Chiral three-body forces and neutron- rich matter

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Astrophysical Transients: Multi-messenger Probes of Nuclear Physics

In collaboration with: J. Lattimer, C. Pethick, A. Schwenk

Overview RG Summary Extras Physics Resolution Forces Filter Coupling Traditional "hard" NN interactions **Overview RG Summary Extras Physics Resolution Forces Filter Coupling**

- constructed to fit low-energy nucleon-nucleon scattering data
- in account to the row of the large in the literature.
"NINI interactions contain repulsive count Repulsive core ⁼[⇒] large high-*^k* (! 2 fm−1) components • "hard" NN interactions contain repulsive core at small relative distance in large nucleus [≈] 1 fm−¹ [≈] 200 MeV but . . . rd = interactions contain repulsive core at sm−
I
- **strong coupling between low and high-momentum components, hard to solve!** ween low and nigh-mc

- $H_\lambda = U_\lambda H U^\dagger_\lambda \quad \text{ with the resolution parameter } \ \lambda$ • goal: generate unitary transformation of "hard" Hamiltonian
- basic idea: change resolution in small steps

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\mathbf{s}: \ \frac{dH_{\lambda}}{d\lambda} = [\eta_{\lambda}, H_{\lambda}]
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- elimination of coupling between low- and high momentum components, calculations much easier
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations
- RG transformation also changes three-body (and higher-body) interactions

Chiral three-nucleon forces (leading order)

• large uncertainties in 2π coupling constants at present:

$$
c_1 = -0.9^{+0.2}_{-0.5}\ ,\ c_3 = -4.7^{+1.5}_{-1.0}\ ,\ \ c_4 = 3.5^{+0.5}_{-0.2}
$$

leads to theoretical uncertainties in many-body observables

 \bullet c_D and c_E have to be determined in A \geq 3 systems

Chiral 3N interaction as density-dependent two-body interaction

(2) construct effective density-dependent NN interaction

Basic idea: Sum one particle over occupied states in the Fermi sea

(3) combine with free-space NN interaction

combinatorial factor c depends on type of diagram

Equation of state: Many-body perturbation theory central quantity of interest: energy per particle *E/N* $H(\lambda) = T + V_{NN}(\lambda) + V_{3N}(\lambda) + ...$

- "hard" interactions require non-perturbative summation of diagrams
- with low-momentum interactions much more perturbative
- inclusion of 3N interaction contributions crucial

Properties of the effective interaction $V_{\rm 3N}$

General momentum dependence:

 $V_{\rm 3N} = V_{\rm 3N}(\mathbf{k},\mathbf{k'},\mathbf{P})$

- P-dependence much weaker than k, k' -dependence!
- neglect P-dependence, set ${\bf P} = {\bf 0}$
- matrix elements have the same form like free-space NN interaction matrix elements

• straightforward to include in existing many-body schemes

$(\Lambda_{3N} = 2.0 \,\text{fm}^{-1})$ Properties of the effective interaction $\overline{V}_{\rm 3N}$

Equation of state of pure neutron matter

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence

Equation of state of pure neutron matter

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence
- good agreement with other approaches (different NN interactions)

Neutron matter: Symmetry energy

$$
E(\rho, \alpha = 1) = -a_V + \frac{K_0}{18\rho_0^2} (\rho - \rho_0)^2 + S_2(\rho)
$$

$$
S_2(\rho) = a_4 + \frac{p_0}{\rho_0^2} (\rho - \rho_0)
$$

- uncertainties in c_i couplings lead to uncertainties in symmetry energy
- given the experimental constraint $a_4 = 30 \pm 4 \,\text{MeV}$ smaller absolute values of c_3 seem to be preferred from our results

Constraints on the nuclear equation of state (EOS) LETTER onstraints on the nuclear equation of state (FOS) $\mathsf{\color{red} \cup}$ onstraints on the nuclear equation of state (EOS) \blacksquare pointing towards Earth, in yellow. At orbital phase \sum

0 10 0

A two-solar-mass neutron star measured using Shapiro delay $C1$ Δ

P. B. Demorest¹, T. Pennucci², S. M. Ransom¹, M. S. E. Roberts³ & J. W. T. Hessels^{4,5}

Credit: NASA/Dana Berry **Credit: NASA/Dana Berry**

 $\mathbf \zeta$ \blacksquare $\frac{1}{2}$ $\mathsf{Im}\,$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

dP

 $\boldsymbol{\varGamma}$

 J_{∞}

 C

 σ

⁼ [−]*GM*!

 \mathbf{u}

standard x² fit produce similar uncertainties.

what times pulses should arrive at Earth, taking into account pulsar and the Earth, taking into account pulsar

timescales of the orbital period or less. Additional discussion of the

Companion mass, M2 (M()

parameters, with MCMC error estimates, are given in Table 1. Owing to 1. Owing the United States, and the United States, and the United States, and the United Sta

two-dimensional posterior probability density function (PDF) in the M2–i plane, computed from a histogram of MCMC trial values. The ellipses show 1s

dr

Structure of a neutron star is determined by Tolm eimer-Volkov (TOV) equation: period, producing a set of average pulse profiles, or flux-versus-rotaarrival using standard procedures, with a typical uncertainty of ,1 ms. We use the measured arrival times to determine the measured arrival times to determine key physical para-----comprehensive timing model that accounts for every rotation of the neutron star over the time spanned by the fit. The model predicts at the fit. The model predicts at the model pr Epoch of ascending node (MJD) 52,331.1701098(3) star is datarmingd l אנטו וא טכנכווווווכט נ $R = \sum_{i=1}^n (T_i \cap \{1\})$ olkov (TOV) equatio Inclination angle 89.17(2)u the high significance of this detection, our MCMC procedure and a \mathcal{C}^{11} 0.1 0.1 0.0 0.4 \mathcal{C}^{11} o.5 0.4 \mathcal{C}^{11} $\overline{}$

$$
\frac{dP}{dr} \qquad \qquad + \frac{P}{\epsilon c^2} \bigg[1 + \frac{4\pi r^3 P}{Mc^2} \bigg] \left[1 - \frac{2GM}{c^2 r} \right]^{-1} \qquad \qquad \bigg\vert \qquad \qquad \bigg\vert^{22.0 \, \rho_c}
$$

c dient: energy density $\epsilon = \epsilon(P)$ The long-term data determine model parameters (for example spindown rate and astrometry) with characteristic timescales longer than a few weeks, whereas the new data best constraints on the prov density $\epsilon = \epsilon(P)$ * These quantities vary stochastically on >1-d timescales. Values presented here are the averages for our GUPPI data set. \mathcal{C} dient energy density $\epsilon = 4$ Orbital phase (turns) $\mathbf{X}^{\mathbf{H}}$ $\sum_{i=1}^n \sigma_i = \sigma(T_i)$ ϵ \sim ϵ \sim ϵ \sim ϵ m_{α} and m_{α} parameters with α ISITY $\epsilon = \epsilon(P)$ companion mass and orbital inclination, fully described inclination, fully described in \mathcal{L} dient: energy c

Neutron star radius constraints

Problem: Solution of TOV equation requires EOS up to very high densities. Radius of a typical NS (M~1.4 M $_{\odot}$) theoretically not well constrained. \odot

But: Radius of NS is relatively insensitive to high density region.

incorporation of beta-equilibrium: neutron matter \longrightarrow neutron star matter

Neutron star radius constraints

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But: Radius of NS is relatively insensitive to high density region.

without 3N forces EOS differs significantly from crust EOS around $\rho_0/2$

- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint after incorporating crust corrections: 10*.*7 − 13*.*4 km

Constraints on neutron star equations of state

 $1.97M_{\odot}$ neutron star and causality constrain nuclear EOS at high densities

Constraints on neutron star equations of state

very stiff EOSs lead to low central densities in typical neutron stars

Conclusions

• derivation of density-dependent effective NN interactions from 3N interactions

- effective NN interaction efficient to use and accounts for 3N effects in neutron and nuclear matter to good approximation
- good agreement with empirical symmetry energy and nuclear saturation properties
- constraints for the neutron star EOS and radii of neutron stars

Equation of state of symmetric nuclear matter **Equation of state of symmetric nuclear in**

- mpirical saturation at $n_S \sim 0.16 \, \text{fm}^{-3}$ and $\emph{E}_{\rm binding}/N \sim -1$ • empirical saturation at $n_S \sim 0.16\, {\rm fm}^{-3}$ and $\,E_{\rm binding}/N \sim -16\, {\rm MeV}$
- nuclear saturation delicate due to cancellations of large kinetic and
- steeds large wave exceptions: Potential energy contributions
Potential energy contributions nd the contraction delicate due to cancellations of large kin iear saturation gelicate que to cancellations c
ntial energy contributions
- 3N forces are essential! Here: fit 3NF couplings to few-body systems: refitial crici₈₇ coriti ibutions
RN forces are essential¹ Here∙fit RNF couplings to few-ho **Dick Furnstahl RG in Nuclear Physics**

 $E_{\rm 3H} = -8.482 \, \rm MeV$ and $r_{\rm ^4He} = 1.95 - 1.96 \, \rm fm$

Equation of state of symmetric nuclear matter

- saturation point consistent with experiment, without new free parameters
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions

Hierarchy of many-body contributions

• binding energy results from cancellations of much larger kinetic and potential energy contributions

- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter [∼] ¹*/*³

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Basics concepts of chiral effective field theory

- choose effective degrees of freedom: here nucleons and pions
- short-range physics captured in few short-range couplings
- separation of scales: $Q \ll \Lambda_h$, breakdown scale Λ_h ~500 MeV
- power-counting: expand in powers Q/Λ_h
- systematic: work to desired accuracy, obtain error estimates

Plan: Use EFT interactions as input to RG evolution.

In collaboration with:

