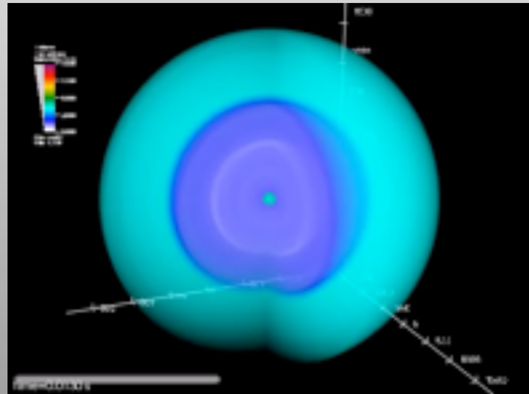
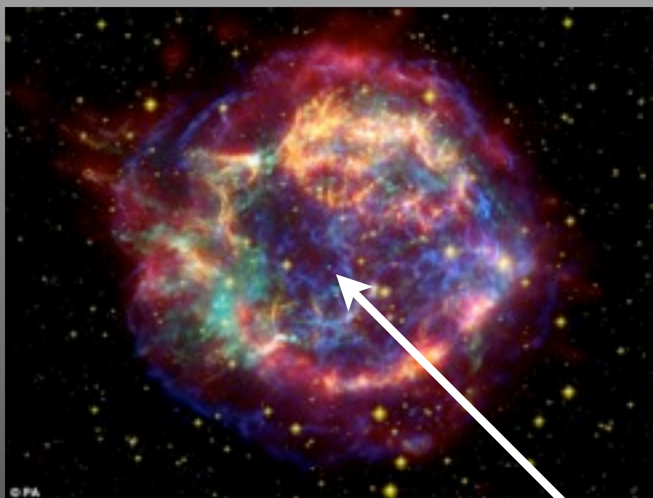
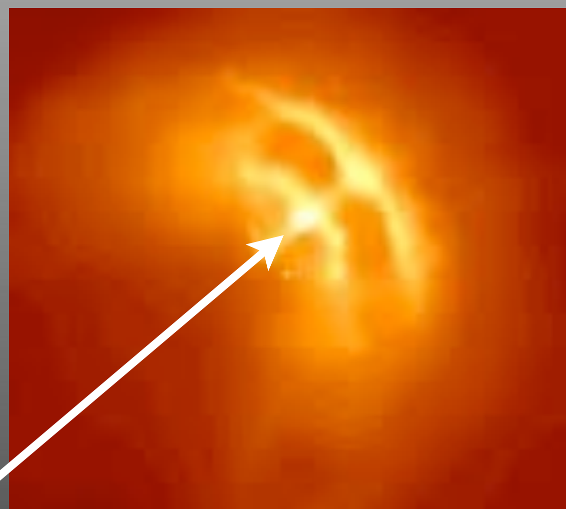
Magnetar Flares: Starquakes or Stellar Flares?

Maxim Lyutikov (Purdue)

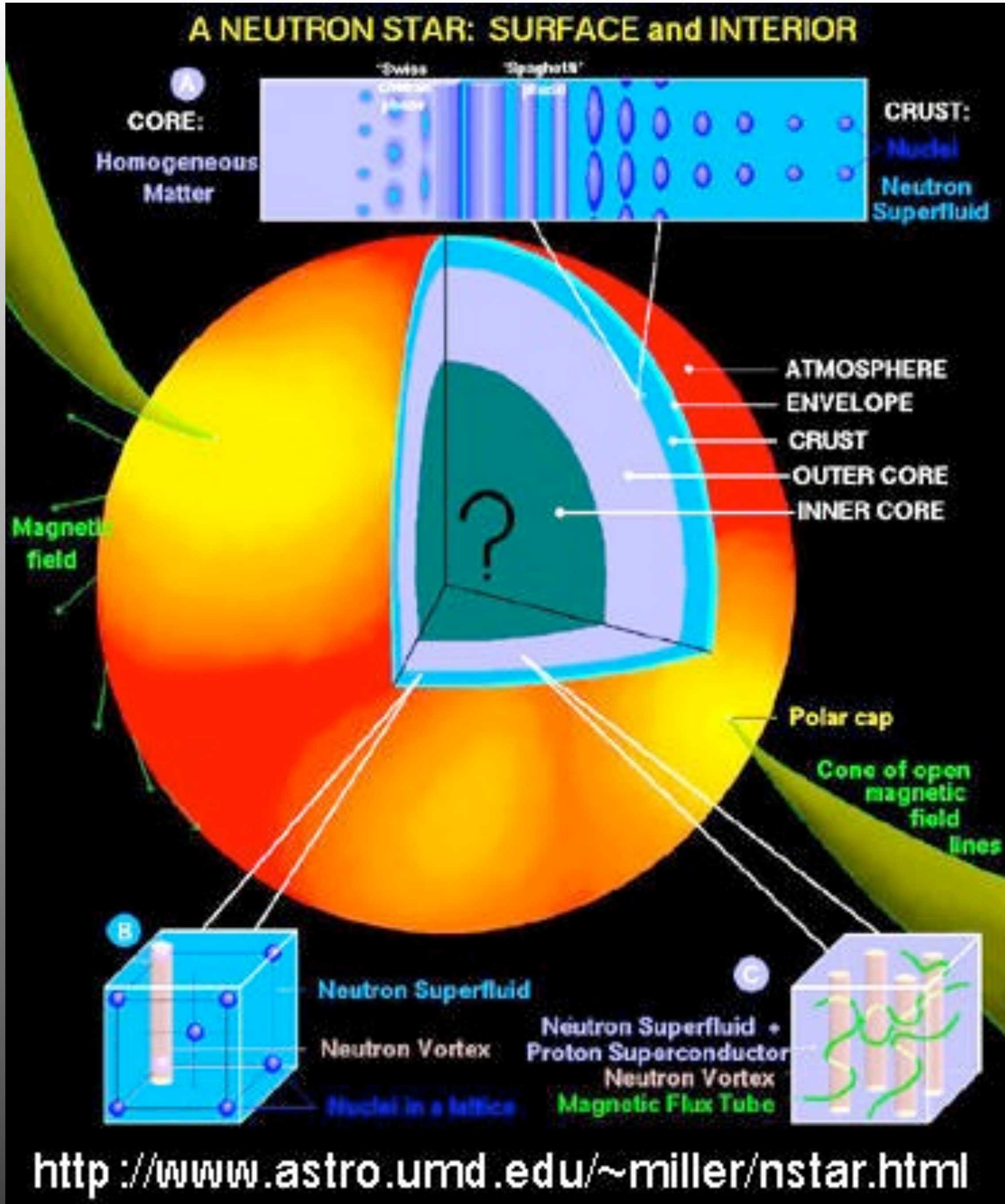
Neutron stars are born in supernova explosions





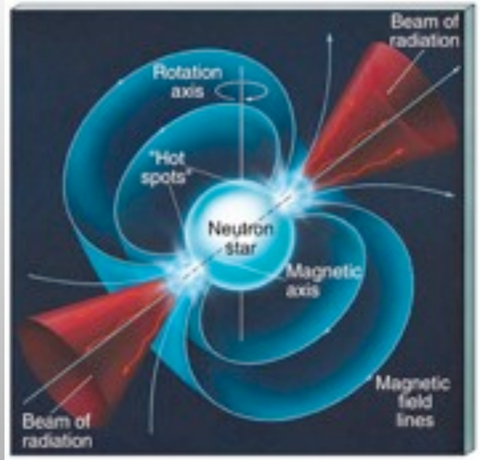


NS


A NEUTRON STAR: SURFACE and INTERIOR
CORE:
Homogeneous Matter
CRUST:
Nuclei
Neutron Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic field
Polar cap
Cone of open magnetic field lines
Neutron Superfluid
Neutron Vortex
Neutron Superfluid + Proton Superconductor
Neutron Vortex
Magnetic Flux Tube
http://www.astro.umd.edu/~miller/nstar.html

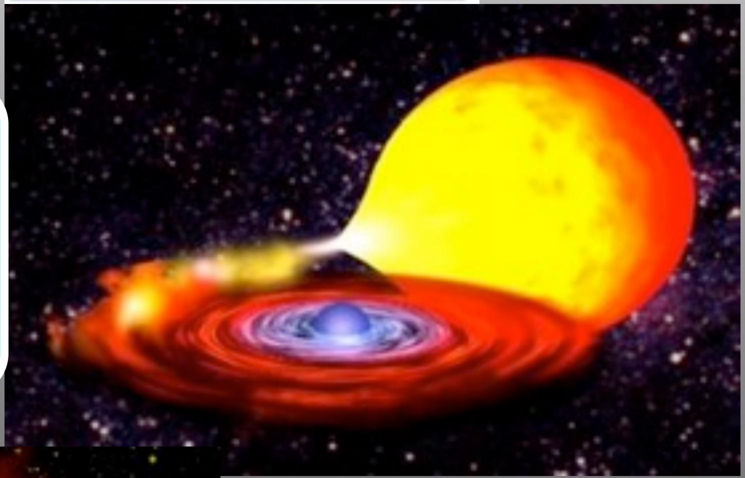
Observations of NS

- **Pulsars**: 1800 pulsars observed in radio
- The youngest seen also at higher energies
- Mostly isolated
- Typical rotation periods: 1.5 ms – 5 s



Accreting NSs: several hundreds in High Mass and Low Mass X-ray binaries

- May be transients
- Typical rotation periods 0.1-1000 s



3. **CCO**: thermal spectra, T~ keV

4. Magnetars

Magnetars

Old definition:

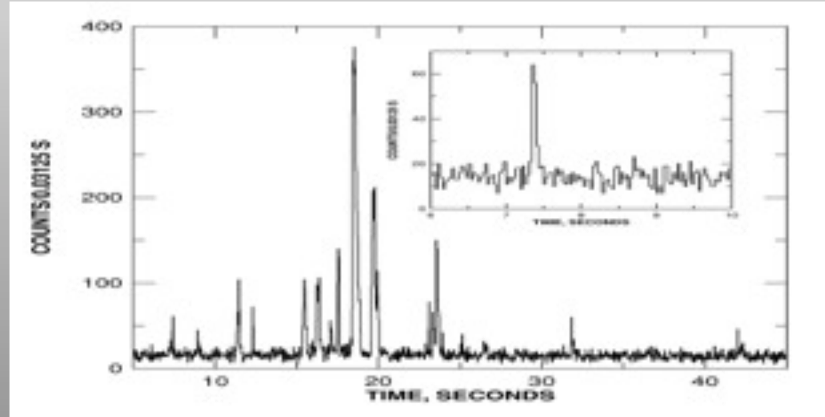
- isolated,
- youngish ~ 10^4 yrs,
- L_X >> E_{dot} (not rotationally powered)
- SGRs
- AXP

Now:

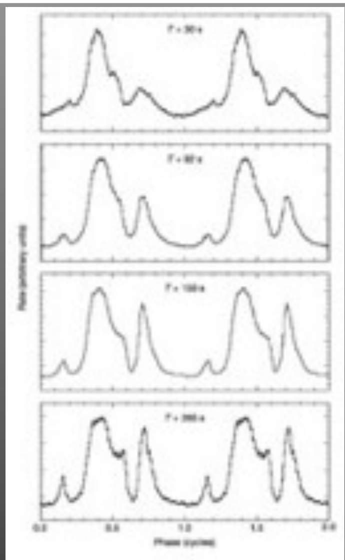
- magnetar-like bursts from young PSR J1846-0258 (Gavriil et al 2008)
- Low B-field magentar SGR 0418+5729, t~24 Myr (Rea et al. 2010)
- **Perhaps all NSs show magnetar-like behavior to some extent**

Magnetars A: Soft Gamma Ray repeaters (SGRs)

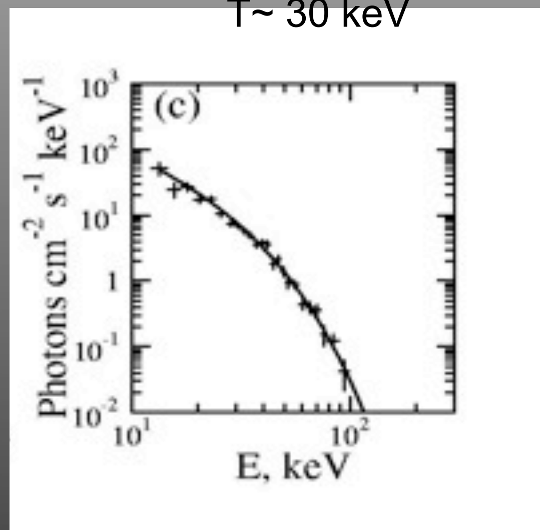
- **SGRs** emit short (< 1 s) repeating bursts of hard X / soft gamma- rays with soft spectrum $L_x \sim 10^{40} - 10^{41}$ erg/s (Super Eddington for a NS)
- Pulsations 2.6 – 8 sec
- Isolated



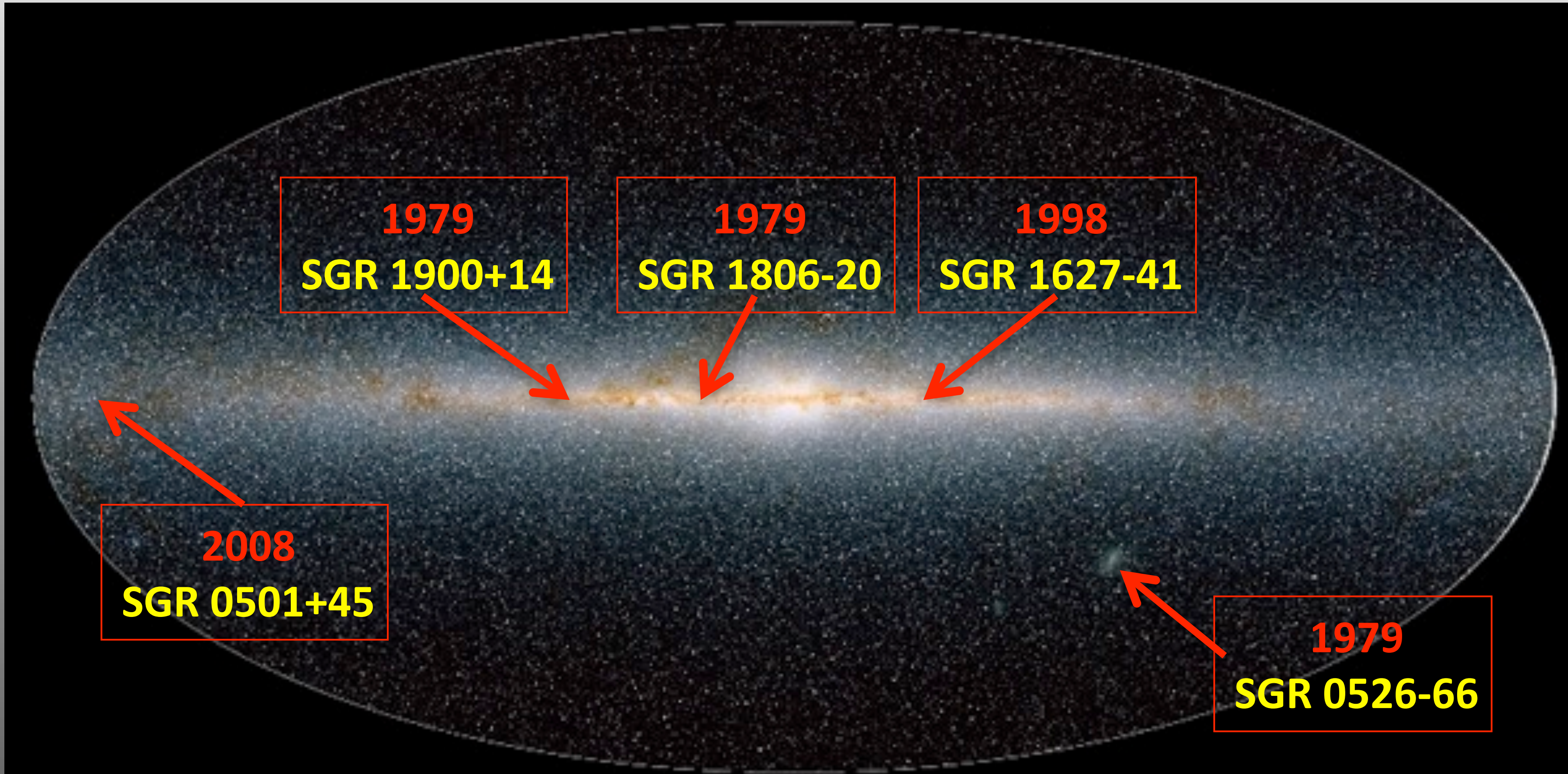
Hurley et al. 1999



T~ 30 keV



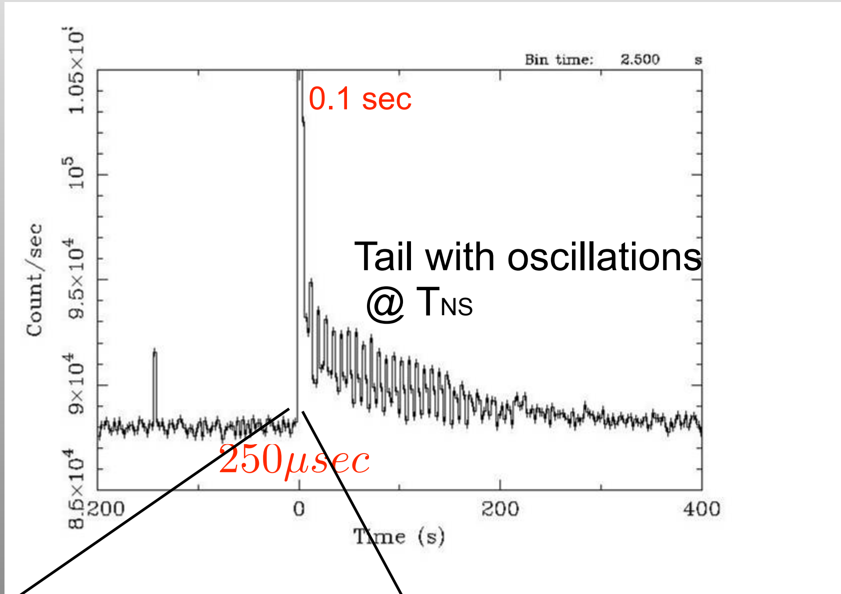
Location and discovery date of the 5 SGRs

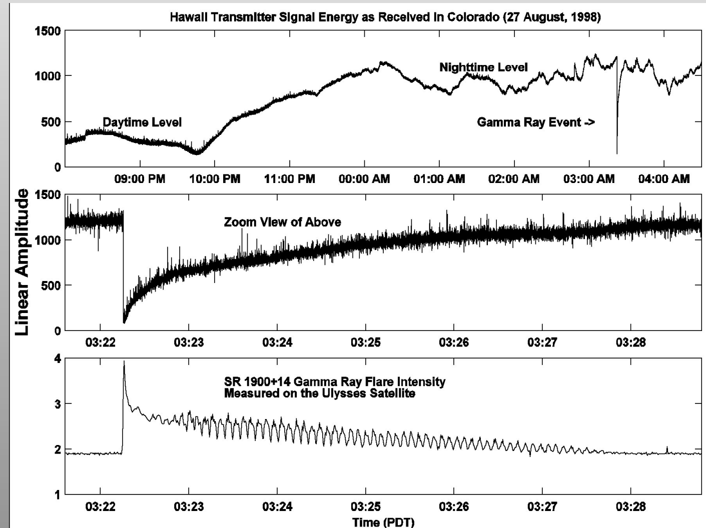


S.Mereghetti

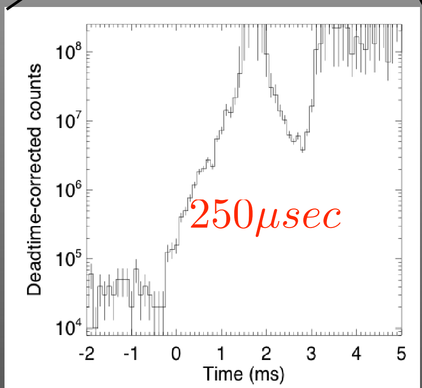
- **SGRs** were initially confused with GRBs.

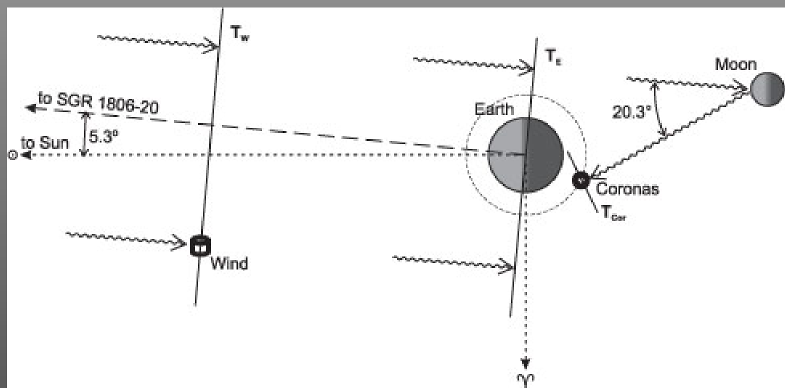
SGR Giant flares: $L_{\text{peak}} \sim 10^{47} erg/s$

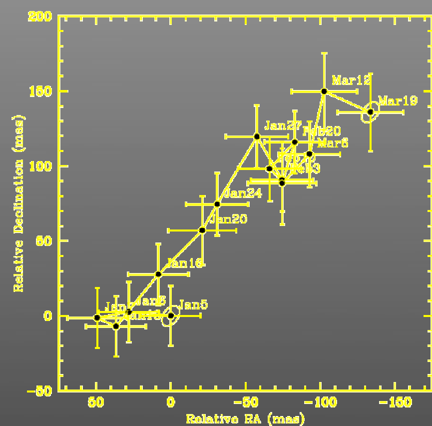




Night ionosphere became like a day







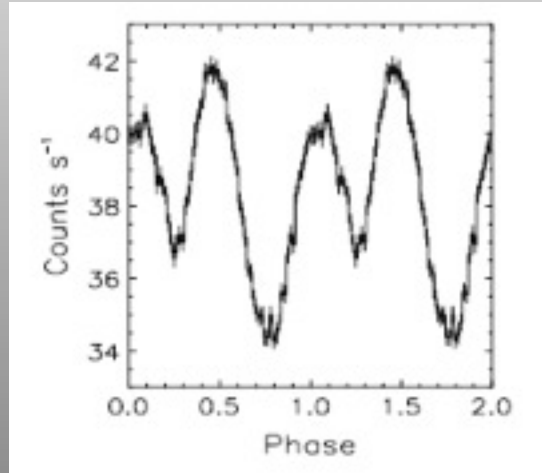
Rise time $250\mu s$

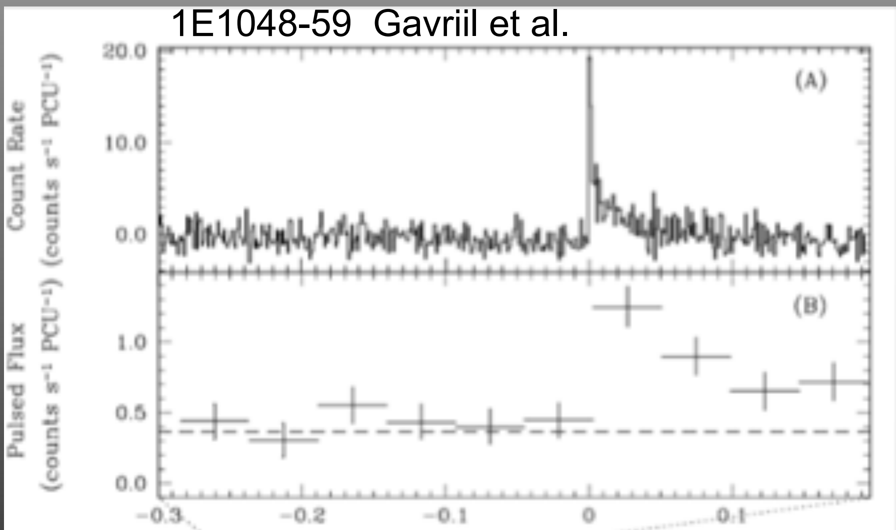
Compton reflection from the Moon

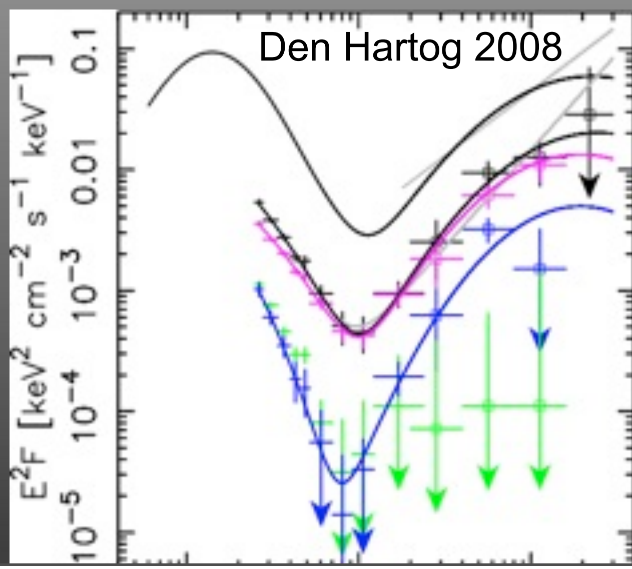
“CME” is seen

Magnetars B: Anomalous X-ray pulsars (AXPs)

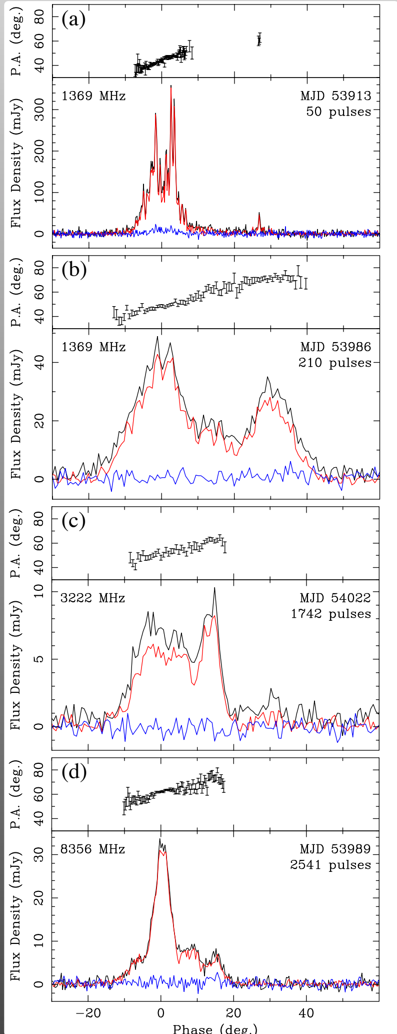
-Persistent L_x ~ 10^{34} - 10^{36} erg s^{-1}
- soft X-ray spectrum (kT~0.5 keV)
+ hard tail up to 200 keV
- 3 are in Supernova Remnants
- 3 are transients
- Also bursting



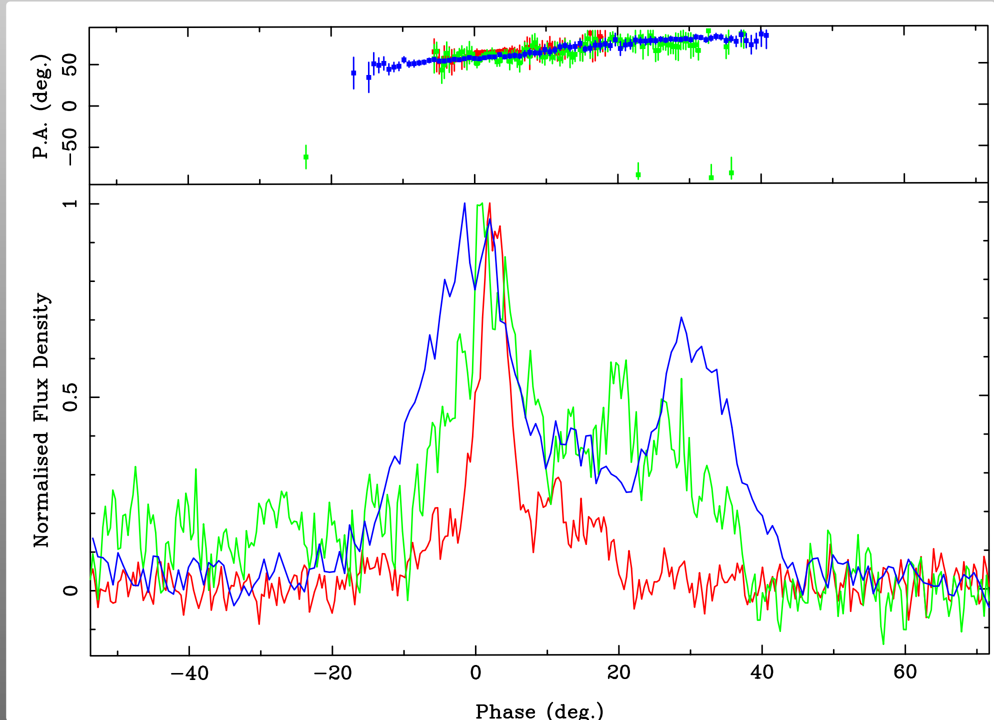




Radio emission: high variable, extending to very high freq.



Camilo et al



Brightest pulsar at 40 GHz

(Classic) Magnetars are powered by dissipation of superstrong B-field, B~10^{14-15} G

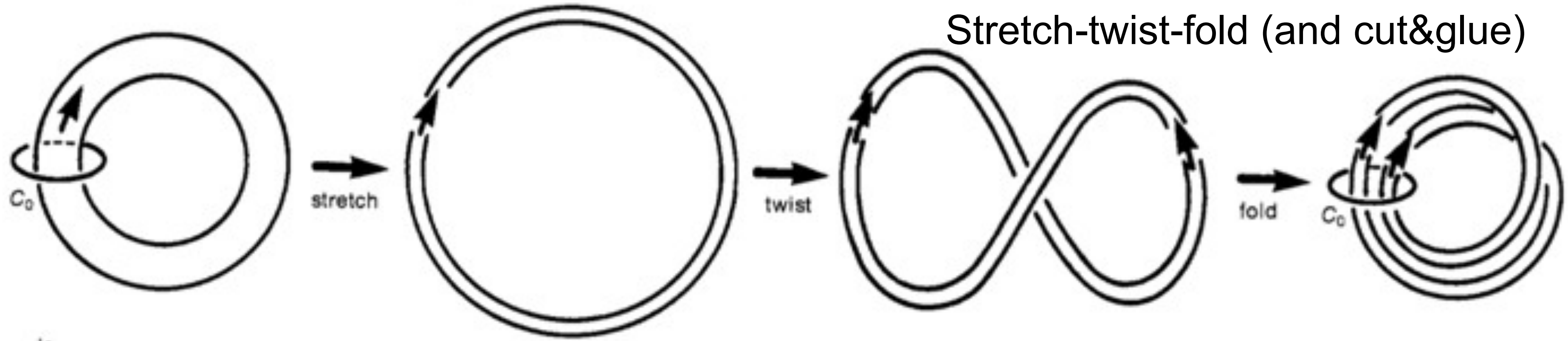
- $L_X = 10^{34}$ - 10^{36} erg s^{-1} > 100 $L_{spindown}$, $L_{spindown} = I\,\Omega\dot{\Omega}$ (not rotationally powered)
- Spin periods P = 5 – 12 s - slow
- Characteristic ages 3 10 ^{3} -- 4 10^{5} yr

Thompson & Dunkan

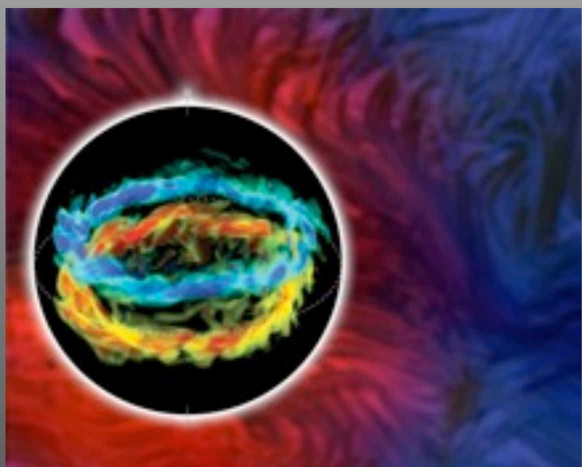
- From spindown $I\Omega\dot{\Omega} \sim B^2 R_{NS}^2 c \left(\frac{\Omega R_{NS}}{c}\right)^4$
- From flare energetics: $E_{\rm flare} \sim E_{\rm tail} \sim B^2 R_{NS}^3$

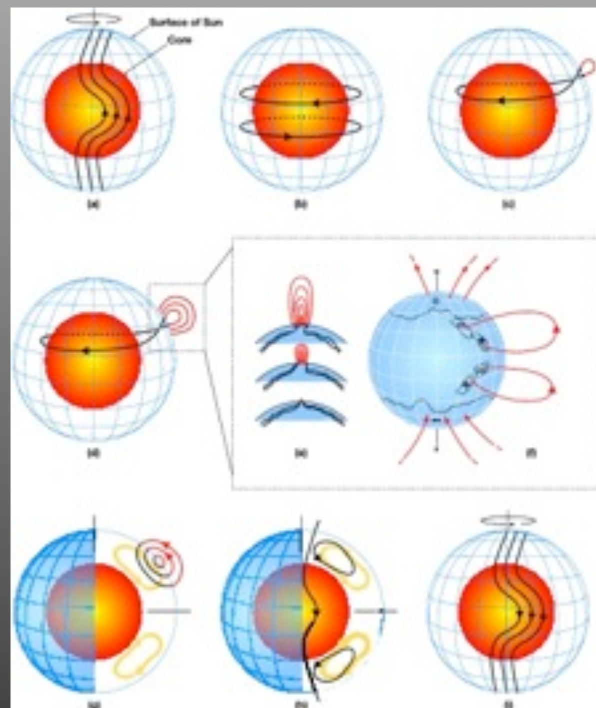
} $B \sim 10^{14} - 10^{15} G$

Origin of B-field: dynamo



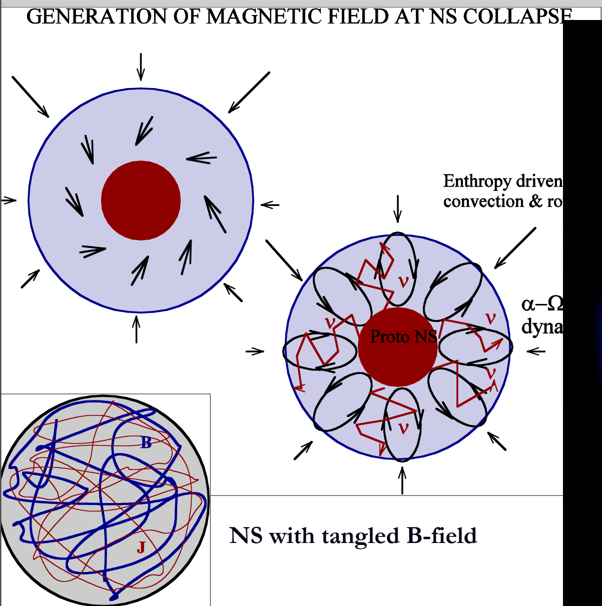
Solar dynamo

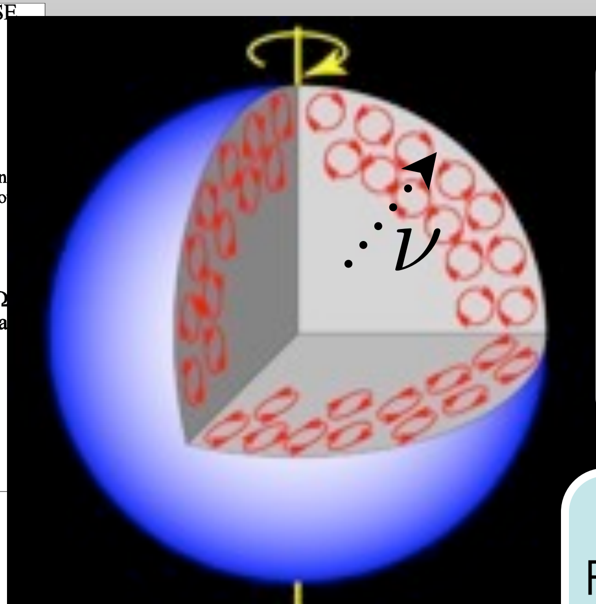




- Note: time scale for B-field generation, 11 yrs, unrelated to flares onset, ~ 1min.

Magnetar dynamo: only ~ 10 sec after birth.





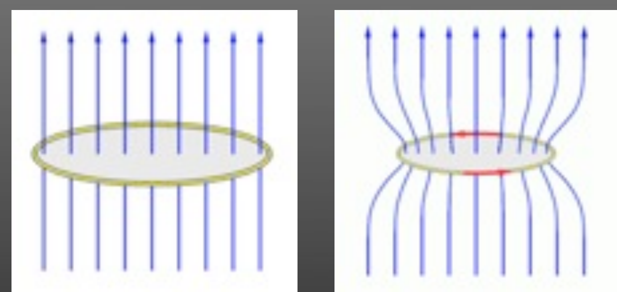
- In magnetars, convection time scale, ~ msec, is << than rotation (?),

- Rossby number R~ $P_{rot}/t_{conv} > 1$; typically alpha-omega dynamo needs R < 1.

Just compressed B-field? Probably not: magnetic flux >> flux of most magnetized WDs

Field may be limited to outer layers,

- Turbulence is most efficient at tau ~ 1
- Buoyancy will bring B-field "up"
- will be later locked in a crust - assumption of the model.



B-field relaxes to MHD equilibrium (10-100 secs)

Turbulence dies out @ ~ 10 sec,

NS relaxes to an MHD equilibrium.

B-field must be a combination of toroidal and poloidal field, otherwise unstable

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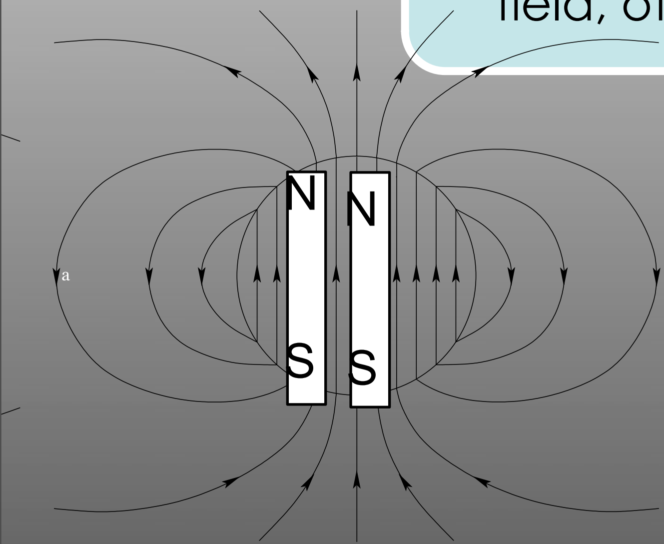

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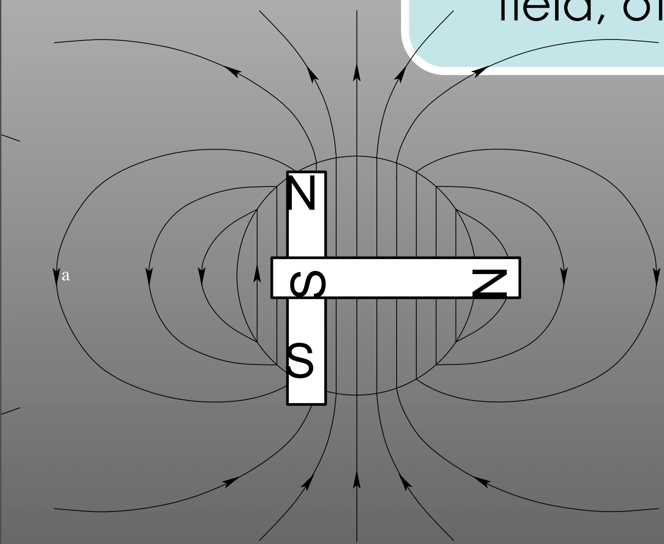

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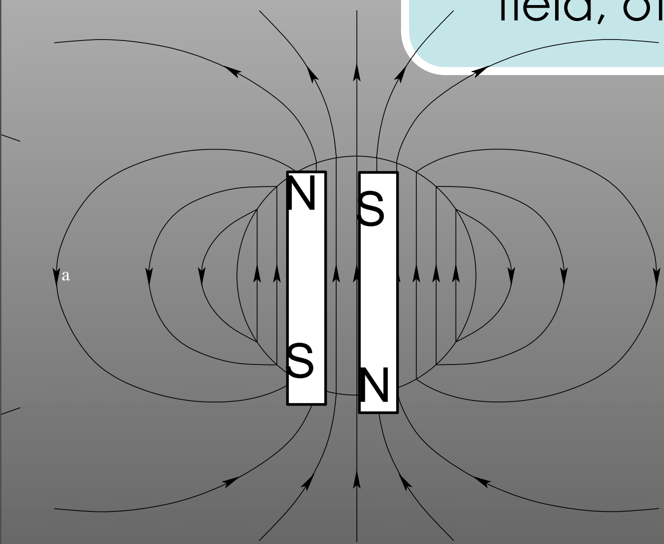

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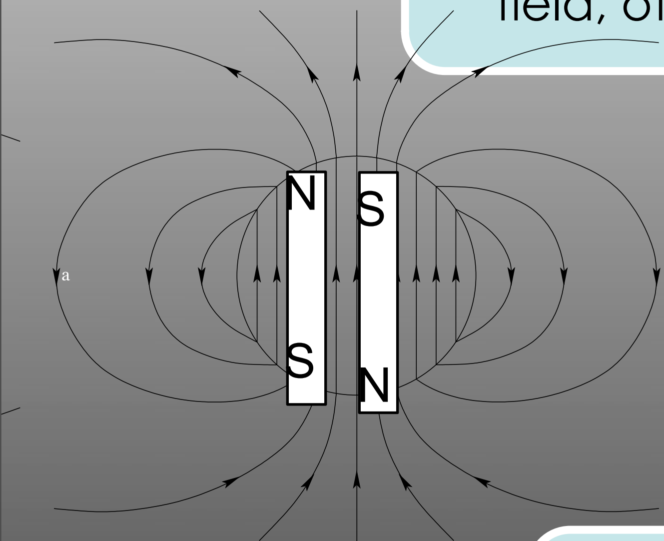
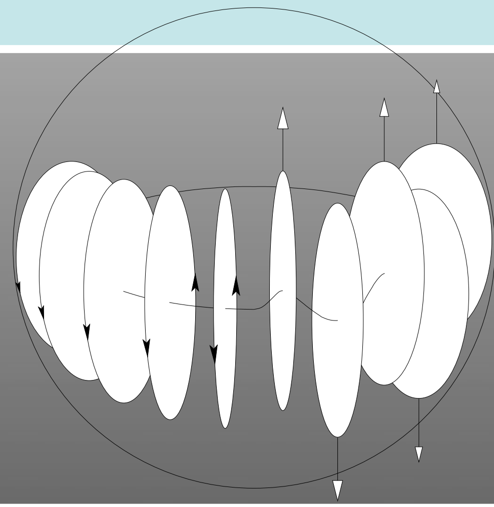


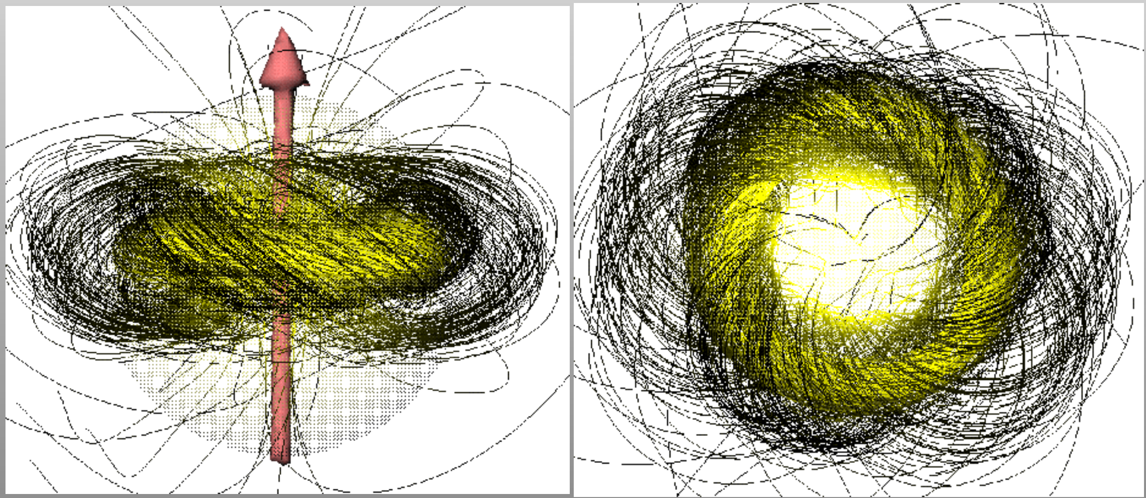


Need toroidal B-field to stabilize poloidal and vice versa

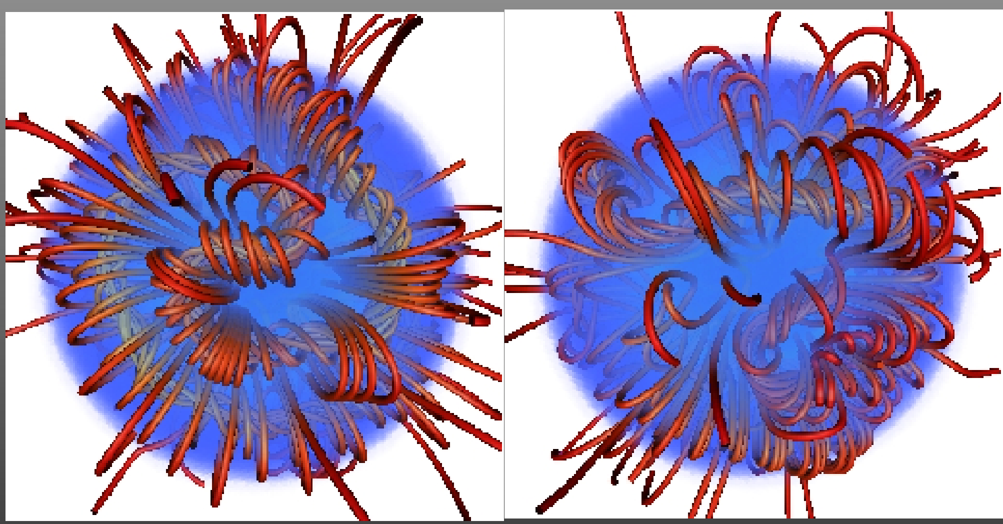
(Prendergast, Dungey, Fawley & Ruderman, Braithwaite)

Poloidal-toroidal equilibria of fluid stars

(Braithwaite)



large toroidal field - stable



small toroidal field - unstable

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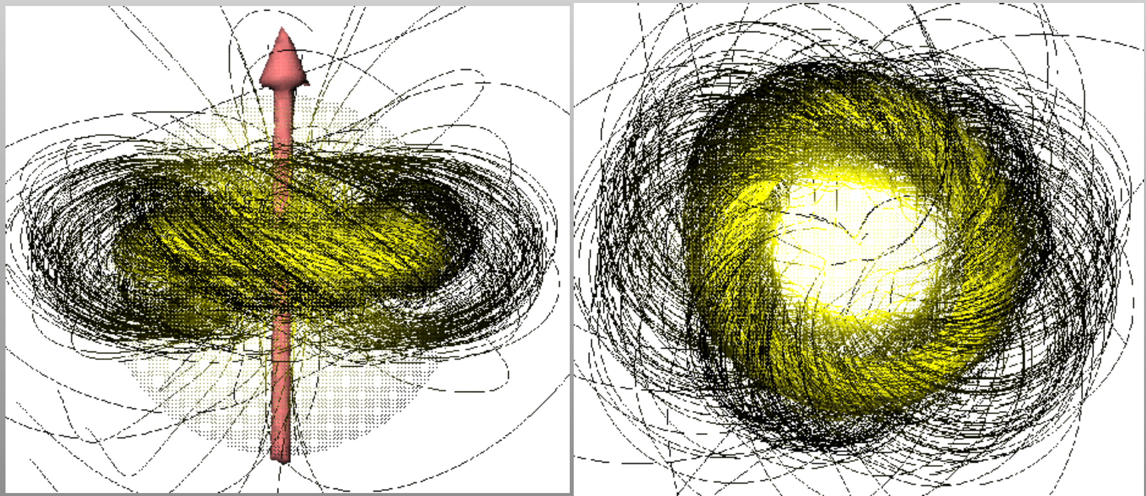

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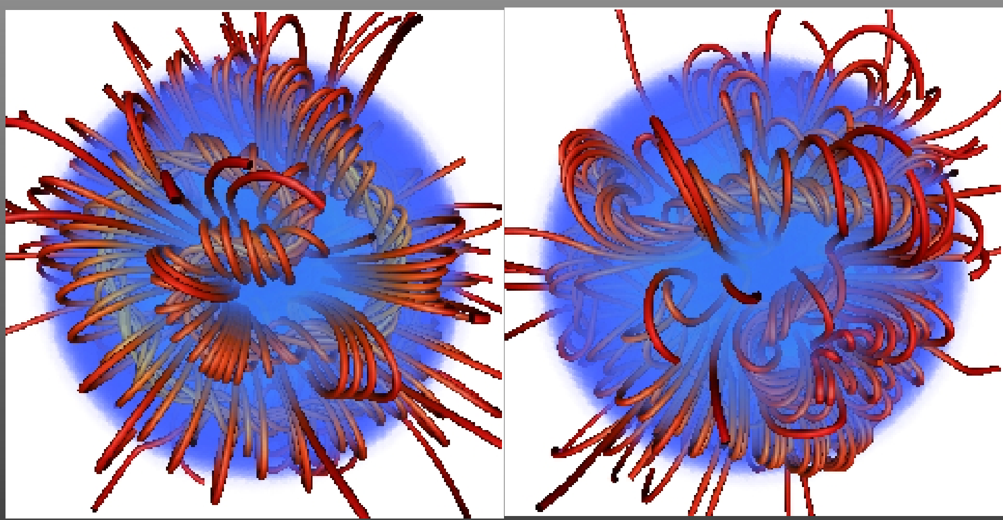
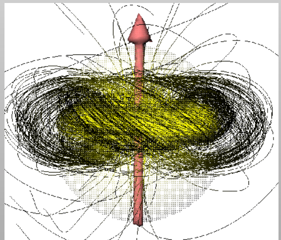


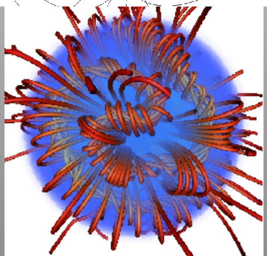
small toroidal field - unstable

Ask me later about mathematical structure of these solutions

III. @ ~ 100 sec crust freezes, dynamics is Electron MHD (EMHD)

Kingsep, 1989





3. Crust freezes, t ~ 100 sec
(no shear tresses at freezing)

EMHD: After freezing, ions form a fixed lattice, electrons flow as fluid, velocity= current: J = - n e v

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} = \frac{\mathbf{J}}{ne} \times \mathbf{B} \;, \quad \mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c}{4\pi e} \nabla \times \left(\frac{\nabla \times \mathbf{B}}{n} \times \mathbf{B} \right)$$

Note: no inertia!

- Electrons flow as an inertialess fluid
- Hall-dominated plasma

MHD equilibrium is, generally, not EMHD equilibrium

MHD: $$\mathbf{J} \times \mathbf{B} = \nabla p + \rho \nabla \Phi$$

EMHD: $$\mathbf{E} = -\mathbf{v} \times \mathbf{B} = \frac{\mathbf{J}}{ne} \times \mathbf{B}$$

$$\nabla \times \frac{\mathbf{J} \times \mathbf{B}}{\rho} = -\frac{\nabla p \times \nabla \rho}{\rho^2}$$

$$\nabla \times \frac{\mathbf{J} \times \mathbf{B}}{n} = 0$$

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Non-barotropic EoS

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mu-gradient

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Freezing of MHD equilibrium results in non-equilibrium EMHD state

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- Whistler waves are launched. Whistlers exert shear stress on the crust an may break it.
- Not clear what state a Hall plasma wants to achieve

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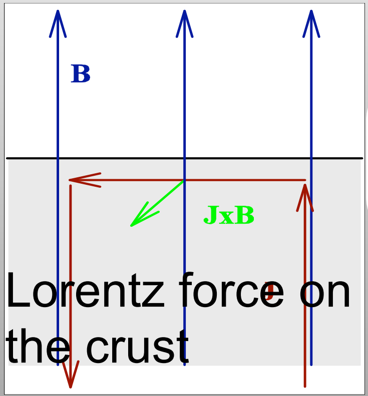
4. Shear stresses build on Hall time $\tau_H = \frac{L^2 \omega_p^2}{c^2 \omega_B} = 4 \times 10^3 L_4^2 B_{14}^{-1}$ yrs

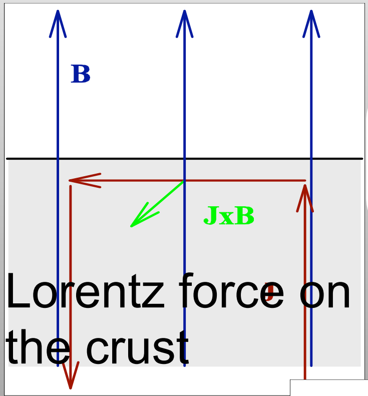
Magnetar years



Lorentz force on the crust

5. If shear stress due to Lorentz force is strong enough, it breaks the crust

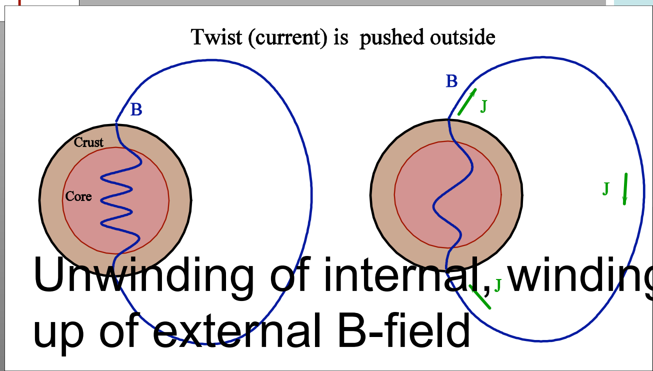
Magnetar years



Lorentz force on the crust

5. If shear stress due to Lorentz force is strong enough, it breaks the crust

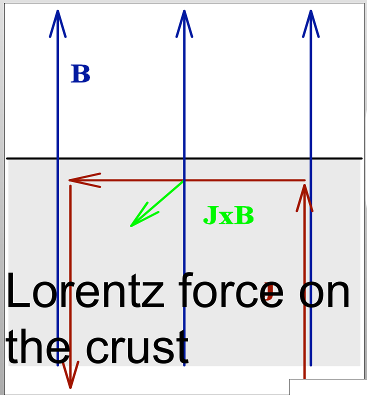
6. Unwinding of internal B-field can be
   a. sudden (cracking)
   b. plastic (magnetospheric instability of B-field)



Unwinding of internal, winding up of external B-field

Cannot lift the crust, only rotate

Magnetar years



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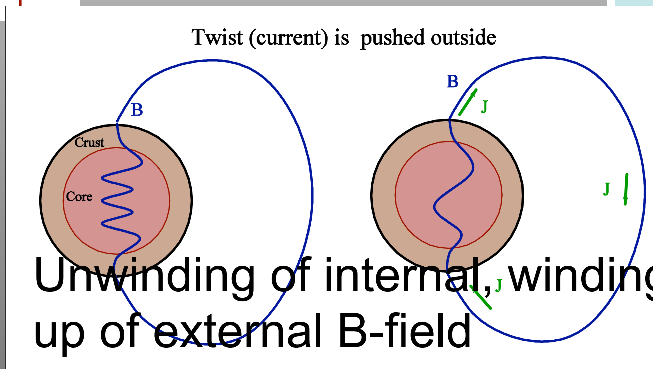

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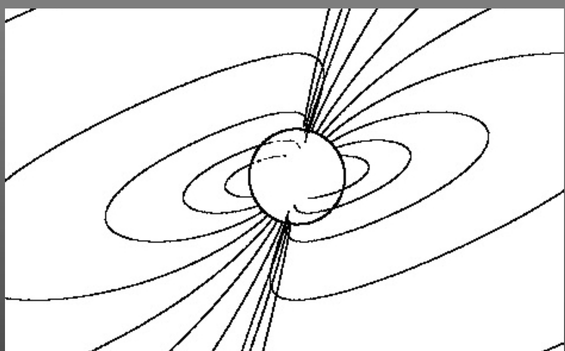
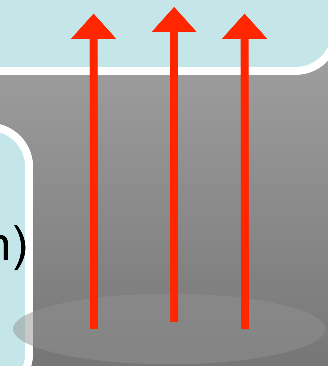


7. High energy emission/flares are generated in the twisted magnetosphere

Production of burst and (giant) flares

1. At freezing, no shear stresses: newborn stars are not magnetars
2. Stresses build on Hall time scale

$$\tau_H = \frac{L^2 \omega_p^2}{c^2 \omega_B} = 4 \times 10^3 L_4^2 B_{14}^{-1} \,\mathrm{yrs}$$



Flares: release of crustal stresses

- **cracking (star-quake)**(Thompson & Dunkan)
- **plastic (a la Solar flares & CMEs)** (Lyutikov)

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Sensitive to parameters, can vary highly

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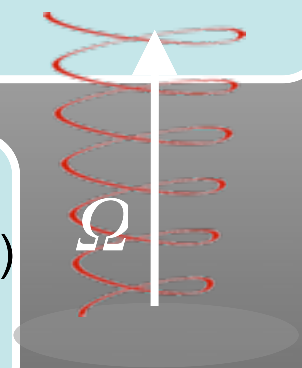
Sensitive to parameters, can vary highly



Flares: release of crustal stresses

- **cracking (star-quake)**(Thompson & Dunkan)
- **plastic (a la Solar flares & CMEs)** (Lyutikov)

Production of burst and (giant) flares

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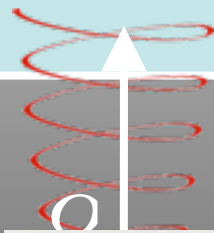

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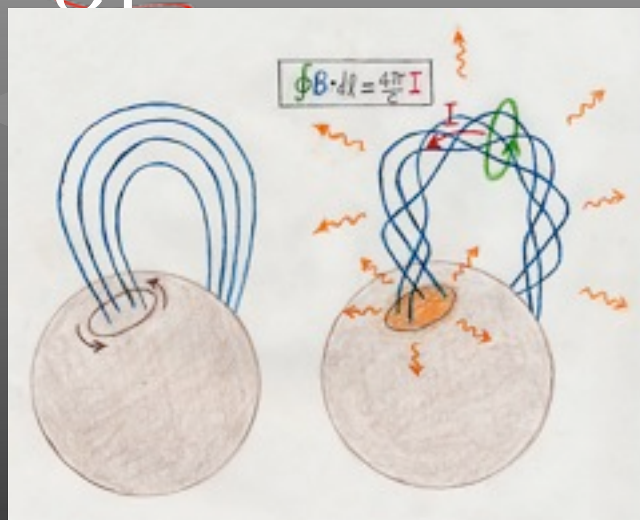

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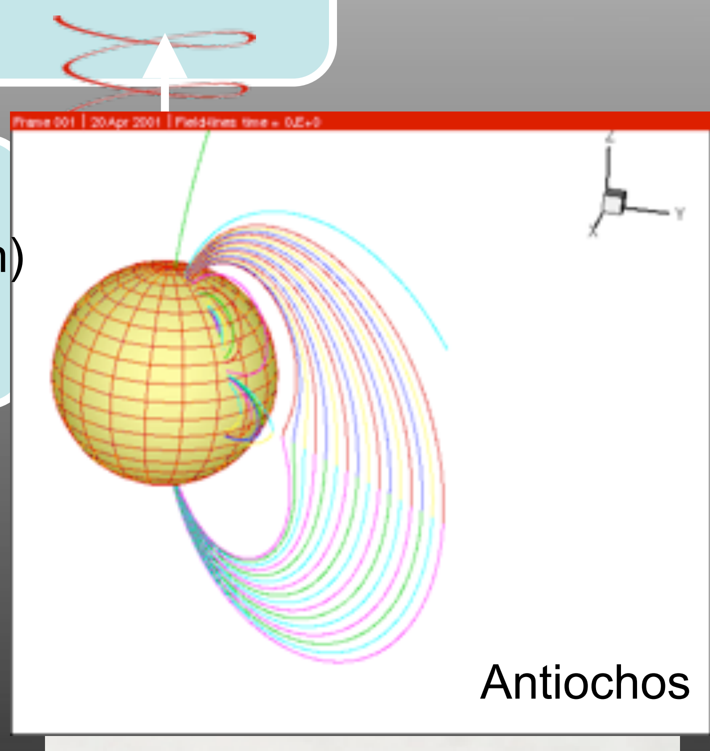

Dynamics of magnetic field-induced cracking *(with Yu. Levin)*

- Not clear if the crust is brittle or plastic?
  - Brittle fracture needs voids at the tip: $v_{shear} > v_s$
  - Not satisfied in NS crusts (or deep Earth quakes)
  - Horowitz: very fast shearing.
- If crust is plastic: no fast cracks
- Let's **assume** that fracture is brittle.

  Critical stress

$$\frac{B_0 B_x}{\mu} \approx \frac{c_A^2}{c_{el}^2} = \sigma_0 = 10^{-5} - 10^{-2}$$

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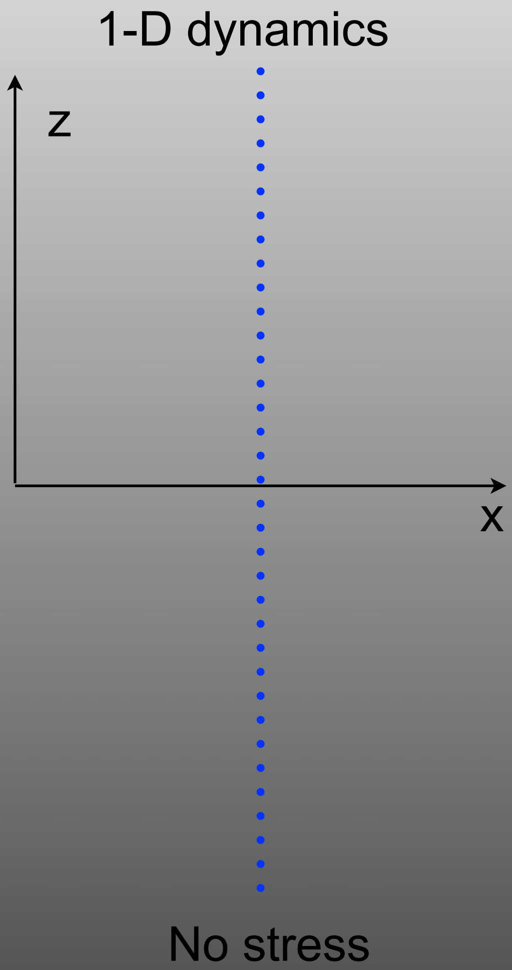
Critical stress

$$\frac{B_0 B_x}{\mu} \approx \frac{c_A^2}{c_{el}^2} = \sigma_0 = 10^{-5} - 10^{-2}$$

Main point: even if the crust is brittle, magnetic crack cannot release a lot of energy quickly enough

Ideal magneto-elastic medium



$$\begin{cases} \rho\ddot{\zeta} = B_{z,0}B_x'/(4\pi) + \mu\zeta'' \\ \dot{B}_x = B_{z,0}\dot{\zeta}' \end{cases}$$

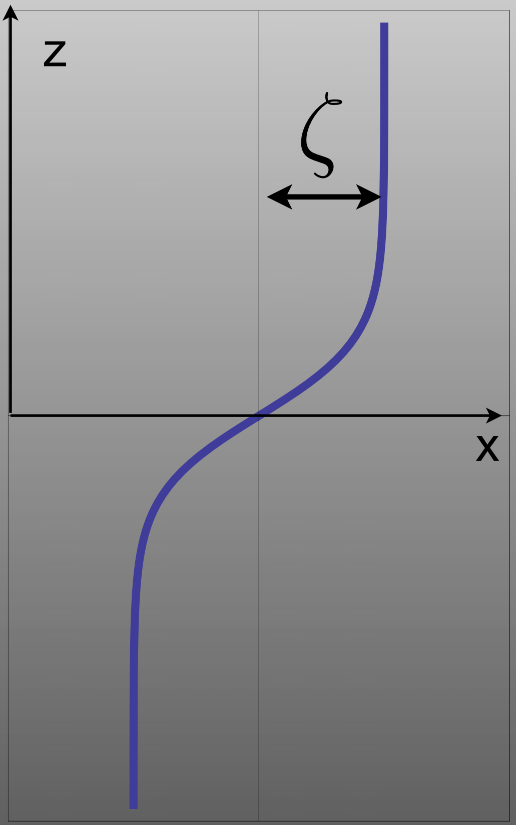
$$\ddot{\zeta} - (c_A^2 + c_{el}^2)\partial_z^2\zeta = 0$$

$$c_{\mathrm{el}} = \sqrt{\mu/\rho}$$

$$c_A = B_z/\sqrt{4\pi\rho}$$

Ideal magneto-elastic medium

1-D dynamics



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"Usual" cracking

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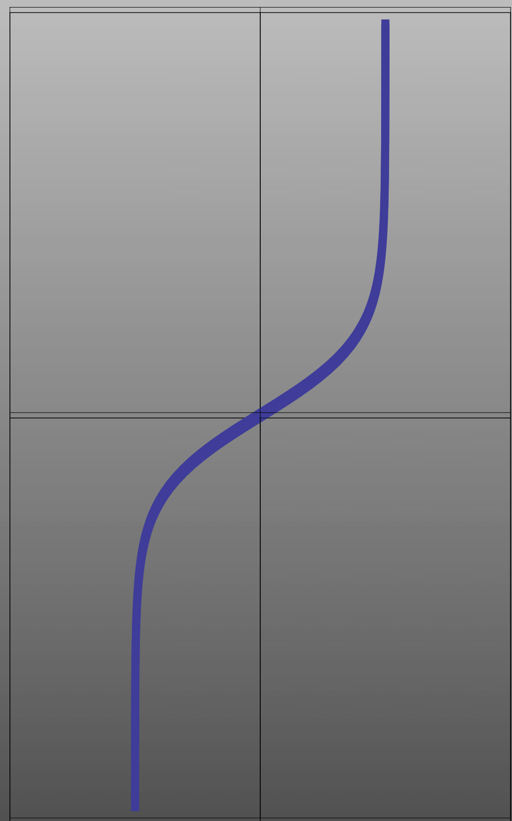

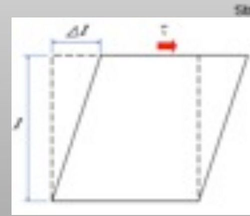

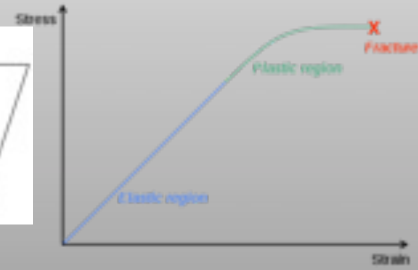

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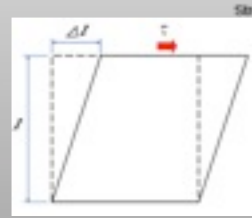

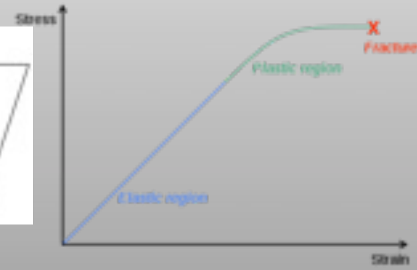

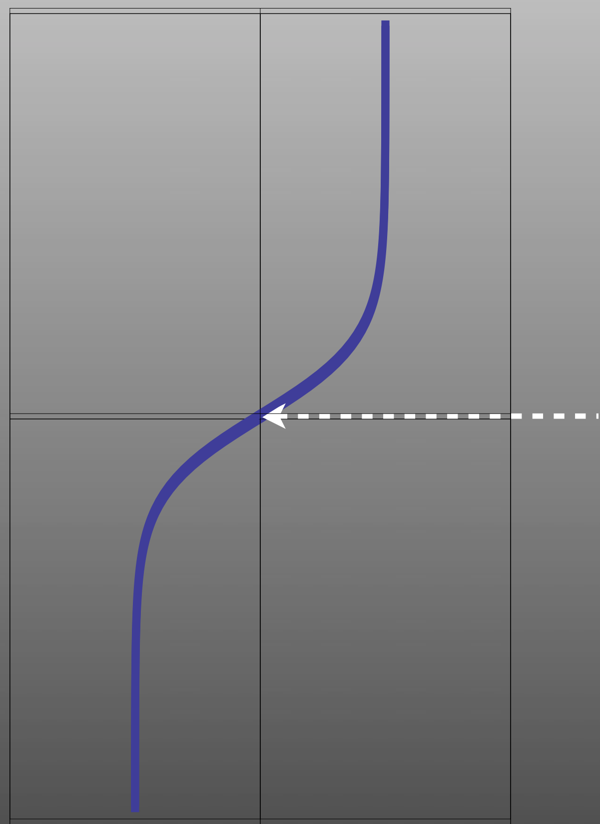

maximal stress $\sigma \rightarrow \max$

“Usual” cracking

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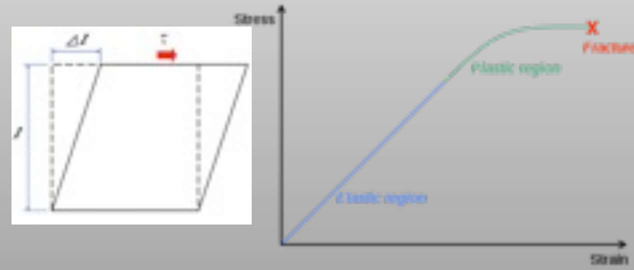

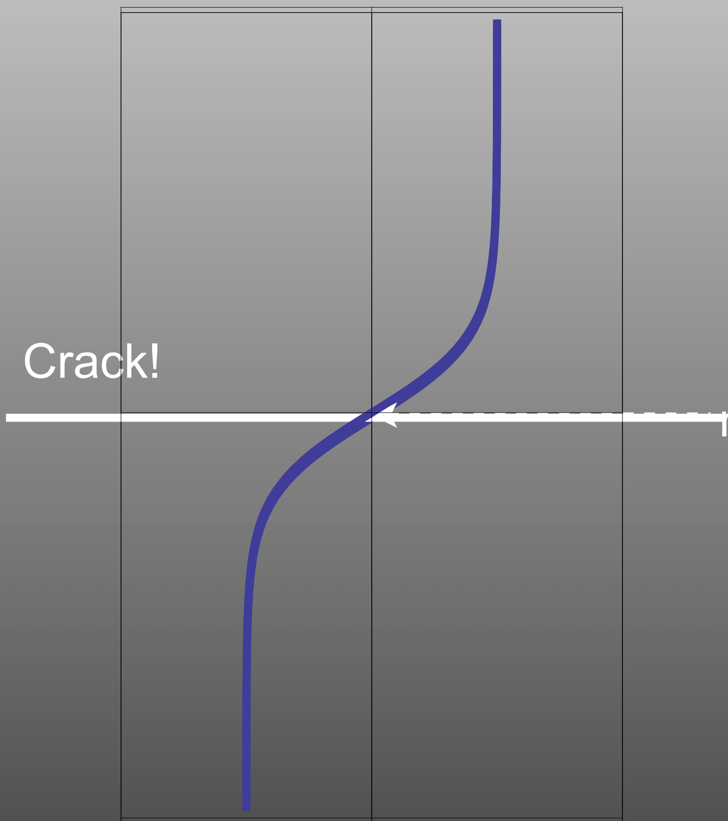

$$\sigma \to \max$$

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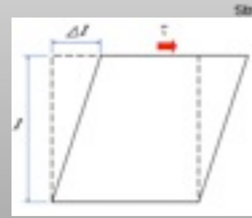

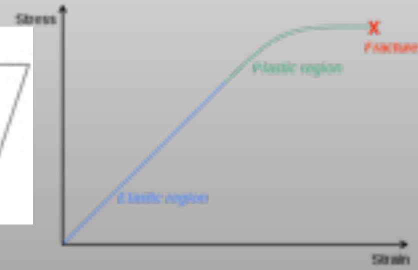

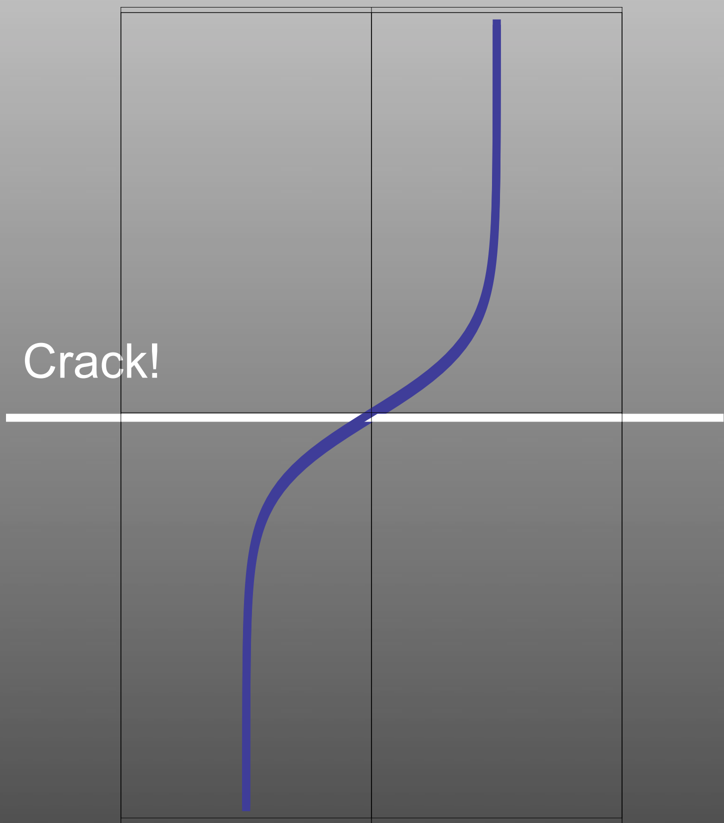

Large σ gradients

$$\sigma \neq 0$$

$$\sigma = 0$$

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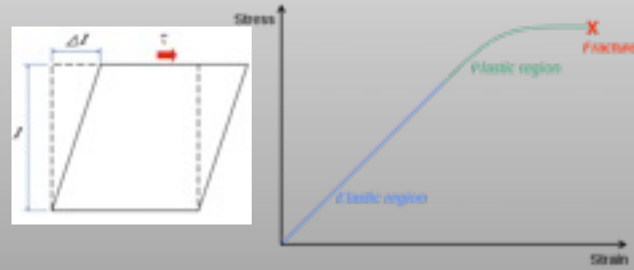

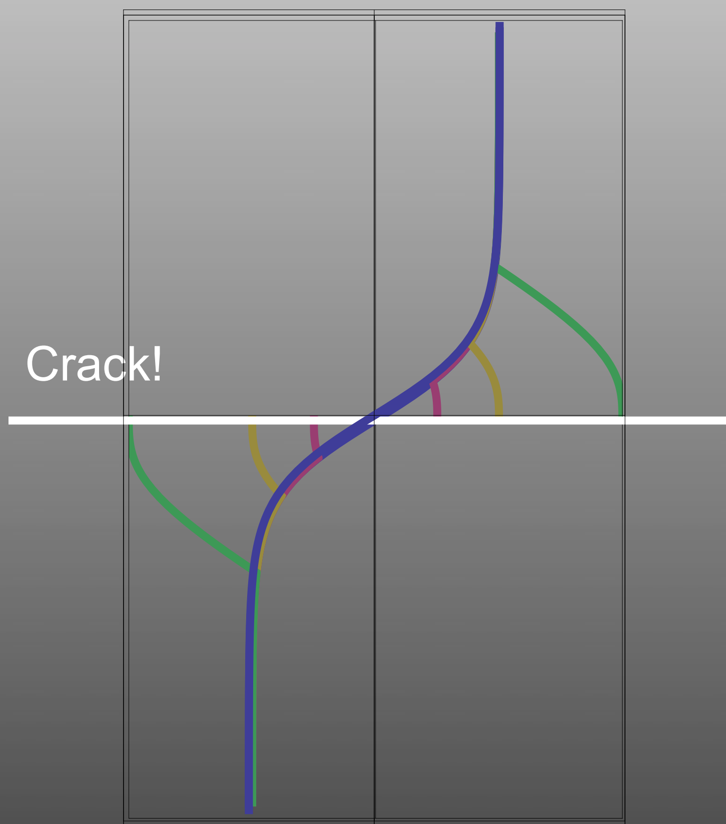

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- Rarefaction wave propagates
- finite velocity @ t=+0,

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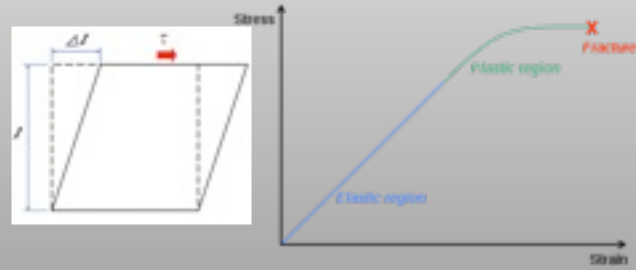

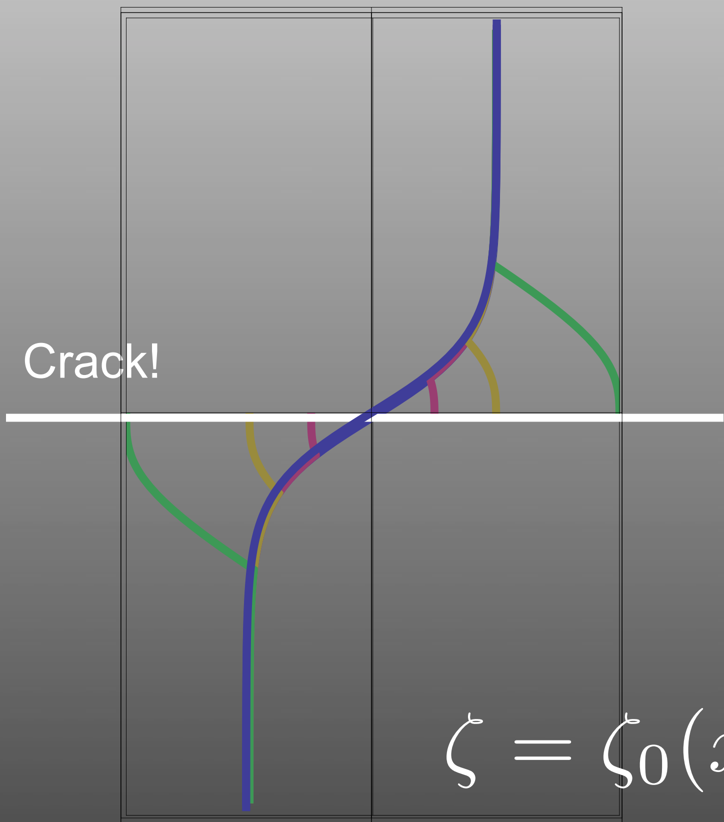

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$$\zeta = \zeta_0(x) - (x - vt)\zeta_0'(0)$$

"Usual" cracking

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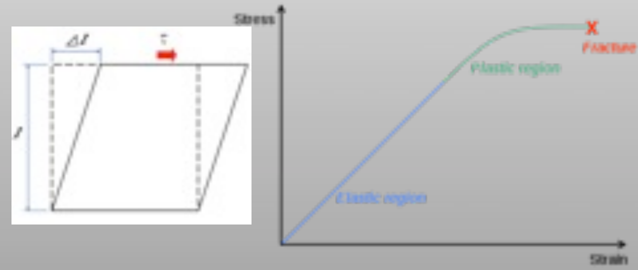
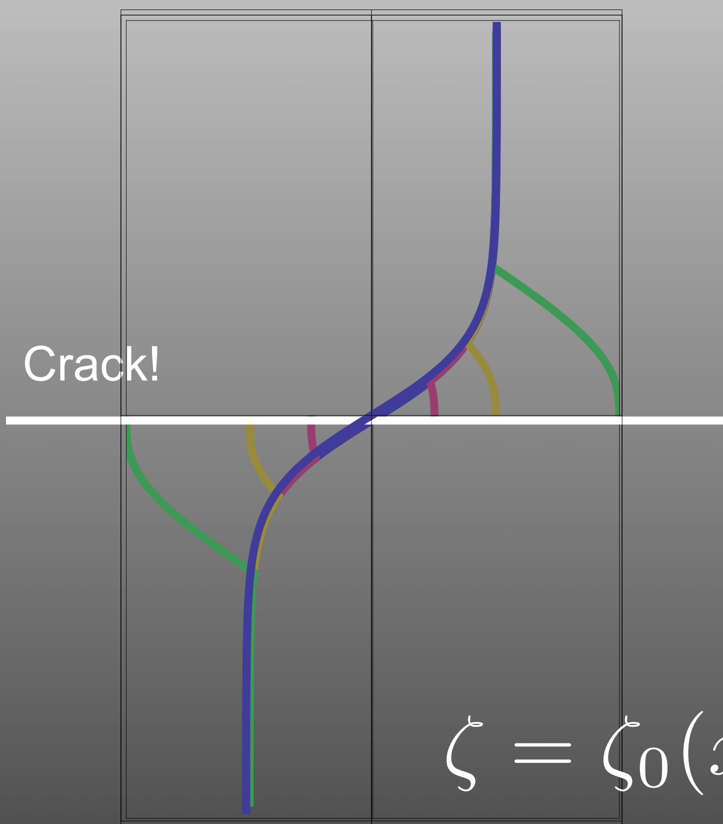
Magnetic field is torn!





Large σ gradients

$$\sigma \neq 0$$

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- Rarefaction wave propagates
- finite velocity @ t=+0,

$$\zeta = \zeta_0(x) - (x - vt)\zeta_0'(0)$$

- Infinitely large Bz

$$\partial_t B_x = \partial_z (B_z v_x) = B_z v_0 \delta(z)$$

Magnetic cracking

Additional condition: continuity of B-field: must take resistivity into account

$$\rho\ddot{\zeta} = B_{z,0}B'_x/(4\pi) + \mu\zeta''$$

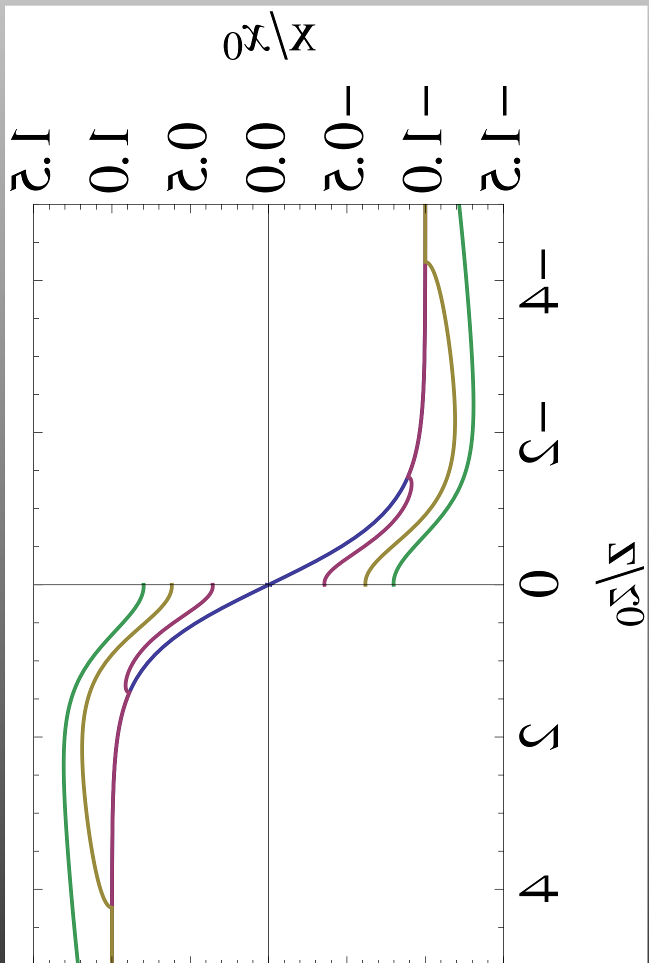
$$\dot{B}_x = B_{z,0}\dot{\zeta}' + \eta_{\rm res}B''_x$$

$$\left(\partial_t - \eta\partial_z^2\right)\left(\partial_t^2 - c_s^2\partial_z^2\right)\zeta = v_A^2\partial_t\partial_z^2\zeta$$

resistive wave   shear wave

Solve using Laplace transform, in the limit eta -> 0
No in-going shear or resistive wave, zero stress and continuous B-field at z=0

$$\zeta(z,t) = \zeta_0(z) + \zeta 0'(0)\left(\frac{2\sqrt{\eta}c_{\mathrm{el}}^3\sqrt{t-\frac{z}{c_t}}\,\Theta\left(t-\frac{z}{c_t}\right)}{\sqrt{\pi}{c_A}^2 c_t} + \frac{2\sqrt{\eta t}c_{\mathrm{el}}e^{-\frac{z^2c_t^2}{4t\eta c_{\mathrm{el}}^2}}}{\sqrt{\pi}c_t} - zErfc\left(\frac{zc_t}{2\sqrt{\eta t}c_{\mathrm{el}}}\right)\right)$$



Amplitude of shear wave is negligibly small

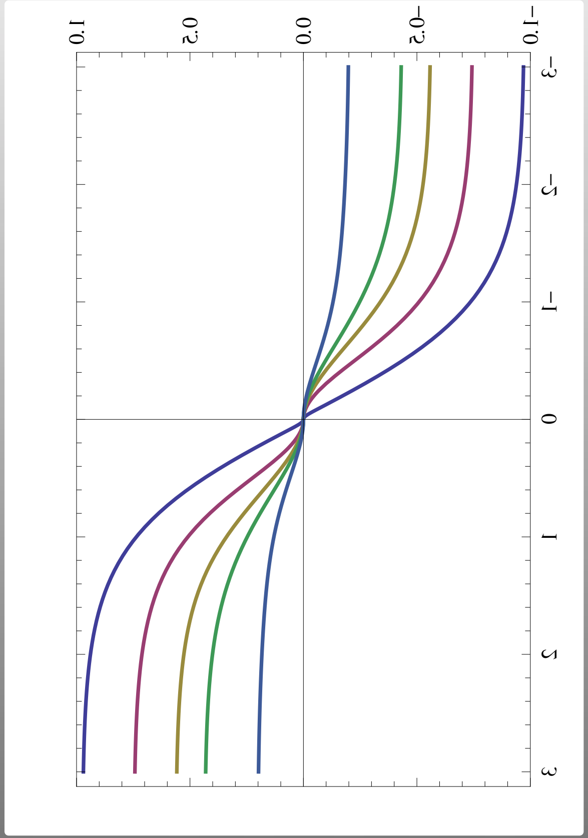
$$\propto \sqrt{\eta}$$

Evolution proceeds on resistive time scale

$$v(z=0) = \frac{c_{\mathrm{el}}c_t}{c_A^2}\sqrt{\frac{\eta}{\pi t}}\zeta_0{}'(0)$$

Crack evolution is slow, on resistive time

- Amplitude of shear wave is negligibly small $\propto \sqrt{\eta}$
- Evolution proceeds on resistive time scale

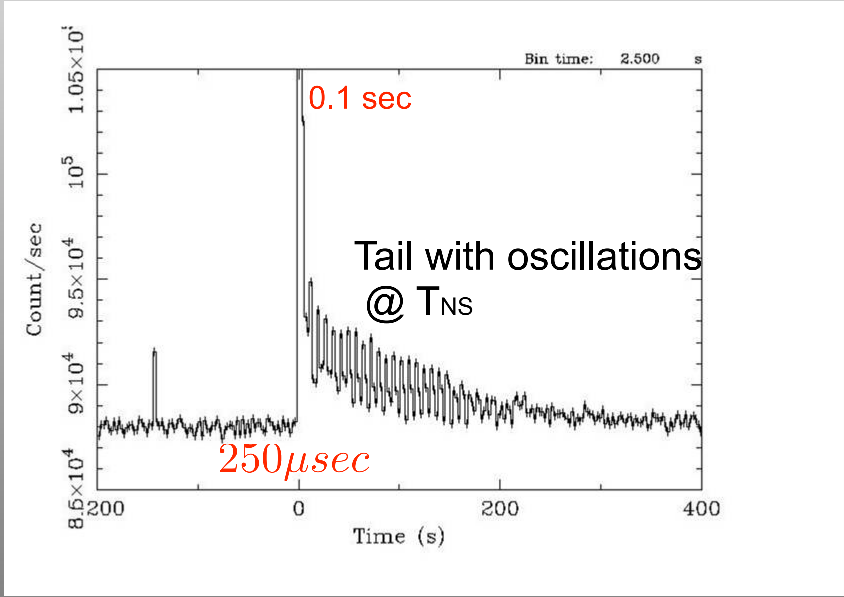


B-field line, (neglecting the small shear wave)

Note: B-field is continuous

Even if crust allows cracking, the post-crack evolution proceeds on slow, resistive time-scale. Only B-field energy within the crack is released (not within the shear wave-affected volume).

Where energy is stored before the flare, crust or magnetosphere?

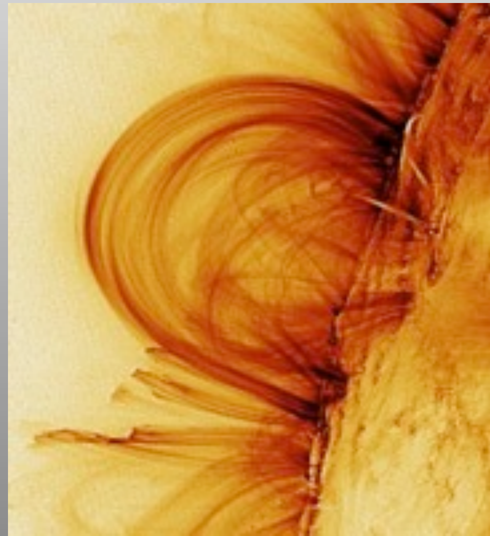


Shear time scale ~ 0.1 sec, but magnetic crack are not sudden, they are slow!

1. Flare rise time: 250 mu-sec: magnetospheric, ~ 10 R_{NS}/c= 10 R_{NS}/v_A
2. Flare energy stored and dissipated in the magnetosphere
3. Similar to Solar flares and Coronal Mass ejections

"Solar flares" paradigm of magnetars

? ~

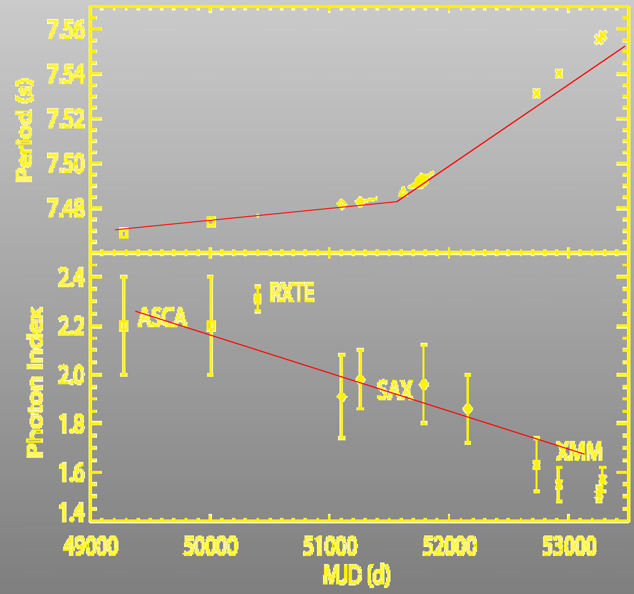


- Magnetic field is generated inside the star by a dynamo mechanism
- Non-potential (current-carrying) field is pushed outside
- Instability of twisted (current-carrying) fields leads to magnetic dissipation: flares (generating sometimes CME-like ejections)
- Radio emission: from active regions (~Solar type III radio bursts)

Pre- and post-flare evolution

1. Main prediction of the "Solar flare" model for magnetars:
   - Before flare: larger current → larger persistent luminosity, **harder spectra**, larger spindown
   - Post-Flares: twist is smaller → spectra softer, profile simpler
2. Giant flares:
   - Aug 27 giant flare of SGR 1900+14: Simpler profile, Spectrum: power-law index 1.9 → 2.5 (Woods et al)
   - Dec 2004 flare of 1806
     - Before the flare:
     - Spindown increased
     - Spectrum hardened
   - After the flare
     - Pulsed fraction decreased(10%→ 3%),
     - Spindown decreased
     - Spectrum softened

(Mereghetti et al, Tiego et al)



All in agreement with "Solar flare model"

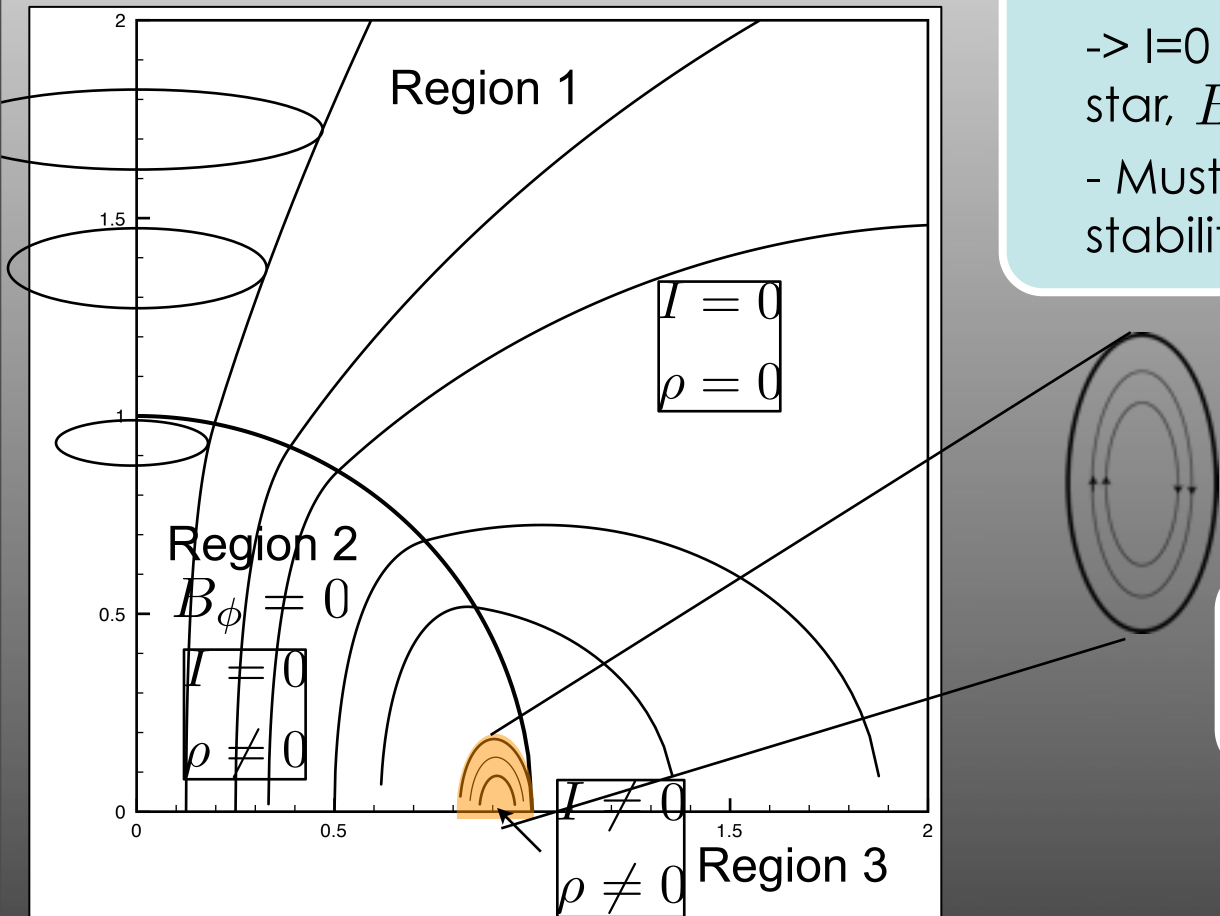
Overall evolution of B-fields in NS crusts remain unclear

- Initial MHD state (eg. torus or twisted)
- Stability of various EMHD configurations
- There are indications that EMHD configurations
- are generically unstable
- Not clear what state EMHD system wants to achieve
- Turbulent cascade vs non-local (in k space) formation of current sheets (and ensuring resistive decays)
- Statistics of stresses (flares) for a given statistics of field fluctuations?

Things to remember

- Crust may not need to be cracking to produce flares: plastic deformations may do
- Magnetically-induced cracks cannot release a large amount of energy in short time
- B-field is dissipated outside in Solar flares-like events. Electric currents are pushed out through crustal deformations
- Activity peaks at the end of magnetar phase

Axisymmetric B-fields in fluid stars



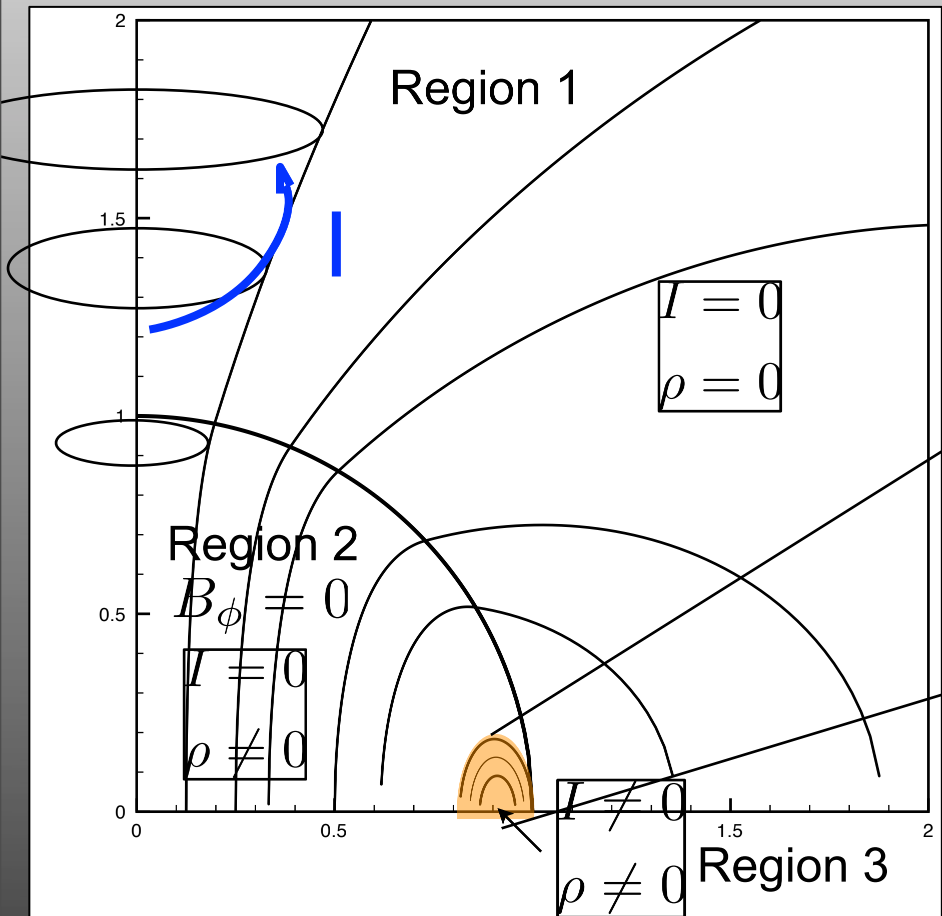
- Poloidal current I=I(P)

-> I=0 on field lines leaving the star, $B_\phi = 0$ outside

- Must be non-zero inside for stability, limited to some region

B poloidal - given
B toroidal - given (=0)
Demand no current sheets

Axisymmetric B-fields in fluid stars



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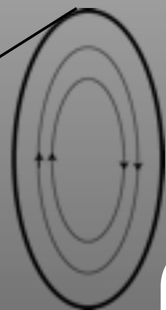

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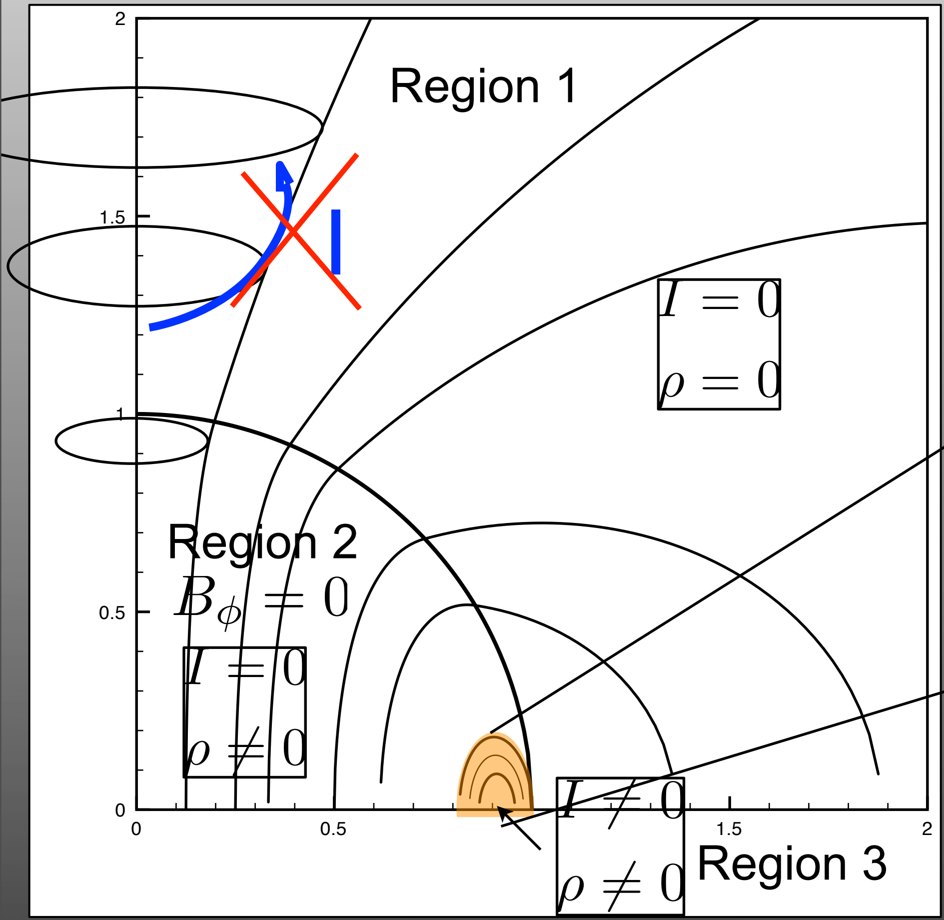

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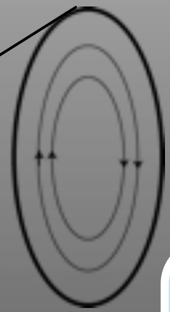

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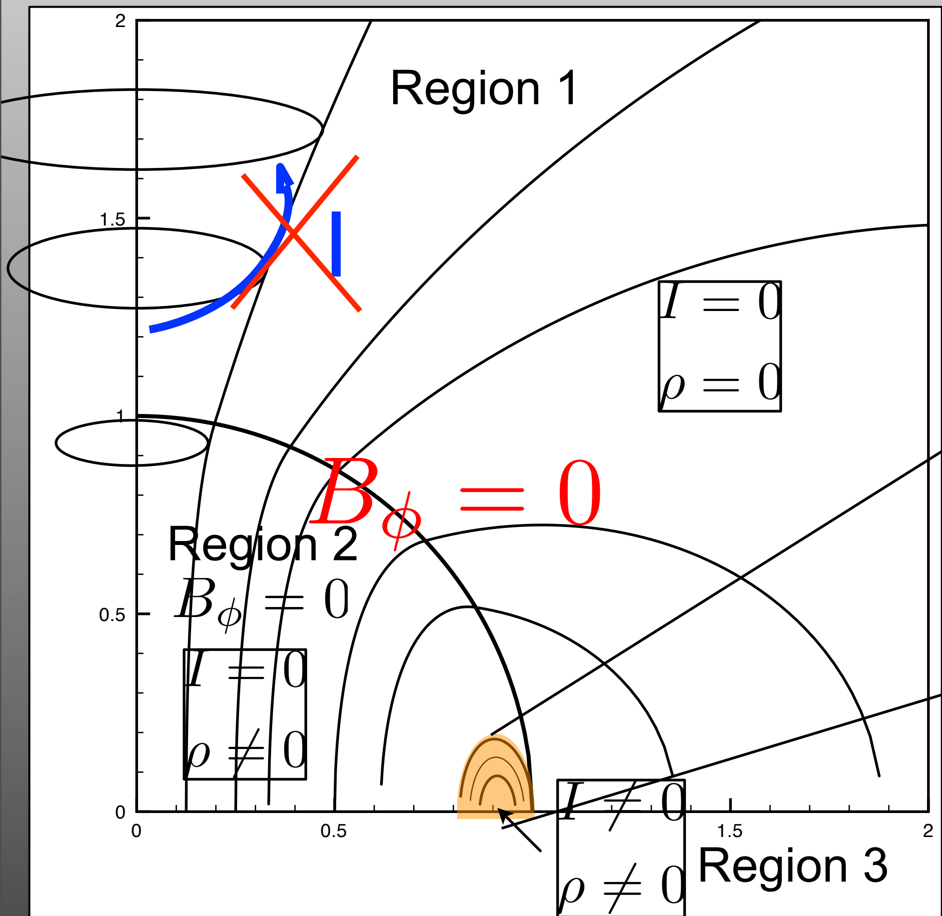

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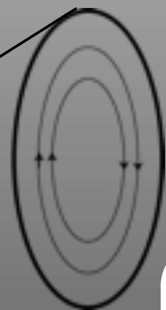

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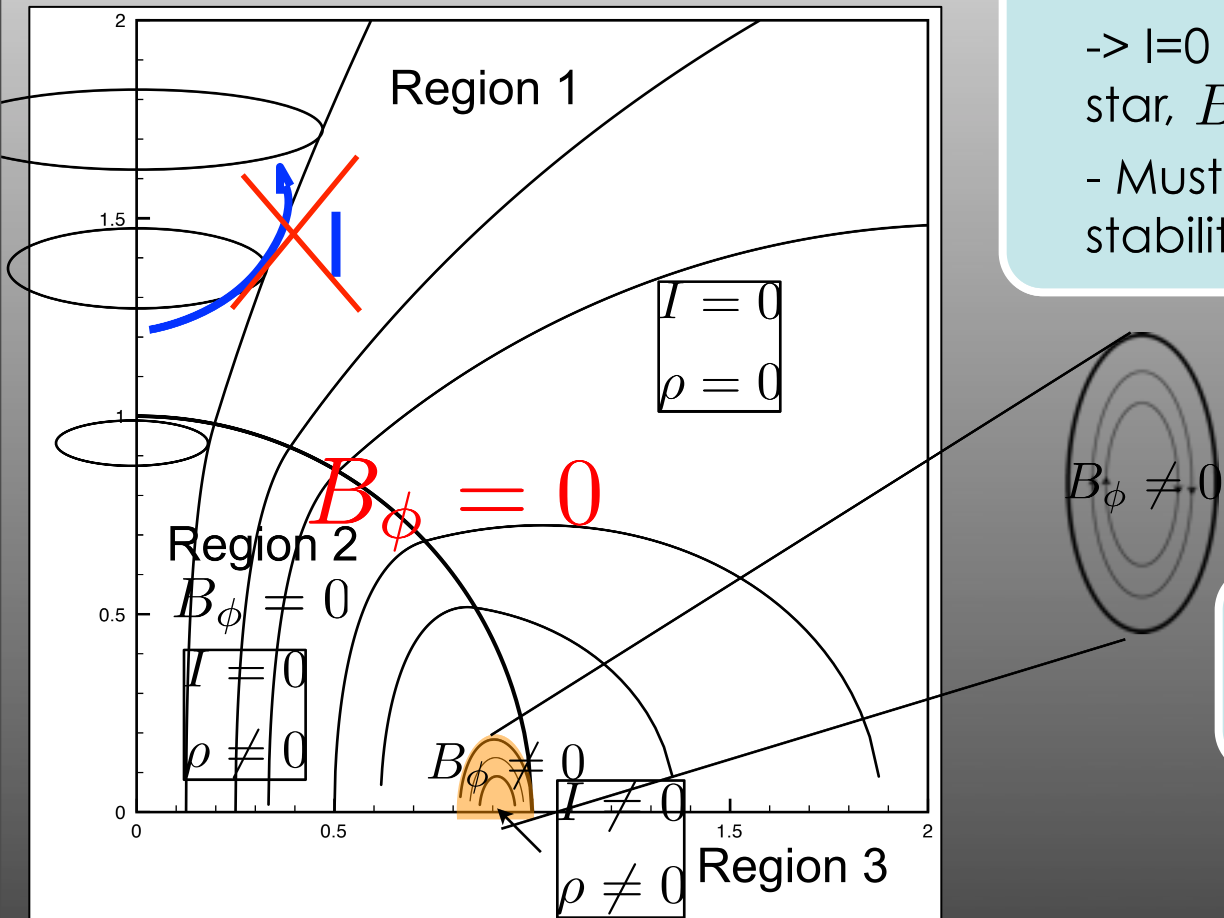

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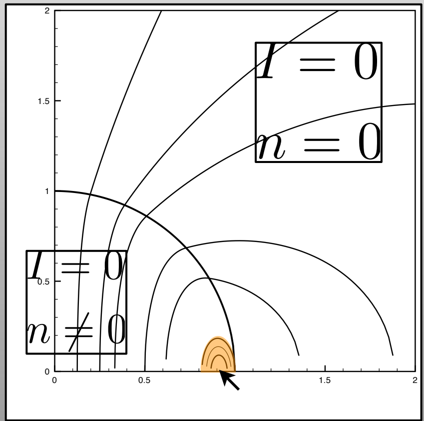
Trapped toroidal field



$$\partial_r^2 P + \frac{\sin\theta}{r^2}\partial_\theta\left(\frac{1}{\sin\theta}\partial_\theta P\right) + 4I(P)I'(P) = F(P)nr^2\sin^2\theta$$

Unknown a priori distributions of poloidal currents & pressure

$$\mathbf{B}_p = \frac{\nabla P \times \mathbf{e}_\phi}{r\sin\theta}$$

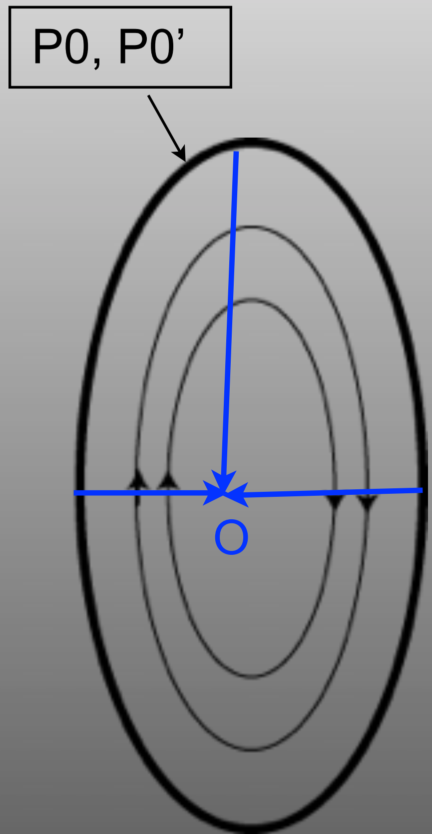
$$\mathbf{B}_\phi = \frac{2I(P)}{r\sin\theta}$$

Need to find an equation and its solution that satisfies over-determined boundary conditions, given P and P' on the border.

An elliptical equation with both Neumann and Dirichlet boundary conditions and having two unknown functions of the solution I(P) and F(P).

- We devised a procedure to simultaneously construct flux function P and unknown functions I(P) and F(P).

Over-determined problem (?!)



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Known P0, P0', I=I(P0), F=F(P0) on the boundary. Can find P''.

By taking derivatives, can find $P^{(n)}$ as function of $I(P_0)...I(P_0)^{(n-2)}$ and $F(P_0)...F(P_0)^{(n-2)}$ (these are numbers to be determined). Require smooth convergence at O.

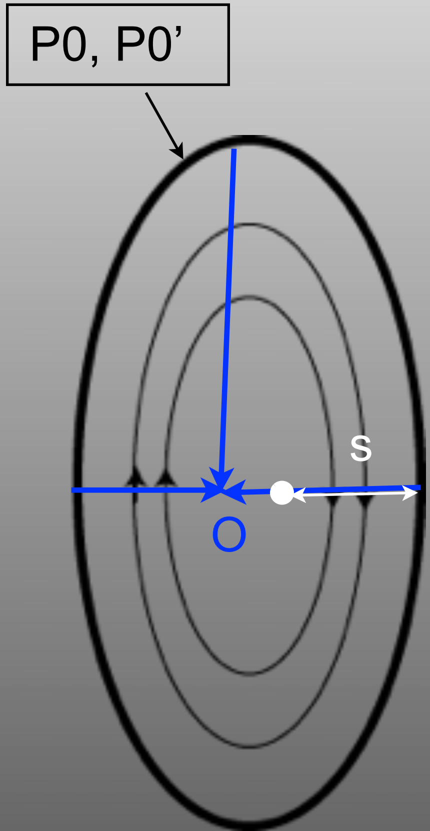
$$I = \sum_i I(P_0)^{(i)}\frac{(P-P_0)^i}{i!}$$

$$F = \sum_i F(P_0)^{(i)}\frac{(P-P_0)^i}{i!}$$

$$P = P_0 + P's + \sum_i C_i\frac{s^i}{i!}$$

Determine simultaneously expansion of I, F and P in terms of P-P0

Over-determined problem (?!)



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