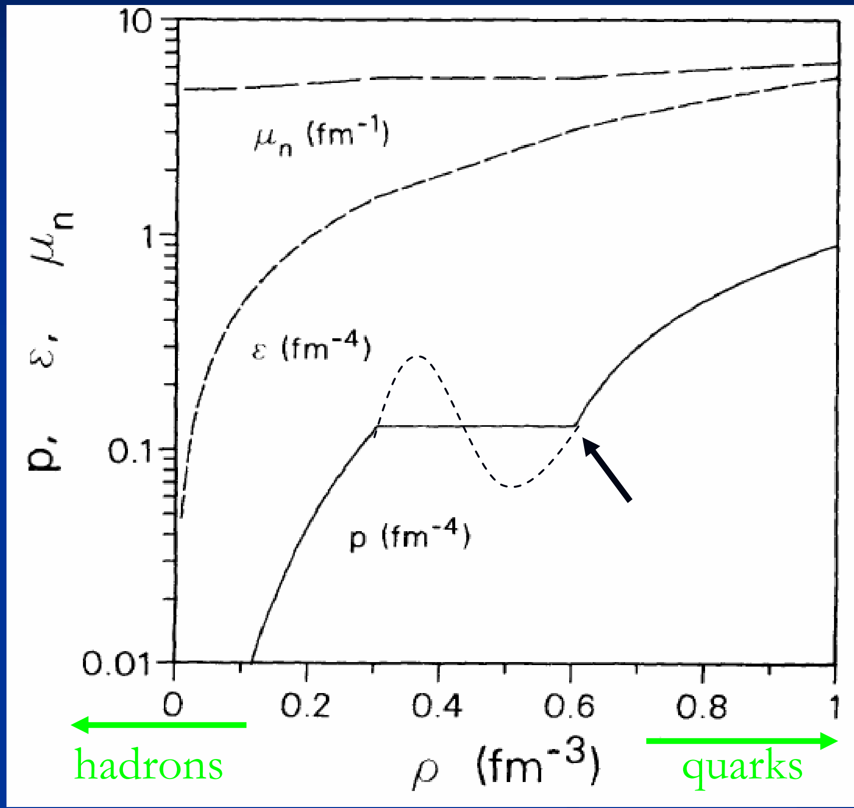
$P_1=P_2$ (mechanical equilibrium)

E.g. , $\mu_n=\mu_u+\mu_d$ (chemical eq.)

- System with one conserved charge
  - Maxwell construction
  - NS profile $\rho(r)$ has discontinuity
- Charge and baryon number conservation
  - Chemical beta equilibrium:

    E.g. : d(*or* s) ↔ u + e^- + $\overline{\nu_e}$
  - Global charge neutrality
  - NS profile $\rho(r)$ is continuous
  - $dP/d\rho$ discontinuities

In NS matter we can have desired EOS's by constructing its derivative



"Pressure, energy density, and chemical potentials as a function of [redacted] baryon density when there is [redacted] one conserved charge."
(Glendenning 1992)

Scheme for generating EOS's

0. Use low density "known" EOS to set starting point
1. Recast the hypothetical EOS as speed of sound squared

   $c_s^2(h)=dP/d\rho$ vs h (where $dh=dp/(p+\rho)$) in the unknown region
2. Since $c_s^2(h)$ is chosen to be piecewise, at each next step in h we can generate a linear piece of curve with deviation in its slope from the previous step: "tree"

$$\alpha_{\text{next slope}} = \alpha_{\text{previous slope}} \,(1+\delta\, j/N_\alpha)$$

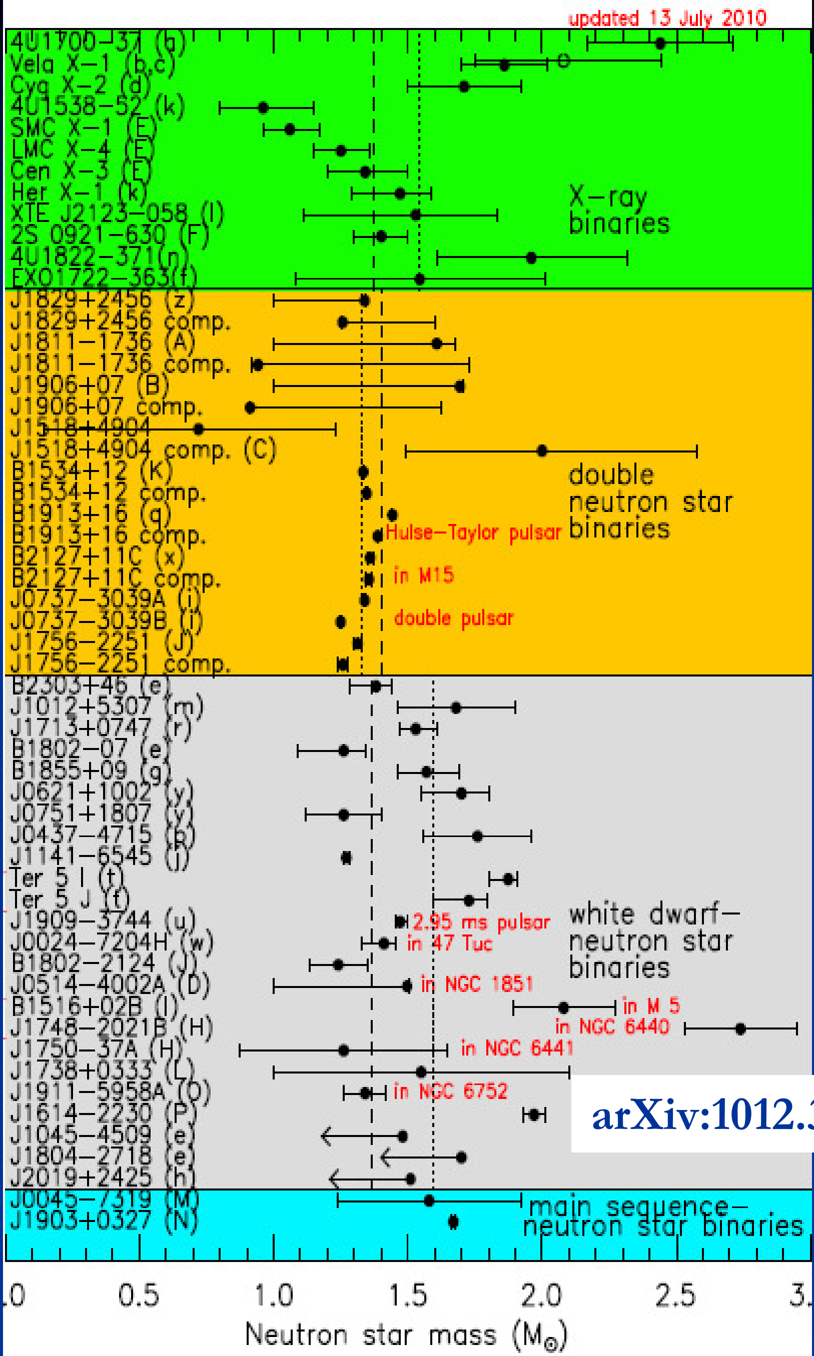
   where

   δ – relative deviation

   $j = -N_\alpha, N_\alpha - 1, \ldots, 0, \ldots, N_\alpha$ – branch index

   $2N_\alpha+1$ – branching [movie]
3. For every piecewise $c_s^2(h)$, calculate corresponding $P(\rho)$
4. **Now I am checking adoptive grid algorithm: $c_s^2(\ln[h])$**
5. **Also into using faster Monte Carlo scheme**

J.M. Lattimer, NIC XI, Heidelberg, 20/07/10



Masses are well measured in binary systems but radii are not precisely known.

The other systems, in which the radiation radius R_∞ is measured, do not have reliable masses.

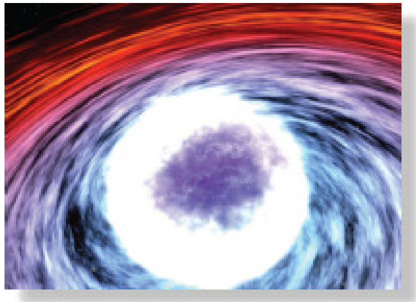
Object	R_∞ (km)	D (kpc)	$kT_{eff,\infty}$ (eV)	Ref.
Omega Cen (Chandra)	13.5 ± 2.1	$5.36 \pm 6\%$	66^{+4}_{-5}	Rutledge et al. ('02)
Omega Cen (XMM)	13.6 ± 0.3	$5.36 \pm 6\%$	67 ± 2	Gendre et al. ('02)
M13 (XMM)	12.6 ± 0.4	$7.80 \pm 2\%$	76 ± 3	Gendre et al. ('02)
47 Tuc X7 (Chandra)	$14.5^{+1.6}_{-1.4}$ ($1.4\ M_\odot$)	$5.13 \pm 4\%$		Rybicki et al. ('05)
M28 (Chandra)	$14.5^{+6.9}_{-3.8}$	$5.5 \pm 10\%$	90^{+30}_{-10}	Becker et al. ('03)
[illegible] 0748-676 (Chandra)	13.8 ± 1.8 ($2.10 \pm 0.28\ M_\odot$)	9.2 ± 1.0		Ozel ('06)

$$R_\infty = R/\sqrt{1-(2GM/c^2R)}\ ; \qquad F = 4\pi T_{eff}^4 (R_\infty/D)^2$$

Atmospheric (sometimes magnetic) modeling required.

arXiv:1012.3208v1

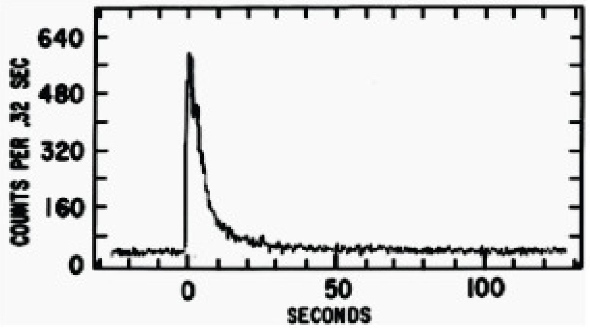
Thermonuclear X-ray Bursts



Accretion on neutron star

Unstable nuclear burning of accreted matter on the neutron star surface causes type I (thermonuclear) X-ray bursts.

Burst light curve



Rise time ≈ 0.5 - 5 seconds
Decay time ≈ 10 - 100 seconds
Recurrence time ≈ hours to day
Energy release in 10 seconds ≈ 10^{39} ergs

↓

Sun takes more than a week to release this energy.

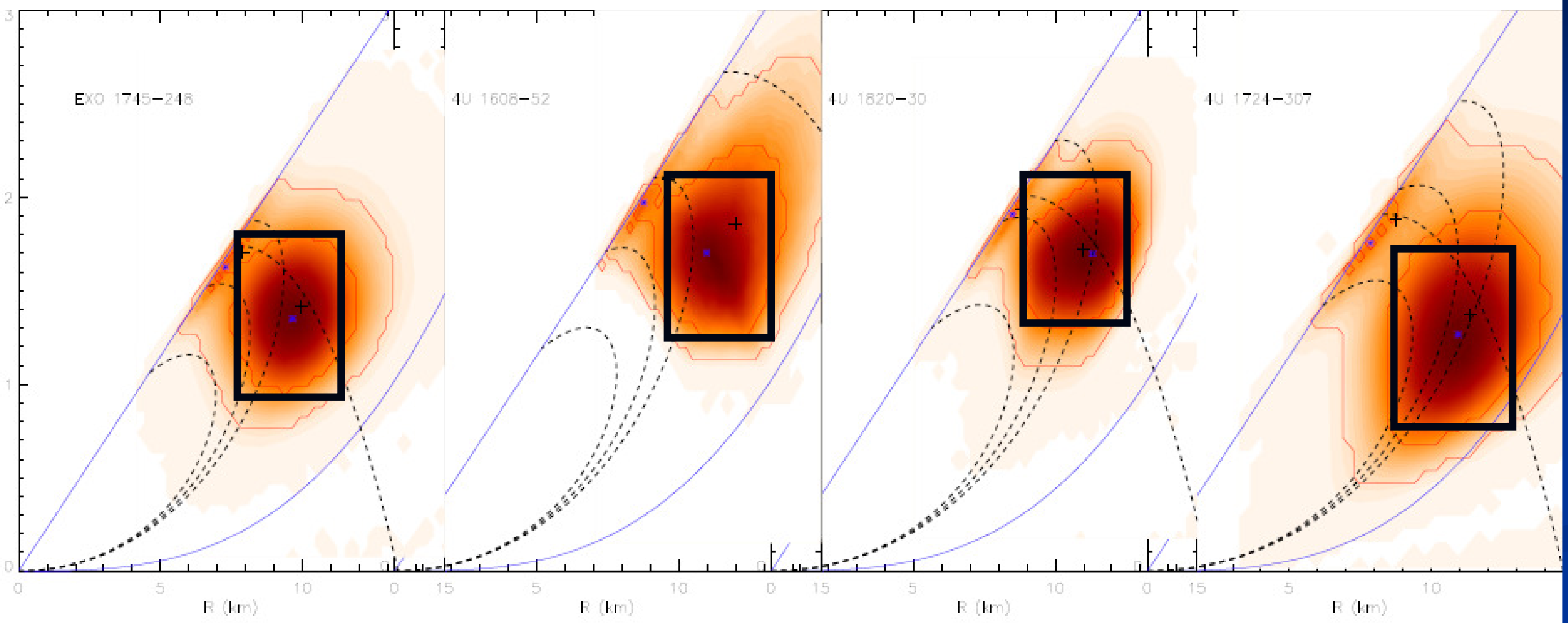
Why is *unstable* burning needed?

Energy release:
Gravitational ≈ 200 MeV / nucleon
Nuclear ≈ 7 MeV / nucleon

Accumulation of accreted matter for hours → Unstable nuclear burning for seconds ⇒ Thermonuclear X-ray burst.

Sudip Bhattacharyya
NASA's Goddard Space Flight Center

M & R from X-Ray Bursts

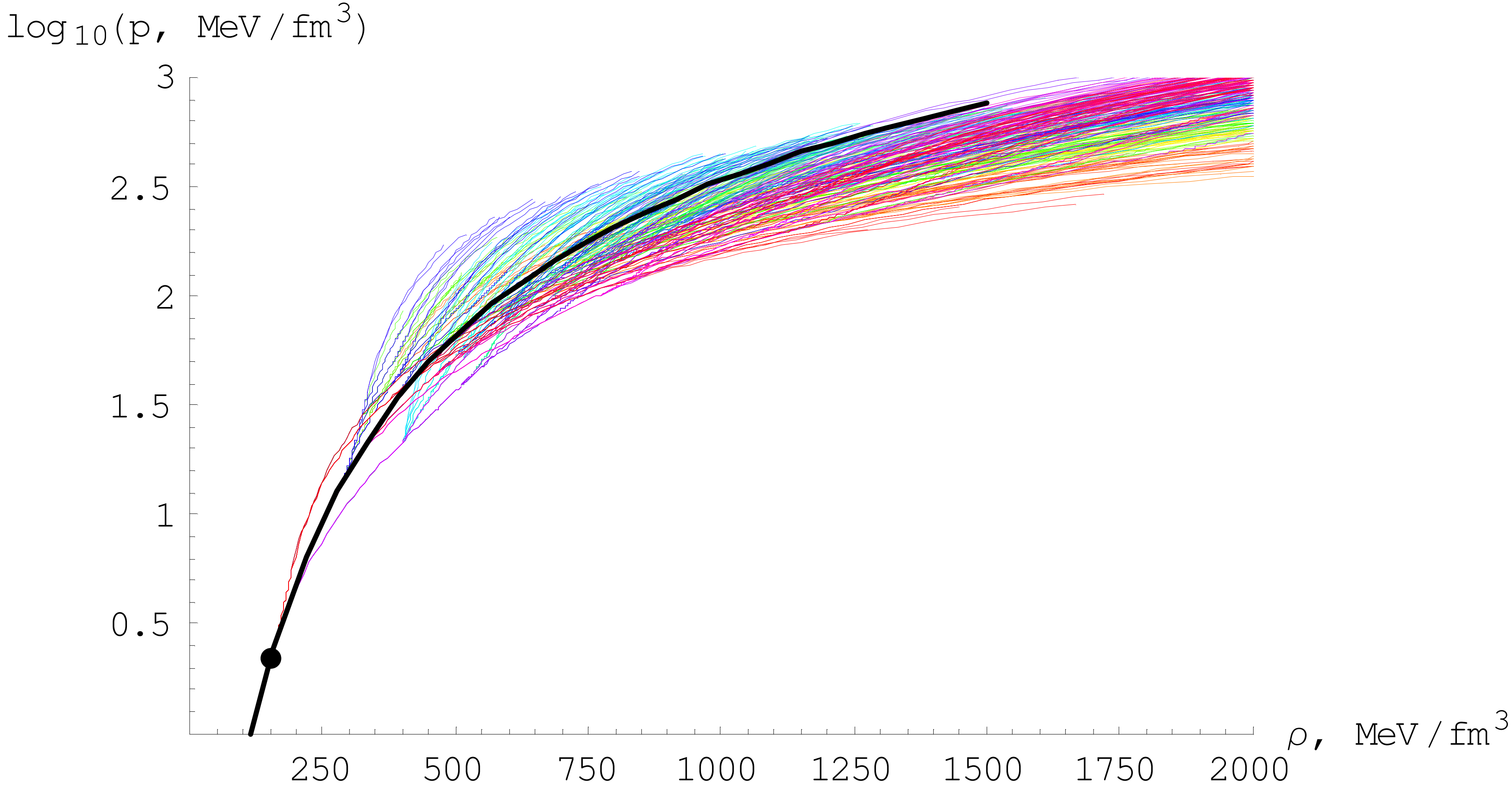


Sources of observational data:

4U 1745-248: Özel et al. (2009)
4U 1608-522: Güver et al. (2010)
4U 1820-30: Güver et al. (2010)
4U 1724-307: Suleimanov et al. (2010)

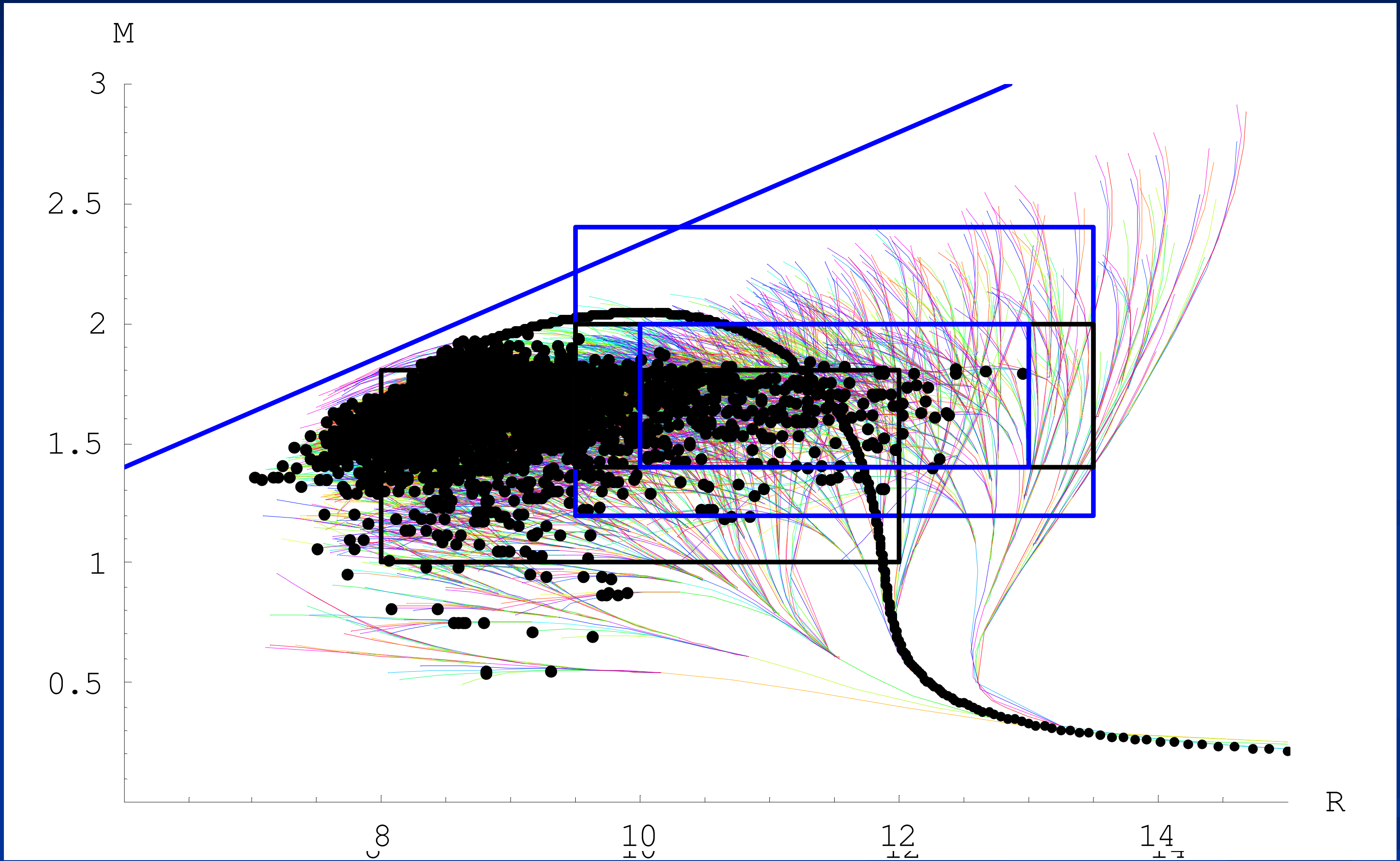
Jim Lattimer (2010)

Generating EOS with "seed" SLY4 ($n < n_0 = 0.16\ fm^{-3}$)

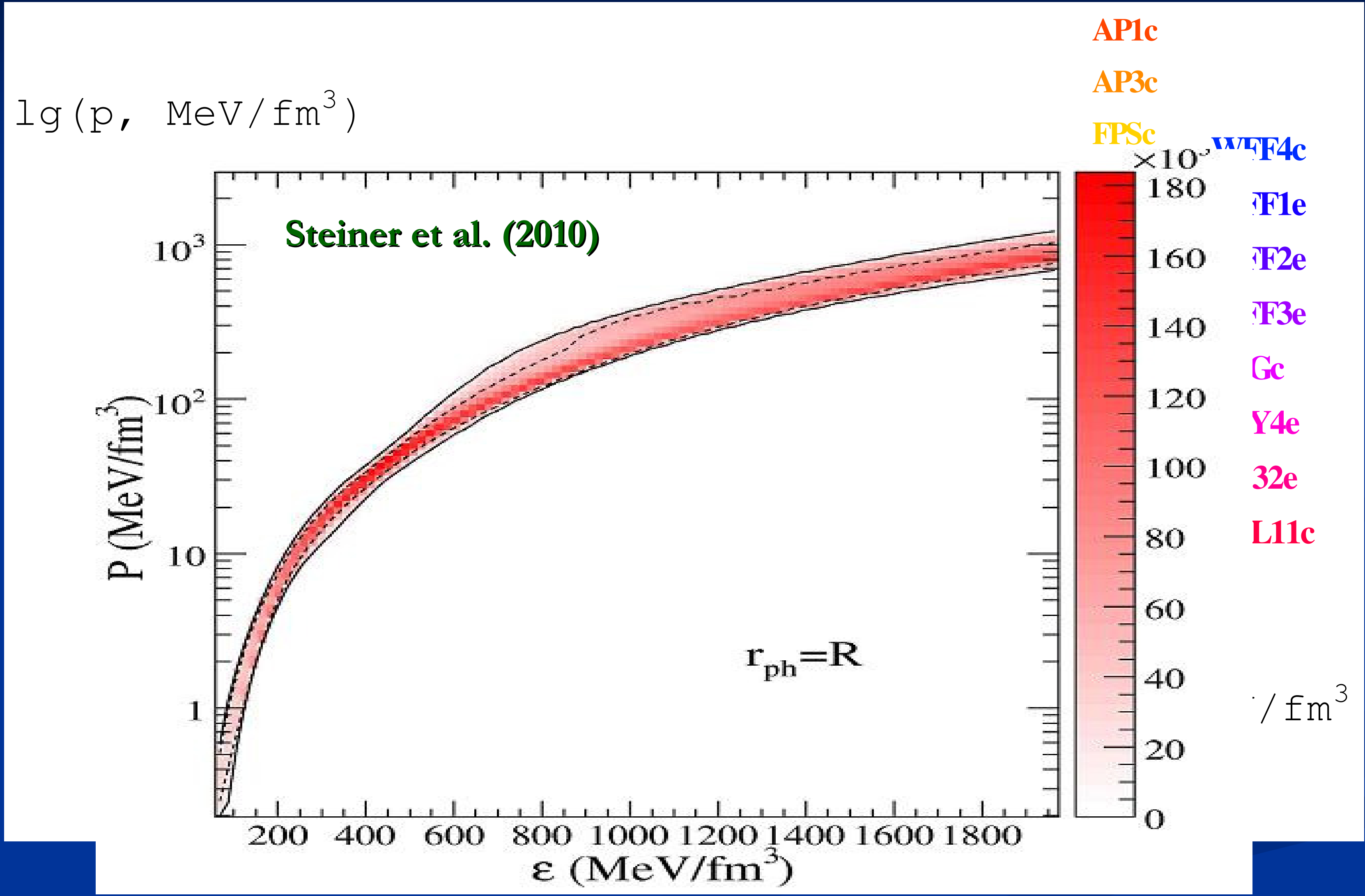


[movie]

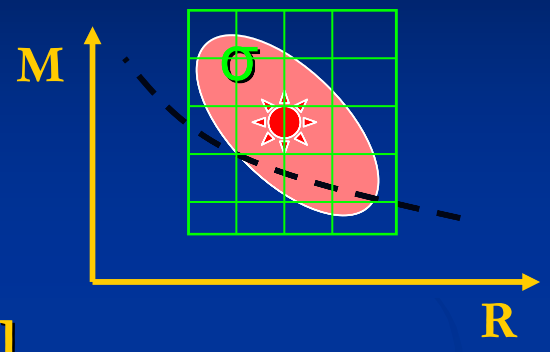
Generating EOS with "seed" SLY4 ($\rho < \rho_0$=0.16)



The band vs model EOSs and Andrew's work.

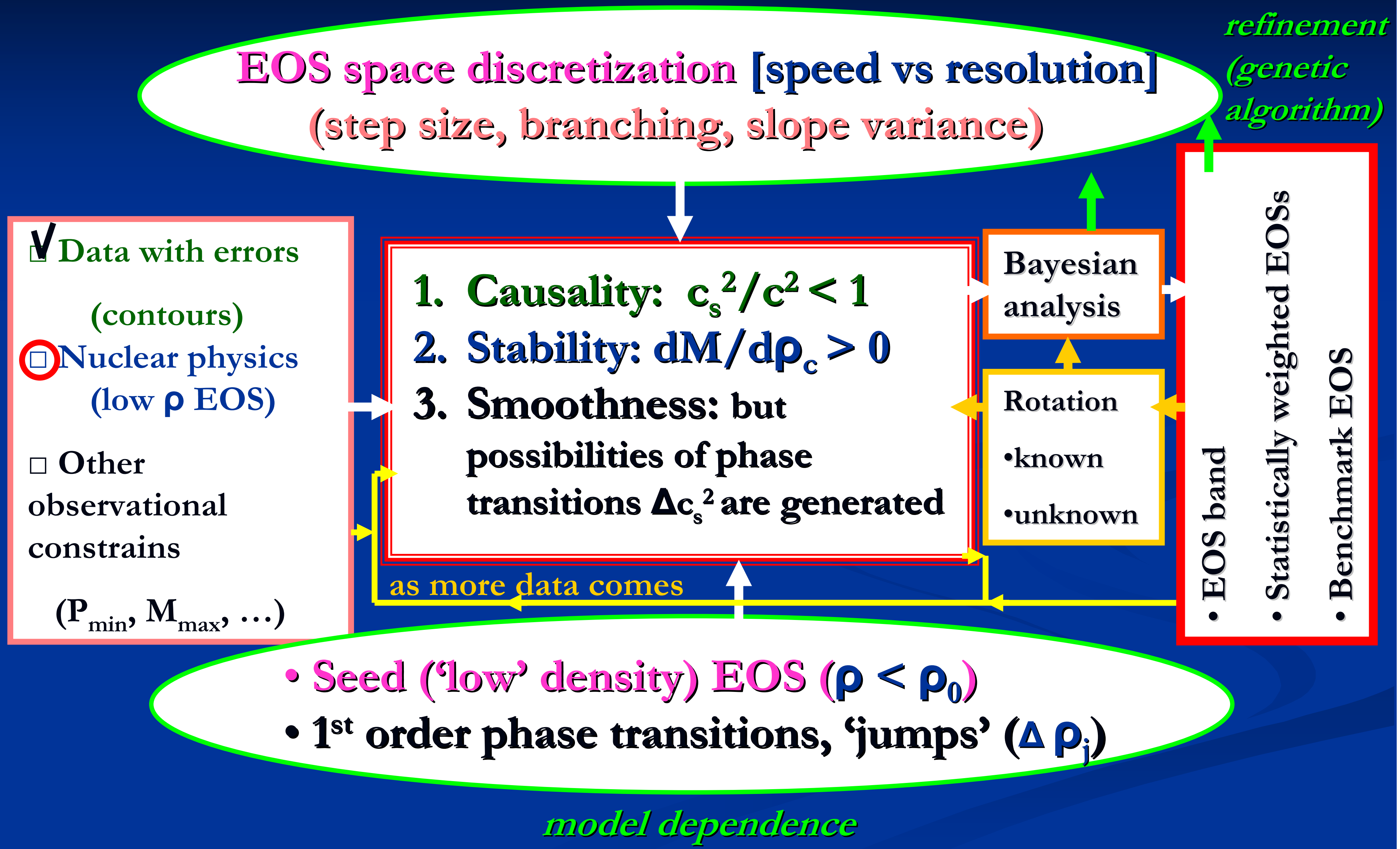


Sequential Bayesian analysis



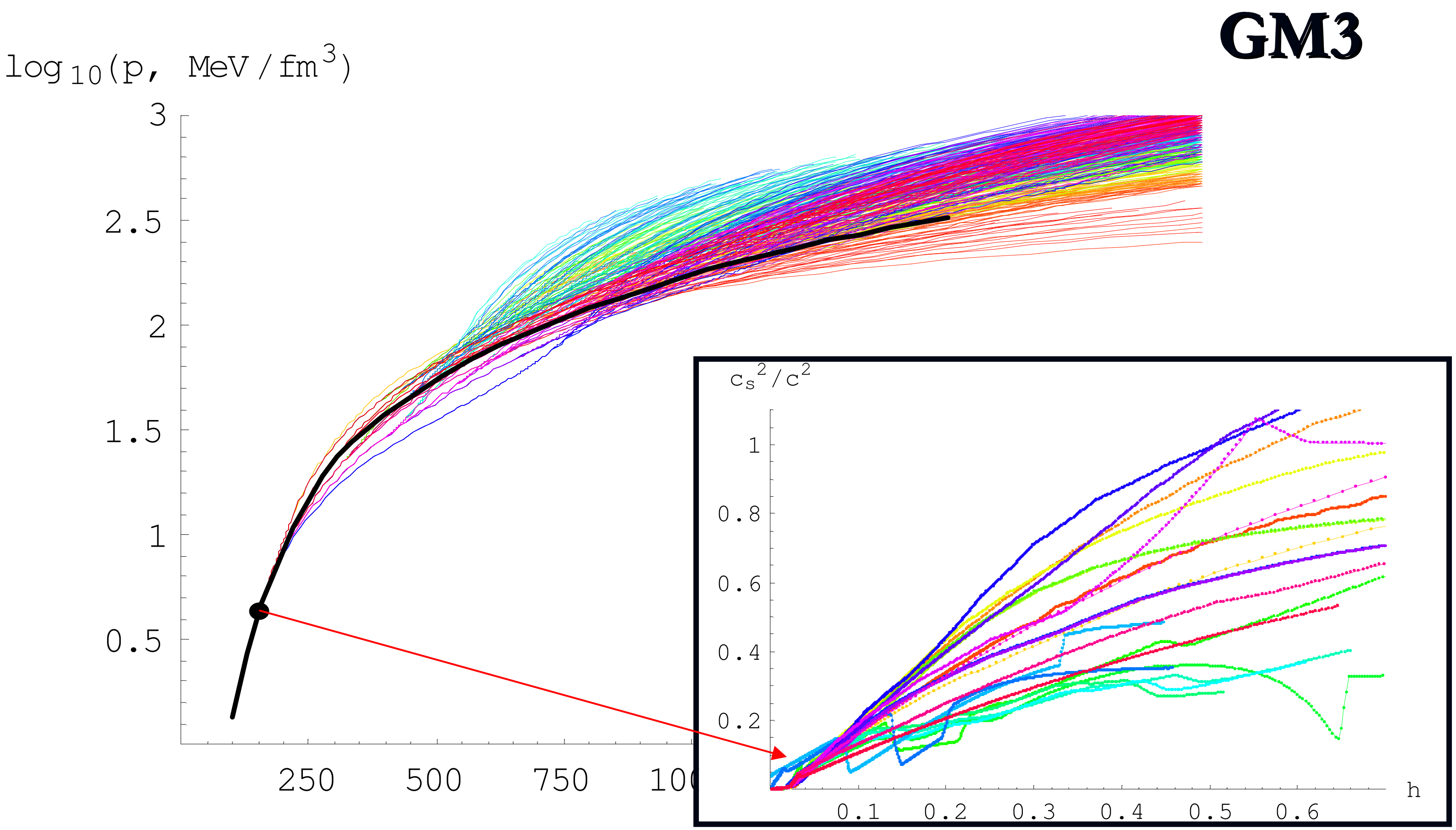
1. $P(H_i) = 1/(\text{total number of curves})$
2. *Likelihood:* $P(D|H_i) = ?$
   1. **integral of σ along the curve**
   2. **maximum[**crossed σ along the **curve]**
   3. division into smaller boxes and then proper curve counting
3. *Normalization:* $P(D) = \sum P(D|H_i)\, P(H_i)$
4. *Prior:* $P(H_i) \leftarrow P(H_i|D) = P(D|H_i)\, P(H_i)/P(D)$
5. If several (M & R)'s (e.g. data D) available, go to step 2.
6. Every curve acquires a weight $P(H_i)$

The TOOL

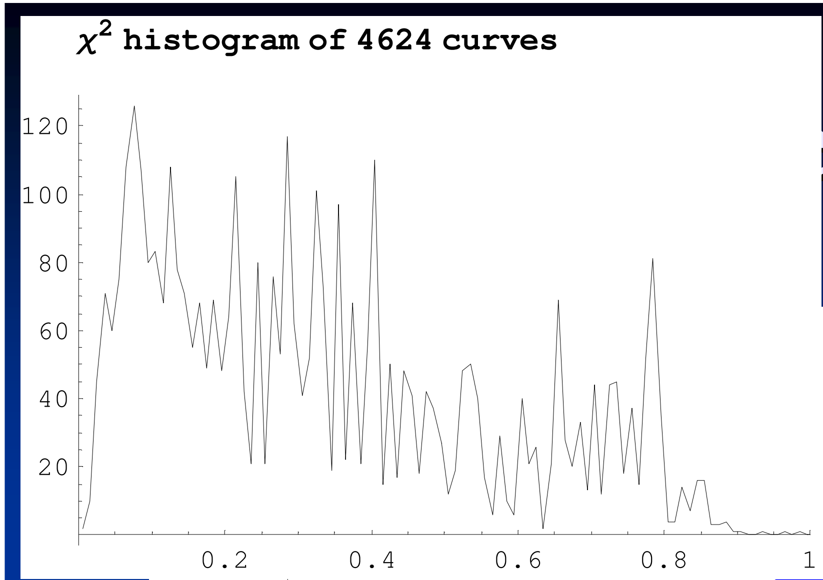

EOS space discretization [speed vs resolution]
(step size, branching, slope variance)
refinement (genetic algorithm)
√ Data with errors (contours)
□ Nuclear physics (low ρ EOS)
□ Other observational constrains
(P_{min}, M_{max}, …)
1. Causality: $c_s^2/c^2 < 1$
2. Stability: $dM/d\rho_c > 0$
3. Smoothness: but possibilities of phase transitions Δc_s^2 are generated
Bayesian analysis
Rotation
•known
•unknown
• EOS band
• Statistically weighted EOSs
• Benchmark EOS
as more data comes
• Seed ('low' density) EOS ($\rho < \rho_0$)
• 1st order phase transitions, 'jumps' ($\Delta \rho_j$)
model dependence

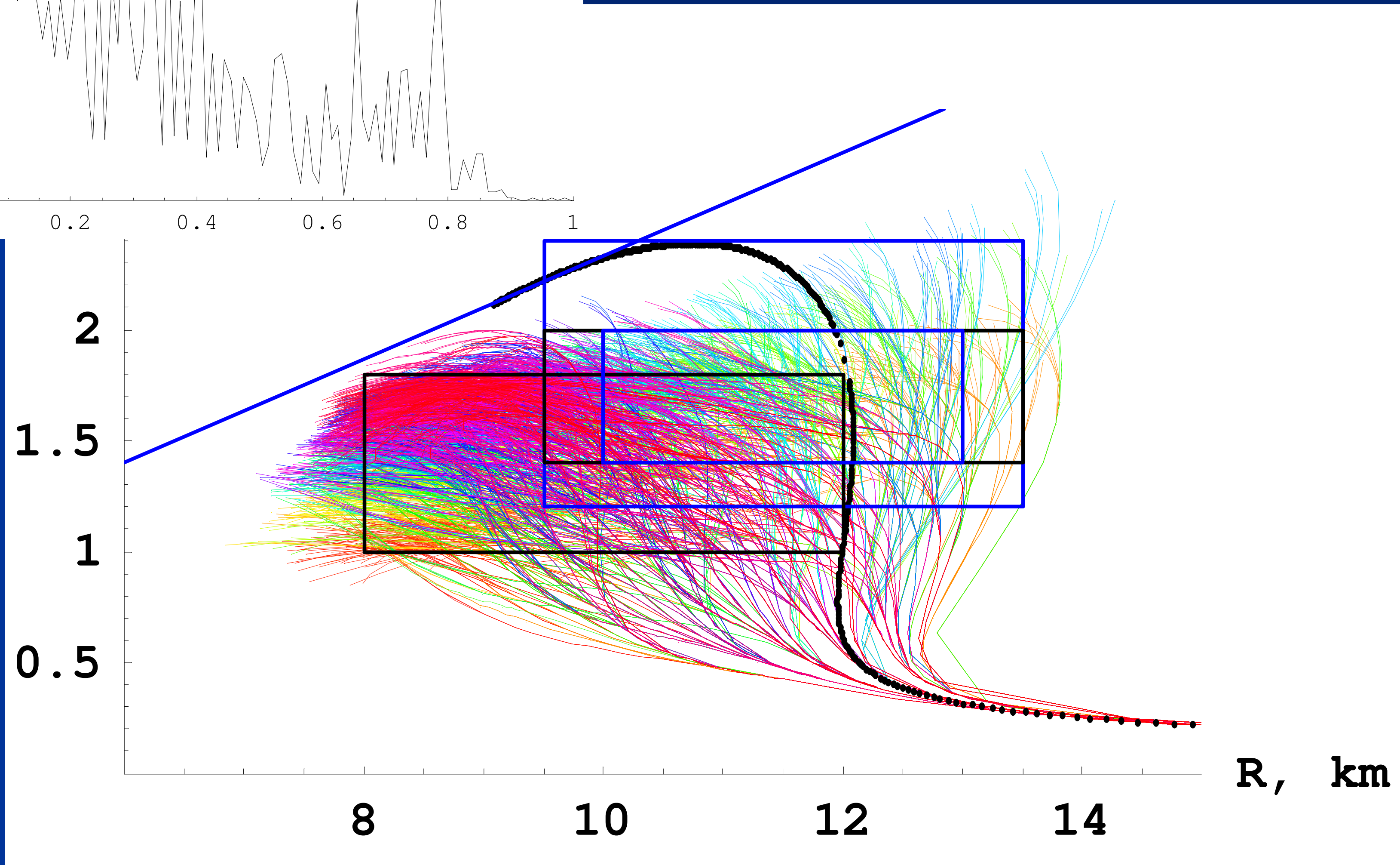
Dependence on seed EOS

$$\delta p(\rho) \approx \delta p(\rho_o) + \int_{\rho_o}^{\rho} c_s^2 d\rho \approx \begin{cases} \delta p(\rho_o) & \text{for } \rho \geq \rho_o, \\ \int_{\rho_o}^{\rho} c_s^2 d\rho \gg \delta p(\rho_o) & \text{for } \rho \gg \rho_o. \end{cases}$$

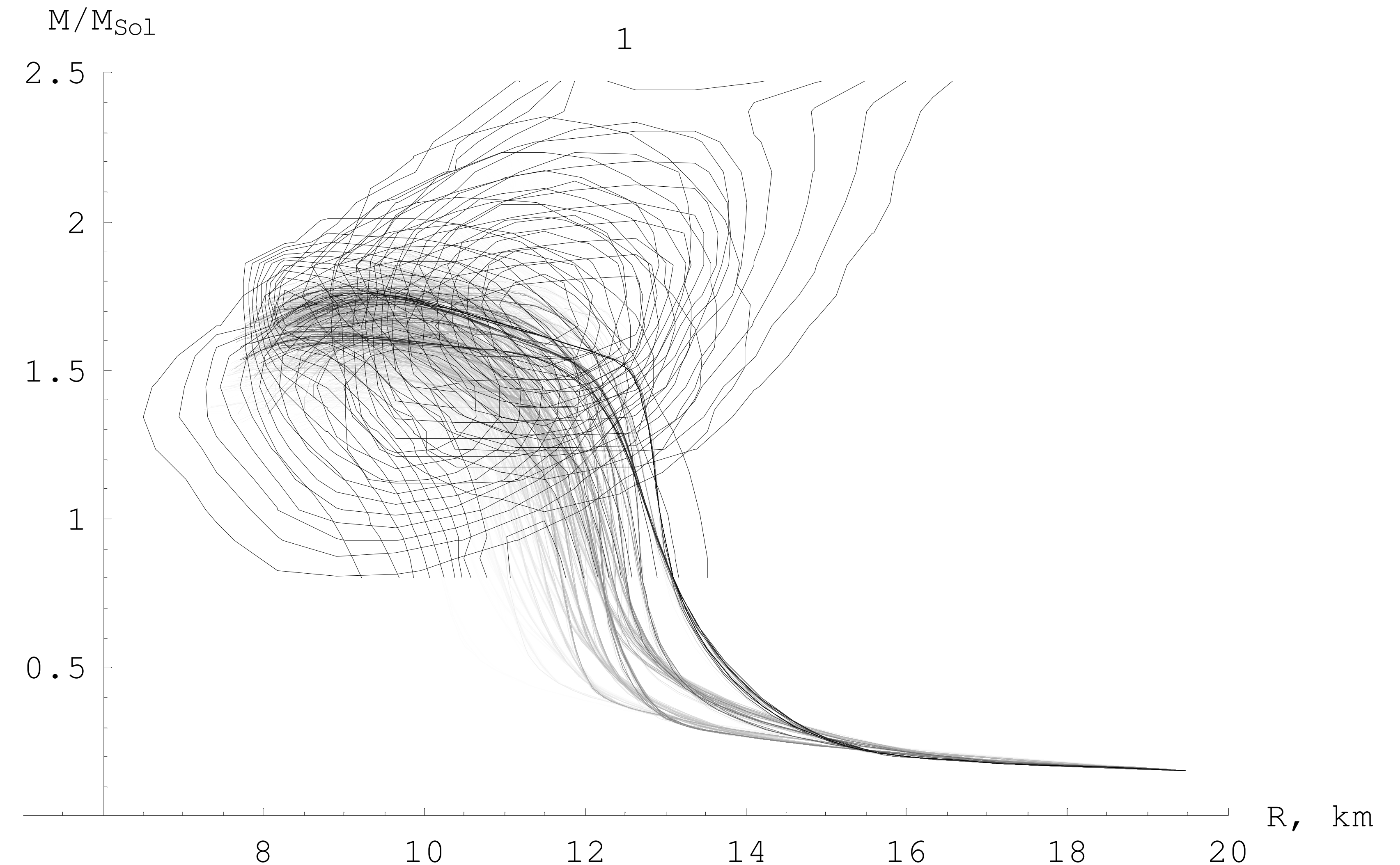


…e with boxes (AP3)

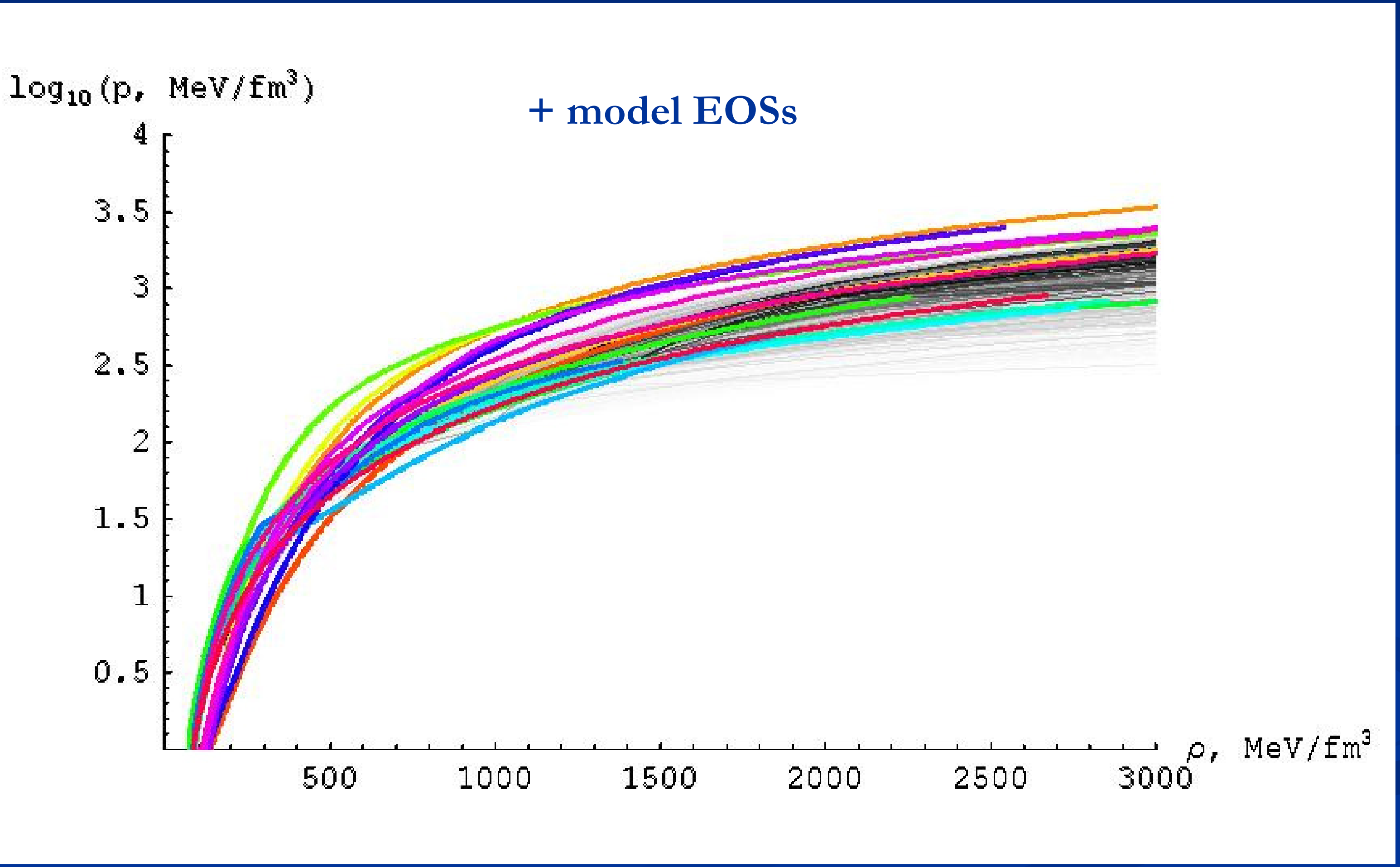




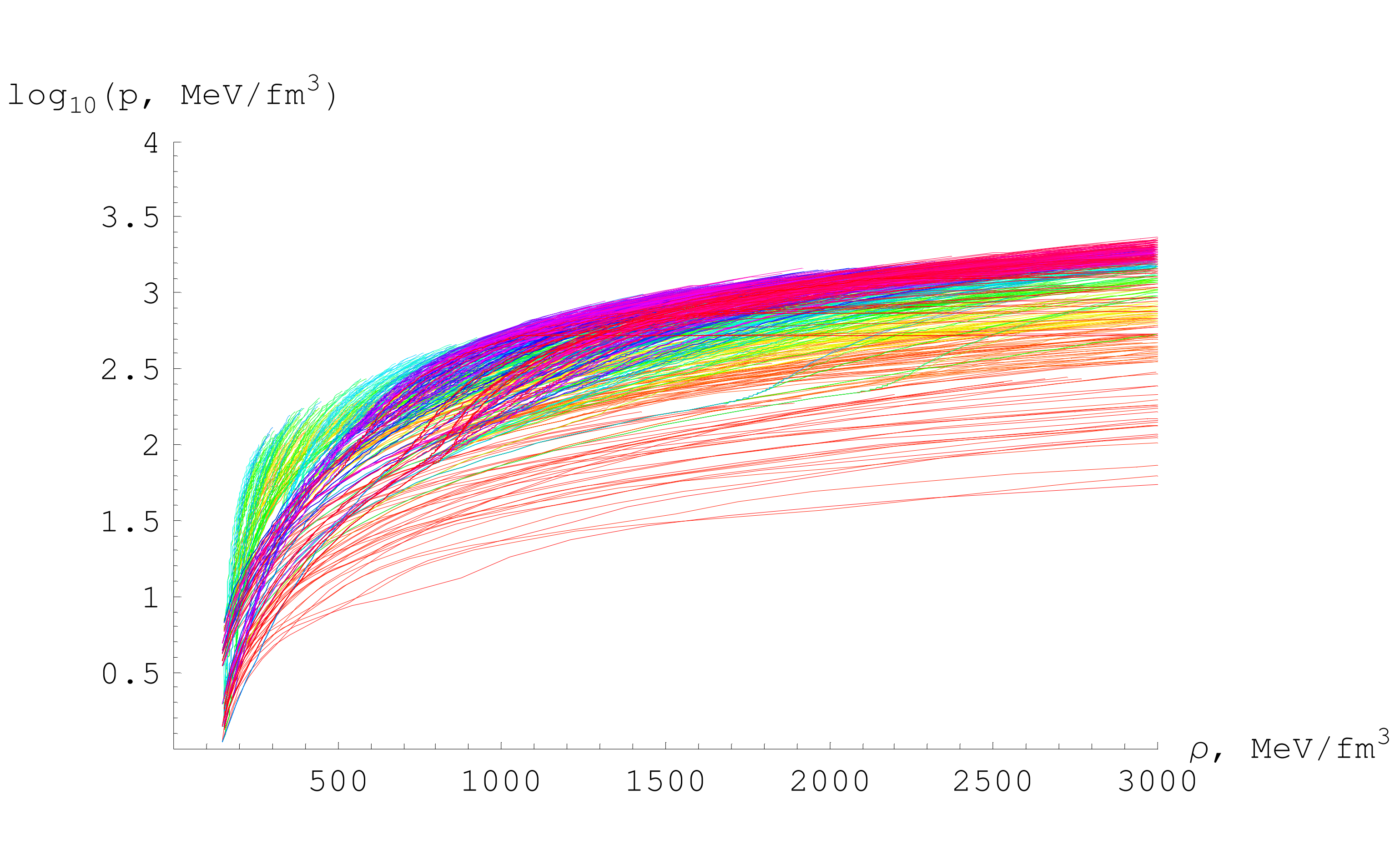
Bayesian analysis with contours (AP3)



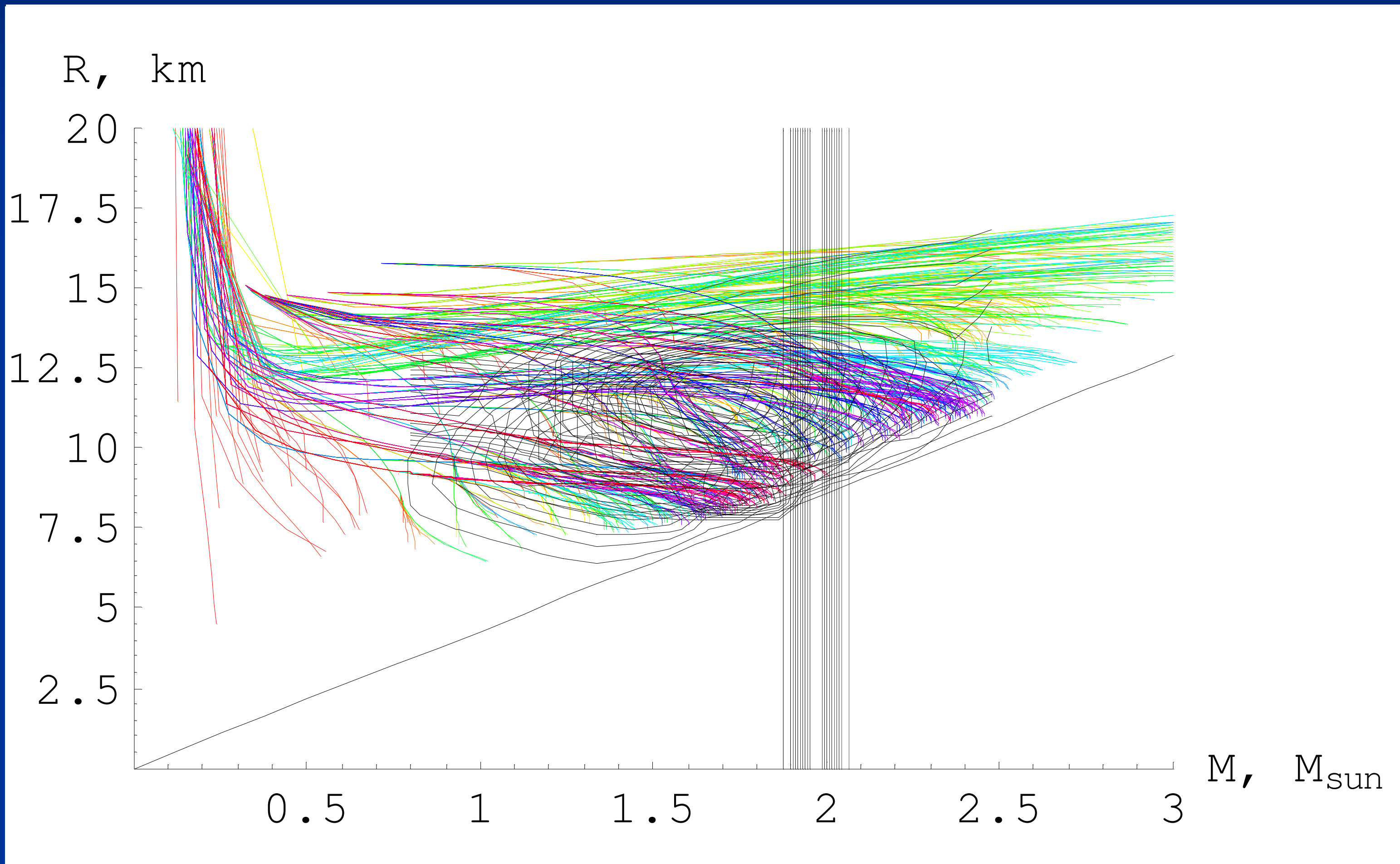
Resulting EOS band



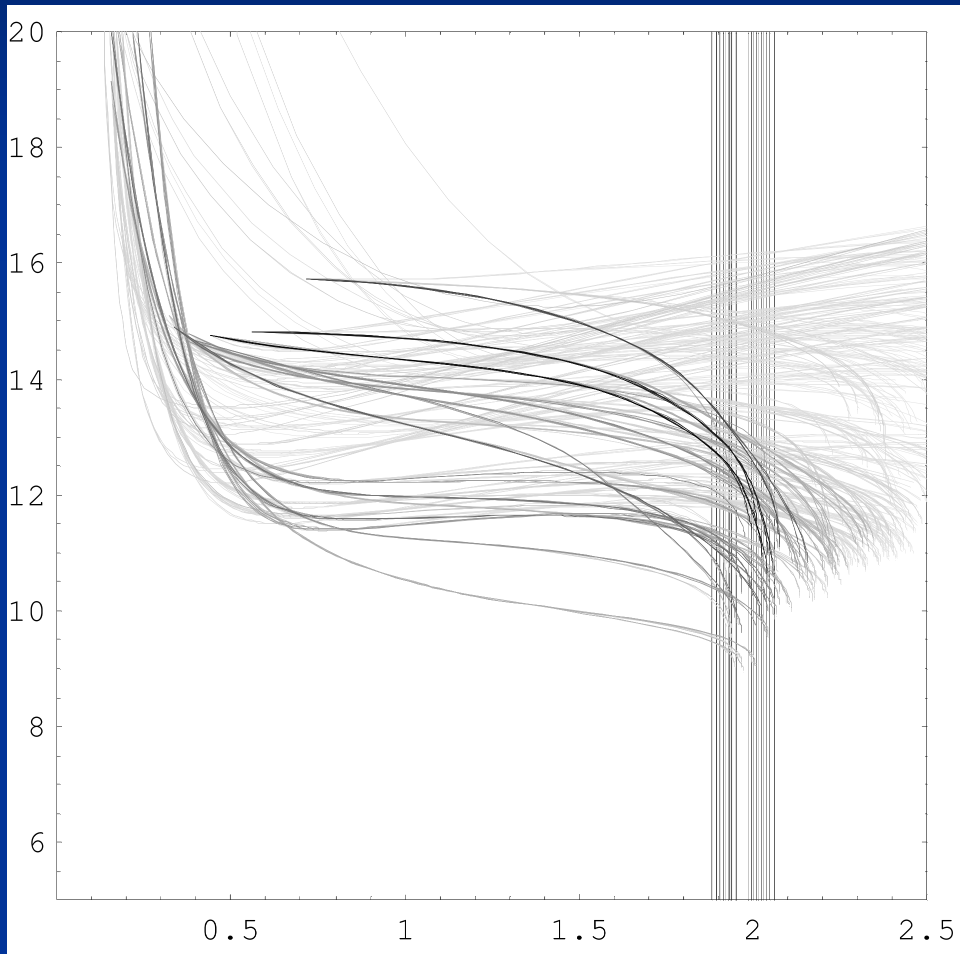
Many seed EOSs



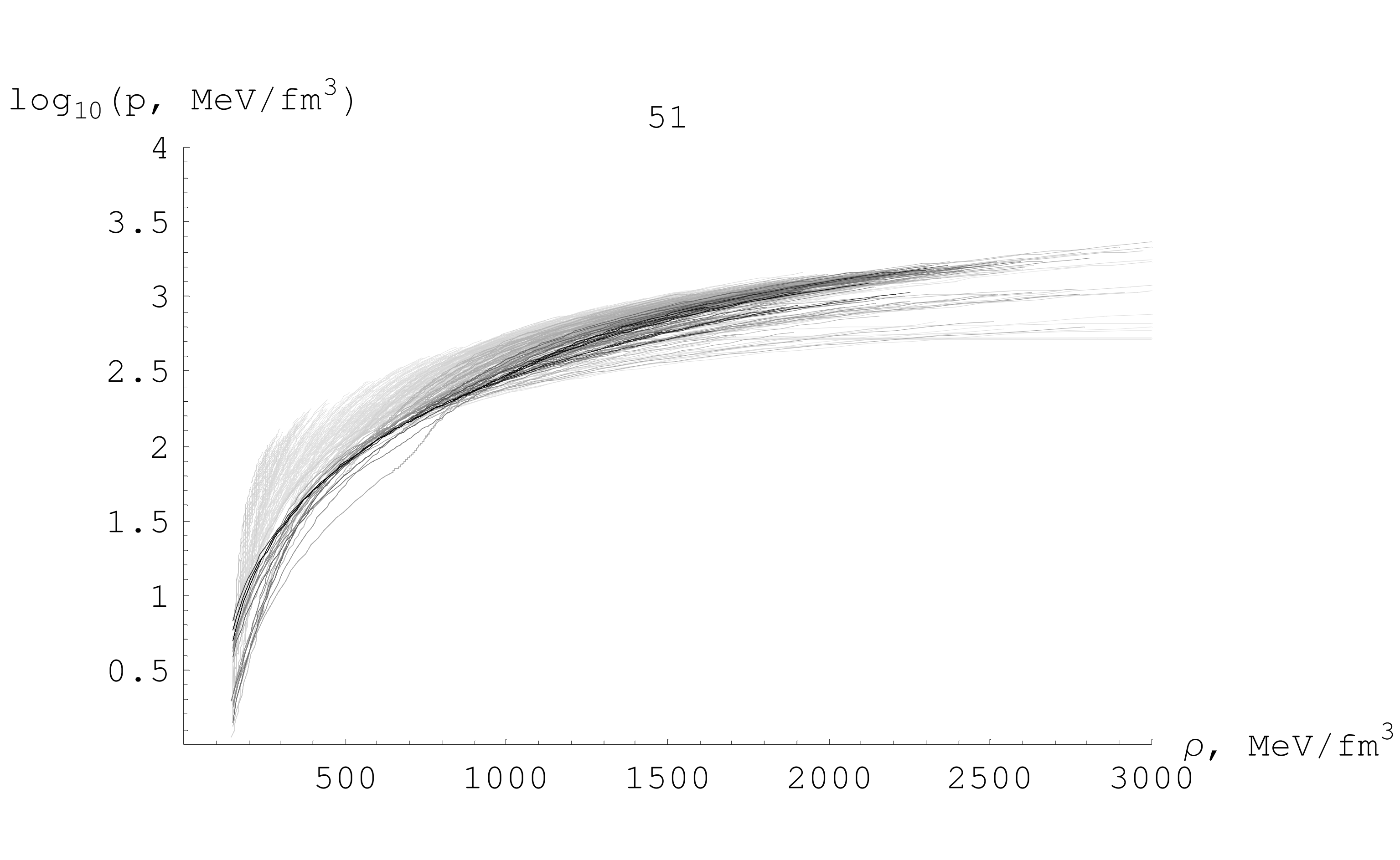
With many seed EOSs



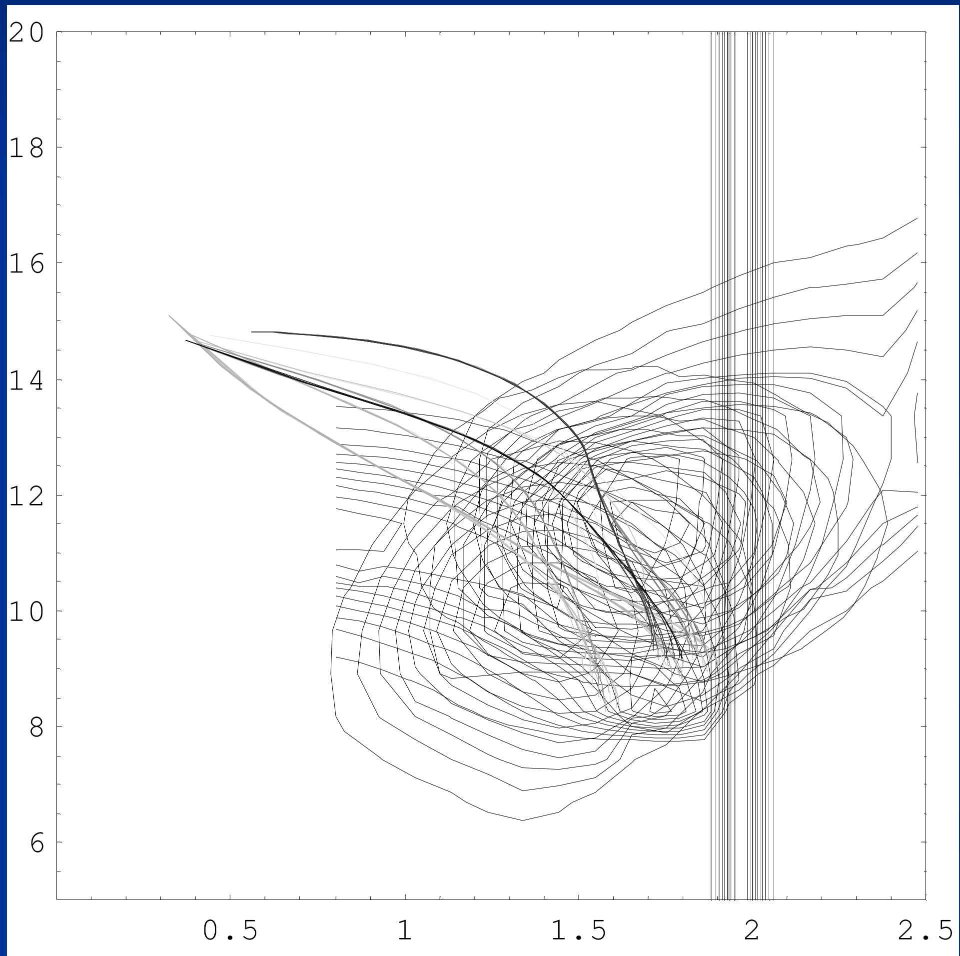
Only 2 solar mass star, R-M

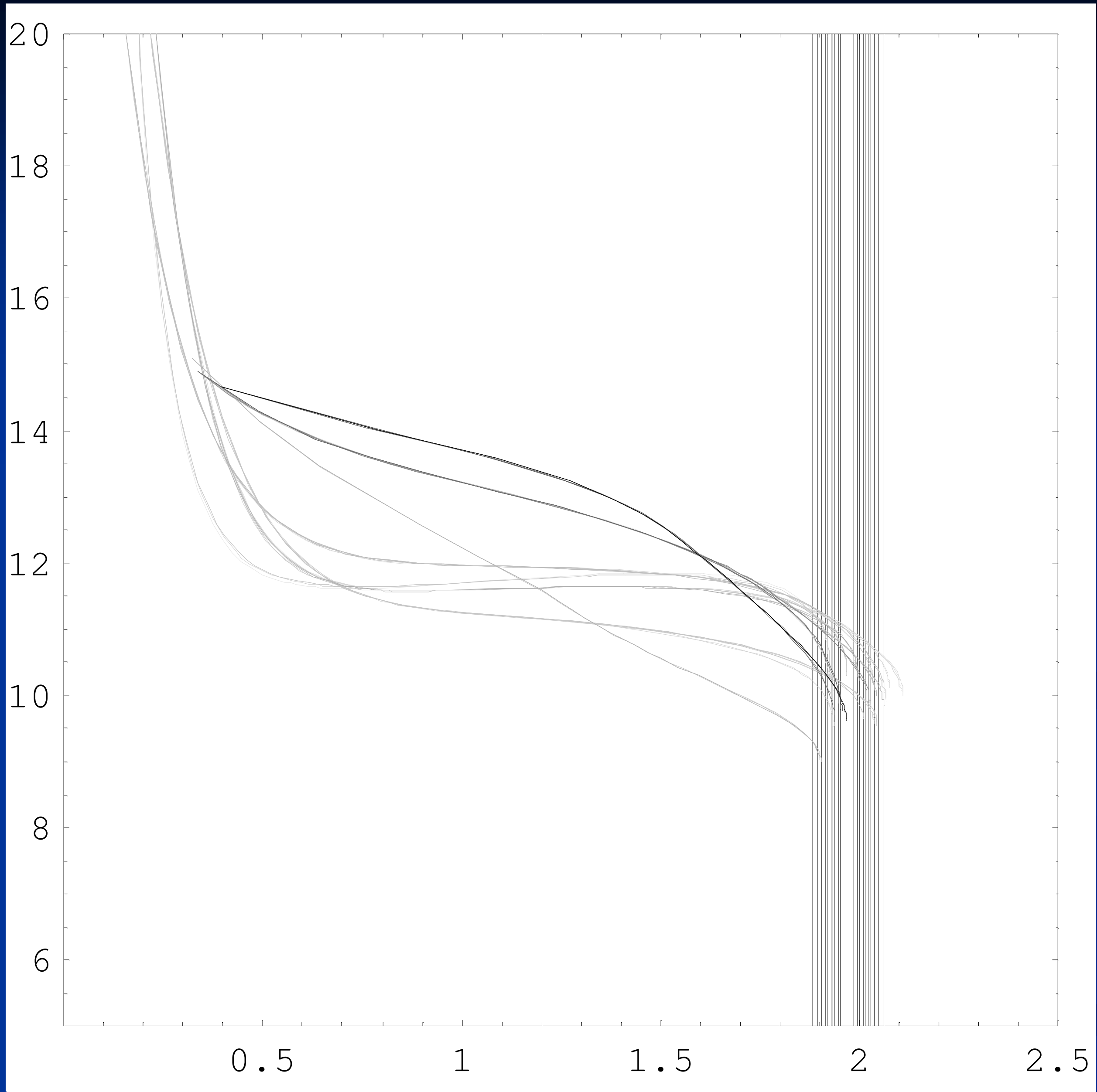


Only 2 solar mass star, EOSs

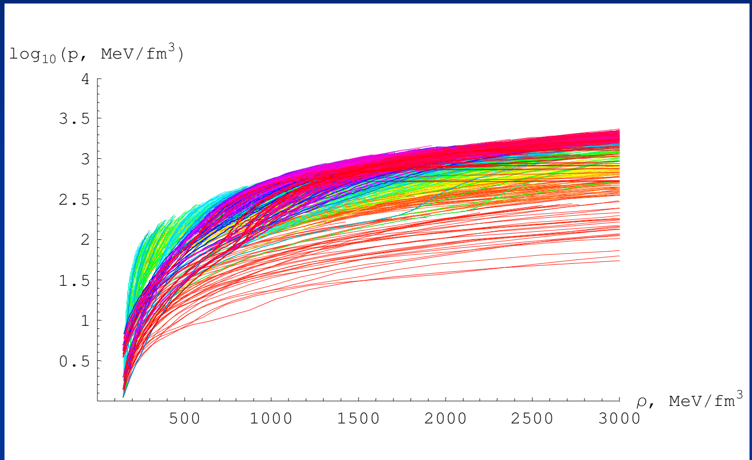


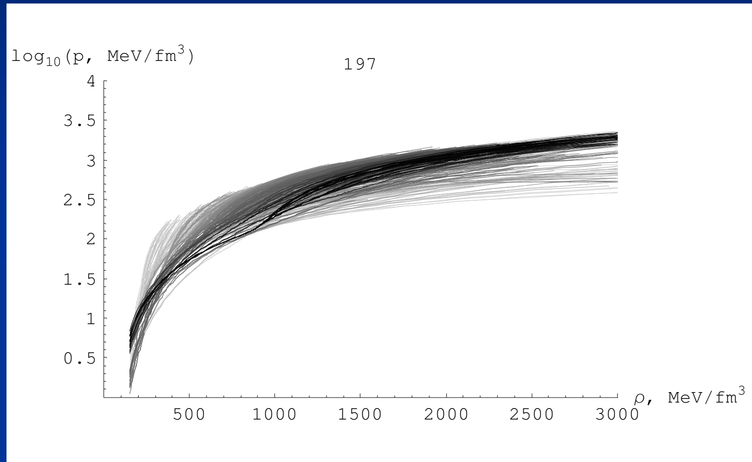
Together, 2 solar mass and 5 stars

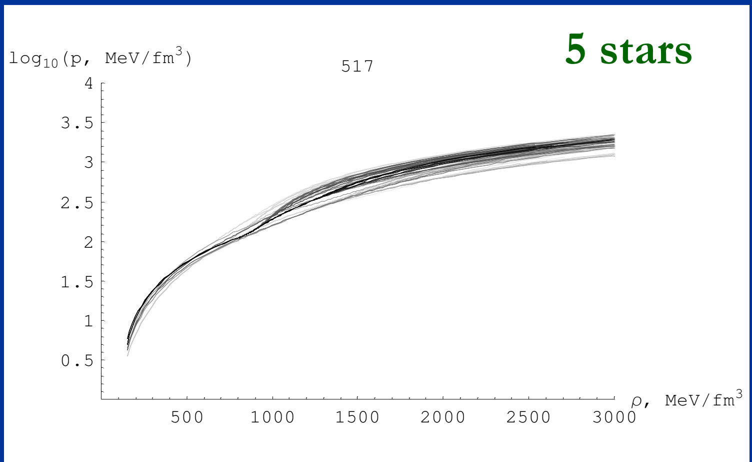


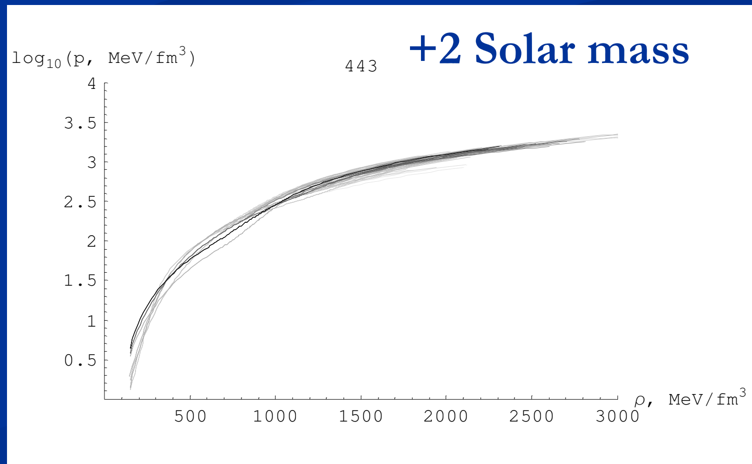

20
18
16
14
12
10
8
6
0.5
1
1.5
2
2.5

Together, 2 solar mass and 5 stars









Relative probability of seed EOSs

5 stars

0.241338	ms2c
0.231626	ms00c
0.148553	GM2c
0.140887	pal32e
0.0937868	GM1c
0.0908382	GM3c
0.052972	PAL11c
0	WFF4c
0	WFF3e
0	WFF2e
0	WFF1e
0	SLY4e
0	psc
0	PCLc
0	MPA1c
0	FPSc
0	ENGc
0	AP3c
0	AP1c

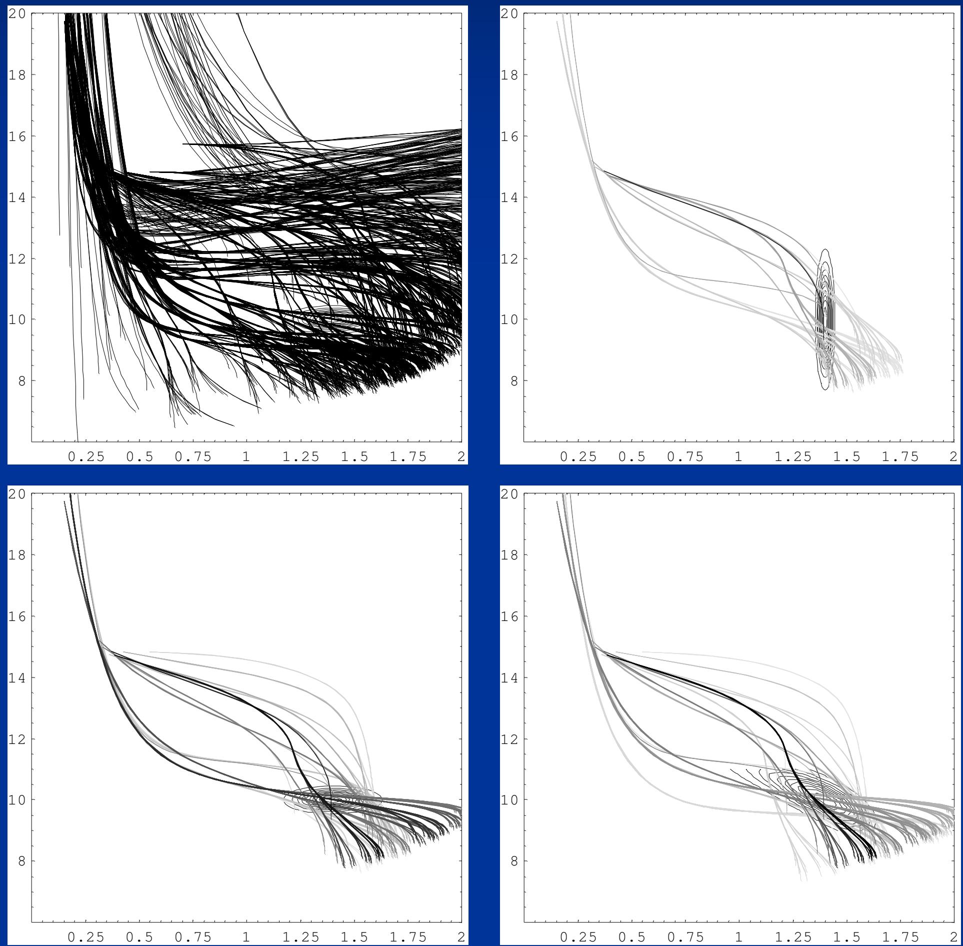
5 stars + 2 solar mass

0.338644	GM1c
0.205141	PCLc
0.110367	WFF3e
0.101353	PAL11c
0.0902202	AP3c
0.078366	FPSc
0.0759085	WFF1e
0	WFF4c
0	WFF2e
0	SLY4e
0	psc
0	pal32e
0	ms2c
0	ms00c
0	MPA1c
0	GM3c
0	GM2c
0	ENGc
0	AP1c

2 solar only

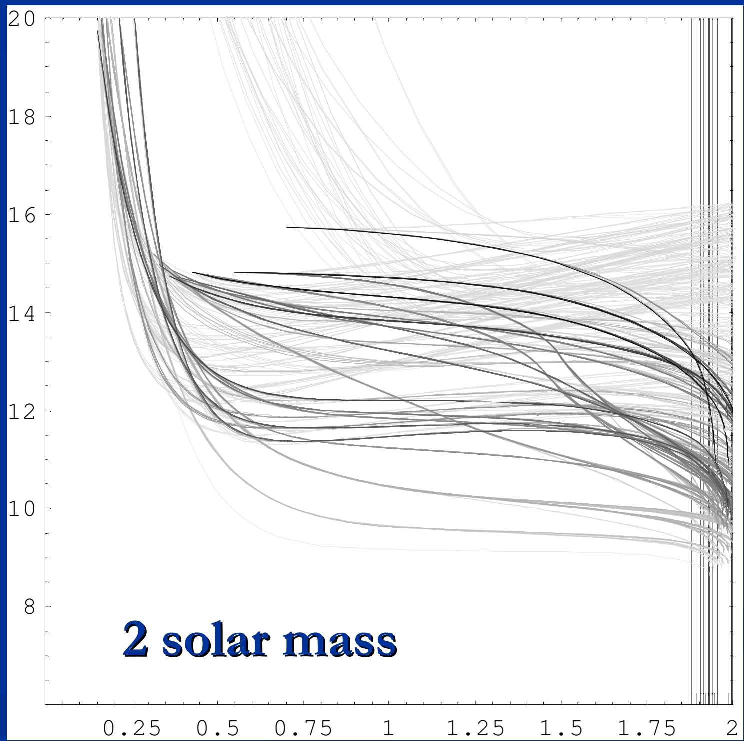
0.109664	ms2c
0.101387	ms00c
0.0808058	psc
0.067935	PCLc
0.0659151	WFF3e
0.0604734	AP3c
0.0554514	GM1c
0.0527658	ENGc
0.0509865	pal32e
0.0469431	FPSc
0.046008	WFF4c
0.0457392	GM2c
0.0450535	GM3c
0.0449179	MPA1c
0.0318951	SLY4e
0.028861	PAL11c
0.0280517	WFF1e
0.0188124	AP1c
0.0183353	WFF2e

Turning an ellipse

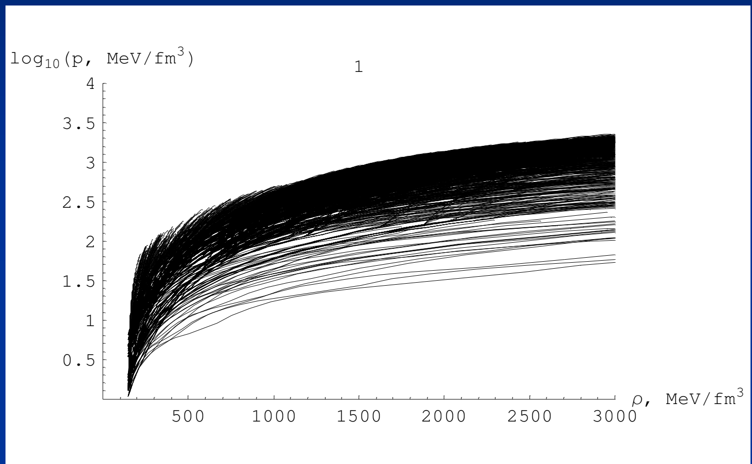


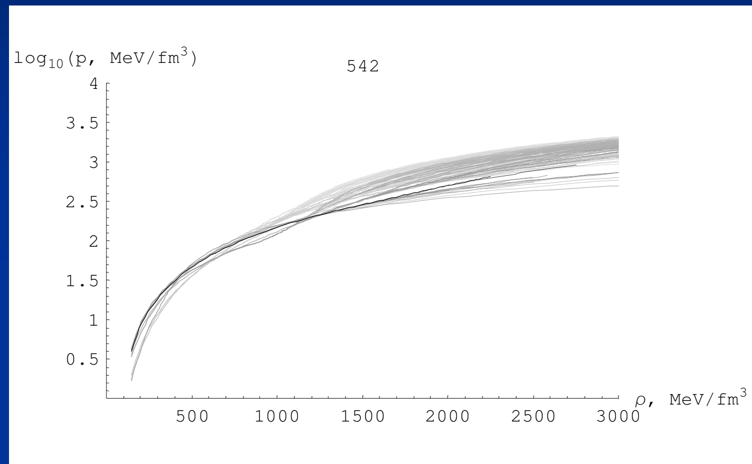
Typical star of 1.4 solar mass

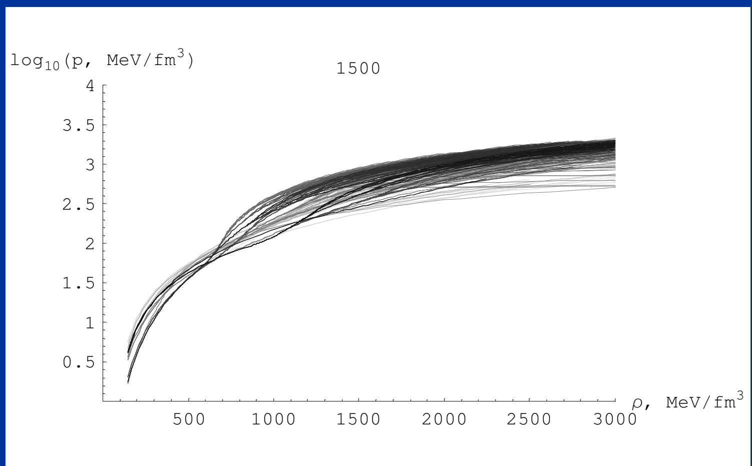
and 10 km radius.

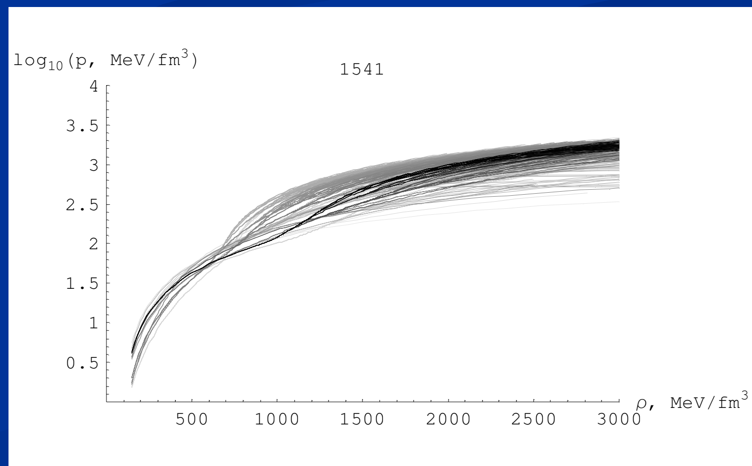


Turning an ellipse

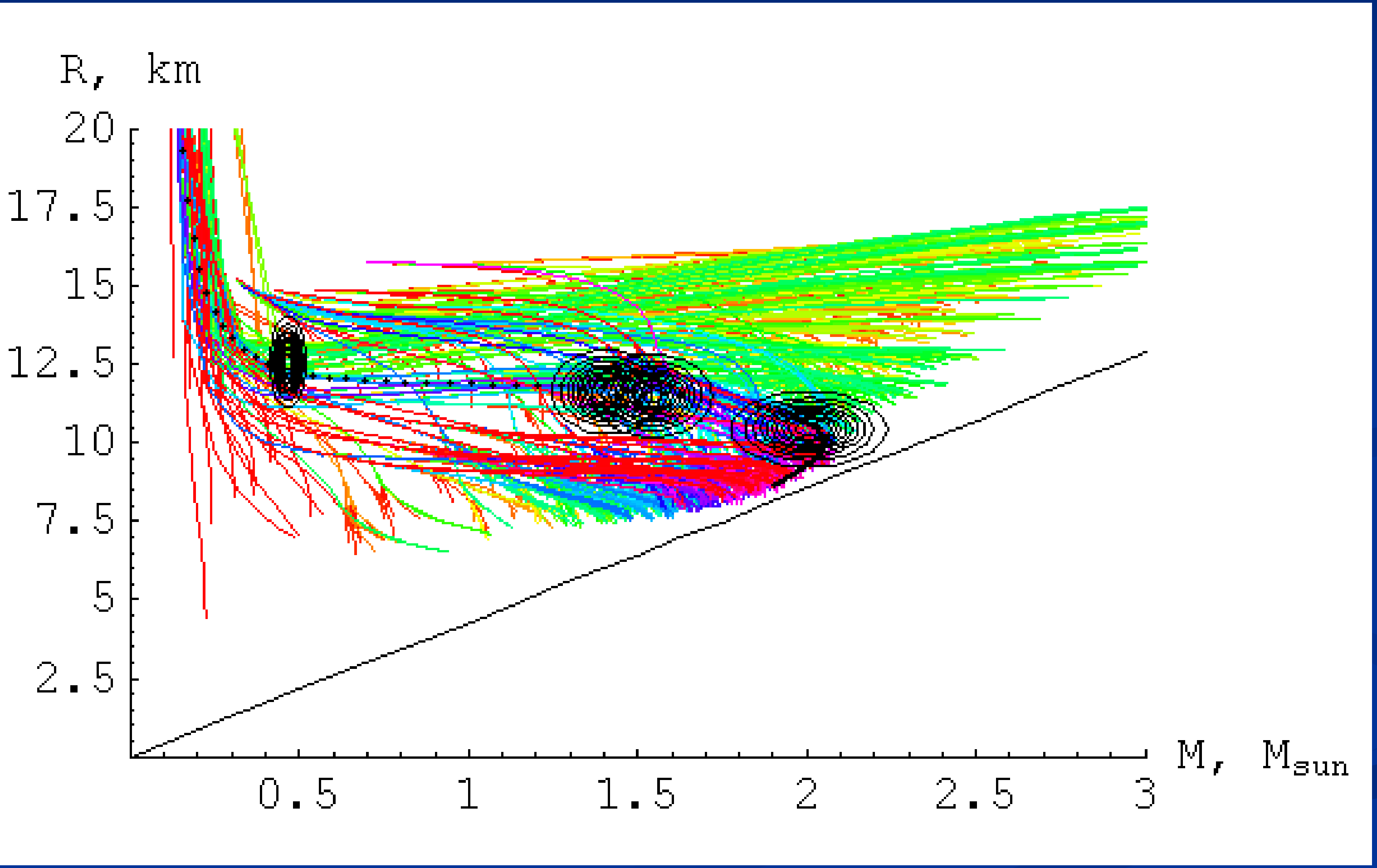


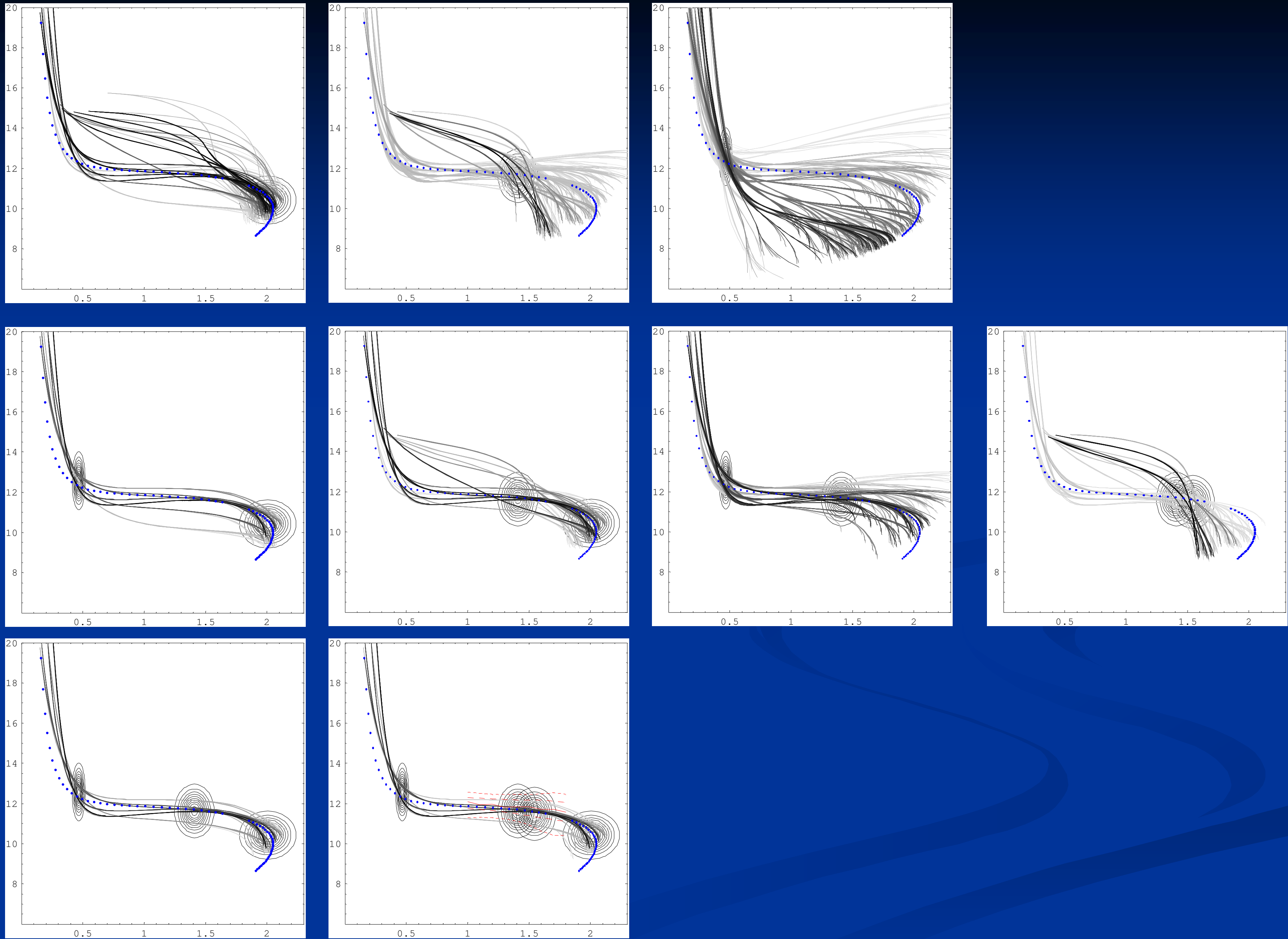


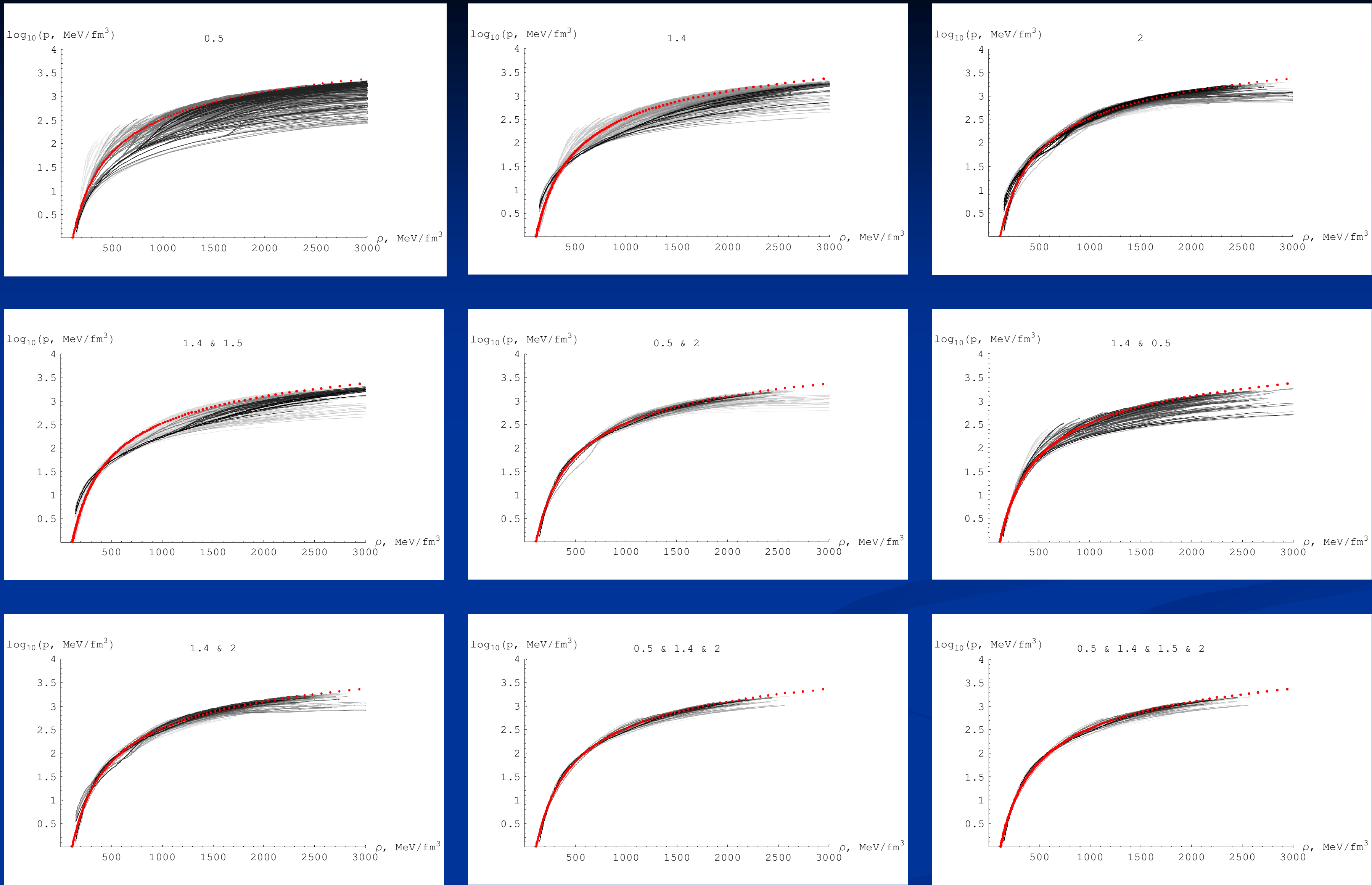




Playing with 4 stars (SLY)




20
18
16
14
12
10
8
0.5
1
1.5
2



log10(p, MeV/fm3)
ρ, MeV/fm3
0.5
1.4
2
1.4 & 1.5
0.5 & 2
1.4 & 0.5
1.4 & 2
0.5 & 1.4 & 2
0.5 & 1.4 & 1.5 & 2

2 solar

0.0821313	SLY4e
0.0812719	GM1c
0.0796758	WFF3e
0.0792627	AP3c
0.0788332	ms00c
0.0785774	PCLc
0.0775472	WFF4c
0.0759901	MPA1c
0.0733188	ms2c
0.0515601	PAL11c
0.0508611	FPSc
0.039268	GM3c
0.0375714	GM2c
0.026417	WFF1e
0.0255262	pal32e
0.0235602	psc
0.0195336	WFF2e
0.019094	ENGc
0	AP1c

1.4 & 2 solar

0.130332	SLY4e
0.122328	AP3c
0.11927	WFF3e
0.115924	WFF4c
0.107259	PAL11c
0.0808789	MPA1c
0.0639204	ms2c
0.0574244	PCLc
0.0493676	FPSc
0.0441501	GM2c
0.0366995	GM1c
0.0354675	WFF1e
0.0213773	WFF2e
0.0156012	ENGc
0	psc
0	pal32e
0	ms00c
0	GM3c
0	AP1c

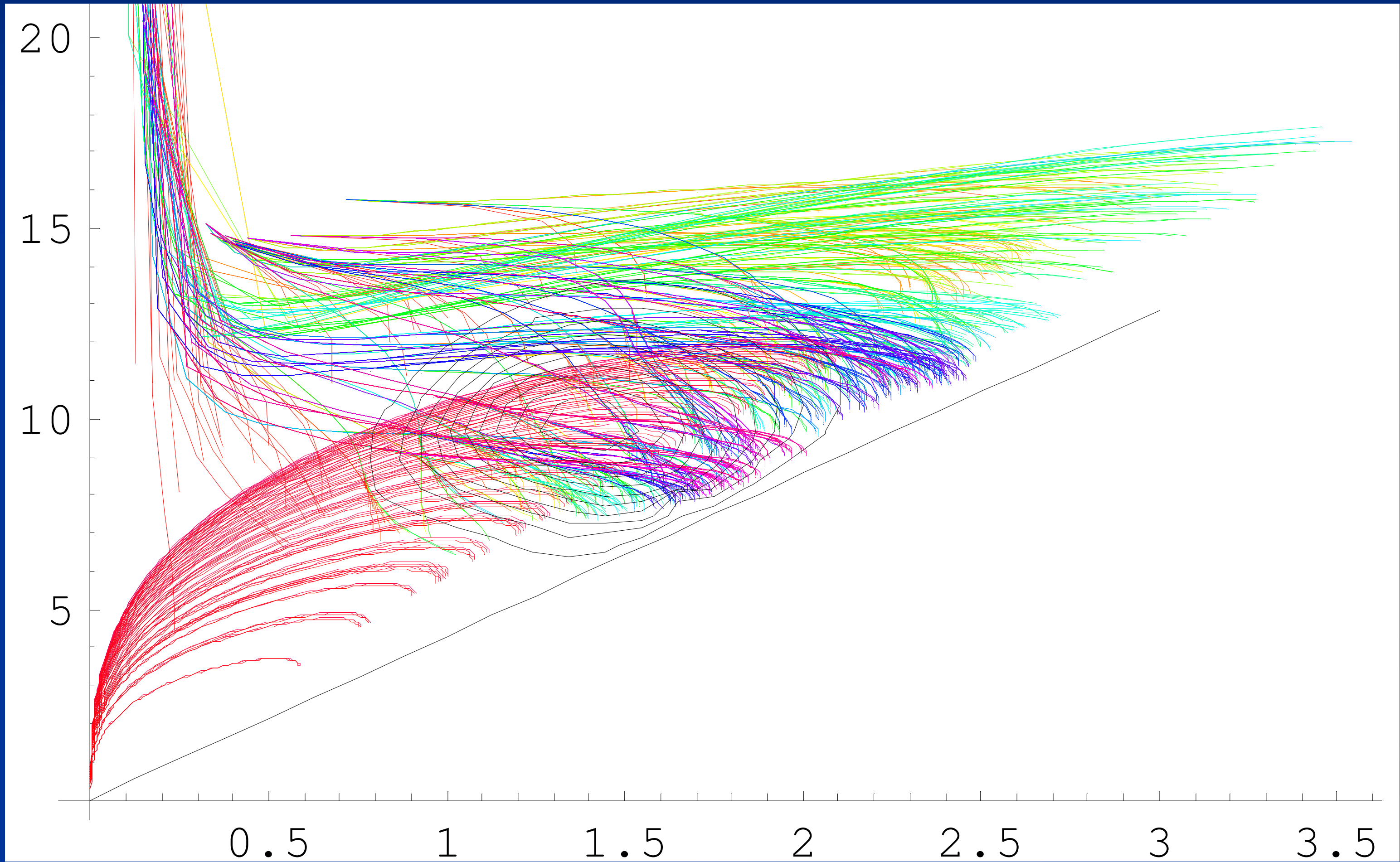
Next steps being taken

- **Adaptive grid with jumps. Generation of complementary EOSs by TOV inversion schemes (M-R plane).**
- Inclusion of more constraints.
  - Rotational period, redshift, B.E., Love number…
  - Data from nuclear experiments
  - Cooling and bursting data
- Sequential data analysis as more individual stars available.
- Strange quark matter stars, very deferent seed EOS.

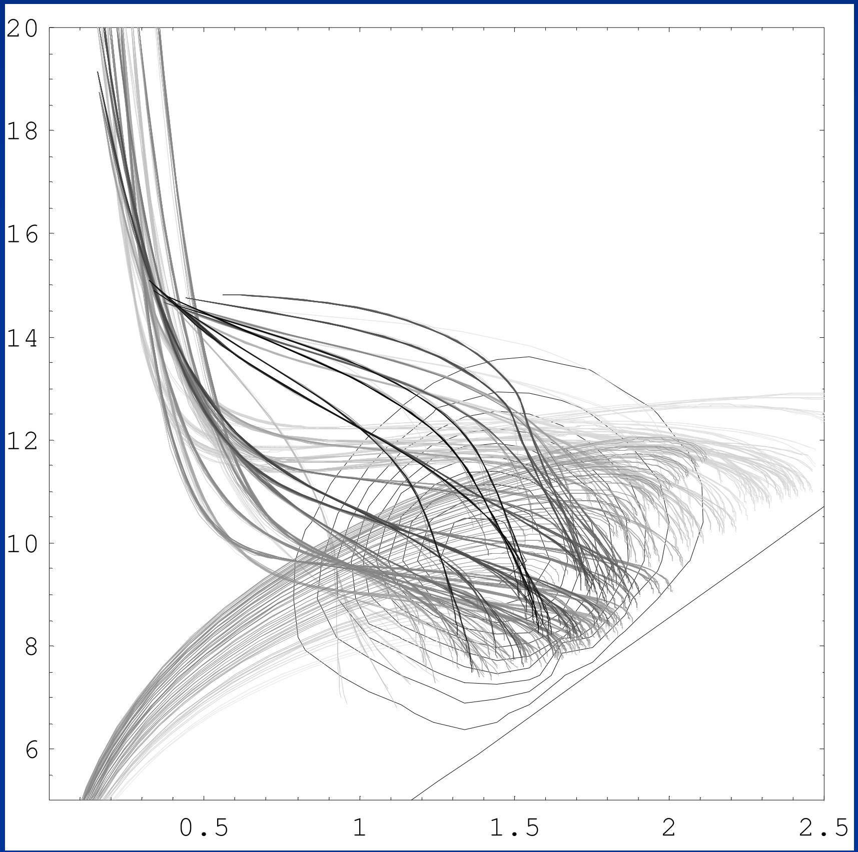
Summary

- The way to use available observations of several stars to determine a EOS is being tested and developed.
- Schemes to generate EOS with incorporation of observational errors is constructed on the basis of sequential Bayesian analysis. It uses EOS expressed as the speed of sound $c_s(h)$ and the variable h and complementary inversion scheme from M-R into EOS.
- Scheme is tested on 5 stars from X-ray burst data and produced reasonable and consistent band of EOSs.
- Additional theoretical constrains and phase transitions can be easily implemented.
- New measurements of several individual stars is expected to get us closer and closer to pinpointing benchmark EOS.

SQM vs NS

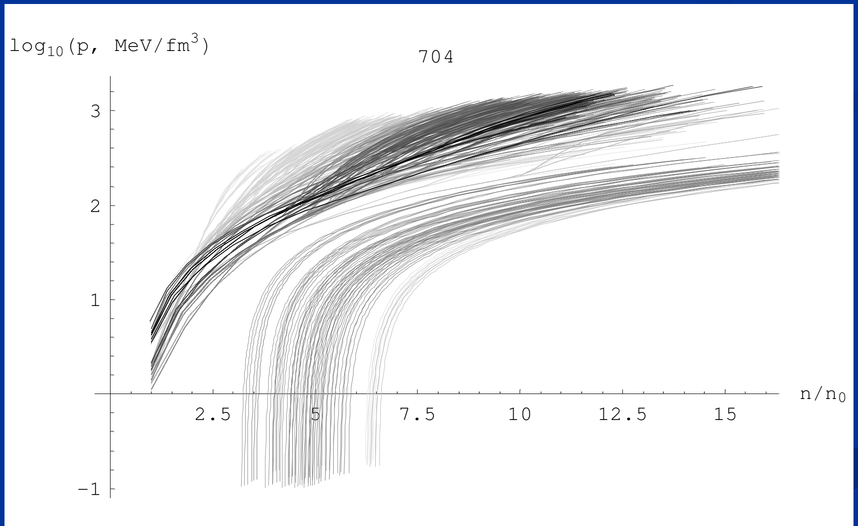


SQM vs NS

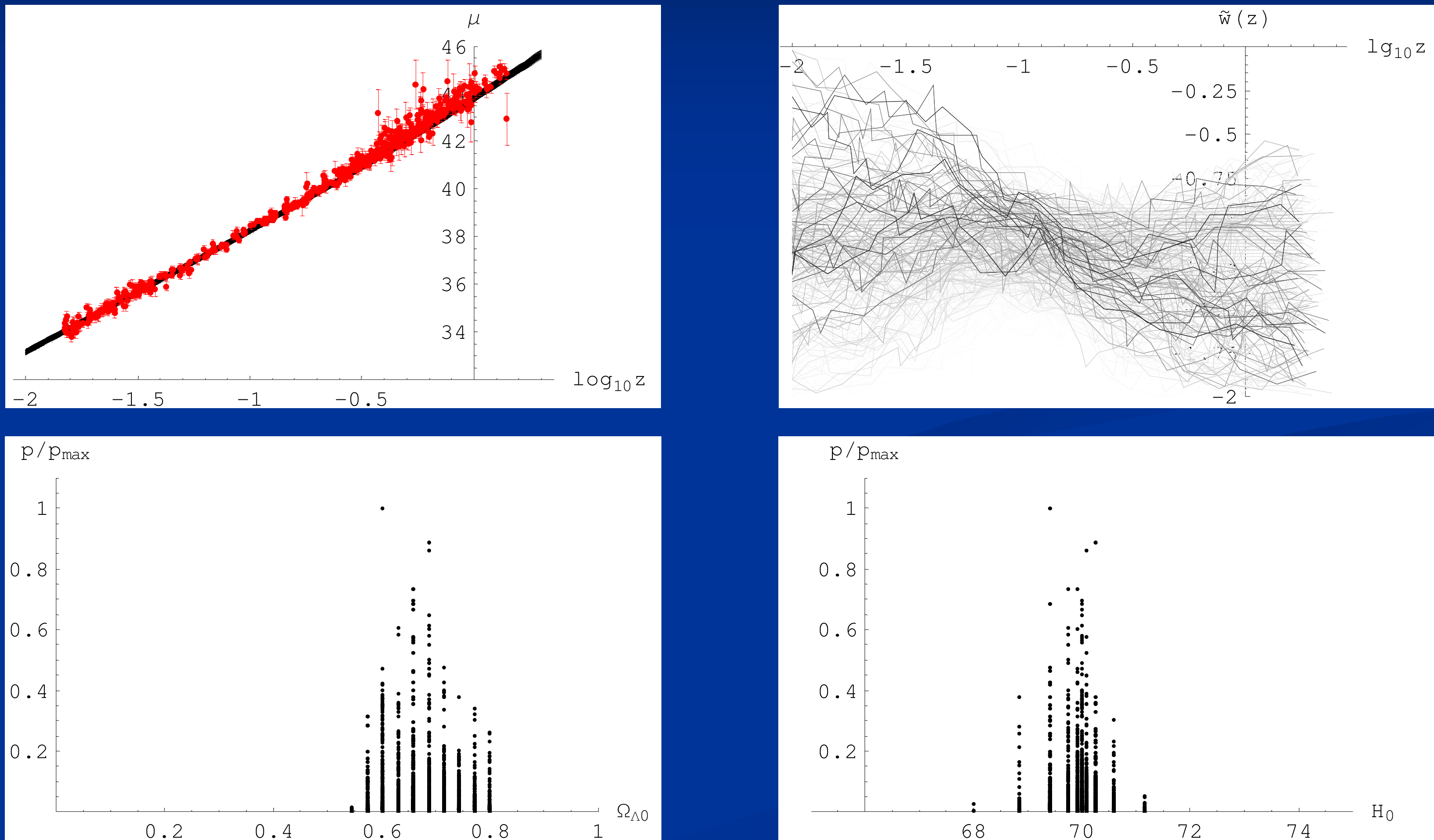


SQM EOS is given by MIT bag model with 3 parameters:

B, α_s and m_s



Reconstructing DE EOS



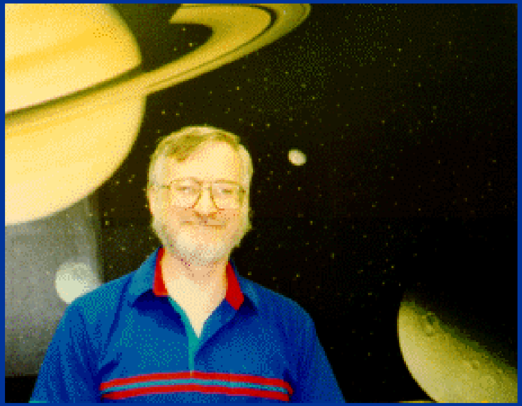
Acknowledgments

- Prof. Madappa Prakash

  Ohio University

- Prof. Jim Lattimer

  Stony Brook University



- Dr. Andrew Steiner

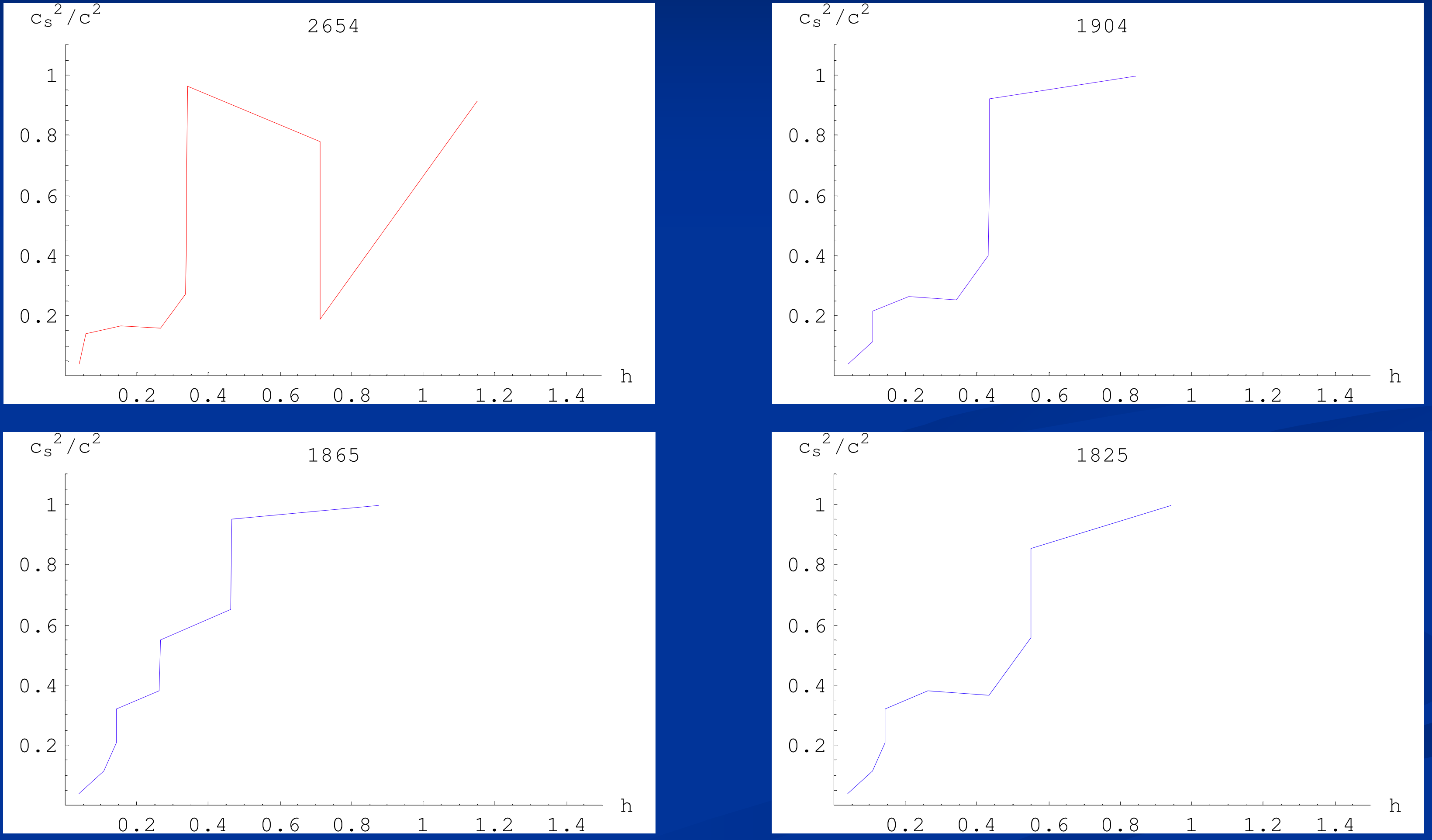
  INT

The end

Thank you!

Questions?

Examples of generated "phase transition"



EQUATIONS OF STATE

Symbol	Reference	Approach	Comp.
FP	Friedman & Pandharipande	Variational	np
PS	Pandharipande & Smith	Potential	$n\pi^0$
WFF(1-3)	Wiringa, Fiks & Fabrocine	Variational	np
AP(1-4)	Akmal & Pandharipande	Variational	np
MS(1-3)	Müller & Serot	Field Theoretical	np
MPA(1-2)	Muther, Prakash & Ainsworth	Dirac-Brueckner HF	np
ENG	Engvik et al.	Dirac-Brueckner HF	np
PAL(1-6)	Prakash, Ainsworth & Lattimer	Schematic Potential	np
GM(1-3)	Glendenning & Moszkowski	Field Theoretical	npH
GS(1-2)	Glendenning & Schaffner-Bielich	Field Theoretical	npK
PCL(1-2)	Prakash, Cooke & Lattimer	Field Theoretical	npHQ
SQM(1-3)	Prakash, Cooke & Lattimer	Quark Matter	Q (u, d, s)
HS	Haensel, Salgado & Bonazzola	Crust, Ref. [65]	Z,e,n
BPS	Baym, Pethick & Sutherland	Crust, Ref. [66]	Z,e,n

Table 7.1: Approach refers to the underlying theoretical technique. Composition (Comp.) refers to strongly interacting components (n=neutron, p=proton, Z=nucleus, H=hyperon, K=kaon, Q=quark); all models include leptonic contributions. The original table and references can be found in [13].

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ Rhoades & Ruffini (1974), Hartle (1978)
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)\ \mathrm{km}$

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_{central} < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2\ \mathrm{g\ cm^{-3}}$

  Lattimer & Prakash (2005)
- $P_{min} \simeq 0.74(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96 \pm 0.03(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}\ \mathrm{ms}$

  (empirical) Lattimer & Prakash (2004)
- $\epsilon_{central} > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2\ \mathrm{g\ cm^{-3}}$ (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

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Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Pulse Shape Modulations*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

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Upper Limit from LIGOII

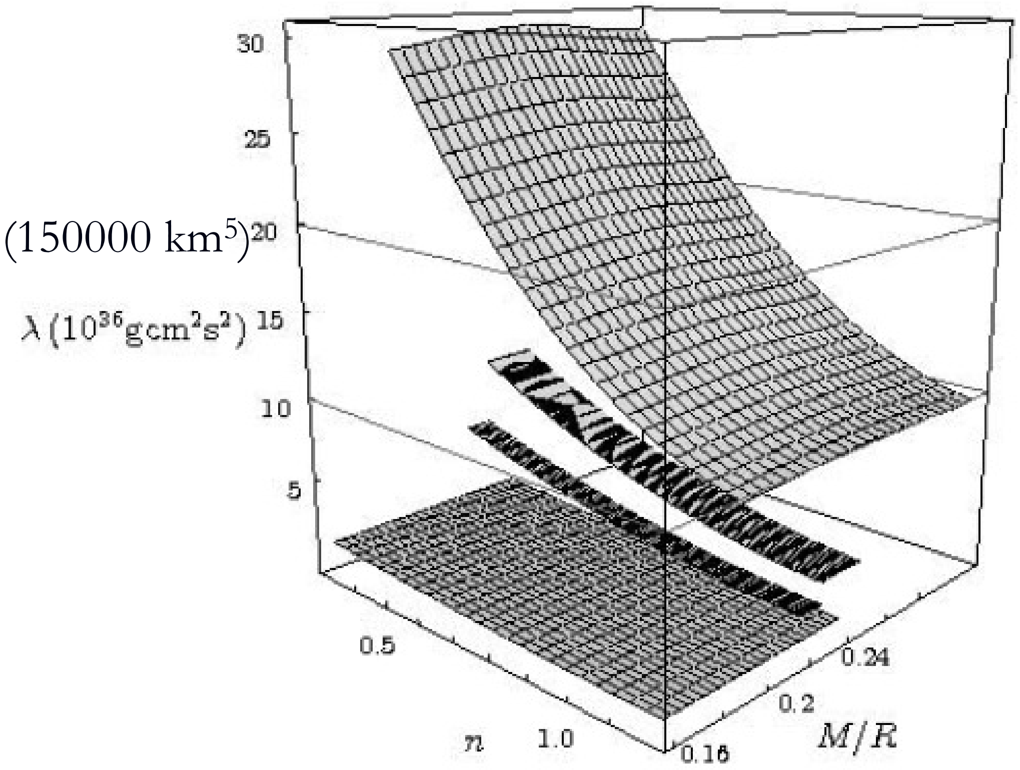
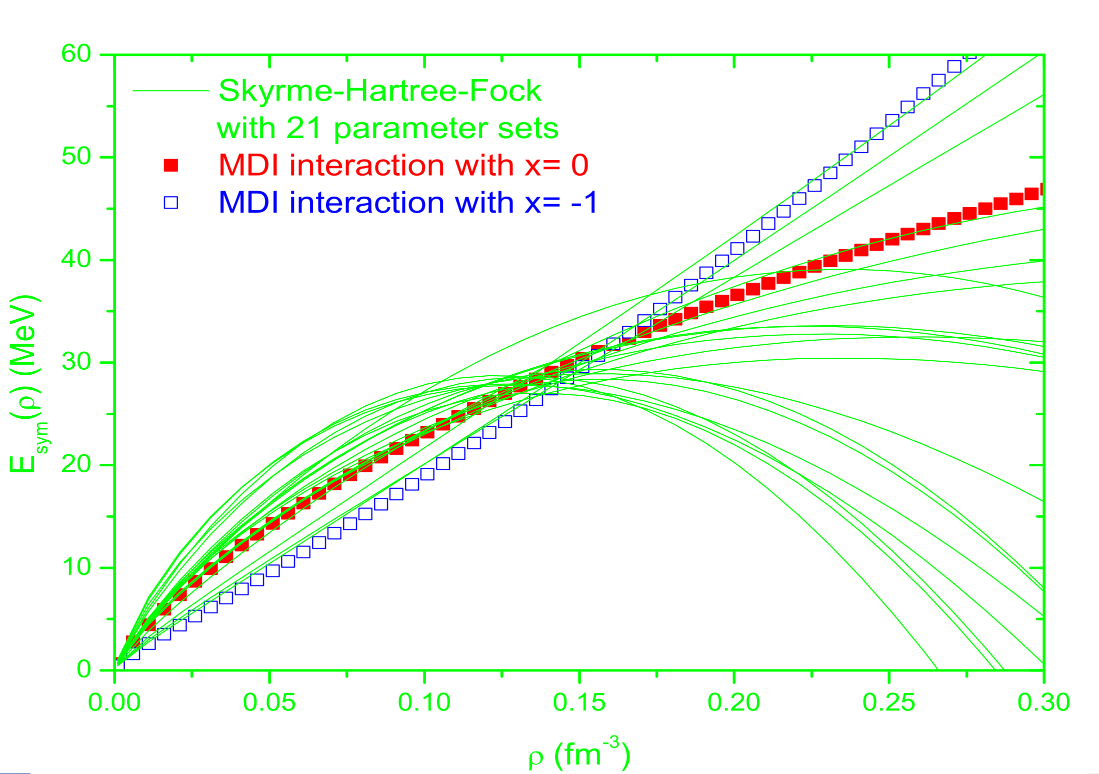


FIG. 3.—Range of Love numbers for the estimated NS parameters from X-ray observations. *Top to bottom sheets*: EXO 0748-676, ωCen, M13, and NGC 2808. For an inspiral of two 1.4 $M_\odot$ NSs at a distance of 50 Mpc, LIGO II detectors will be able to constrain λ to $\lambda \leq 20.1 \times 10^{36}$ g cm^2 s^2 with 90% confidence (Flanagan & Hinderer 2008).

The Nuclear (A)Symmetry Energy



- Energy cost to create an asymmetry (δ) between neutrons and protons

$$E_{sym} = \frac{1}{2n}\frac{d^2\epsilon}{d\delta^2}$$

$$\delta = 1 - 2\frac{n_p}{n_p + n_n} = 1 - 2x$$

- Structure of nuclei & neutron stars determined by the energy & pressure of beta-stable nucleonic matter

$$E(n,x) \simeq E(n,0.5) + E_{sym}(n)(1-2x)^2 + \cdots$$

$$P(n,x) \simeq n^2[E'(n,0.5) + E'_{sym}(n)(1-2x)^2] + \cdots$$

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Courtesy M. Prakash

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