Oscillations of hot, young neutron stars

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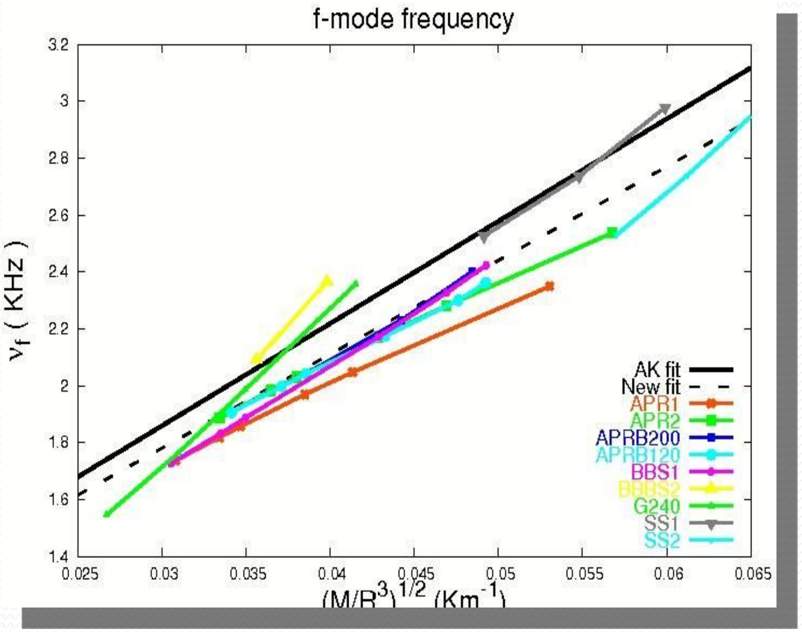
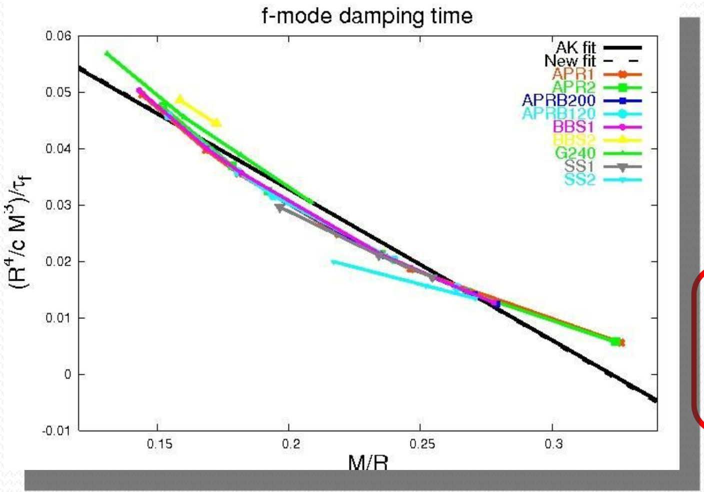
Why should we study stellar oscillations ?

When a NS is perturbed by some external or internal event, it can be set into non-radial, damped oscillations, the quasi-normal modes (**QNM**) which produce GW emission. The detection of these signals by GW detectors (AdvLIGO, AdvVirgo) will allow to measure the oscillation frequencies and damping times of the QNM, which carry information on the structure and EoS of a NS.

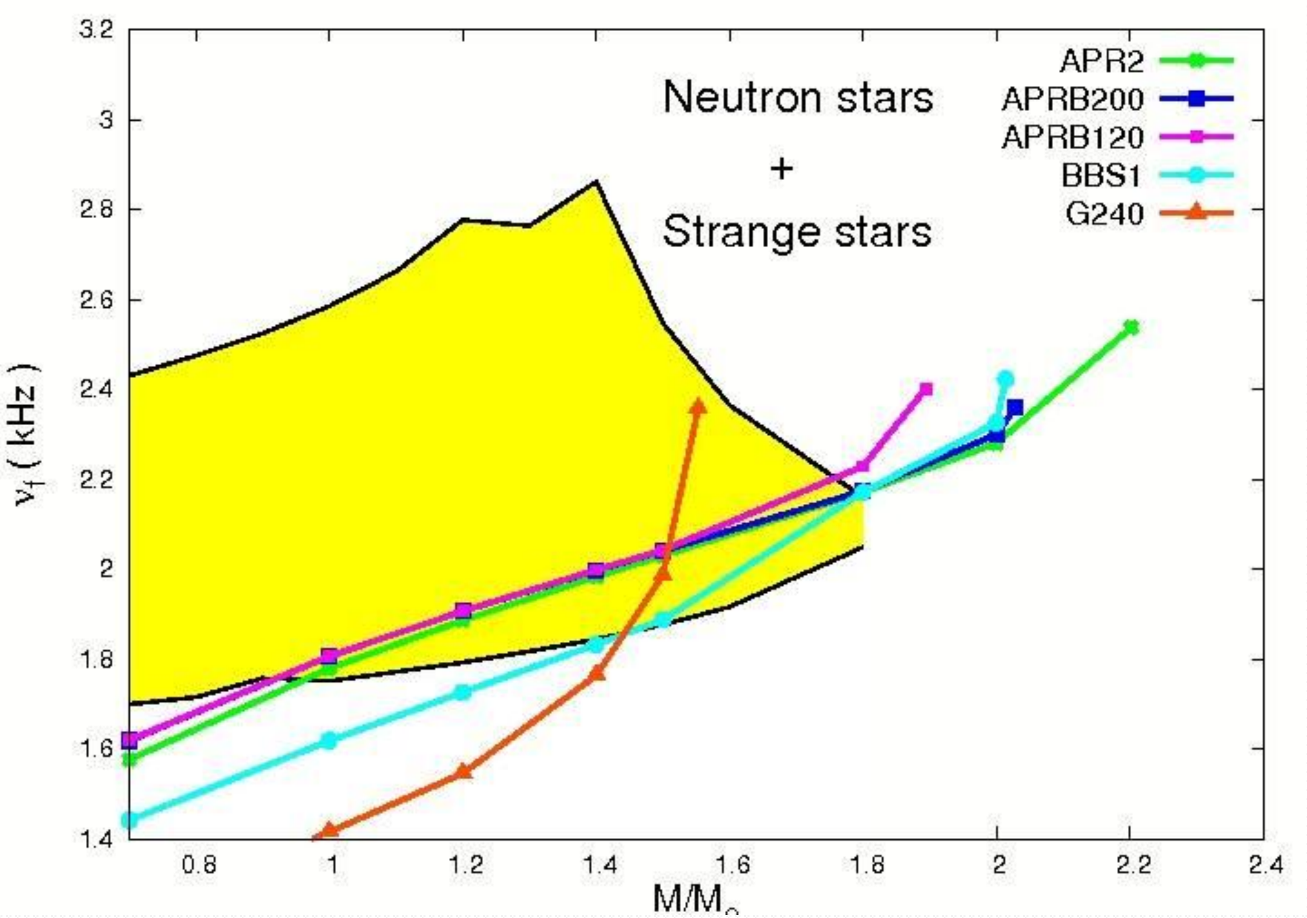
(Andersson & Kokkotas '98; Benhar et al., '04, '07)





- To infer the value of the NS mass M and radius R
- To discriminate between different possible EoS

Is the emitting source a NS or a strange star ?



Stellar perturbations : some historical remarks

The equations describing the non radial perturbations of a non rotating star in GR were firstly derived in

- Thorne & Campolattaro, *1967, ApJ* **149**, *591; ApJ* **152**, *673*
- Thorne, *1969, ApJ* **158**, *1; ApJ* **158**, *997*
- Price & Thorne, *1969, ApJ*, **155**, *163*
- Campolattaro & Thorne, *1970,* ApJ **159**, *847*
- Chandrasekhar & Ferrari, *1990, 1991, 1992* Proc. R. Soc. Lond.

The theory was completed by Lindblom & Detweiler (1983), who brought the analytic framework to a form suitable for the numerical integration, and for the determination of the complex frequencies of the *quasi normal modes.*

A main difference between the Newtonian and the relativistic theory is that in GR the oscillations are damped by the emission of gravitational waves.

The QNM of Compact Stars

- The perturbed spacetime metric is expanded in tensor spherical harmonics, as

$$
\begin{aligned}
ds^2 = &-e^{\psi}\left(1 + r^{\ell} H_0^{\ell m} Y_{\ell m} e^{i\omega t}\right) dt^2 \\
&+ e^{\lambda}\left(1 - r^{\ell} H_2^{\ell m} Y_{\ell m} e^{i\omega t}\right) dr^2 \\
&- 2i\omega r^{\ell+1} H_1^{\ell m} Y_{\ell m} e^{i\omega t} dt dr \\
&+ r^2\left(1 - r^{\ell} K^{\ell m} Y_{\ell m} e^{i\omega t}\right)\left(d\vartheta^2 + \sin^2\theta d\varphi^2\right)
\end{aligned}
$$

ω is the frequency, $Y_{lm}(\theta,\varphi)$ are the scalar spherical harmonics, and $H_i^{lm}(r)$, $K^{lm}(r)$ describe the metric perturbations with polar parity. $\psi(r)$, $\lambda(r)$ are found by solving the TOV equations.

- Einstein's equations, linearized in the perturbations, yield a system of 4 first-order differential equations (the Lindblom-Detweiler eqs.) for some perturbation functions, plus some algebraic relations which allow to compute the remaining functions in terms of the others.

☞ The EoS has to be specified (to close the system) !

☞ A solution only exists for a discrete set of (complex) values of the frequency : the *quasi-normal modes* of the star

$$\omega = 2\pi\nu + \frac{i}{\tau_{GW}}$$

(Cowling 1942)

Classification of the quasi-normal modes

g-modes: main restoring force is the buoyancy force due to entropy and/or composition gradients. A cold NS does NOT have g-modes.

f-mode: the fundamental mode. It is related to a global oscillation of the fluid. In cold NS ν is about 1-3 kHz, τ ~ 1 sec, hence the most efficient in radiating GW. Both in Newtonian theory and in GR, the frequency scales with the star average density.

$$\omega_f = \sqrt{\frac{2\ell(\ell+1)}{2\ell+1}\cdot\left(\frac{M}{R^3}\right)}$$

p-modes: main restoring force is the pressure. In cold NS ν is about a few thousands Hz, and τ ~ 20 s

w-modes: pure space-time modes (only in GR), ν~ many kHz, rapid damping

r-modes : the main restoring force is the Coriolis force (only for rotating stars).

There is a lot of physics to explore!

The modes frequency depends on the equation of state (EOS) of matter in the interior, on the rotation rate, on the phase of life the star is going through, namely on whether it is old and cold, or young and hot.

g-modes are strongly related to the thermodynamical properties of the star and appear if there are thermal or composition gradients.

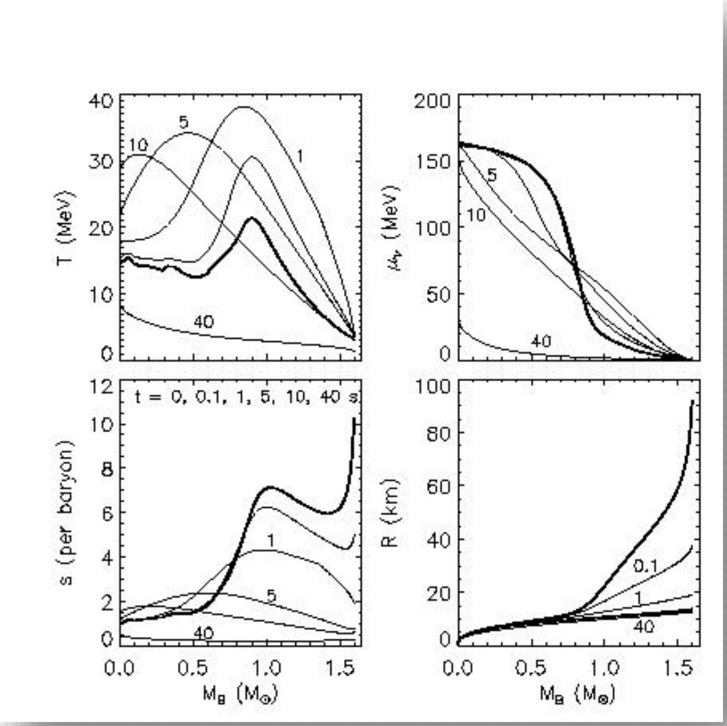
In a hot PNS, g-modes do appear!

The first minute of a PNS life : a qualitative description

t: 50-100 ms after core bounce

Hot shocked envelope (s~4-6) and unshocked core (s~1), in which neutrinos are trapped. Extensive neutrino losses from the mantel reduce the lepton pressure, and the mantle contracts.

At this stage the PNS has R~20-30 km.



t ~ 1-10 s after core bounce

Quasi-stationary evolution begins. Diffusion of high-energy neutrinos from the core to the surface generates a large amount of heat in the star, with T ~ some tens of MeV. The entropy increases in the core and decreases in the envelope, up to a nearly isentropic configuration.

t ~ 15 s after core bounce

Lepton poor PNS but still hot. Neutrino diffusion cools the star, until their mean free path becomes ~ R, and T=1-5 MeV → a NS is born (up to about 1 minute).

Pons, Reddy, Prakash, Lattimer, Miralles , ApJ 1999, 2000

We want to study how the frequencies and the damping times of the QNMs change when 0.1 s < t < 50 s.

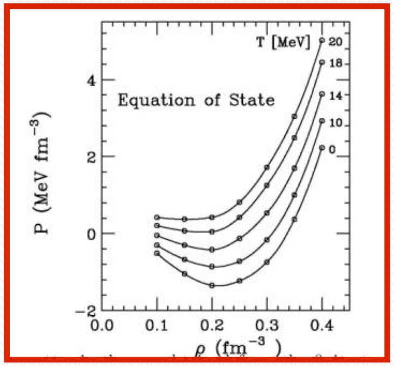
- *take a series of snapshots at different stages*
- *choose an entropy profile*
- *solve the equations of the stellar perturbations.*

☞ The EoS of dense matter is a microscopic one, derived in the BHF approach at finite T.

EoS of nuclear matter at finite T

(Microscopic approaches)

Variational method — **B. Friedman & V. R. Pandharipande, Nucl. Phys. A361, 502 (1981)**



T_c~18-20 MeV.

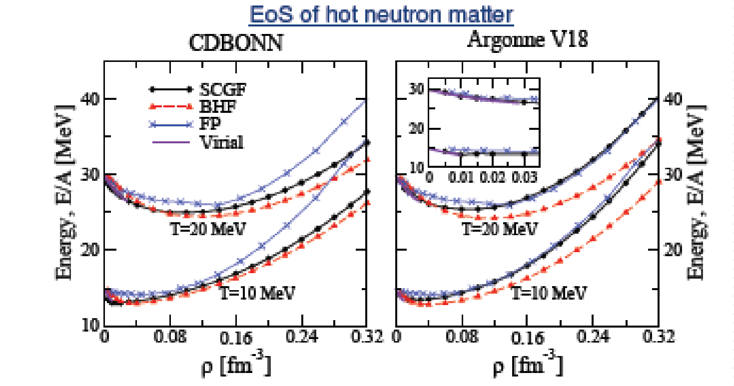
BHF — **M. Baldo & L. Ferreira, PRC 59, 682 (1999)**
Baldo, Burgio, Schulze, A&A (2006,2009, 2010)

Chiral Perturbation Theory — **S.Fritsch, N.Kaiser, W.Weise, PLB 545, 73 (2002)**

DBHF — **B. Ter Haar & R. Malfliet , Phys. Rep. 149, 207 (1987)**
H.Huber, F.Weber, M.K.Weigel , PRC 57, 3484,(1998)

$$T_c \approx 10\ MeV$$

SCGF



Rios, Polls & Vidana, Phys. Rev. C79, 025802 (2009)

Brueckner-Hartree-Fock theory at finite T

Bloch & De Dominicis
NP 10, 509 (1959)

- Two-body scattering matrix $K(W)$

$$\langle 12|K(W)|34\rangle = \langle 12|V|34\rangle + \mathrm{Re}\sum_{3'4'}\langle 12|V|3'4'\rangle \frac{\left[1-n^{FD}(3')\right]\left[1-n^{FD}(4')\right]}{W-E_{3'}-E_{4'}+i\varepsilon}\langle 3'4'|K(W)|34\rangle$$

- Single-particle energy E_i & single-particle potential $U(k)$

$$E_i = \frac{k_i^2}{2m_i} + U(k_i)$$

$$U(1) = \sum_2 n^{FD}(2)\langle 12|K(W)|12\rangle_A$$

- Auxiliary chemical potentials $\tilde{\mu}$

$$n_i = \sum_k n_i^{FD}(k) = \sum_k \frac{1}{1+e^{\beta(E_i(k)-\tilde{\mu}_i)}}$$

Grand-canonical potential density ω

$$\omega = -\sum_k \left[\frac{1}{\beta} \ln(1+e^{-\beta(E_k-\tilde{\mu})}) + n(k)U(k) \right] + \frac{1}{2}\int \frac{dW}{2\pi} e^{\beta(2\tilde{\mu}-W)} Tr_2\left(\arctan\left[K(W)\pi\delta(H_0-W)\right]\right)$$

Free energy density

$$f = \omega + n\tilde{\mu}$$

.... and then all thermodynamic quantities

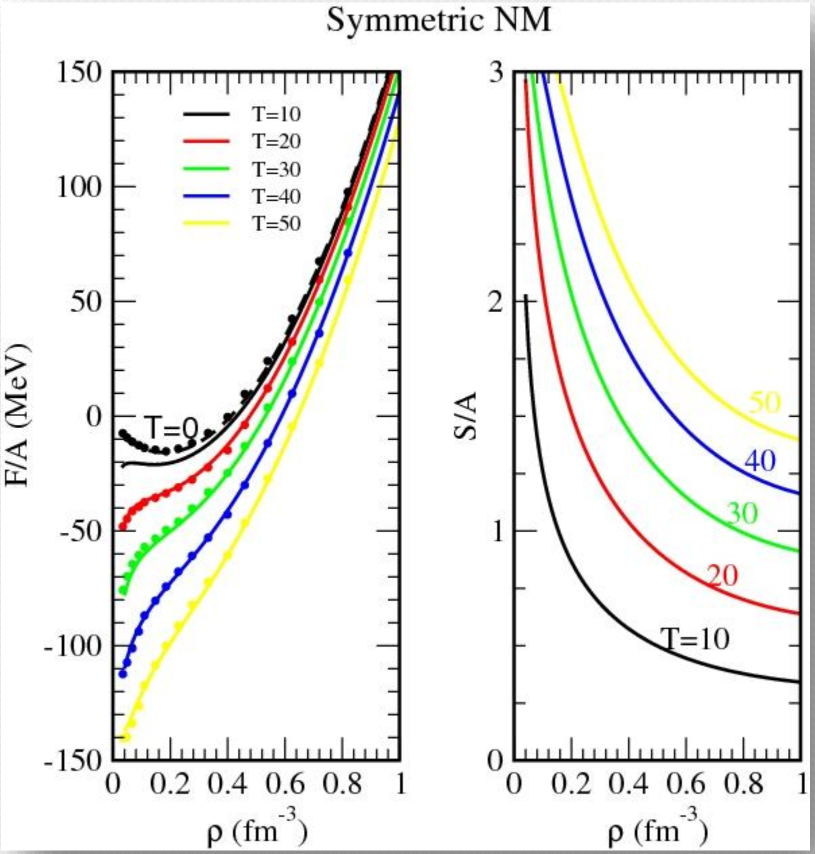
$$\mu = \left(\frac{\partial f}{\partial n}\right)_T, P = n^2\left(\frac{\partial (f/n)}{\partial n}\right)_T, s = -\frac{1}{n}\frac{\partial f}{\partial T}, \varepsilon = f + Tns$$

The only input required : <u>nucleon-nucleon interaction V</u>

Argonne v_{18} + three-body force (Urbana IX model)

......reduced to an effective two-body force ......
see Hans-Josef talk

EoS at finite T : results for SNM



$$\frac{F}{A}(\rho,T) = -(137+157*t^2)\rho + 308*\rho^{1.82}$$
$$+207*t^2*\log\rho$$
$$+(-47.5*t^2+71*t^{2.4})/\rho - 4.8,$$
$$with\, t = T/100$$

- Typical Van der Waals behavior, with Tc=19 MeV and $\rho_c \sim \rho_o/3$
- T=0: the usual NM saturation curve
- Similar parametrization for PNM

see : F.B. and H.-J.Schulze, A&A **518**, A17 (2010)

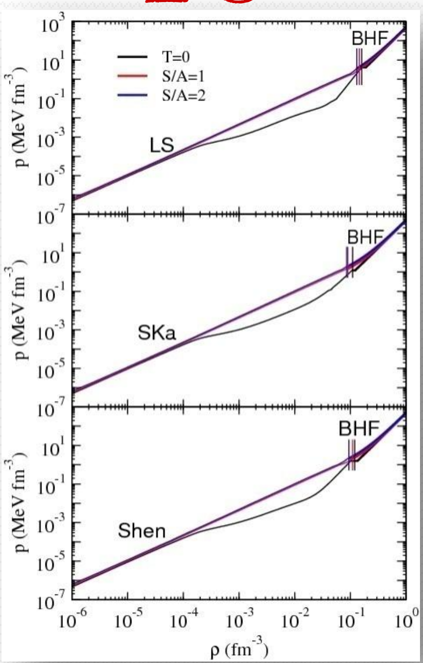
Low density regime

- Finite nuclei not included in BHF.
- Other EoS needed @ $n<n_0/2$.

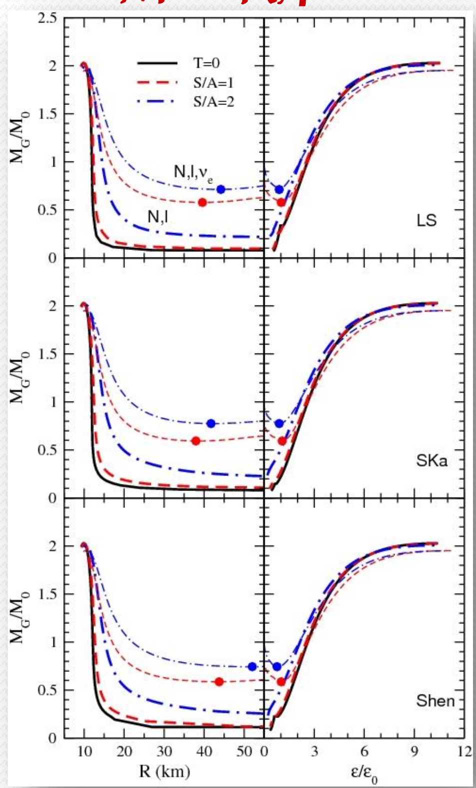
Lattimer-Swesty : LS, (K=370); Ska, (K=263)

Shen, (K=281)

EoS



M vs. R, ρ



		minimum mass			maximum mass		
		$M/M_\odot$	R (km)	ρ_c/ρ_0	$M/M_\odot$	R (km)	ρ_c/ρ_0
untrapped T=0	LS				2.03	9.86	10.55
	SKa				2.03	9.86	10.42
	Shen				2.03	9.93	10.42
trapped S/A=1	LS	0.58	40	1.02	1.95	10.2	11.34
	SKa	0.60	38	1.08	1.95	10.2	11.20
	Shen	0.58	44	1.02	1.95	10.3	11.20
trapped S/A=2	LS	0.70	44	0.90	1.95	10.7	10.85
	SKa	0.77	42	0.90	1.95	10.8	10.70
	Shen	0.75	52	0.77	1.95	10.8	10.80

$\mathbf{M_{max}}$ ~ 2 M_0, independent on the low density EoS

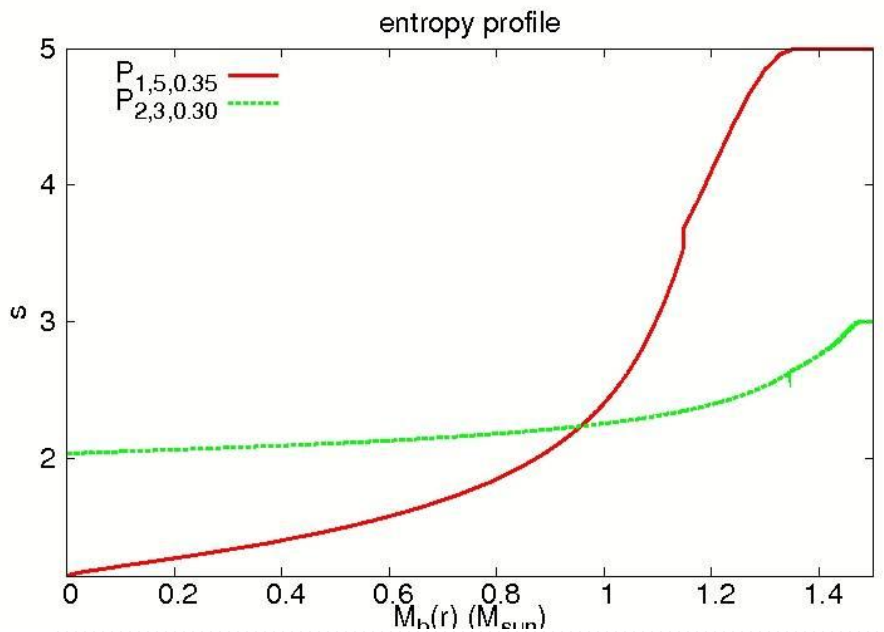
$\mathbf{M_{min}}$ ~ 0.6-0.77 M_0. Shen : larger Radius.

The PNS model

- Neutrino trapped beta-stable matter, imposing chemical equilibrium, charge neutrality and lepton number conservation. Below a threshold density $\rho \sim 10^{-6}\ \mathrm{fm}^{-3}$, matter is untrapped.
- Entropy profile $s(\rho)$ made by two pieces, one in the core s_c and one in the envelope s_e, joined smoothly with a cubic interpolation, according to the qualitative picture of the first minute of a PNS life.

TABLE I. Stellar models with fixed baryonic mass $M_B = 1.5 M_\odot$ corresponding to different entropy profiles and lepton fractions. The gravitational mass M, radius R, central temperature T^c, and central neutrino fraction x_ν^c are tabulated for each profile.

s_c	s_e	Y_e	$M/M_\odot$	R (km)	T^c (MeV)	x_ν^c
1.0	5.0	0.38	1.43	31.5	19.8	0.052
1.0	5.0	0.35	1.43	30.3	20.2	0.041
1.0	5.0	0.28	1.42	29.4	21.2	0.020
1.5	4.5	0.33	1.43	24.6	30.3	0.035
1.5	4.5	0.32	1.42	24.4	30.5	0.031
2.0	4.0	0.32	1.43	21.5	40.5	0.033
2.0	4.0	0.30	1.43	21.3	41.0	0.027
2.0	3.0	0.30	1.42	16.5	41.2	0.026
2.0	3.0	0.28	1.41	16.4	41.8	0.021
2.0	2.0	0.28	1.41	14.5	41.6	0.020
2.0	2.0	0.23	1.40	14.1	42.9	0.010
1.0	1.0	0.23	1.37	12.5	20.2	0.007
1	1	$x_\nu = 0$	1.36	12.2	20.9	0.000
$T = 0$			1.35	11.9	0.00	0.000



Results : Dependence on the entropy profile

Stellar models with constant M_{bar}=1.5 and Y_e=0.32

Scor	Senv	ΔS	Tc	R	νg	τg	νf	τf	νp	τp
1.0	5.0	4	20.6	29.6	906	6.27	1194	4.42	1528	0.75
1.5	4.5	3	30.5	24.3	910	42.9	1346	0.76	1845	0.55
1.0	4.0	3	20.2	18.4	870	793	1741	0.27	2574	0.99
2.0	4.0	2	40.5	21.5	669	2 x 10^3	1449	0.45	2097	0.72
2.0	3.0	1	40.7	16.8	492	6 x 10^5	1714	0.25	2977	1.64

- *g*-mode : the frequency mainly depends on the entropy jump $\Delta s = s_{env} - s_{core}$. As the entropy jump decreases, the damping time of the first g-mode increases dramatically : τ=6.27 for $P_{1,5,0.32}$ to τ=6x10^5 for $P_{2,3,0.32}$
- *f*-mode : ν_f and τ_f increase with decreasing R (different scaling laws from the T=0 case). Similarly for the *p*-mode (damping time insensitive to changes of the entropy profile).

Results : Dependence on the lepton fraction

Fixed entropy profile, P1,5 →

Stellar models with constant M_{bar}=1.5

Ye	R	M	νg	τg	νf	τf	νp	τp
0.38	31.5	1.43	863	6.78	1116	9.75	1415	1.0
0.36	30.6	1.43	883	6.62	1147	6.83	1463	0.89
0.32	29.6	1.42	906	6.25	1194	4.44	1527	0.75
0.3	29.4	1.42	910	5.99	1209	4.01	1543	0.73
0.28	29.4	1.42	908	5.71	1216	3.96	1546	0.72

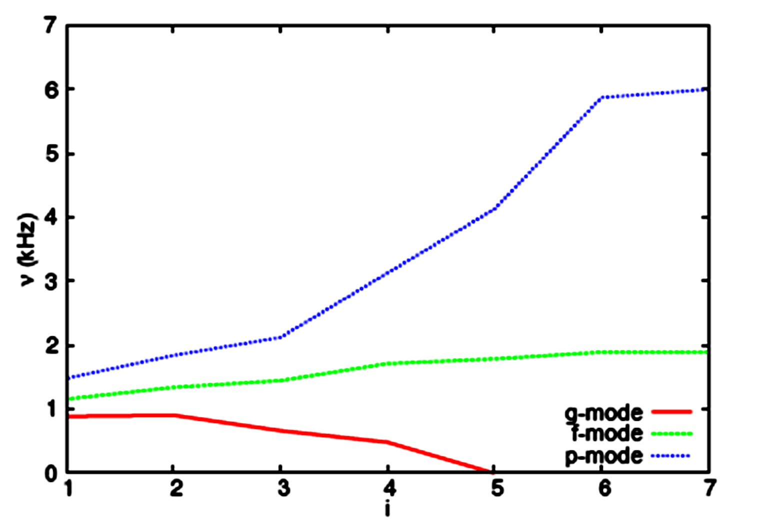
- The radius R is a slightly decreasing function of the lepton fraction. The *f*-and *p*-mode behave similarly to the previous case.
- The dependence on the lepton fraction is much weaker than that on the entropy profile.

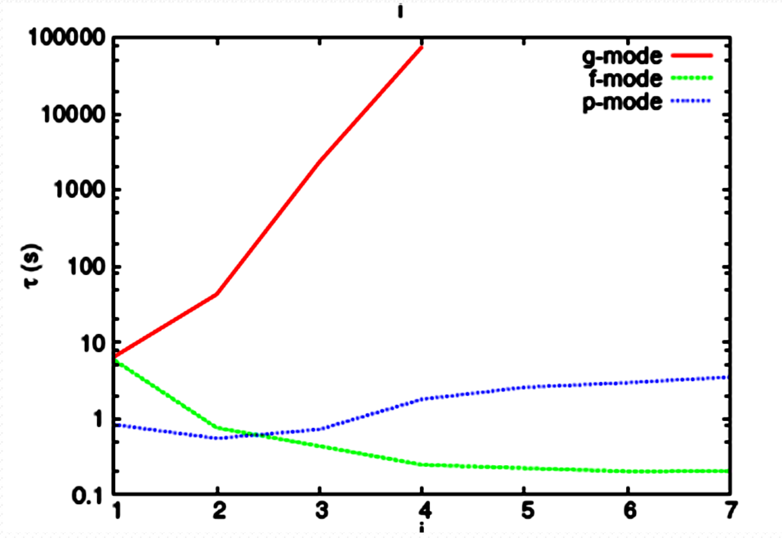
Change of the QNM's with the PNS evolution

M_{bar}=1.5

i	Score	Senv	Ye	R	νg	τg	νf	τf	νp	τp
1	1.	5.	0.35	30.3	890	6.54	1162	5.89	1484	0.84
2	1.5	4.5	0.32	24.4	910	42.9	1346	0.76	1845	0.55
3	2.	4.	0.3	21.3	667	2.3 x 10^3	1452	0.44	2125	0.73
4	2.	3.	0.28	16.4	485	7.6 x 10^4	1717	0.25	3133	1.8
5	2.	2.	0.23	14.1	0	-----	1790	0.23	4134	2.59
6	1.	1.		12.2	0	-----	1896	0.21	5879	2.98
7	0.	0.		11.9	0	----	1896	0.21	6006	3.52

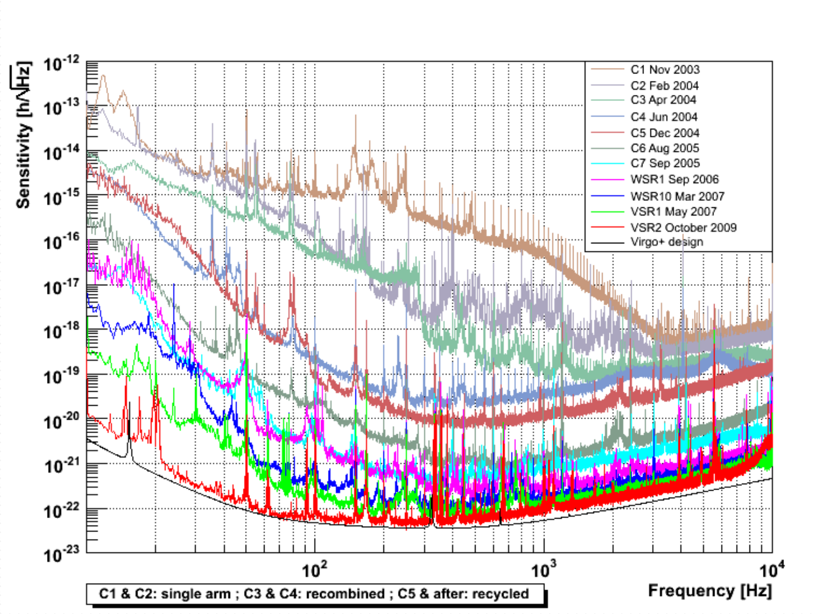
- The frequencies of the lowest order modes at earlier times (i=1,2) cluster in a small region around 1 kHz.
- In the same configurations, the damping times of the *f*- and *g*-modes become similar, and the *p*-mode damping time can become smaller than that of the *f*-mode.





These results are good for GW detection by Advanced LIGO/Virgo :

- because the f-mode has smaller frequency, falling in a more sensitive region
- because the g-mode oscillation has a small damping time



Conclusions

While it is well known that for cold NS

- It is more likely to detect the f-mode, more efficient in radiating GW

For QNM of hot, young NS at earlier times

- The f-mode frequencies are smaller → better for LIGO/Virgo detection
- The *g*-mode frequency is related to the entropy gradient, thus detecting this mode and its evolution we'd learn about thermodynamics of early PNS
- *g*–modes are similar to *f*-mode, both in frequency (few hundreds of Hz) and damping time (some seconds). So, in PNS's both modes are competitive as far as GW emission is concerned.
- In the first phases, damping times are smaller than dissipative time scales associated to neutrino processes. Hence, if the star has mechanical energy to dissipate, it will do it through the *g* and *f* modes

• The results are qualitatively similar to what obtained in Ferrari, Miniutti, Pons, MNRAS **342,** 629 (2003), where a RMF EoS was used.

• The main difference with our results : our frequencies are larger , and a smaller g-mode damping time.