Triples Counting and Three-Nucleon Forces in Light Nuclei

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WORK WITH

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WORK NOT POSSIBLE WITHOUT EXTENSIVE COMPUTER RESOURCES

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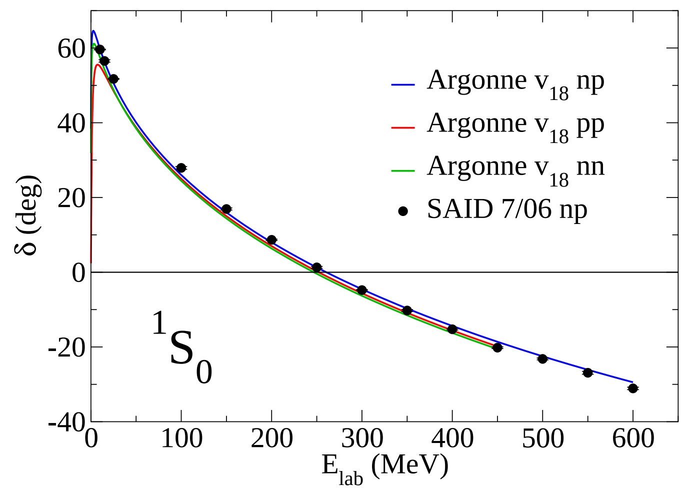
NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, m_n-m_p effects included

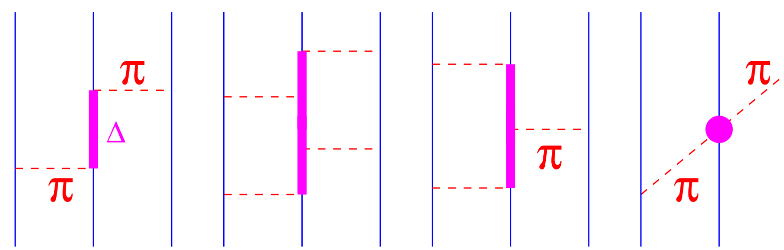
Argonne v$_{18}$: $v_{ij} = v^{\gamma}_{ij} + v^{\pi}_{ij} + v^{I}_{ij} + v^{S}_{ij} = \sum v_p(r_{ij}) O^p_{ij}$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with χ^2/d.o.f.=1.1



Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana IX $V_{ijk}(+) = V^{2\pi P}_{ijk} + V^{U}_{ijk} + (V^{X}_{ijk} + V^{2\pi S}_{ijk})$



- Fujita-Miyazawa 2π P-wave term
- short-range repulsion for matter saturation
- (intermediate-range term and 2π S-wave)
- correspond roughly to c_3+c_4, c_E, (c_D, c_1) terms of χEFT.

Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)

QUANTUM MONTE CARLO

Variational Monte Carlo (VMC): construct Ψ_V that

- Are fully antisymmetric and translationally invariant
- Have cluster structure and correct asymptotic form
- Contain non-commuting 2- & 3-body operator correlations from v_{ij} & V_{ijk}
- Are orthogonal for multiple J^π states
- Minimize $E_V = \langle \Psi_V | H | \Psi_V \rangle \geq E$

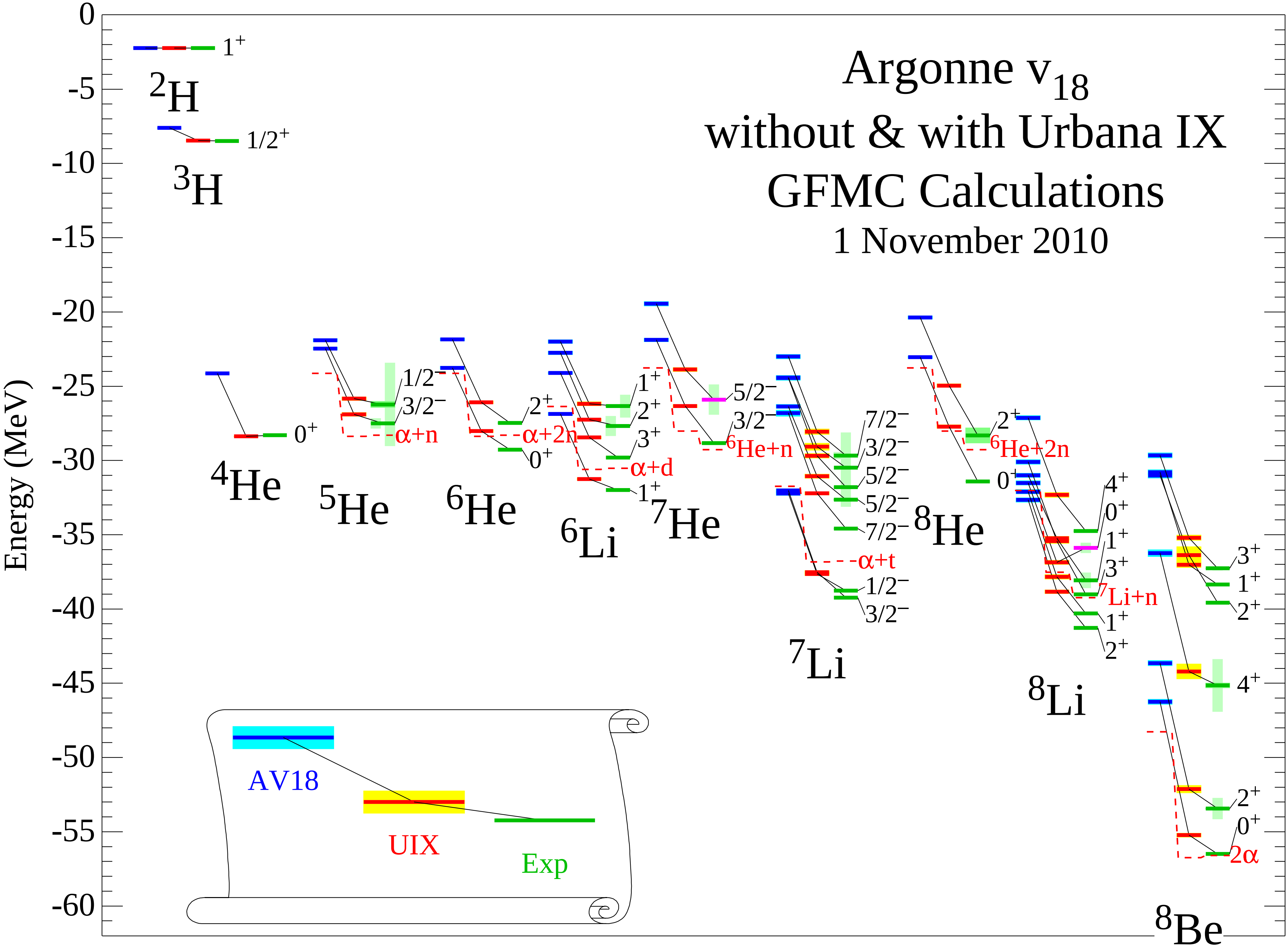
These are $\sim 2^A \binom{A}{Z}$ component spin-isospin vectors in $3A$ dimensions

Green's function Monte Carlo (GFMC): project out the exact eigenfunction

- $\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \Psi_n \Rightarrow \Psi_0$ at large τ
- Propagation done stochastically in small time slices $\Delta\tau$
- Exact $\langle H \rangle$ for local potentials; mixed estimates for other $\langle O \rangle$
- Constrained-path propagation controls fermion sign problem for $A \geq 8$
- Multiple excited states for same J^π stay orthogonal

Many tests demonstrate 1–2% accuracy for realistic $\langle H \rangle$

Pudliner, Pandharipande, Carlson, Pieper, & Wiringa, PRC **56**, 1720 (1997)
Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)
Pieper, Varga, & Wiringa, PRC **66**, 044310 (2002)
Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)



Argonne v18
without & with Urbana IX
GFMC Calculations
1 November 2010
Energy (MeV)
2H
3H
4He
5He
6He
6Li
7He
7Li
8He
8Li
8Be
α+n
α+2n
α+d
6He+n
α+t
6He+2n
7Li+n
2α
AV18
UIX
Exp

Counting Nucleon Triples

Triples can have $S = \frac{1}{2}$ or $\frac{3}{2}$ and $T = \frac{1}{2}$ or $\frac{3}{2}$. The appropriate projection operators are:

$$P_{S=3/2,1/2} = \frac{1}{2} \pm \frac{1}{6} \sum_{cyc} \sigma_i \cdot \sigma_j \quad ; \quad P_{T=3/2,1/2} = \frac{1}{2} \pm \frac{1}{6} \sum_{cyc} \tau_i \cdot \tau_j$$

The triples can have all three particles in the s-shell (sss), or two-particles in the s-shell and one in the p-shell (ssp), etc. For uncorrelated (unc) wave functions, the numbers of each type are determined by the symmetry characteristics $^{2S+1}$L[n] of the state. When tensor correlations are present (cor), S is not conserved and some triples will be promoted from $S = \frac{1}{2}$ to $S = \frac{3}{2}$.

	^{4}He ^{1}S[4]		^{6}He ^{1}S[42]				
ST	sss	cor	sss	ssp	spp	unc	cor
$\frac{1}{2}\ \frac{1}{2}$	4	3.35	4	4	$^4/_3$	9.33	8.3
$\frac{3}{2}\ \frac{1}{2}$		0.65		4		4.00	5.0
$\frac{1}{2}\ \frac{3}{2}$				4	$^8/_3$	6.67	6.1
$\frac{3}{2}\ \frac{3}{2}$							0.6
total	4	4	4	12	4	20	20

	^{8}He ^{1}S[422]						^{8}Be ^{1}S[44]					
ST	sss	ssp	spp	ppp	unc	cor	sss	ssp	spp	ppp	unc	cor
$\frac{1}{2}$ $\frac{1}{2}$	4	8	$^{16}/_{3}$		17.33	15.7	4	8	8	4	24	21.8
$\frac{3}{2}$ $\frac{1}{2}$		8	$^{8}/_{3}$		10.67	12.3		8	8		16	18.2
$\frac{1}{2}$ $\frac{3}{2}$		8	$^{32}/_{3}$	4	22.67	20.7		8	8		16	14.1
$\frac{3}{2}$ $\frac{3}{2}$			$^{16}/_{3}$		5.33	7.3						1.9
total	4	24	24	4	56	56	4	24	24	4	56	56

POTENTIAL TERMS

The two-nucleon potential one-pion-exchange (OPE) can be written as:

$$v_{ij}^{\pi} = A_{\pi} X_{ij} (\tau_i \cdot \tau_j) \equiv A_{\pi} [Y(\mu r) \sigma_i \cdot \sigma_j + T(\mu r) S_{ij}] (\tau_i \cdot \tau_j)$$

$$Y(\mu r) = \frac{e^{-\mu r}}{\mu r} \xi(r) \qquad T(\mu r) = \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) Y(\mu r) \xi(r)$$

where $\xi(r)$ is a short-range form factor.

The two Urbana IX three-nucleon potential terms are the P-wave two-pion exhange (TPE) which has anticommutator and commutator terms:

$$V_{ijk}^{2\pi P} = A_{2\pi}^{P} \left(\sum_{cyc} \{X_{ij}, X_{ik}\} \{\tau_i \cdot \tau_j, \tau_i \cdot \tau_k\} + \frac{1}{4} [X_{ij}, X_{ik}] [\tau_i \cdot \tau_j, \tau_i \cdot \tau_k] \right)$$

and the phenomenological short-range repulsive term:

$$V_{ijk}^{U} = A^{U} \sum_{cyc} T^2(\mu r_{ij}) T^2(\mu r_{ik})$$

The additional terms we evaluate in perturbation are a mixed short-range/OPE term:

$$V_{ijk}^{X} = A^{X} \sum_{cyc} [T^2(\mu r_{ij}) X_{ik} (\tau_i \cdot \tau_k) + X_{ij} (\tau_i \cdot \tau_j) T^2(\mu r_{ik})]$$

and the S-wave TPE:

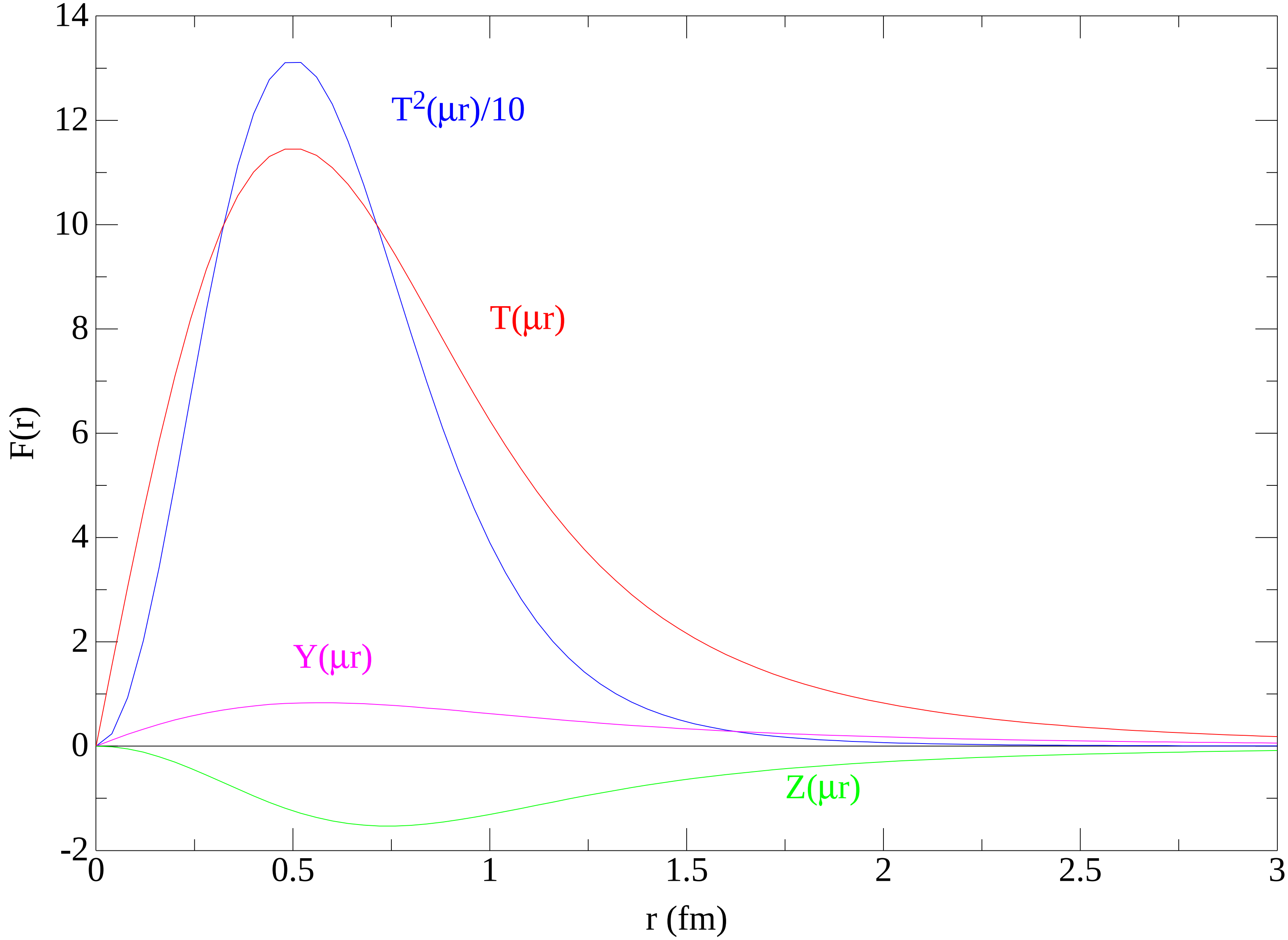
$$V_{ijk}^{2\pi S} = A_{2\pi}^{S} \sum_{cyc} [Z(\mu r_{ij}) Z(\mu r_{ik})\, \sigma_j \cdot \hat{\mathbf{r}}_{ij}\, \sigma_k \cdot \hat{\mathbf{r}}_{ik}\, \tau_j \cdot \tau_k]$$

$$Z(\mu r) = \frac{\mu r}{3} [Y(\mu r) - T(\mu r)]$$

Strengths of coefficients A in MeV:

$A_{2\pi}^{P}$	A^{U}	A^{X}	$A_{2\pi}^{S}$
-0.0293	0.0048	-0.0048	-1.00

Functional Forms



V_{ijk} Expectation Values

^{4}He

		$V^{2\pi A}$	$V^{2\pi C}$	$V^{2\pi S}$	V^{U}	V^{X}
$T=\frac{1}{2}$	$SS'=\frac{1}{2}\ \frac{1}{2}$	−2.35	−1.52	0.53	4.50	0.70
	$SS'=\frac{3}{2}\ \frac{1}{2}+\frac{1}{2}\ \frac{3}{2}$	−5.01	−2.79	−1.02		4.52
	$SS'=\frac{3}{2}\ \frac{3}{2}$	−0.03	−0.12	−0.04	0.91	0.01
	total	−7.39	−4.43	−0.53	5.41	5.23

^{6}He

		$V^{2\pi A}$	$V^{2\pi C}$	$V^{2\pi S}$	V^{U}	V^{X}
$T=\frac{1}{2}$	$SS'=\frac{1}{2}\ \frac{1}{2}$	−2.51	−1.65	0.62	4.80	0.74
	$SS'=\frac{3}{2}\ \frac{1}{2}+\frac{1}{2}\ \frac{3}{2}$	−5.72	−3.17	−1.19		5.04
	$SS'=\frac{3}{2}\ \frac{3}{2}$	−0.00	−0.15	0.04	1.21	0.17
$T=\frac{3}{2}$	$SS'=\frac{1}{2}\ \frac{1}{2}$	0.05		0.02	0.25	0.02
	$SS'=\frac{3}{2}\ \frac{1}{2}+\frac{1}{2}\ \frac{3}{2}$	−0.05		0.00		0.02
	$SS'=\frac{3}{2}\ \frac{3}{2}$	−0.00		−0.00	0.02	0.00
	total	−8.23	−4.97	−0.51	6.28	5.99

^{8}He

		$V^{2\pi A}$	$V^{2\pi C}$	$V^{2\pi S}$	V^{U}	V^{X}
$T=\frac{1}{2}$	$SS'=\frac{1}{2}\ \frac{1}{2}$	−2.85	−1.89	0.75	5.55	0.82
	$SS'=\frac{3}{2}\ \frac{1}{2}+\frac{1}{2}\ \frac{3}{2}$	−7.27	−3.90	−1.58		6.22
	$SS'=\frac{3}{2}\ \frac{3}{2}$	−0.05	−0.25	0.16	1.88	0.52
$T=\frac{3}{2}$	$SS'=\frac{1}{2}\ \frac{1}{2}$	0.07		0.05	0.81	0.06
	$SS'=\frac{3}{2}\ \frac{1}{2}+\frac{1}{2}\ \frac{3}{2}$	−0.16		−0.00		0.08
	$SS'=\frac{3}{2}\ \frac{3}{2}$	0.03		−0.01	0.08	0.00
	total	−10.23	−6.04	−0.63	8.32	7.70

^{8}Be

		$V^{2\pi A}$	$V^{2\pi C}$	$V^{2\pi S}$	V^{U}	V^{X}
$T=\frac{1}{2}$	$SS'=\frac{1}{2}\ \frac{1}{2}$	−5.02	−3.30	1.20	9.69	1.48
	$SS'=\frac{3}{2}\ \frac{1}{2}+\frac{1}{2}\ \frac{3}{2}$	−11.14	−6.18	−2.24		10.03
	$SS'=\frac{3}{2}\ \frac{3}{2}$	0.02	−0.31	0.01	2.33	0.25
$T=\frac{3}{2}$	$SS'=\frac{1}{2}\ \frac{1}{2}$	0.11		0.04	0.26	0.02
	$SS'=\frac{3}{2}\ \frac{1}{2}+\frac{1}{2}\ \frac{3}{2}$	−0.06		0.00		0.03
	$SS'=\frac{3}{2}\ \frac{3}{2}$	0.00		−0.01	0.02	0.00
	total	−16.09	−9.79	−1.00	12.30	11.81

OBSERVATIONS

- Tensor part of $V^{2\pi P}$ dominates – about 2/3 comes from connecting $S{=}\frac{1}{2} \Leftrightarrow S'{=}\frac{3}{2}$ triples
- $T{=}\frac{3}{2}$ triples contribute $<$0.5% of $V^{2\pi}$, $<$2% of V^{X}, but up to 10% of V^{U}
- $V^{2\pi C}$/ $V^{2\pi A}$= 0.60±0.01 in all cases
- $V^{2\pi S}$/ $V^{2\pi P}$= 0.042±0.003 in all cases
- V^{X}/ V^{U}= 0.95±0.02 in all cases
- All components scale the same for T=0,1,2 nuclei
- No combination will solve the Deficit Problem

	^{4}He	^{6}He	^{8}He	^{8}Be
⟨AV18+UIX⟩ − Expt	−0.1	2.4	3.7	1.3
⟨UIX⟩	−6.4	−6.9	−8.0	−13.6

ALTERNATIVES

- Illinois V_{ijk} models fix light nuclei energies by adding three-pion ring term, but it is too attractive in neutron matter
- Alternate V_{ijk}^{LS} and $V_{ijk}^{LS\tau}$ terms are being investigated
- A_y Problem in Nd scattering remains an outstanding issue