Chiral three-body forces and neutron- rich matter

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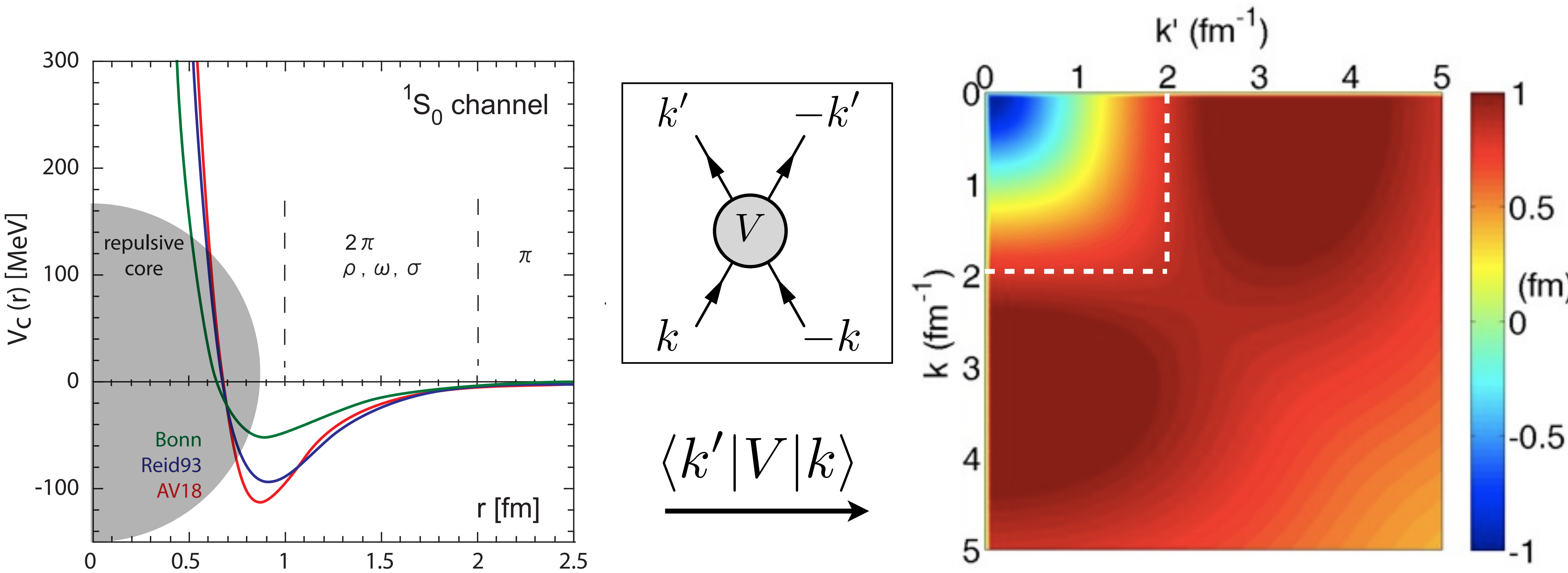
Seattle, August 1, 2011

**Astrophysical Transients:
Multi-messenger Probes of Nuclear Physics**

THE OHIO STATE UNIVERSITY

In collaboration with:
J. Lattimer, C. Pethick, A. Schwenk

Traditional “hard” NN interactions



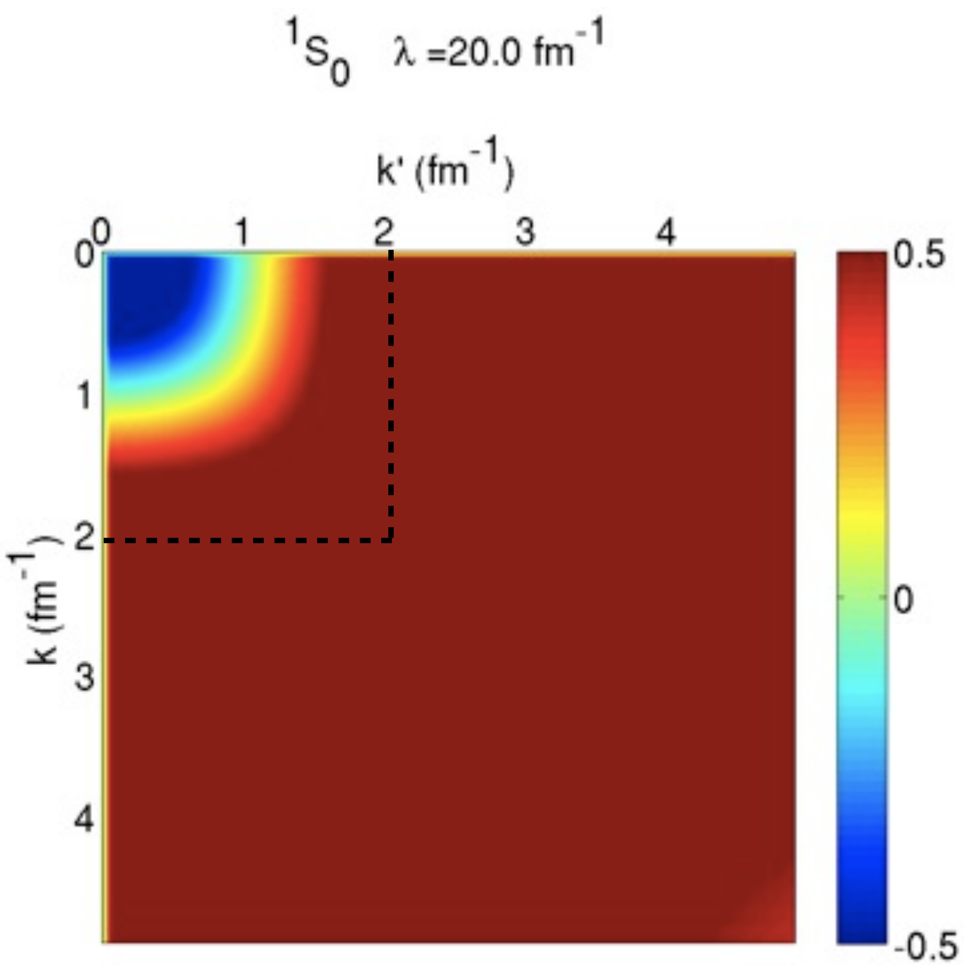
- constructed to fit low-energy nucleon-nucleon scattering data
- “hard” NN interactions contain repulsive core at small relative distance
- strong coupling between low and high-momentum components, hard to solve!

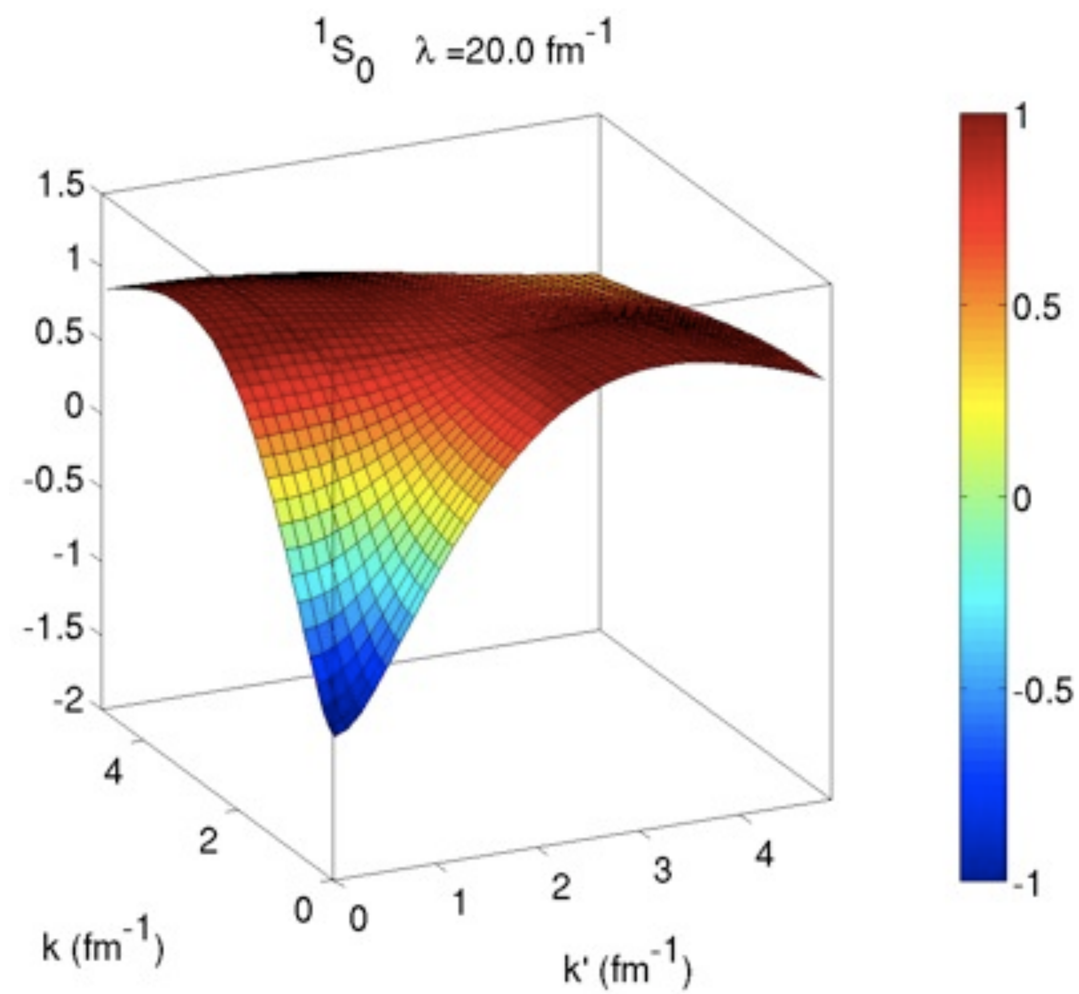
Low-momentum interactions: The (Similarity) Renormalization Group

- goal: generate unitary transformation of "hard" Hamiltonian

$$H_\lambda = U_\lambda H U_\lambda^\dagger \quad \text{with the resolution parameter } \lambda$$

- basic idea: change resolution in small steps: $\dfrac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$





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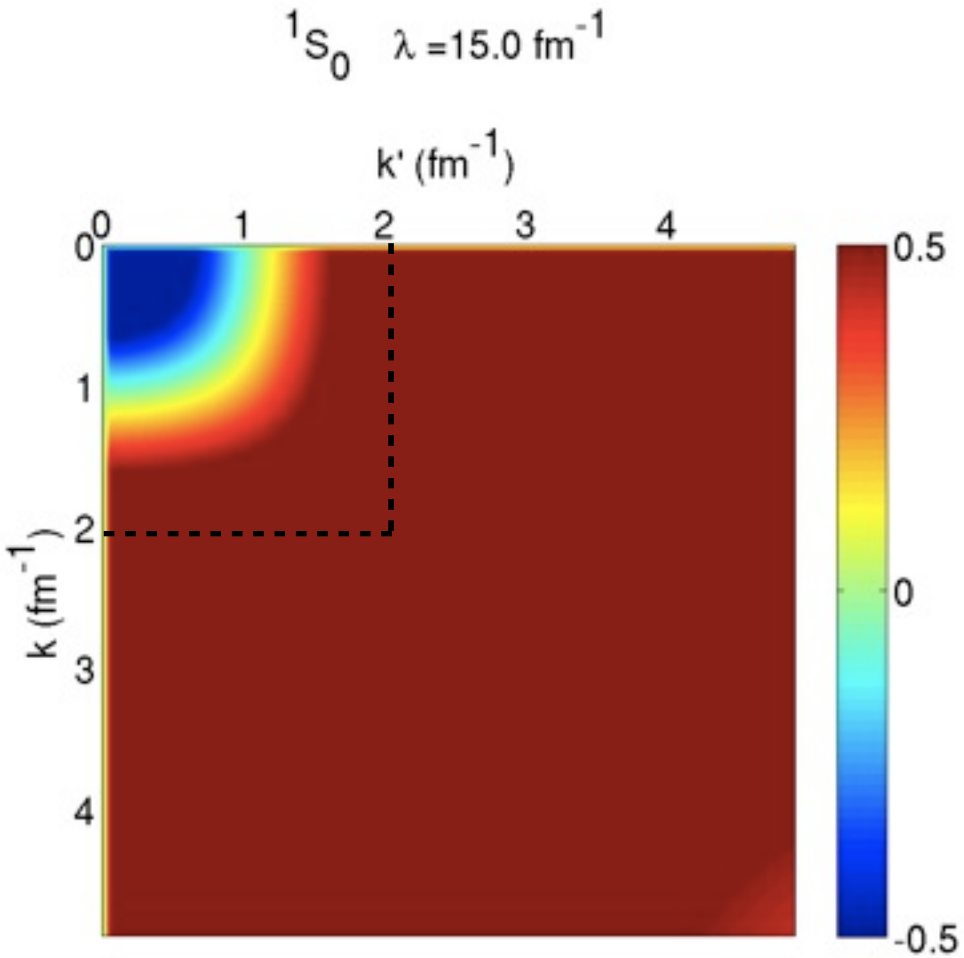

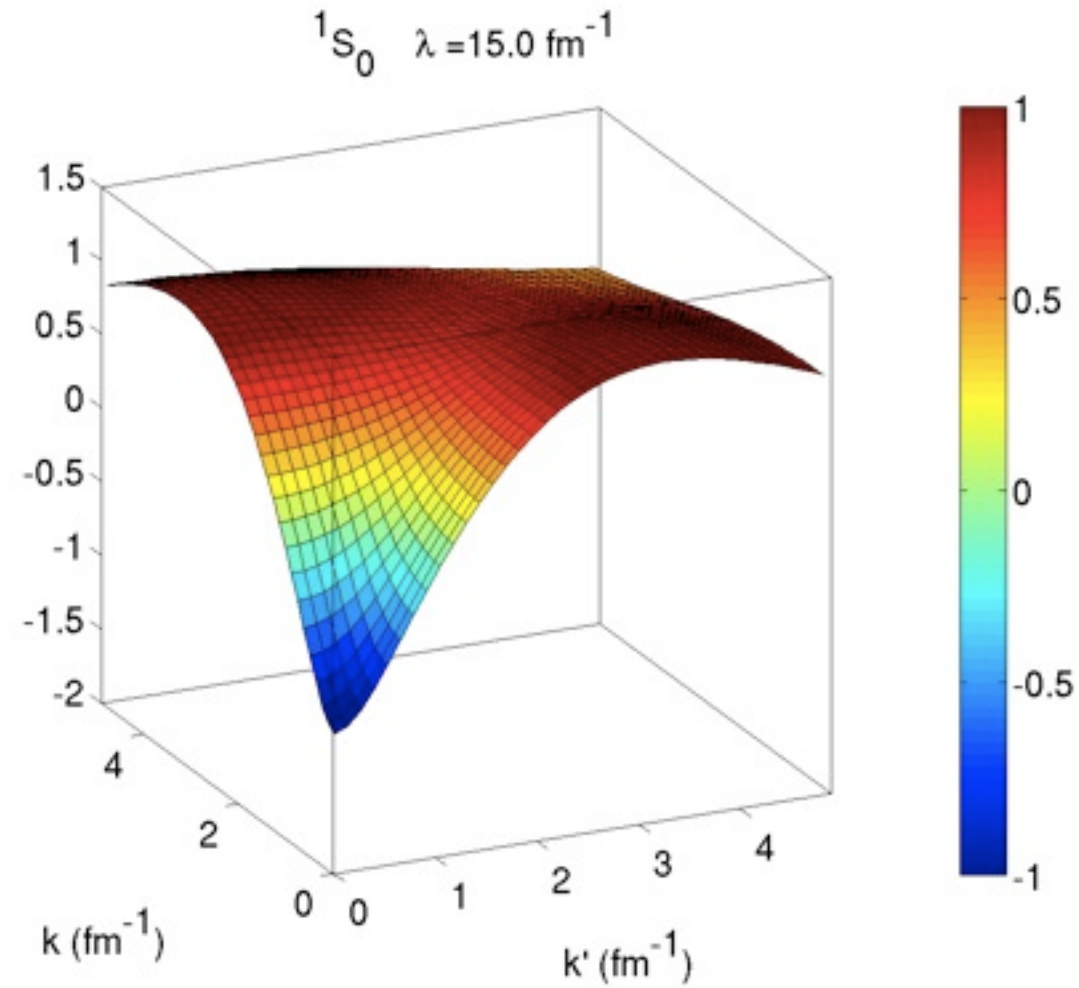

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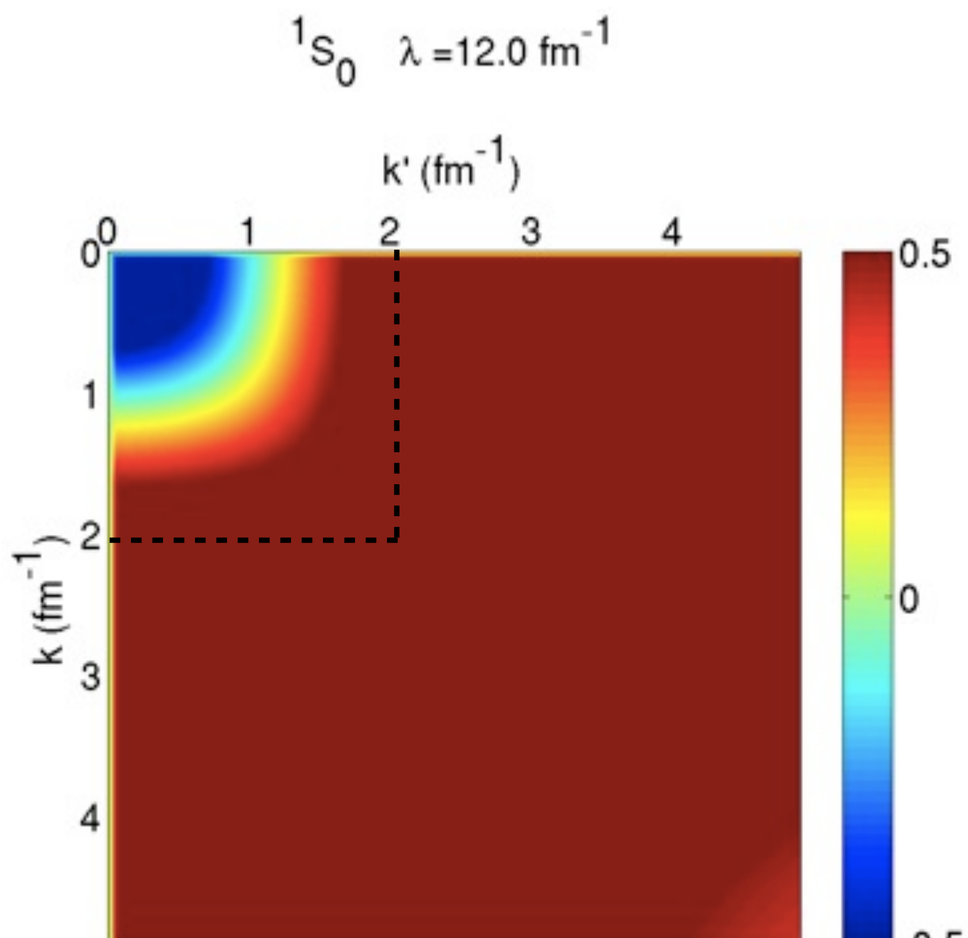

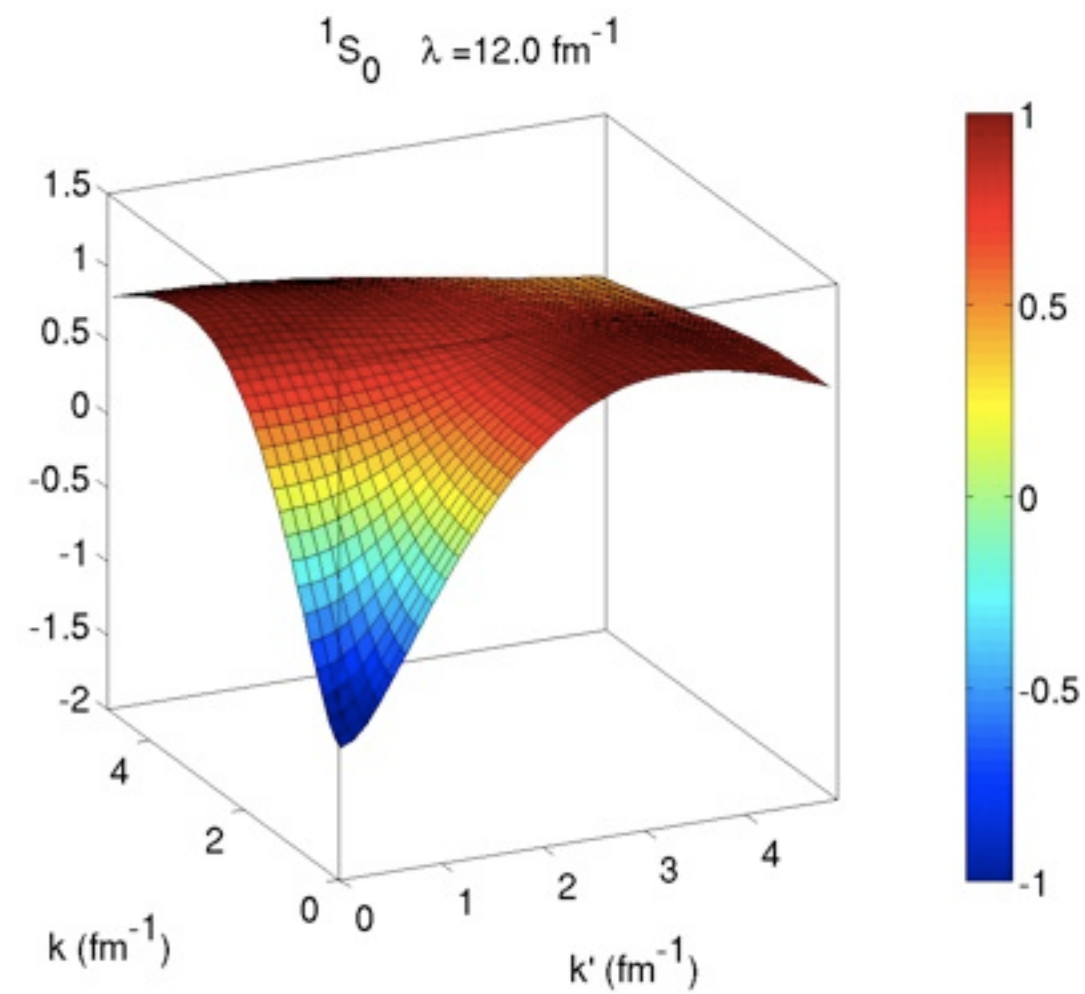

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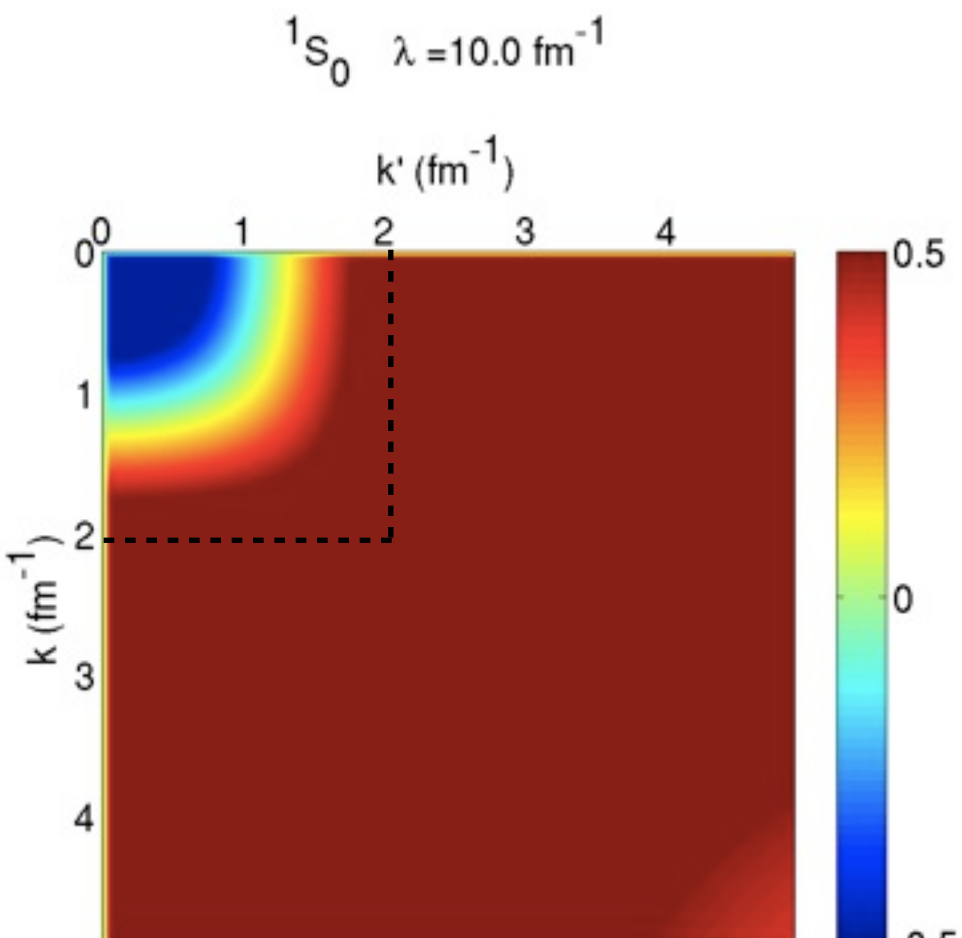

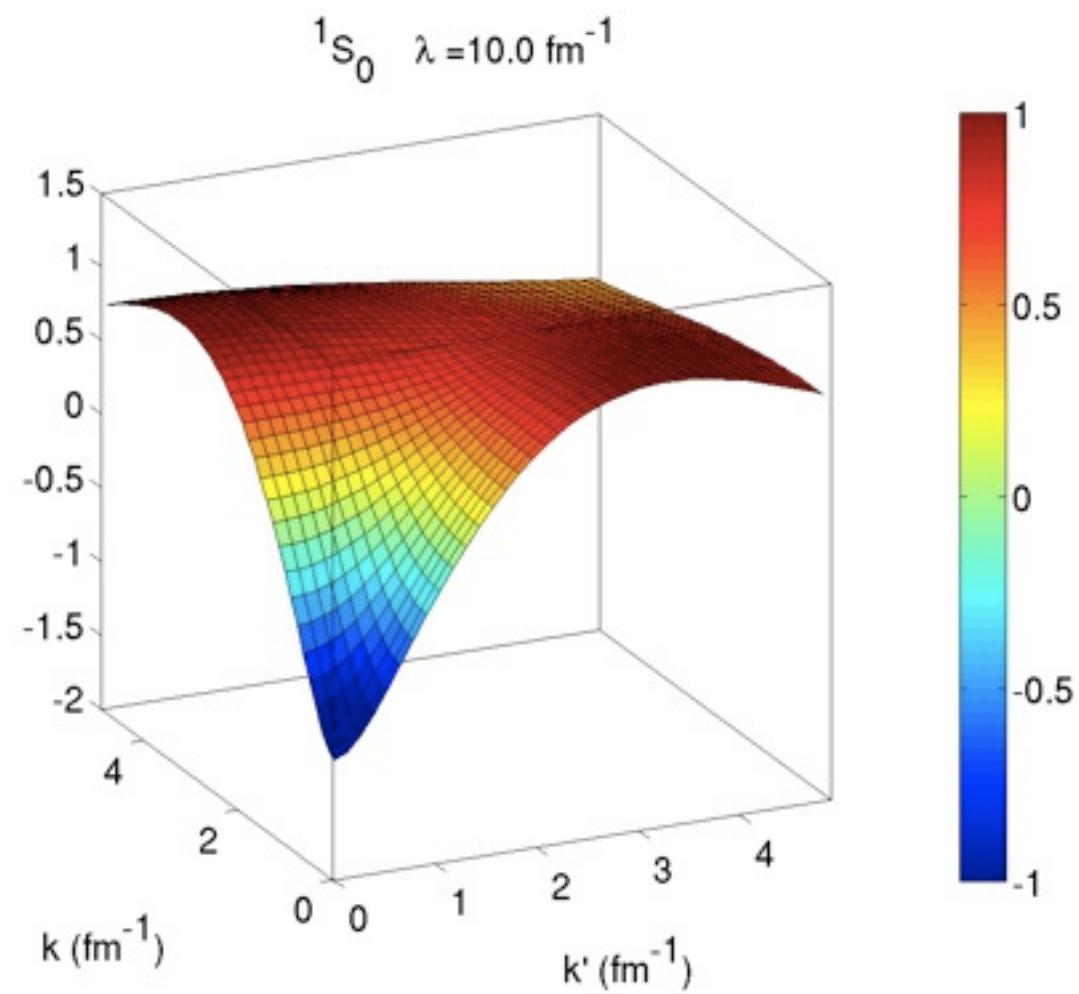

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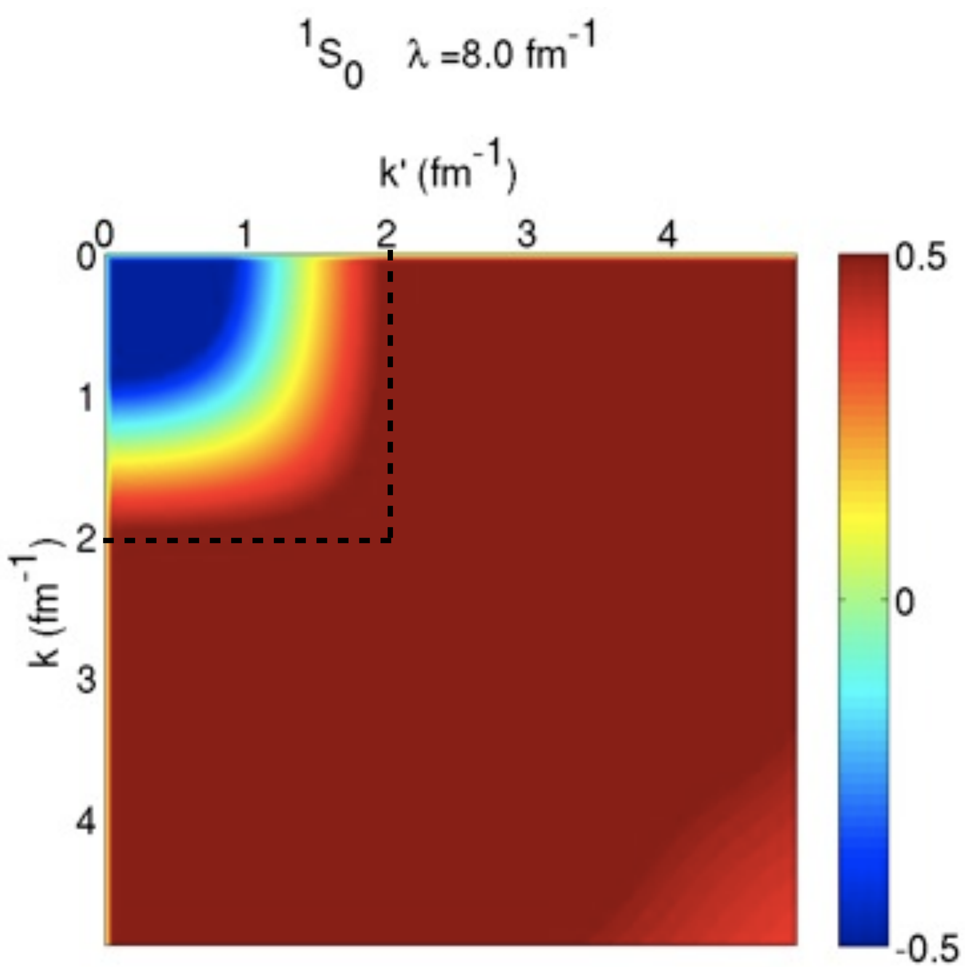

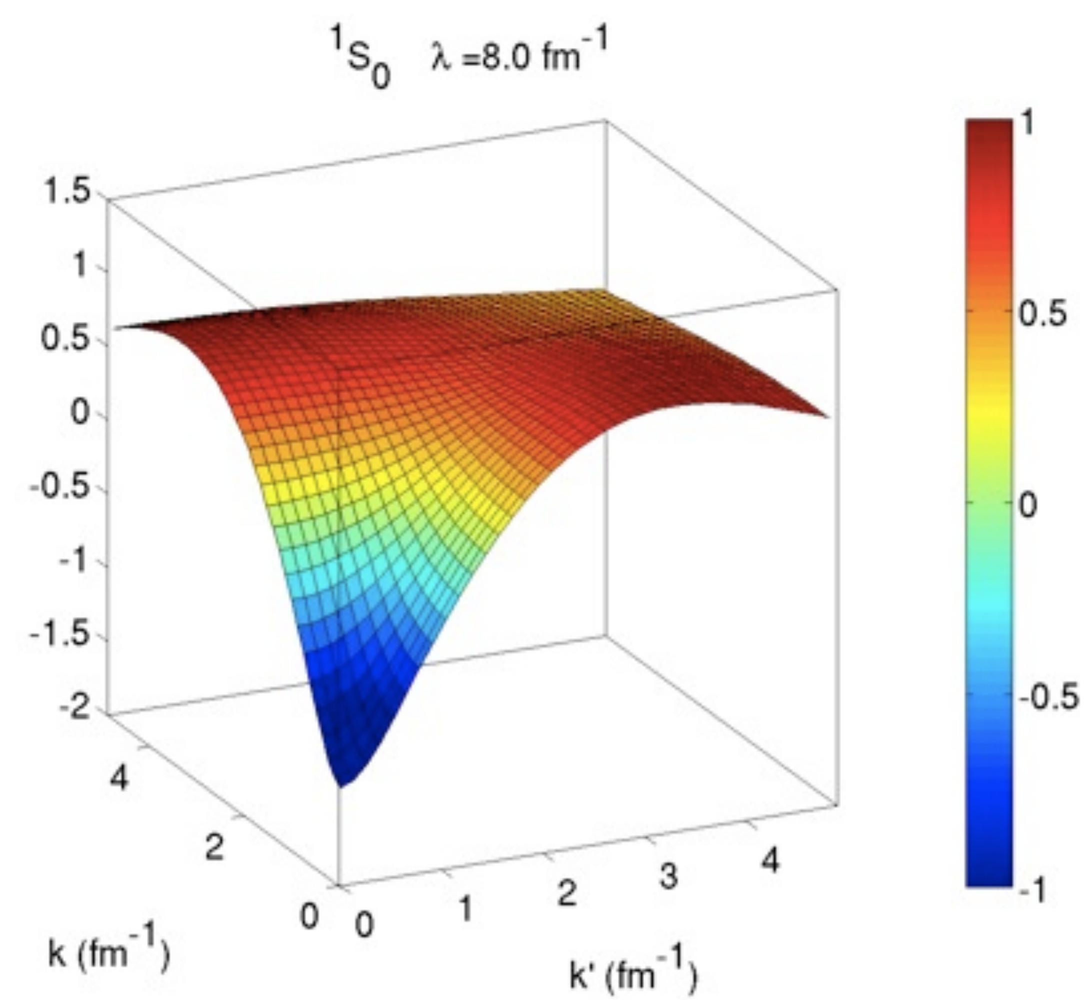

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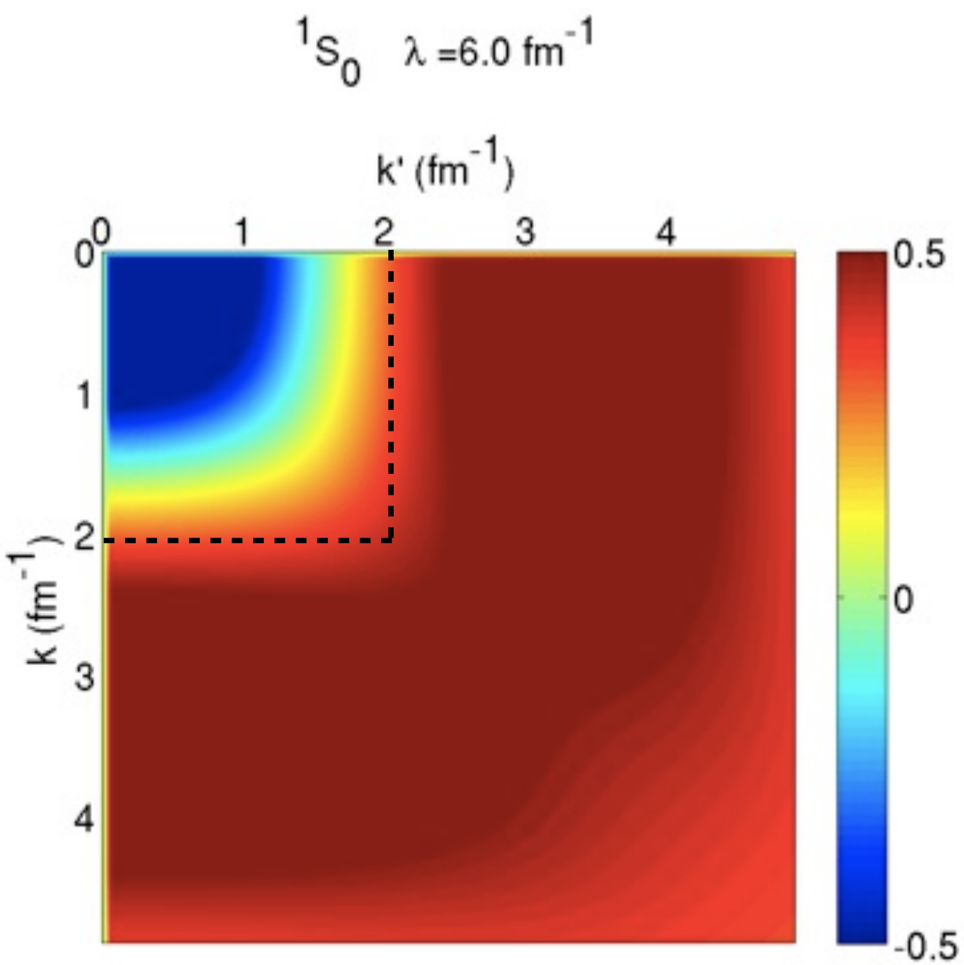

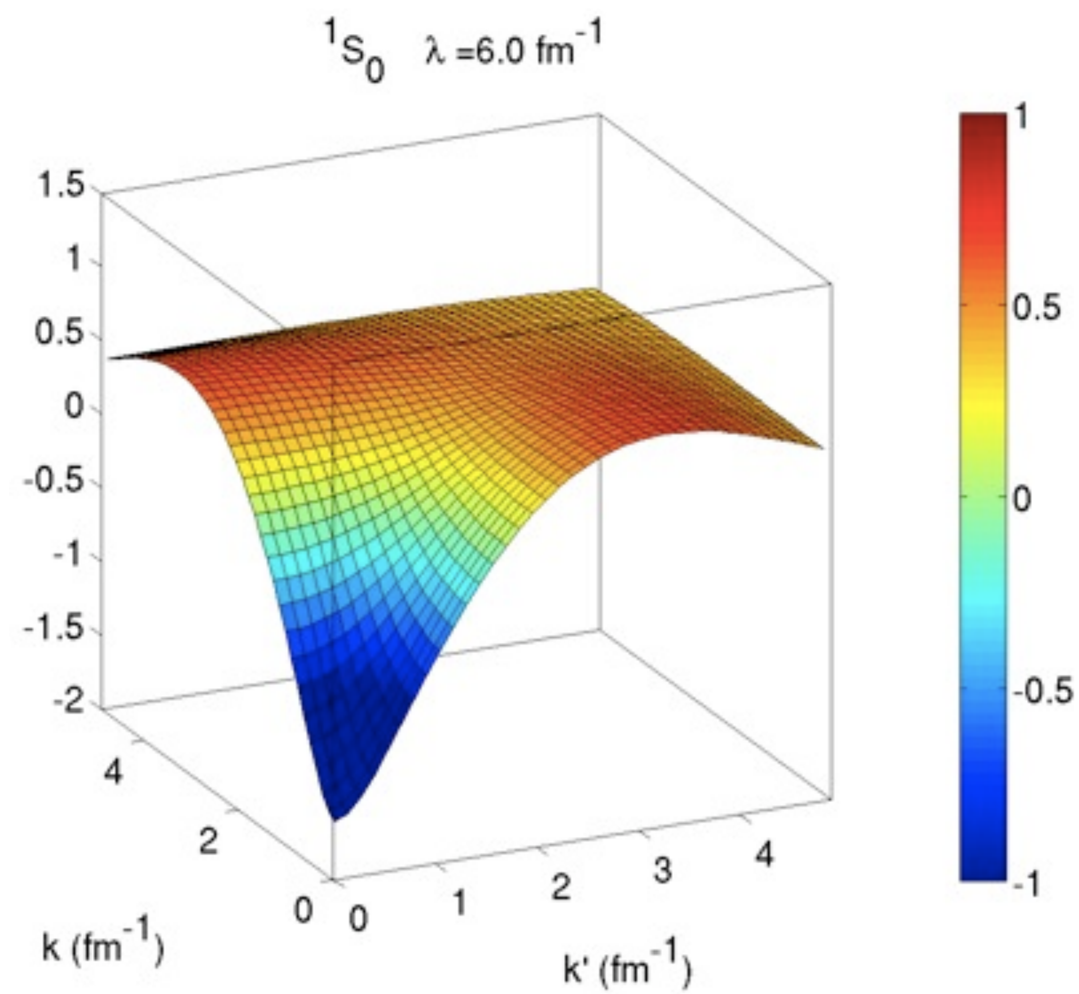

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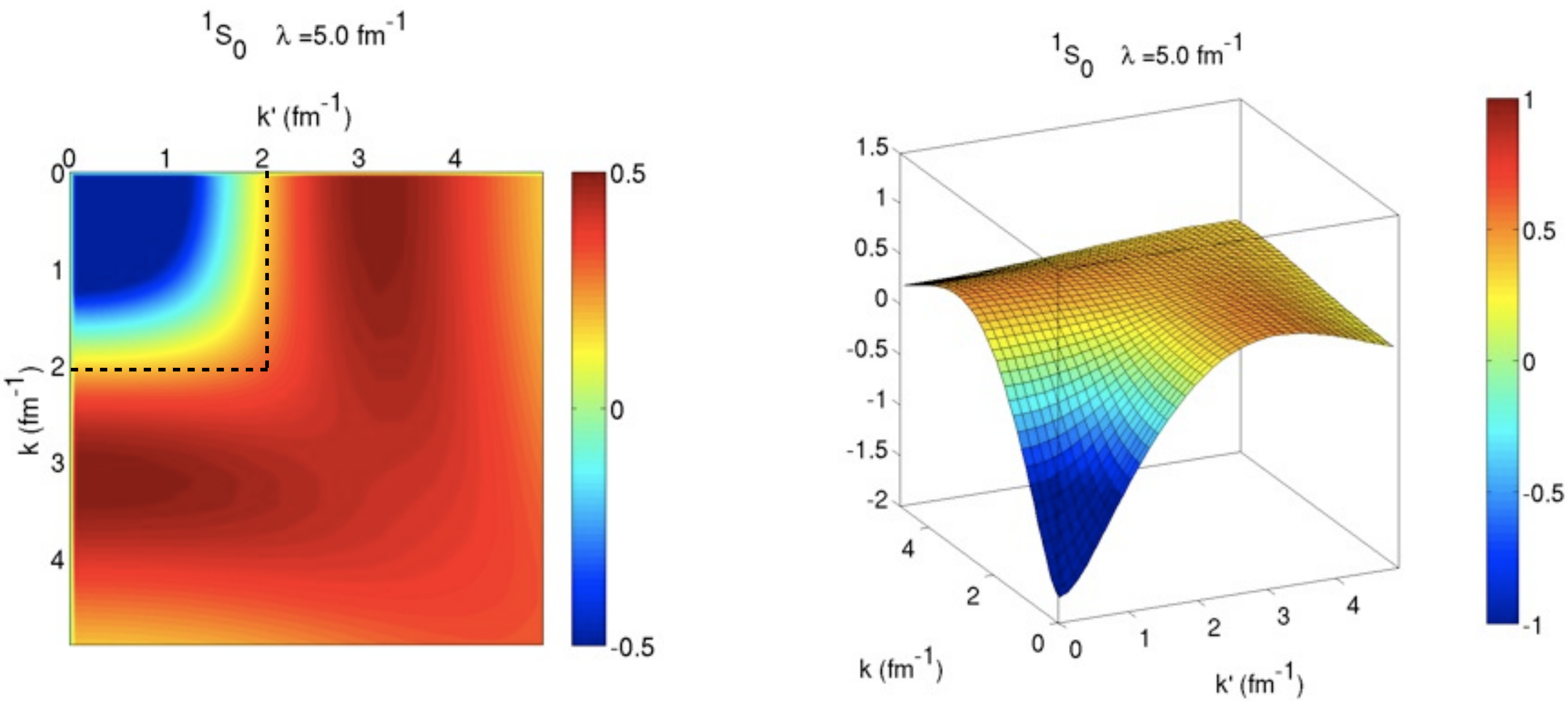

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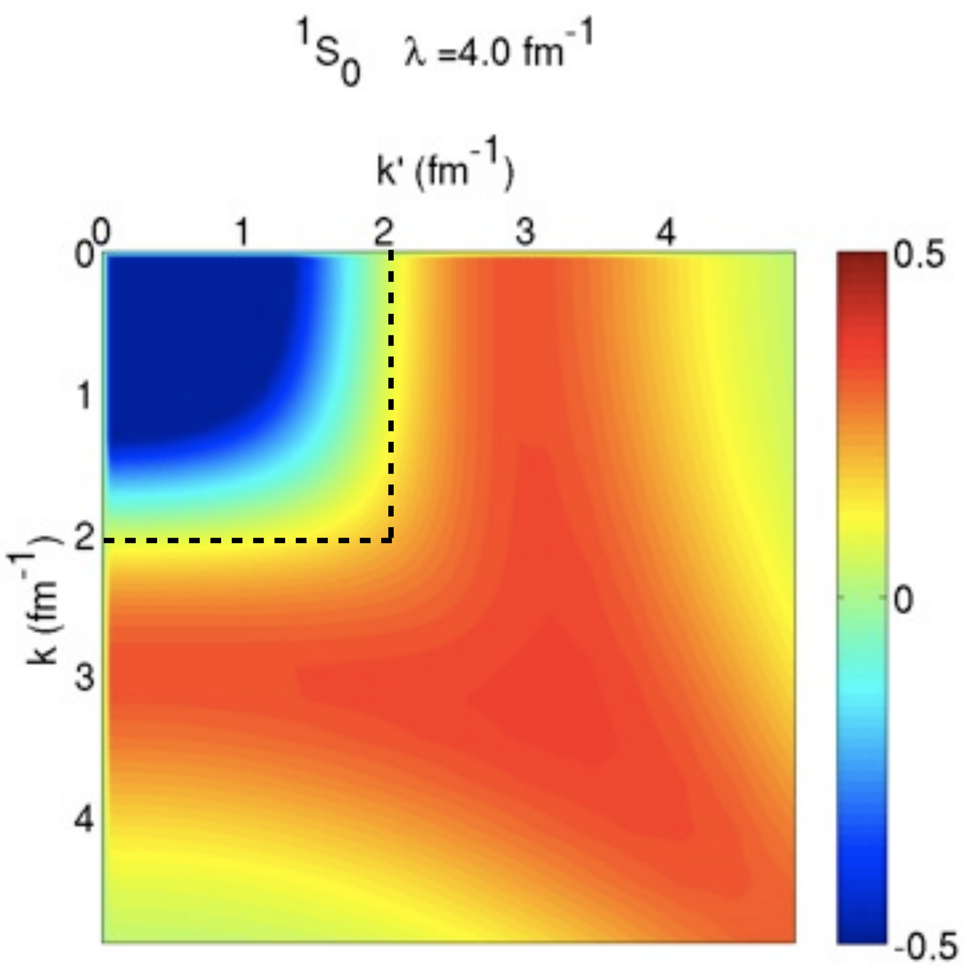

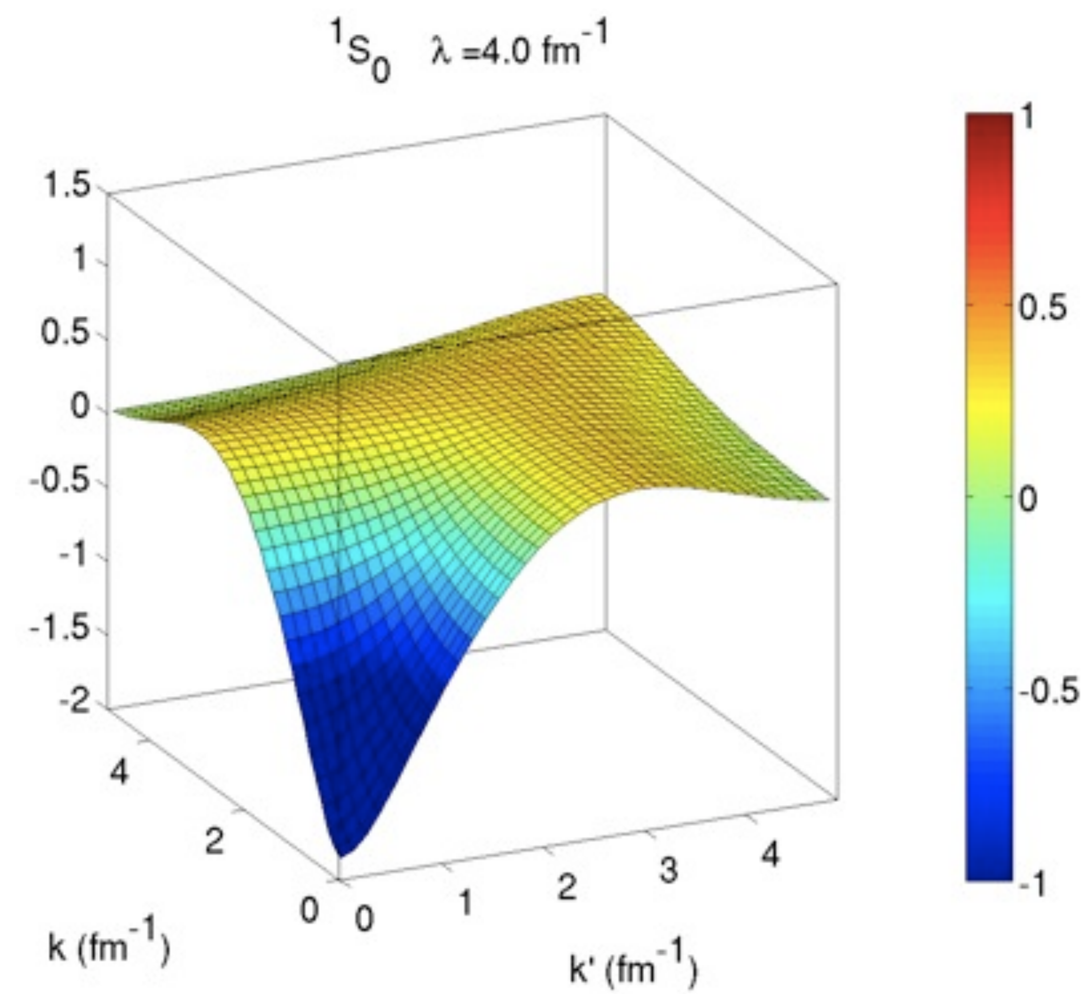

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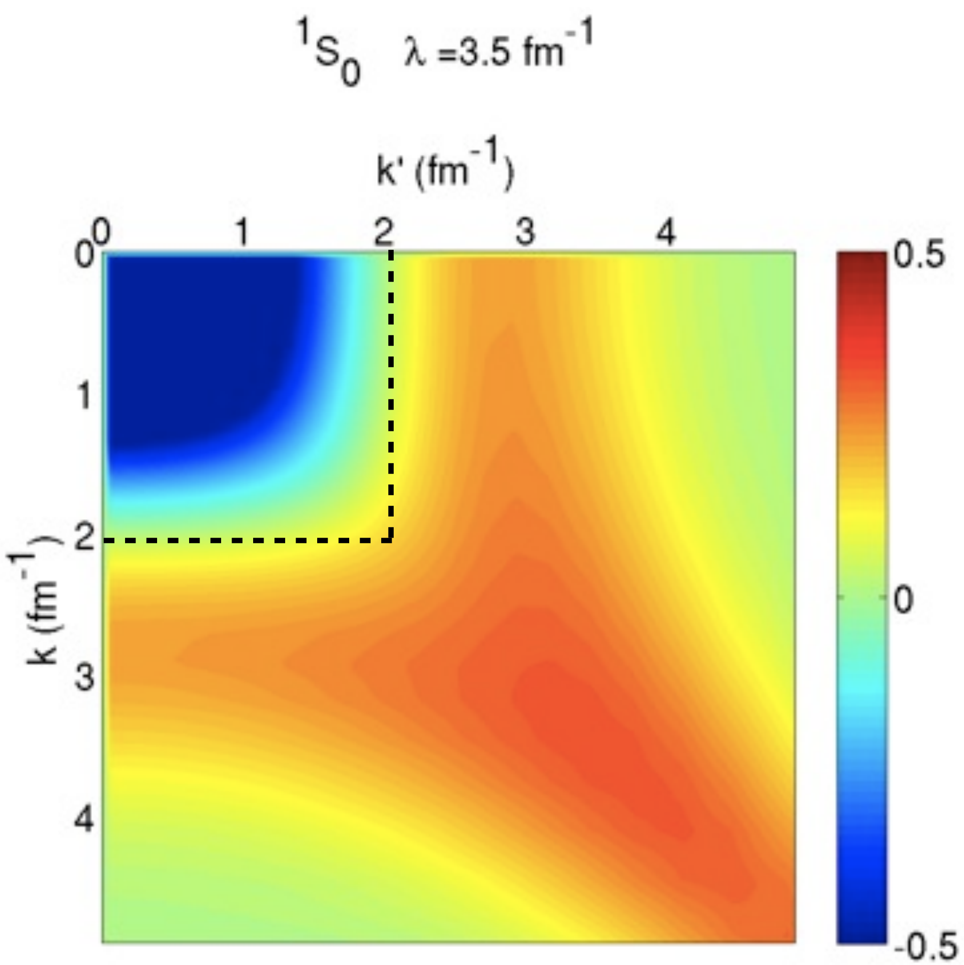

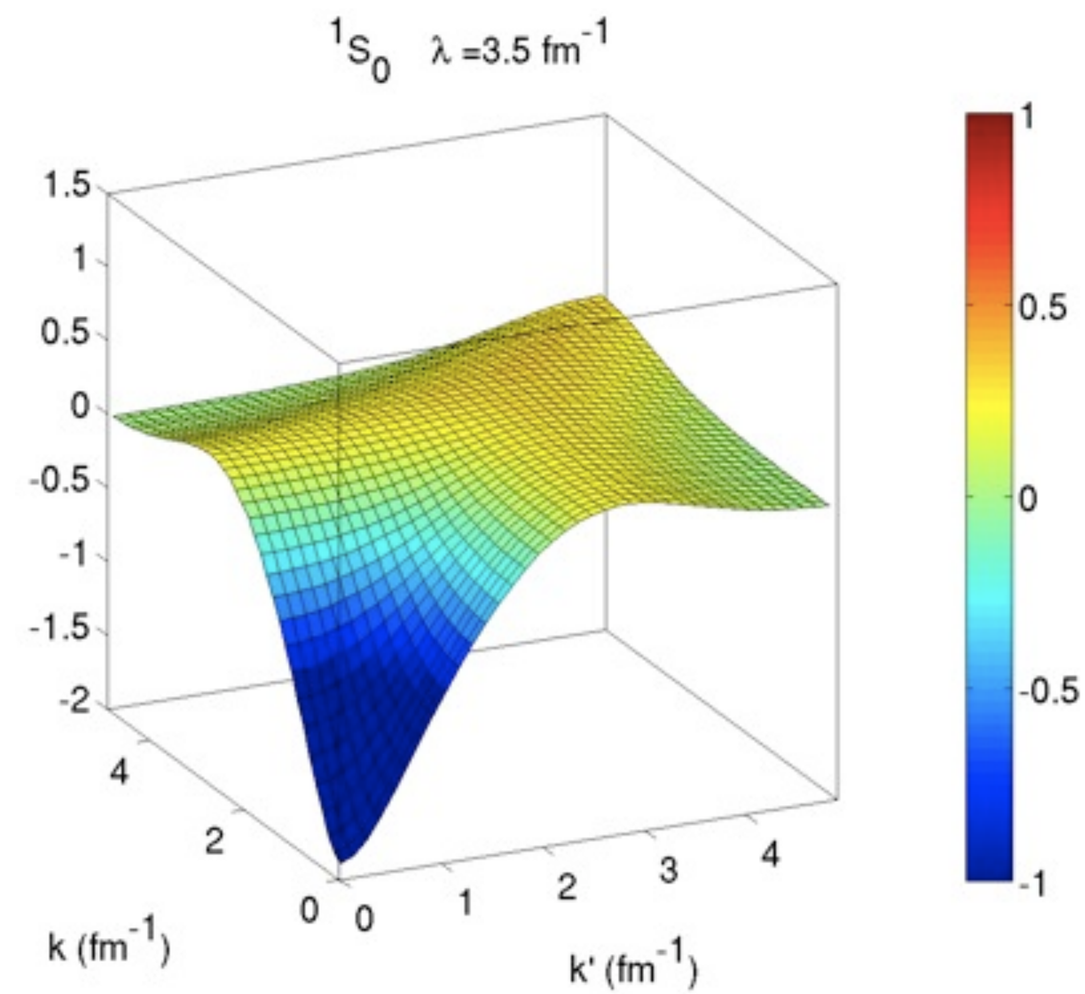

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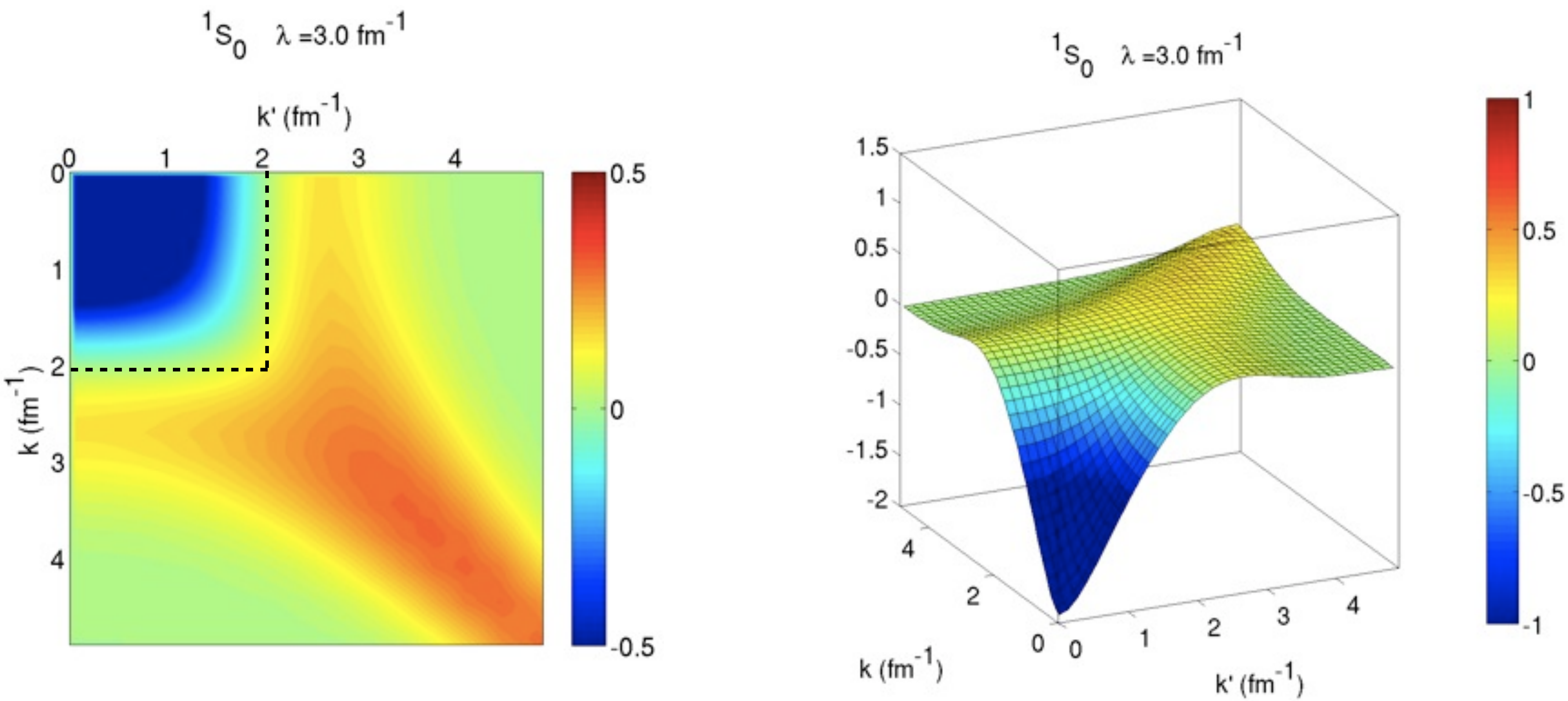

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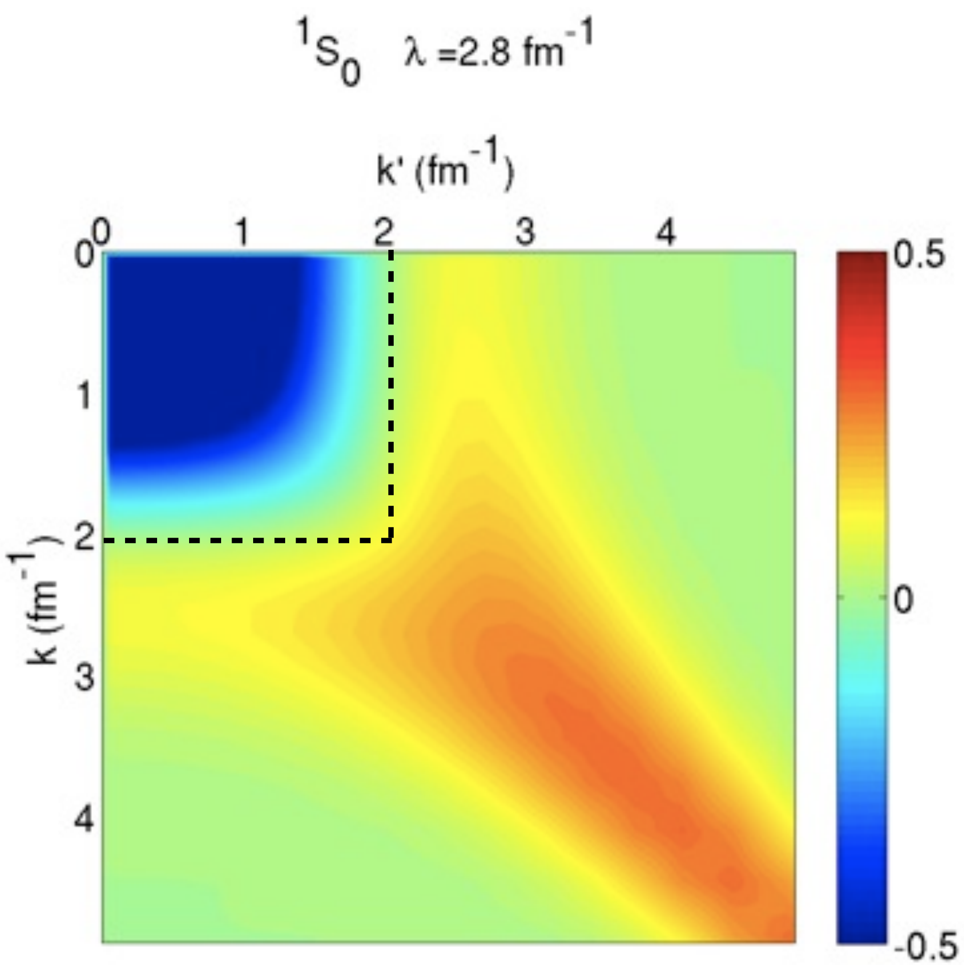

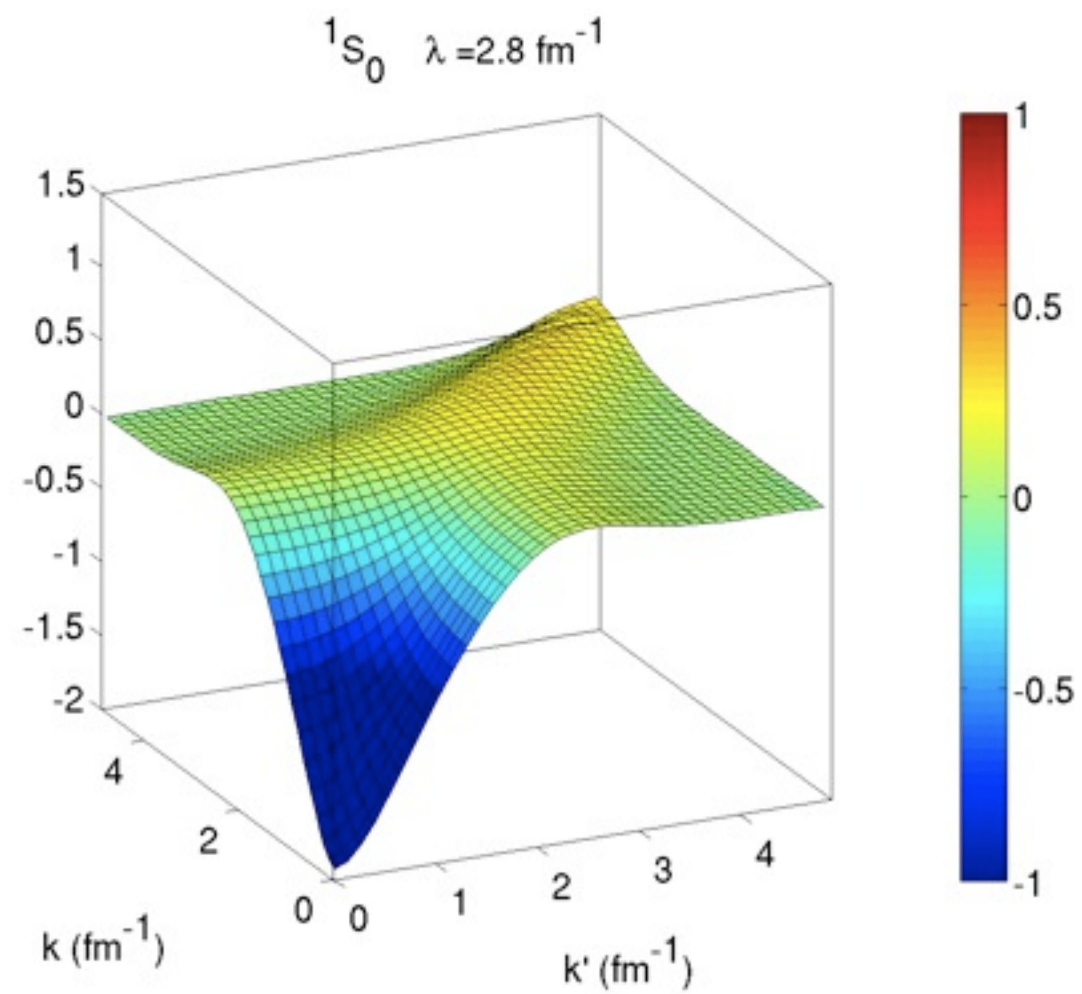

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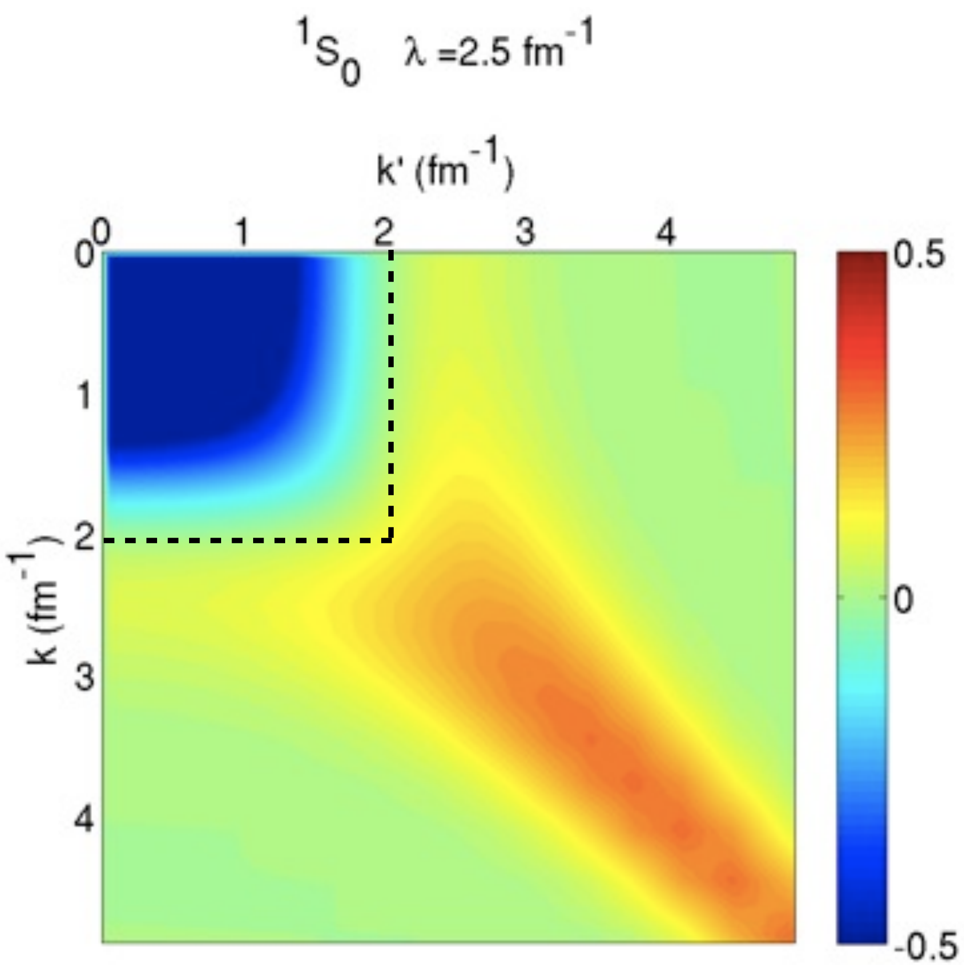

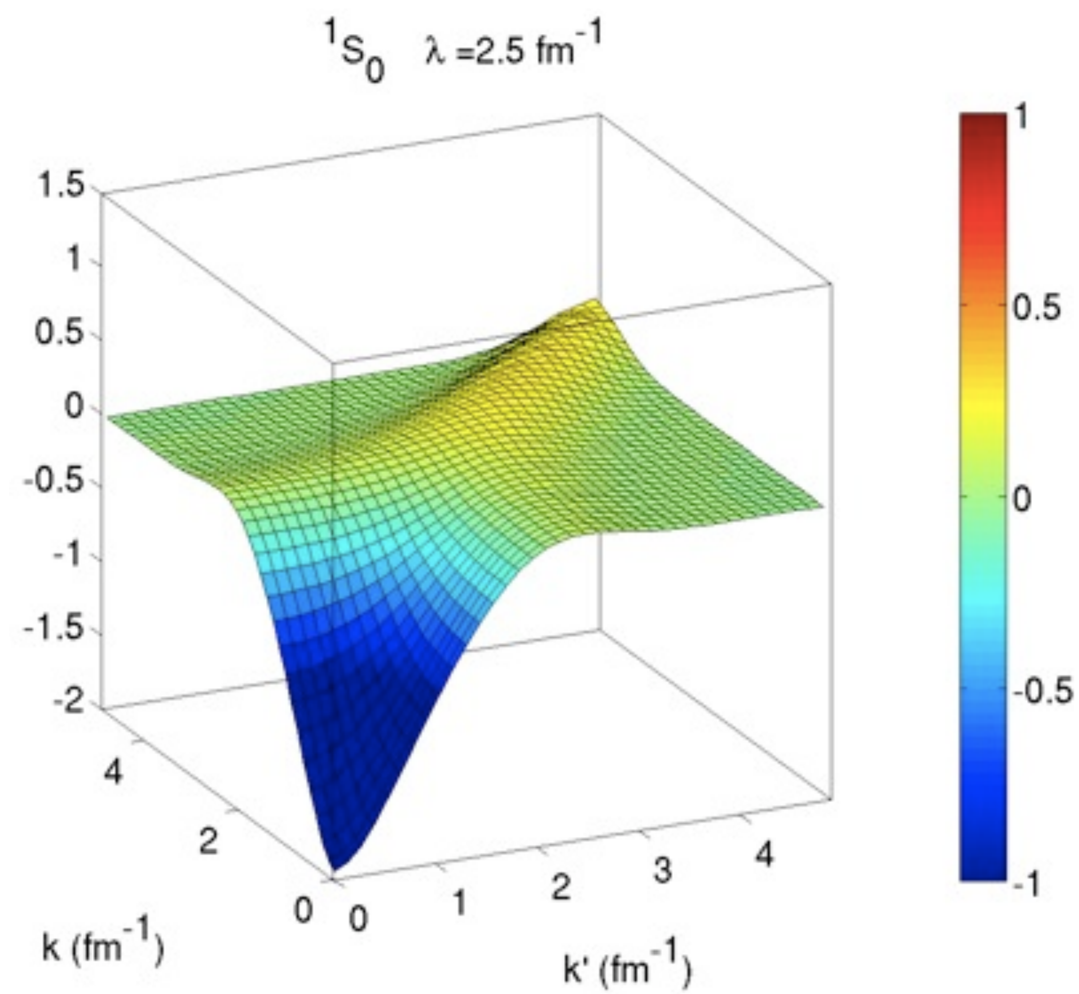

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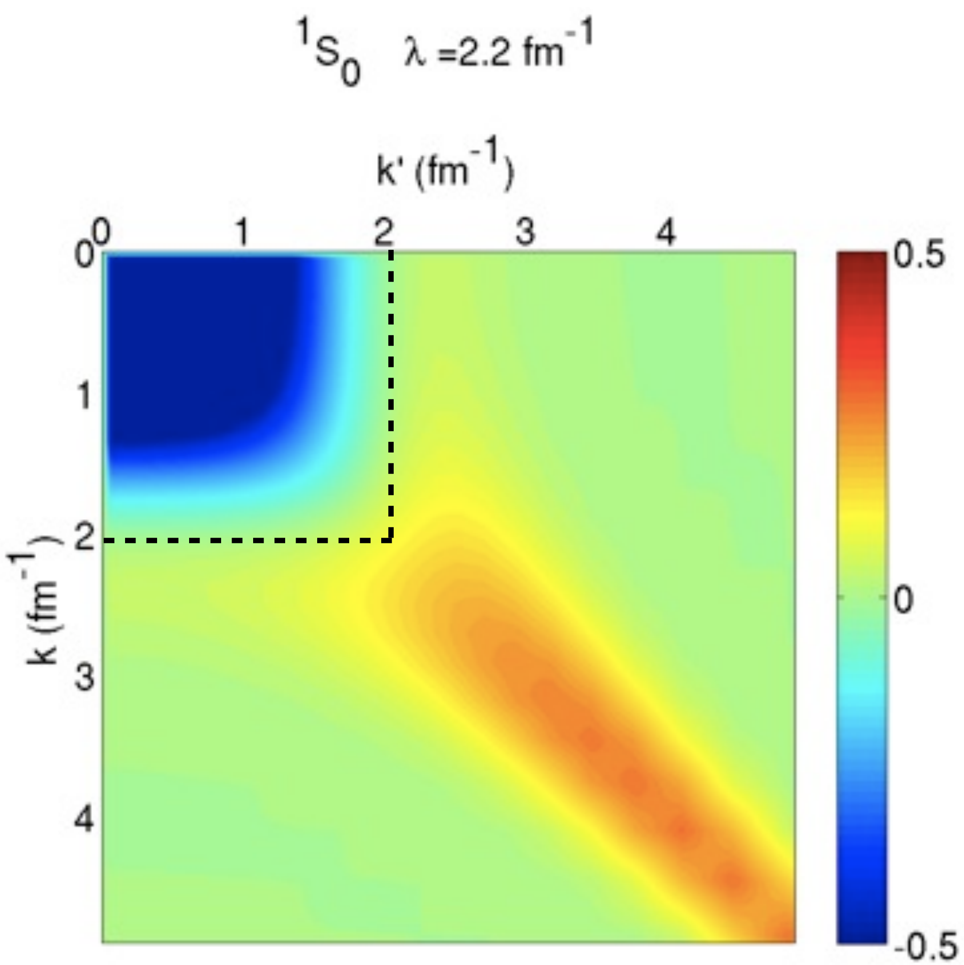

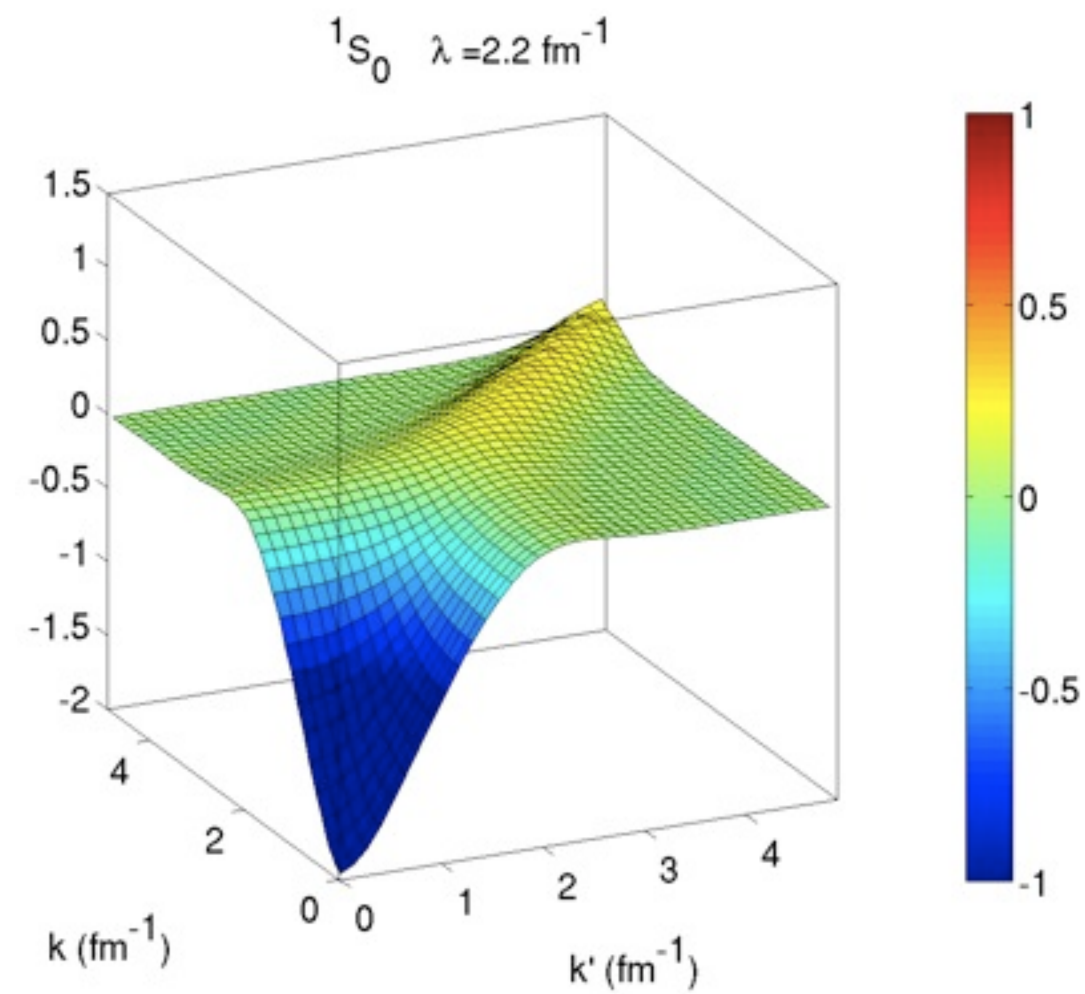

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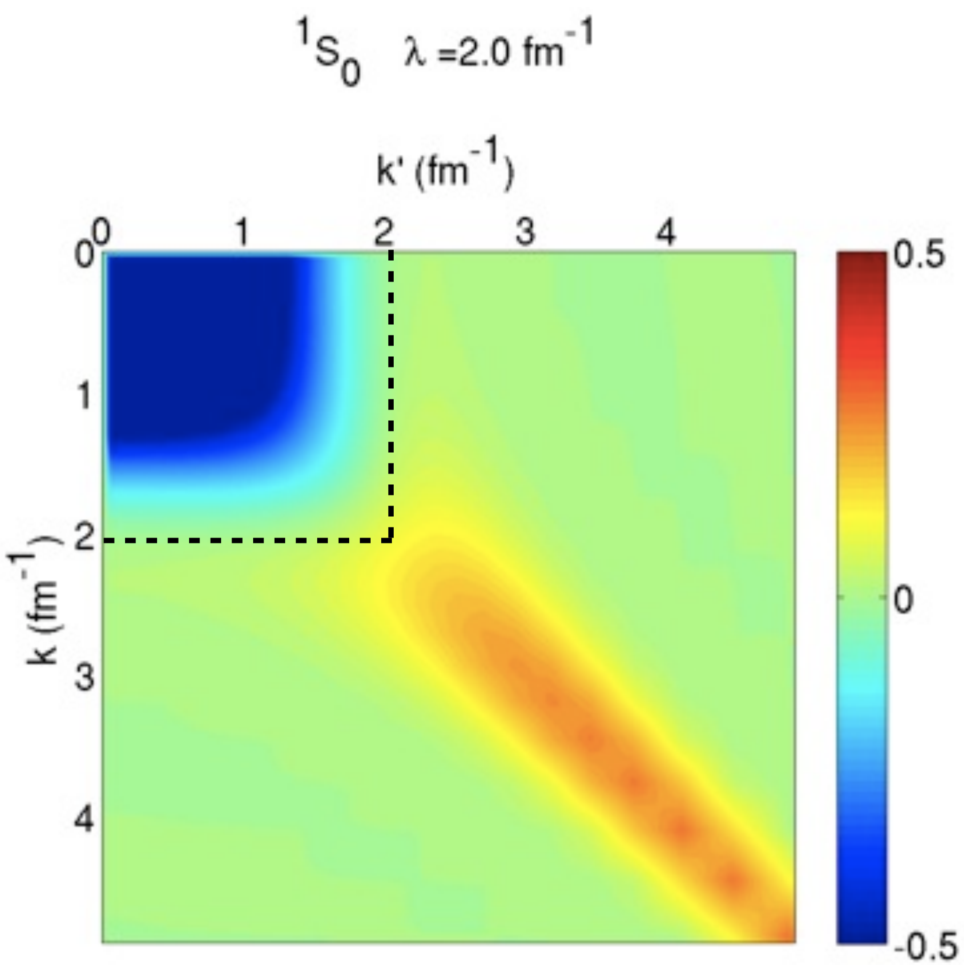

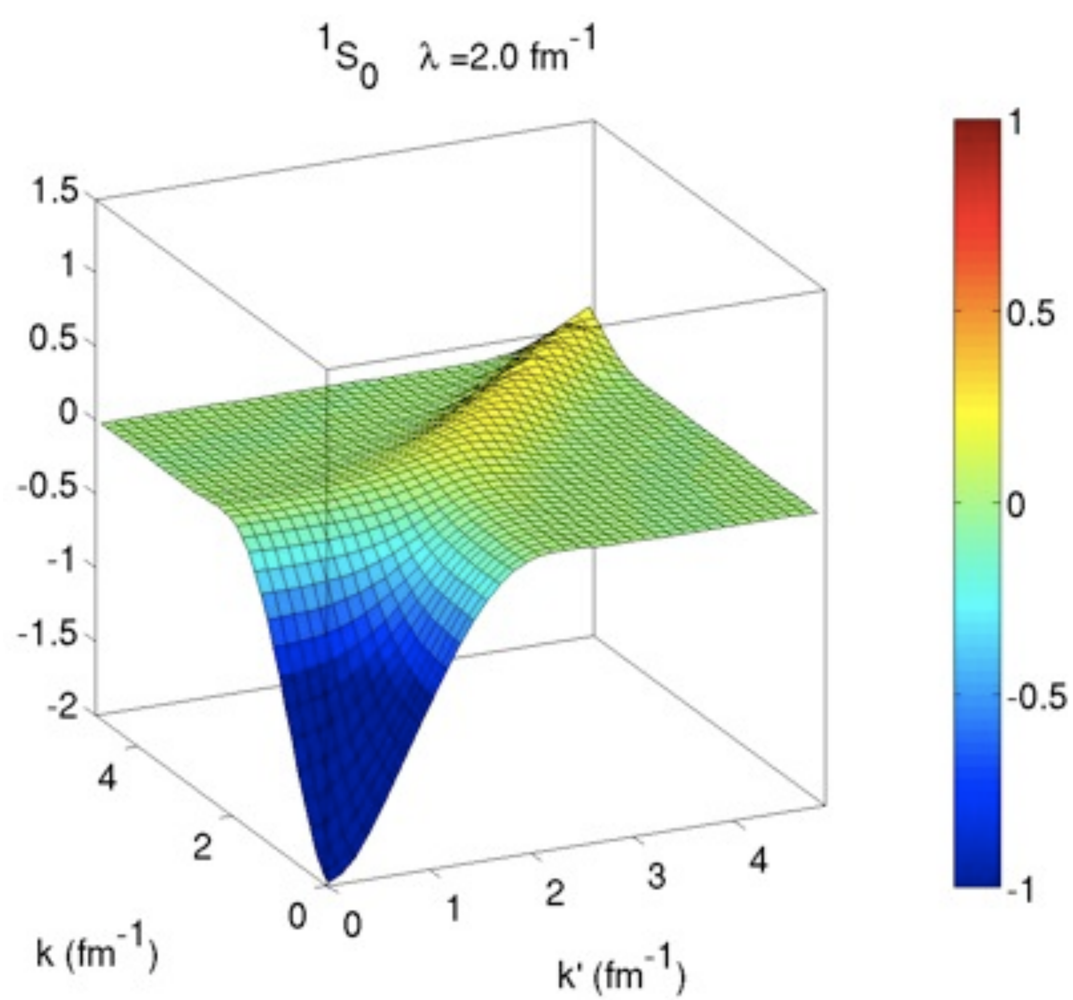

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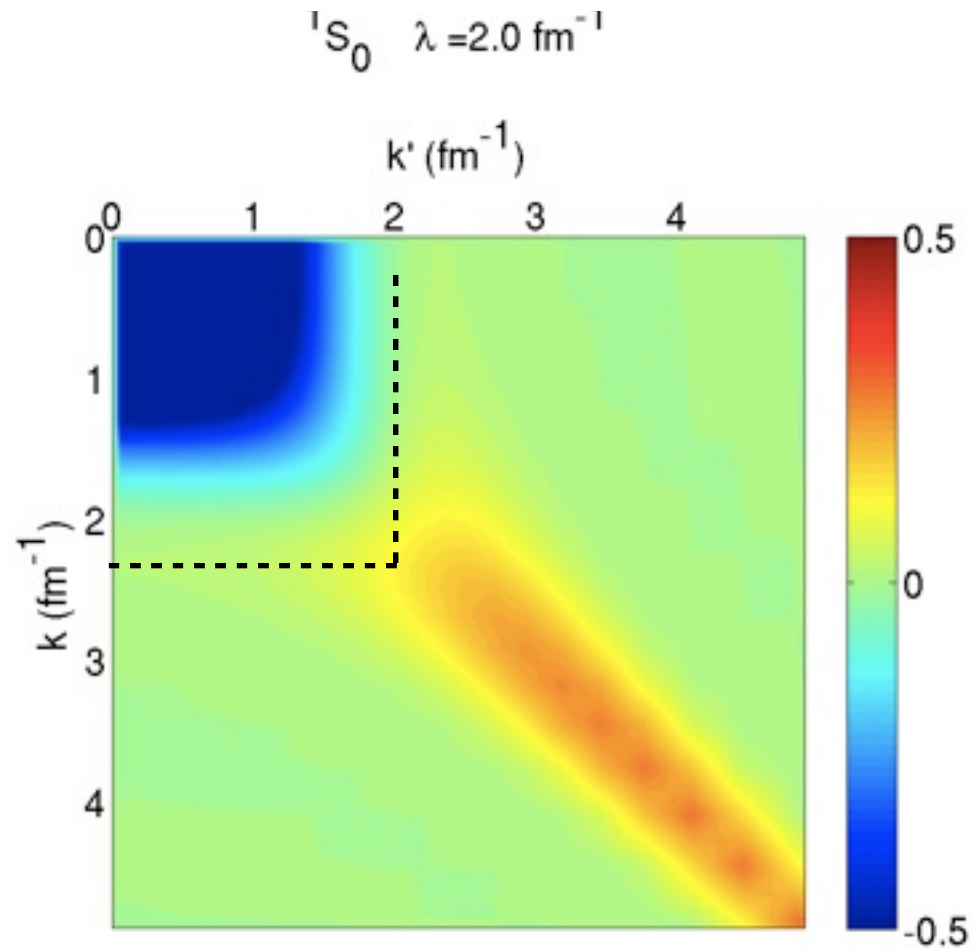
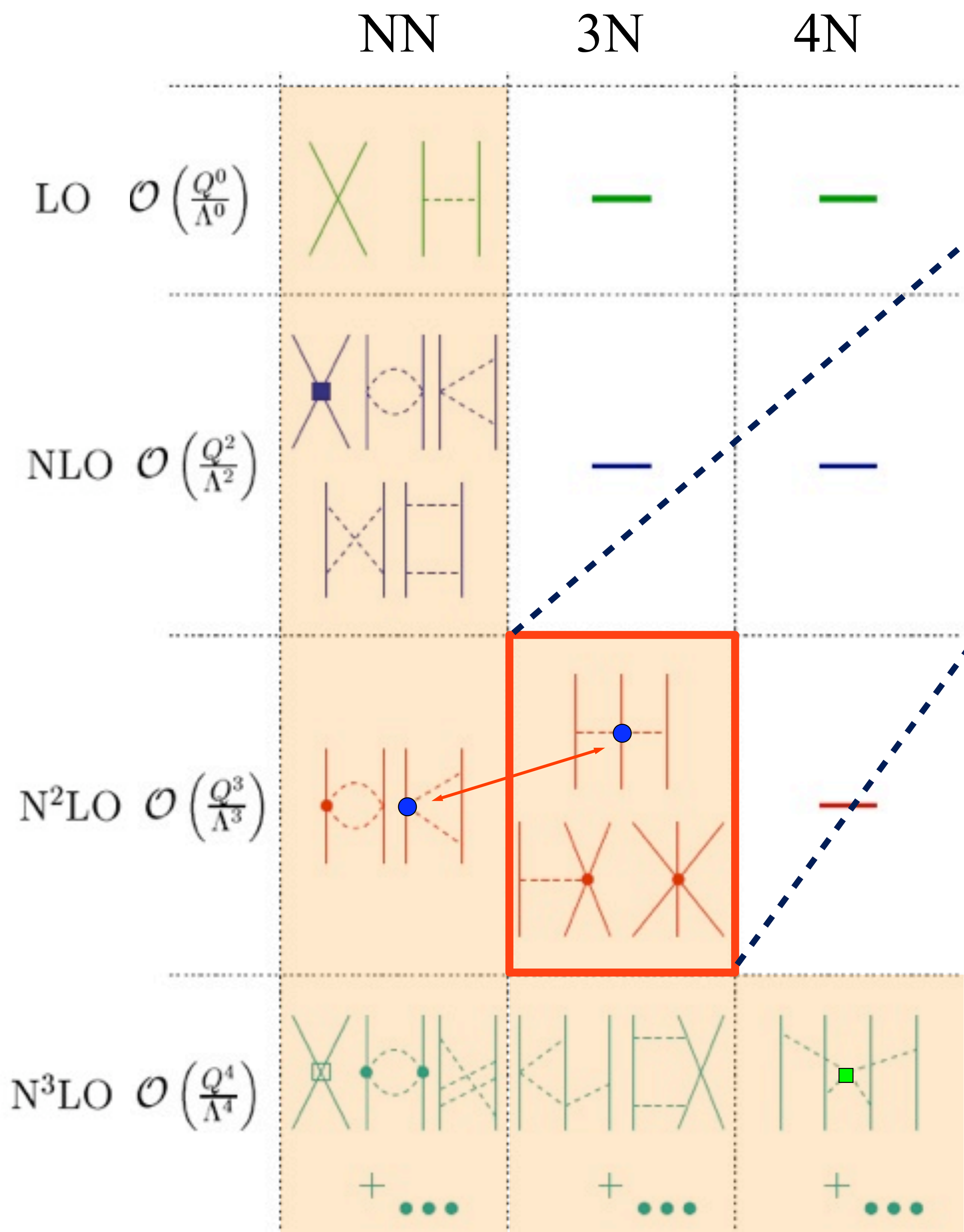
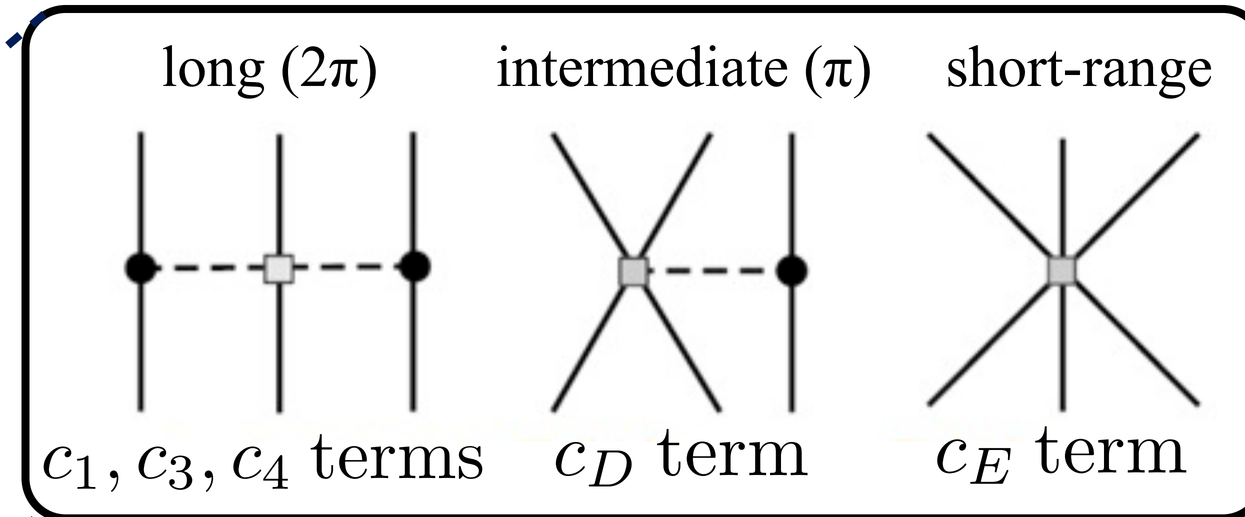


- elimination of coupling between low- and high momentum components, calculations much easier
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations
- RG transformation also changes three-body (and higher-body) interactions

Chiral three-nucleon forces (leading order)





- large uncertainties in 2π coupling constants at present:

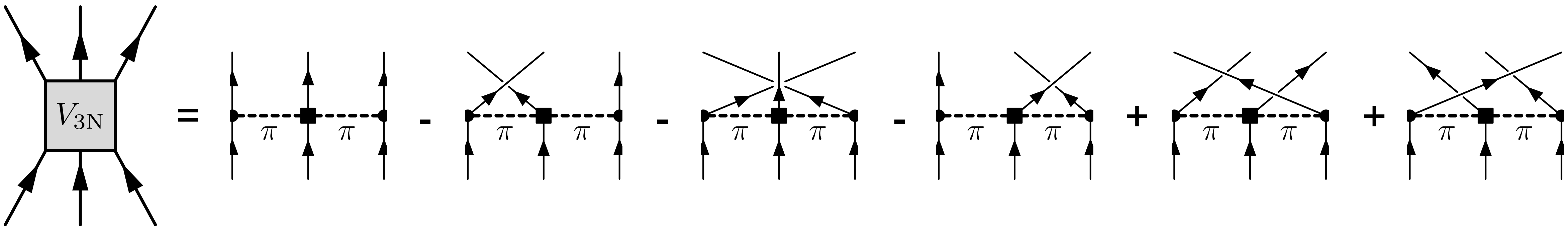
$$c_1 = -0.9^{+0.2}_{-0.5}\ ,\ c_3 = -4.7^{+1.5}_{-1.0}\ ,\ c_4 = 3.5^{+0.5}_{-0.2}$$

leads to theoretical uncertainties in many-body observables

- c_D and c_E have to be determined in $A \geq 3$ systems

Chiral 3N interaction as density-dependent two-body interaction

(1) calculate antisymmetrized 3N interaction



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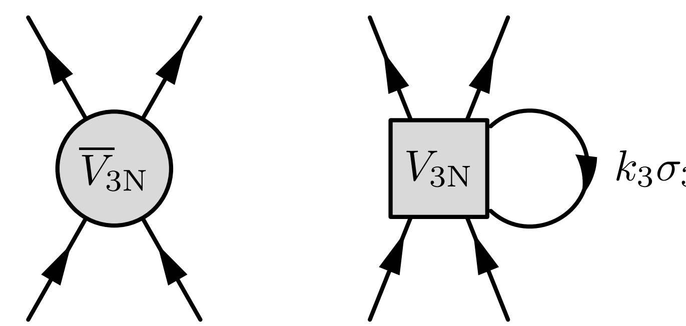
(2) construct effective density-dependent NN interaction

Basic idea:
Sum one particle over occupied states in the Fermi sea



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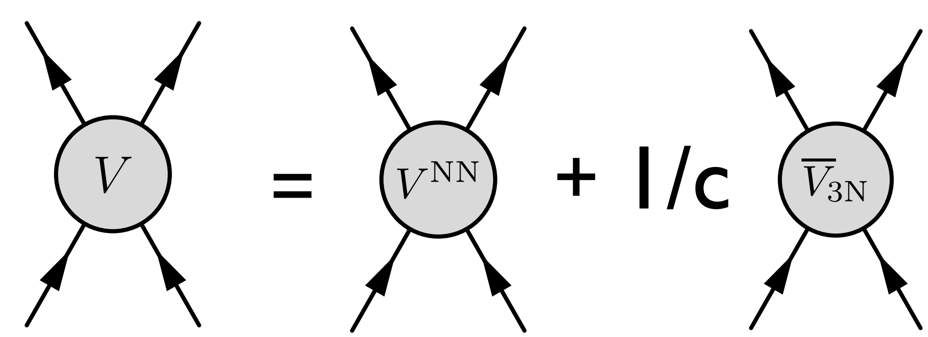
(3) combine with free-space NN interaction

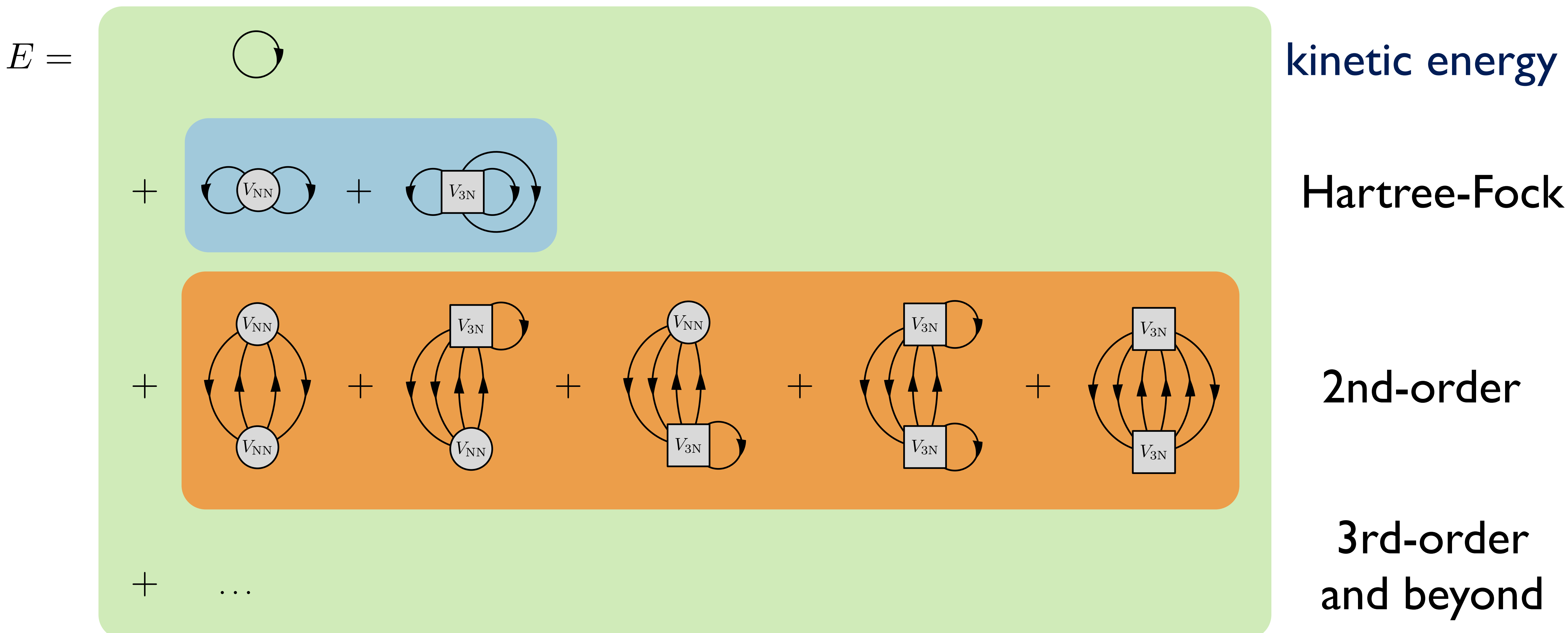
combinatorial factor c depends on type of diagram



Equation of state: Many-body perturbation theory

central quantity of interest: energy per particle E/N

$$H(\lambda) = T + V_{\rm NN}(\lambda) + V_{\rm 3N}(\lambda) + \ldots$$

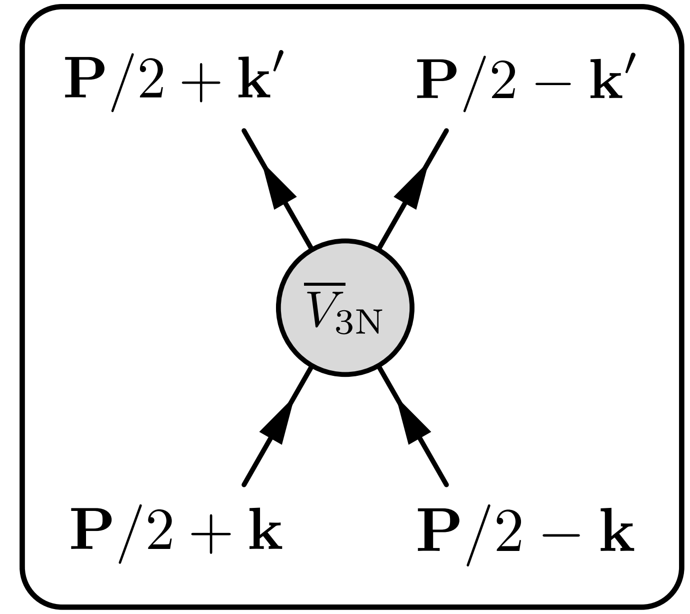


- "hard" interactions require non-perturbative summation of diagrams
- with low-momentum interactions much more perturbative
- inclusion of 3N interaction contributions crucial

Properties of the effective interaction $\overline{V}_{3N}$

General momentum dependence:

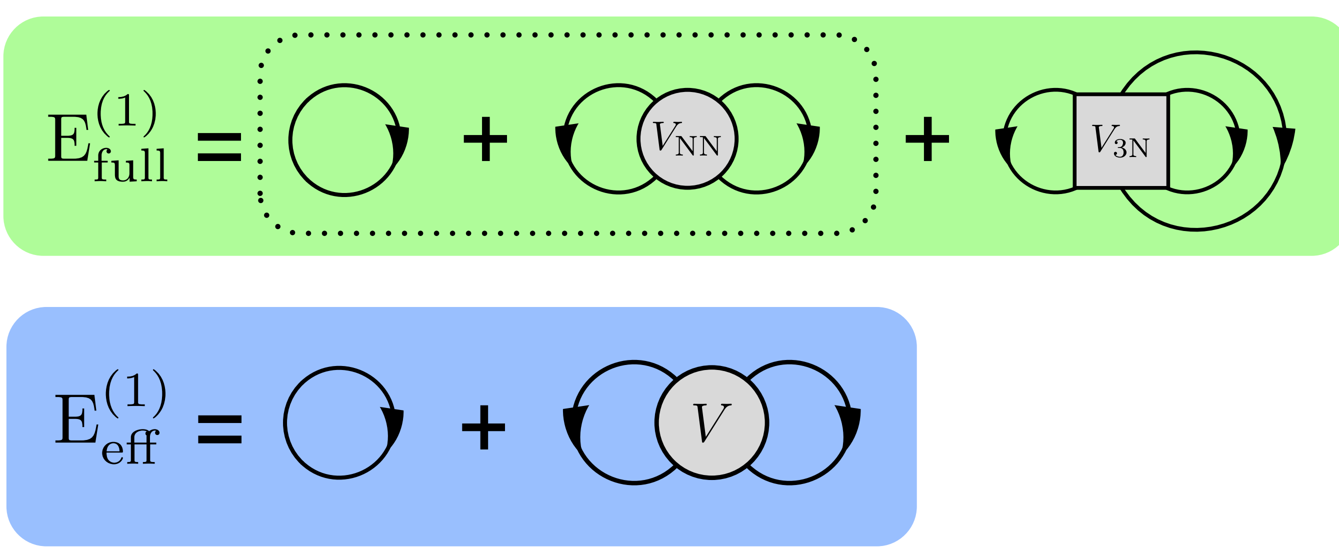
$$\overline{V}_{3N} = \overline{V}_{3N}(\mathbf{k}, \mathbf{k}', \mathbf{P})$$

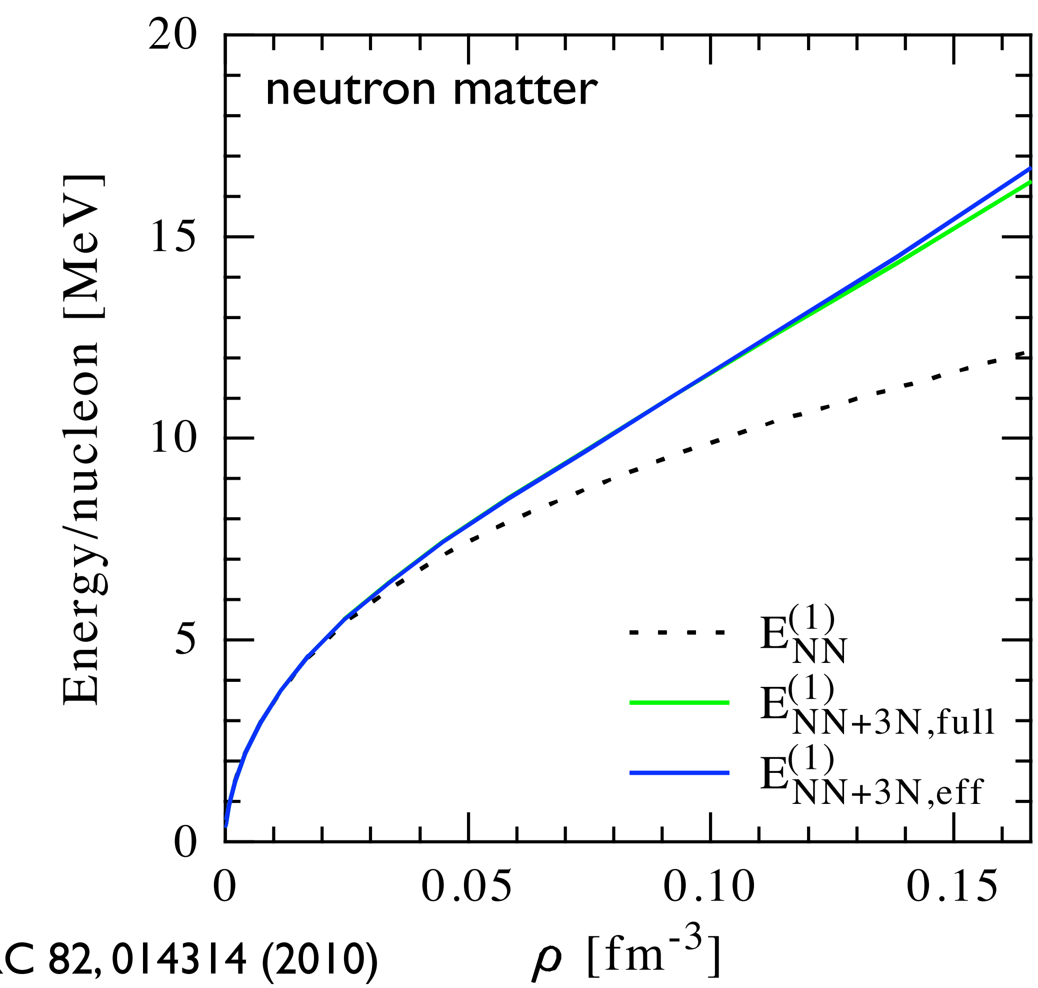


- $\mathbf{P}$-dependence much weaker than $\mathbf{k}, \mathbf{k}'$-dependence!
- neglect $\mathbf{P}$-dependence, set $\mathbf{P} = \mathbf{0}$
- matrix elements have the same form like free-space

NN interaction matrix elements

- straightforward to include in existing many-body schemes

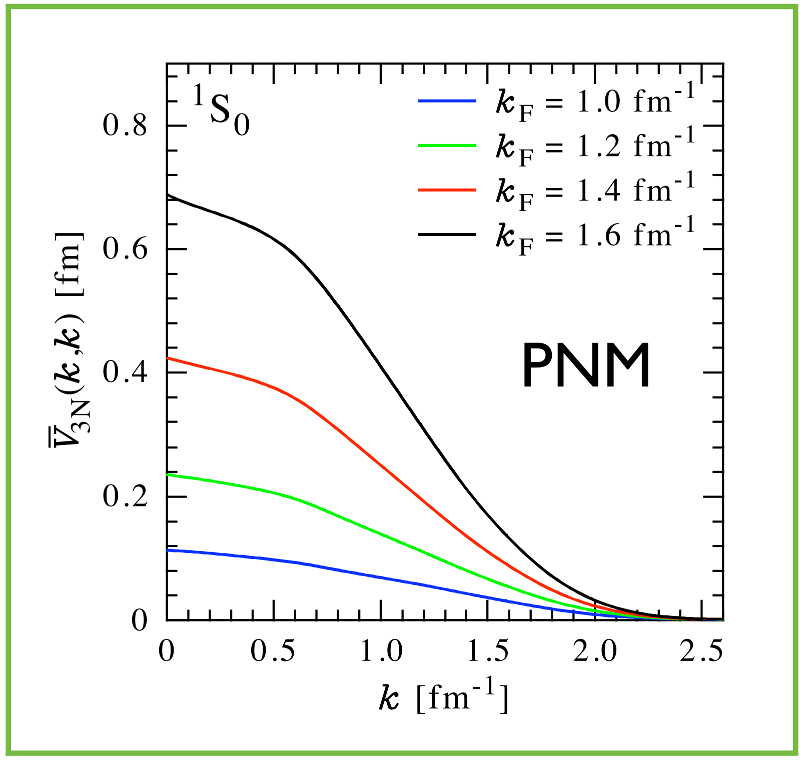


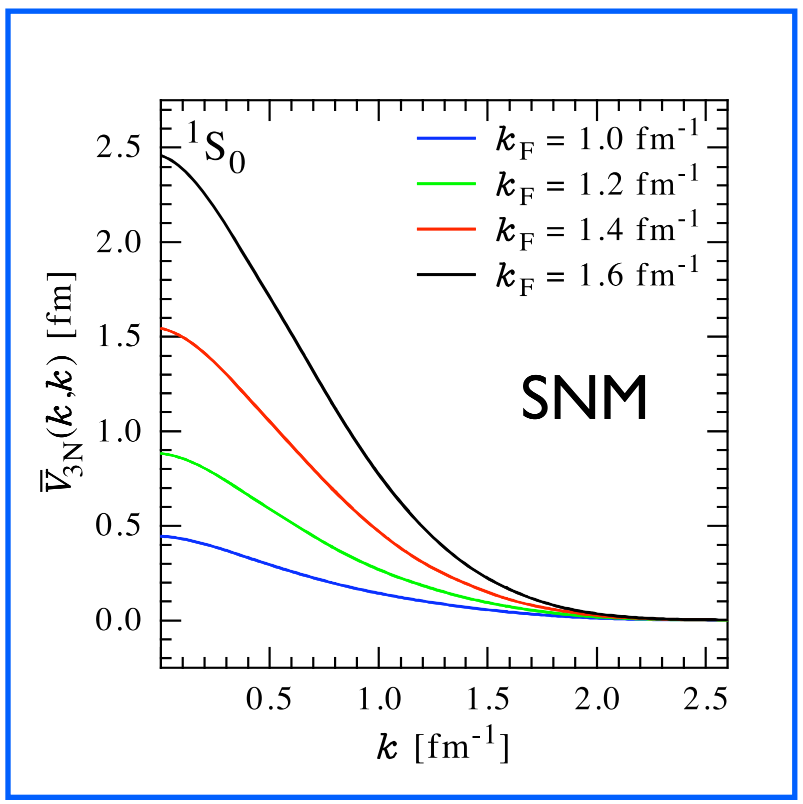


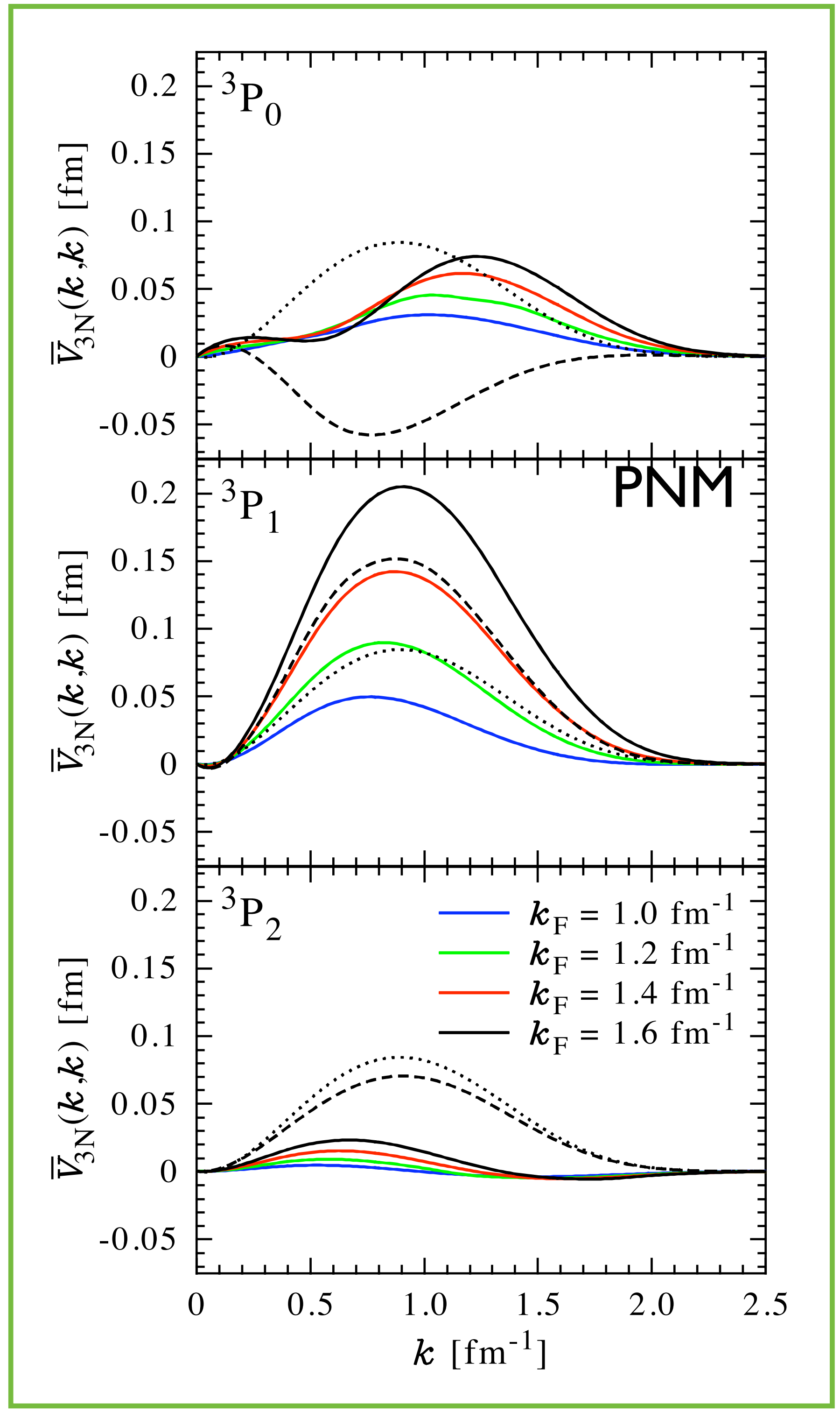
KH and Schwenk, PRC 82, 014314 (2010)

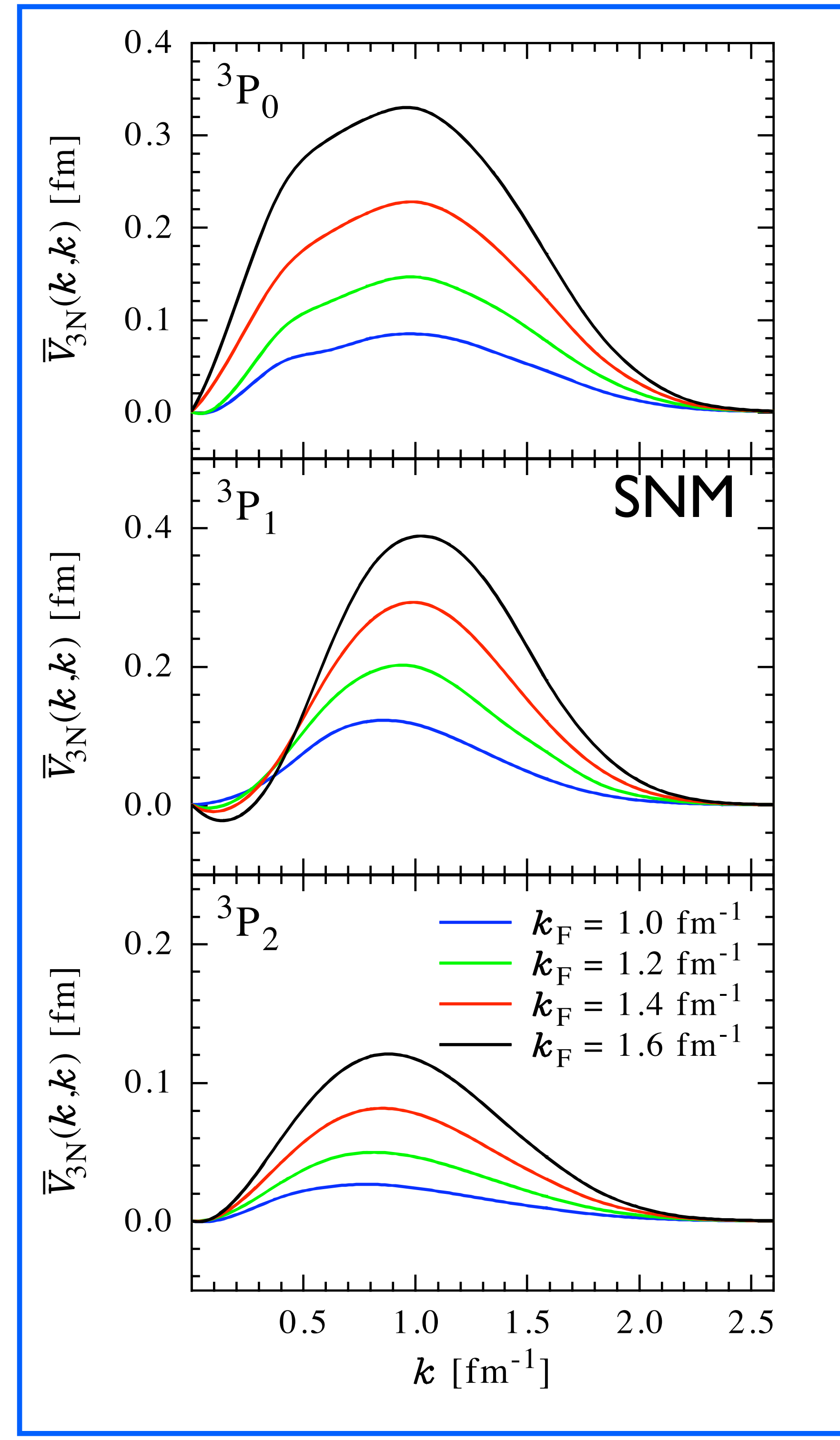
Properties of the effective interaction $\overline{V}_{3N}$

$(\Lambda_{3N} = 2.0\,\mathrm{fm}^{-1})$

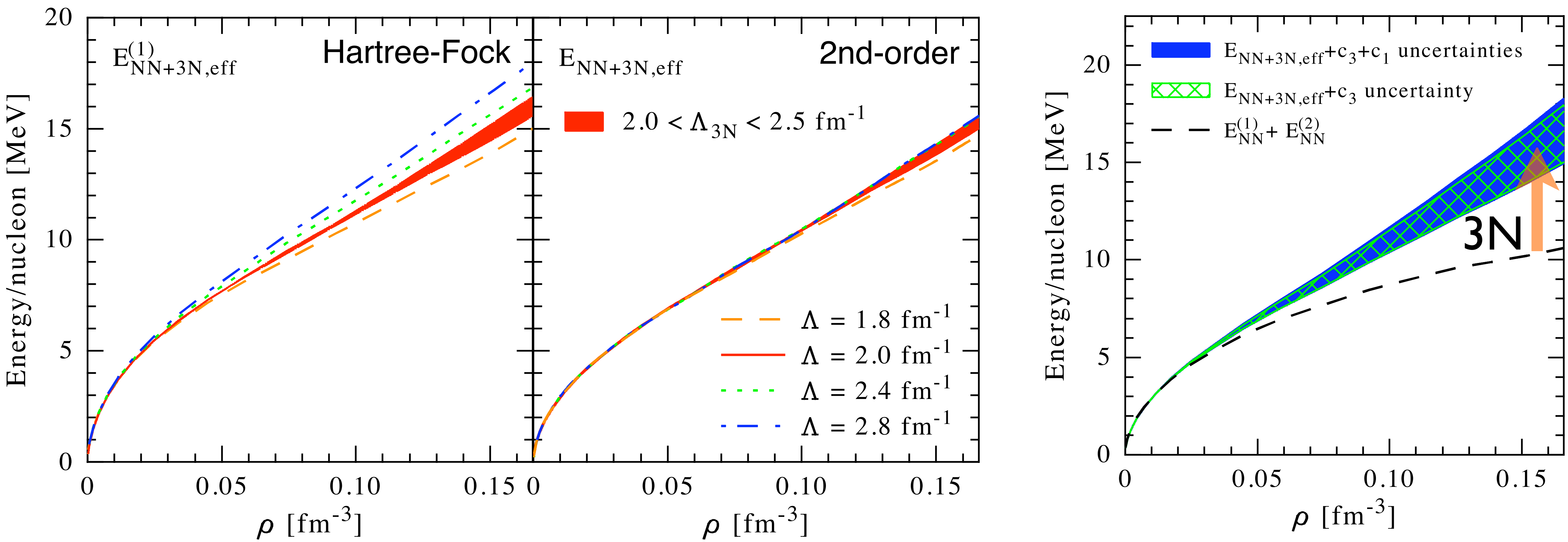








Equation of state of pure neutron matter



KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence

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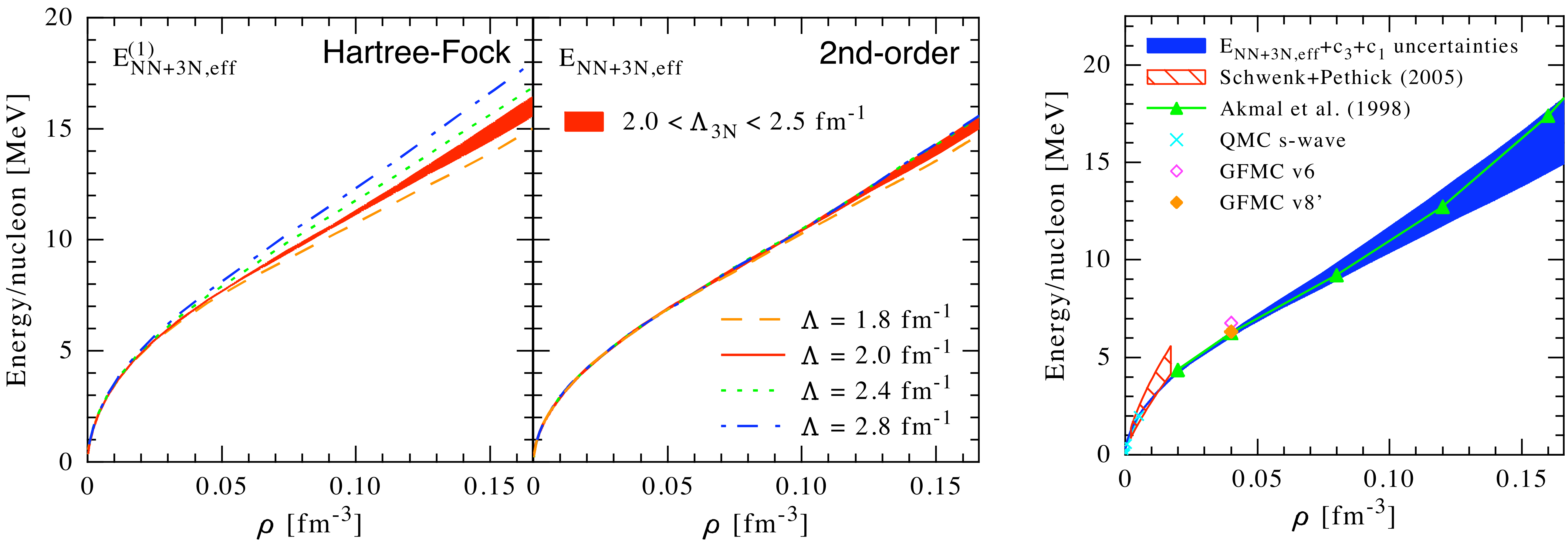

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- variation due to 3N input uncertainty much larger than resolution dependence
- good agreement with other approaches (different NN interactions)

Neutron matter: Symmetry energy

$$E(\rho, \alpha = 1) = -a_V + \frac{K_0}{18\rho_0^2}(\rho - \rho_0)^2 + S_2(\rho)$$

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_0^2}(\rho - \rho_0)$$

c_1 [GeV]	c_3 [GeV]	a_4 [MeV]	p_0 [MeV fm^{-3}]
-0.81	-3.2	31.7	$2.4/2.5$
-0.81	-5.7	33.7	$2.9/3.0$
-0.7	-3.2	31.7	$2.4/2.5$
-1.4	-5.7	34.5	$3.3/3.4$

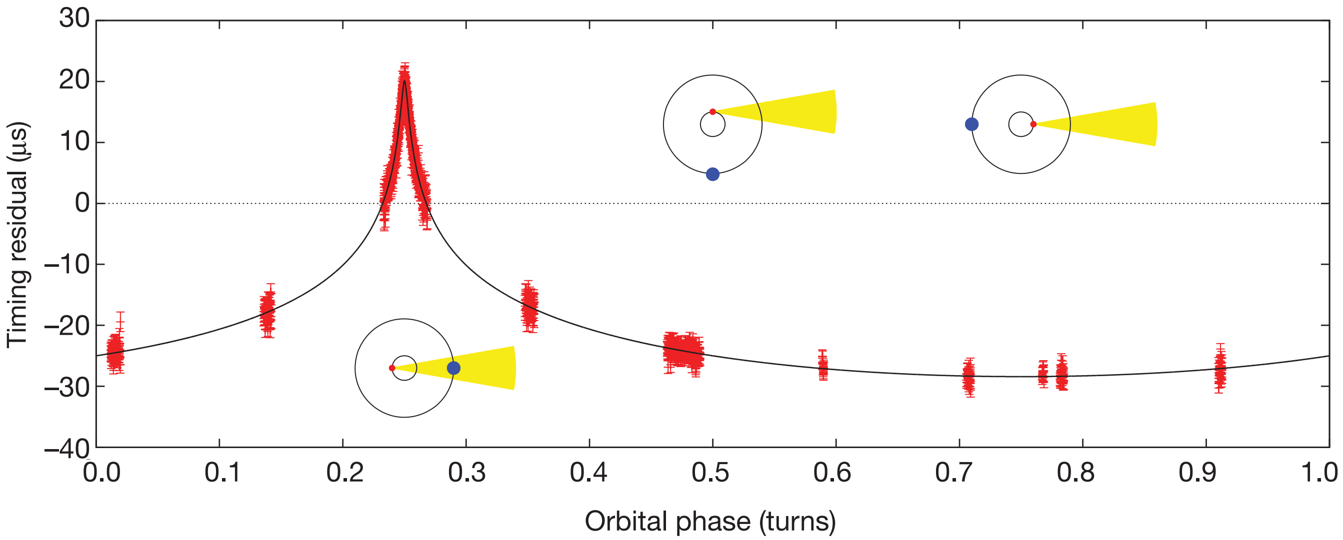
- uncertainties in c_i couplings lead to uncertainties in symmetry energy
- given the experimental constraint $a_4 = 30 \pm 4\,\mathrm{MeV}$ smaller absolute values of c_3 seem to be preferred from our results

Constraints on the nuclear equation of state (EOS)

nature

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]



Demorest et al., Nature 467, 1081 (2010)

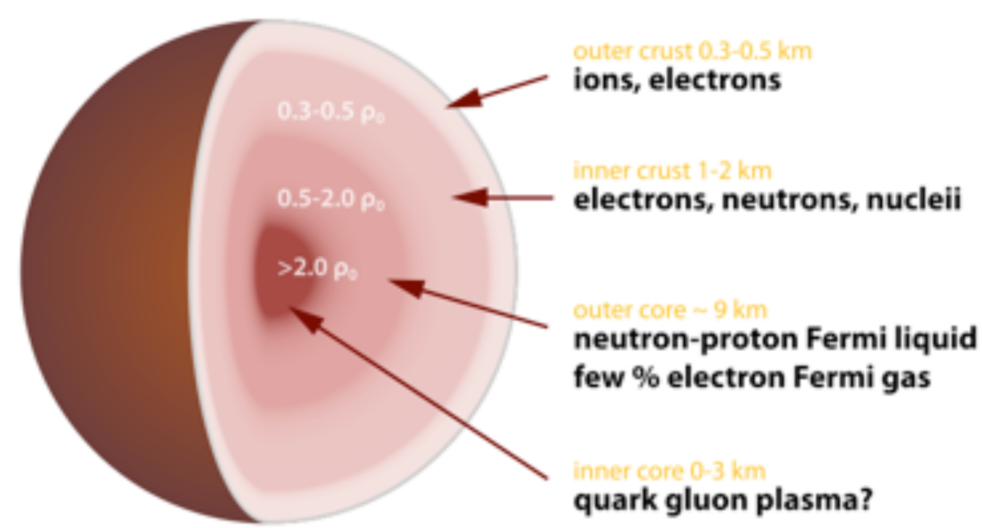
$$M_{\max} = 1.65 M_{\odot} \rightarrow 1.97 \pm 0.04\, M_{\odot}$$

Credit: NASA/Dana Berry

Structure of a neutron star is determined by Tolman-Oppenheimer-Volkov (TOV) equation:

$$\frac{dP}{dr} = -\frac{GM\epsilon}{r^2}\left[1+\frac{P}{\epsilon c^2}\right]\left[1+\frac{4\pi r^3 P}{Mc^2}\right]\left[1-\frac{2GM}{c^2 r}\right]^{-1}$$

crucial ingredient: energy density $\epsilon = \epsilon(P)$



Neutron star radius constraints

Problem: Solution of TOV equation requires EOS up to very high densities.
Radius of a typical NS (M~1.4 $M_\odot$) theoretically not well constrained.

But: Radius of NS is relatively insensitive to high density region.

---

incorporation of beta-equilibrium: neutron matter ⟶ neutron star matter

parametrize piecewise high-density extensions of EOS:

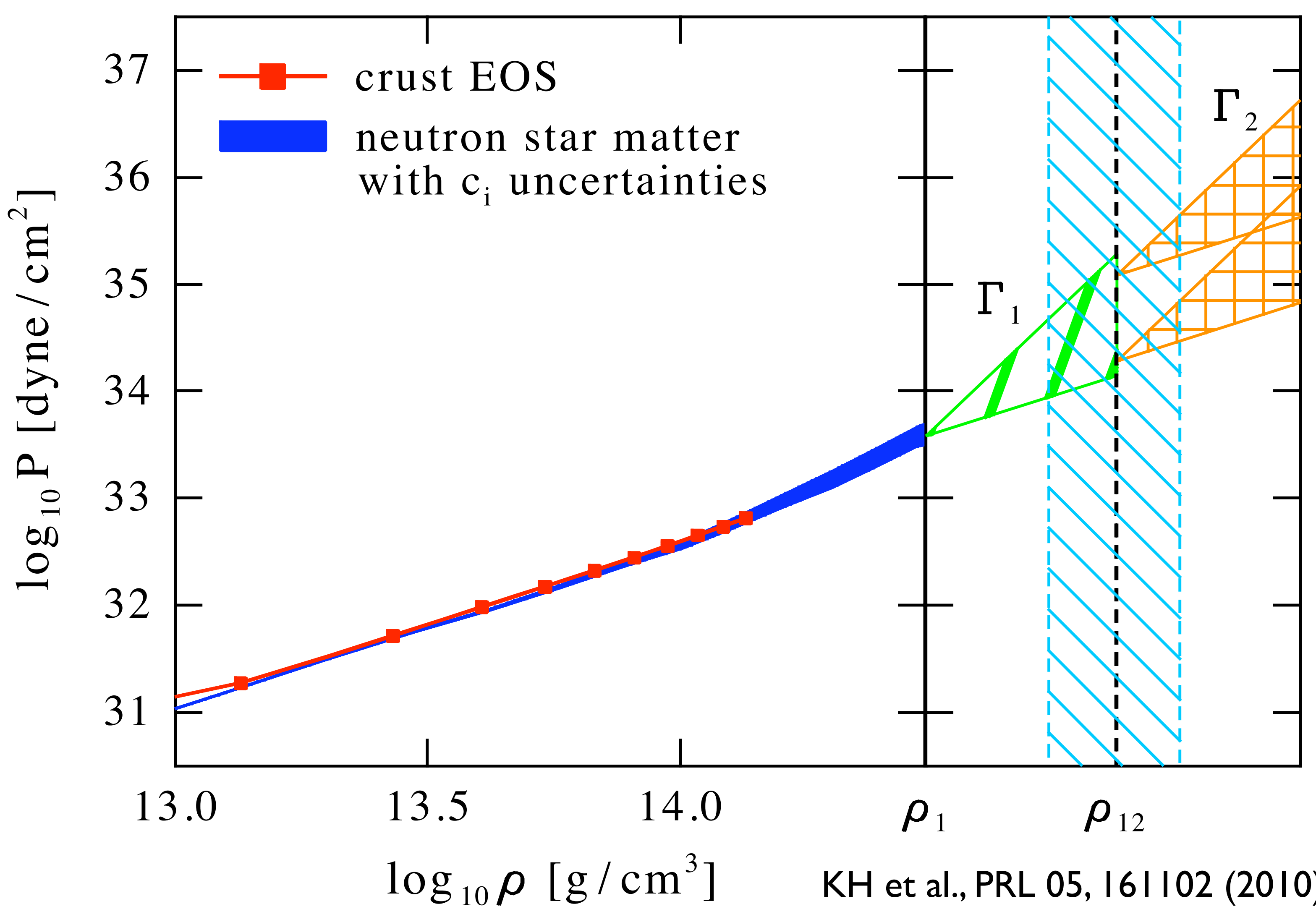
- use polytropic ansatz

$$p \sim \rho^\Gamma$$

- range of parameters

$$\Gamma_1, \rho_{12}, \Gamma_2$$

limited by physics



KH et al., PRL 05, 161102 (2010)

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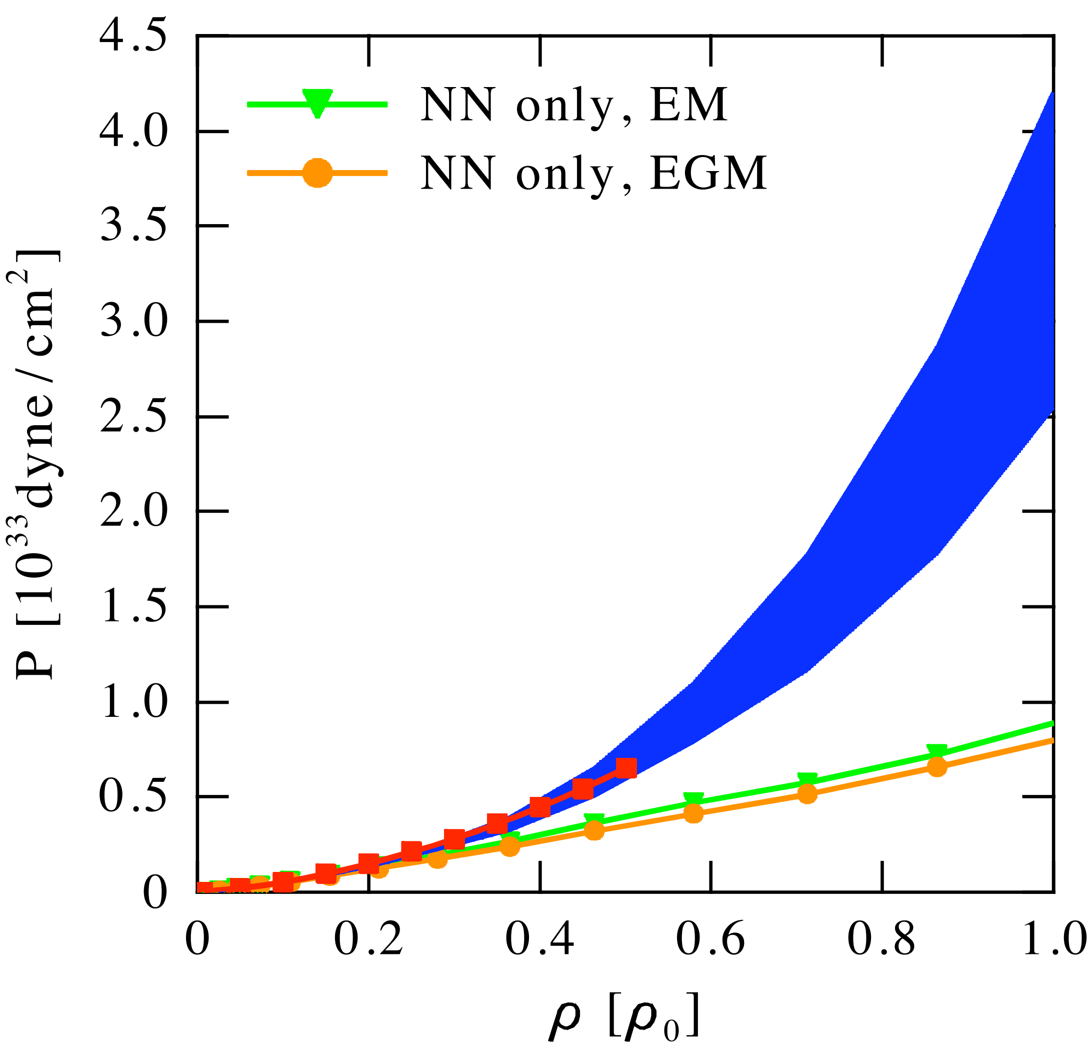
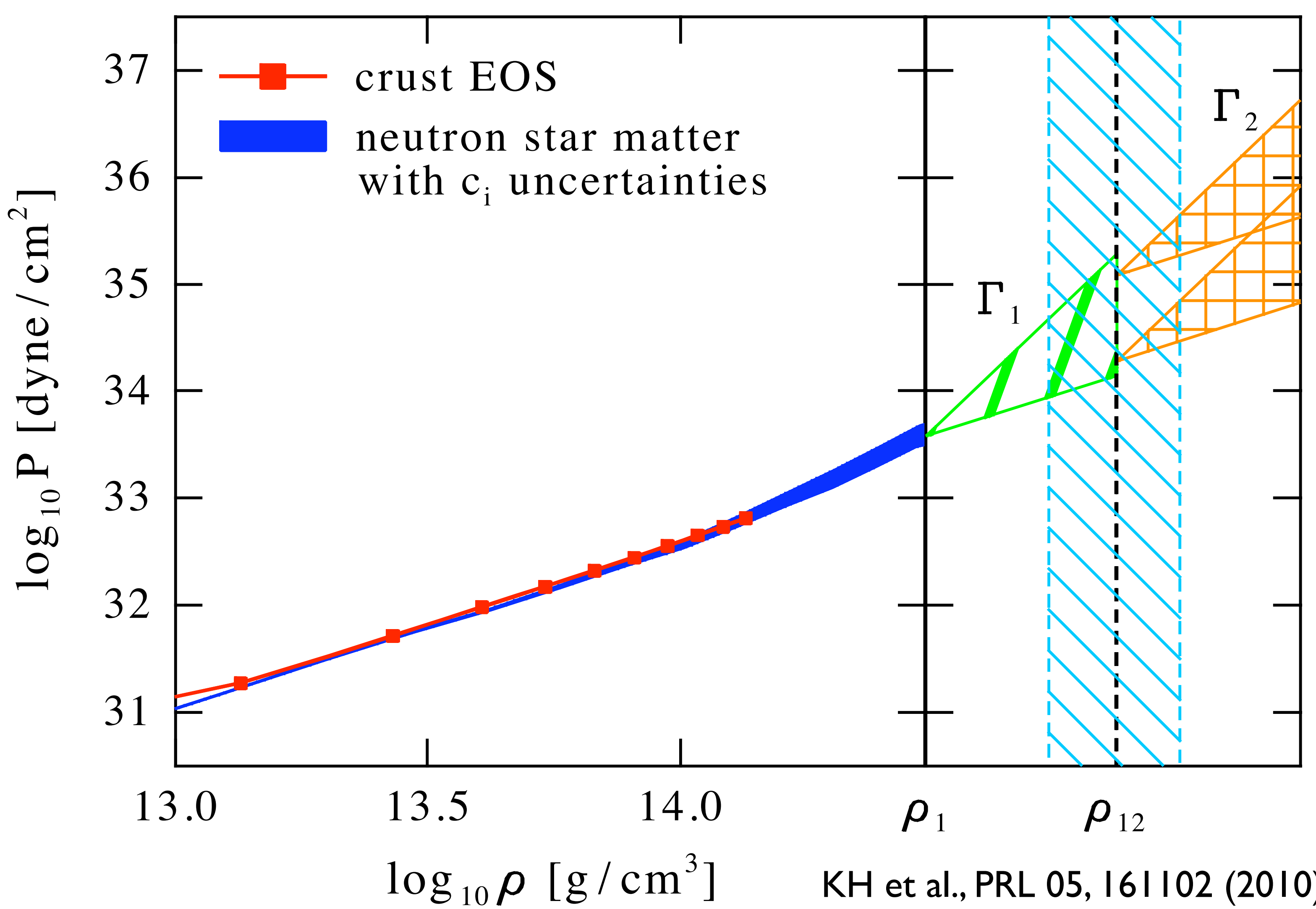




KH et al., PRL 05, 161102 (2010)

without 3N forces EOS differs significantly from crust EOS around $\rho_0/2$

Neutron star radius constraints

use the constraints:

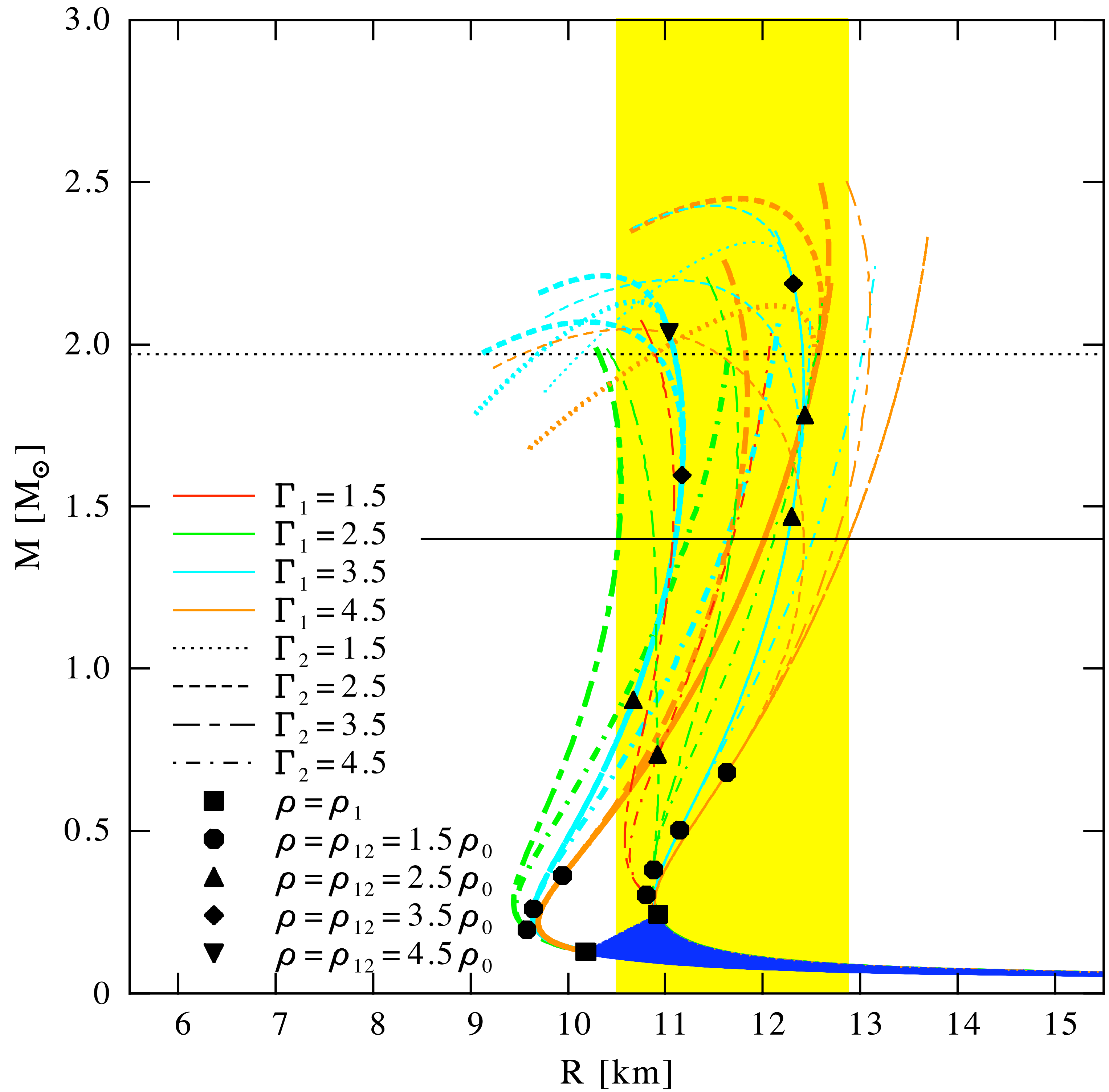
recent NS observation

$$M_{\max} > 1.97\, M_\odot$$

causality

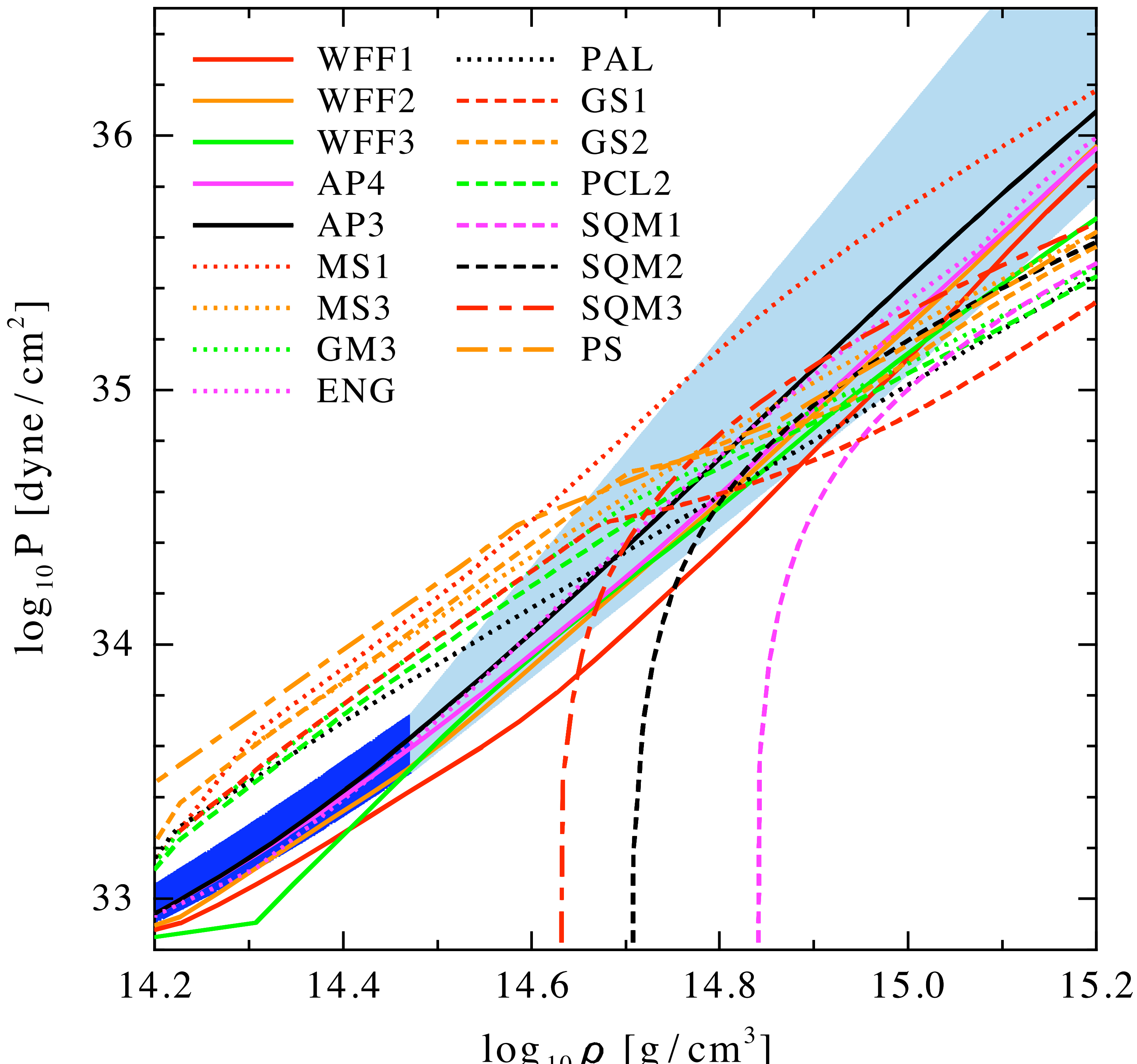
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

cf. KH et al., PRL 05, 161102 (2010)



- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint after incorporating crust corrections: $10.7 - 13.4\,\mathrm{km}$

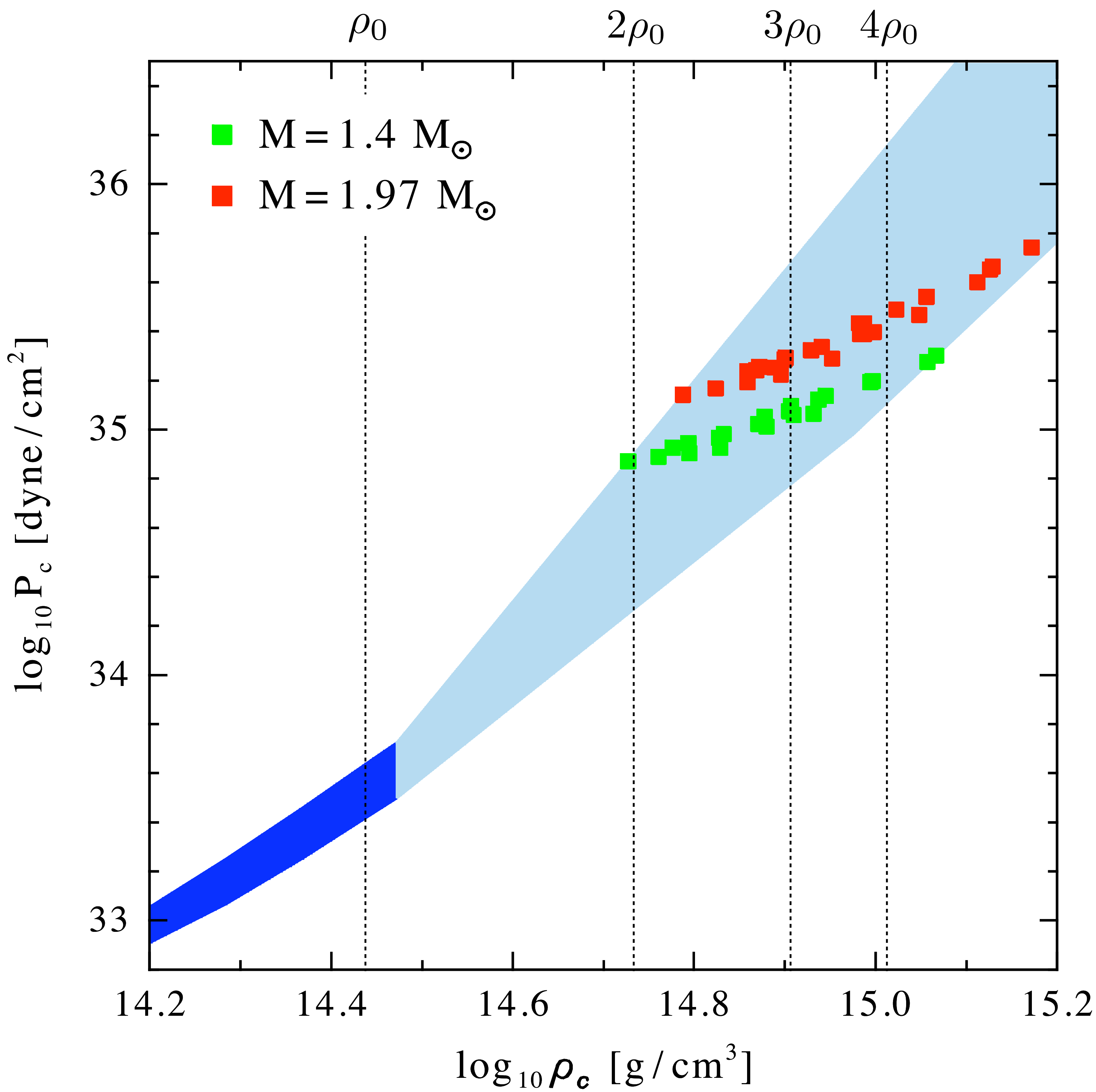
Constraints on neutron star equations of state



cf. KH et al., PRL 05, 161102 (2010)

$1.97 M_\odot$ neutron star and causality constrain nuclear EOS at high densities

Constraints on neutron star equations of state



very stiff EOSs lead to low central densities in typical neutron stars

Conclusions

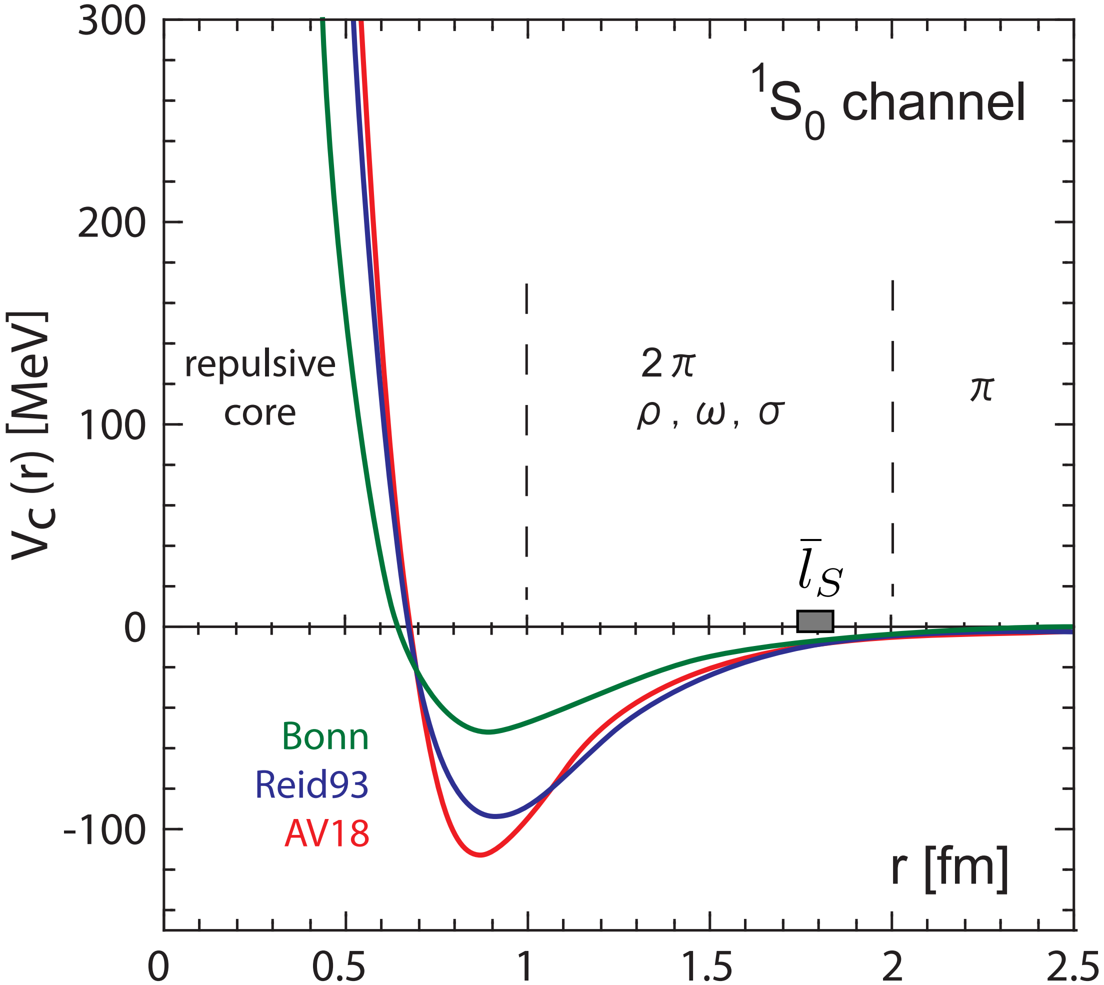
• derivation of density-dependent effective NN interactions from 3N interactions

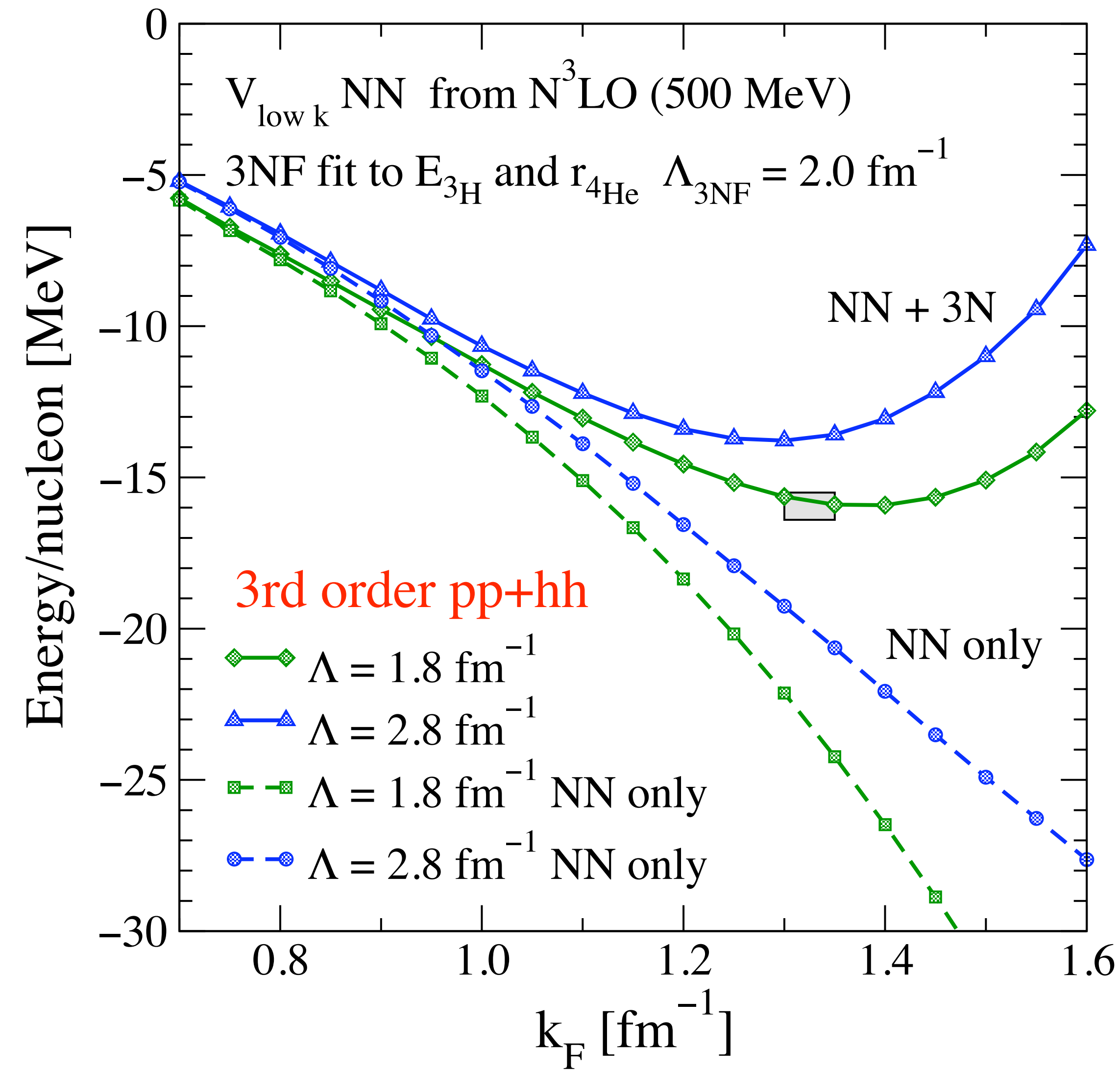
• effective NN interaction efficient to use and accounts for 3N effects in neutron and nuclear matter to good approximation

• good agreement with empirical symmetry energy and nuclear saturation properties

• constraints for the neutron star EOS and radii of neutron stars

Equation of state of symmetric nuclear matter

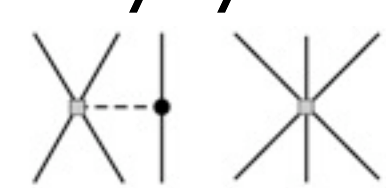




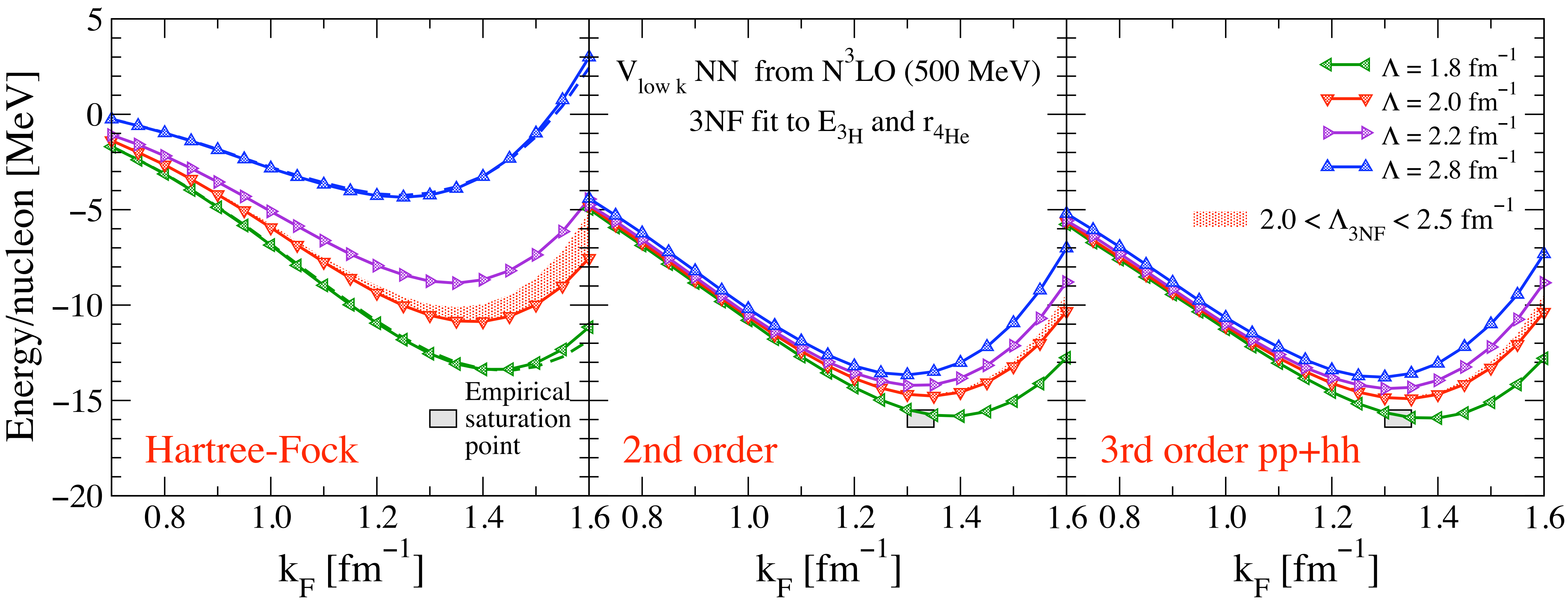
KH et al., PRC(R) 83, 031301 (2011)

- empirical saturation at $n_S \sim 0.16\,\text{fm}^{-3}$ and $E_{\text{binding}}/N \sim -16\,\text{MeV}$
- nuclear saturation delicate due to cancellations of large kinetic and potential energy contributions
- 3N forces are essential! Here: fit 3NF couplings to few-body systems:

$$E_{^3\text{H}} = -8.482\,\text{MeV} \quad \text{and} \quad r_{^4\text{He}} = 1.95 - 1.96\,\text{fm}$$



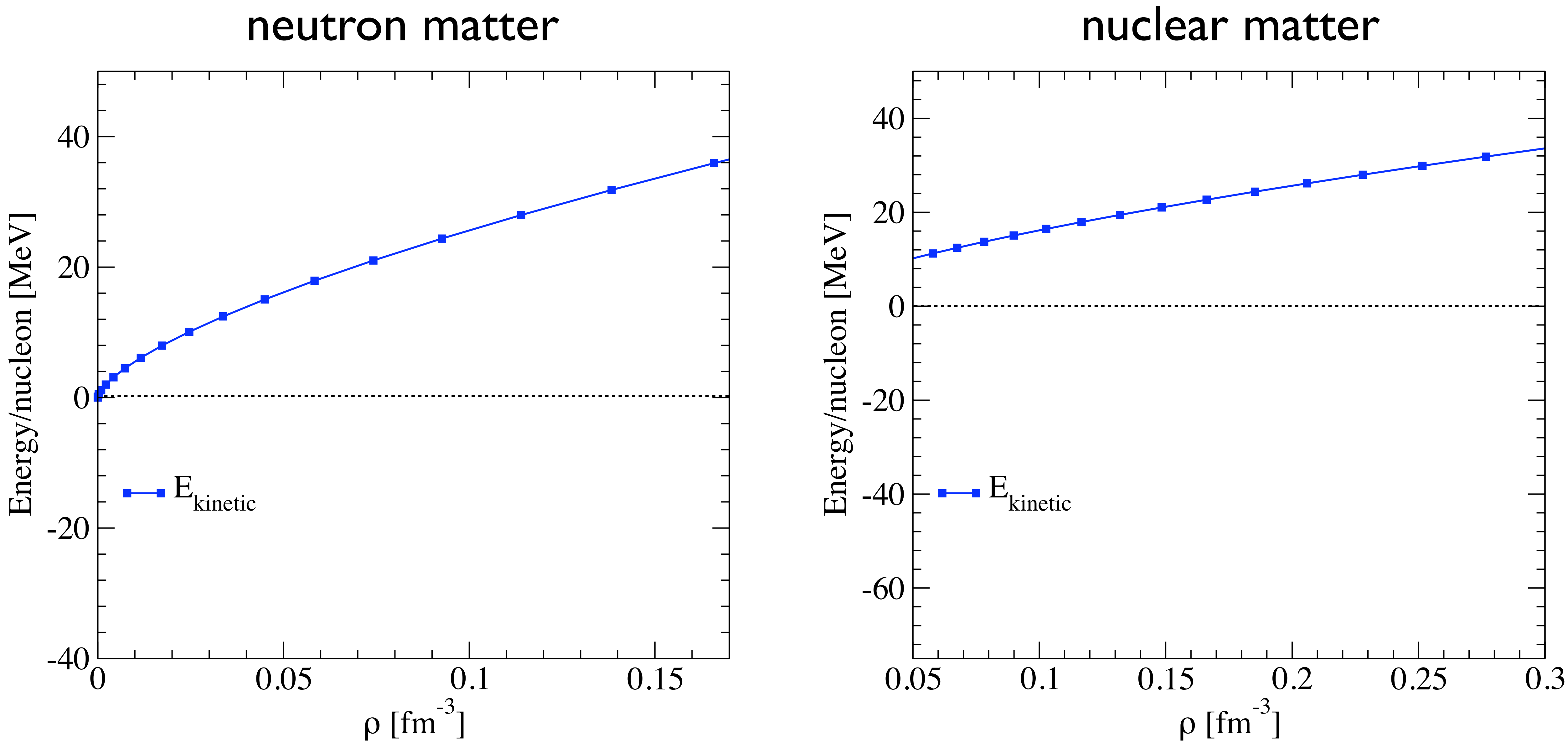
Equation of state of symmetric nuclear matter



KH et al., PRC(R) 83, 031301 (2011)

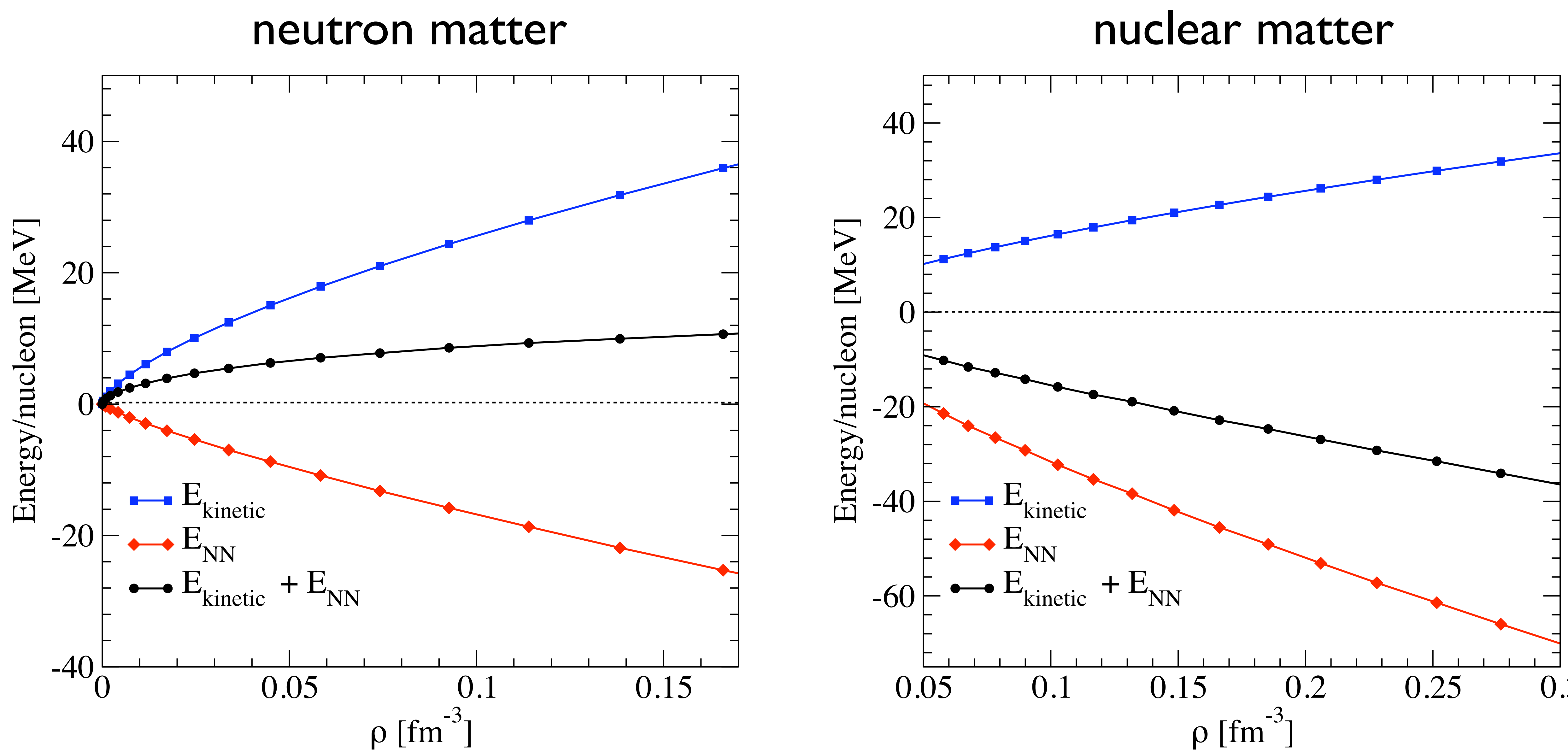
- saturation point consistent with experiment, without new free parameters
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions

Hierarchy of many-body contributions



- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

Hierarchy of many-body contributions



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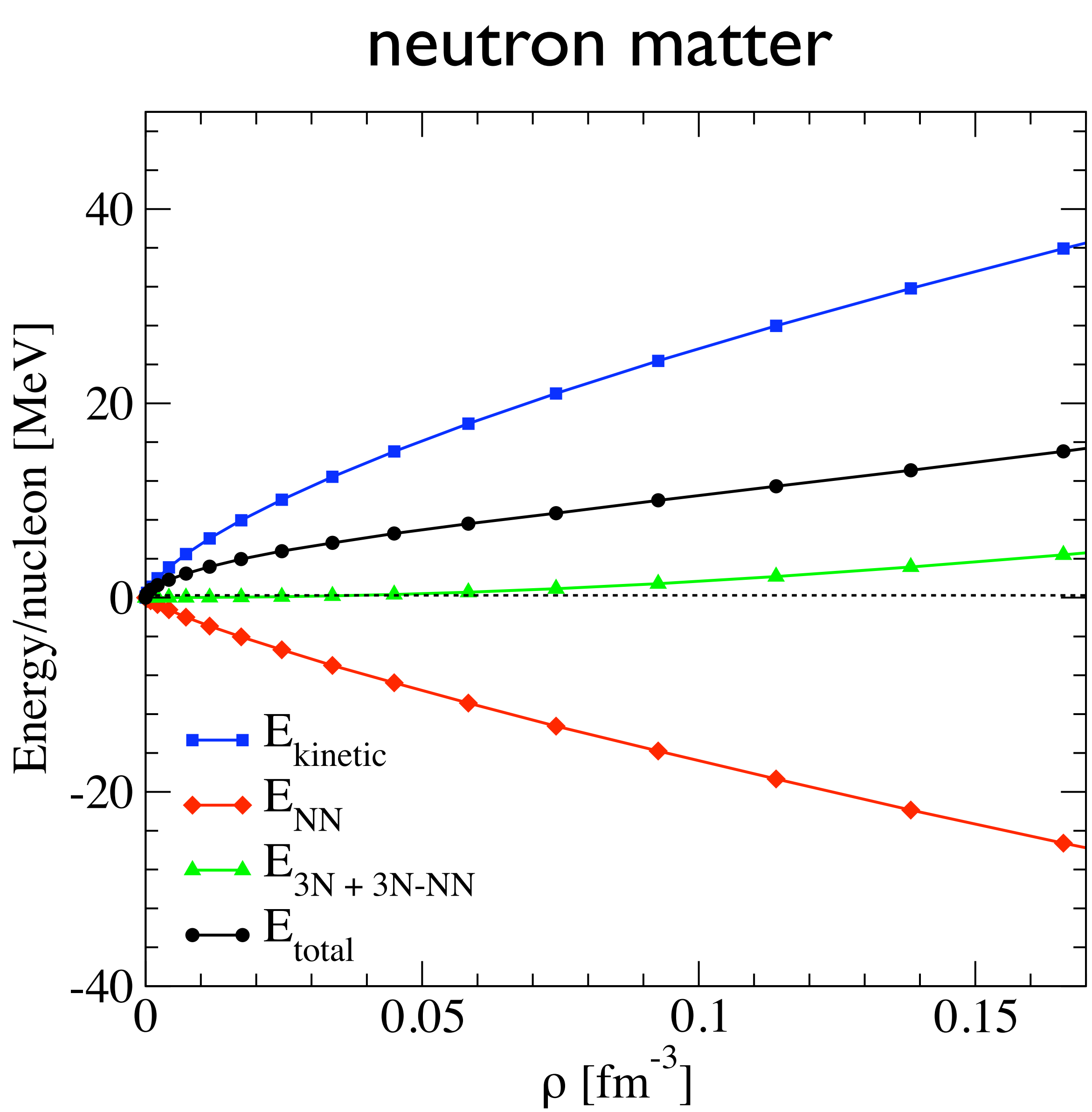

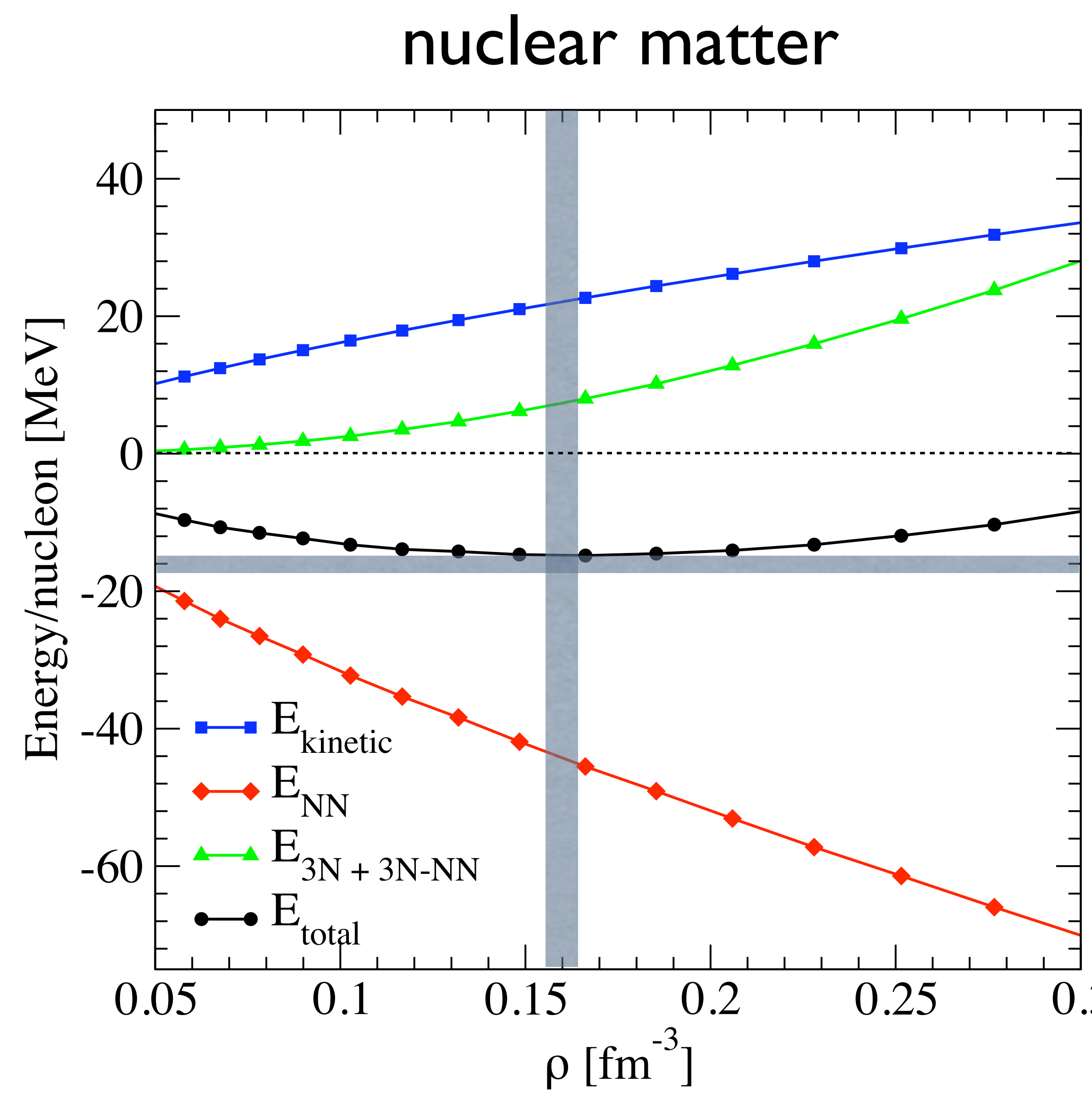

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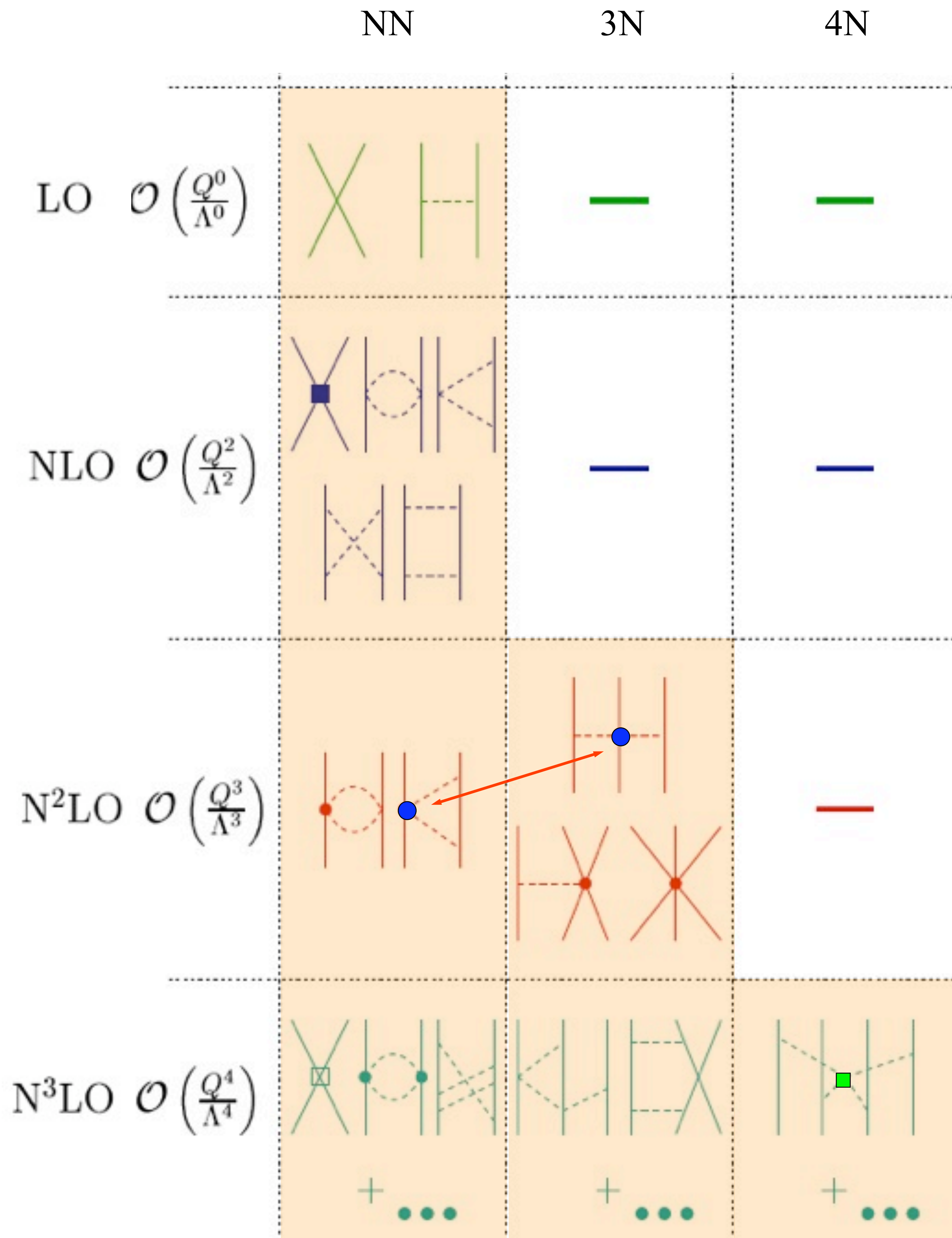
Basics concepts of chiral effective field theory

- choose effective degrees of freedom: here nucleons and pions
- short-range physics captured in few short-range couplings
- separation of scales: $Q << \Lambda_b$, breakdown scale Λ_b~500 MeV
- power-counting: expand in powers Q/Λ_b
- systematic: work to desired accuracy, obtain error estimates

Plan: Use EFT interactions as input to RG evolution.



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