QMC Calculations of Light Nuclei
– More Than Just Energies –

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WORK WITH

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WORK NOT POSSIBLE WITHOUT EXTENSIVE COMPUTER RESOURCES

Argonne Laboratory Computing Resource Center (Fusion)

Argonne Math. & Comp. Science Division (SiCortex)

Argonne Leadership Computing Facility (BlueGene/P)

Argonne NATIONAL LABORATORY

Physics Division

Work supported by US DOE Office of Nuclear Physics and UNEDF SciDAC

GOALS

Understand nuclei at the level of elementary interactions between individual nucleons, including

- Binding energies, excitation spectra, relative stability, widths
- Densities, electroweak transitions, cluster-cluster overlaps & spectroscopic factors
- Low-energy NA & AA' scattering, astrophysical reactions, asymptotic normalizations

REQUIREMENTS

- Two-nucleon potentials that accurately describe elastic NN scattering data
- Consistent three-nucleon potentials and electroweak current operators
- Precise methods for solving the many-nucleon Schrödinger equation

RESULTS

- Quantum Monte Carlo methods can evaluate realistic Hamiltonians accurate to $\sim$1–2%
- About 100 states calculated for $A \leq 12$ nuclei in good agreement with experiment
- ^{5}He = $n\alpha$ scattering and low-energy electroweak astrophysical reactions
- Applications to elastic & ineleastic e, π scattering, $(e, e'p)$, (d, p) reactions, etc.

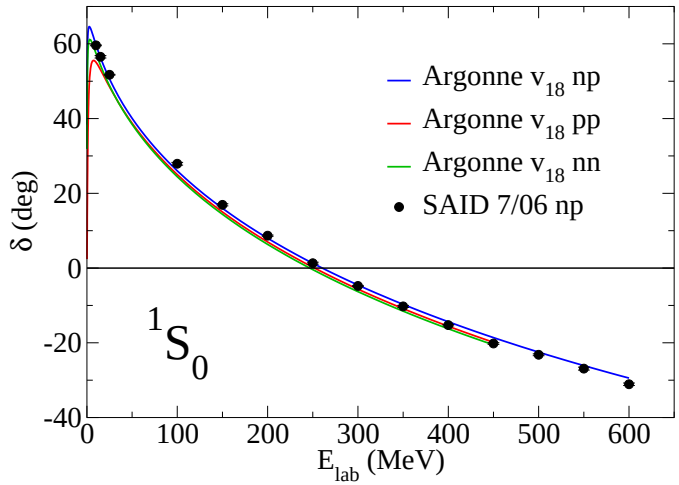
NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, m_n-m_p effects included

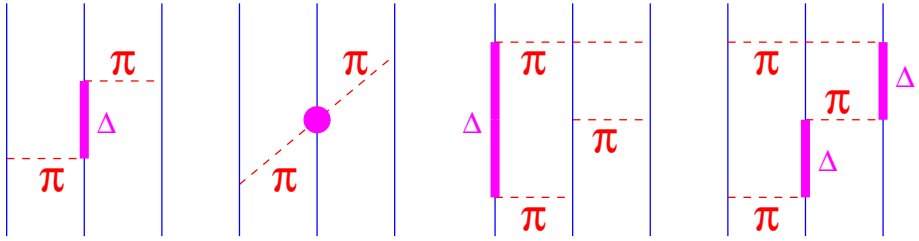
Argonne v$_{18}$: $v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{I} + v_{ij}^{S} = \sum v_p(r_{ij}) O_{ij}^{p}$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with χ^2/d.o.f.=1.1



Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^{R}$



- Urbana has standard 2π P-wave + short-range repulsion for matter saturation
- Illinois adds 2π S-wave + 3π rings to provide extra T=3/2 interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei

Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)
Pieper, AIP CP **1011**, 143 (2008)

QUANTUM MONTE CARLO

Variational Monte Carlo (VMC): construct Ψ_V that

- Are fully antisymmetric and translationally invariant
- Have cluster structure and correct asymptotic form
- Contain non-commuting 2- & 3-body operator correlations from v_{ij} & V_{ijk}
- Are orthogonal for multiple J^π states
- Minimize $E_V = \langle \Psi_V | H | \Psi_V \rangle \geq E$

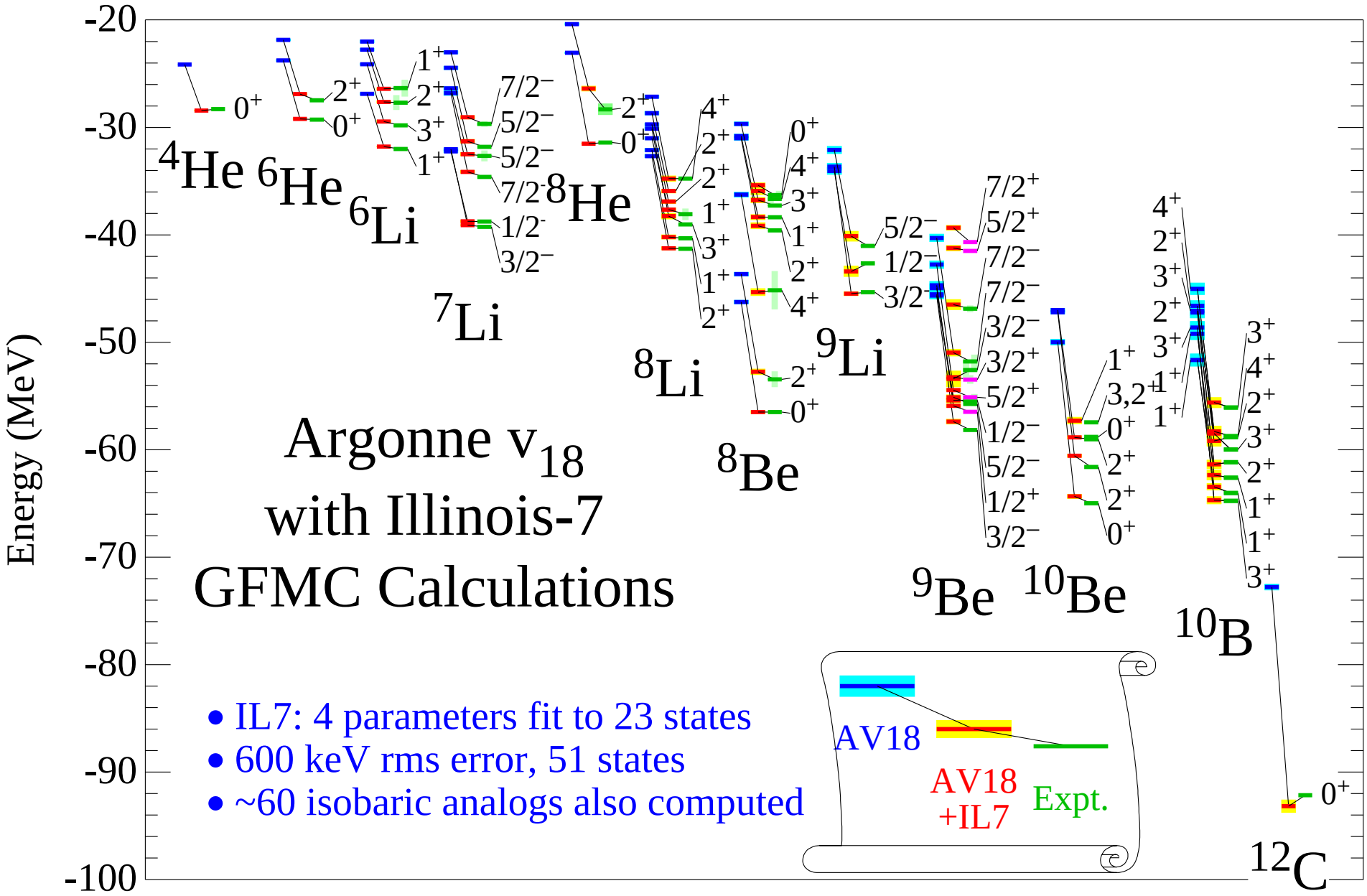
These are $\sim 2^A \binom{A}{Z}$ component spin-isospin vectors in $3A$ dimensions

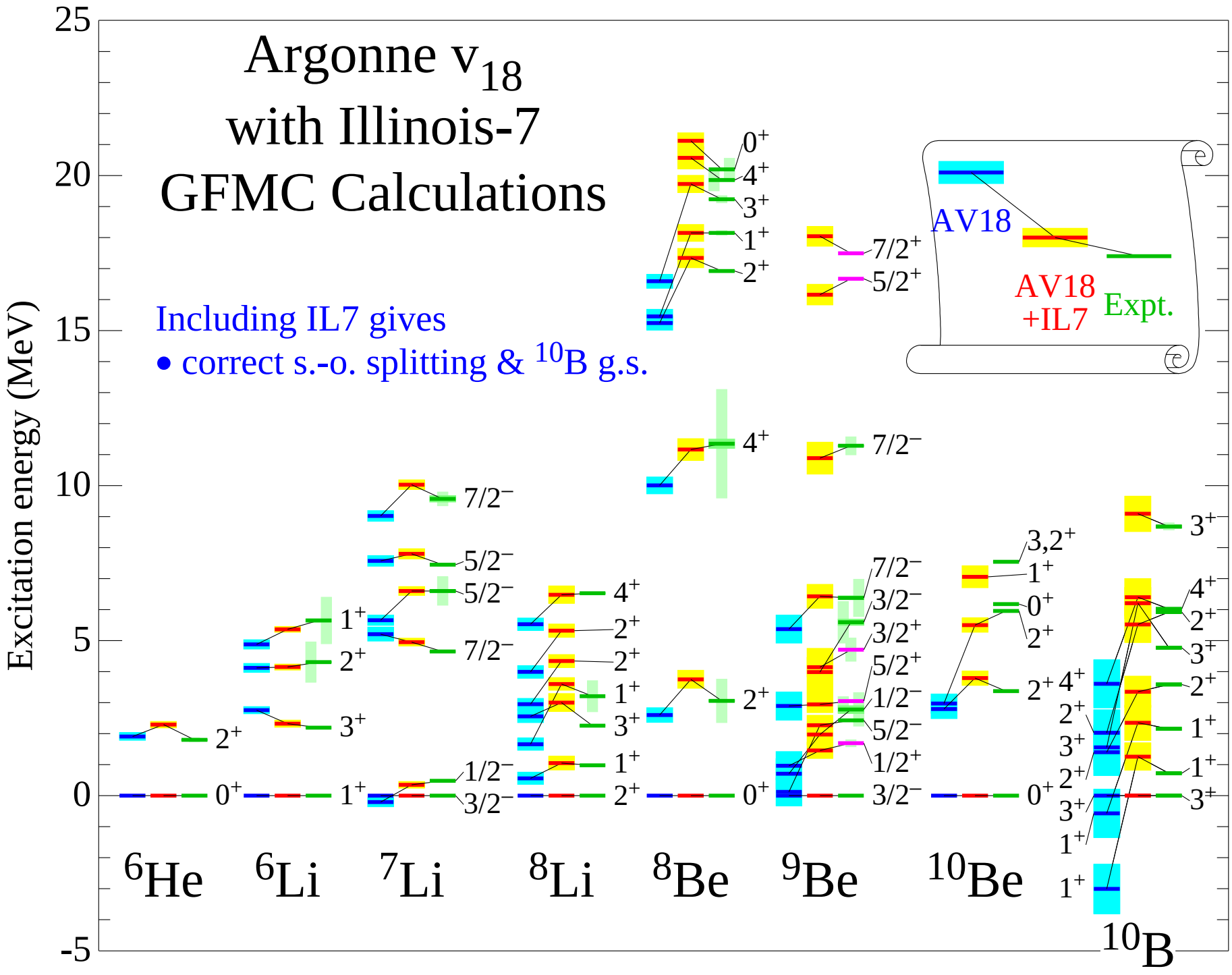
Green's function Monte Carlo (GFMC): project out the exact eigenfunction

- $\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \Psi_n \Rightarrow \Psi_0$ at large τ
- Propagation done stochastically in small time slices $\Delta\tau$
- Exact $\langle H \rangle$ for local potentials; mixed estimates $\langle \Psi_V | O | \Psi(\tau) \rangle$ for other operators
- Constrained-path propagation controls fermion sign problem for $A \geq 5$
- Multiple excited states for same J^π stay orthogonal

Many tests demonstrate 1–2% accuracy for realistic $\langle H \rangle$

Pudliner, Pandharipande, Carlson, Pieper, & Wiringa, PRC **56**, 1720 (1997)
Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)
Pieper, Varga, & Wiringa, PRC **66**, 044310 (2002)
Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)


Energy (MeV)
-20
-30
-40
-50
-60
-70
-80
-90
-100
^{4}He
^{6}He
^{6}Li
^{7}Li
^{8}He
^{8}Li
^{8}Be
^{9}Li
^{9}Be
^{10}Be
^{10}B
^{12}C
Argonne v$_{18}$
with Illinois-7
GFMC Calculations
• IL7: 4 parameters fit to 23 states
• 600 keV rms error, 51 states
• ~60 isobaric analogs also computed
AV18
AV18
+IL7
Expt.

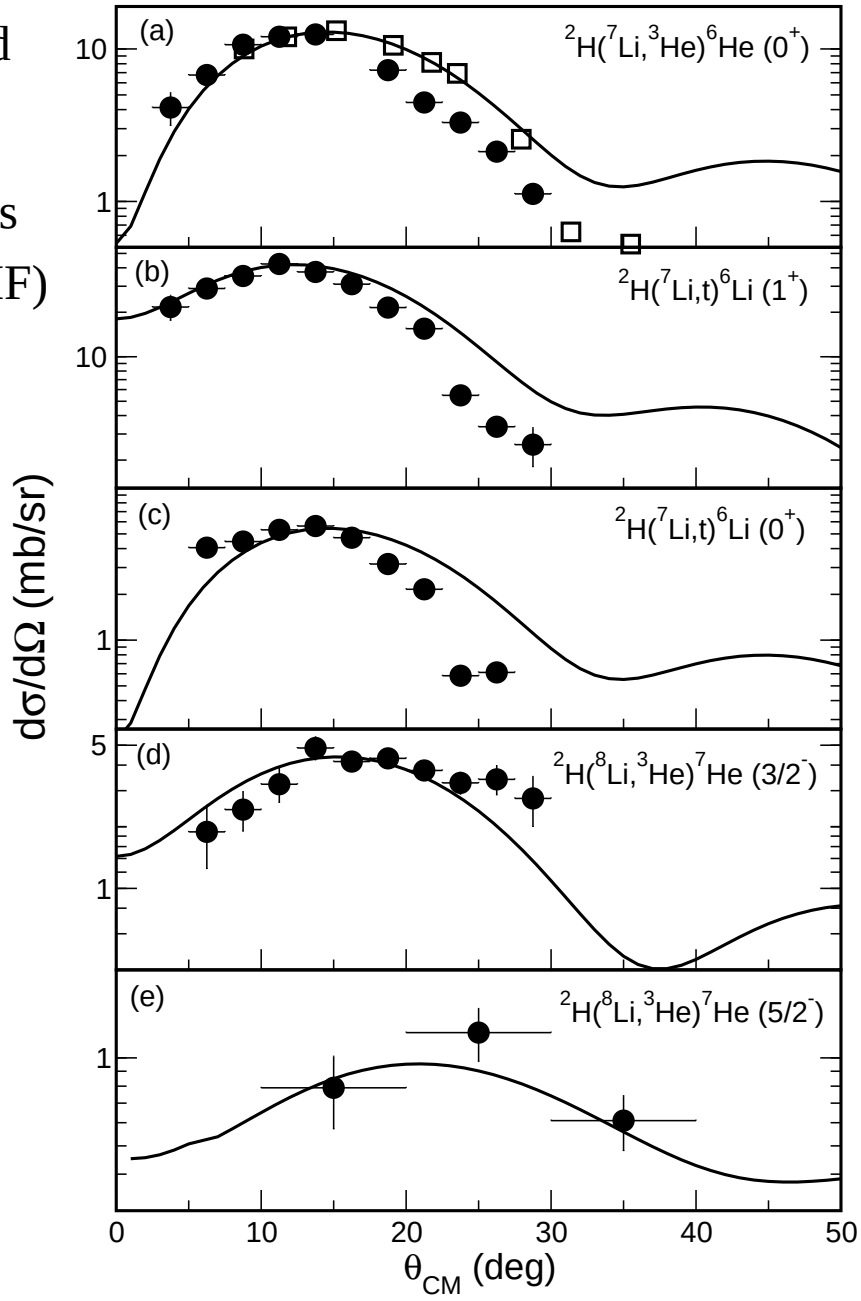

Argonne v18
with Illinois-7
GFMC Calculations
Including IL7 gives
• correct s.-o. splitting & 10B g.s.
AV18
AV18
+IL7
Expt.
Excitation energy (MeV)
25
20
15
10
5
0
-5
6He
6Li
7Li
8Li
8Be
9Be
10Be
10B

Applications to Light-ion Reactions

The availability of radioactive-ion beams has renewed interest in reactions like (d,p) in inverse kinematics

We have helped analyze a number of RIB experiments such as $d(^8\mathrm{Li},p)^9\mathrm{Li}$ (ATLAS) & $d(^9\mathrm{Li},t)^8\mathrm{Li}$ (TRIUMF)

- PTOLEMY DWBA calculations for transfer
- (d,p) vertex from AV18
- (d,t), $(^8\mathrm{Li},^9\mathrm{Li})$, etc. vertices computed as A-body overlaps using VMC $\langle \Psi_V(A\text{-}1)|a|\Psi_V(A)\rangle$
- Norm is spectroscopic factor
- Absolute prediction for $d\sigma/d\Omega$
- Good predictions of n-knockout from $^{10}\mathrm{Be}$ and $^{10}\mathrm{C}$ (NSCL)



Macfarlane & Pieper, PTOLEMY, ANL-76-11, Rev. 1 (1978)
Wuosmaa *et al.*, PRL **94**, 082502 (2005) + ...
Kanungo *et al.*, PLB **660**, 26 (2008)
Grinyer *et al.*, PRL **106**, 162502 (2011)

ONE-NUCLEON OVERLAPS IN VMC/GFMC

For antisymmetric and translationally invariant parent $\Psi_A(\alpha)$ and daughter $\Psi_{A-1}(\gamma)$ wave functions, with $\alpha \equiv [J^\pi_A, T_A, T_{z_A}]$, $\gamma \equiv [J^\pi_{A-1}, T_{A-1}, T_{z_{A-1}}]$, and single-nucleon quantum numbers $\nu \equiv [l, s, j, t, t_z]$, the translationally invariant overlap function is:

$$R(\alpha, \gamma, \nu; r) = \sqrt{A} \left\langle \left[\Psi_{A-1}(\gamma) \otimes \mathcal{Y}(\nu)(\hat{r}')\right]_{J_A, T_A} \left| \frac{\delta(r - r')}{r^2} \right| \Psi_A(\alpha) \right\rangle$$

where $\mathcal{Y}(\nu)(\hat{r}') = \left[Y_l(\hat{r}') \otimes \chi_s\right]_j \chi_t$ and $|\Psi_{A-1}(\gamma)|^2 = 1$, $|\Psi_A(\alpha)|^2 = 1$.

The corresponding spectroscopic factor is the norm of the overlap:

$$S(\alpha, \gamma, \nu) = \int |R(\alpha, \gamma, \nu; r)|^2 r^2 dr$$

Overlap functions R satisfy a one-body Schrödinger equation with appropriate source terms. Asymptotically, at $r \to \infty$, these source terms contain core-valence Coulomb interaction at most, and hence for parent states below core-valence separation thresholds:

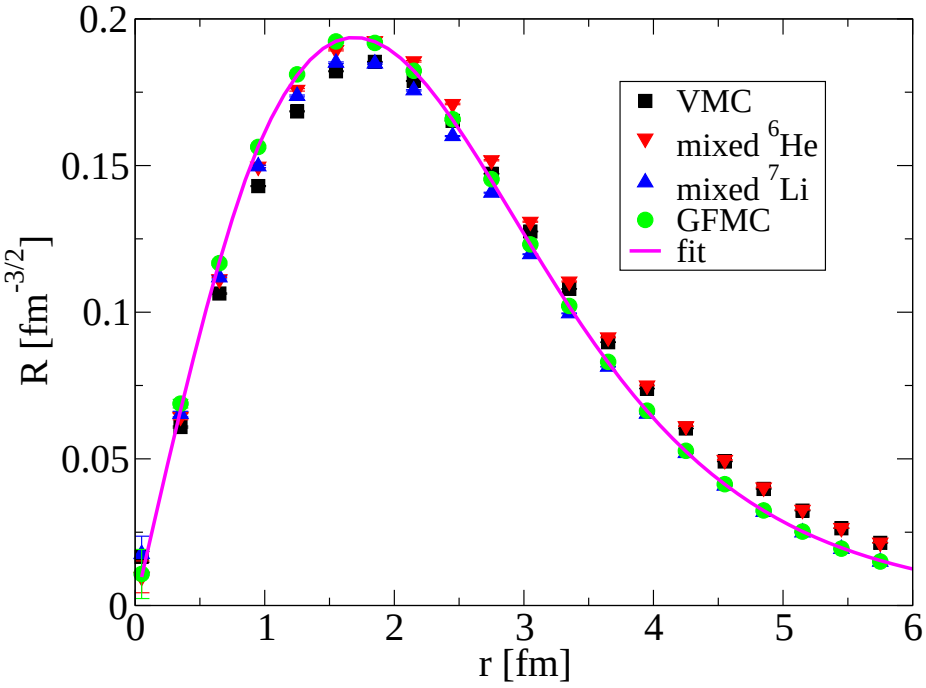
$$R(\alpha, \gamma, \nu; r) \xrightarrow{r \to \infty} C(\alpha, \gamma, \nu) \frac{W_{-\eta, l+1/2}(2kr)}{r},$$

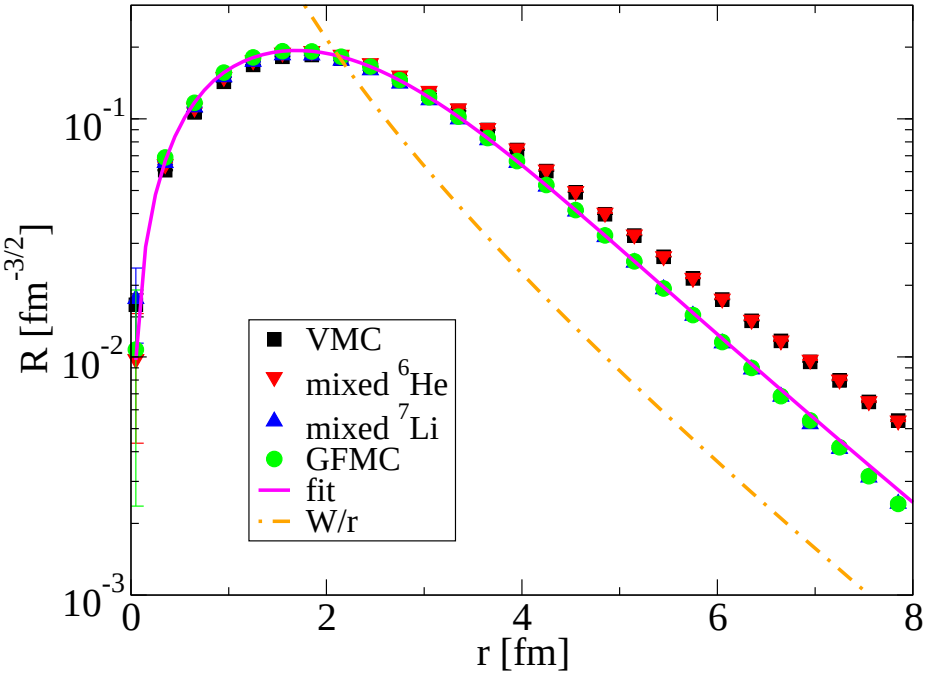
where $W_{-\eta, l+1/2}(2kr)$ is a Whitakker function with $k = \sqrt{2\mu B}/\hbar$, B is the separation energy, and $C(\alpha, \gamma, \nu)$ is the asymptotic normalization coefficient or ANC.

GFMC evaluation of R is by extrapolation requiring two mixed estimates minus the VMC result:

$$R(\alpha,\gamma,\nu;r;\tau) \approx \langle R(\alpha,\gamma,\nu;r;\tau)\rangle_{M_A} + \langle R(\alpha,\gamma,\nu;r;\tau)\rangle_{M_{A-1}} - \langle R(\alpha,\gamma,\nu;r)\rangle_V ,$$

where M_A denotes a mixed estimate where parent $\Psi_A(\alpha;\tau)$ has been propagated in GFMC and M_{A-1} is a mixed estimate where daughter $\Psi_{A-1}(\gamma;\tau)$ has been propagated.





Imaginary time evolution of overlaps in the $p_{3/2}$ channel of the overlap $\langle {}^6\mathrm{He} + p | {}^7\mathrm{Li}\rangle$

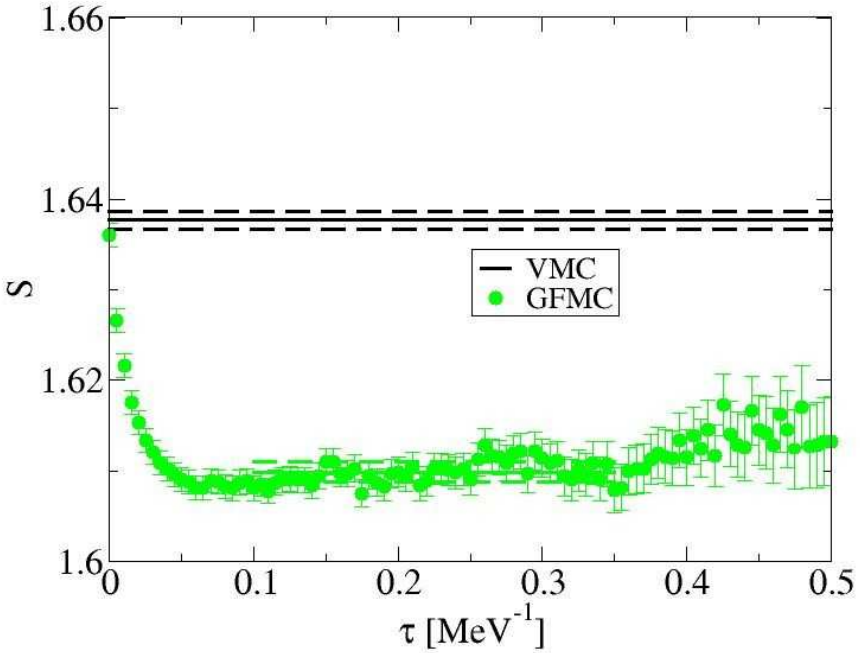
A convenient parametrization for input to PTOLEMY or other direct reaction code is provided by fitting a single-particle potential to reproduce the overlap R:

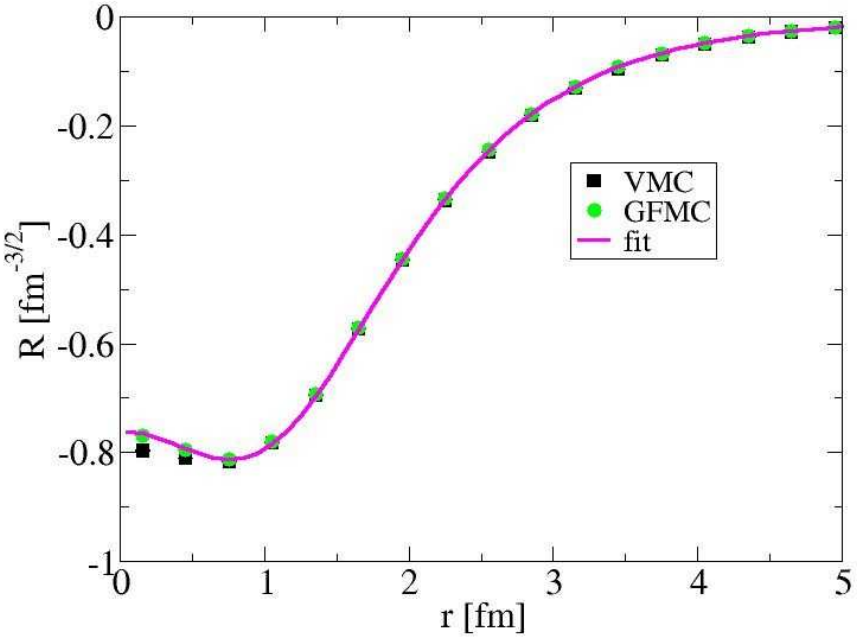
$$V(r) = V_{WS}\left[\frac{1}{1+\exp((r-R_{WS})/a_{WS})} - \beta\exp(-(r/\rho)^2)\right] + \left(4\vec{l}\cdot\vec{s}\right)\frac{V_{so}}{r}\frac{d}{dr}\left[\frac{1}{1+\exp((r-R_{so})/a_{so})}\right] + V_{Coul}$$

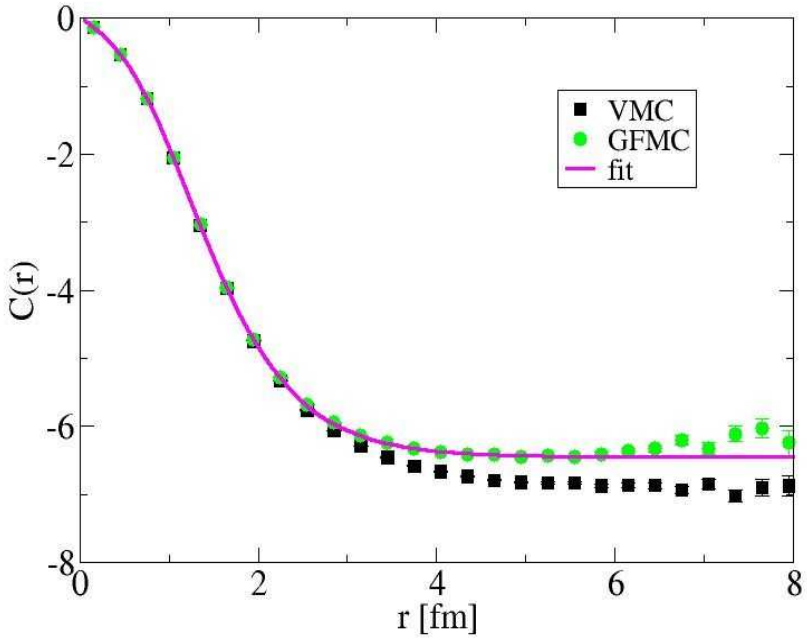
The potential parameters are adjusted to minimize χ^2 under the constraint that the overlap tail falls off with the correct core-valence separation energy B. This helps in extraction of the ANCs.

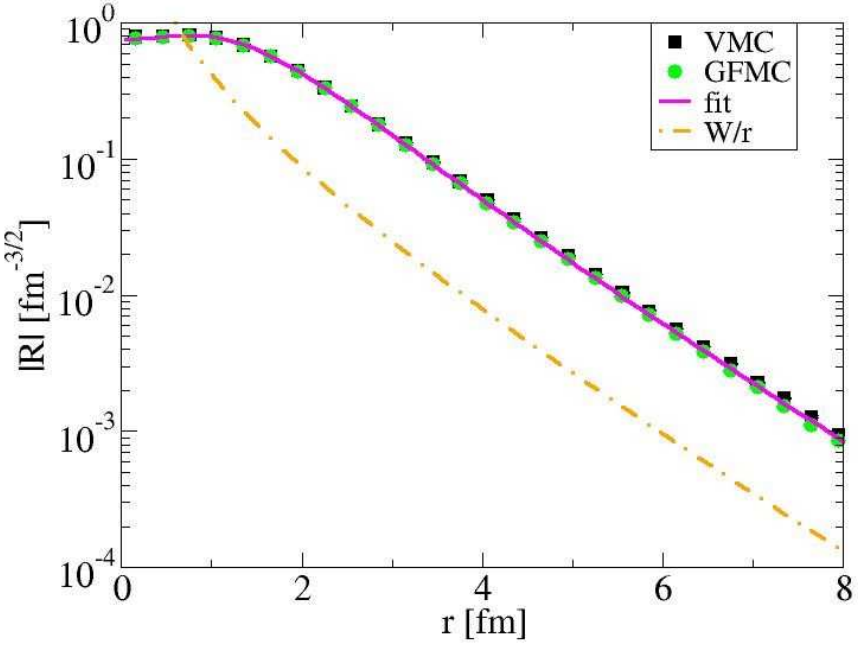
Brida, Pieper, & Wiringa, in preparation

Example: (^{3}H,^{4}He), $s_{1/2}$ overlap

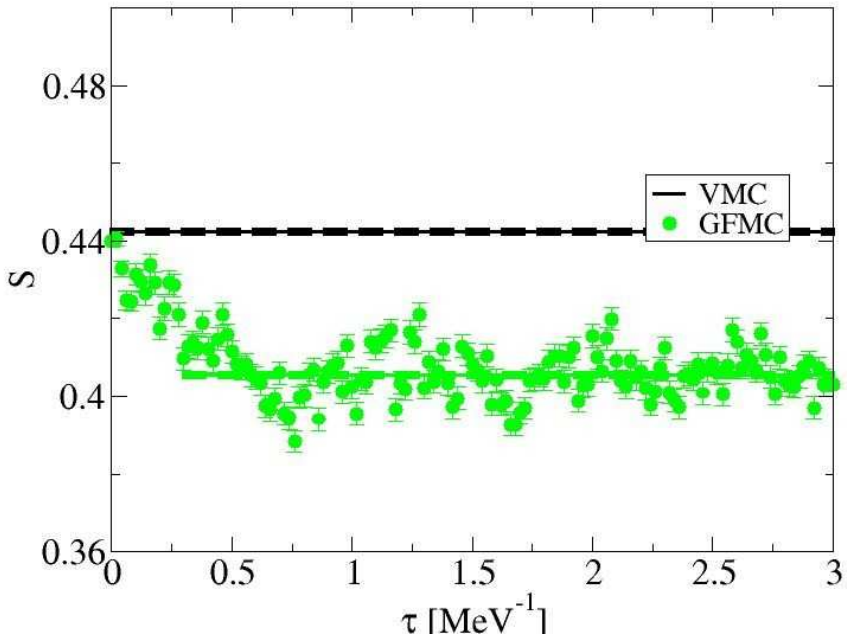


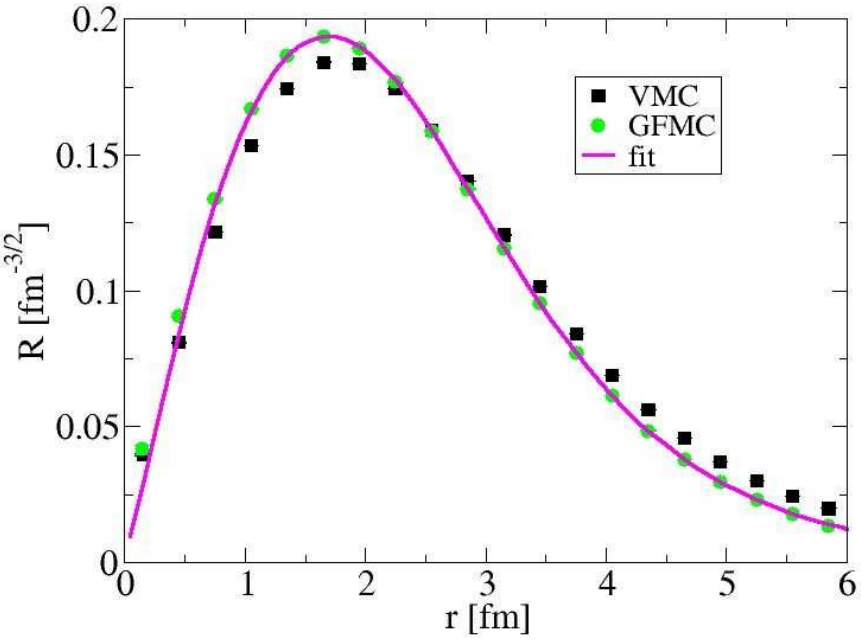


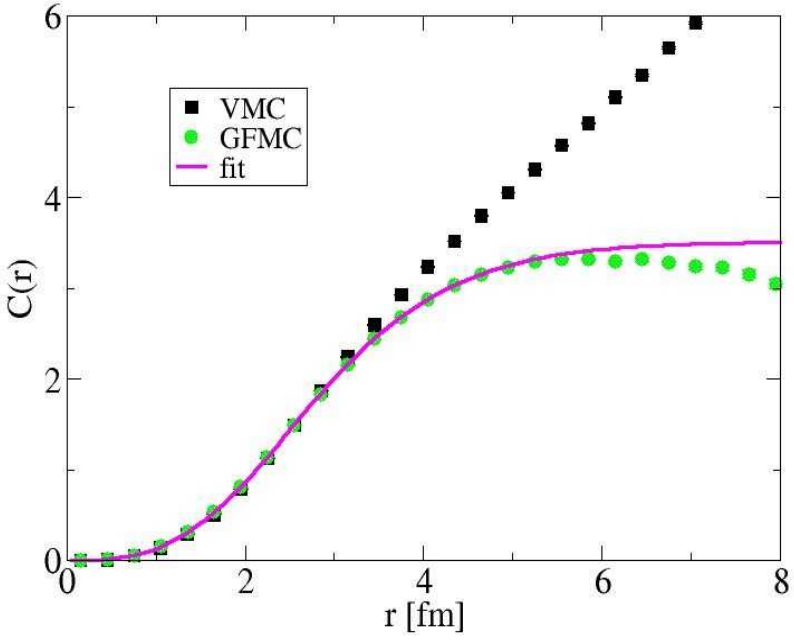


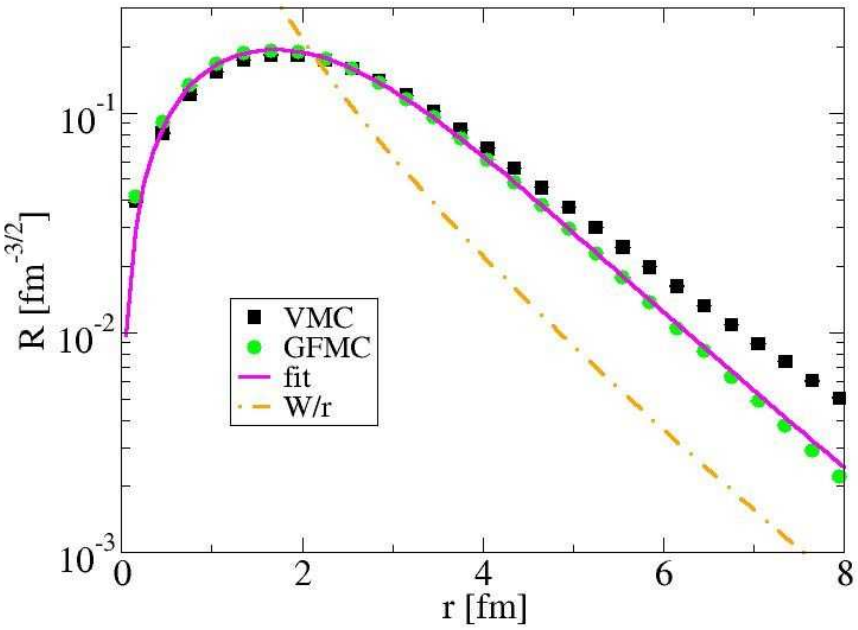


Example: (^{6}He,^{7}Li), $p_{3/2}$ overlap









GFMC separation energies

A	A-1	*B* [MeV]	
		GFMC	exp
^{3}H	^{2}H	6.24	6.26
^{3}He	^{2}H	5.49	5.49
^{4}He	^{3}H	19.96	19.81
	^{3}He	20.71	20.58
^{7}Li(g.s.)	^{6}He(g.s)	9.88	9.98
	^{6}Li(g.s)	7.15	7.25
	^{6}Li(3+)	9.49	9.44
	^{6}Li(0+)	10.65	10.81
^{7}Li(1/2-)	^{6}Li(g.s)	6.95	6.77
^{7}Be(g.s.)	^{6}Li(g.s)	5.69	5.61
	^{6}Li(3+)	8.03	7.79
	^{6}Li(0+)	9.18	9.17

- total energies are within 1% around exp. values
- sep. energies are within 3% around exp. values
- use exp. sep. energies for ANCs accounting for uncertainties

s-shell: spectroscopic factors

A	A-1	channel	S	
			GFMC	exp
^{3}H	^{2}H	$s_{1/2}$	1.30	
		$d_{3/2}$	0.0224	
^{3}He	^{2}H	$s_{1/2}$	1.31	
		$d_{3/2}$	0.0221	
^{4}He	^{3}H	$s_{1/2}$	1.61	1.4-1.6
	^{3}He	$s_{1/2}$	1.60	

GFMC systematic uncertainties are 2-3% or less

exp. values: Pandharipande et al, RMP 69 (1997) 981

s-shell: ANCs

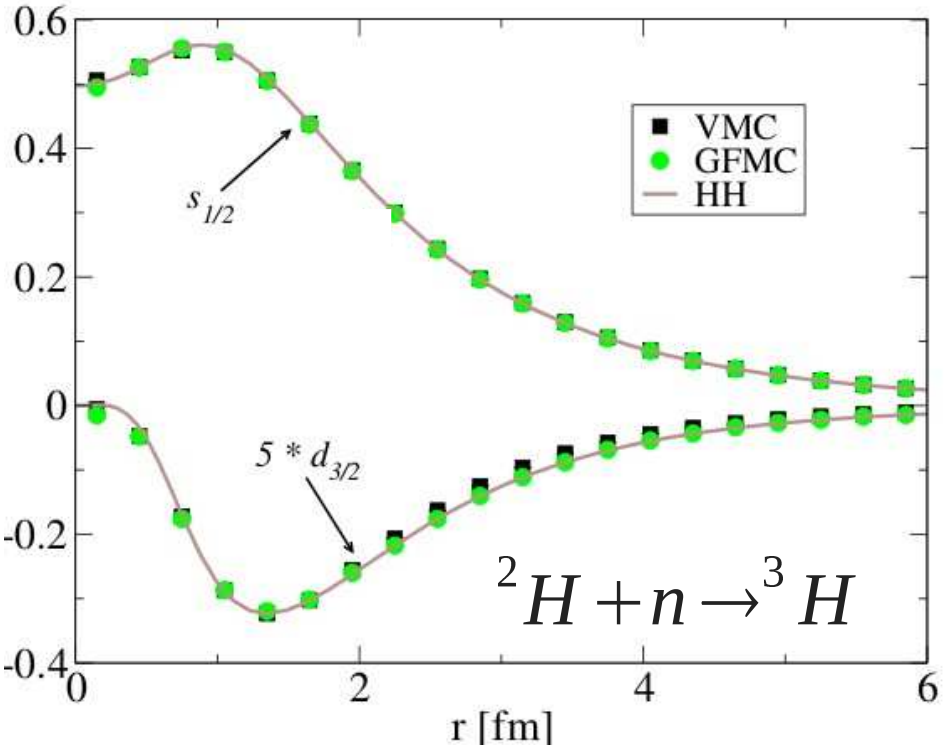
A	A-1	channel	*ANC* [fm$^{-1/2}$]	
			GFMC	exp
^{3}H	^{2}H	$s_{1/2}$	2.14	2.11(3), 2.07(2), 1.87(14)
		$d_{3/2}$	-0.0848	
		$d_{3/2}/s_{1/2}$	-0.0396	-0.0418(15)
^{3}He	^{2}H	$s_{1/2}$	2.10	2.10(16), 1.76(11)
		$d_{3/2}$	-0.0762	
		$d_{3/2}/s_{1/2}$	-0.0363	-0.0389(42)
^{4}He	^{3}H	$s_{1/2}$	6.45	7.36(19), 6.70(50), 5.44(15)
	^{3}He	$s_{1/2}$	6.45	6.77(51), 6.52(49)

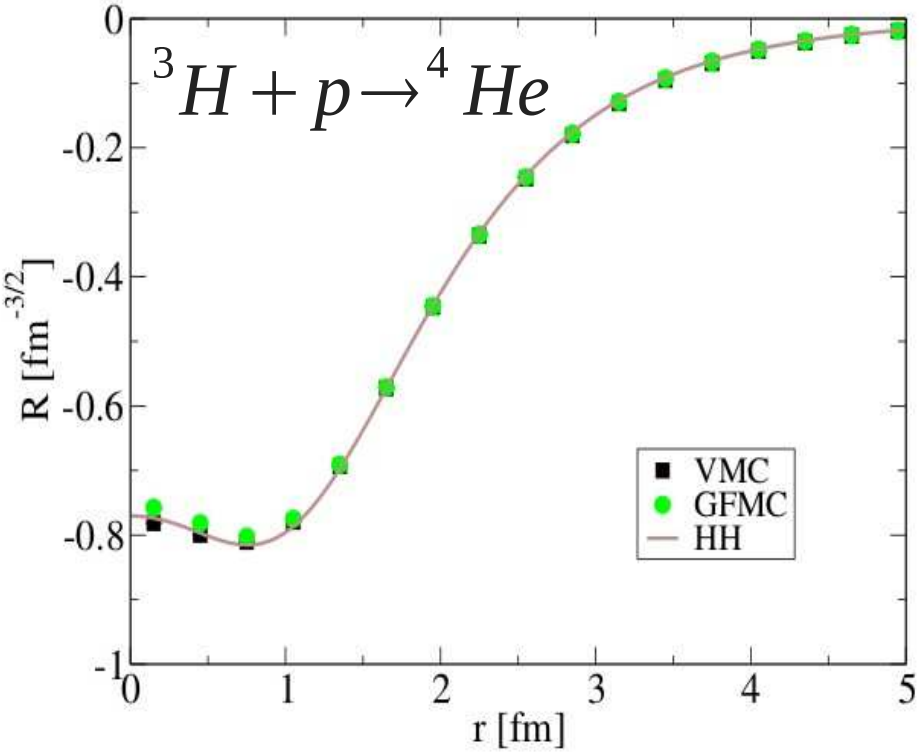
GFMC systematic uncertainties are 5% or less

exp. values: Timofeyuk (2010), Girard (1979), Locher (1978, Purcell (2010), Blimov (1985, Blokhintsev (1977)

s-shell: comparing with HH

potential: Av18+UIX





	channel	GFMC	HH
S	$s_{1/2}$	1.30	1.30
	$d_{3/2}$	0.0223	0.0225
ANC	$s_{1/2}$	2.14	~2.12
	$d_{3/2}$	-0.0842	

	channel	GFMC	HH
S	$s_{1/2}$	1.61	1.60
ANC	$s_{1/2}$	6.49	~6.48

p-shell: spectroscopic factors

A	A-1	channel	S	
			GFMC	exp
^{7}Li(g.s.)	^{6}He(g.s)	$p_{3/2}$	0.406	0.44(6), 0.42(4)
	^{6}Li(g.s)	$p_{1/2}$	0.230	
		$p_{3/2}$	0.438	
		$\sum S$	0.668	0.74(11), 073(5)
	^{6}Li(3+)	$p_{3/2}$	0.435	0.72(14), 0.58(13)
	^{6}Li(0+)	$p_{3/2}$	0.203	0.19(3)
^{7}Li(1/2-)	^{6}Li(g.s)	$p_{1/2}$	0.060	
		$p_{3/2}$	0.759	
		$\sum S$	0.819	1.15, 0.90(9)

SFs for overlaps with ^{7}Be(g.s) are close to ^{7}Li(g.s)
Shell-model SFs quenched by as much as 40%.

GFMC systematic uncertainties are 2-3% or less

exp. values: Wuosmaa (2008), Lapikas (1999), Ju (2010),
Li (1969), Towner (1969), Schiffer (1967)

p-shell: ANCs

A	A-1	channel	ANC [fm$^{-1/2}$]	
			GFMC	*exp*
^{7}Li(g.s.)	^{6}He(g.s)	$p_{3/2}$	3.52	2.48
	^{6}Li(g.s)	$p_{1/2}$	1.73	
		$p_{3/2}$	2.29	
		$\sqrt{\sum ANC^2}$	2.87	1.26-2.82
	^{6}Li(3+)	$p_{3/2}$	3.50	2.06-3.00
	^{6}Li(0+)	$p_{3/2}$	2.39	1.71-2.62
^{7}Li(1/2-)	^{6}Li(g.s)	$p_{1/2}$	0.57	
		$p_{3/2}$	2.85	
		$\sqrt{\sum ANC^2}$	2.91	

ANCs for overlaps with ^{7}Be(g.s) are close to ^{7}Li(g.s)

GFMC systematic uncertainties are 5% or less

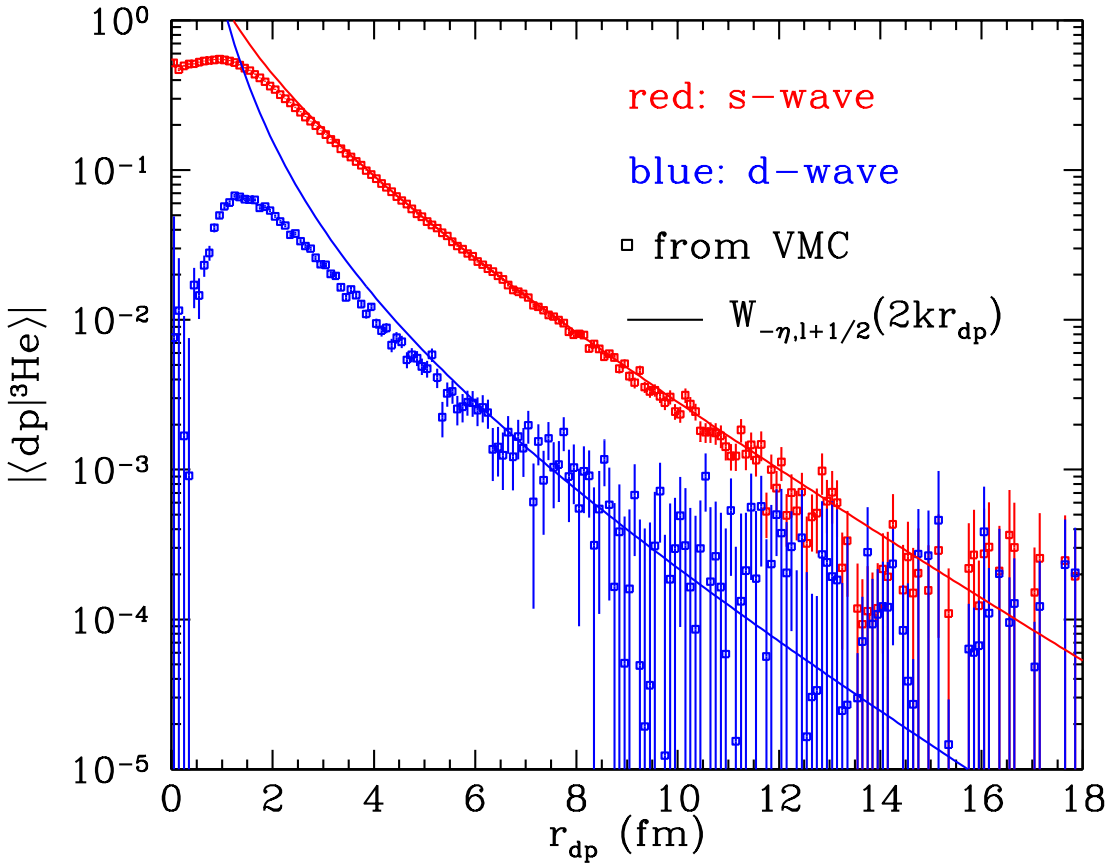
exp. values: Gulamov (1995), Bekbaev (1991)

ALTERNATE ROUTE TO ANCS

The VMC wave functions account fairly well for short-range correlations but may have poor asymptotic behavior, particularly in p-shell.

Fitting $C = rR(r)/W(2kr)$ is generally difficult because long-range shapes can be wrong, and Monte Carlo sampling of the tails is difficult.

In the s-shell, however, it works ok:



INTEGRAL REALTION FOR THE ANC

There is a better way than explicit overlaps, ideally suited to QMC methods

Consider the A-body wave function Ψ_A and its overlap with Ψ_{A-1} plus a final proton with separation energy B.

Write the Schrödinger equation as

$$(H - E)\,\Psi_A = 0$$

and expand H and E into parts internal to Ψ_{A-1} and parts involving the last particle

$$(H_{\rm int} + T_{\rm rel} + U_{\rm rel} + V_C - V_C - E_{\rm int} + B)\,\Psi_A = 0$$

Then

$$\begin{aligned} \Psi_A \quad = \quad & -\left[T_{\rm rel} + V_C + B\right]^{-1} (U_{\rm rel} - V_C)\,\Psi_A \\ & -\left[T_{\rm rel} + V_C + B\right]^{-1} (H_{\rm int} - E_{\rm int})\,\Psi_A \end{aligned}$$

The second line is zero since $(H_{\rm int} - E_{\rm int})\Psi_{A-1} = 0$

Rewriting the Green's function $[T_{\rm rel} + V_C + B]^{-1}$ in terms of special functions turns

$$\Psi_A = -\left[T_{\rm rel} + V_C + B\right]^{-1} \left(U_{\rm rel} - V_C\right) \Psi_A$$

into

$$\Psi^\dagger_{A-1}\chi^\dagger Y^\dagger_{lm}\Psi_A = \frac{2\mu}{k\hbar^2 w}\mathcal{A}\int \frac{M_{-\eta,l+\frac{1}{2}}(2kr_<)W_{-\eta,l+\frac{1}{2}}(2kr_>)}{r_<\, r_>} \times\Psi^\dagger_{A-1}\chi^\dagger Y^\dagger_{lm}(\hat{\mathbf{r}}_{cc})\left(U_{\rm rel} - V_C\right)\Psi_A d\mathbf{R}$$

so at large radius

$$C_{lj} = \frac{2\mu}{k\hbar^2 w}\mathcal{A}\int \frac{M_{-\eta,l+\frac{1}{2}}(2kr_{cc})}{r_{cc}}\Psi^\dagger_{A-1}\chi^\dagger Y^\dagger_{lm}(\hat{\mathbf{r}}_{cc})\left(U_{\rm rel} - V_C\right)\Psi_A d\mathbf{R}$$

$M_{-\eta,l+\frac{1}{2}}(2kr)$ is the "other" Whittaker function, irregular at $r \to \infty$. Here U_{rel} is

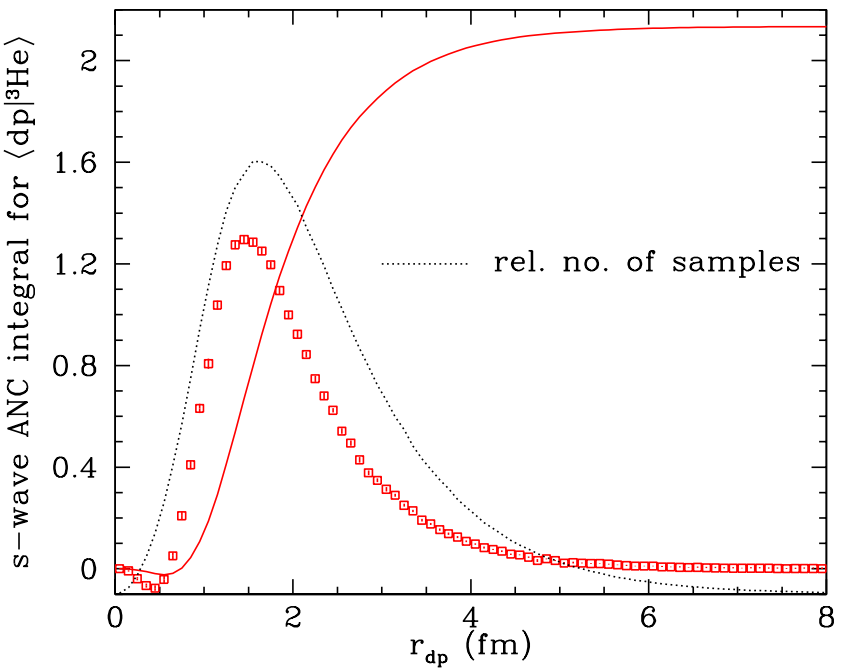
$$U_{\rm rel} = \sum_{i<A} v_{iA} + \sum_{i<j<A} V_{ijA}$$

and at large separation of the last nucleon, $U_{\rm rel} \to V_C$, so $(U_{\rm rel} - V_C) \to 0$.
This makes the integrand terminate at ~ 7 fm for AV18+UIX.

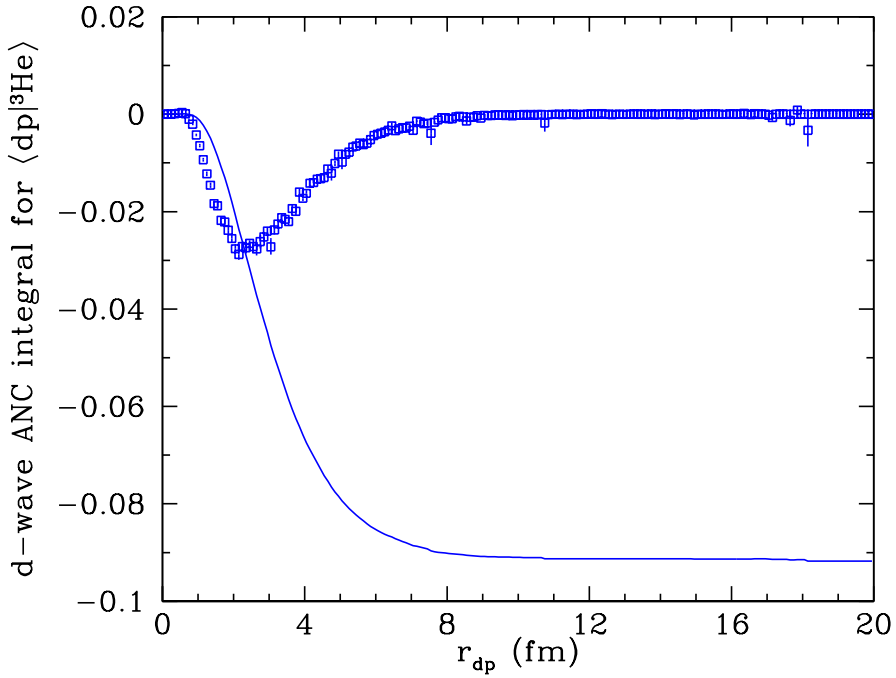
This is great for QMC methods, which are good at integration over the wave function interior, but bad in the tails.

ANC: $^3\mathrm{He} \to dp$

s-wave ANC integrand & integral



d-wave ANC integrand & integral



Points are Monte-Carlo sampled integrand; solid curves are cumulative integrals

For $^3\mathrm{He} \to dp$, we get $C_s^{dp} = 2.131(8)\ \mathrm{fm}^{-1/2}$, $C_d^{dp} = -0.0927(10)\ \mathrm{fm}^{-1/2}$

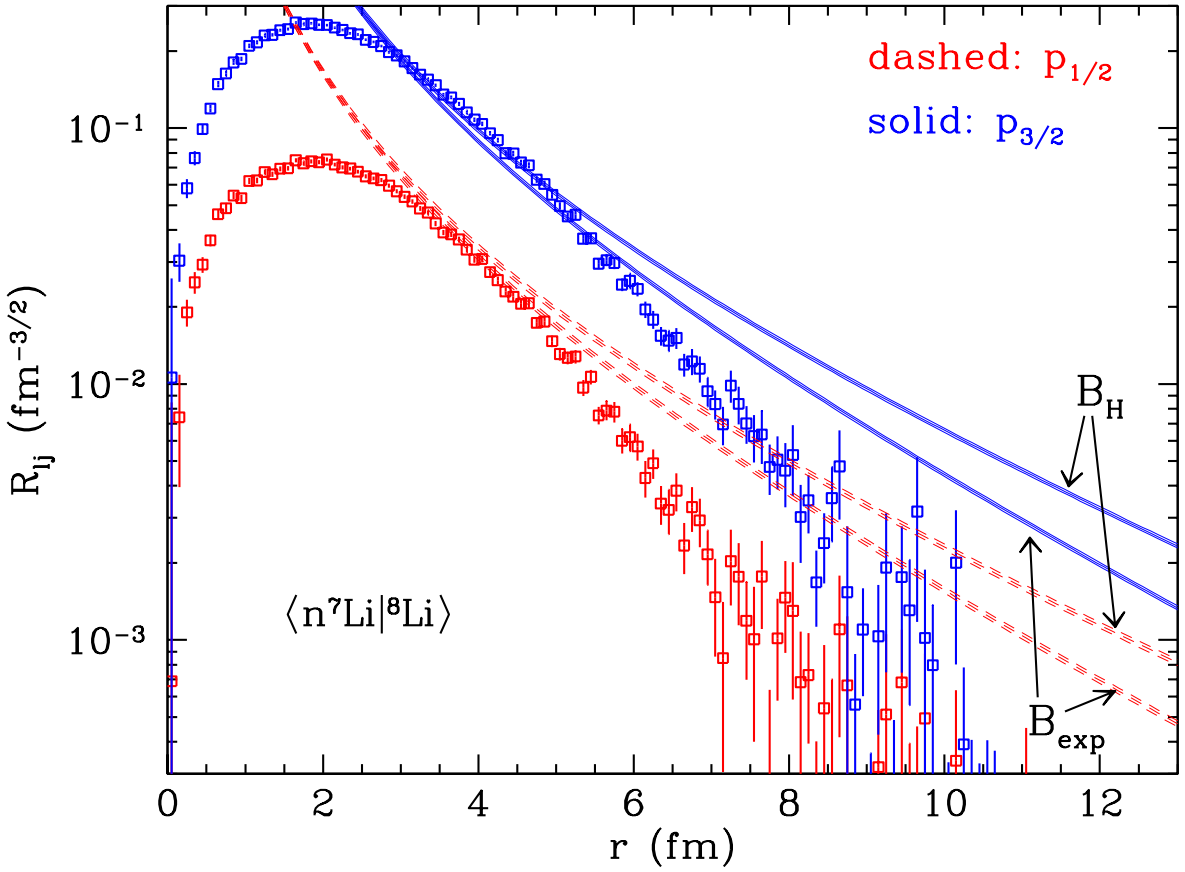
Corresponding HH values are 2.16 & -0.0865

Corresponding GFMC values are 2.10 & -0.0794

C_d^{dp} converges just where sampling gets sparse in the explicit overlap

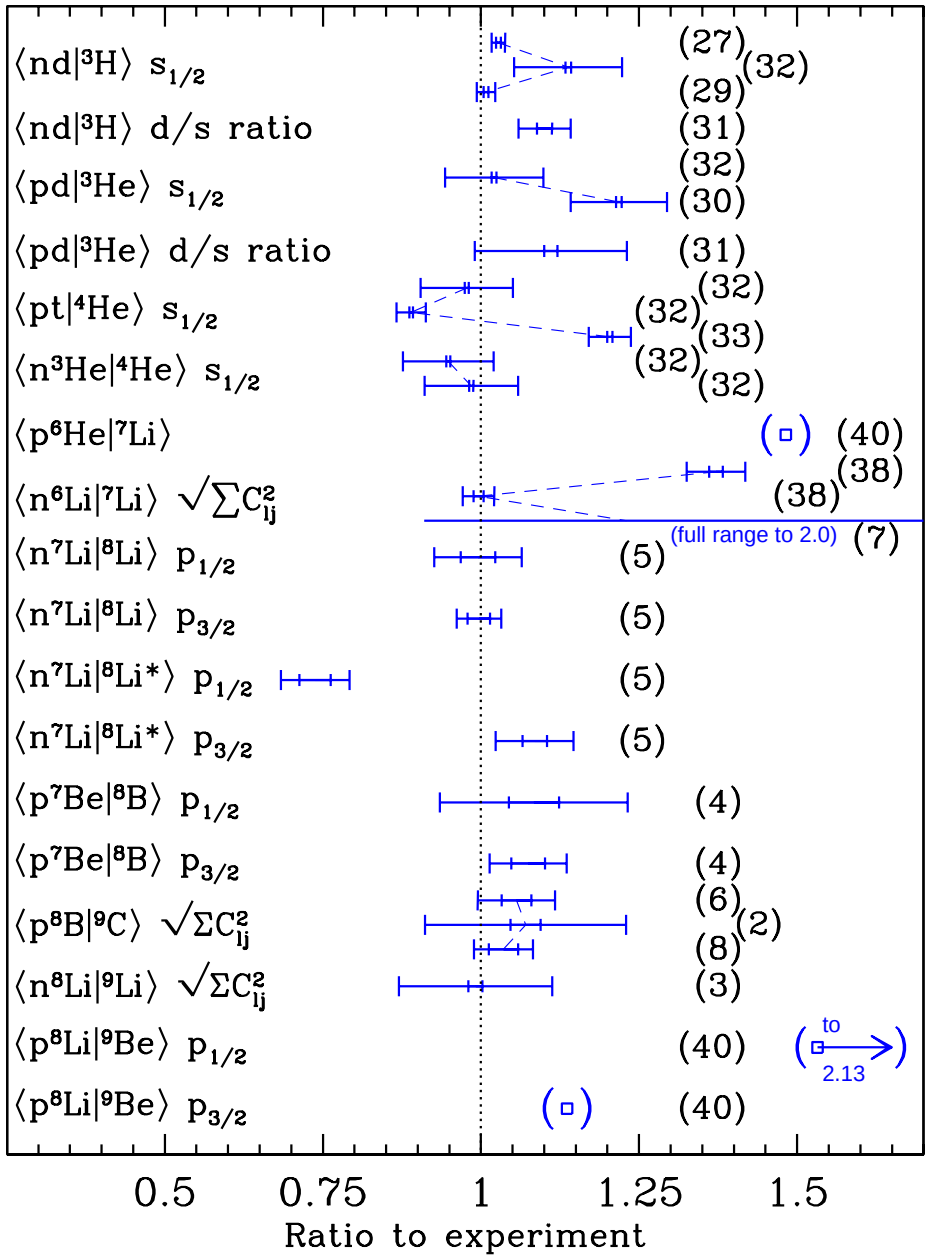
ANC: $^8\mathrm{Li} \rightarrow ^7\mathrm{Li} + n$

Here is a case where fitting to VMC samples is impossible, but the integral method using the laboratory separation energy works beautifully:



ANC (fm^{-1})	VMC: AV18+UIX binding	VMC: Lab binding	Experiment
$C^2_{p\,1/2}$	0.029(2)	0.048(3)	0.048(6)
$C^2_{p\,3/2}$	0.237(9)	0.382(14)	0.384(38)

RESULTS FOR ONE-NUCLEON REMOVAL $3 \leq A \leq 9$



- Small error bars are VMC statistics
- Large ones are "experimental"
- Sensitivity to wave function construction seems weak but hard to quantify
- $A \leq 4$ clearly dominated by systematics, also old
- With a few exceptions, these are the first *ab initio* ANCs in $A > 4$

Nollett and Wiringa, PRC **83**, 041001(R) (2011)

QMC FOR CONTINUUM STATES

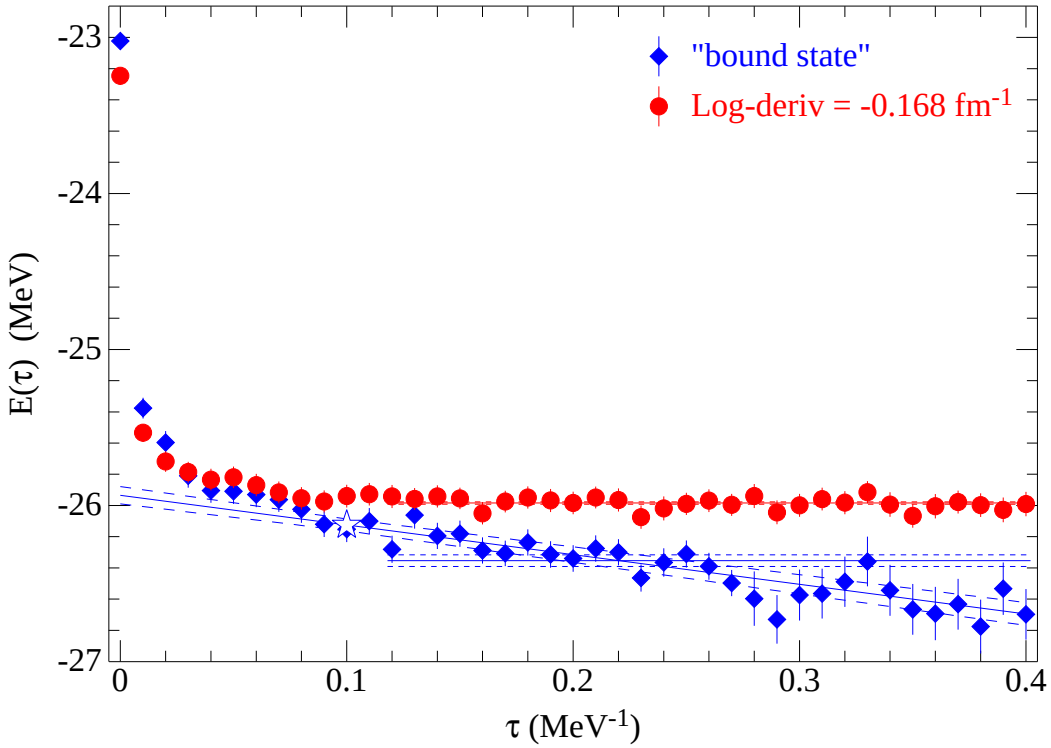
We generally treat nuclei as particle-stable systems – probably good for energies of narrow resonances, but no prediction for widths. For wide states we need full scattering solution.

METHOD

- Pick a logarithmic derivative, χ, at some large boundary radius ($R_B \approx 9$ fm)
- GFMC propagation, using method of images to preserve χ at R, finds $E(R_B, \chi)$
- Phase shift, $\delta(E)$, is function of R_B, χ, E
- Repeat for a number of χ until $\delta(E)$ is mapped out
- need E accurate to $\sim 1/3\%$

Example for $^5\mathrm{He}(\frac{1}{2}^-)$

- "Bound-state" boundary condition does not give stable energy; Decaying to n+^{4}He threshold
- Scattering boundary condition produces stable energy.



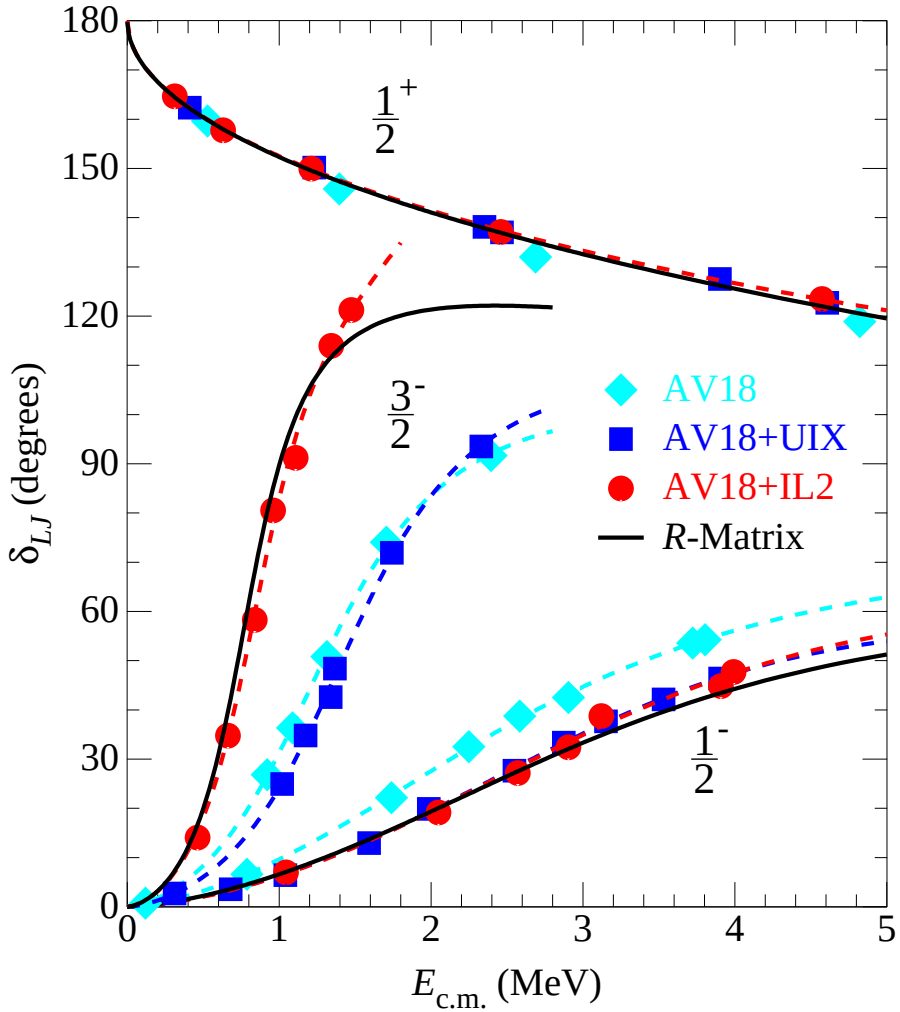
^{5}He AS n+^{4}He SCATTERING

Black curves: Hale phase shifts from R-matrix analysis up to $J = \frac{9}{2}$ of data

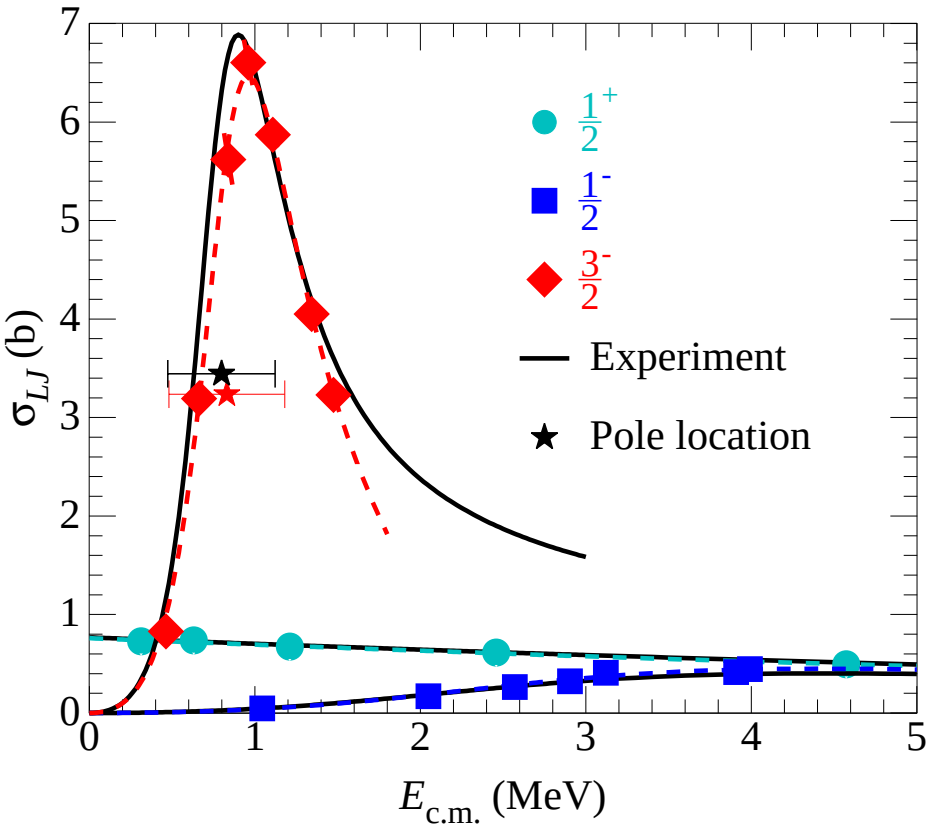
AV18 with no V_{ijk} underbinds ^{5}He($3/2^-$) & overbinds ^{5}He($1/2^-$)

AV18+UIX improves ^{5}He($1/2^-$) but still too small spin-orbit splitting

AV18+IL2 reproduces locations and widths of both P-wave resonances



AV18+IL2



Nollett, Pieper, Wiringa, Carlson, & Hale, PRL **99**, 022502 (2007)

WIDTHS AS ANCS

Mapping $E(\gamma) \rightarrow \delta(E)$ is laborious, requires many solutions, and sensitive to calculational precision

Reliable widths of narrow states will be difficult by this method

But widths are closely related to ANCs, so maybe there's a cheap way to estimate them

An unbound wave function at large radius looks like

$$\psi(r \rightarrow \infty) \propto F_l(kr)\cos\delta + G_l(kr)\sin\delta$$

so that at resonance ($\delta = 90^\circ$; as our pseudobound states should have)

$$\psi(r \rightarrow \infty) = C_{lj}\phi_1\phi_2 G_l(kr)$$

The flux per unit time through the surface is $|C_{lj}|^2 v = \frac{\hbar k}{\mu}|C_{lj}|^2$

From $\Gamma \simeq \hbar/\tau$, we get $\Gamma \simeq \frac{\hbar^2 k}{\mu}|C_{lj}|^2$

One could also consider Gamow's decaying complex-energy states and getthe same answer

WIDTHS AS ANCS

The relation

$$\psi(r \to \infty) = C_{lj}\phi_1\phi_2 G_l(\eta, kr)$$

for resonant states is mathematically almost the same as

$$\psi(r \to \infty) = C_{lj}\phi_1\phi_2 W_{-\eta, l+\frac{1}{2}}(2kr)$$

for bound states

You can get from one to the other by considering the bound k and η as $\pm i$ times their scattering-state counterparts

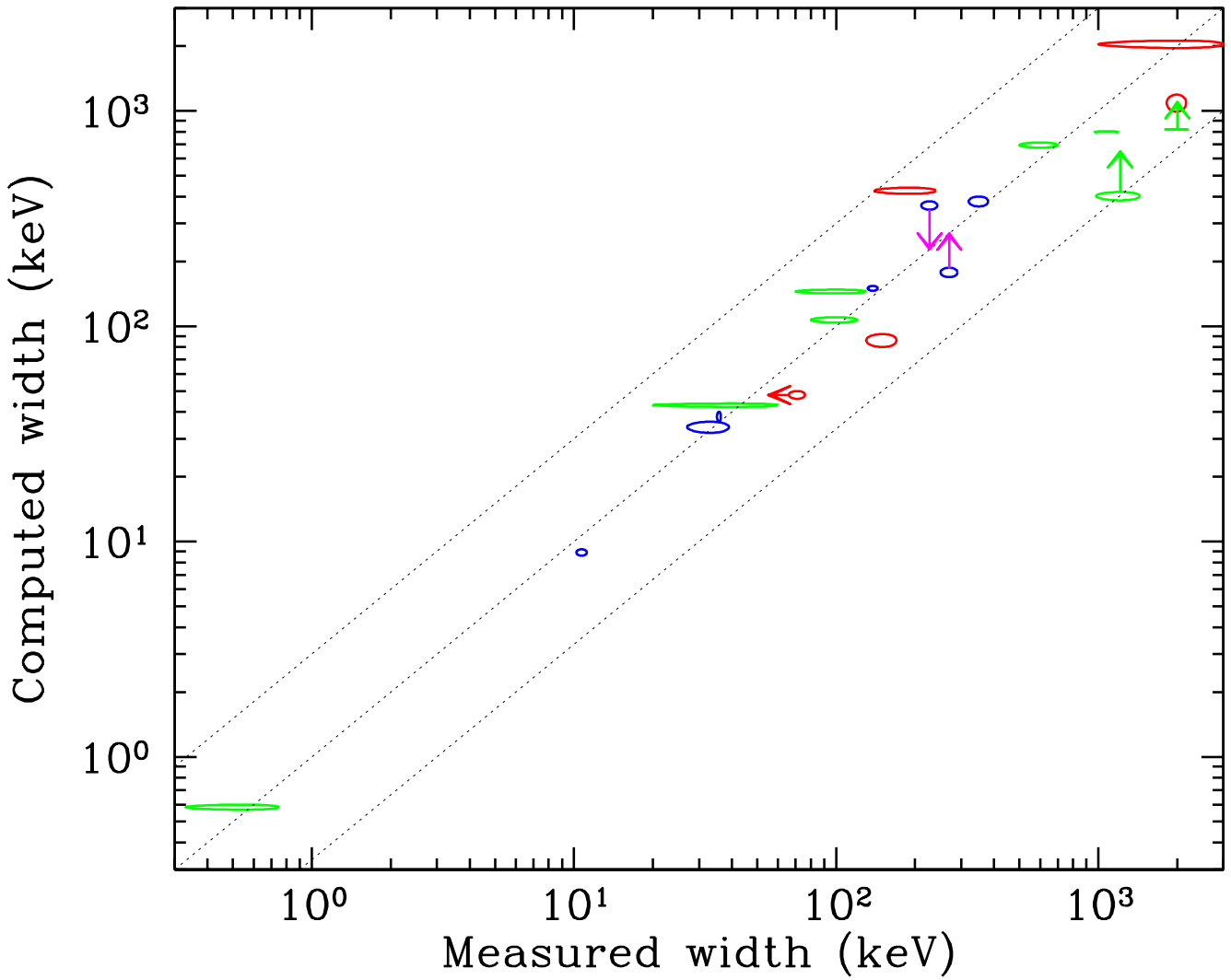
The integral method also applies to resonant states, except that now F_l appears in the integral instead of $M_{-\eta, l+\frac{1}{2}}$

This is used as a mathematical tool to get the asymptotics right in α and p decays (e.g. Esbensen & Davids (2000) deformed proton emitters)

TESTING THE INTEGRAL RELATION FOR Γ

The integral estimate should apply to states that are in some sense narrow

Here are low-lying states in $A \leq 9$ with width mainly/all in nucleon emission



A=7

A=8

A=9

Arrows: uncomputed channels or T mixing

Nollett & Wiringa, in preparation

Lots of widths come out close to experiment

Widths not close to experiment generally have some unaccounted-for width (e.g. α or 3-body channel) or isospin mixing, or are broad

Pseudobound ^{5}He states yield wildly unreasonable widths, probably because they're very broad

Width integral appears to be better than just using the Wigner limit and S_{lj}

FUTURE WORK

GFMC one-nucleon overlaps for $A \geq 8$ nuclei

Two-nucleon overlaps like $\langle \alpha nn | {}^6\mathrm{He} \rangle$ and $\langle \alpha d | {}^6\mathrm{Li} \rangle$ under development – first VMC calculations made

Generalize integral method for ANCs and widths to α and other cluster breakups

Utilize integral method with GFMC wave functions and IL7 potential

Integral method may provide way of extracting surface amplitudes from GFMC for coupled-channel scattering problems

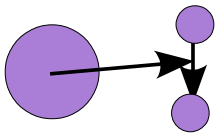
Still need to do full scattering calculations for wide states like ${}^8\mathrm{Be}(2^+, 4^+)$ but integral method will probably give best estimate of width of ${}^8\mathrm{Be}(0^+)$

Example: 2N overlaps for ^{6}He

2N overlaps in the past:

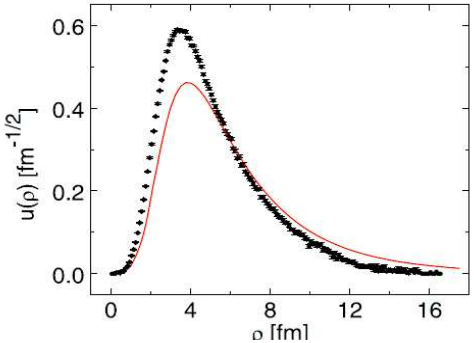
three-body calculations

- 3-body w.f.s are the 2N overlaps
- inert core, simple interactions, …



- microscopic calculations
  - I.Brida and F.M.Nunes, NPA 847(2010) 1

    phenomenological N-N forces
  - spec. factors greater by 30%-70% than in 3-body



Present:

VMC:

- fully ab-initio
- preliminary VMC results:

Future:

GFMC