Radius and Mass Determinations from Neutron Star Observations

J.M. Lattimer

Department of Physics & Astronomy
Stony Brook University

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Collaborators: E. Brown (MSU), K. Hebeler (OSU), D. Page (UNAM), C.J. Pethick (NORDITA), M. Prakash (Ohio U.), A. Schwenk (TU Darmstadt), A. Steiner (MSU)

Astrophysical Transients: Multi-messenger Probes of Nuclear Physics
Institute for Nuclear Theory

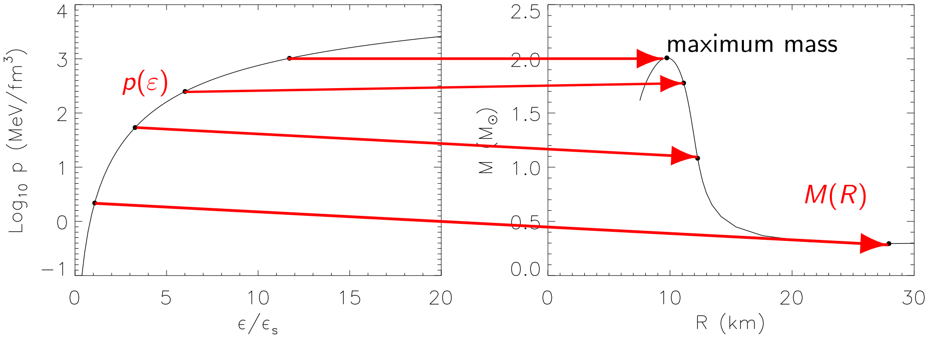
Outline

- Neutron Star Limits from General Relativity and Causality
- Mass Measurements
  - 2 $M_\odot$ Neutron Stars?
  - Limits to the Extent of Quark Matter
- Neutron Star Radii
  - Relation to the Nuclear Symmetry Energy
  - Thermal Emission from Cooling Neutron Stars
  - Photospheric Radius Expansion X-Ray Bursters
- The Universal Mass-Radius Relation and the Neutron Star EOS
  - Consistency with Neutron Matter Expectations
  - Implications for Other Laboratory Constraints

Neutron Star Structure

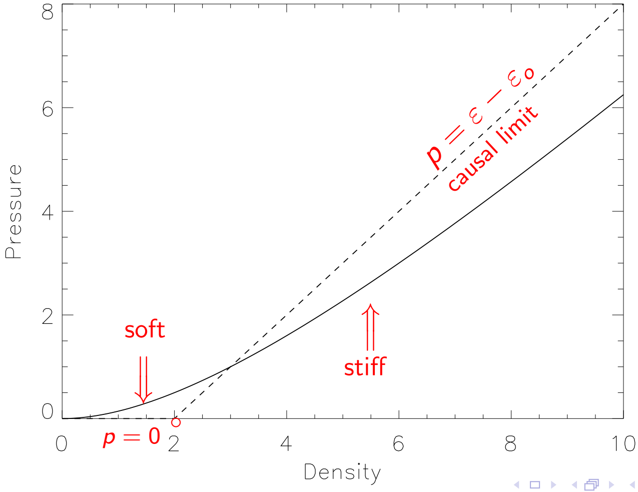
Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi pr^3)(\varepsilon+p)}{r(r-2Gm/c^2)}$$
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Extreme Properties of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ε_0 is the only EOS parameter

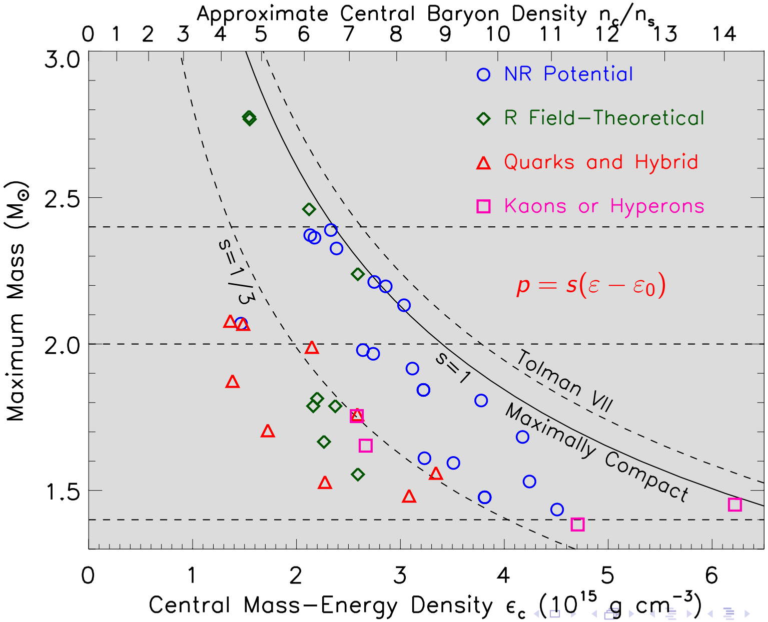
The TOV solutions scale with ε_0

$w = \varepsilon/\varepsilon_0$
$y = p/\varepsilon_0$
$x = r\sqrt{G\varepsilon_0}/c^2$
$z = m\sqrt{G^3\varepsilon_0}/c^2$

Extreme Properties of Neutron Stars

- $M_{max} = 4.1 \ (\varepsilon_s/\varepsilon_0)^{1/2} \ \mathrm{M}_\odot$ (Rhoades & Ruffini 1974)
- $M_{B,max} = 5.41 \ (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2} \ \mathrm{M}_\odot$
- $R_{min} = 2.82 \ GM/c^2 = 4.3 \ (M/\mathrm{M}_\odot) \ \mathrm{km}$
- $\mu_{B,max} = 2.09$ GeV
- $\varepsilon_{c,max} = 3.034 \ \varepsilon_0 \simeq 51 \ (\mathrm{M}_\odot/M_{largest})^2 \ \varepsilon_s$
- $p_{c,max} = 2.034 \ \varepsilon_0 \simeq 34 \ (\mathrm{M}_\odot/M_{largest})^2 \ \varepsilon_s$
- $n_{B,max} \simeq 38 \ (\mathrm{M}_\odot/M_{largest})^2 \ n_s$
- $\mathrm{BE}_{max} = 0.34 \ M$
- $P_{min} = 0.74 \ (\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10 \ \mathrm{km})^{3/2} \ \mathrm{ms} = 0.20 \ (M_{sph,max}/\mathrm{M}_\odot) \ \mathrm{ms}$

Maximum Energy Density in Neutron Stars



Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

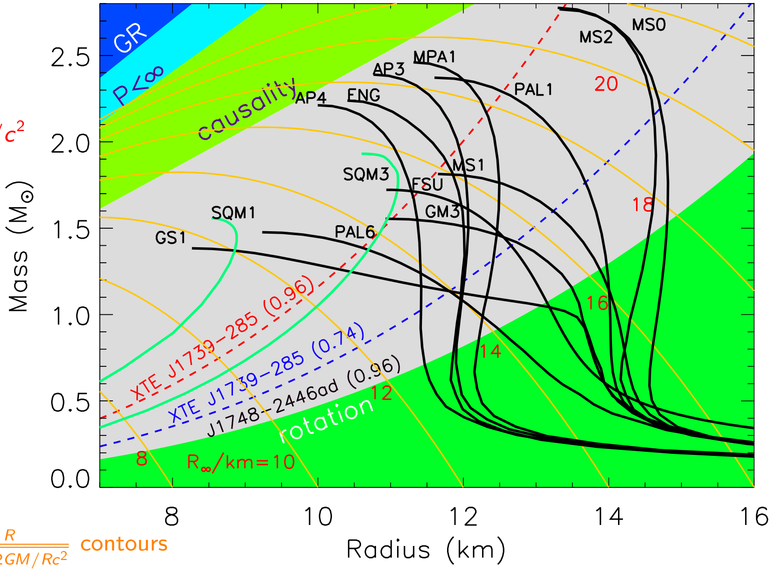
causality:

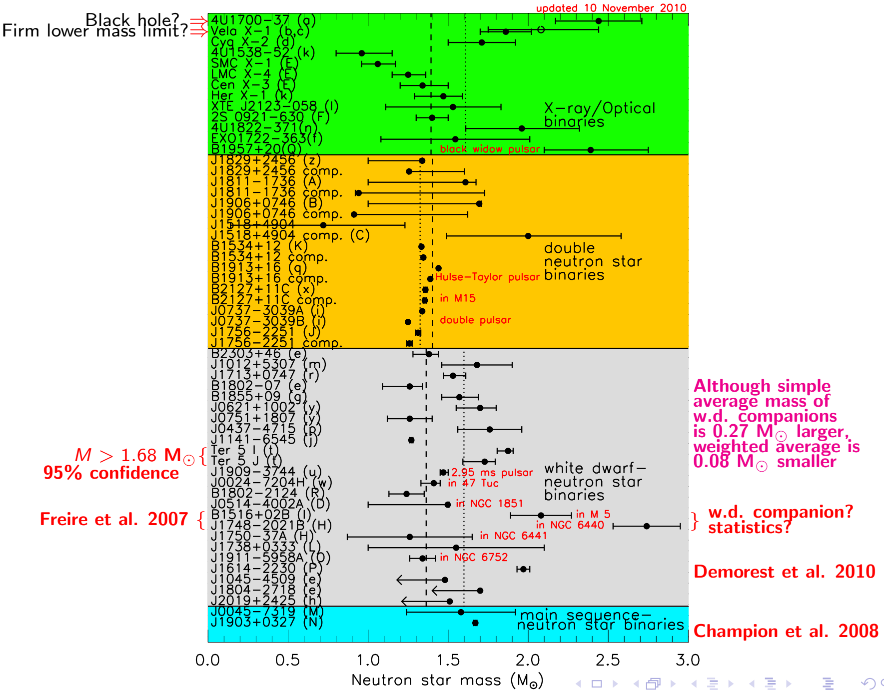
$R \gtrsim 2.9GM/c^2$

— normal NS

— SQS

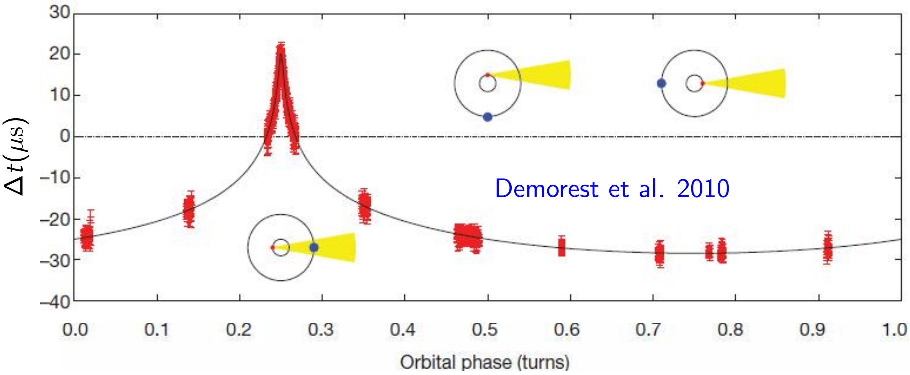
— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours




updated 10 November 2010
Black hole?
Firm lower mass limit?
4U1700−37 (a)
Vela X−1 (b,c)
Cyg X−2 (d)
4U1538−52 (k)
SMC X−1 (E)
LMC X−4 (E)
Cen X−3 (E)
Her X−1 (k)
XTE J2123−058 (l)
2S 0921−630 (F)
4U1822−371(n)
EXO1722−363(f)
B1957+20(Q)
black widow pulsar
X−ray/Optical binaries
J1829+2456 (z)
J1829+2456 comp.
J1811−1736 (A)
J1811−1736 comp.
J1906+0746 (B)
J1906+0746 comp.
J1518+4904
J1518+4904 comp. (C)
B1534+12 (K)
B1534+12 comp.
B1913+16 (q)
B1913+16 comp.
Hulse−Taylor pulsar
B2127+11C (x)
B2127+11C comp.
in M15
J0737−3039A (i)
J0737−3039B (i)
double pulsar
J1756−2251 (J)
J1756−2251 comp.
double neutron star binaries
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802−07 (e)
B1855+09 (g)
J0621+1002 (y)
J0751+1807 (y)
J0437−4715 (p)
J1141−6545 (j)
Ter 5 I (t)
Ter 5 J (t)
J1909−3744 (u)
2.95 ms pulsar
J0024−7204H (w)
in 47 Tuc
B1802−2124 (R)
J0514−4002A (D)
in NGC 1851
B1516+02B (I)
in M 5
J1748−2021B (H)
in NGC 6440
J1750−37A (H)
in NGC 6441
J1738+0333 (L)
J1911−5958A (O)
in NGC 6752
J1614−2230 (P)
J1045−4509 (e)
J1804−2718 (e)
J2019+2425 (h)
white dwarf−neutron star binaries
J0045−7319 (M)
J1903+0327 (N)
main sequence−neutron star binaries
$M > 1.68$ M$_\odot$
95% confidence
Freire et al. 2007
Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.08 M$_\odot$ smaller
w.d. companion? statistics?
Demorest et al. 2010
Champion et al. 2008
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass (M$_\odot$)

PSR J1614-2230

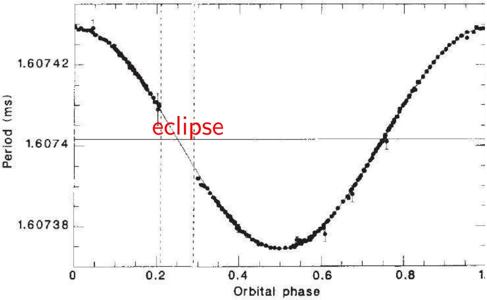
3.15 ms pulsar in 8.69d orbit with 0.5 $M_\odot$ white dwarf companion.
Shapiro delay tightly confines the edge-on inclination: $\sin i = 0.99984$
Pulsar mass is 1.97 ± 0.04 $M_\odot$
Distance > 1 kpc, $B \simeq 1.8 \times 10^8$ G

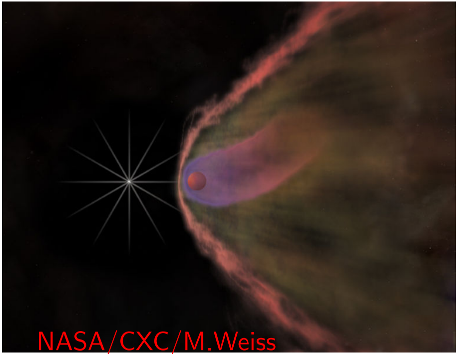


Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with a $M_c \sim 0.03$ $M_\odot$ companion.
Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe.
It is believed the companion is ablated by the pulsar leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe.
Ablation by pulsar leads to eventual disappearance of companion.
The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.

pulsar radial velocity





NASA/CXC/M.Weiss

Implications of Maximum Masses

$M_{max} > 2\ \mathrm{M}_\odot$

- Upper limits to energy density, pressure and baryon density:
  - $\varepsilon < 13.1\varepsilon_s$
  - $p < 8.8\varepsilon_s$
  - $n_B < 9.8n_s$
- Lower limit to spin period: $P > 0.56$ ms
- Lower limit to neutron star radius: $R > 8.5$ km
- Upper limits to energy density, pressure and baryon density in the case of a quark matter core ($s = 1/3$):
  - $\varepsilon < 7.7\varepsilon_s$
  - $p < 2.0\varepsilon_s$
  - $n_B < 6.9n_s$

$M_{max} > 2.4\ \mathrm{M}_\odot$

- Upper limits to energy density, pressure and baryon density:
  - $\varepsilon < 8.9\varepsilon_s$
  - $p < 5.9\varepsilon_s$
  - $n_B < 6.6n_s$
- Lower limit to spin period: $P > 0.68$ ms
- Lower limit to neutron star radius: $R > 10.4$ km
- Upper limits to energy density, pressure and baryon density in the case of a quark matter core ($s = 1/3$):
  - $\varepsilon < 5.2\varepsilon_s$
  - $p < 1.4\varepsilon_s$
  - $n_B < 4.6n_s$

$p \simeq Kn^{\gamma}$

$\gamma = d \ln p / d \ln \; n \sim 2$

$R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$

$R \propto p_f^{1/2} n_f^{-1} M^0$

$(1 < n_f / n_s < 2)$

Wide variation:

$1.2 < \frac{p(n_s)}{\mathrm{MeV\ fm}^{-3}} < 7$

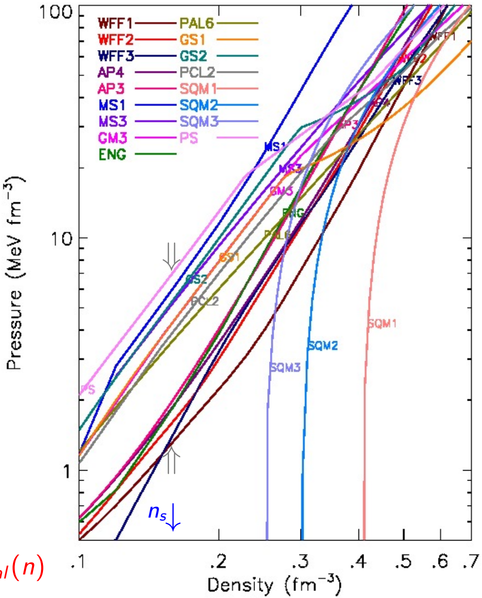
GR phenomenological result

(Lattimer & Prakash 2001)

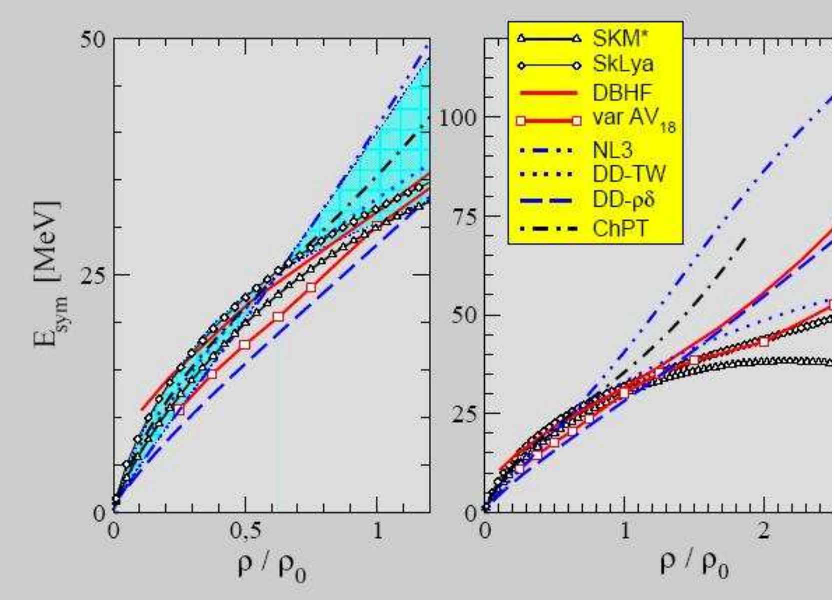
$R \propto p_f^{1/4} n_f^{-1/2}$

$p_f = n^2 dE_{sym}/dn$

$E_{sym}(n) = E_{neutron}(n) - E_{symmetrical}(n)$



The Uncertain $E_{sym}(n)$



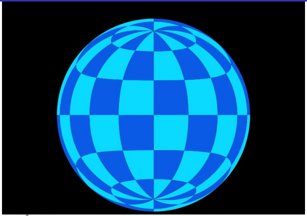
C. Fuchs, H.H. Wolter, EPJA 30(2006) 5

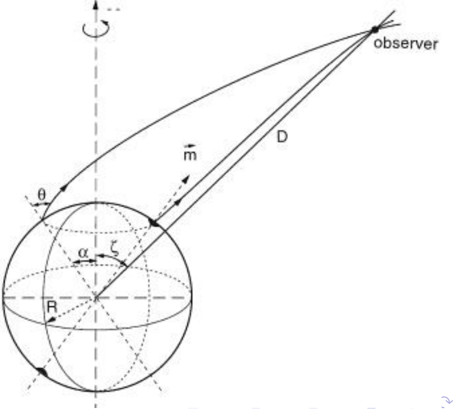
Radiation Radius

- The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

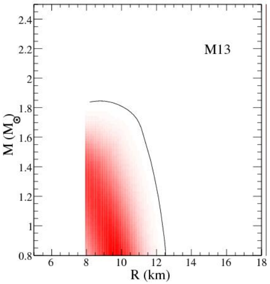
$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

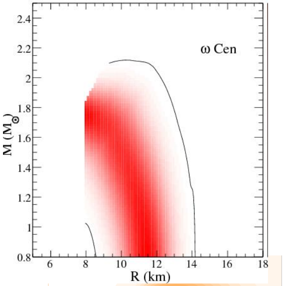
- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii:
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)



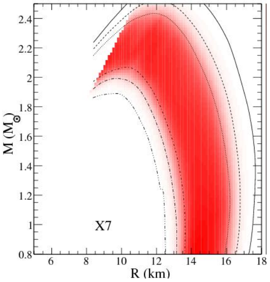


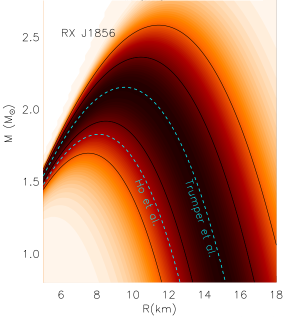
Inferred M-R Probability Estimates from Thermal Sources



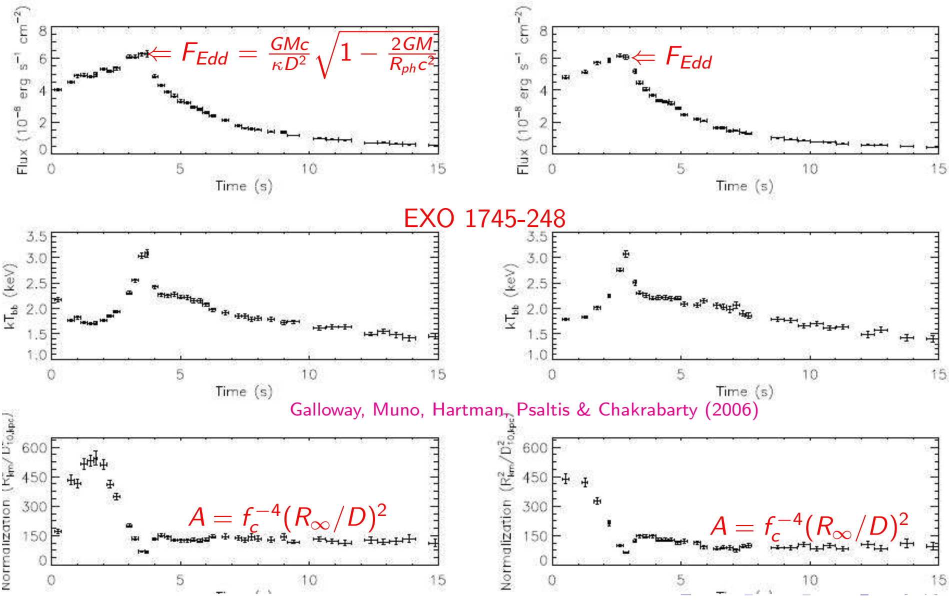


Steiner, Lattimer & Brown 2010





Photospheric Radius Expansion X-Ray Bursts



Systematics with $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} = \frac{GMc}{\kappa D^2}\sqrt{1-2\beta}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm^2 g^{-1}}$$

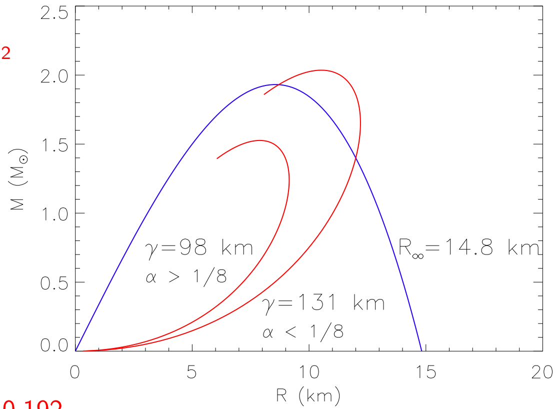
$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta(1-2\beta)$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)^{3/2}}$$

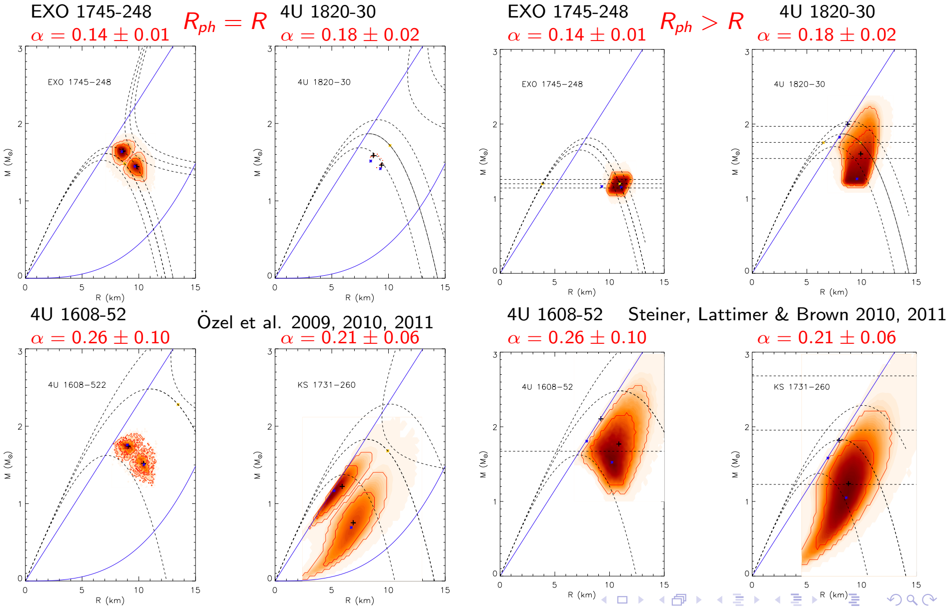
$$\beta = \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\alpha}$$

$$R_\infty = \alpha\gamma$$



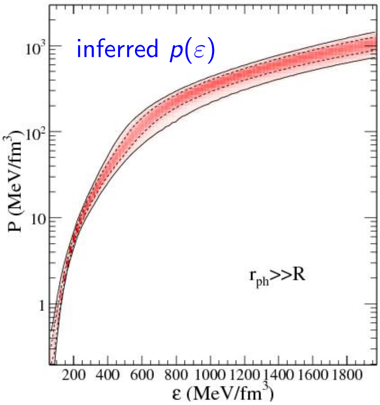
If $R_{ph} >> R$, $\alpha < 1/\sqrt{27} \simeq 0.192$

$M - R$ Probability Estimates from PRE Bursts

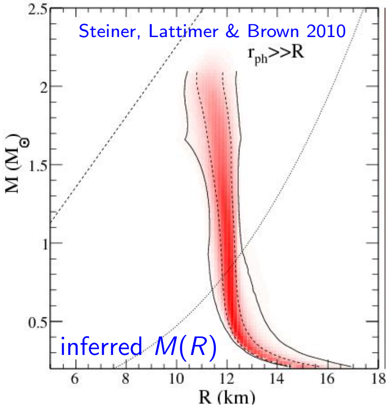


Bayesian TOV Inversion

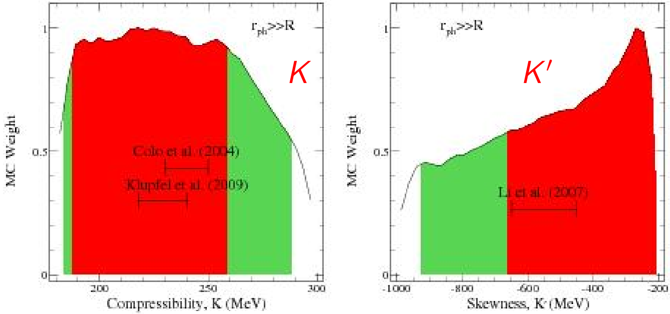
- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: Polytropic EOS with n_2



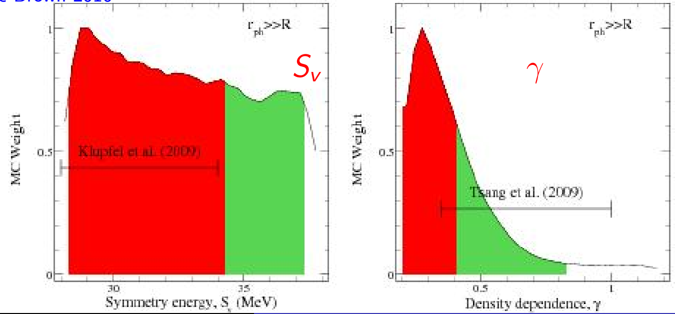
- EOS parameters $(K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2)$ uniformly distributed
- M and R probability distributions for 7 neutron stars treated equally.



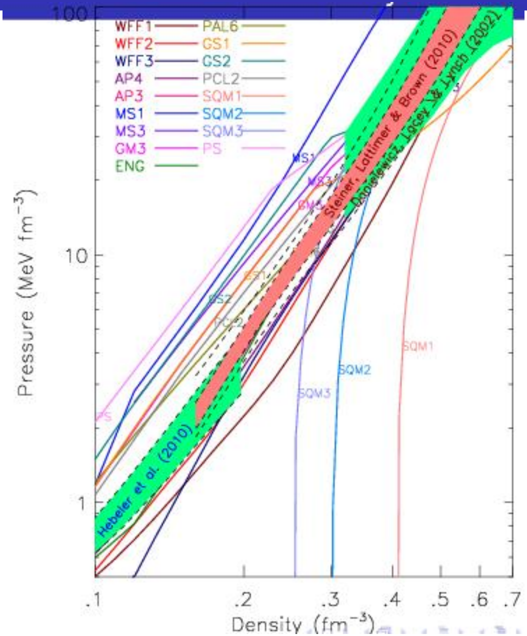
Inferred Model EOS Parameters

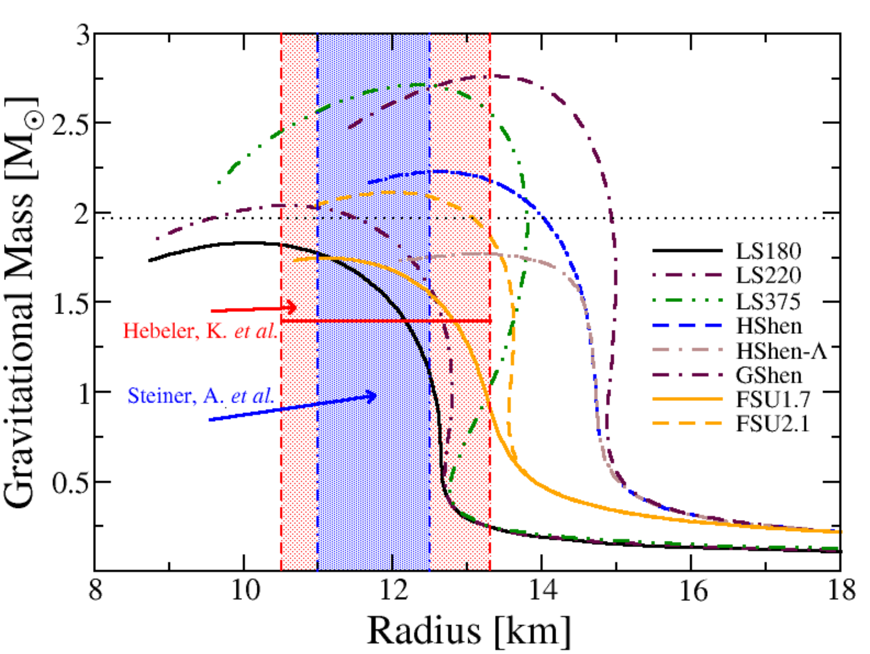


Steiner, Lattimer & Brown 2010

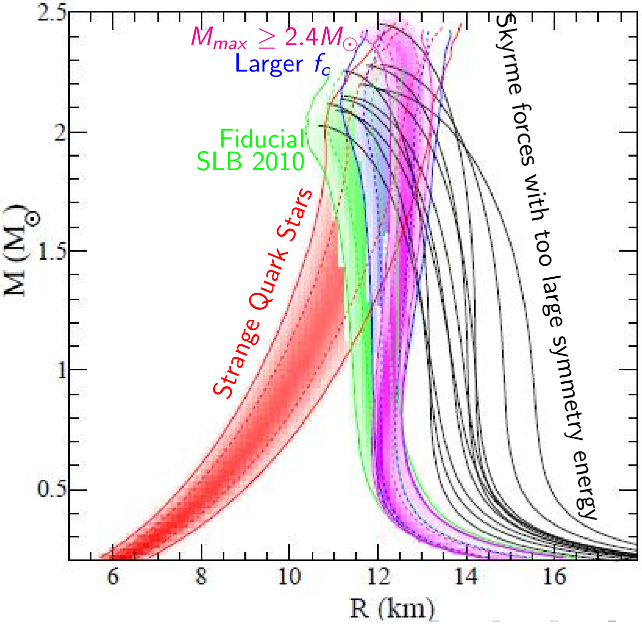


Consistency with Neutron Matter and Heavy-Ion Collisions

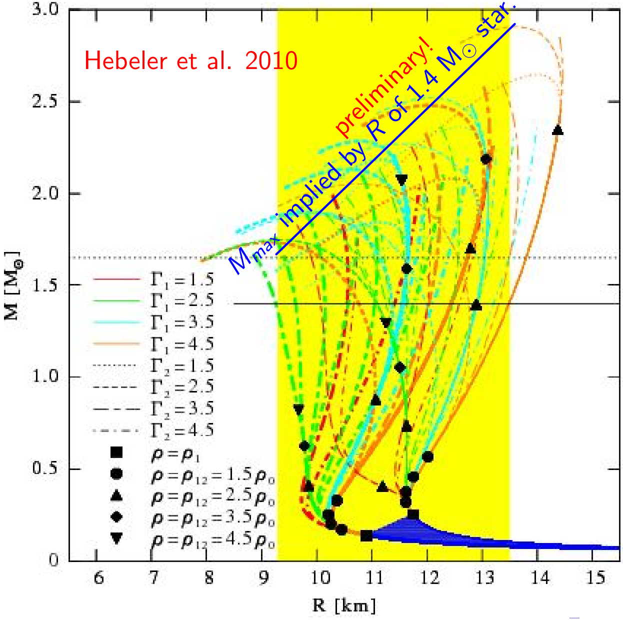



Gravitational Mass [M☉]
Radius [km]
3
2.5
2
1.5
1
0.5
8
10
12
14
16
18
Hebeler, K. et al.
Steiner, A. et al.
LS180
LS220
LS375
HShen
HShen-Λ
GShen
FSU1.7
FSU2.1

With More Extreme Assumptions



Radius and Maximum Mass Limits



Neutron Matter and the Symmetry Energy

- Fits to nuclear binding energies result in a strong, nearly linear, correlation between volume and surface symmetry energy coefficients of the liquid droplet model.
- This correlation is dependent on the nature of the liquid droplet model and how it treats the interaction between the Coulomb effects on the nuclear surface, and does not translate directly into a correlation between S_v and $L = 3(dS_v/d\ln n)_{n_s}$.
- Finite nucleus models, such as Thomas-Fermi and Hartree or Hartree-Fock, for a particular nuclear interaction, can be fit to binding energies to obtain the correlation between S_v and L.
- Neutron matter studies (Hebeler & Schwenk; Carlson et al.) indicate that E_{sym} and $(dE_{sym}/d\ln n)_{n_s}$ are also correlated.
- Comparing these correlations could constrain the properties of the symmetry energy. It could be dependent on the nature of the nuclear interaction model, but this has not been thoroughly explored.