Inner crust composition and transition densities

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Motivation and Outline

Motivation:

How uncertain is

- Inner crust composition
- Extent of various pasta phases
- Transition densities

due to uncertainties in

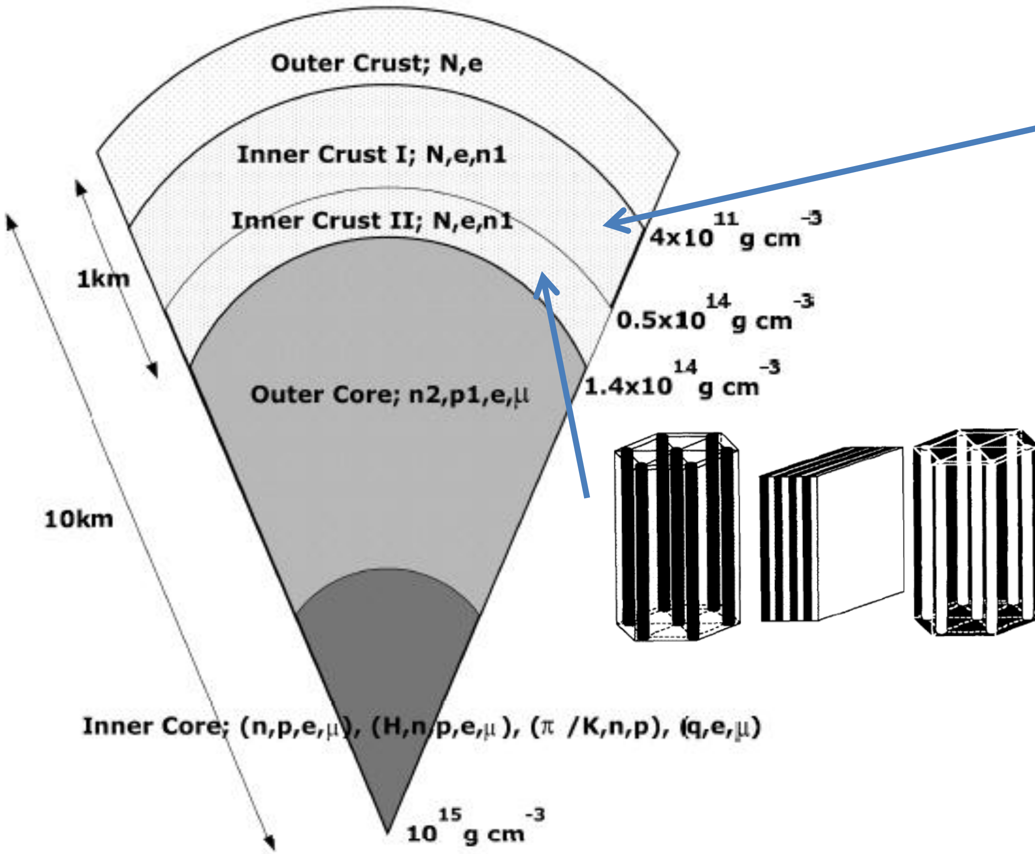
- Nuclear physics
- Crustal model

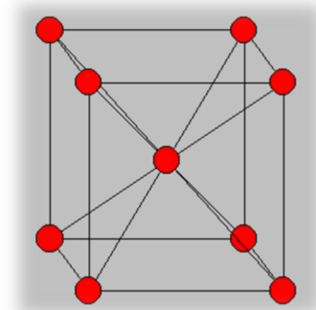
?

Outline:

A simple crust model: compressible liquid drop
Nuclear physics constraints
Range of crustal properties
A more sophisticated crust model

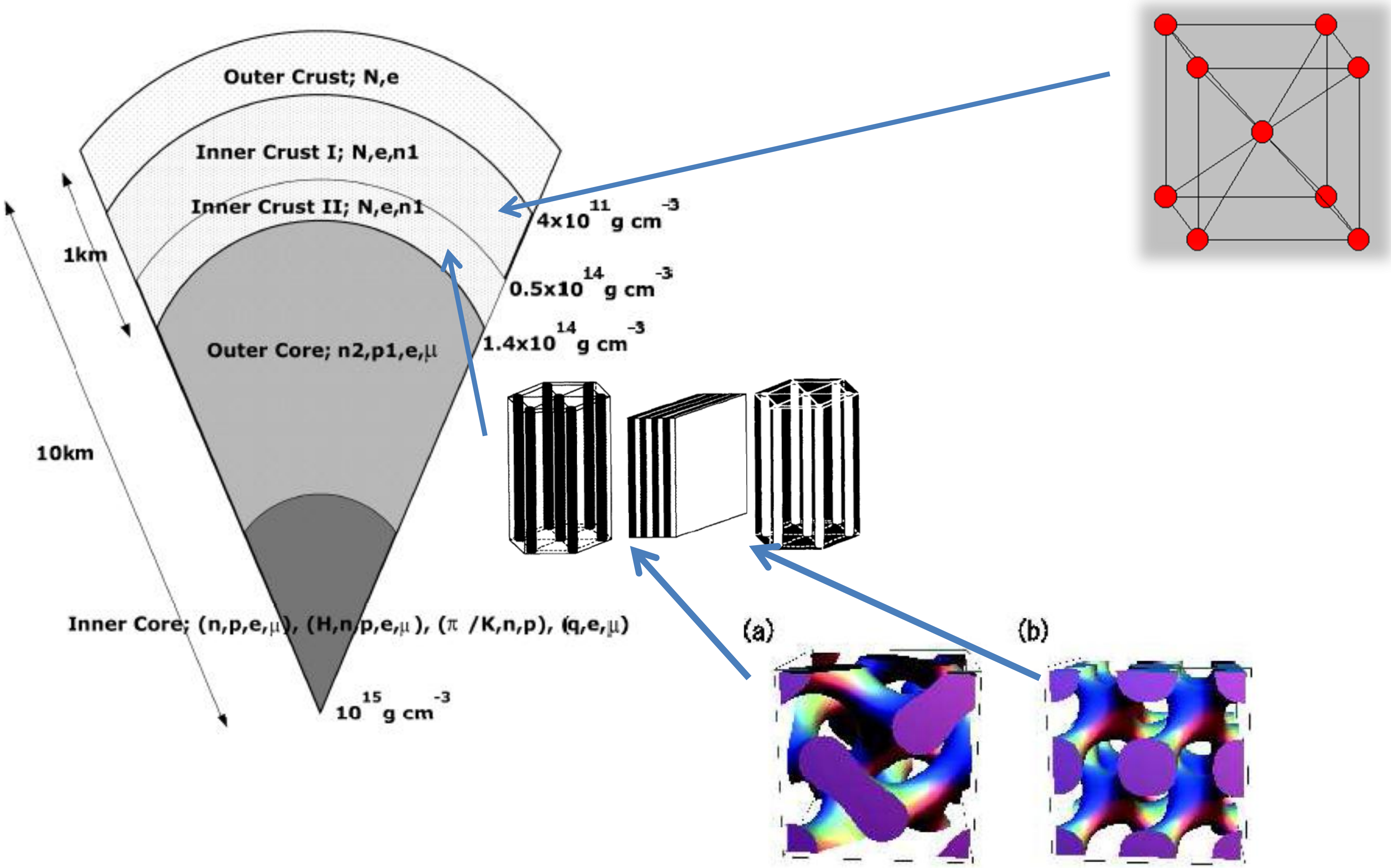
Compressible Liquid Drop Model (CLDM)





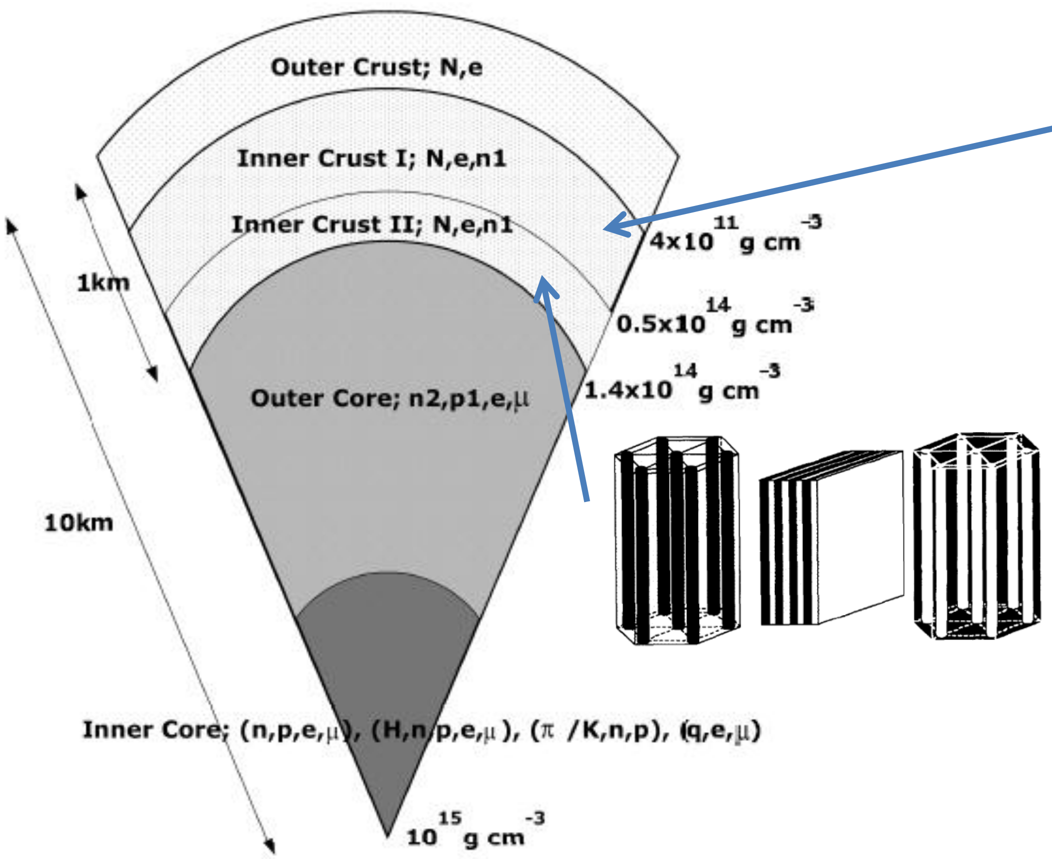
Shear modulus?
Pinning?
Bubbles: ‘free’ protons
 - direct Urca
 (Gusakov, Yakovlev, Haensel, Gnedin 2004)

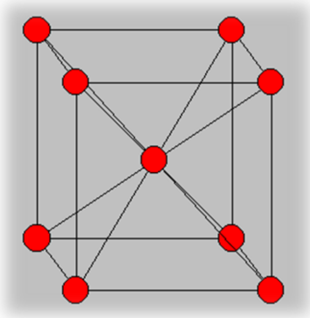
Compressible Liquid Drop Model (CLDM)



Nakazato, Oyamatsu, Yamada 2009

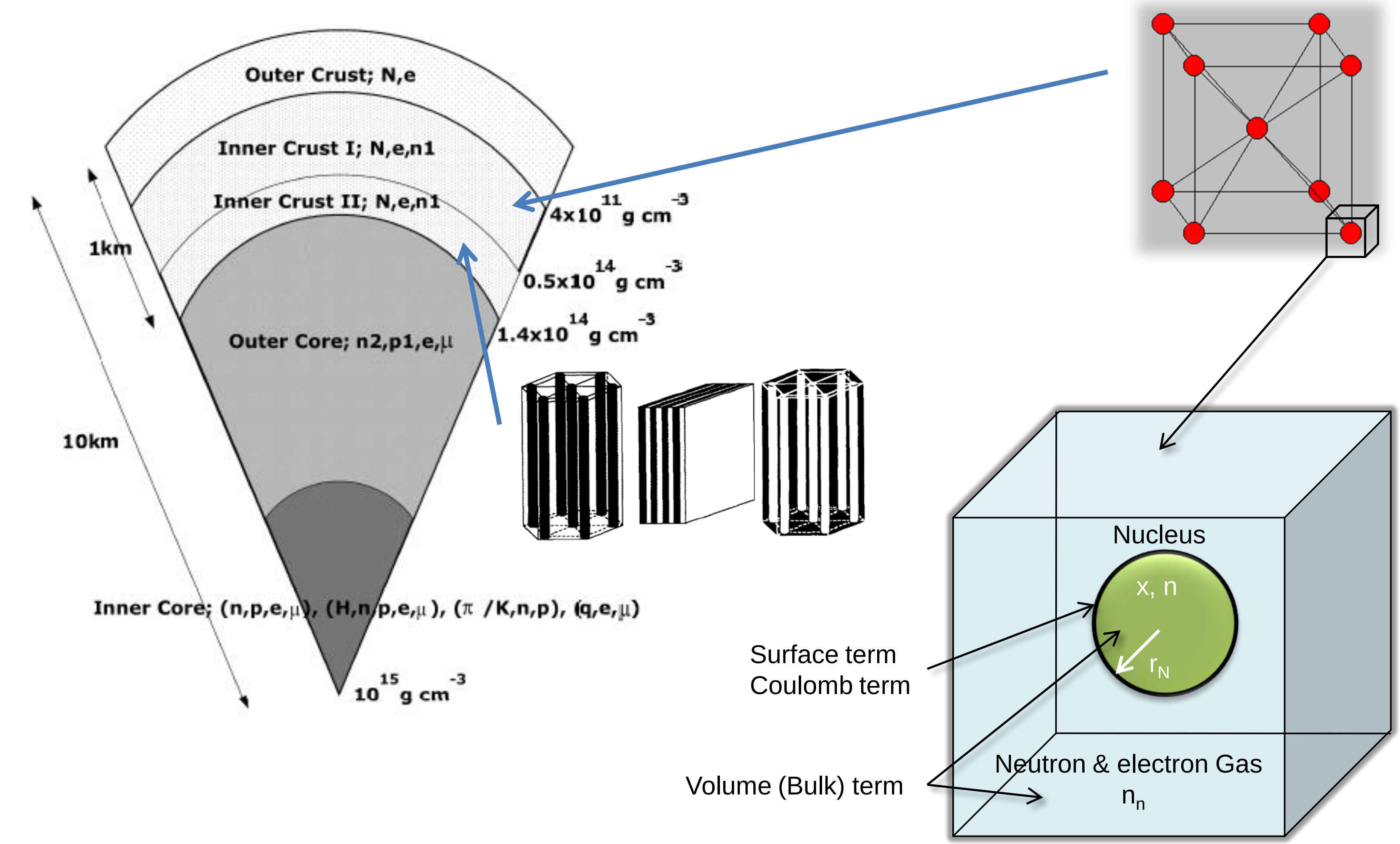
Compressible Liquid Drop Model (CLDM)





Compressible Liquid Drop Model (CLDM)

$$\varepsilon_{\rm cell}(r_{\rm c},x,n,n_{\rm n}) = v\left[nE(n,x)+\varepsilon_{\rm exch}+\varepsilon_{\rm thick}\right]+u\varepsilon_{\rm surf}+u\varepsilon_{\rm Coul}+(1-v)n_{\rm n}E(n_{\rm n},0)+\varepsilon_{\rm e}(n_{\rm e})$$



G. Baym, H.A. Bethe and C.J. Pethick, Nucl. Phys. **A175**, 225 (1971)

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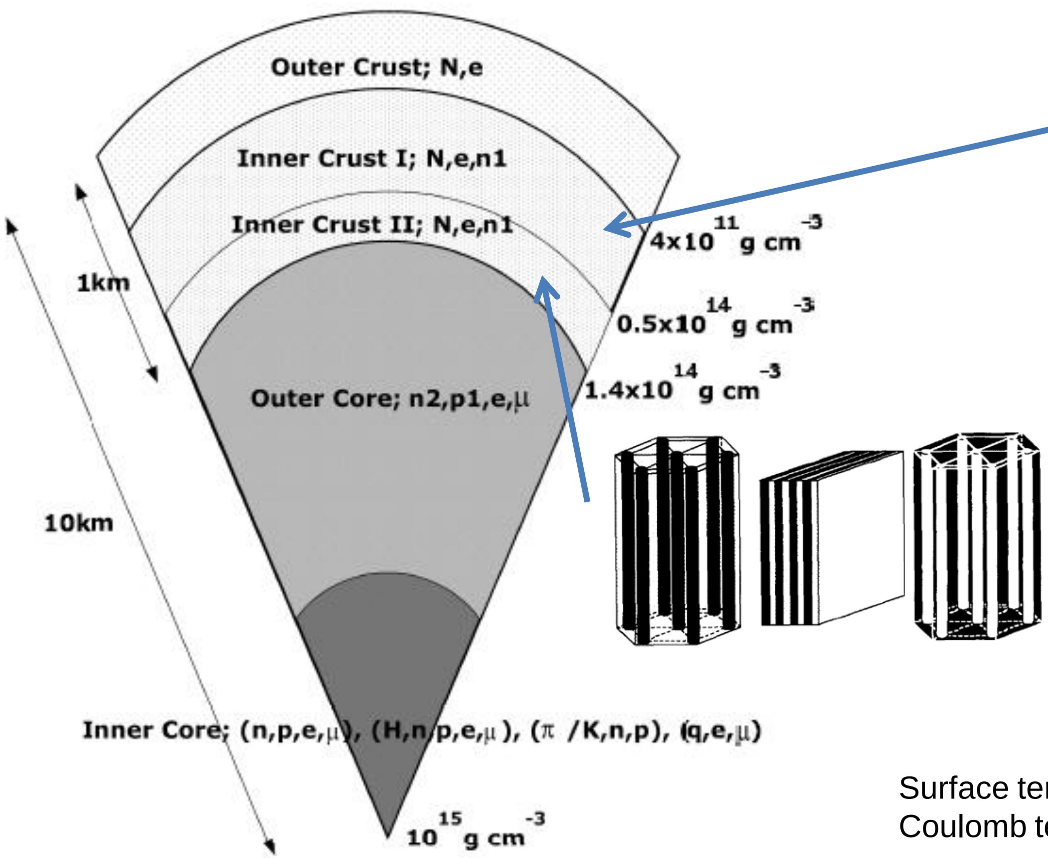

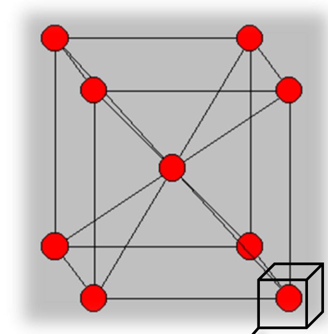

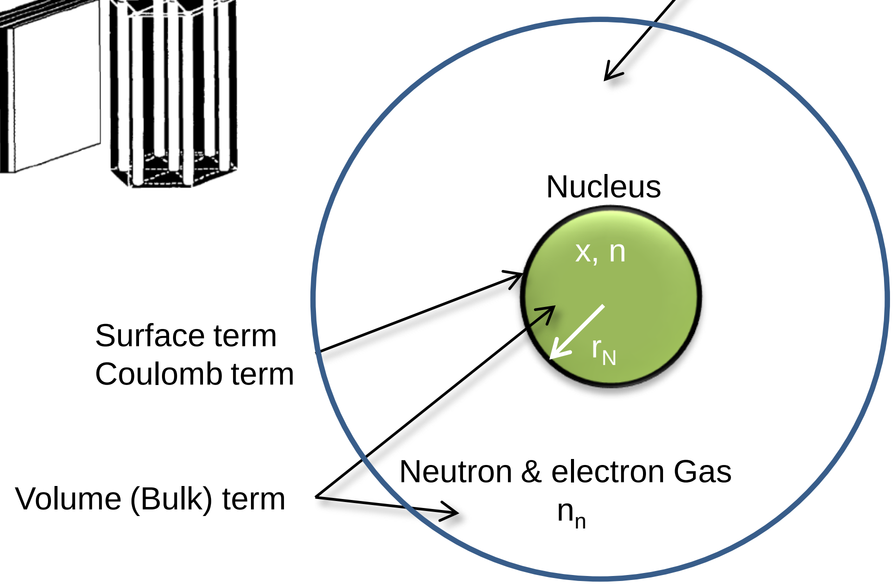

Wigner-Seitz approximation: unit cell is replaced by one which has the same geometry as the nuclear cluster

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PROS:

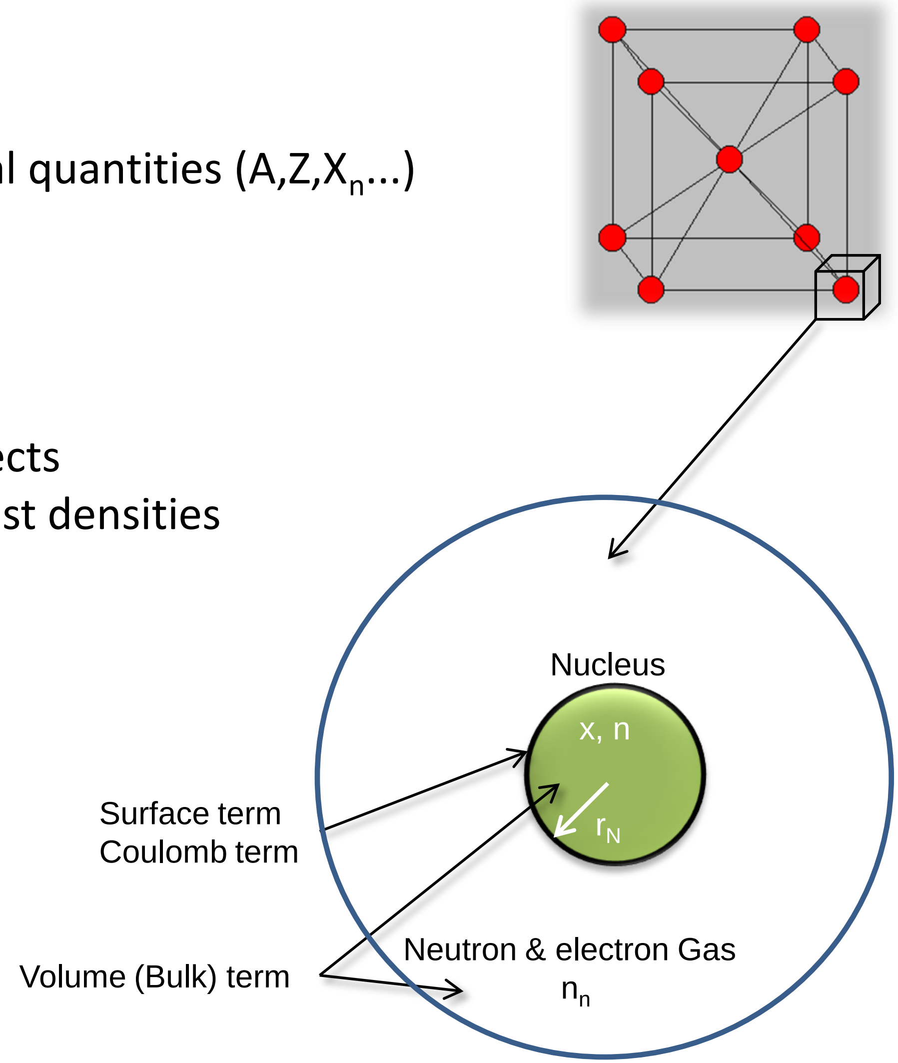
- Physically transparent
- Easy and quick to calculate compositional quantities (A,Z,X_n...)

for use in macroscopic NS models

CONS:

- Semi-classical, macroscopic; no shell effects
- WS approximation not good at the highest densities

of the inner crust.



Wigner-Seitz approximation: unit cell is replaced by one which has the same geometry as the nuclear cluster

Compressible Liquid Drop Model (CLDM)

PROS:
- Physically transparent
- Easy and quick to calculate compositional quantities (A,Z,X_n...)

for use in macroscopic NS models
- Lots of CLDM crust models out there: which one to use?

CONS:
- Semi-classical, macroscopic; no shell effects
- WS approximation not good at the highest densities

of the inner crust.
- Exactly how wrong is CLDM near the crust-core transition?

G. Baym, H.A. Bethe and C.J. Pethick, Nucl. Phys. **A175**, 225 (1971)

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J.M. Lattimer, C.J. Pethick, D.G. Ravenhall and D.Q. Lamb, Nucl. Phys. **A432**, 646 (1985)

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C.P. Lorenz, D.G. Ravenhall and C.J. Pethick, Phys. Rev. Lett. **70**, 4, 379 (1993)

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F. Douchin, P. Haensel and J. Meyer, Nucl. Phys. **A665**, 419 (2000)

A.W. Steiner, Phys. Rev. **C77**, 035805 (2008)

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Uniform nuclear matter EoS

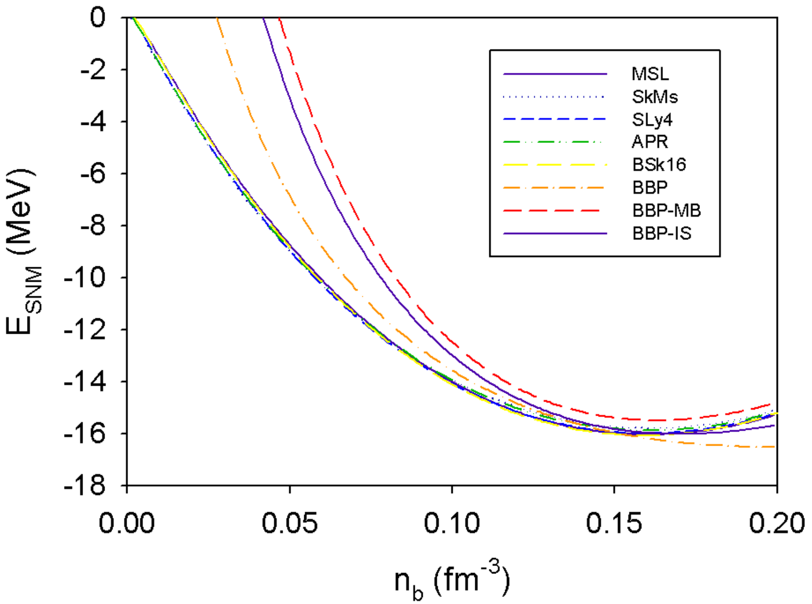
Surface energy

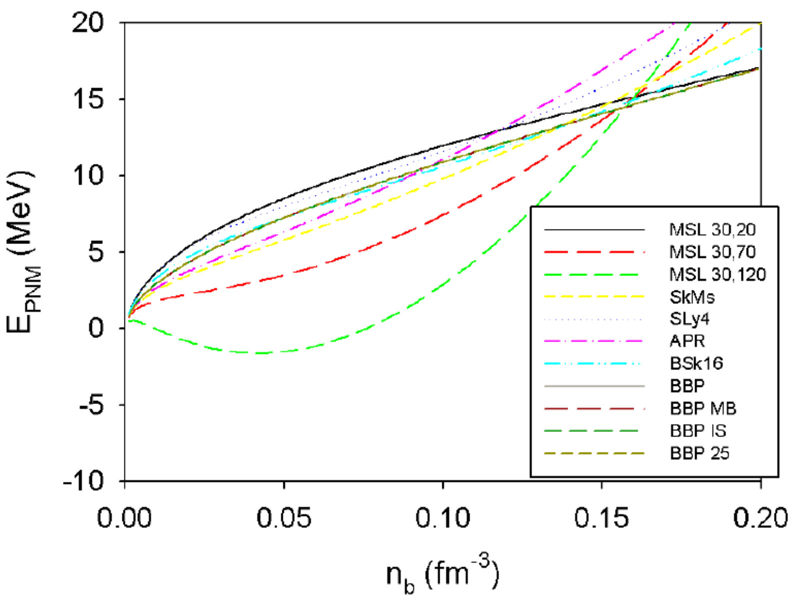
Nuclear Matter EoS

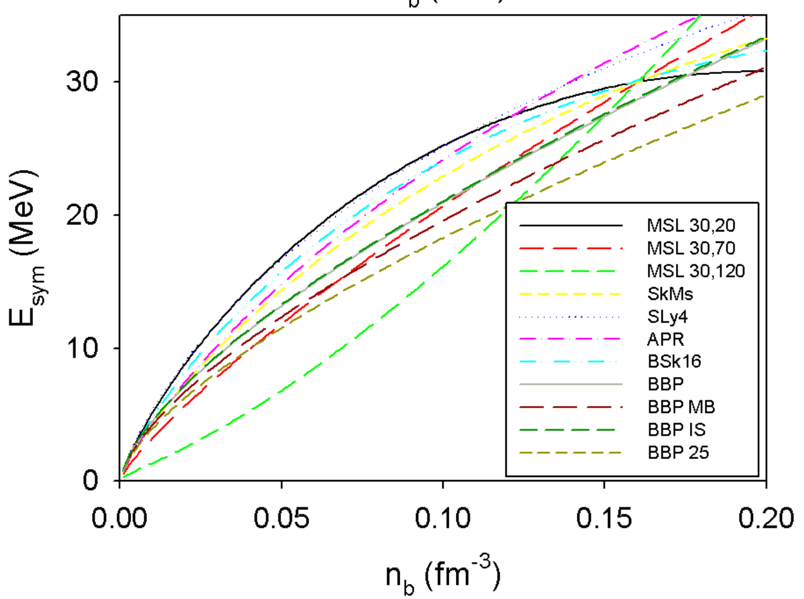
$$E(n,x) = E_0(n) + S(n)\delta^2 + \ldots \qquad \delta = 1 - 2x$$

$$S(n) = J + L\chi + \frac{K_{sym}}{2}\chi^2 + \ldots \qquad \chi = \frac{n - n_s}{3n_s}$$

$$E_{\mathrm{PNM}}(n) \approx E_0(n) + S(n)$$







Nuclear Matter EoS

$$E(n,x) = E_0(n) + S(n)\delta^2 + \ldots \qquad \delta = 1 - 2x$$

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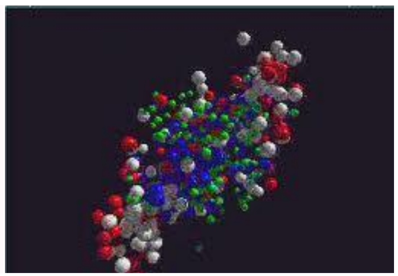

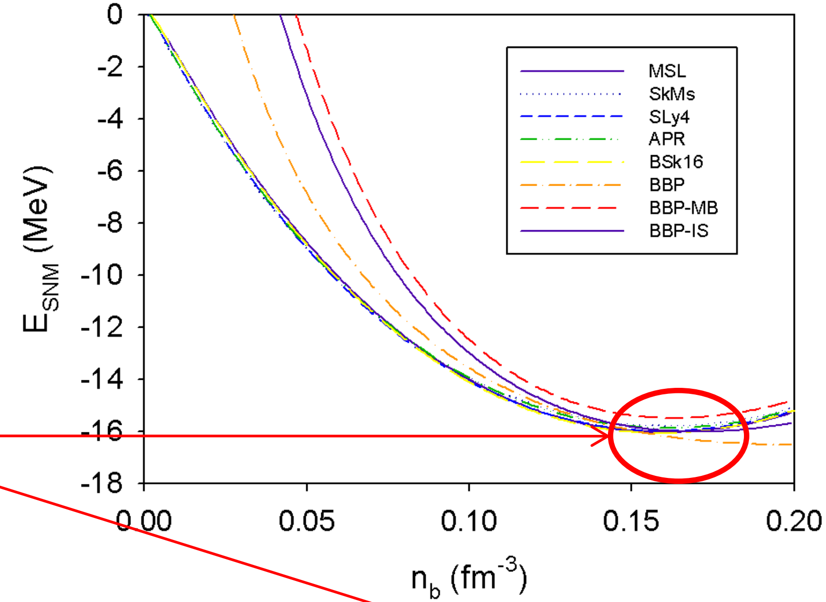

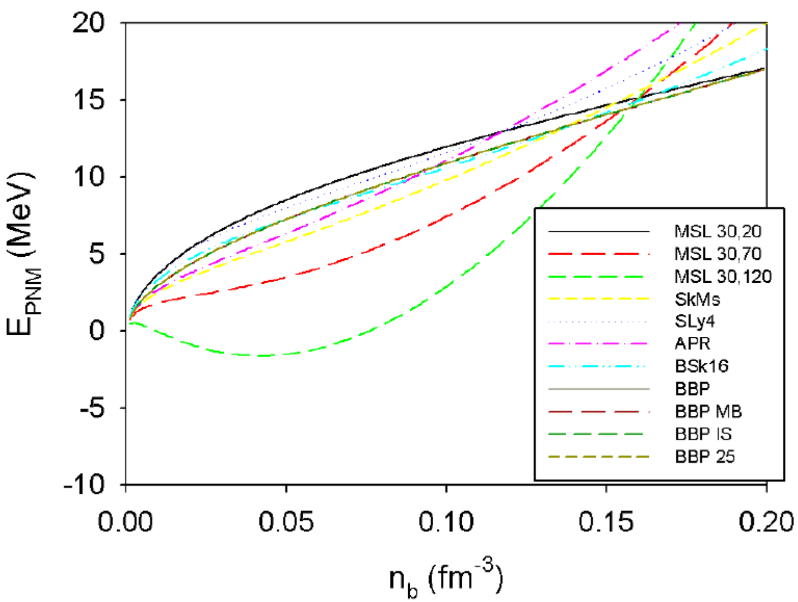

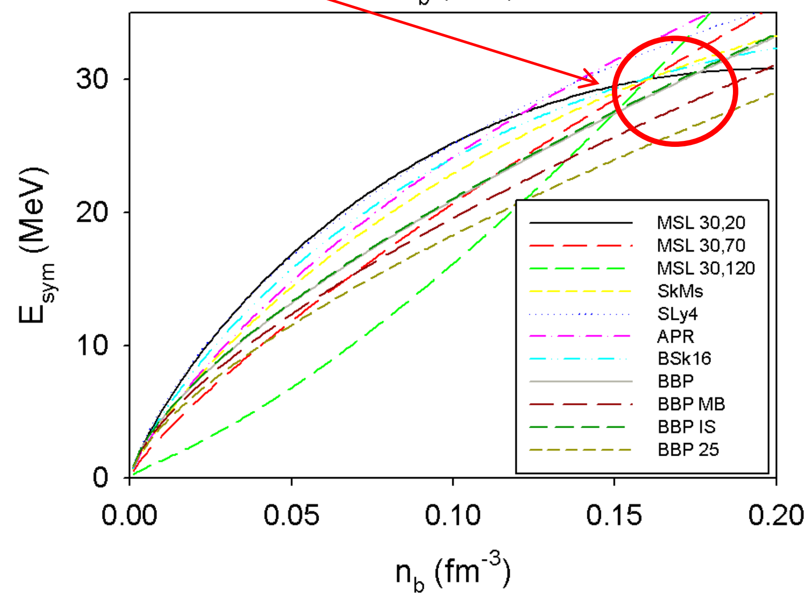

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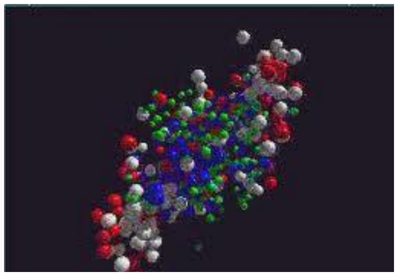
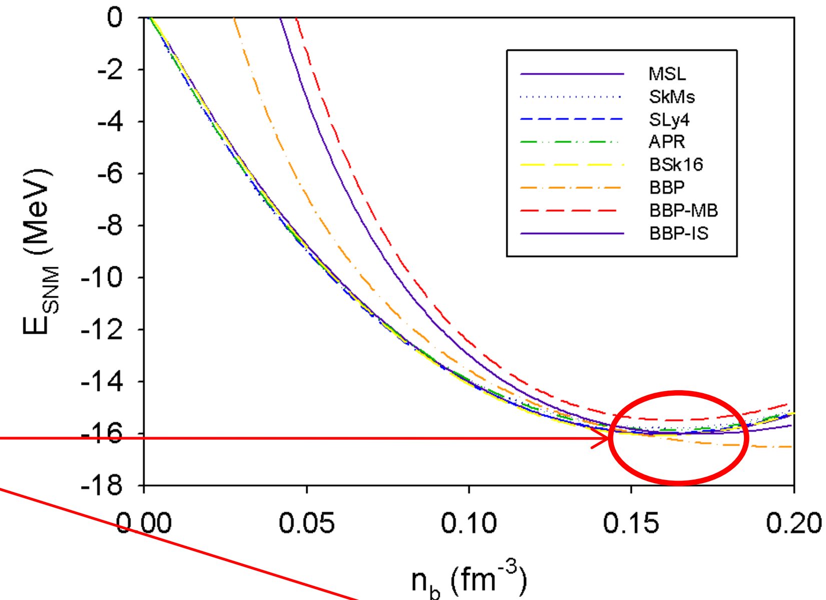
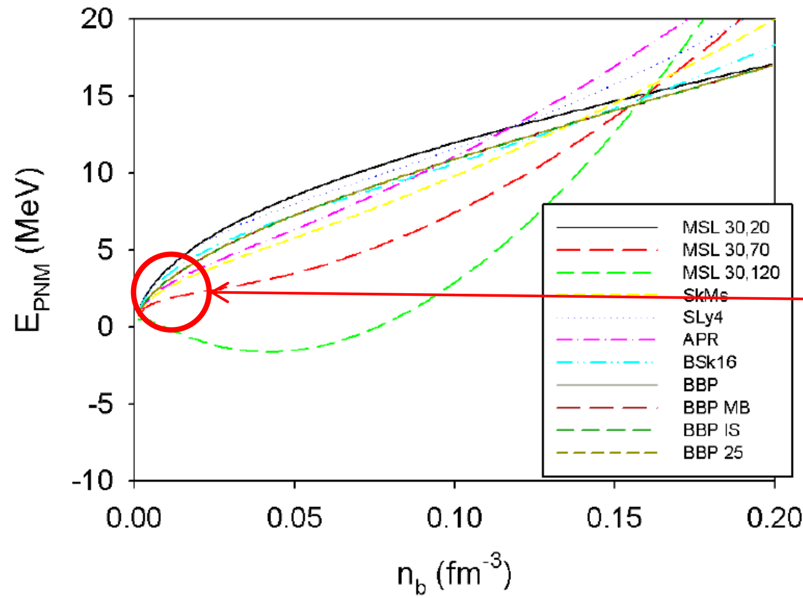
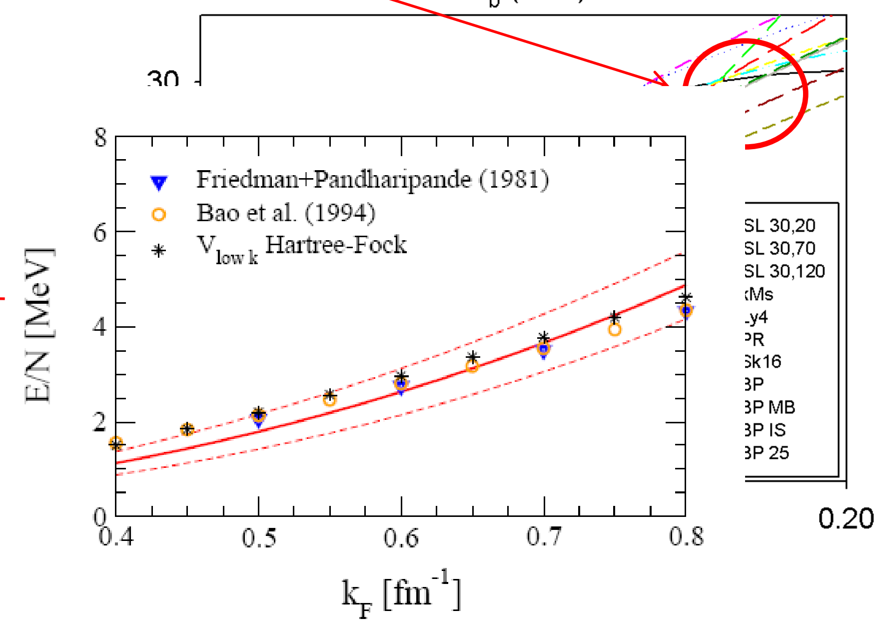
$$E_{\rm PNM}(n) \approx E_0(n) + S(n)$$





A. Schwenk and C.J. Pethick, Phys. Rev. Lett. **95**, 160401

Nuclear Experimental Constraints on L

isospin diffusion in heavy ion collisions involving ^{112}Sn and ^{124}Sn

1 62< *L* <107 MeV Bao-An Li, Lei-Wen Chen and Che Ming Ko, Phys. Rep. 464, 113 (2008)

2 45< *L* <103 MeV M.B. Tsang, Yingzun Zhang, P. Danielewicz, M. Famiano, Zhuxia Li, W.G. Lynch and A.W. Steiner, PRL 102, 122701 (2009)

Isoscaling from multifragmentation reactions

8 *L*≈ 66MeV D.V. Shetty, S.J. Yennello, G.A. Souliotis, Phys. Rev. C76, 024606 (2007)

Pygmy dipole resonance

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Surface symmetry energies of nuclei over a wide range of masses

5 75< *L* < 115 MeV P. Danielewicz and J. Lee, AIPC Conf. Proc. 947, 301 (2007)

Neutron skins of a wide mass range of nuclei

6 25< *L* <100 MeV M. Centelles, X. Roca-Maza, X. Vinas and M. Warda, PRL 102, 122502 (2009)

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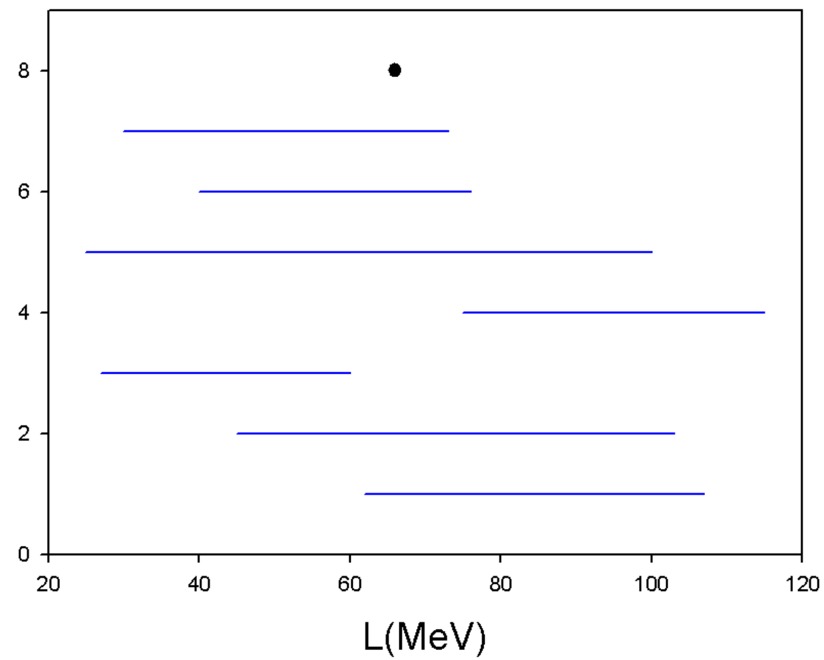

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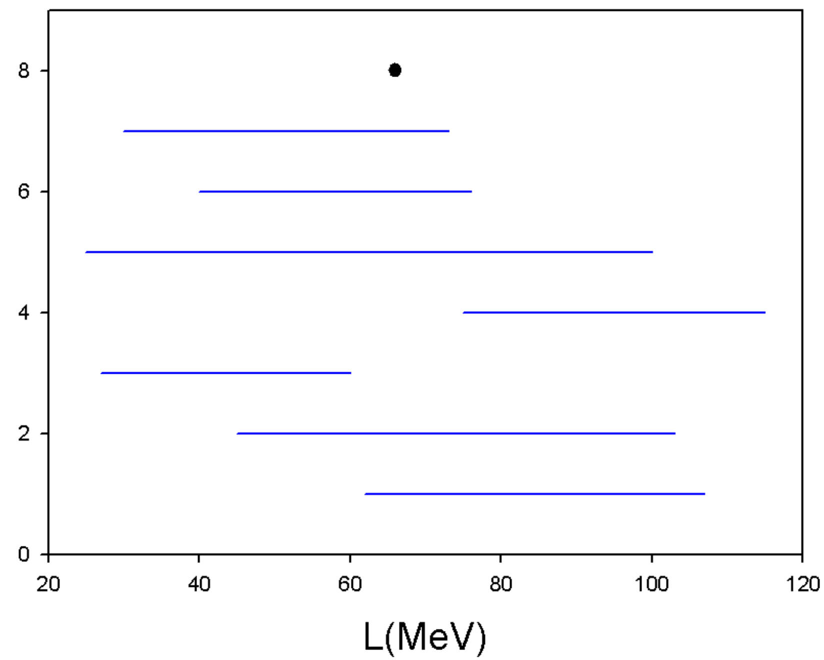


25 < J < 35 MeV
From, e.g., mass models

Nuclear Matter EoS

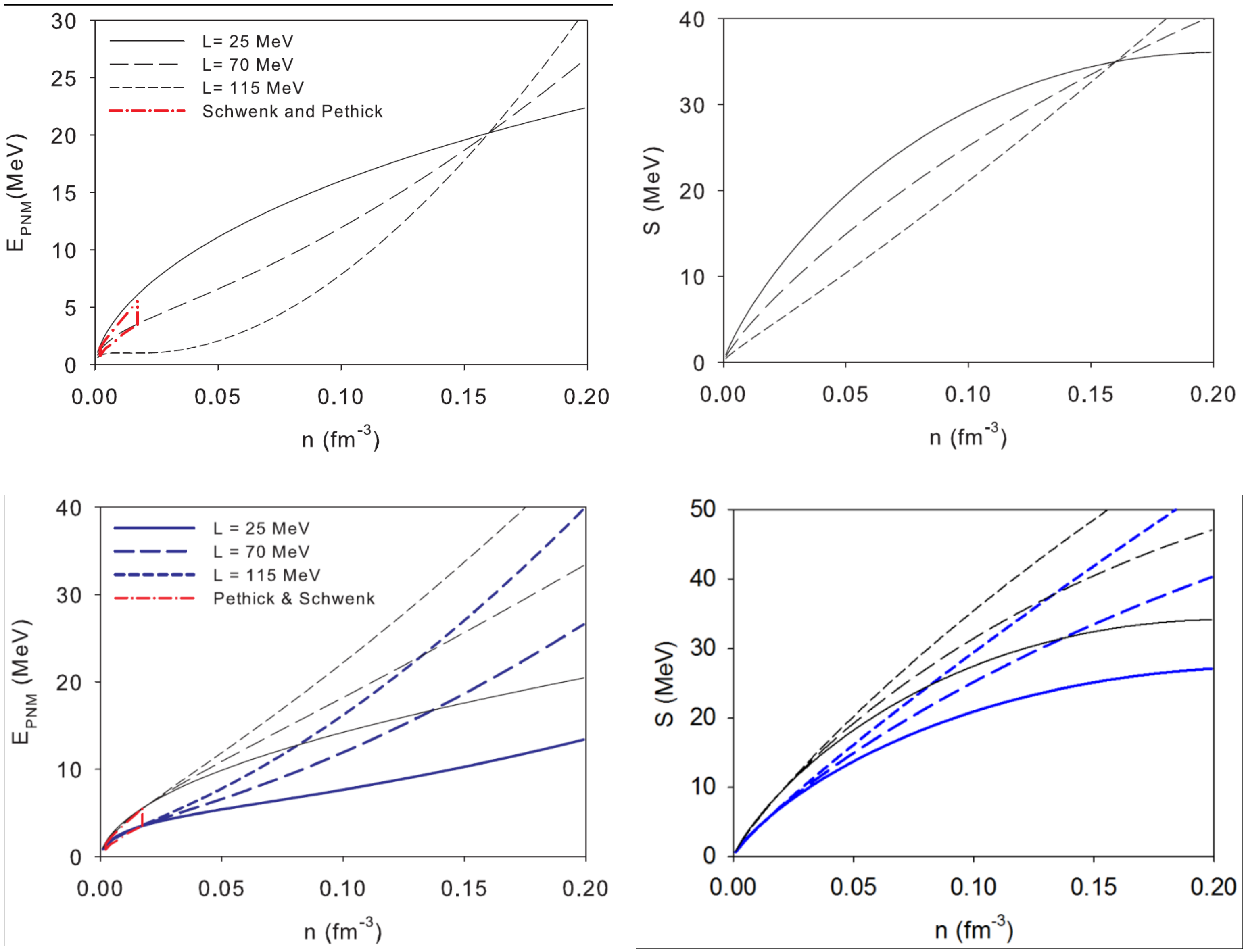
MSL EoS

$$E^{MSL}(n,\delta) = \frac{\eta}{n}\left(\frac{\hbar^2}{2m_n^*}n_n^{5/3} + \frac{\hbar^2}{2m_n^*}n_p^{5/3}\right) + \frac{\alpha}{2}\frac{n}{n_0} + \frac{\beta}{\sigma+1}\frac{n}{n_0} + E_{sym}^{loc}(n)\delta^2$$

$$E_{sym}^{loc}(n) = (1-y)E_{sym}^{loc}(n_0)\frac{n}{n_0} + yE_{sym}^{loc}(n_0)\left(\frac{n}{n_0}\right)^{\gamma_{sym}}$$

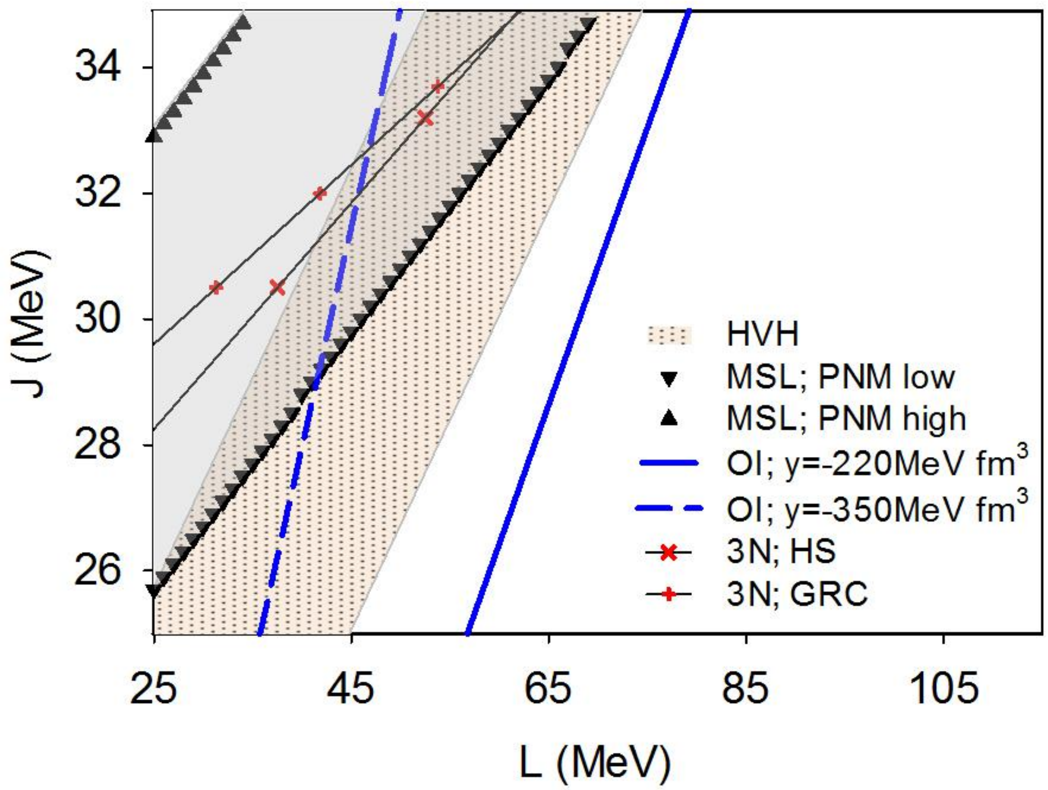
Chen, Cai, Ko, Xu, Chen, Ming 2009

Nuclear Matter EoS



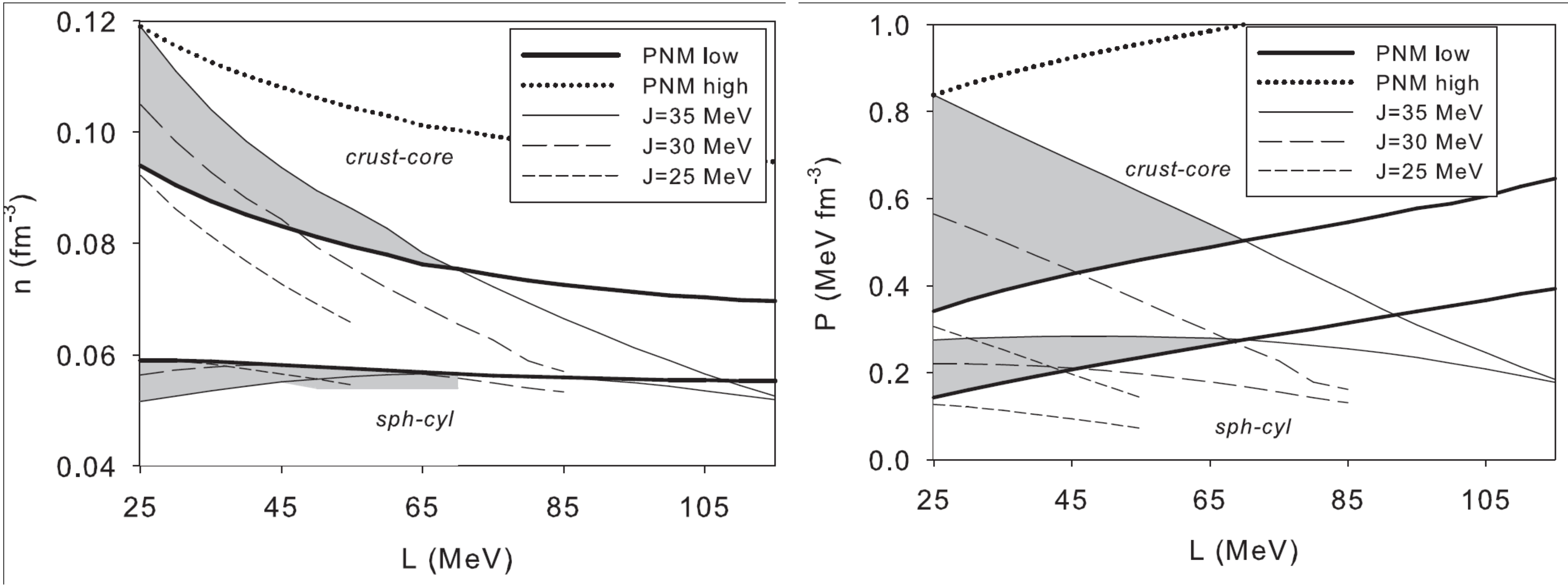
Nuclear Matter EoS

Demanding consistency with low density PNM gives a phenomenological correlation between the symmetry energy and its slope at saturation; other studies show similar correlations



HVH: Xu, Li Chen 2010
OI: Oyamatsu, Iida 2005
HS: Hebeler, Lattimer, Pethick, Schwenk 2010
GRC: Gandolfi, Carlson, Reddy 2011

Inner crust: transition densities and pressures



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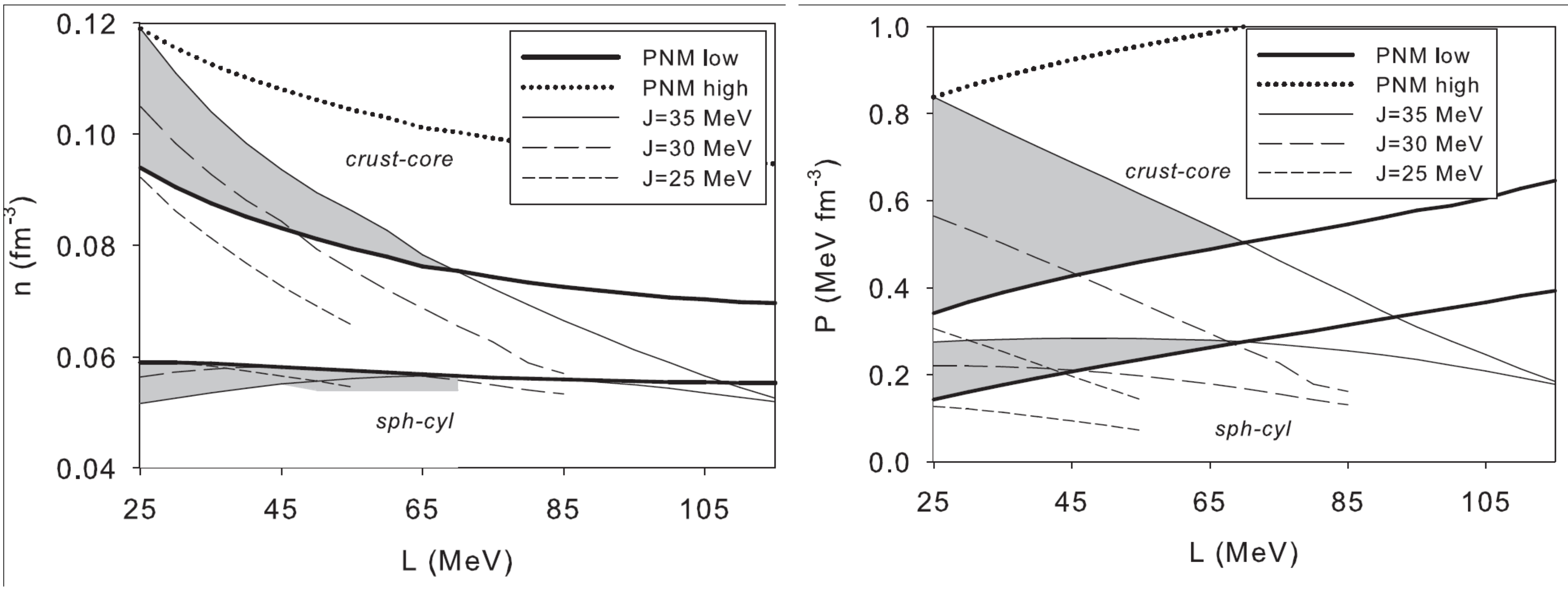
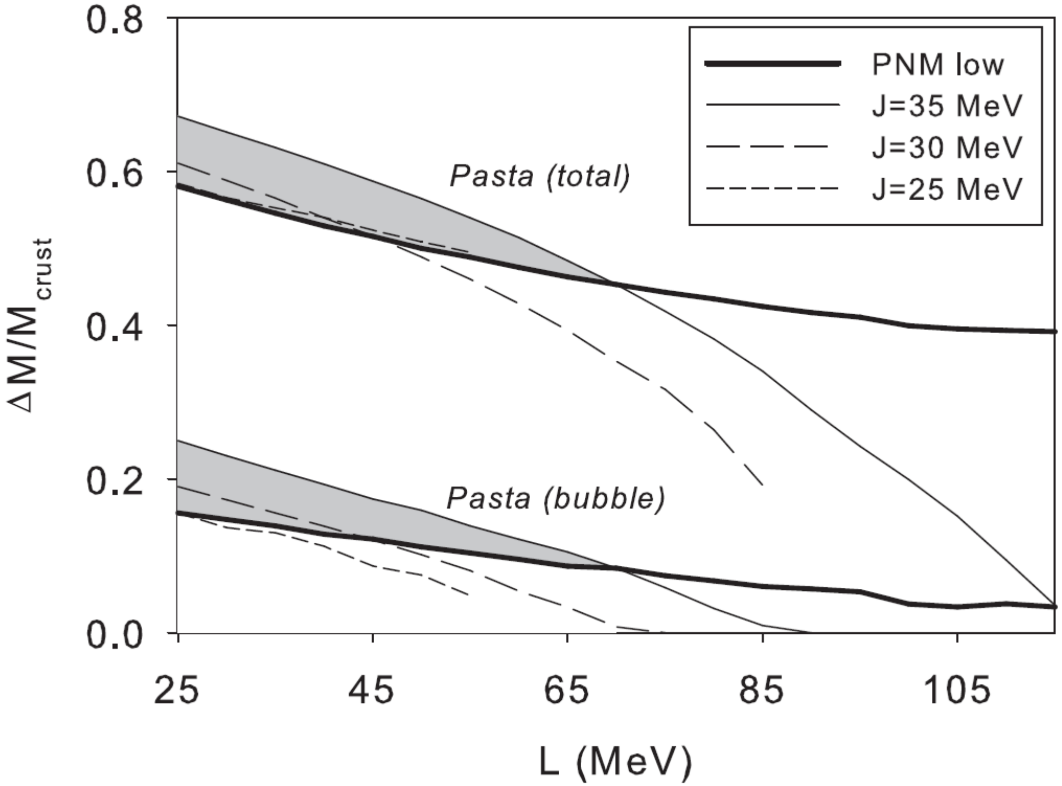


$$\frac{\Delta M_{\rm pasta}}{\Delta M_{\rm crust}} \approx 1 - \frac{P_{\rm sph-pasta}}{P_{\rm crust-core}}; \qquad \frac{\Delta M_{\rm bubble}}{\Delta M_{\rm crust}} \approx 1 - \frac{P_{\rm bubble}}{P_{\rm crust-core}}$$

(c.f. Lorenz, Ravenhall, Pethick 1993)

Inner crust: mass fractions of pasta layers



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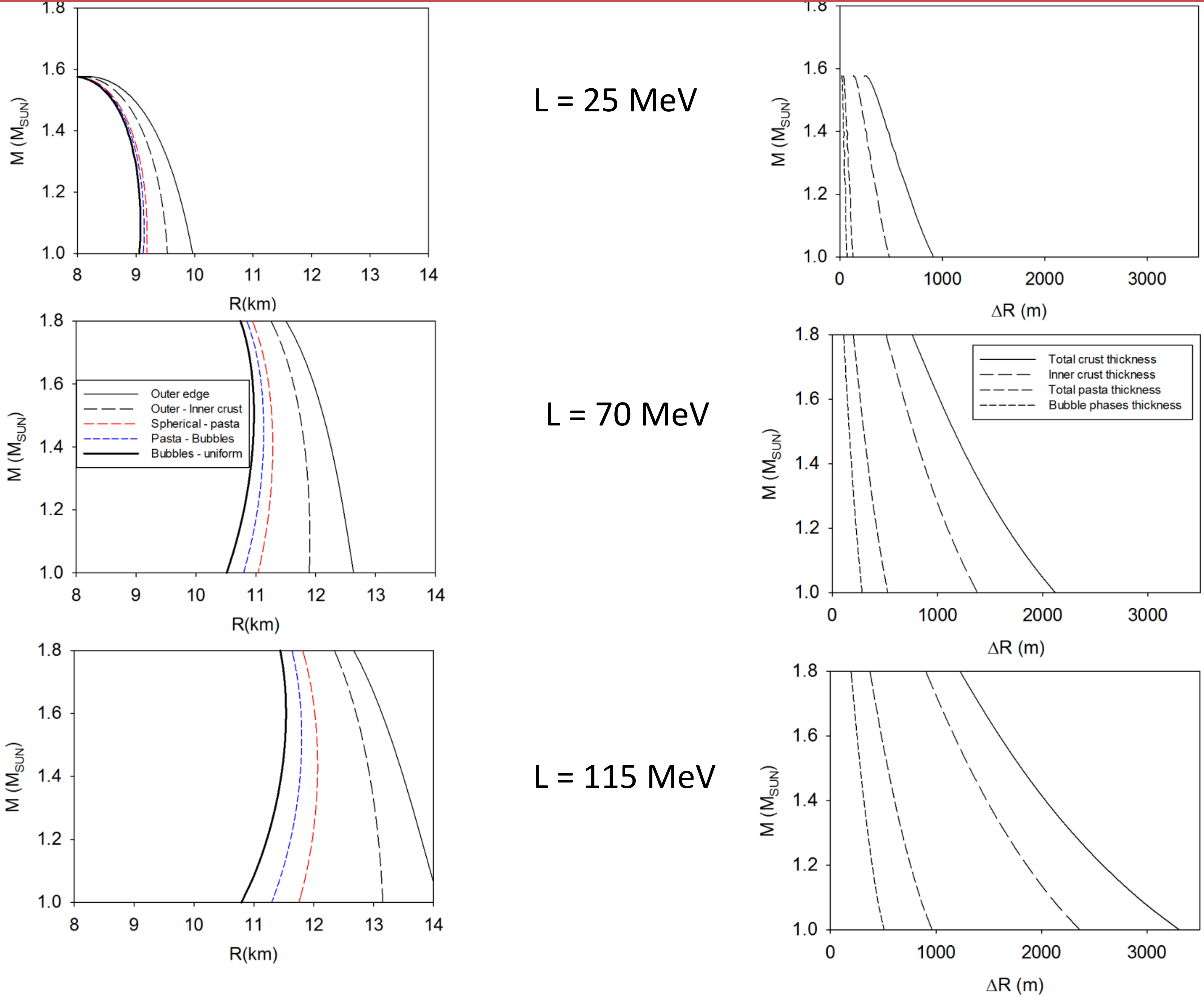
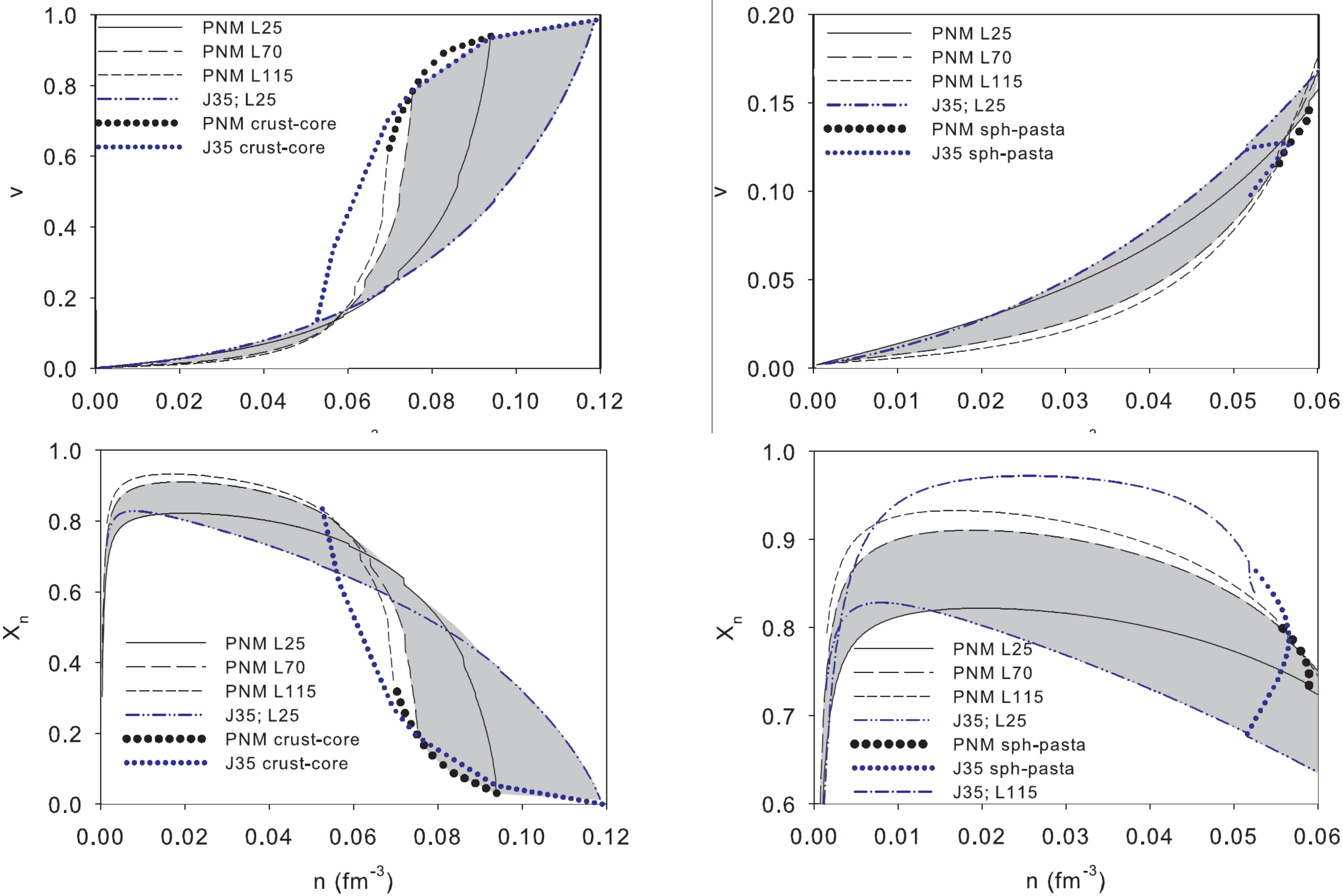
(c.f. Lorenz, Ravenhall, Pethick 1993)

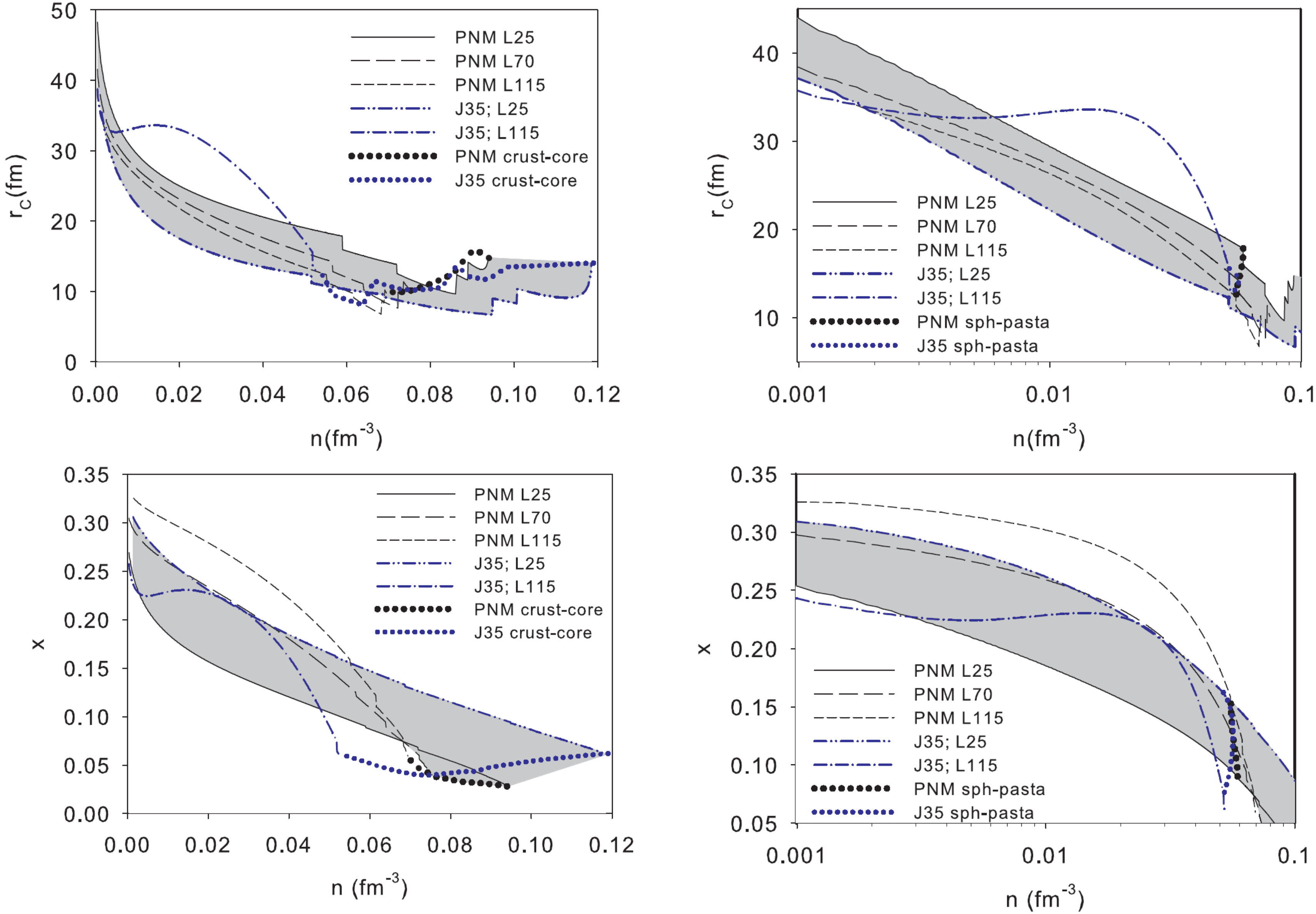
Inner crust: spatial extent of crust layers



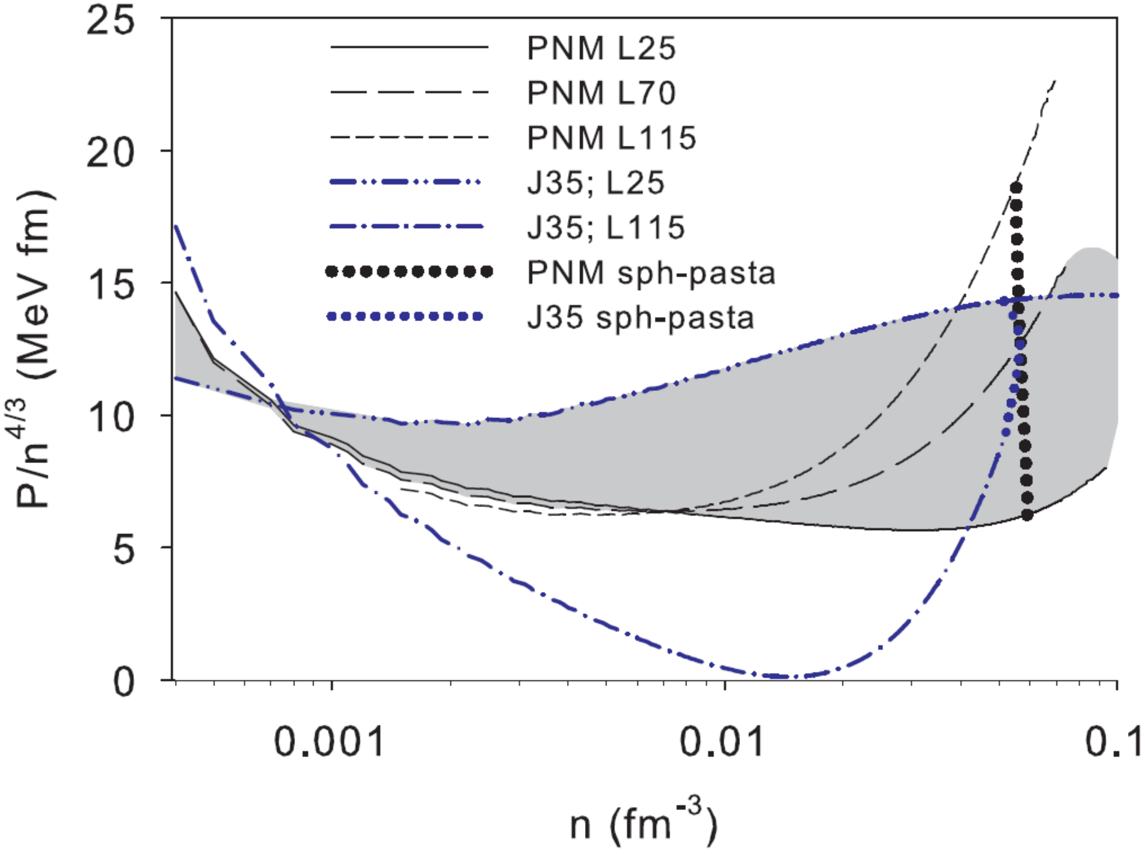
Inner crust: volume fraction of clustered matter; number fractions of 'dripped neutrons'



Inner crust: WS cell sizes and proton fractions

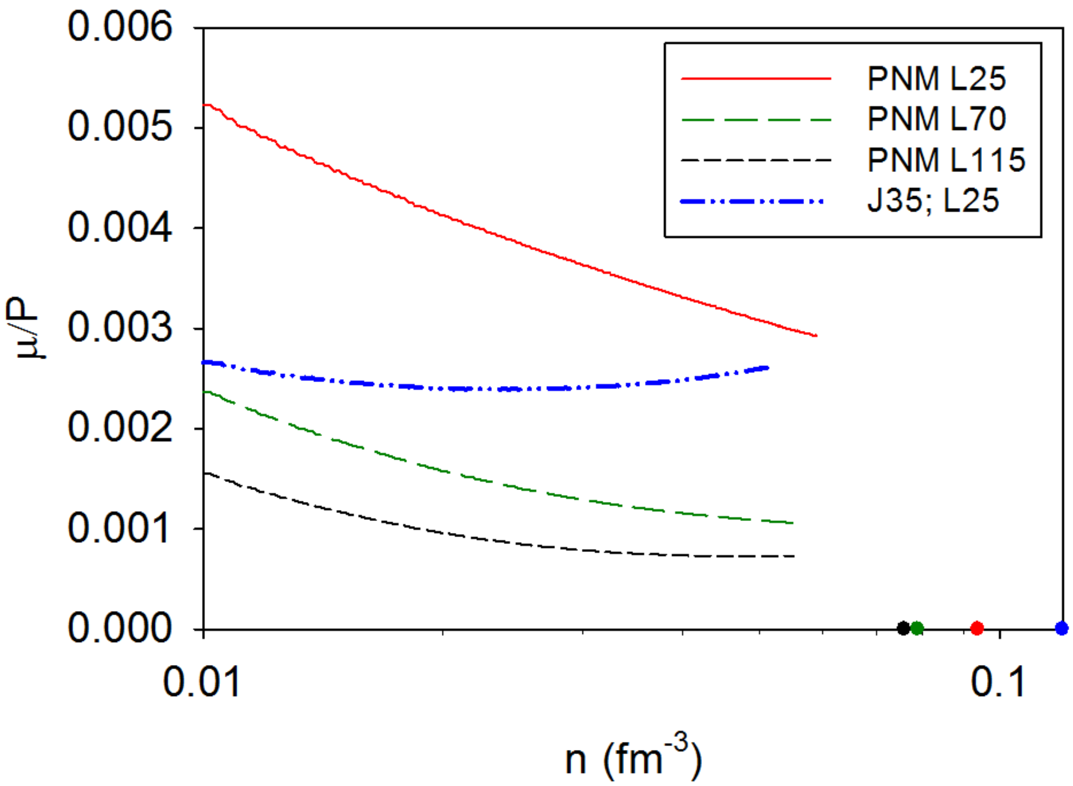


Inner crust: pressure



Inner crust: shear modulus

$$\mu = 0.1106 \frac{n_i (Ze)^2}{a}$$



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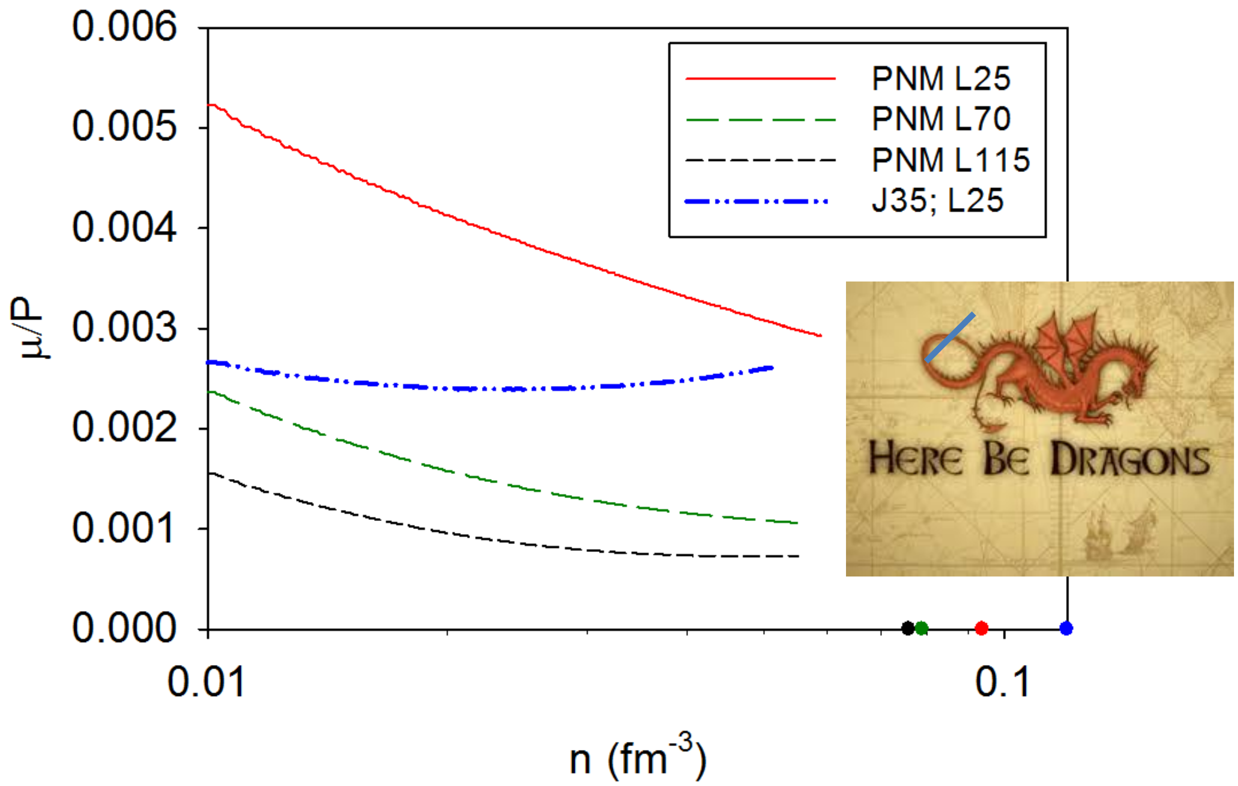
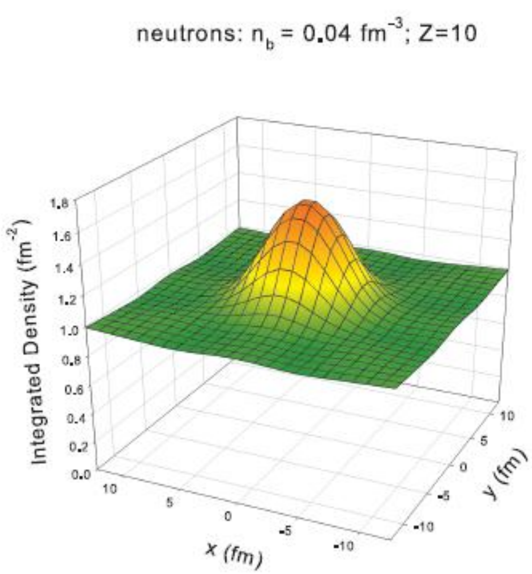
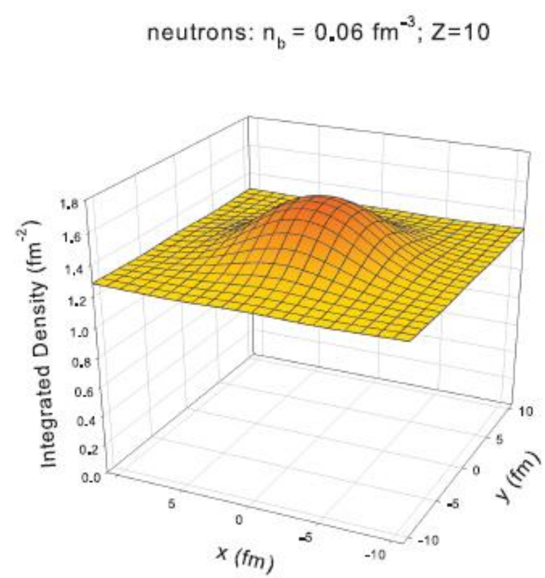
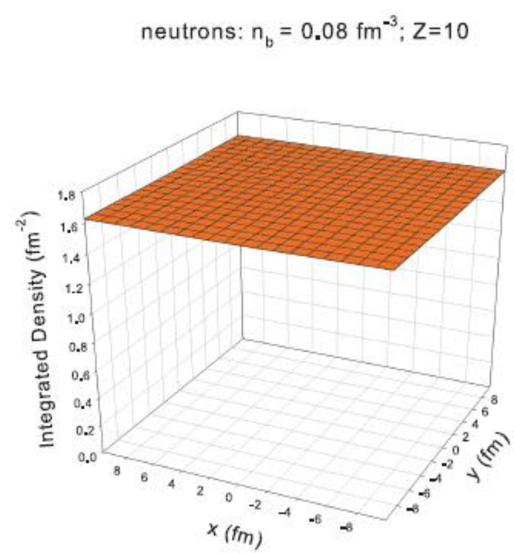


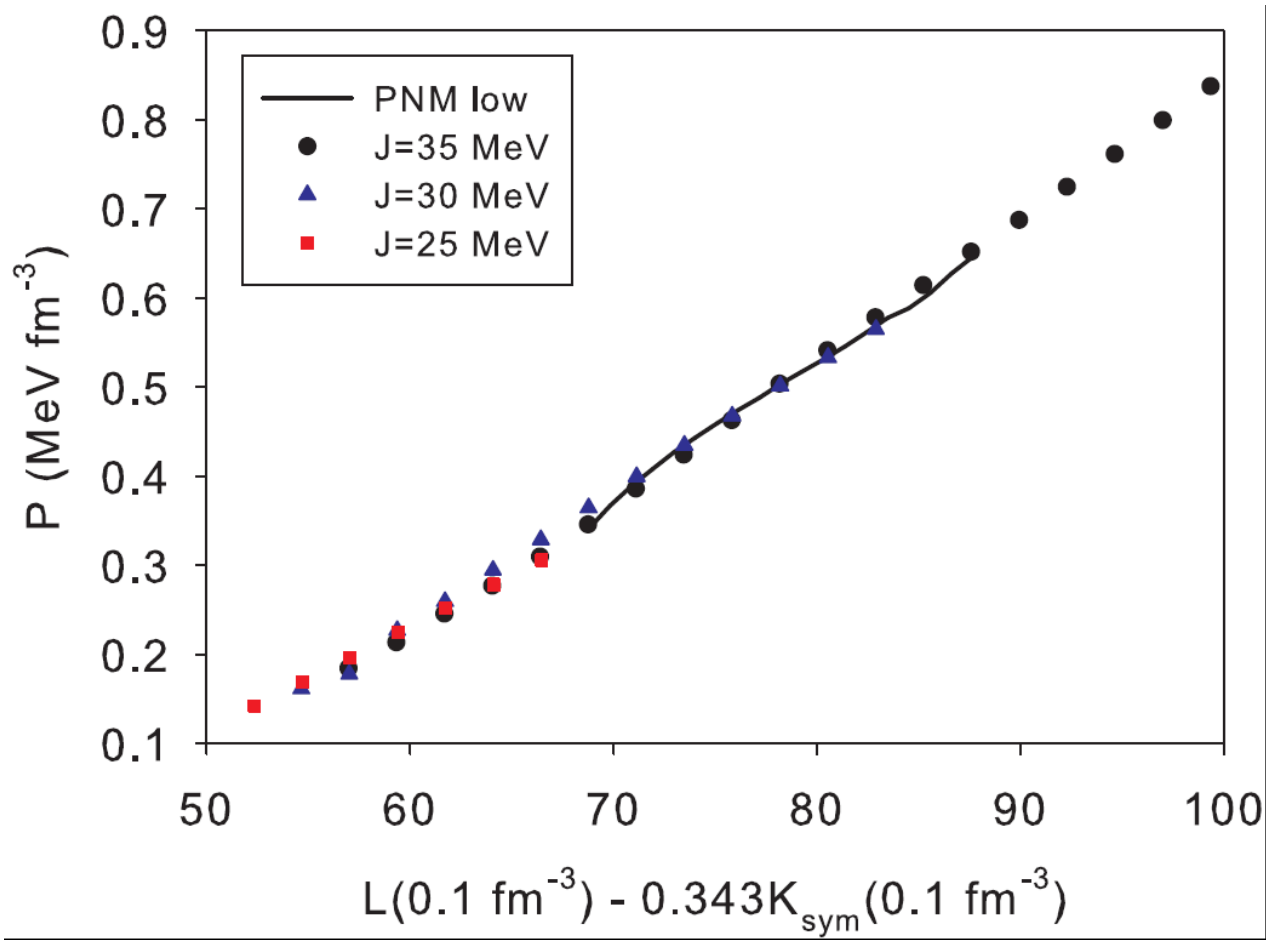
CONCLUSIONS and FUTURE

- Significant variation in crustal properties within liquid drop model
- Consistent treatment of core and crust EoS important
- Need to assess where liquid drop model breaks down
  - comparison with more microscopic models (e.g. 3DHF)









PNM low
J=35 MeV
J=30 MeV
J=25 MeV
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
P (MeV fm^-3)
50
60
70
80
90
100
L(0.1 fm^-3) - 0.343K_sym(0.1 fm^-3)