Astrophysical Transients: Multi-Messenger Probes of Nuclear Physics
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# A low energy theory of the neutron star crust

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arXiv:1102.5379 [nucl-th], with S. Reddy and R. Sharma

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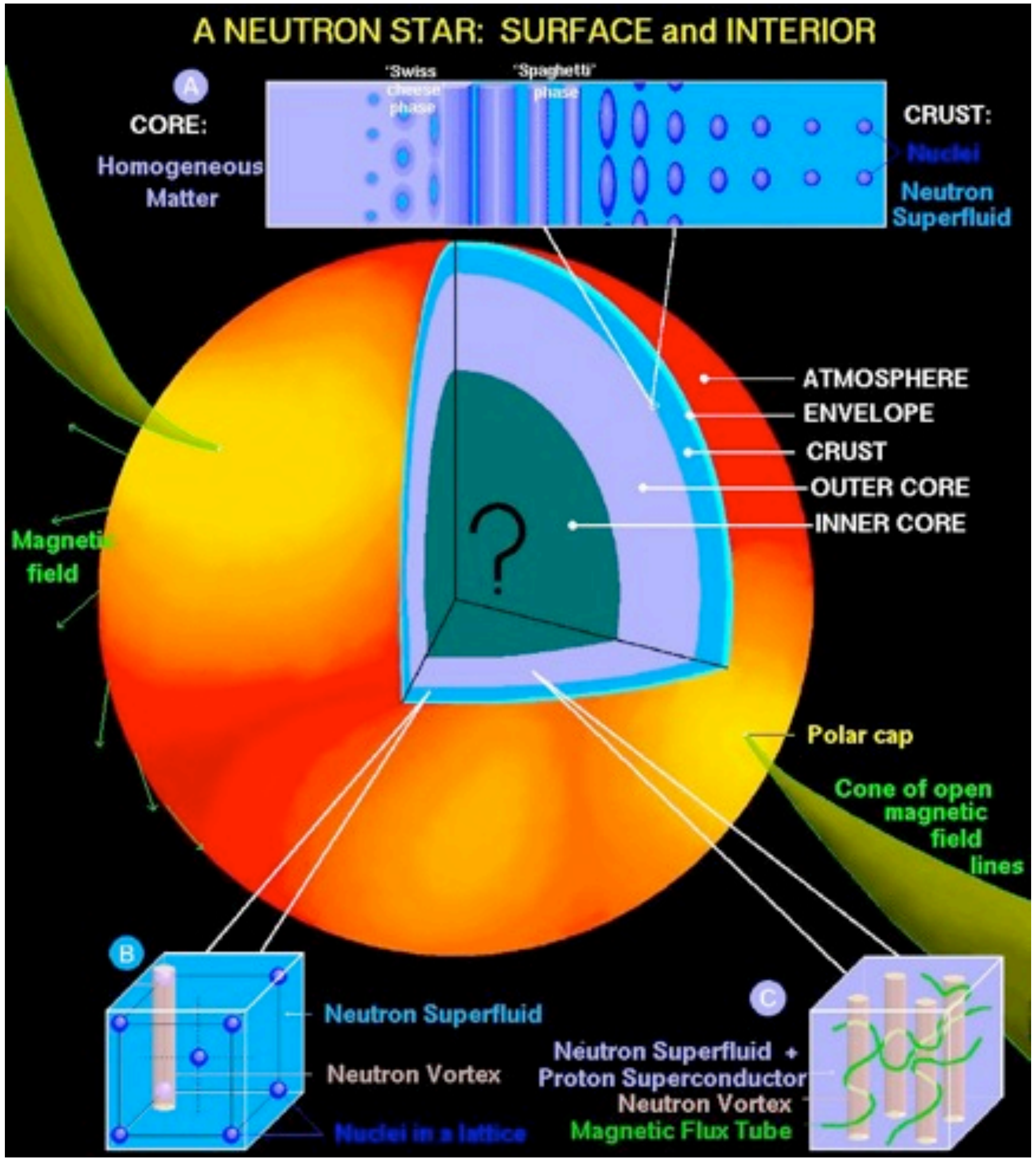
VC, Deborah Aguilera, Jose Pons, Sanjay Reddy, Rishi Sharma

# Prelude

- Transport properties of the crust play a central role in many transient phenomena (e.g. crustal heating and relaxation in accreting NS, or excitation of shear modes in magnetars during giant flares).
- Transport in the crust ← microphysics of the crust

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# Outline

- Introduction: microphysics of the neutron star inner crust
- EFT for lattice and superfluid phonons
    - Symmetries
    - “Thermodynamic matching” → identify LECs
- Applications:
    - mixing of lattice and superfluid modes
    - superfluid heat conduction

# Introduction

# The crust

- Impressionistic view:

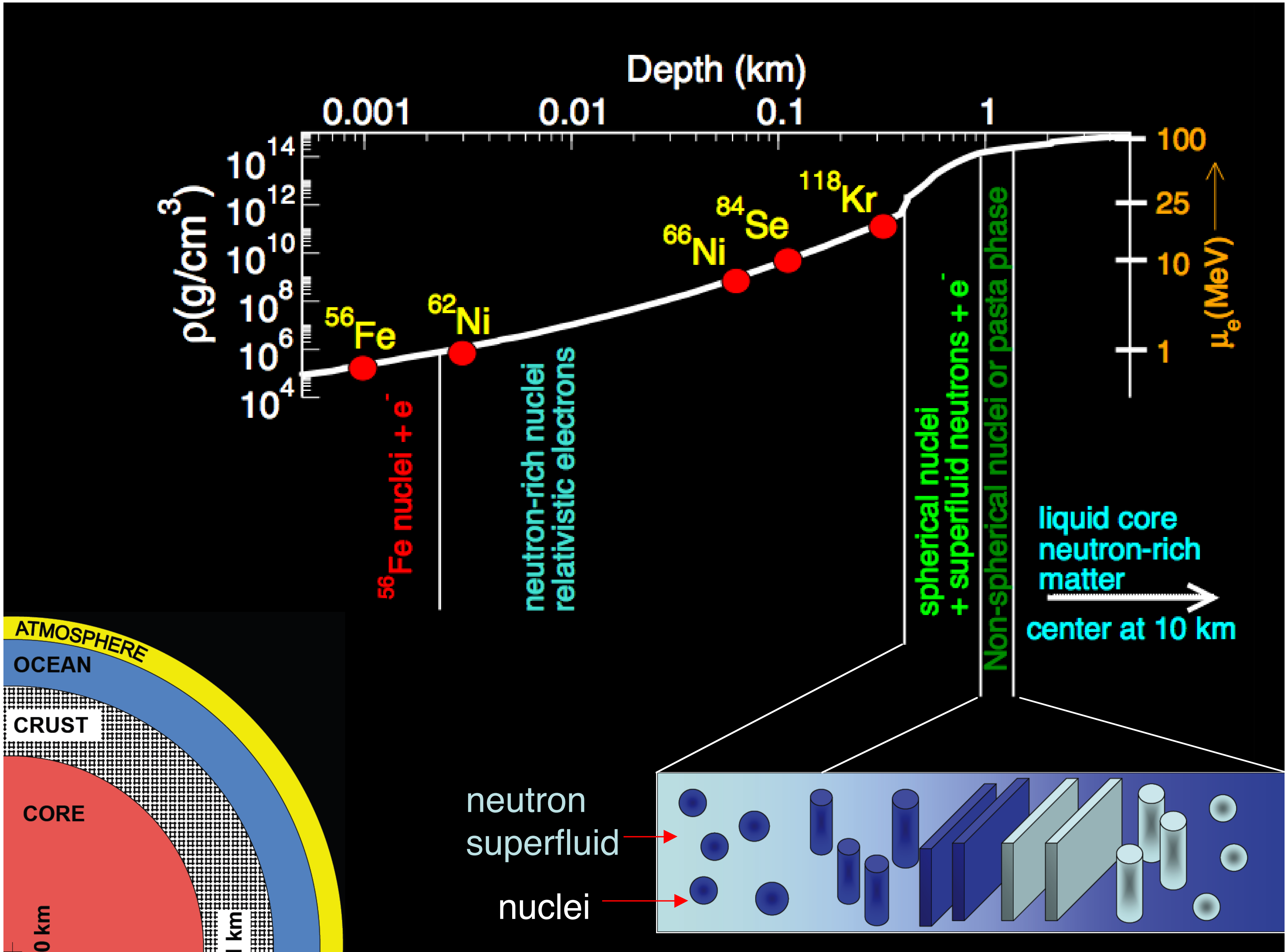
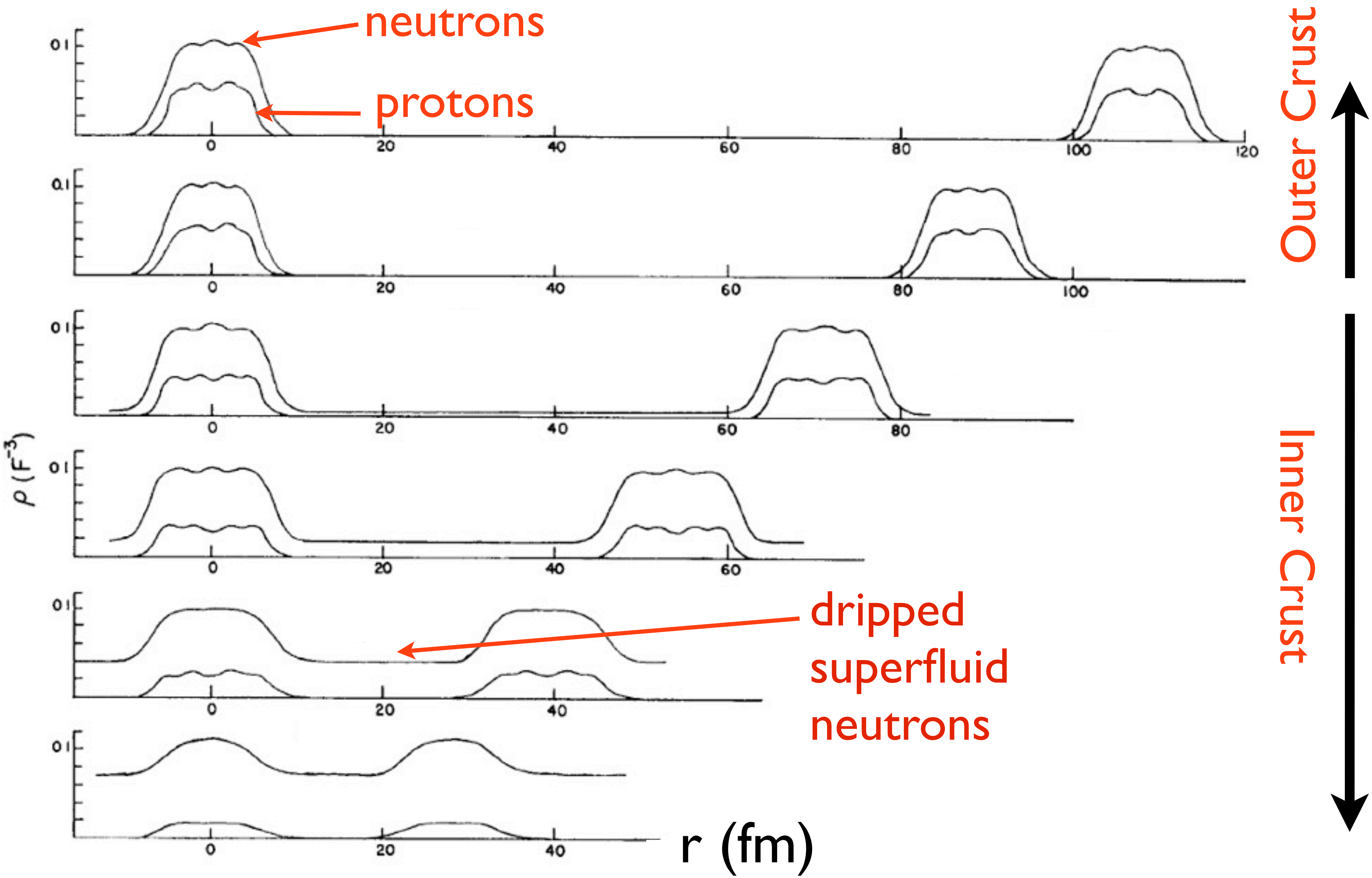



Figure: courtesy of S. Reddy

Negele & Vautherin (1973)

- More quantitative picture

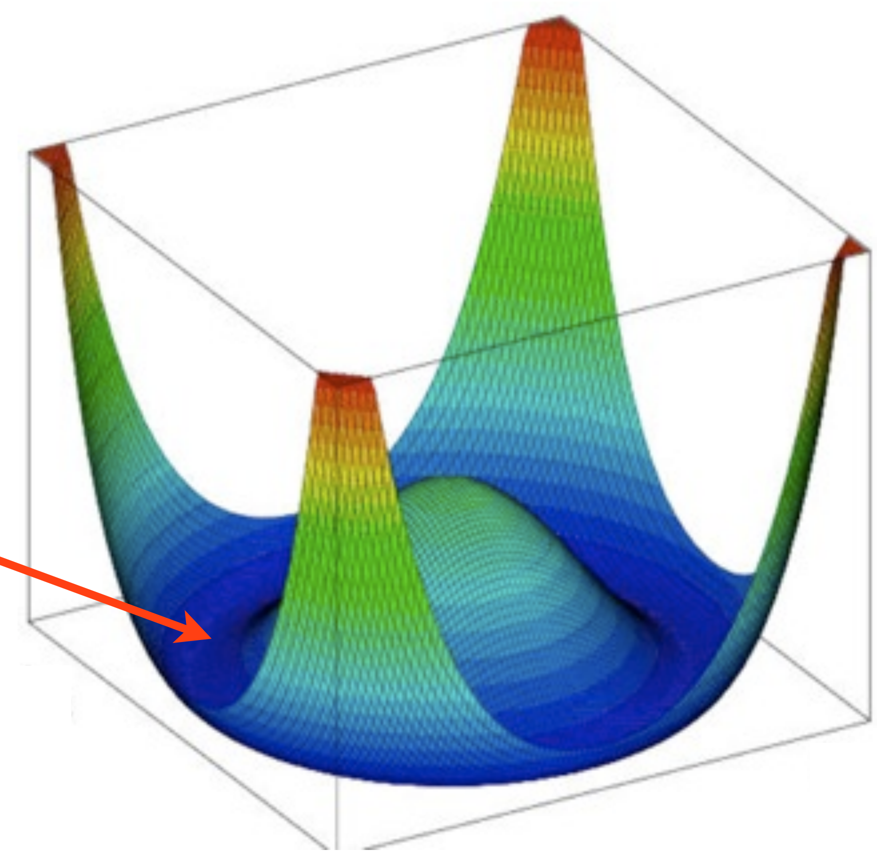


- Ground state
  - Lattice (cluster) structure: spontaneously breaks translation invariance
  - Neutron superfluid: spontaneously breaks $U(1)_n$ number symmetry

$$\langle \psi_\uparrow(r)\psi_\downarrow(r)\rangle = |\Delta| \ \exp\left(-2i\ \theta\right)$$

- Expect 3+1 Goldstone Bosons: collective excitations along the "valley" of degenerate minima

$$E(k) \xrightarrow{k \to 0} 0$$



# Relevant Temperature Scales in the Crust

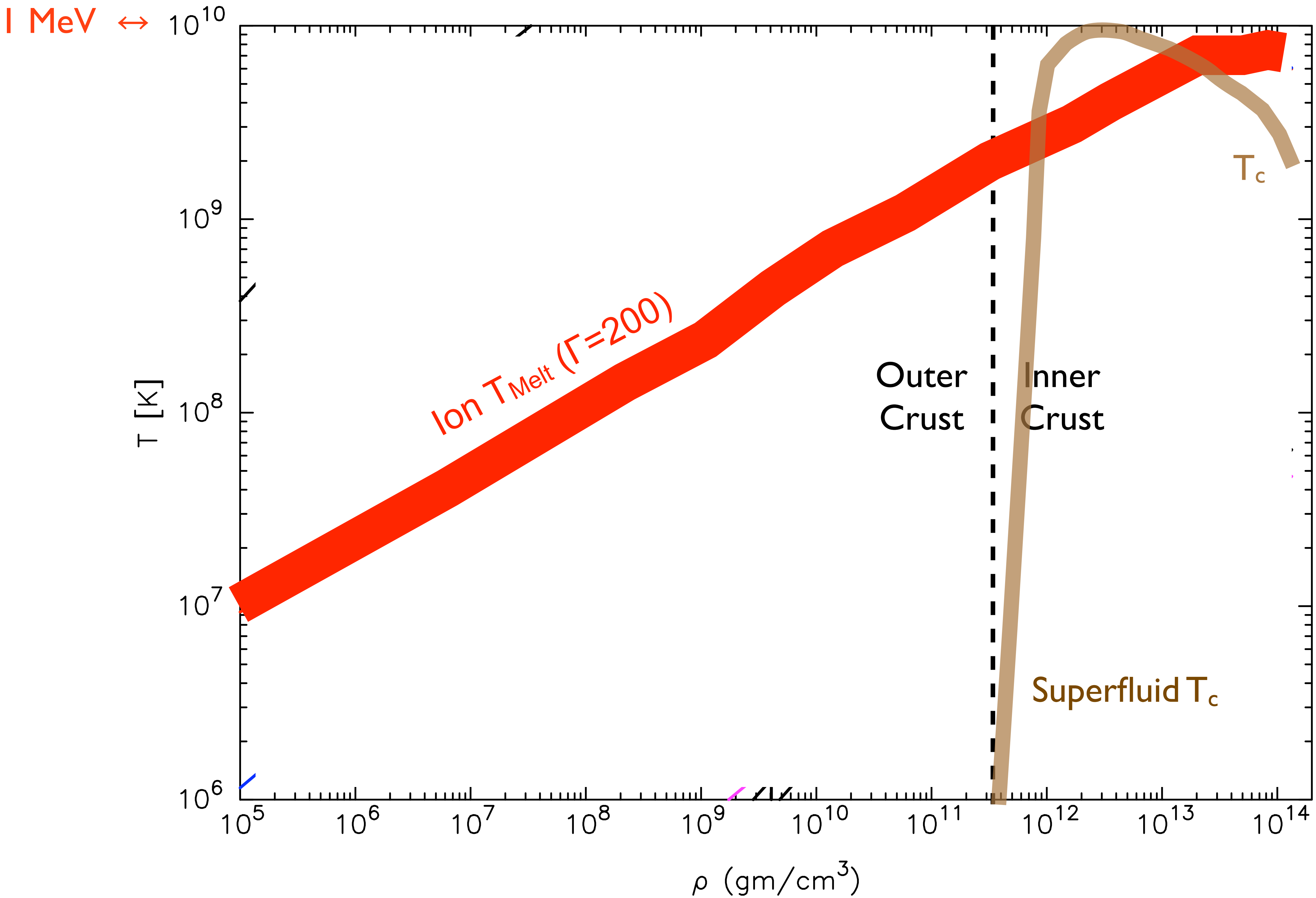



Figure: courtesy of Sanjay Reddy

# Relevant Temperature Scales in the Crust

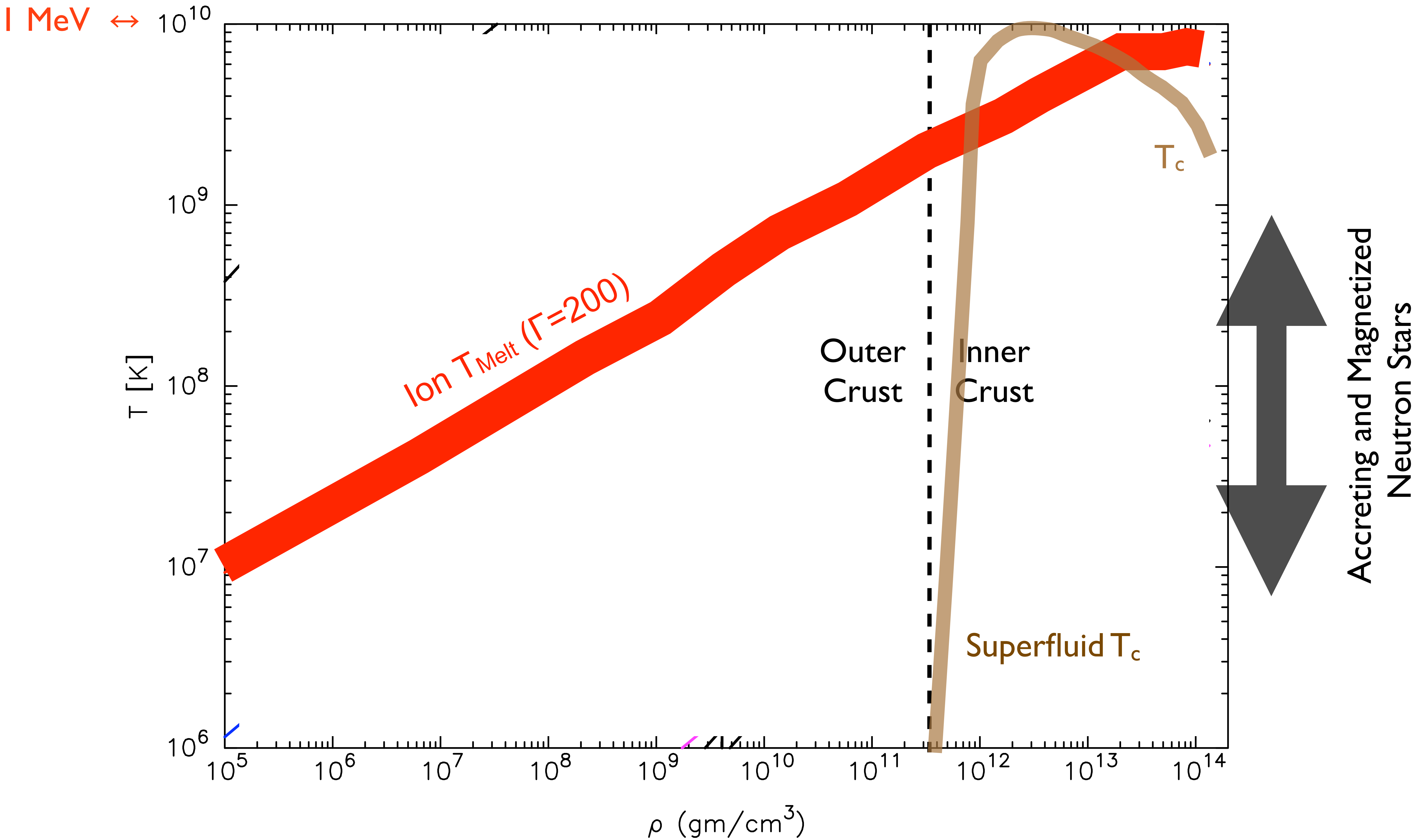
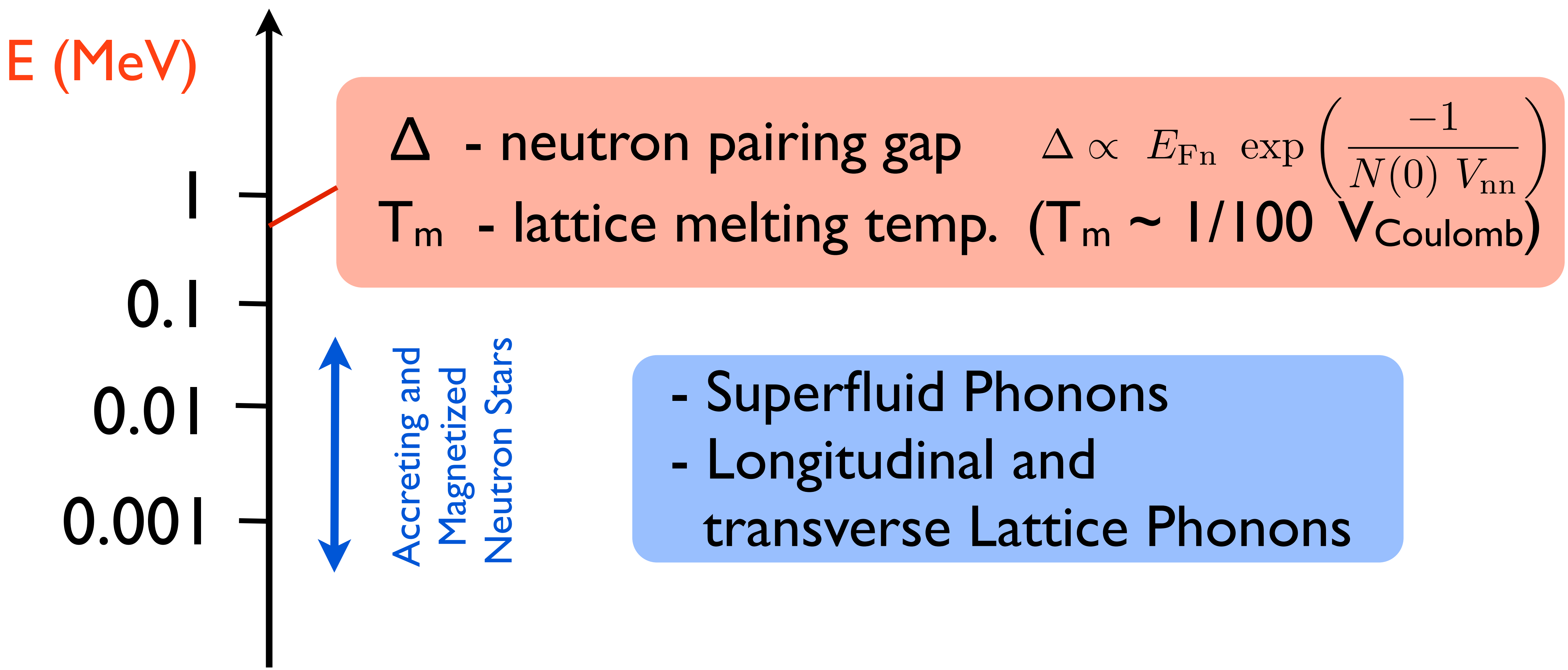



Figure: courtesy of Sanjay Reddy

- GB (and electrons) are the only relevant degrees of freedom in the inner crust for T < $10^9$ K ~ 100 keV

- Separation of scales:




- Describe physics with a low-energy effective theory of phonons

# EFT for lattice and superfluid phonons

# Low energy EFT

- GB Fields:

$$\xi^{a=1..3}(\mathbf{r},t) \longleftrightarrow$$

Displacement field:
deformation about ground state

$$e^{i\xi^a(x)P^a}\,|\Omega\rangle$$

$$\phi(\mathbf{r},t) \longleftrightarrow$$

Phase of superfluid condensate

$$\langle \Psi_{n(\downarrow)}\Psi_{n(\uparrow)}\rangle = |\Delta|\; e^{-2i\phi}$$

- (Nonlinear) transformations under broken symmetries

$$\xi_b(x) \to \xi'_b(x') = \xi_b(x) + a_b \quad (\leftarrow \quad x_b \to x'_b = x_b + a_b \ )$$

$$\phi \to \phi + \theta_n \quad (\leftarrow \quad \Psi_n \to \Psi_n\; e^{-i\theta_n} \ )$$

- Only derivative couplings allowed in $L_{eff}$: expansion in p/Λ

- Quadratic Lagrangian invariant under $U(1)_n$ and $T_a$

$$\mathcal{L} = \frac{f_\phi^2}{2}(\partial_0\phi)^2 - \frac{v_\phi^2 f_\phi^2}{2}(\partial_i\phi)^2$$

Superfluid phonons

$$+ \frac{\rho}{2}\partial_0\xi^a\partial_0\xi^a - \frac{1}{4}\mu(\xi^{ab}\xi^{ab}) - \frac{K}{2}(\partial_a\xi^a)(\partial_b\xi^b)$$

Lattice phonons
(longitudinal and transverse)

$$\xi^{ab} = (\partial_a\xi^b + \partial_b\xi^a) - \tfrac{2}{3}\partial_c\xi^c\delta^{ab}$$

$$+ g_{\rm mix} f_\phi \sqrt{\rho}\, \partial_0\phi\partial_a\xi^a + \cdots$$

n-p interaction → mixing of longitudinal lattice phonons and superfluid phonons

- Phenomenology (transport) ↔ LECs ($f_\phi$, $v_\phi$, $\rho$, $\mu$, $K$, $g_{mix}$)

Reliable calculation of transport properties requires reliable knowledge of these key non-perturbative parameters

- Quadratic Lagrangian invariant under $U(1)_n$ and $T_a$

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n-p interaction → mixing of longitudinal lattice phonons and superfluid phonons

- Phenomenology (transport) ↔ LECs ($f_\phi$ , $v_\phi$, $\rho$, $\mu$, $K$, $g_{\rm mix}$)
- Drawbacks of this description:
  - ★ Not all symmetries of underlying theory manifest
  - ★ Nature of LECs and relation to underlying theory obscure

# External fields method

Gasser-Leutwyler '84
...
Son-Wingate '05

- In the underlying theory, introduce external fields coupled to conserved currents:

U(1)n $J_\mu^{(n)} \leftrightarrow A_\mu^n$ U(1)p $J_\mu^{(p)} \leftrightarrow A_\mu^p$ Ta $T_{\mu\nu} \leftrightarrow g_{\mu\nu}$

$$\mathcal{L} \to \sqrt{-g}\left(\mathcal{L} + J_\mu^{(n)} A^{n\mu} + J_\mu^{(p)} A^{p\mu}\right)$$

# External fields method

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$$\mathcal{L} \to \sqrt{-g}\left(\mathcal{L} + J_\mu^{(n)} A^{n\mu} + J_\mu^{(p)} A^{p\mu}\right)$$

1. Modified action invariant under <u>local</u> U(1)$_{n,p}$ and T$_a$ (general coordinate transformations ⊃ Poincare and Galilei group)

U(1)$_n$

$$\Psi_n(x) \to \Psi'_n(x) = \exp(-i\theta_n(x))\Psi_n(x)$$
$$A_\mu^n(x) \to A'^n_\mu(x) = A_\mu^n(x) - \partial_\mu \theta^n(x) \, ,$$

T$_a$

$$x^\mu \to x'^\mu = x^\mu + a^\mu(x)$$
$$g^{\mu\nu}(x) \to g'^{\mu\nu}(x') = g^{\rho\sigma}(x)\frac{\partial x'^\mu}{\partial x^\rho}\frac{\partial x'^\nu}{\partial x^\sigma}$$
$$\psi(x) \to \psi'(x') = \psi(x)$$

# External fields method

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U(1)$_n$ $J_\mu^{(n)} \leftrightarrow A_\mu^n$ U(1)$_p$ $J_\mu^{(p)} \leftrightarrow A_\mu^p$ T$_a$ $T_{\mu\nu} \leftrightarrow g_{\mu\nu}$

$$\mathcal{L} \to \sqrt{-g}\left(\mathcal{L} + J_\mu^{(n)} A^{n\mu} + J_\mu^{(p)} A^{p\mu}\right)$$

2. Corresponding "gauge" invariance of the partition function

$$Z[A_\mu^n, A_\nu^p, g_{\mu\nu}] = e^{iW[A_\mu^n, A_\mu^p, g_{\mu\nu}]} = \int [d\Psi_n][d\Psi_p] e^{i\mathcal{S}[\Psi_n, \Psi_p, A_\mu^n, A_\mu^p, g_{\mu\nu}]}$$

$$Z[A + \delta A, g + \delta g] = Z[A, g]$$

- Response to slowly varying external fields (low-energy EFT)

$$Z[A^n_\mu, A^p_\mu, g_{\mu\nu}] = \int [d\Psi_n][d\Psi_p] e^{i\mathcal{S}[\Psi_n, \Psi_I, A^n_\mu, A^p_\mu, g_{\mu\nu}]}$$

$$\rightarrow \int [d\phi][d\xi^a] e^{i\mathcal{S}_{\text{eff}}[\phi, \xi^a, A^n_\mu, A^p_\mu, g_{\mu\nu}]}$$

"Integrate out" high frequency modes

- Response to slowly varying external fields (low-energy EFT)

$$Z[A^n_\mu, A^p_\mu, g_{\mu\nu}] = \int [d\Psi_n][d\Psi_p] e^{iS[\Psi_n, \Psi_I, A^n_\mu, A^p_\mu, g_{\mu\nu}]}$$

$$\rightarrow \int [d\phi][d\xi^a] e^{iS_{\rm eff}[\phi, \xi^a, A^n_\mu, A^p_\mu, g_{\mu\nu}]}$$

- "Gauge invariance" of $Z[A^n, A^p, g]$ → strong constraints on $L_{\rm eff}$.
  To leading order in power counting [$\partial^m \varphi^n \sim O(p^{m-n})$] only 3 building blocks:

$$\begin{aligned} X &= g^{\mu\nu} D_\mu\phi D_\nu\phi \\ W^a &= g^{\mu\nu} D_\mu\phi \partial_\nu z^a \\ H^{ab} &= g^{\mu\nu} \partial_\mu z^a \partial_\nu z^b \end{aligned}$$

$$D_\mu\phi(x) = \partial_\mu\phi(x) + A^n_\mu(x)$$

$$z^a(x) = x^a - \xi^a(x)$$

↓

$$\mathcal{L}_{\rm eff} = \mathcal{L}_0(X, W^a, H^{ab}) \; + \; \dots$$

# "Thermodynamic" Matching

- $\mathcal{L}_0 \leftrightarrow$ thermodynamic potential at *constant* external fields

$$-\Omega[\bar{A}^n, \bar{A}^p, \bar{g}] \;=\; \mathcal{L}_0\left(X = \bar{A}^n_\mu \bar{A}^{n\mu}, W^a = \bar{A}^{na}, H^{ab} = \bar{g}^{ab}\right)$$

$$\frac{W[\bar{A}^n, \bar{A}^p, \bar{g}]}{VT}$$

evaluated at $\phi_0 = \xi_0^a = 0$

$$A^n_\mu(x) = \bar{A}^n_\mu \;=\; (\mu_n + m_n, \mathbf{A}_i)$$

$$A^p_\mu(x) \;=\; \bar{A}^p_\mu \;=\; (\mu_p + m_p, \mathbf{0})$$

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}$$

- Known result for superfluid: $\mathcal{L}_0\,(X) = P\,(\mu_n \equiv \sqrt{X} - m_n)$

Son '02 , Son-Wingate '05

# “Thermodynamic” Matching

- $\mathcal{L}_0 \leftrightarrow$ thermodynamic potential at *constant* external fields

$$-\Omega[\bar{A}^n, \bar{A}^p, \bar{g}] \;=\; \mathcal{L}_0\left(X = \bar{A}^n_\mu \bar{A}^{n\mu}, W^a = \bar{A}^{na}, H^{ab} = \bar{g}^{ab}\right)$$

$$\frac{W[\bar{A}^n, \bar{A}^p, \bar{g}]}{VT}$$

evaluated at $\phi_0 = \xi_0^a = 0$

- Flavor of (new) derivation:
  - ★ Evaluate $Z[A^n, A^p, g]$: saddle point + fluctuations (loops)
  - ★ Use low-energy EFT power counting: Weinberg ’79 Gasser-Leutwyler ’84
    loop expansion $\leftrightarrow$ gradient expansion in external fields

# Implications for LECs

- $\mathcal{L}_0 \leftrightarrow$ thermodynamic potential at *constant* external fields

$$-\Omega[\bar{A}^n, \bar{A}^p, \bar{g}] \;=\; \mathcal{L}_0\left(X = \bar{A}^n_\mu \bar{A}^{n\mu}, W^a = \bar{A}^{na}, H^{ab} = \bar{g}^{ab}\right)$$

- In principle: calculate $\Omega[A^n, A^p, g]$ with non-perturbative method → map out functional dependence of $\mathcal{L}_0$ on $X, W^a, H^{ab}$

- In practice: LECs related to derivatives of $\Omega$ w.r.t. $\bar{A}^{n,p}, \bar{g}$

$$\frac{\partial \mathcal{L}_0}{\partial(X, W^a, H^{ab})}\bigg|_{\phi=\xi^a=0} \leftrightarrow \frac{\partial \Omega}{\partial(A^{n0}, A^{na}, \bar{g}^{ab})}$$

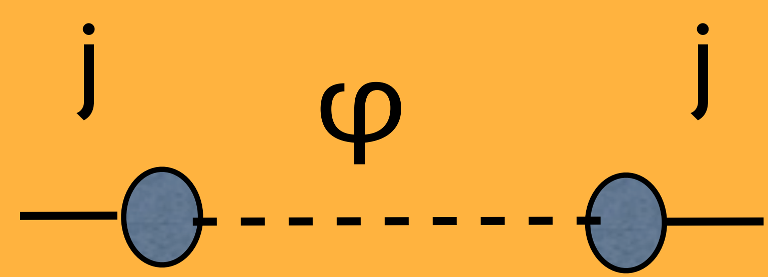
Related to correlation functions of currents at p → 0 $\langle J^n_\mu J^n_\nu \rangle$ $\langle J^n_\mu T_{\alpha\beta} \rangle$ $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$

# Implications for LECs

- $\mathcal{L}_0 \leftrightarrow$ thermodynamic potential at *constant* external fields

- Long-wavelength external fields can only excite GB modes. Response to these external probes knows about LECs

- Correlation functions of broken symmetry currents are dominated at low-momentum by GB exchange (know LECs)




$\partial(X, W^a, H^{ab})\,|_{\phi=\xi^a=0}$ $\partial(A^{n0}, A^{na}, \bar{g}^{ab})$

Related to correlation functions of currents at p → 0 $\langle J^n_\mu\, J^n_\nu \rangle$ $\langle J^n_\mu\, T_{\alpha\beta} \rangle$ $\langle T_{\mu\nu}\, T_{\alpha\beta} \rangle$

- Back to quadratic Lagrangian: identify LECs

$$\mathcal{L} = \frac{f_\phi^2}{2}(\partial_0\phi)^2 - \frac{v_\phi^2 f_\phi^2}{2}(\partial_i\phi)^2$$

$$f_\phi^2 = -\frac{\partial^2\Omega}{\partial\bar{A}_0^n\partial\bar{A}_0^n} \longrightarrow \frac{\partial n_n}{\partial\mu_n} \qquad \langle J_0^n J_0^n\rangle$$

$$v_\phi^2 f_\phi^2 = -\frac{1}{3}\eta^{ab}\frac{\partial^2\Omega}{\partial\bar{A}_a^n\partial\bar{A}_b^n} = \frac{n_n - n_b}{m_n} \longrightarrow \frac{n_f}{m_n} \qquad \langle J_a^n J_b^n\rangle$$

- Unlike pure superfluid, $v_\phi^2 f_\phi^2 \neq \frac{n_n}{m_n}$

- Additional term due to $W^a W^a$ coupling (relative velocity)

- Interpret $n_b$ as number density of neutrons bound or entrained by proton clusters (superfluid current $\propto n_n - n_b$)

- Back to quadratic Lagrangian: identify LECs

$$\mathcal{L} = \frac{f_\phi^2}{2}(\partial_0\phi)^2 - \frac{v_\phi^2 f_\phi^2}{2}(\partial_i\phi)^2$$

$$+ \frac{\rho}{2}\partial_0\xi^a\partial_0\xi^a - \frac{1}{4}\mu(\xi^{ab}\xi^{ab}) - \frac{K}{2}(\partial_a\xi^a)(\partial_b\xi^b)$$

Symmetries dictate appearance of $n_b$ (consistently with interpretation)

$$\rho = \langle T^{00}\rangle + \frac{m_n}{3}\eta^{ab}\frac{\partial^2\Omega}{\partial\bar{A}_a^n\partial\bar{A}_b^n} \longrightarrow (n_p + n_b)\, m_n$$

$$K,\ \mu \quad \leftrightarrow \quad \frac{\partial^2\Omega}{\partial\bar{g}_{ab}\partial\bar{g}_{cd}} \quad \leftrightarrow \quad \langle T_{ab}\, T_{cd}\rangle$$

- Compression and shear modulus of n-p system
- External metric ~ strain tensor of deformed configurations

- Back to quadratic Lagrangian: identify LECs

$$\mathcal{L} = \frac{f_\phi^2}{2}(\partial_0\phi)^2 - \frac{v_\phi^2 f_\phi^2}{2}(\partial_i\phi)^2$$

$$+ \frac{\rho}{2}\partial_0\xi^a\partial_0\xi^a - \frac{1}{4}\mu(\xi^{ab}\xi^{ab}) - \frac{K}{2}(\partial_a\xi^a)(\partial_b\xi^b)$$

$$+ g_{\mathrm{mix}} f_\phi \sqrt{\rho}\, \partial_0\phi\partial_a\xi^a + \cdots$$

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$$g_{\mathrm{mix}} = \frac{1}{f_\phi\sqrt{\rho}}\eta^{ab}\left[\frac{\partial^2\Omega}{\partial\bar{A}_a^n\partial\bar{A}_b^n} + m_n\frac{\partial^2\Omega}{\partial\bar{A}_0^n\partial\bar{g}_{ab}^n}\right]$$

$$\longrightarrow \frac{1}{f_\phi\sqrt{\rho}}\left[n_b - n_p\frac{\partial n_n}{\partial n_p}\right]$$

current-current n-p interaction

density-density n-p interaction

- Back to quadratic Lagrangian: identify LECs

$$\mathcal{L} = \frac{f_\phi^2}{2}(\partial_0\phi)^2 - \frac{v_\phi^2 f_\phi^2}{2}(\partial_i\phi)^2$$

Discuss implications

- Kinetic mixing of phonons
- Mixing-induced superfluid dissipation
- No self-consistent treatment yet -- rely on models for LECs

$$\longrightarrow \quad \frac{1}{f_\phi\sqrt{\rho}}\left[n_b - n_p\frac{\partial n_n}{\partial n_p}\right]$$

current-current n-p interaction

density-density n-p interaction

# Applications

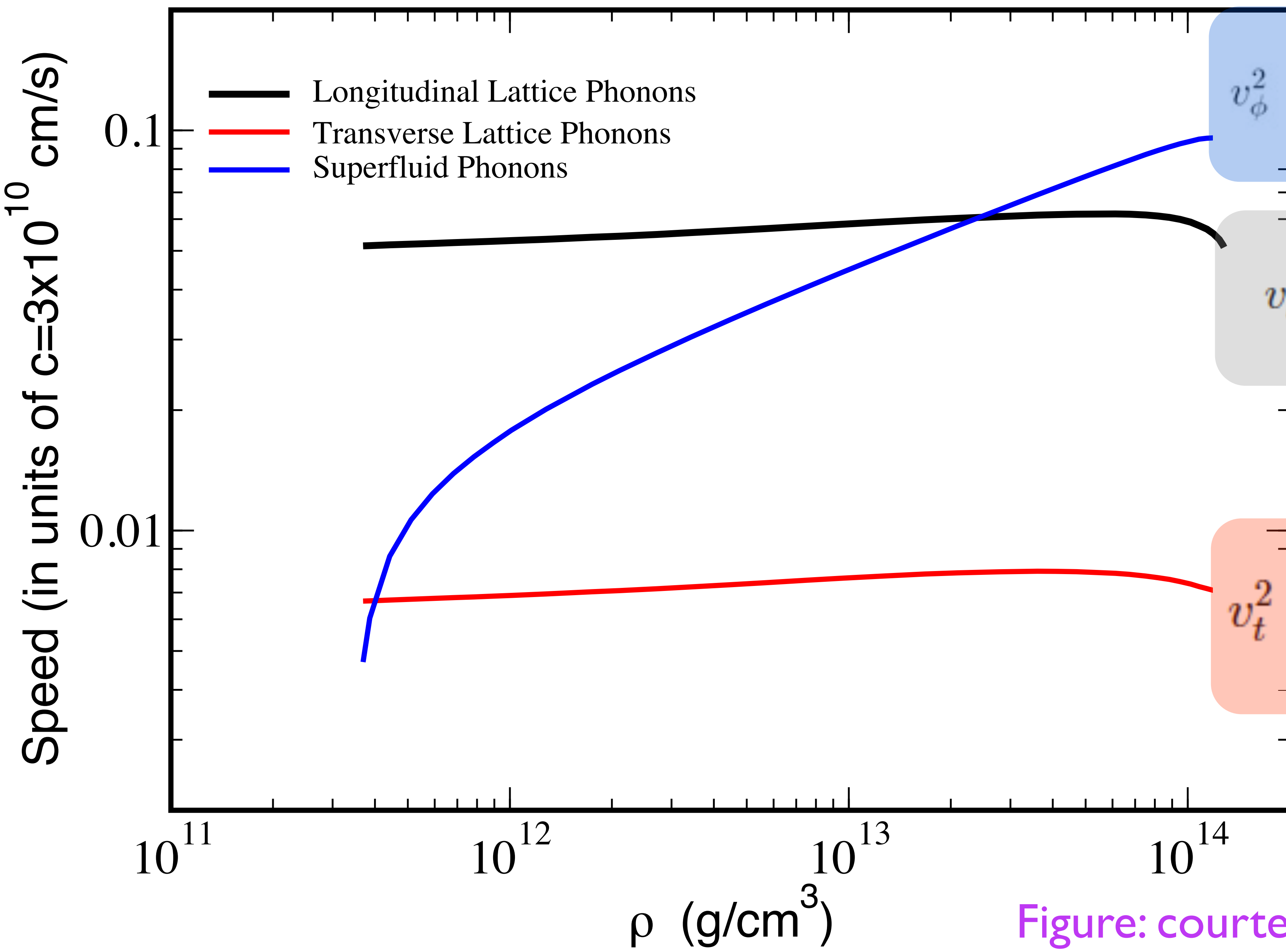
# Kinetic mixing

- Uncoupled low-energy modes

$$\omega_{sPh}(q) = v_\phi \, q$$

$$\omega_{lPh}(q) = v_l \, q$$

$$\omega_{tPh}(q) = v_t \, q$$




$$v_\phi^2 = \frac{n_f}{m_n f_\phi^2} \simeq \left(\frac{k_{Fn}}{\sqrt{3}\, m_n}\right)^2$$

$$v_l^2 = \frac{K + 4/3\mu}{(n_p + n_b)\, m_n}$$

$$v_t^2 = \frac{\mu}{(n_p + n_b)\, m_n}$$

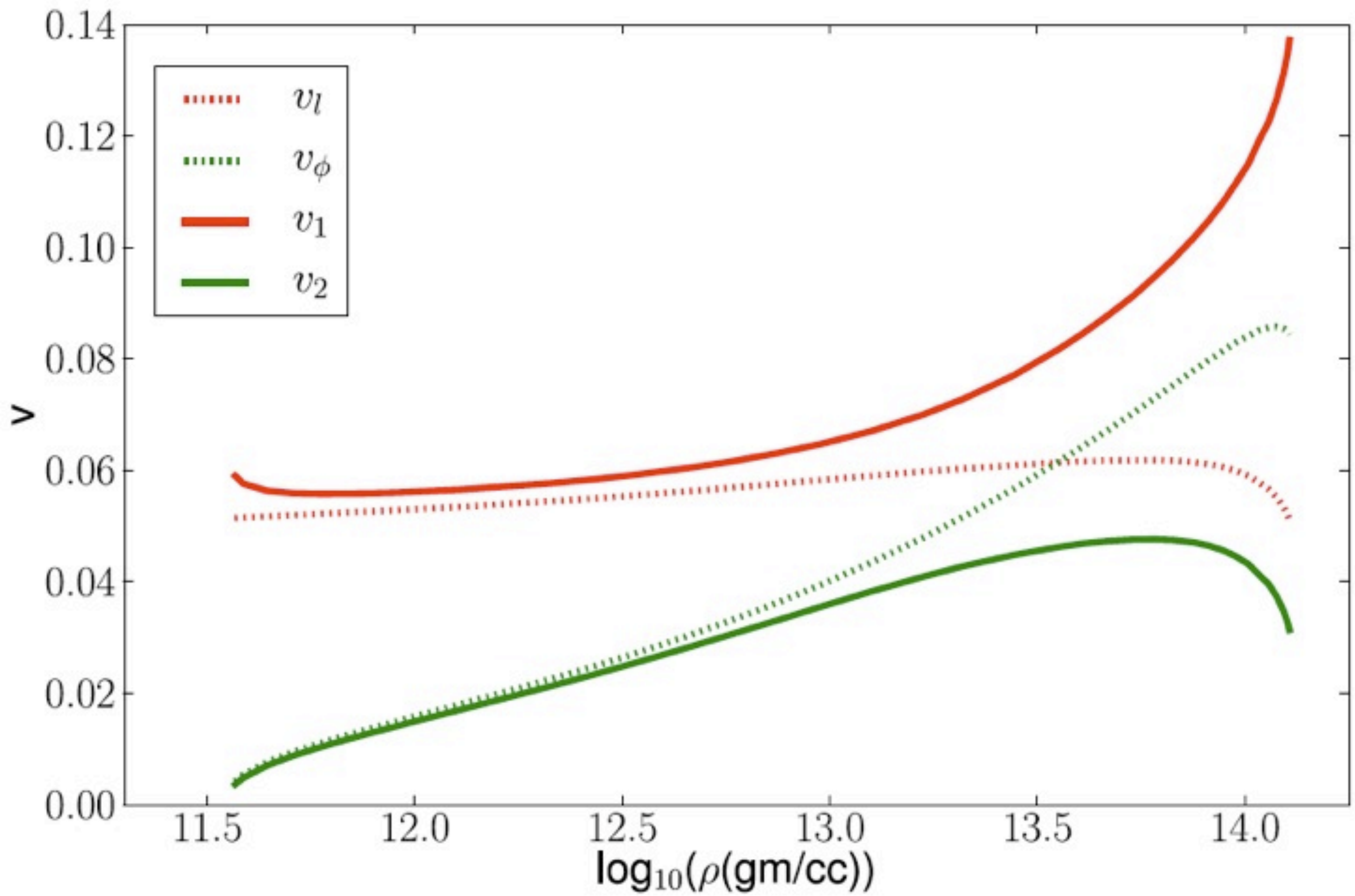
Figure: courtesy of Sanjay Reddy

- Turn on $g_{mix}$: velocities of the two eigenmodes

$$g_{\text{mix}} = \frac{1}{f_\phi \sqrt{\rho}} \left[ n_b - n_p \frac{\partial n_n}{\partial n_p} \right]$$

$\sim 10^{-2}$ Chamel '05

$\leq 10^{-3}$ Aguilera et al '09

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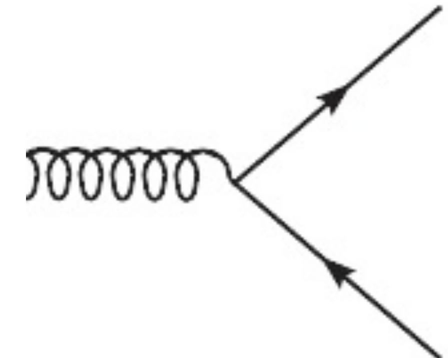
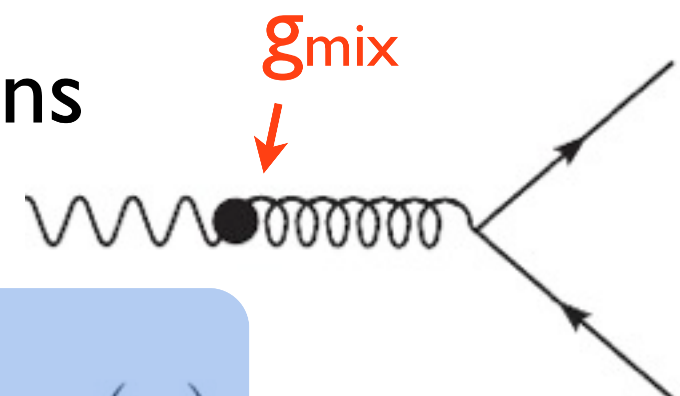
# Superfluid heat conduction

- Electron-ion interactions damp lattice phonons

$$\mathcal{L}_{\mathrm{el-lph}} = \frac{1}{f_{\mathrm{el-ph}}} \, \partial_i \xi_i \, \psi_e^\dagger \, \psi_e$$



- Mixing-induced damping of superfluid phonons




$$\lambda_{\mathrm{abs}}(\omega) = \frac{v_\phi^2}{g_{\mathrm{mix}}^2} \, \frac{1 + (1-\alpha^2)^2 \, (\omega \, \tau_{\mathrm{lPh}})^2}{\alpha \, (\omega \, \tau_{\mathrm{lPh}})^2} \, \lambda_{\mathrm{lPh}}(\omega)$$

$$\alpha = v_l / v_\phi$$

Away from resonance $\lambda_{\mathrm{sPh}} \simeq 10^5 \, \lambda_{\mathrm{lPh}}$

Significant contribution to heat conduction $\kappa = \frac{1}{3} \, C_v \times v \times \lambda$

- Important in magnetized neutron stars, where electron conduction transverse to magnetic field is suppressed

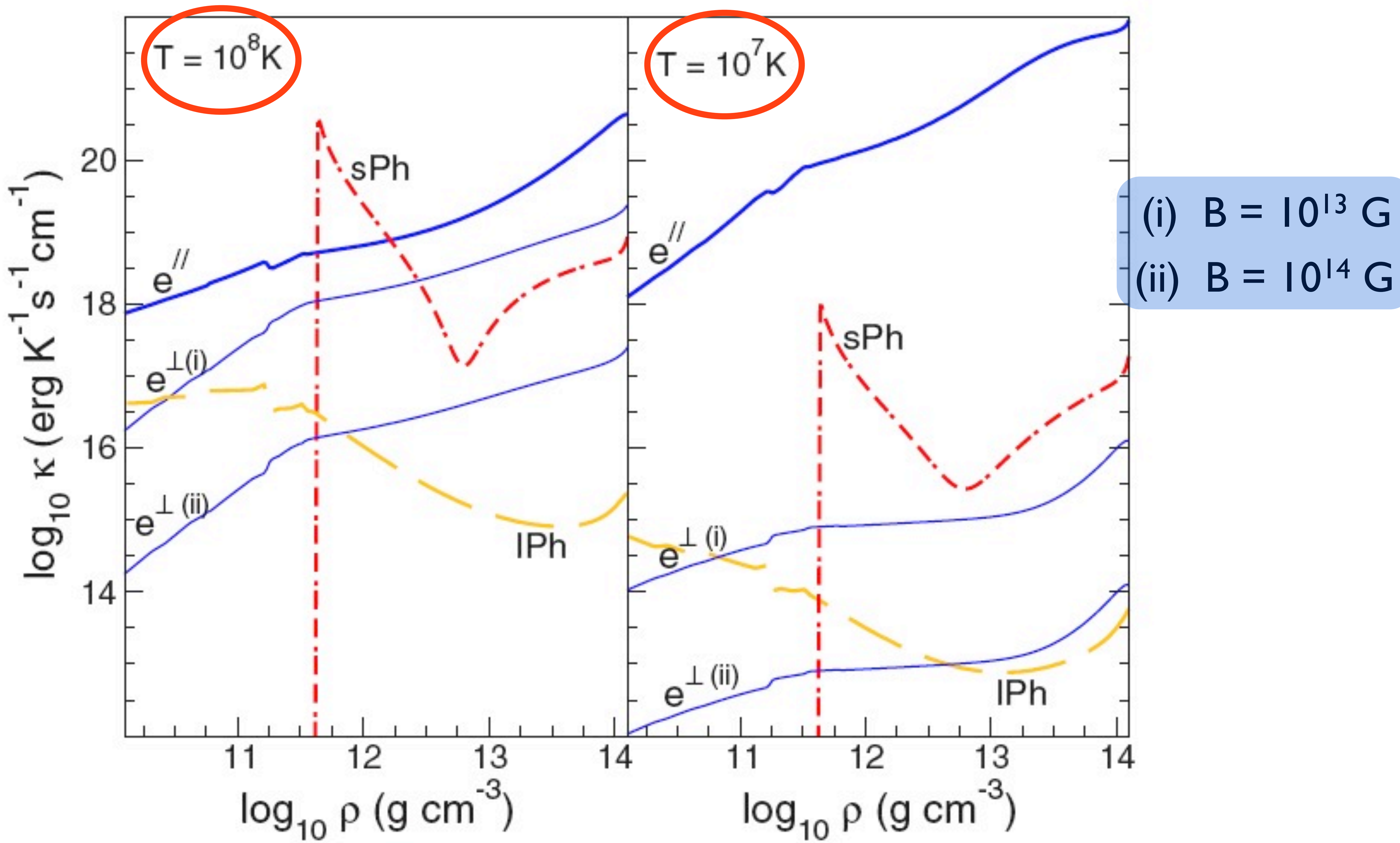
$$\kappa_{\perp} = \frac{\kappa_{||}}{1 + (\omega_g \ \tau_e)^2} \qquad \omega_g = \frac{eB}{\mu_e}$$

$$\kappa_{||} = \kappa_{\rm el}(B = 0)$$

$\tau_{\rm e}$ = Collision time

Canuto and Ventura (1977)
Uripin & Yakovlev (1980)

- Important in magnetized neutron stars, where electron conduction transverse to magnetic field is suppressed

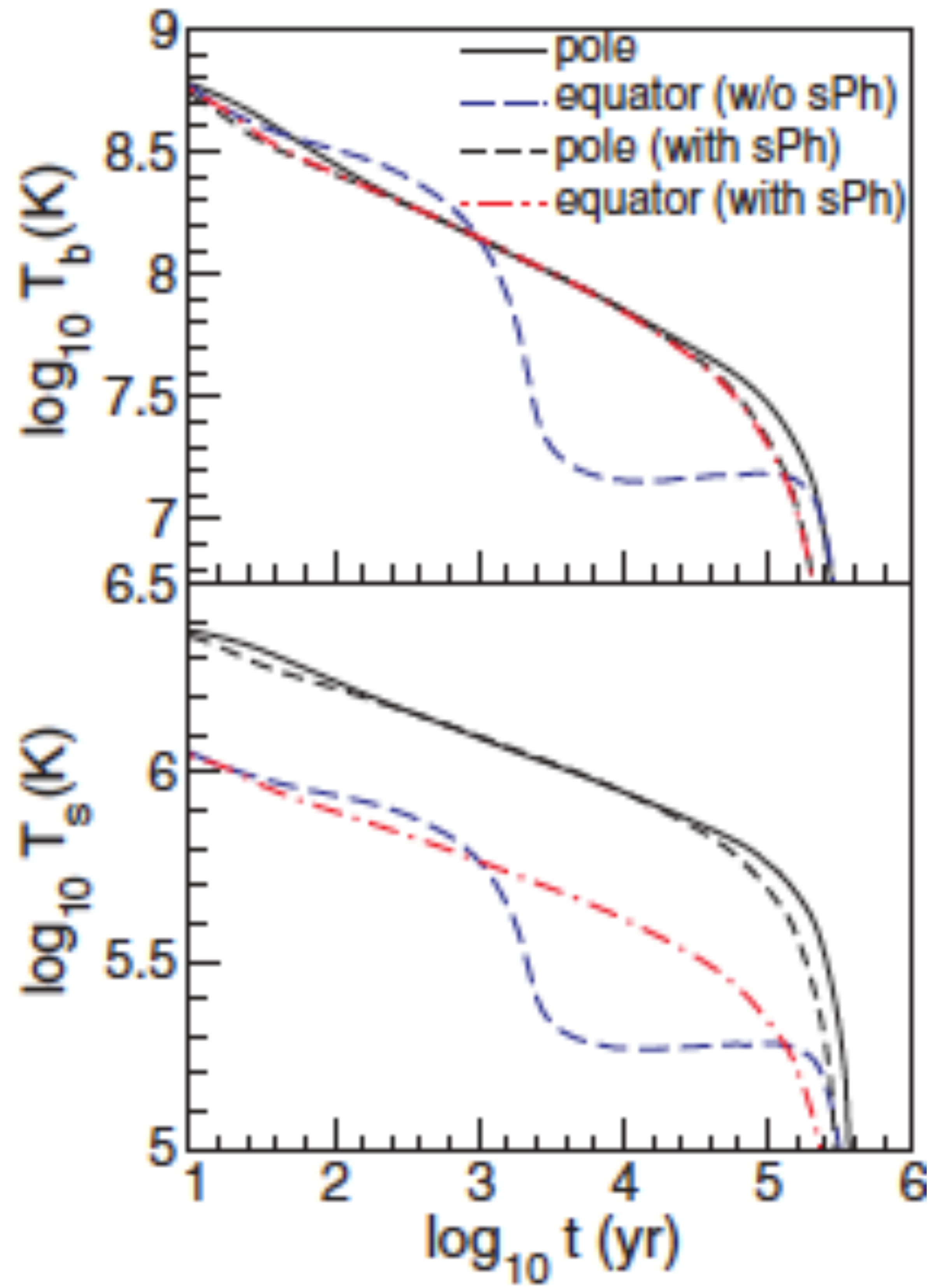



Aguilera-VC-Pons-Reddy-Sharma 09

- Impact on cooling curves of magnetized NS: superfluid heat conduction erases temperature anisotropy in the inner crust

at neutron drip →

surface →

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# Conclusions

- Low energy EFT describing phases of matter that spontaneously break translations and particle number
- Describes the NS inner crust, up to T ~ $10^9$ K ~ 100 keV
- LECs can be obtained from thermodynamic derivatives (or small p behavior of current correlation functions): formal matching calls for consistent non-perturbative calculations
- LECs → transport coefficients and thermal properties → connection with observables
    - modifications to phonon speeds
    - superfluid heat conduction (qualitatively new effect)