Bulk viscosity and the damping of neutron star oscillations

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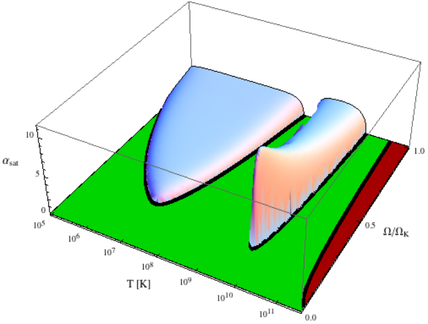
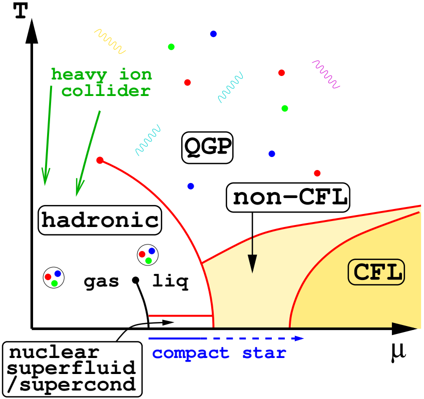
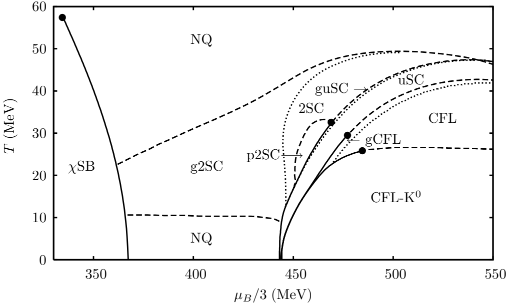


M. Alford, S. Mahmoodifar,
K. Schwenzer, arXiv:1103.3521

QCD phase diagram



NJL model, uniform phases only



Warringa, `hep-ph/0606063`

But there are also non-uniform phases, such as the crystalline ("LOFF"/"FFLO") phase. (Alford, Bowers, Rajagopal, `hep-ph/0008208`)

Signatures of color superconductivity in compact stars

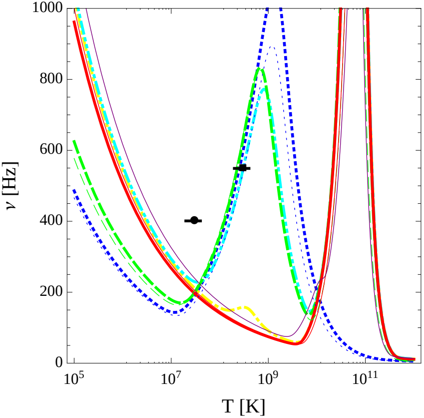
Pairing energy — affects Equation of state. Effects may be subtle.
(Alford, Braby, Paris, Reddy, `nucl-th/0411016`, Kurkela, Romatschke, Vuorinen, `arXiv:0912.1856`)

Gaps in quark spectra and Goldstone bosons — affect Transport properties:
emissivity, heat capacity, viscosity (shear, bulk), conductivity (electrical, thermal)...

1. Gravitational waves: r-mode instability, shear and bulk viscosity
2. Glitches and crystalline ("LOFF") pairing
3. Cooling by neutrino emission, neutrino pulse at birth

Constraints from r-modes

Regions above curves are forbidden because viscosity is too low to hold back the *r*-modes.



Neutron star, nuclear matter

Hybrid star, medium quark matter core

Hybrid star, large quark matter core

Quark star

LMXB data: Aql X-1 (square), SAX J1808.4-3658 (circle).

Damping by crust is not included.

Alford, Mahmoodifar, Schwenzer `arXiv:1012.4883`

Saturation amplitude of r-mode

Spindown from unstable region is determined by r-mode saturation amplitude $\alpha_{\rm sat}$; $\alpha_{\rm sat}$ is determined by nonlinear effects:

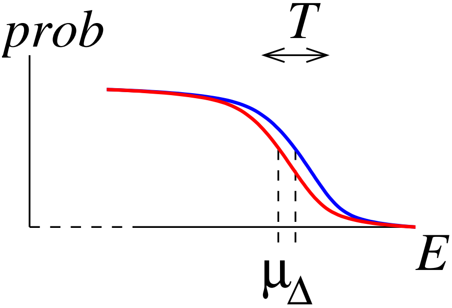
- Nonlinear GR calculation. Above amplitude $\alpha \sim 0.01$, r-mode may decay to short-wavelength daughter modes. Relies on extrapolation from calculations at artifically large gravitational forcing.
  (Lin and Suen, `gr-qc/0409037`)
- Models of coupling of r-mode to a few inertial modes.
  Saturates r-mode at $\alpha \sim 10^{-4}$ to 10^{-2} (dep on T and Ω).
  (Bondarescu, Teukolsky, Wasserman, `arXiv:0809.3448`)
- Suprathermal enhancement of bulk viscosity at high amplitude.
  Uses fully calculable nonlinearity in beta-equilibration process. Crust contributions not yet included; neglecting them, we find that well within the instability region, static saturation occurs at $\alpha_{\rm sat} \gtrsim 1$; near the edges $\alpha_{\rm sat}$ goes to zero.
  (Alford, Mahmoodifar, Schwenzer, `arXiv:1103.3521`)

Subthermal vs Suprathermal

Some quantity "Δ" goes out of equilibrium (eg $S - D$ in quark matter).
In equilibrium, $\mu_\Delta = 0$.

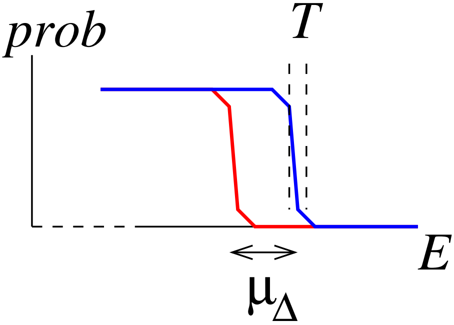
Subthermal

$\mu_\Delta \ll T \ll \mu$



Suprathermal

$T \ll \mu_\Delta \ll \mu$



(Madsen, Phys. Rev. D46,3290 (1992); Reisenegger, Bonacic, `astro-ph/0303454`)

Calculating bulk viscosity

- Compression at freq ω, so density of conserved charge oscillates as $n(t) = \bar{n} + \delta n \sin(\omega t)$
- One quantity "Δ" goes out of equilibrium (eg $S - D$ in quark matter). In equilibrium, $\mu_\Delta = 0$.
- EoS is characterized by susceptibilities B, C.

$$\zeta = -\frac{1}{\pi}\frac{\bar{n}}{\delta n}\frac{C}{B}\int_0^\tau \mu_\Delta(t)\cos(\omega t)dt$$

Bulk visc arises from component of μ_Δ that lags behind the driving oscillation by a phase of 90°; $\mu_\Delta(t)$ is given by

$$\frac{d\mu_\Delta}{dt} = \underbrace{C\omega\frac{\delta n}{\bar{n}}\cos(\omega t)}_{\text{driving osc.}} - \underbrace{\Gamma(\mu_\Delta, T)}_{\text{equilibration}}$$

Re-express this in dimensionless variables:

Computing departure from equilibrium

- Define dimensionless time (i.e. phase) $\varphi = \omega t$
- Define dimensionless departure from equilibrium $\bar{\mu}_\Delta = \mu_\Delta / T$
- Driving coeff $d = \dfrac{C}{T}\dfrac{\delta n}{\bar{n}}$
- Equilibration rate: $\Gamma(\mu_\Delta, T) = \tilde{\Gamma}\, T^\delta\, \gamma(\mu_\Delta / T)$.

  Equilibration coeff $f = \dfrac{B}{\omega}\tilde{\Gamma}\, T^\delta$.

$$\boxed{\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\,\gamma(\bar{\mu}_\Delta)}$$

Dependence on density, EoS, driving amplitude, and temperature is contained in d and f. Dependence of equilibration rate on $\bar{\mu}_\Delta$ is

$$\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta + \chi_1 \bar{\mu}_\Delta{}^3 + \cdots + \chi_N \bar{\mu}_\Delta{}^{2N+1} \qquad (2N = \delta)$$

Suprathermal and subthermal bulk viscosity

Subthermal: assume $\bar{\mu}_\Delta \ll 1$ (i.e. $\mu_\Delta \ll T$), so $\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta$,

$$\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\bar{\mu}_\Delta$$

$$\bar{\mu}_\Delta(\varphi) = -\frac{f\,d}{1+f^2}\cos\varphi + \frac{d}{1+f^2}\sin\varphi$$

$$\zeta_{\rm sub} = \frac{C^2}{B\omega}\frac{f}{1+f^2} = \frac{C^2}{B}\frac{\gamma_{\rm eff}}{\omega^2+{\gamma_{\rm eff}}^2} \qquad (\gamma_{\rm eff} \equiv B\tilde{\Gamma}T^\delta)$$

Suprathermal and subthermal bulk viscosity

Subthermal: assume $\bar{\mu}_\Delta \ll 1$ (i.e. $\mu_\Delta \ll T$), so $\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta$,

$$\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\,\bar{\mu}_\Delta$$

$$\begin{aligned}\bar{\mu}_\Delta(\varphi) &= -\frac{f\,d}{1+f^2}\cos\varphi + \frac{d}{1+f^2}\sin\varphi \\ \zeta_{\rm sub} &= \frac{C^2}{B\omega}\frac{f}{1+f^2} = \frac{C^2}{B}\frac{\gamma_{\rm eff}}{\omega^2+\gamma_{\rm eff}{}^2} \qquad (\gamma_{\rm eff} \equiv B\tilde{\Gamma}T^\delta)\end{aligned}$$

Suprathermal: allow $\bar{\mu}_\Delta \gtrsim 1$ (always assuming $\delta n \ll \bar{n}$),

$$\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\,\bar{\mu}_\Delta\Big(1 + \chi_1\bar{\mu}_\Delta{}^2 + \cdots + \chi_N\bar{\mu}_\Delta{}^{2N}\Big)$$

Now there are nonlinear effects; $\bar{\mu}_\Delta(\varphi)$ may not be harmonic.

The general bulk viscosity

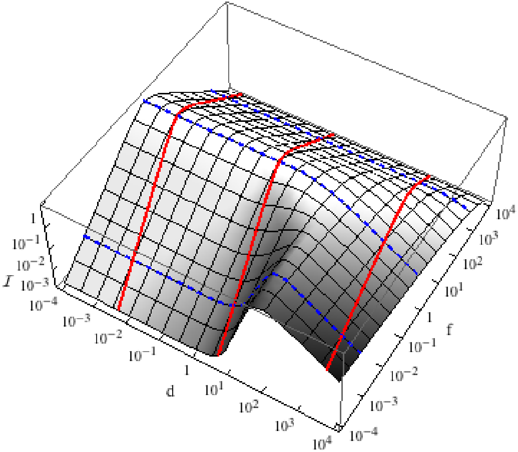
To include the suprathermal regime, we have to solve the diffeq for $\bar{\mu}_{\Delta}(\varphi)$ numerically.

For a given form of matter, we can summarize dependence on driving amplitude and temperature in dimensionless function $\mathcal{I}(d, f)$,

$$\zeta = \frac{C^2}{2\omega B}\mathcal{I}(d, f)$$

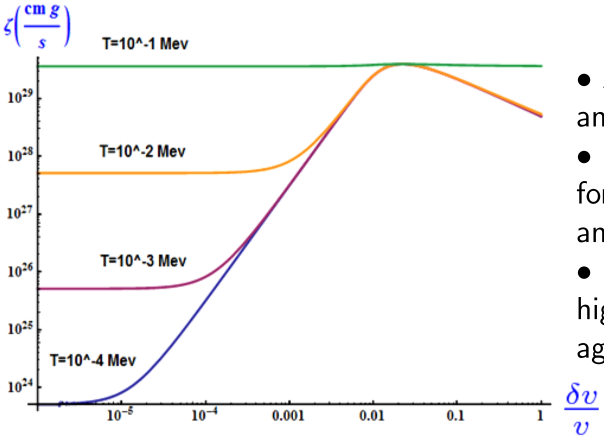
This could then be used to calculate damping time of r-modes.

Nuclear matter, modified Urca



Amplitude dependence of bulk visc

E.g.: Unpaired quark matter: $\gamma(\bar{\mu}_{\Delta}) = \bar{\mu}_{\Delta} + \chi_1 \bar{\mu}_{\Delta}{}^3$



- At low T, suprathermal amplification is huge
- Maximum value is same for all temperatures and amplitudes.
- If amplitude gets too high, bulk viscosity drops again.

Application to r-mode saturation

r-mode stops growing at the "static saturation amplitude" $\alpha_{\rm sat}$, when damping balances gravitational driving:

$$0 = \frac{1}{\tau_{\rm grav}(T,\Omega)} + \frac{1}{\tau_{\rm shear}(T,\Omega)} + \frac{1}{\tau_{\rm bulk}(T,\Omega,\alpha_{\rm sat})}$$

Need to calculate damping times $\tau_{\rm bulk}$ etc, which are global properties of the star and the r-mode, so they depend on the density profile of star, r-mode flow profile, etc.

r-mode saturation: approximate expression

We find an approximate result for the static saturation amplitude, valid far from the instability boundary,

$$\alpha_{\rm sat} = f(\delta)\ \underbrace{\frac{\Lambda_{EW}^{4/\delta} G^{1/\delta}}{\Lambda_{QCD}^{9/\delta}}}_{\text{interactions}}\ \underbrace{\frac{M^{2/\delta}\,\tilde{J}_m^{2/\delta}}{R^{2+(5-2m)/\delta}\Omega^{2-2m/\delta}}}_{\text{star}}\ \underbrace{\frac{1}{\tilde{V}_{m,\delta/2}^{1/\delta}}}_{\text{r}-\text{mode}}$$

- $\tilde{J}$ and $\tilde{V}$ are radial integrals over the star
- This is a conservative estimate: radial integrals exclude the crust, only including $n > 0.25 n_{\rm sat}$
- δ characterizes the equilibration rate

subthermal: $\Gamma(\mu_\Delta, T) = \tilde{\Gamma}\, T^\delta \mu_\Delta$; suprathermal: $\Gamma(\mu_\Delta, T) = \tilde{\Gamma}\chi_N {\mu_\Delta}^{1+\delta}$

- m is the quantum number of the r-mode ($m = 2$ dominates)
- all dependence on Ω is explicit
- in this approximation $\alpha_{\rm sat}$ is indp of T

Likely influence of crust

If we extended our calculation of τ_B, τ_G, τ_S to include the crust, we expect a large additional contribution to bulk viscosity damping.

- r-mode density perturbation is

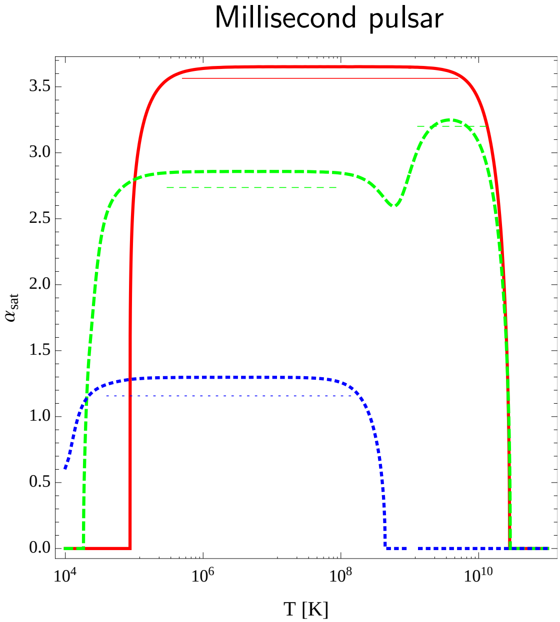
$$\frac{\delta n}{n} \propto \alpha\, r^3 A(r)$$

  where $A(r) = 1/v = d\rho/dp$ grows with r.
  So $\delta n/n$ is largest in the crust.
- Deviation from equilibrium is $\mu_\Delta = C\delta n/n$
- Suprathermal bulk visc due to modified Urca

$$\zeta_{\rm supra} \approx \zeta_{\rm sub}\Bigl(\frac{C}{T}\frac{\delta n}{n}\Bigr)^6$$

So crust is likely to dominate bulk viscosity damping of r-mode, since it place where $\delta n/n$ is largest.

Static saturation amplitude $\alpha_{\rm sat}$

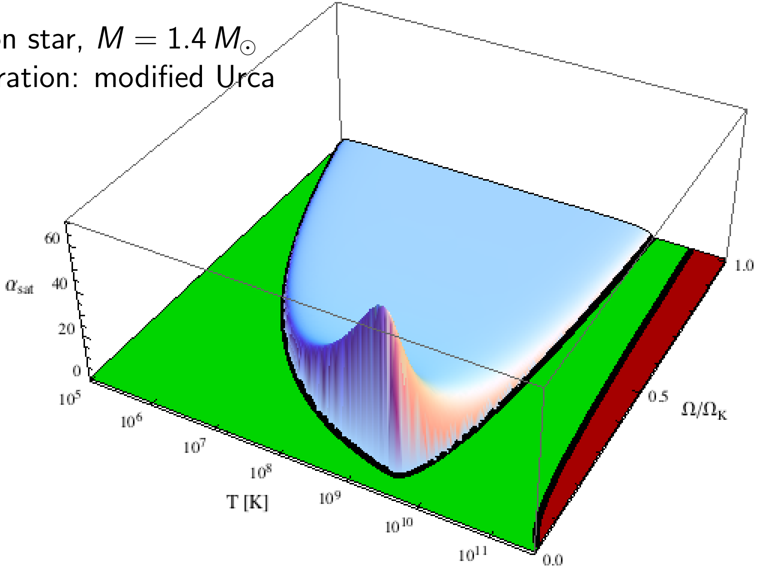


Neutron star
Hybrid star
Quark star

Thin lines show analytic approx.

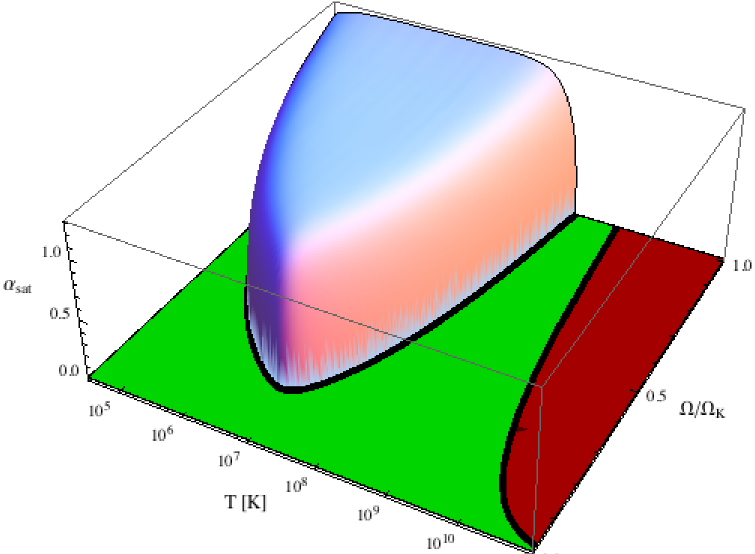
Static saturation amplitude: neutron star

Neutron star, $M = 1.4\,M_{\odot}$
equilibration: modified Urca



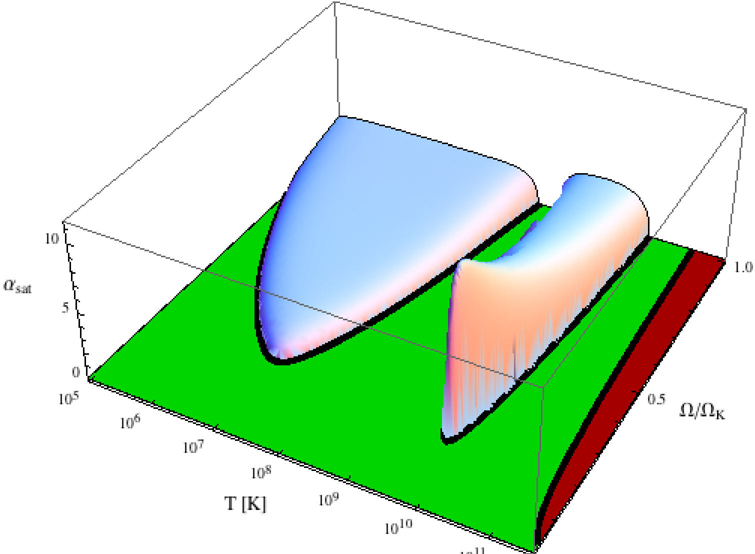
Static saturation amplitude: strange star

Strange star, $M = 1.4\,M_\odot$
equilibration: direct Urca



Static saturation amplitude: hybrid star

Hybrid star, $M = 1.4\,M_\odot$
equilibration: direct Urca



Future directions

Suprathermal bulk viscosity:

- Extend to other phases of quark matter, eg CFL-K0
- Superfluid nuclear matter: suprathermal leptonic viscosity?
- Hyperonic nuclear matter (see Reisenegger)
- Investigate effect of multiple equilibrating quantities

Astrophysics:

- Include crust contributions: might greatly reduce $\alpha_{\rm sat}$.
- Evolution of star: dynamic saturation amplitude may be lower
- Other contributions to r-mode damping: superfluid turbulence? (vortex entanglement, vortex-fluxtube entanglement)
- Back reaction of heavy damping on r-mode profile ("clamping")
- Apply to other modes, eg pulsations, f-modes (which emit grav waves)