Progress in Lattice QCD Calculations of Nucleon Structure

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- Institute of Physics, Taiwan:
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[arXiv:0907.4194 [hep-lat] (Phys. Rev. D81:034507, 2010)]
[arXiv:1001.3620 [hep-lat]]

Ab-initio Calculation of Nucleon Structure

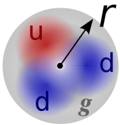
No satisfactory theoretical understanding of nucleon structure

Existing and coming experimental data:

- Proton and neutron size and charge distribution
- Partonic picture of nucleons
- Origin of the nucleon spin: AM of gluons and quarks

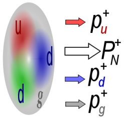
Radius

$\langle r_E^2 \rangle^{p-n} \approx 0.85\ \mathrm{fm}^2$



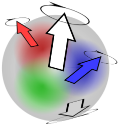
Momentum

$P_N^+ = p_q^+ + p_g^+$



Spin

$\frac{1}{2} = S_q + L_q + J_g$



- Next generation: JLab@12GeV, EIC, ...

1. **QCD on a Lattice**

2. **Lattice Results**
   - Vector form factors
   - Axial form factors
   - Quark Momentum and Angular Momentum

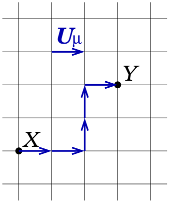
3. **Summary**

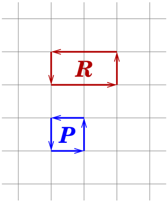
Solving QCD on a Lattice

- Euclidean QFT: $\begin{cases} x^0 \equiv t & \to -ix_4 \equiv -i\tau \\ p^0 \equiv E & \to ip_4 \\ \langle N(t)\bar{N}(0)\rangle & \to e^{-E\tau} \end{cases}$
- Use a lattice regulator

$$A_\mu(x) \to U_{x,\mu} = \mathcal{P}e^{-i\int_x^{x+\hat{\mu}} dx\cdot A}$$

$$S_g[A_\mu] \to (1 - \frac{1}{N_c}\mathrm{ReTr}U_P)$$
$$\sim (F_{\mu\nu})^2 + \mathcal{O}(a^2)$$



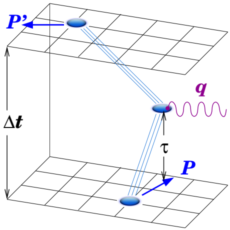


- Sample Prob$[U,\psi,\bar{\psi}] \sim e^{-S[U,\psi,\bar{\psi}]}$ to compute path integral

$$\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathcal{O}\ e^{-S[U,\psi,\bar{\psi}]} \to \frac{1}{N}\sum^{N} \mathcal{O}[U,\psi,\bar{\psi}]$$

- Tune α_S and m_q to reproduce physical observables

Nucleon Matrix Elements



Extract $\langle N(P')|\mathcal{O}|N(P)\rangle$
from 3-pt correlators:

$$\sum_{\vec{x},\vec{y}} e^{-i\vec{P}'\vec{x}+i\vec{q}\vec{y}} \langle N(\Delta t,\vec{x})\,\mathcal{O}(\tau,\vec{y})\,\bar{N}(0)\rangle$$

Signal / noise $\sim e^{-(M_N-\frac{3}{2}m_\pi)\cdot\Delta t}$

Excited states can lead to systematic bias in m.e. :

$$\bar{N}_{\text{lat}}|\Omega\rangle = |N\rangle + C|X\rangle, \quad \Delta M = M_X - M_N,$$

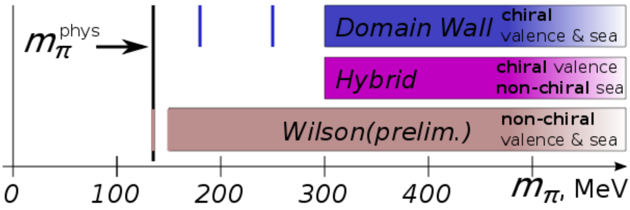
$$\langle N|\mathcal{O}|N\rangle_{\text{lat}} \cong \langle N|\mathcal{O}|N\rangle + |C|^2\cdot\langle X|\mathcal{O}|X\rangle\cdot e^{-\Delta M\cdot\Delta t}$$

Cost of QCD simulations

Solving QCD numerically is hard because

- cost $\sim \frac{1}{m_\pi}$ with light quarks / pions
- physical box size $L \gtrsim \frac{4}{m_\pi}$ to avoid finite volume effects
- lattice size $L_{\text{lat}} = \frac{L}{a}$ grows in the continuum limit $a \to 0$
- chiral symmetry is expensive to preserve in lattice regularization

$$\text{Cost at fixed phys. volume} \sim \left(\frac{L}{\text{fm}}\right)^5 \cdot \left(\frac{\text{fm}}{a}\right)^7 \cdot \left(\frac{\text{MeV}}{m_\pi}\right)$$

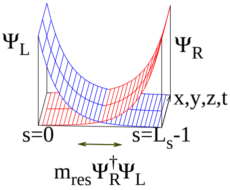


$2+1$ flavors,
$m_u = m_d \ll m_s$
Gauge ensembles:
[RBC/UKQCD, MILC, BMW collaborations]

Chiral Symmetry on a Lattice

Chiral symmetry is important for

- preserving original symmetries of QCD
- removing $\mathcal{O}(a)$ term from the Lagrangian
$$\mathcal{L}_{\text{lat}} = \mathcal{L}_{\text{QCD}} + \cancel{a\,\mathcal{L}^{(5)}} + a^2\,\mathcal{L}^{(6)} + \cdots$$
- simplifying quark operators renormalization



Domain Wall :
5D Fermions have chiral modes on a 4D "defect"

- spatially separate ψ_L and ψ_R in $4+1$ space
- additional dimension $L_5 = 16$ $(L_{space} = 32!)$
- residual breaking $\sim m_{res}\bar{\psi}_L\psi_R$,
$m_{\text{res}} \lesssim 15\% \cdot m_q$

1 QCD on a Lattice

2 **Lattice Results**

- **Vector form factors**
- Axial form factors
- Quark Momentum and Angular Momentum

3 Summary

Electromagnetic Form Factors

$$\langle P' | \bar{q}\gamma^\mu q | P\rangle = \bar{U}(P')\left[F_1^q(Q^2)\gamma^\mu + F_2^q(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]U(P),$$

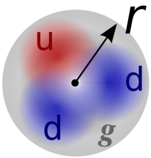
where $q = P' - P$ and $Q^2 = -q^2$

Sachs form factors

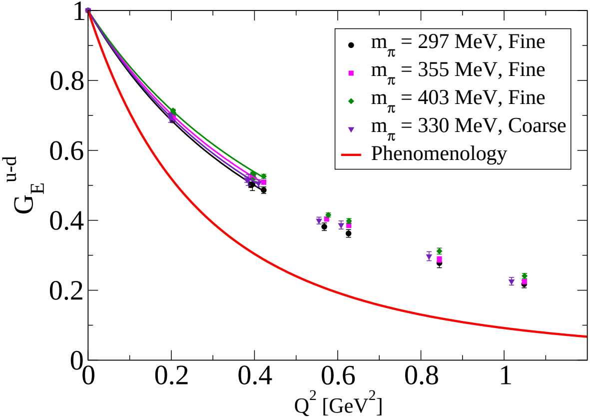
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2), \qquad G_E(0) = Q,$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \qquad G_M(0) = \mu.$$

"Radii"

$$F(Q^2) \approx F(0)\left(1 - \tfrac{1}{6}\langle r^2\rangle \cdot Q^2 + O(Q^4)\right)$$



Electric Form Factor G_E^{u-d} vs. Q^2



Dipole fits

$G(Q^2) \sim \frac{1}{(1+Q^2/M_D^2)^2}$

give

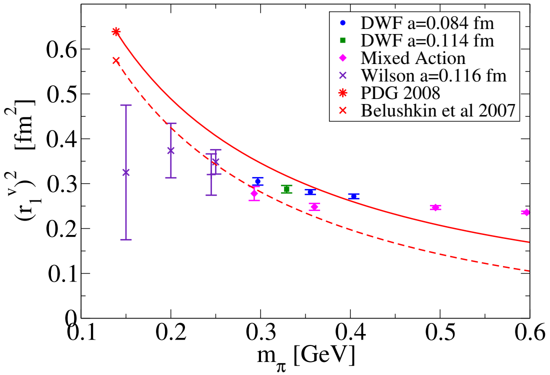
$M_D^2 \approx 0.97 \ldots 1.07$ GeV^2

$(M_D^2)_{\mathrm{exp}} = 0.71$ GeV^2

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \sim \langle N(+\tfrac{\vec{q}}{2}) | Q | N(-\tfrac{\vec{q}}{2}) \rangle$$

Fit to experimental data: [J. J. Kelly '04]

Dirac Radius $\langle r_1^2 \rangle^{u-d}$



Agreement for

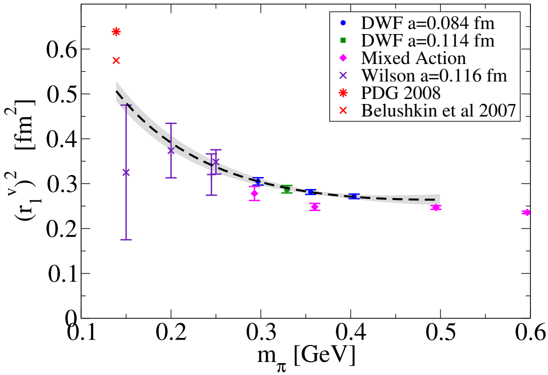
- 2 lattice spacings
- 2 volumes at $m_\pi = 250$ MeV
- 3 LQCD actions

HBχPT+Δ at NLO:
$\delta[(r_1^2)^{u-d}] \sim \log m_\pi$
[V. Bernard et al '98]

Extract $\langle r_1^2 \rangle$ from dipole fits,

$$F_1(Q^2) \sim \frac{1}{(1+\frac{1}{12}\langle r_1^2\rangle Q^2)^2} \approx 1 - \frac{1}{6}\langle r_1^2\rangle \cdot Q^2 + O(Q^4)$$

Dirac Radius $\langle r_1^2 \rangle^{u-d}$



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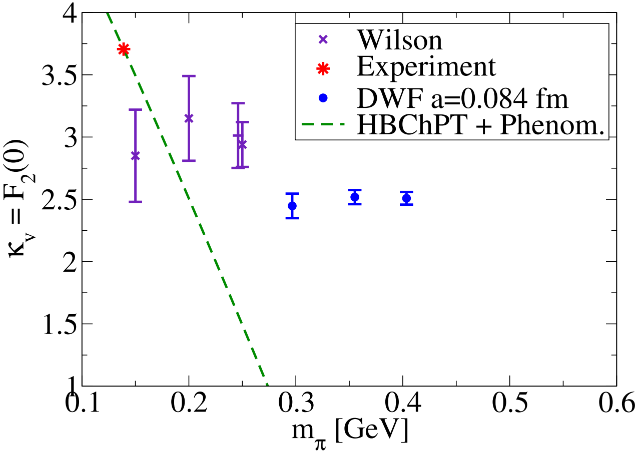
Anomalous Magnetic Moment κ_v



- NLO HBChPT+Δ prediction [V. Bernard et al '98]
- Fit to NLO CBChPT [T. Gail & T. Hemmert '08]

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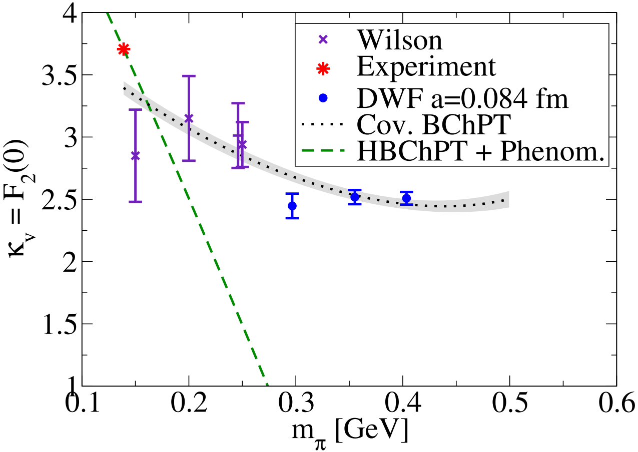

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Axial Form Factors

$$\langle N(p+q)|\bar{q}\gamma^\mu\gamma_5 q|N(p)\rangle = \bar{u}_{p+q}\Big[\gamma^\mu\gamma_5 G_A + \frac{q^\mu}{2M}\gamma_5 G_P\Big]u_p$$

with

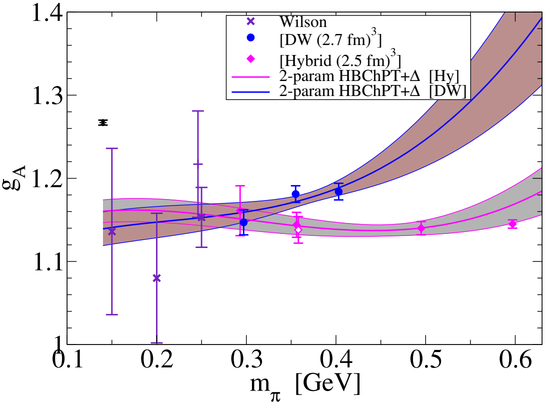
- $G_A(Q^2)$, axial form factor; axial charge $g_A = G_A(0)$
- $G_P(Q^2)$, induced pseudoscalar form factor

Nucleon structure seen by electroweak probes:

- ν, $\bar{\nu}$ quasielastic scattering off protons & nuclei
- charged pion electroproduction $\gamma^* + N \to \pi^a + N'$
- muon capture $p + \mu \to n + \nu_\mu$ $(+\gamma)$

Axial Charge g_A

- Computations are performed with $m_\pi \gtrsim 300$ MeV;
- need low-energy theory to extrapolate results to m_π^{phys}



Experiment 1.267(3)

Fit to HBChPT+Δ

[T. Hemmert et al '03]

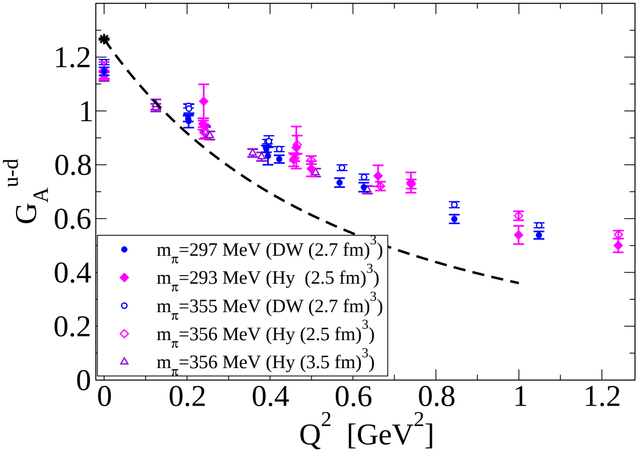
Hybrid 1.160(14)

Domain Wall 1.139(20)

$\approx 10\%$ smaller

- Finite-volume effects?
- Low-energy dynamics?

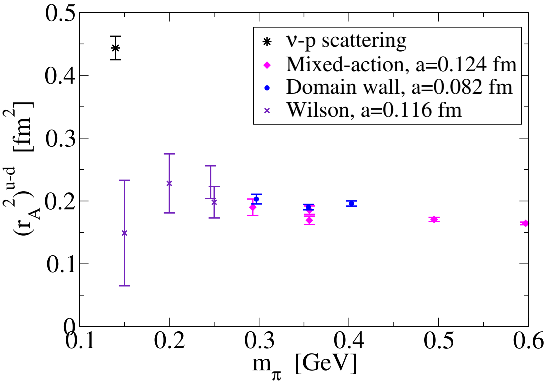
Axial Form Factor G_A^{u-d} vs. Q^2



$$G_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2}$$

$M_A^{\text{exp}} = (1.026 \pm 0.021)$ GeV [PDG 2000]
$M_A^{\text{lat}} \approx 1.5$ GeV

Axial Radius



$$\langle r_A^2 \rangle = -\frac{1}{6G_A}\frac{dG_A}{dQ^2}\Big|_{Q^2=0}$$

(dipole fits, $G_A \sim \frac{g_A}{(1+Q^2/M_A^2)^2}$)

ν-scattering:
$\langle r_A^2 \rangle = (0.665(14)\ \mathrm{fm})^2$

NLO Heavy Baryon ChPT prediction: $\langle r_A^2 \rangle \approx \mathrm{const}$

[V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998)]

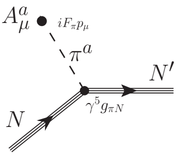
- rapid chiral dynamics beyond NLO?

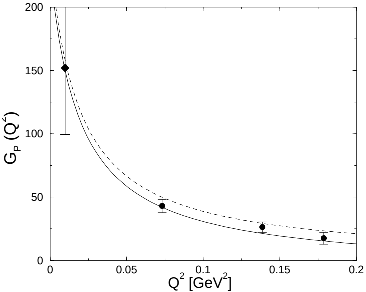
Axial Form Factors and PCAC

Because of partial conservation of A_μ (PCAC),
G_P *is not* independent from G_A:

$\langle P'|\partial_\mu A^\mu|P\rangle \;\to$

$2M_N G_A(Q^2) - \frac{Q^2}{2M_N} G_P(Q^2) \sim \frac{m_\pi^2 F_\pi g_{\pi N}}{m_\pi^2 + Q^2}$





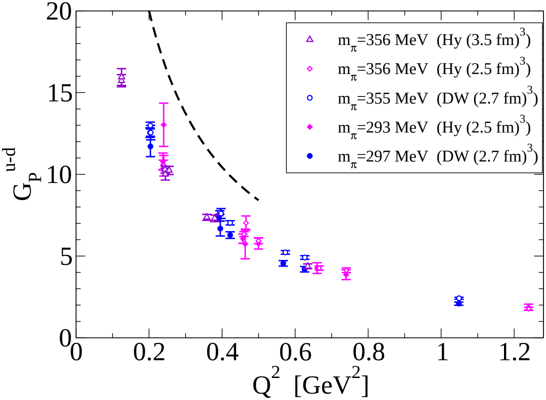
From NLO ChPT:

$$G_P(Q^2) = \frac{2m_N F_\pi g_{\pi N}}{m_\pi^2 + Q^2} - \frac{2}{3} g_A m_N^2 \langle r_A^2 \rangle + \mathcal{O}(m_\pi^2, Q^2)$$

[V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998)]

- important aspect of low-energy *QCD dynamics*

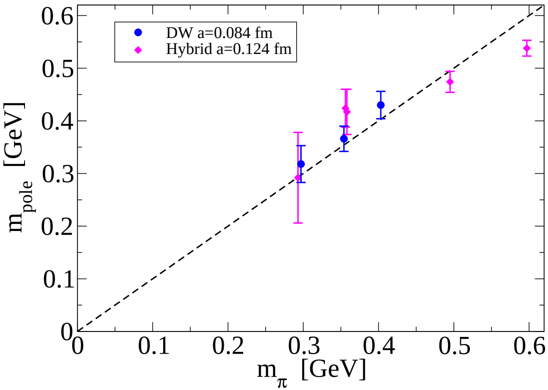
Induced Pseudoscalar G_P^{u-d}



- 2 volumes, $(2.5\ \text{fm})^3$, $(3.5\ \text{fm})^3$
- *Hybrid + Domain Wall*

Phenomenology (dashed curve): $G_P(Q^2) = \frac{4m_N^2 g_A}{m_\pi^2 + Q^2}$

Check Pion-pole Dominance



Fit $\boxed{G_P(Q^2) \Longleftrightarrow \frac{a}{m^2_{\text{pole}}+Q^2} + c}$ for $0.1 \le Q^2 \le 0.5$ GeV^2

1. QCD on a Lattice

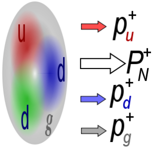
2. **Lattice Results**
   - Vector form factors
   - Axial form factors
   - **Quark Momentum and Angular Momentum**

3. Summary

Nucleon Momentum and Spin from Quarks

Energy-momentum tensor $T_q^{\mu\nu} = \bar{q}\gamma^{\{\mu}\, i\overleftrightarrow{D}^{\nu\}} q$:

$$\langle P' | T_q^{\mu\nu} | P \rangle \longrightarrow \{A_{20}, B_{20}, C_2\}(Q^2)$$

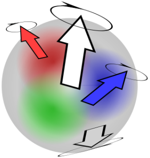


- quark momentum fraction

$$\boxed{\langle x \rangle_q = A_{20}^q(0)}$$

- quark angular momentum [X. Ji '97]:

$$\boxed{J_q = \frac{1}{2}[A_{20}^q(0) + B_{20}^q(0)]}$$



Separating contributions to nucleon spin:

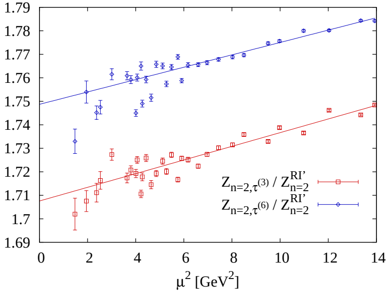
- quark spin $\boxed{\frac{1}{2}\Sigma_q = \langle 1 \rangle_{\Delta q}}$
- quark orbital angular momentum $\boxed{L_q = J_q - \frac{1}{2}\Sigma_q}$
- gluons : the rest $\boxed{J_{\text{glue}} = \frac{1}{2} - \frac{1}{2}\Sigma_q - L_q}$

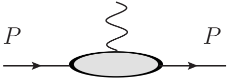
Tensors on a Lattice

- On a lattice, rotational symmetry is broken $O(4) \to H(4)$
- Tensors split into irred. reps. of $H(4)$

E.g., for a rank $n = 2$ tensor there are two components:

$$(\mathbf{4}_1)^{\otimes 2} = \mathbf{1}_1 \oplus \mathbf{6}_1 \oplus \underbrace{\mathbf{6}_3 \oplus \mathbf{3}_1}_{\text{trace=0, symm.}}$$

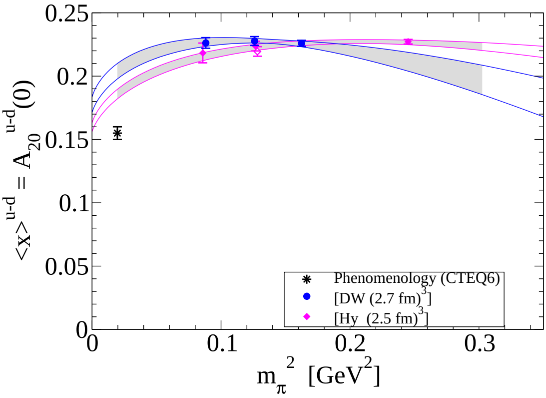




$$Z_{\mathcal{O}}^{\text{scale-indep.}} = \frac{Z_{\mathcal{O}}^{\text{lat}}(a\mu)}{Z_{\mathcal{O}}^{\text{pert}}(\mu)}$$

$$Z_{\mathcal{O}}^{\mathbf{3}_1} / Z_{\mathcal{O}}^{\mathbf{6}_3} \approx 0.98$$

Quark Momentum Fraction $\langle x \rangle_{u-d}$

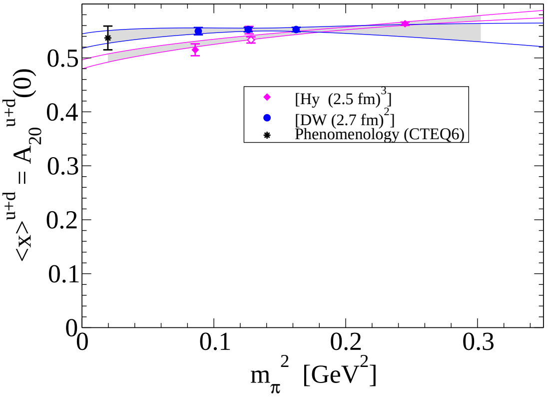


$$\langle x \rangle_q = A_{20}^q(0)$$
$$= \int dx\, x\, (q(x) + \bar{q}(x))$$

ChiPT:
$$\delta[\langle x \rangle_{u-d}] \sim m_\pi^2 \log m_\pi$$

- Results are consistently above the phenomenological value by $15 - 25\%$.

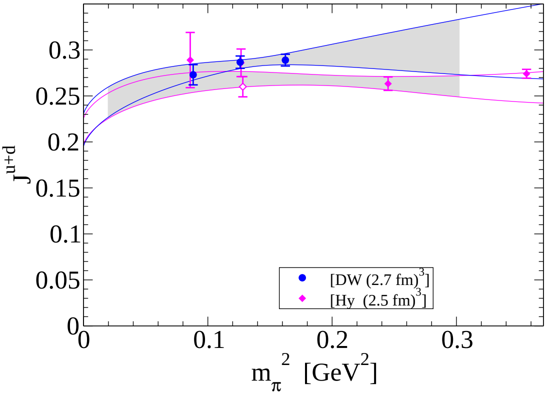
Quark Momentum Fraction $\langle x \rangle_{u+d}$



$$\langle x \rangle_q = A_{20}^q(0) = \int dx\, x\, \big(q(x) + \bar{q}(x)\big)$$

- quarks carry $\approx 1/2$ of boosted nucleon momentum
- qualitative agreement with phenomenology

Quarks Angular Momentum (1): J^{u+d}



Following [X. Ji PRL '97],

$$J_q^3 = \langle N| \int d^3x\, M^{012} |N\rangle,$$

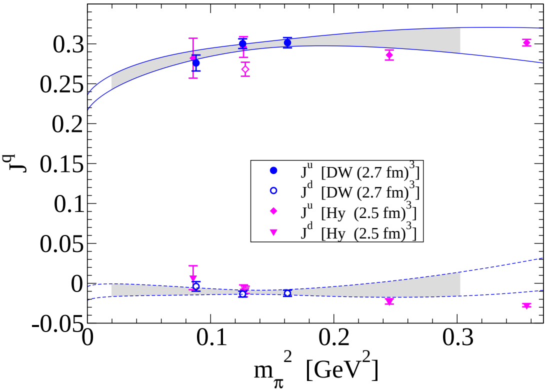
$$M^{\alpha\mu\nu} = x^\mu T_q^{\alpha\nu} - x^\nu T_q^{\alpha\mu}$$

and

$$\boxed{J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]}$$

- Gluon contribution $J^g = \frac{1}{2} - J^q \sim 52\%$ of the nucleon spin
- result agrees with QCD sum rule estimations [Balitsky, Ji (1997)]

Quarks Angular Momentum (2): J^u, J^d



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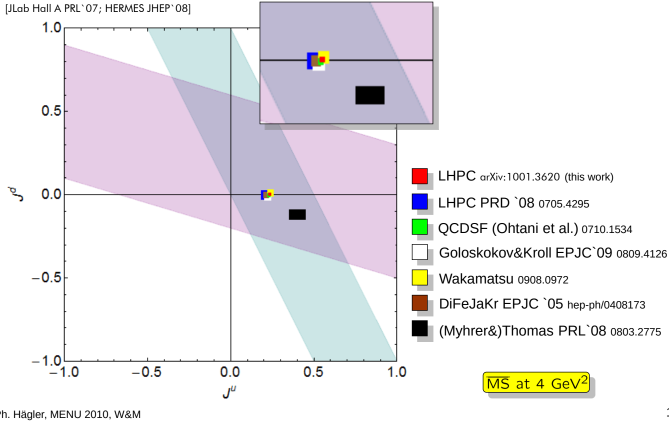
and

$$J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

Most contribution to the nucleon spine comes from u-quarks:

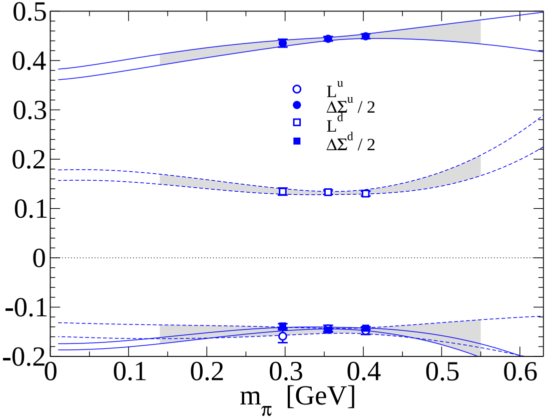
$$|J^d| \ll |J^u|$$

Quark Angular Momentum: p,n-DVCS



- DVCS provides GPD values $\{\mathcal{H},\mathcal{E}\}(x,\xi,t)$ at $x=\pm\xi$
- $J^{u,d} = \frac{1}{2}\int dx x(\mathcal{H}+\mathcal{E})\Big|_{\xi=0,t=0}$ depend strongly on GPD ansatz used

Quark Spin and OAM



$$L^q = J^q - S^q,$$

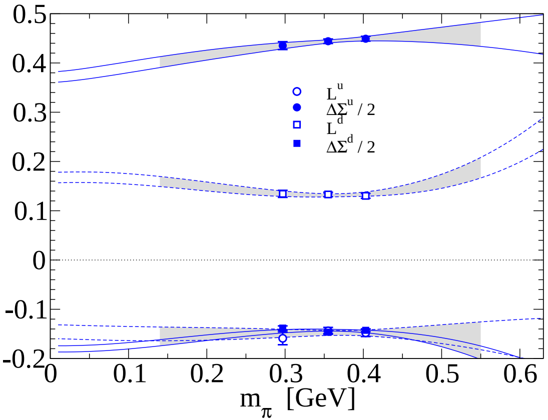
$$S_q = \frac{1}{2}\Delta\Sigma_q = \int dx\,\left(\Delta q(x) + \Delta\bar{q}(x)\right)$$

$$|J^d| \ll \quad |S^d|, |L^d|$$

$$|L^{u+d}| \ll \quad |L^u|, |L^d|$$

In agreement with [Hägler *et al* (2007)]

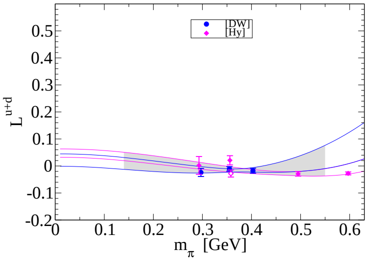
Quark Spin and OAM



$$L^q = J^q - S^q,$$

$$S_q = \frac{1}{2}\Delta\Sigma_q$$

$$= \int dx\,\left(\Delta q(x) + \Delta\bar{q}(x)\right)$$



$$\left|J^d\right| \ll \left|S^d\right|, \left|L^d\right|$$

$$\left|L^{u+d}\right| \ll \left|L^u\right|, \left|L^d\right|$$

$$\left|L^{u+d}\right| \ll \frac{1}{2}$$

In agreement with [Hägler *et al* (2007)]

Summary

- Nucleon structure observables are calculated from the fundamental theory, QCD
  - accurate data with *chiral quarks* down to $m_\pi \gtrsim 300$ MeV
  - preliminary studies with *non-chiral quarks* down to $m_\pi \gtrsim 150$ MeV
- "Nucleon size" observables $\langle r_{1,2}^2 \rangle^v$, $\langle r_A^2 \rangle$ *undershoot* experimental values and weakly depend on m_π *contradicting* the predictions of ChPT
- Quark contributions to nucleon momentum and spin are *in qualitative agreement* with phenomenology
- Peculiar pattern of quark contributions to the nucleon spin:
$$|J^d| \ll \{|S^d|, |L^d|\}, \quad |L^{u+d}| \ll \{|L^u|, |L^d|\}$$
- Lattice calculations *will* constrain experimental GPD fits

Backup slides

- GPDs
- Proton and neutron form factors from experiments
- Dipole fits of lattice vector form factors
- Dirac and Pauli Radii vs ChPT

Generalized Parton Distributions

- Generalized Parton Distributions $\langle P'|\mathcal{O}^{[\gamma^5]}(x)|P\rangle \to \{\mathcal{H},\mathcal{E},\tilde{\mathcal{H}},\tilde{\mathcal{E}}\}(x,\xi,q^2)$,

$$\mathcal{O}^{[\gamma^5]}(x) = \int \frac{d\lambda}{2\pi}\, e^{2i\lambda x}\, \bar{q}_{(-\lambda n)} \Big[\not{n}\, [\gamma^5]\, \mathcal{W}(-\lambda n, \lambda n)\Big] q_{(\lambda n)},$$

  - x is the *longitudinal* momentum fraction
  - q^2 links to the *transverse (impact parameter)* space

- Computed on a lattice using *local* operators

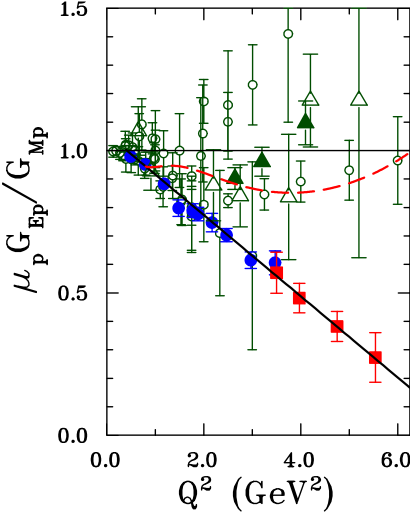
$$\mathcal{O}_n^{[\gamma^5]} = \bar{q}\Big[\gamma_{\{\mu_1}\, [\gamma^5]\, i\overleftrightarrow{D}_{\mu_2} \cdots i\overleftrightarrow{D}_{\mu_n\}}\Big] q$$

corresponding to moments $\mathcal{O}_n = \int dx\, x^n \mathcal{O}(x) \to \bar{q}\gamma^+ (i\overleftrightarrow{D}^+)^n q$
and reducing to Generalized Form Factors
$\langle P'|\mathcal{O}_n|P\rangle \longrightarrow \{A_{ni}, B_{ni}, C_n, \tilde{A}_{ni}, \tilde{B}_{ni}\}(Q^2)$,
e.g. $\int dx\, x^n \mathcal{H}^q(x,0,q^2) = A_{n+1,0}(q^2)$, $\quad \int dx\, x^n \mathcal{E}^q(x,0,q^2) = B_{n+1,0}(q^2)$

- for higher $n \geq 4$, mixing with $n' < n$ occurs $\sim a^{-(n-n')}$

Proton G_E Discrepancy

[M. Vanderhaeghen, Nucl.Phys.A805:210(2008)]



Different results from

- Rosenbluth separation
- Polarization transfer

due to 2γ exchange

[Blunden, Melnitchouk and Tjon, 2003]

[Chen et al, 2004]

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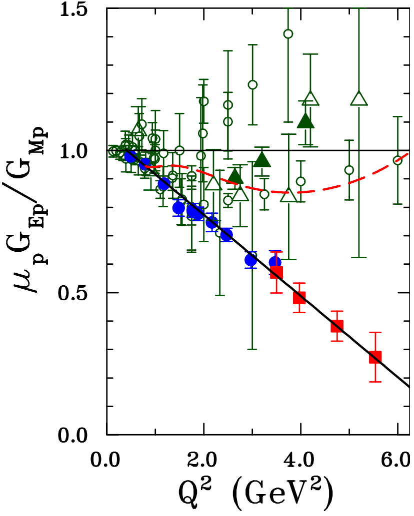

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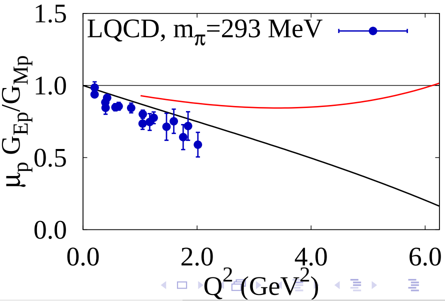
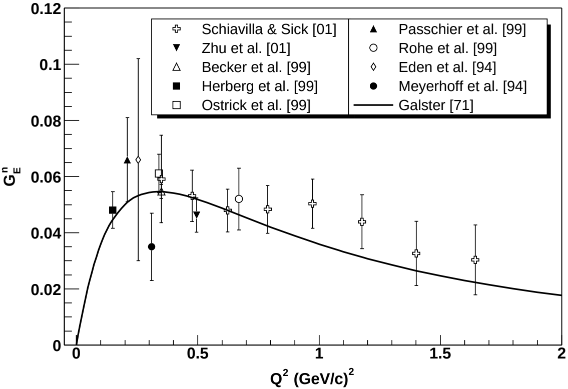
Lattice (systematics unclear!):



Neutron Electric Form Factor



$\langle r^2_{En} \rangle < 0$:

$(+)$ core

$(-)$ surface

$n \longleftrightarrow p + \pi^+$

[H. Gao, Int.J.Mod.Phys.E12:1 (2003)]

- deuterium targets
- thermal neutron scattering: $\langle r^2_{E,n} \rangle = -0.113(3)(4)$ fm^2 [PRL 74, 2427 (1995)]

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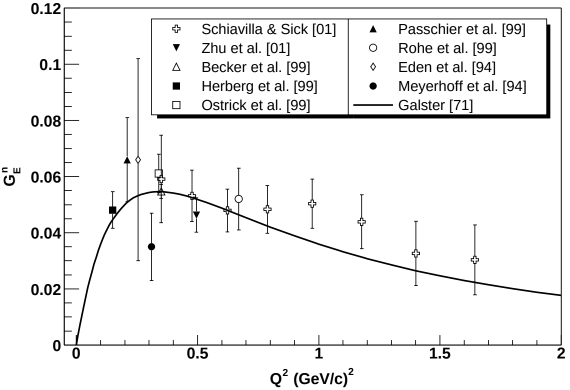

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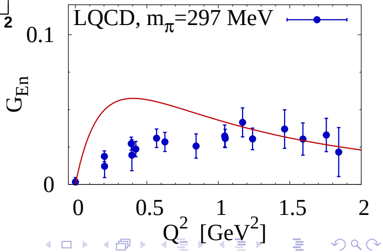
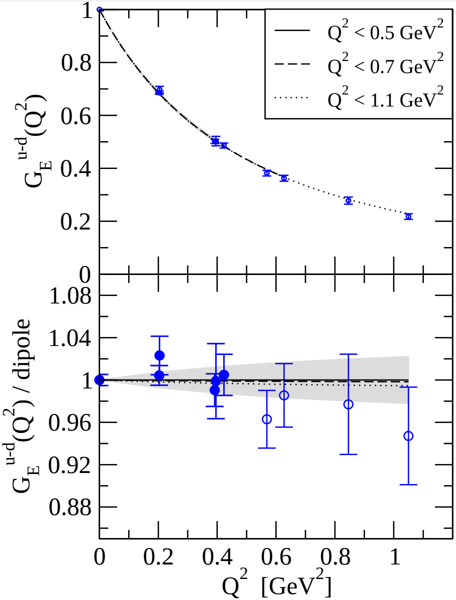


Dipole Fits to G_E^{u-d} ($m_\pi = 297$ MeV)

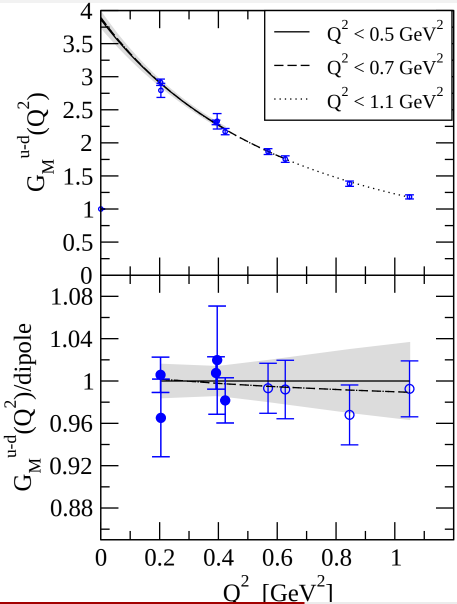


- Dipole fits
  $G_E(Q^2) = \frac{1}{(1+Q^2/M_D^2)^2}$
- Cut-off $Q^2 \le 0.5$ GeV^2
- Results are consistent for higher cutoffs

$M_D^2 = 0.97 \ldots 1.07$ GeV^2

$\left(M_D^2\right)_{\text{exp}} = 0.71$ GeV^2

Dipole Fits to G_M^{u-d} ($m_\pi = 297$ MeV)



- Dipole fits $G_M(Q^2) = \frac{1+\kappa_v}{(1+Q^2/M_D^2)^2}$
- Cut-off $Q^2 \leq 0.5$ GeV^2
- Results are consistent for higher cutoffs

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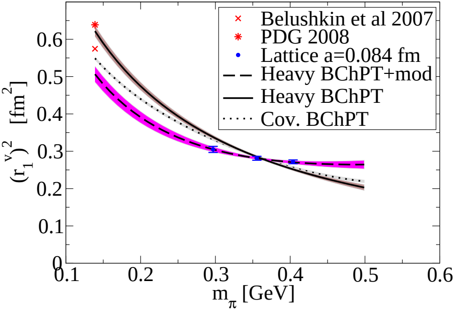
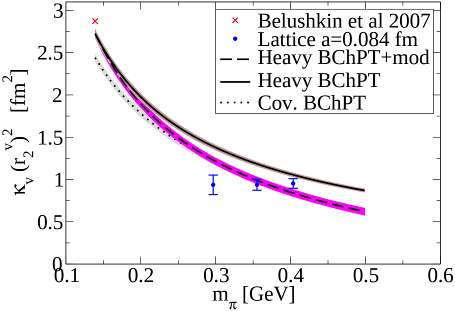
Dirac and Pauli Radii vs. ChPT





Div. "pion cloud" contribution:

$$\text{Dirac} \quad \delta\left[(r_1^v)^2\right] \sim \log m_\pi$$
$$\text{Pauli} \quad \delta\left[\kappa_v (r_2^v)^2\right] \sim \frac{1}{m_\pi}$$

HBChPT+Δ [V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998)]

CBChPT [T. Gail, PhD Thesis (2007)]