Excited State Spectroscopy from Lattice QCD

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Lattice QCD at Zero Temperature, INT, July 5-8

Hadron Spectrum Collaboration

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Baryon spectrum

Meson spectrum

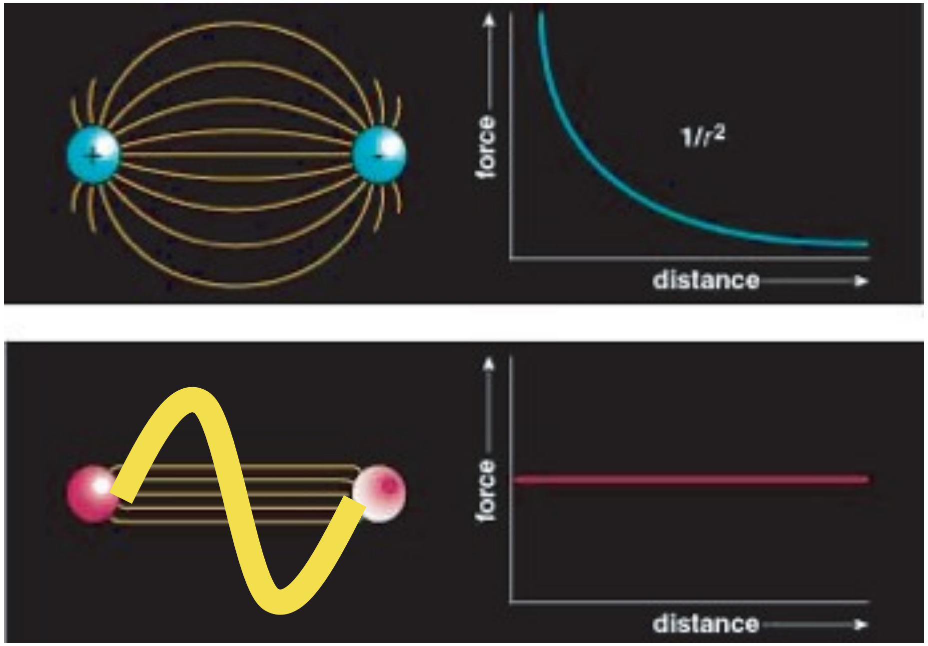
Plan of Talk

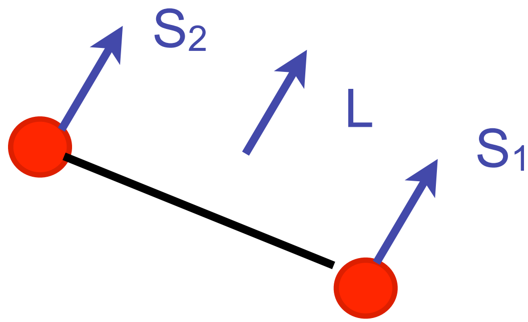
- What are they and why are they interesting?
- Methods
  - variational method, distillation
  - Symmetries on the lattice
  - Interpolating operators - in the continuum, and on the lattice
- Results
  - Isovector Meson Spectrum
  - Low-lying baryon spectrum
  - Isoscalar spectrum
- Challenges
  - Strong decays - phase-shifts and resonance parameters
  - I=2 ππ Momentum-Dependent Phase Shift
- Computational Challenges for Spectroscopy
- Summary

Goals - I

- *Why is it important?*
  - *What are the key degrees of freedom describing the bound states?*
    - *How do they change as we vary the quark mass?*
  - *What is the origin of confinement, describing 99% of observed matter?*
  - *If QCD is correct and we understand it, expt. data must confront ab initio calculations*
  - *What is the role of the gluon in the spectrum – search for exotics*

Goals - II



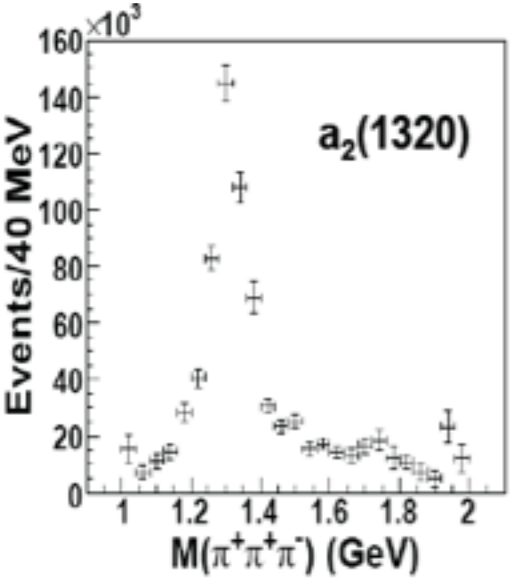


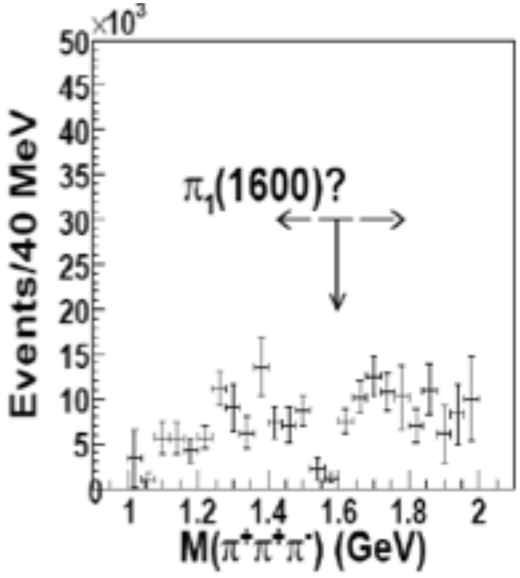
Simple quark model (for neutral mesons) admits only certain values of J^{PC}

$$P = (-1)^{l+1}$$
$$C = (-1)^{l+s}$$

- Exotic Mesons are those whose values of J^{PC} are in accessible to quark model: 0^{+-}, 1^{-+}, 2^{+-}
  - Multi-quark states:
  - Hybrids with *excitations of the flux-tube*
- Study of hybrids: revealing gluonic degrees of freedom of QCD.
- *Glueballs:* purely, or predominantly, gluonic states

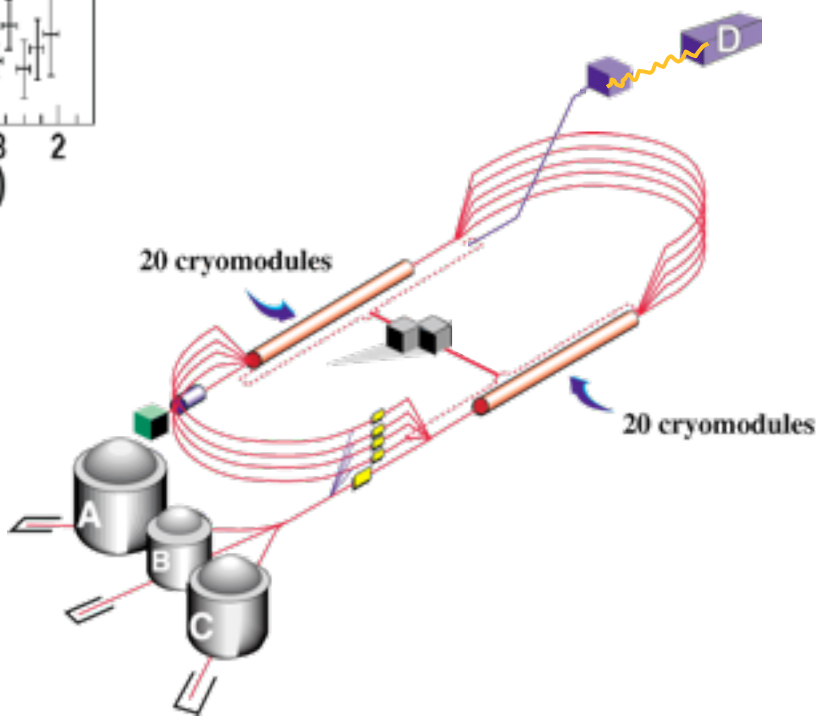
Hybrids - lattice + expt





Beyond "bump hunting"!

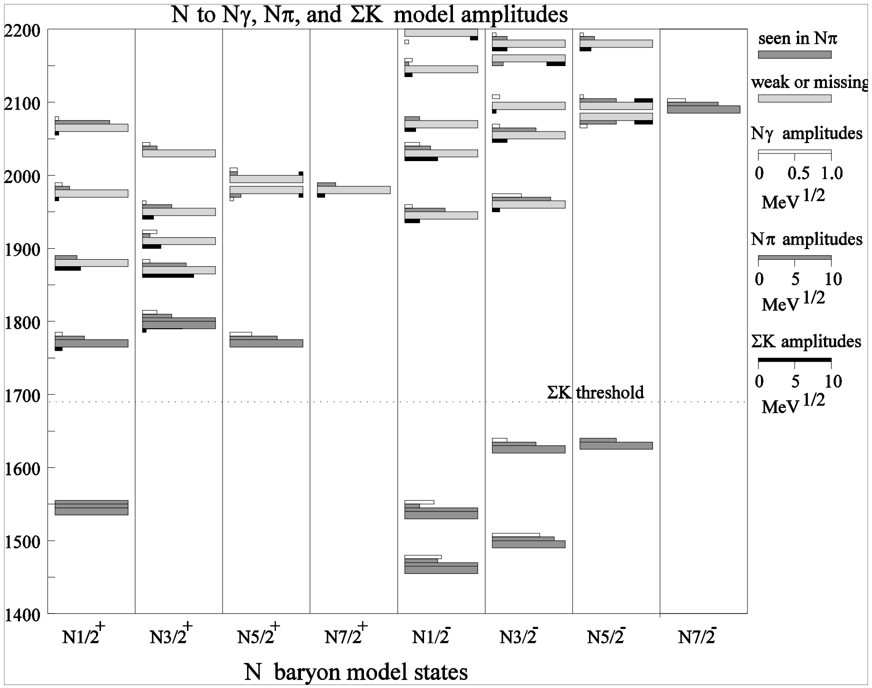
π1(1600) in pion production at BNL



No clear evidence in photoproduction at CLAS

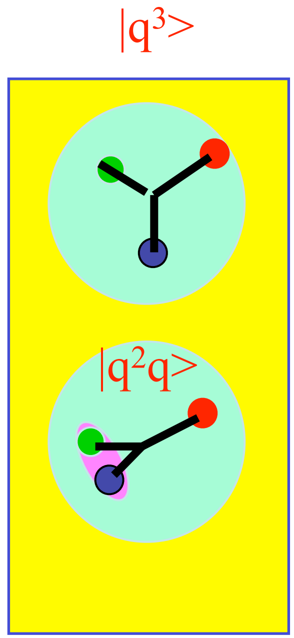
Goals - III

- No baryon "exotics", ie quantum numbers not accessible with simple quark model; but may be hybrids!
- Nucleon Spectroscopy: Quark model masses and amplitudes – states classified by isospin, parity and spin.



Capstick and Roberts, PRD58 (1998) 074011

- Missing, because our pictures do not capture correct degrees of freedom?
- Do they just not couple to probes?



CLAS at JLab

Variational Method

- Construct matrix of correlators

$$C_{\alpha\beta}(t, t_0) = \langle 0 \mid \mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(t_0) \mid 0\rangle$$
$$\longrightarrow \sum_n Z^n_\alpha Z^{n\dagger}_\beta e^{-M_n(t-t_0)}$$

where $\{\mathcal{O}_\alpha\}$ are basis of operators of definite symmetry: P, C and J?

Delineate contributions using variational method: solve

$$C(t)u(t, t_0) = \lambda(t, t_0)C(t_0)u(t, t_0)$$
$$\lambda_i(t, t_0) \rightarrow e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

Challenges

➡ Resolve energy dependence - *anisotropic lattice*
➡ Judicious construction of interpolating operators - *cubic symmetry*

Anisotropic lattices

To appreciate difficulty of extracting excited states, need to understand signal-to-noise ratio in two-point functions. Consider correlation function:

$$C(t) = \langle 0 \mid \mathcal{O}(t)\mathcal{O}(0)^\dagger \mid 0 \rangle \longrightarrow e^{-Et}$$

Then the fluctuations behave as

DeGrand, Hecht, PRD46 (1992)

$$\sigma^2(t) \simeq \left(\langle 0 \mid |\mathcal{O}(t)\mathcal{O}(0)^\dagger|^2 \mid 0 \rangle - C(t)^2 \right) \longrightarrow e^{-2m_\pi t}$$

Signal-to-noise ratio degrades with increasing E - Solution: anisotropic lattice with $a_t < a_s$

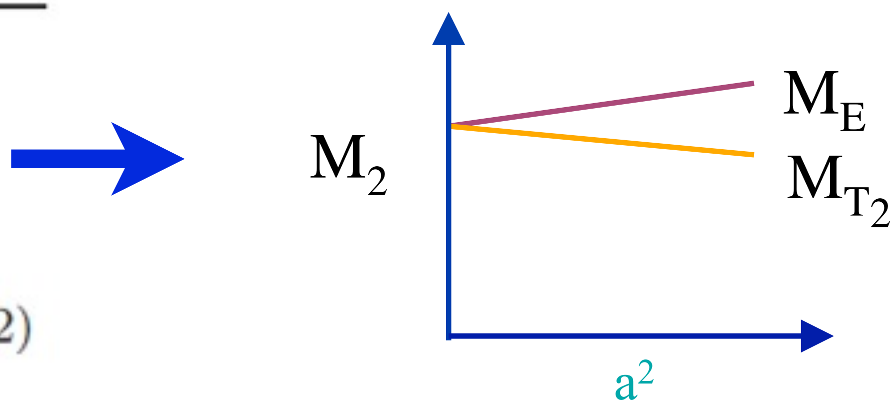
Challenges - II

- States at rest are characterized by their behavior under *rotations - SO(3)*

  Lattice does not possess full symmetry of the continuum - allowed energies characterised by cubic symmetry, or the octahedral point group O_h

  - *24 elements*
  - *5 conjugacy classes/5 irreducible representations*
  - $O_h \times I_s$: rotations + inversions (parity)

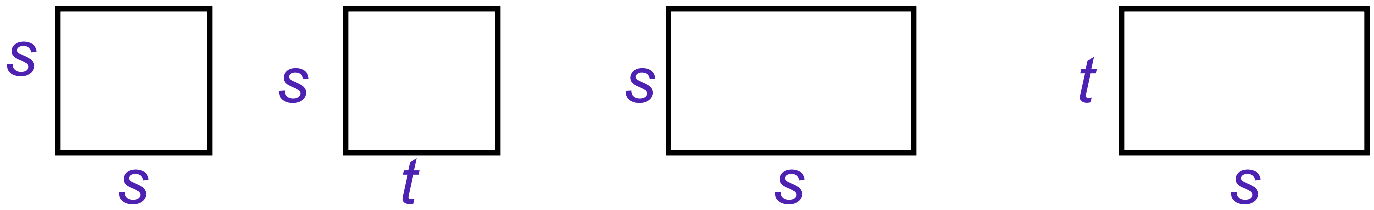
J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$



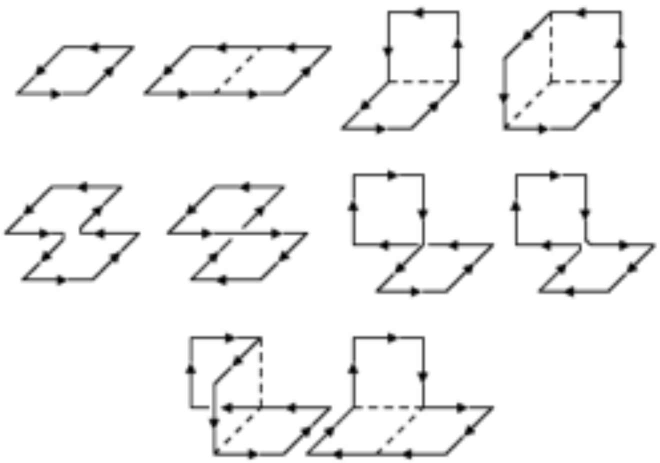
Glueball Spectroscopy - I

Improved anisotropic pure-gauge action Morningstar, Peardon 97,99

$$S[U] = \beta\xi \left\{ \frac{5}{3U_s^4} P_{ss'} + \frac{4}{3\xi^2 u_s^2 u_t^2} P_{st} - \frac{1}{12 u_s^6} R_{ss'} - \frac{1}{12\xi^2 u_s^4 u_t^2} R_{st} \right\}$$



Operators: closed Wilson loops



ξ is bare anisotropy a_s/a_t

Obtain renormalized anisotropy by comparing different Wilson Loops

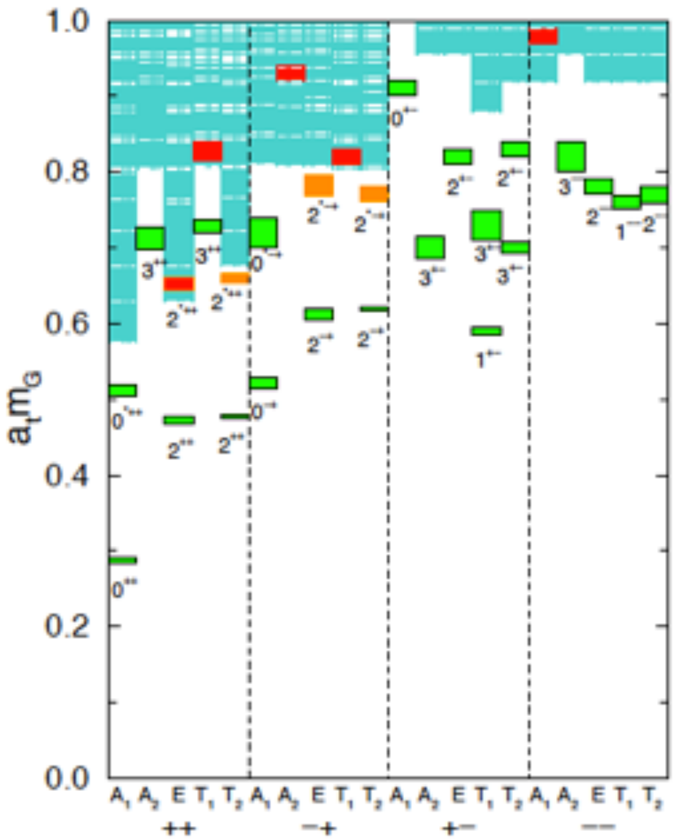
$$W_{xt}(Ia_s, Ja_t) \xrightarrow{J\to\infty} Z_{xt} e^{-Ja_t V(Ia_s,0,0)},$$
$$W_{xy}(Ia_s, Ja_s) \xrightarrow{J\to\infty} Z_{xy} e^{-Ja_s[V(Ia_s,0,0)+V_0]}$$

Ratio at large J gives ξ

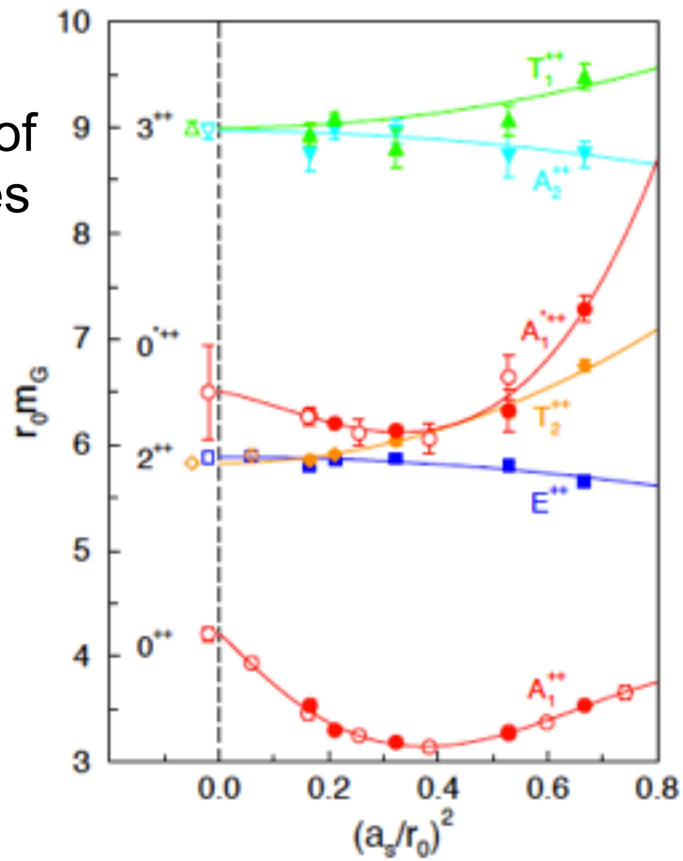
Morningstar, 96

Glueball Spectroscopy - II

$\beta = 2.5 : \ \xi = 5$



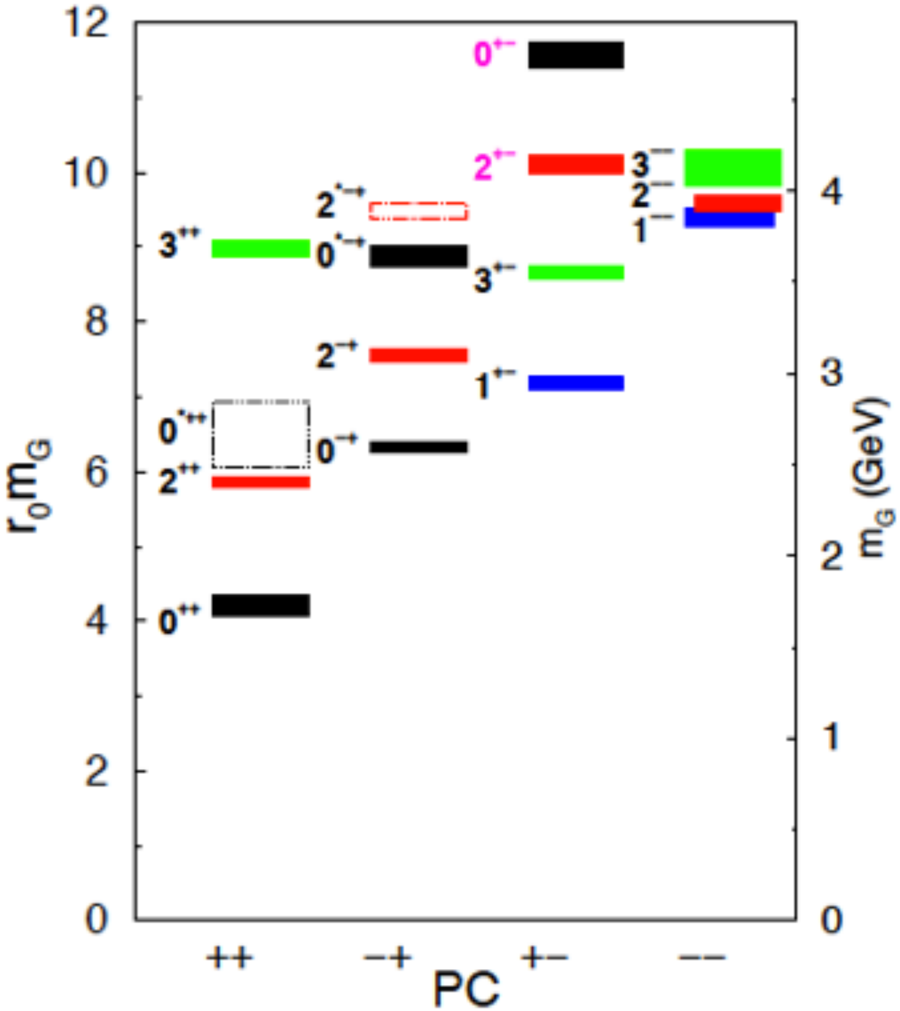
Observe emergence of degeneracies

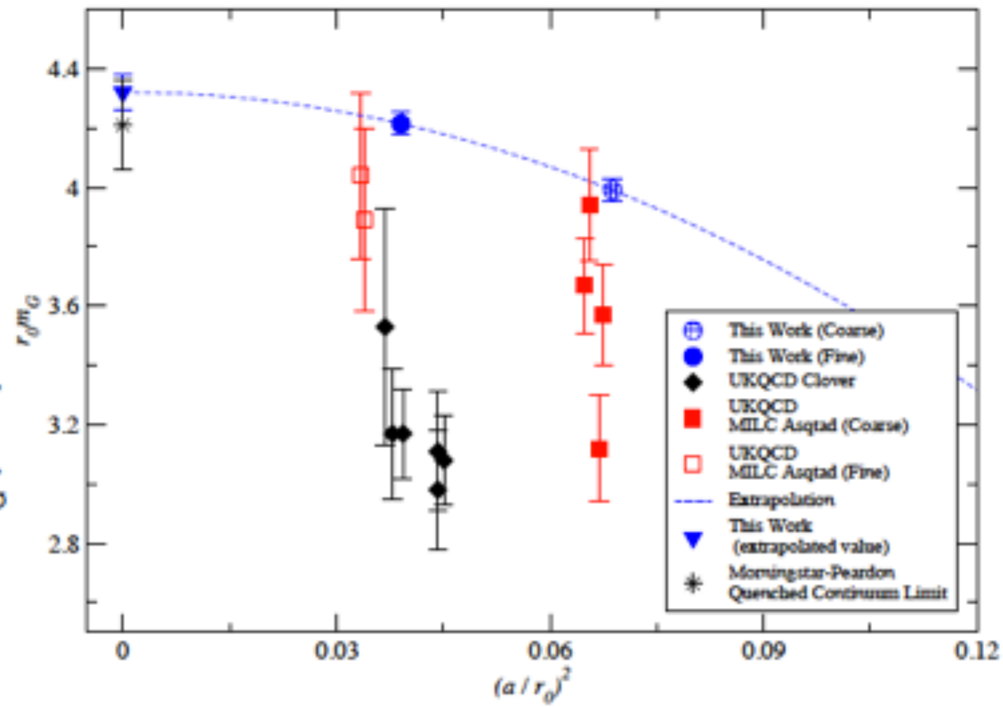


Glueball Spectrum - III

Note that this is the pure Yang-Mills spectrum - not the erroneously named “quenched” glueball spectrum!

UKQCD, C.Richards et al, arXiv:1005.2473





2+1 flavor staggered - can mix with two-pi states!

Meson spectroscopy with Quarks

- Anisotropic lattices - to precisely resolve energies
- Variational method - with sufficient operator basis to delineate states
- Many values of lattice spacing - identification of spin.

Anisotropic fermion action

Edwards, Joo, Lin, PRD78 (2008)

$$
\begin{aligned}
S_G^{\xi}[U] &= \frac{\beta}{N_c \gamma_g} \left\{ \sum_{x,s>s'} \left[\frac{5}{3u_s^4} \mathcal{P}_{ss'} - \frac{1}{12u_s^6} \mathcal{R}_{ss'} \right] + \sum_{x,s} \left[\frac{4}{3u_s^2 u_t^2} \mathcal{P}_{st} - \frac{1}{12u_s^4 u_t^2} \mathcal{R}_{st} \right] \right\} \\
S_F^{\xi}[U, \overline{\psi}, \psi] &= \sum x \overline{\psi}(x) \frac{1}{\tilde{u}_t} \left\{ \tilde{u}_t \hat{m}_0 + \hat{W}_t + \frac{1}{\gamma_f} \sum_s \hat{W}_s - \right. \\
& \left. \frac{1}{2} \left[\frac{1}{2} \left(\frac{\gamma_g}{\gamma_f} + \frac{1}{\xi} \right) \frac{1}{\tilde{u}_t \tilde{u}_s^2} \sum_s \sigma_{ts} \hat{F}_{ts} + \frac{1}{\gamma_f} \frac{1}{\tilde{u}_s^3} \sum_{s<s'} \sigma_{ss'} \hat{F}_{ss'} \right] \right\} \psi(x).
\end{aligned}
$$

Two anisotropy parameters to tune, in gauge and fermion sectors

$$\xi = 3.5$$

$$
\begin{aligned}
\gamma_g &= \xi_0 \\
\gamma_f &= \xi_0 / \nu
\end{aligned}
$$

Dispersion Relation

Anisotropic Clover Generation - I

Tuning performed for three-flavor theory

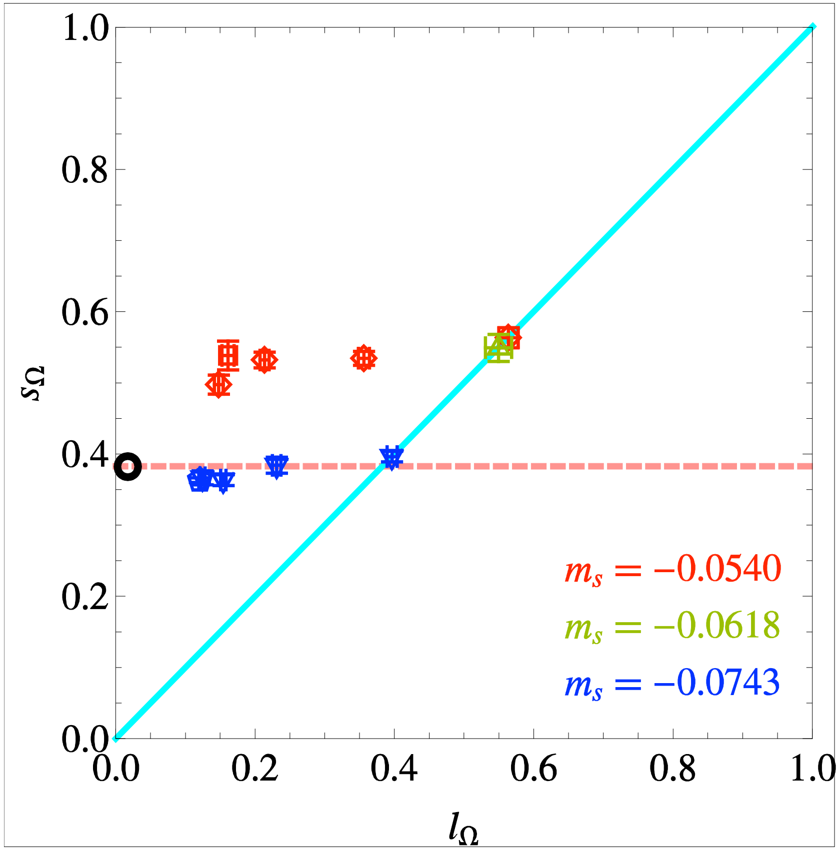
Challenge: setting scale and strange-quark mass

Lattice coupling *fixed*

Proportional to m_s to LO ChPT

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega



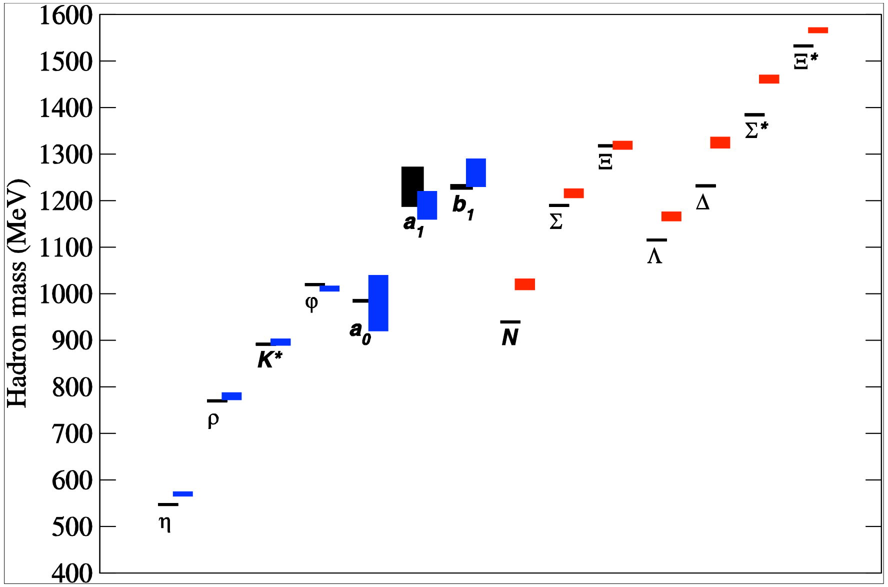
Express physics in (dimensionless) (l,s) coordinates

$$l_X = (9/4)m_\pi^2/m_X^2$$

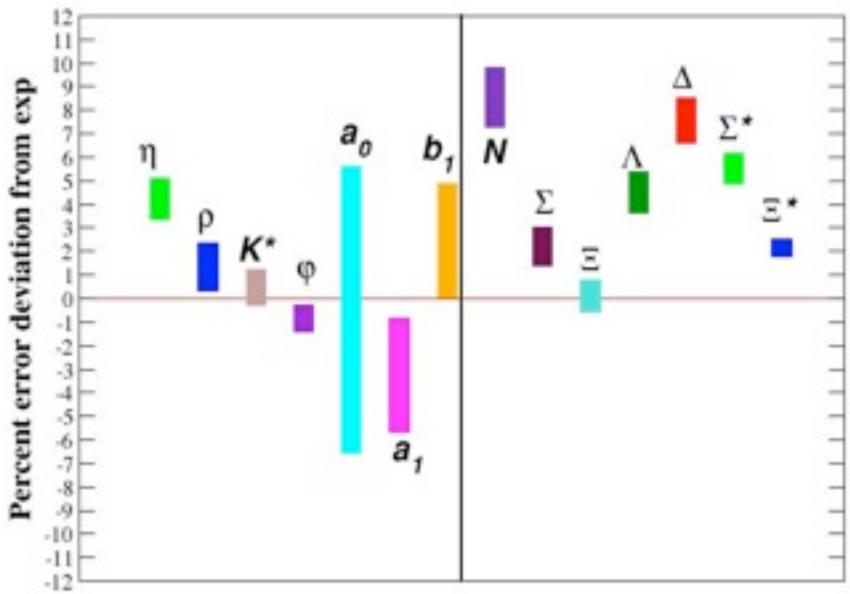
H-W Lin et al (Hadron Spectrum Collaboration), PRD79, 034502 (2009)

Proportional to m_l to LO ChPT

Anisotropic Clover – II



Low-lying spectrum: *agrees with experiment to 10%*



Correlation functions: Distillation

- Use the new "distillation" method.
- Observe $$L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$

Eigenvectors of Laplacian

- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- Meson correlation function

Includes displacements

$$C_M(t,t') = \langle 0 \mid \bar{d}(t')\Gamma^B(t')u(t')\bar{u}(t)\Gamma^A(t)d(t)|0\rangle$$

- Decompose using "distillation" operator as

M. Peardon *et al.*, PRD80,054506 (2009)

$$C_M(t,t') = \mathrm{Tr}\langle \phi^A(t')\tau(t',t)\Phi^B(t)\tau^\dagger(t',t),\rangle$$

where

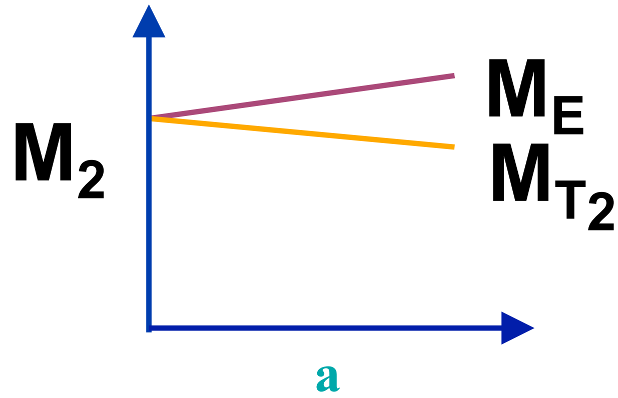
$$\begin{aligned}\Phi^{A,ij}_{\alpha\beta} &= v^{*(i)}(t)[\Gamma^A(t)\gamma_5]_{\alpha\beta}v^{(j)}(t') \\ \tau^{ij}_{\alpha\beta}(t,t') &= v^{*(i)}(t')M^{-1}_{\alpha\beta}(t',t)v^{(j)}(t).\end{aligned}$$

Perambulators

Identification of Spin - I

Problem:

- YM glueball requires data at several lattice spacings
- density of states in each irrep large.



Solution: exploit known continuum behavior of overlaps

- **Construct interpolating operators of *definite* (continuum) JM: O^{JM}**

Starting point

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

$$\bar{\psi}(\vec{x}, t)\Gamma D_i D_j \ldots \psi(\vec{x}, t)$$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x - i\overleftrightarrow{D}_y\right)$$

$$\overleftrightarrow{D}_{m=0} = i\overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x + i\overleftrightarrow{D}_y\right).$$

Identification of spin

Straighforward to project to definite spin: *J = 0, 1, 2*

$$(\Gamma \times D^{[1]}_{J=1})^{J,M} = \sum_{m_1,m_2} \langle 1, m_1; 1, m_2 \big| J, M \rangle \, \bar{\psi} \Gamma_{m_1} \overleftrightarrow{D}_{m_2} \psi.$$

- **Use projection formula to find subduction under irrep. of cubic group - operators are closed under rotation!**

$$\underbrace{O^{[J]}_{\Lambda\lambda}(t,\vec{x})}_{\text{Irrep, Row}} = \frac{d_\Lambda}{g_{O^D_h}} \sum_{R \in O^D_h} \underbrace{D^{(\Lambda)*}_{\lambda\lambda}(R)}_{\text{Irrep of R in }\Lambda} U_R O^{J,M}(t,\vec{x}) \underbrace{U^\dagger_R}_{\text{Action of R}}$$

$$= \sum_M S^{J,M}_{\Lambda,\lambda} O^{J,M}$$

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} \equiv (\Gamma \times D^{[n_D]}_{\dots})^J_{\Lambda,\lambda} =$$

$$\sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \, (\Gamma \times D^{[n_D]}_{\dots})^{J,M} \equiv \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \mathcal{O}^{J,M}$$

Identification of Spin - II

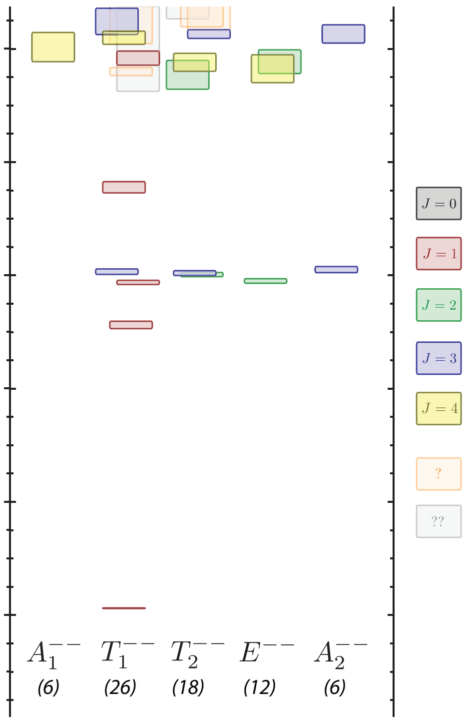
Hadspec collab. (dudek et al), 0909.0200, PRL

Overlap of state onto subduced operators

$$\langle 0 \,|\, O^{J,M} \,|\, J', M' > \;=\; Z_J \delta_{J,J'} \delta_{M,M'}$$

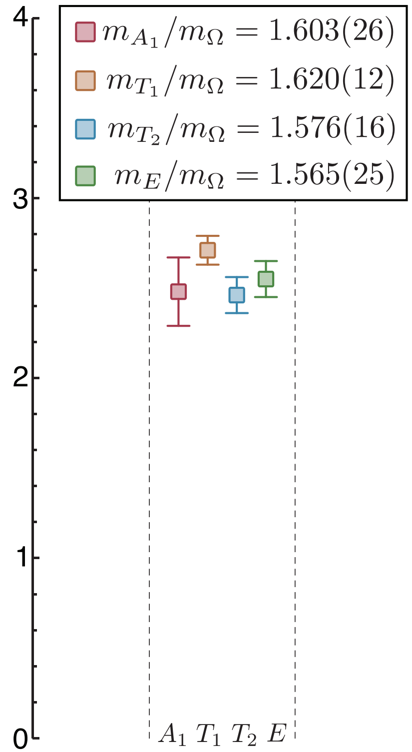
$$\langle 0 \,|\, O^{J}_{\Lambda,\lambda} \,|\, J', M' \rangle \;=\; S^{J,M'}_{\Lambda,\lambda} Z_J \delta_{J,J'}$$

Common across irreps.



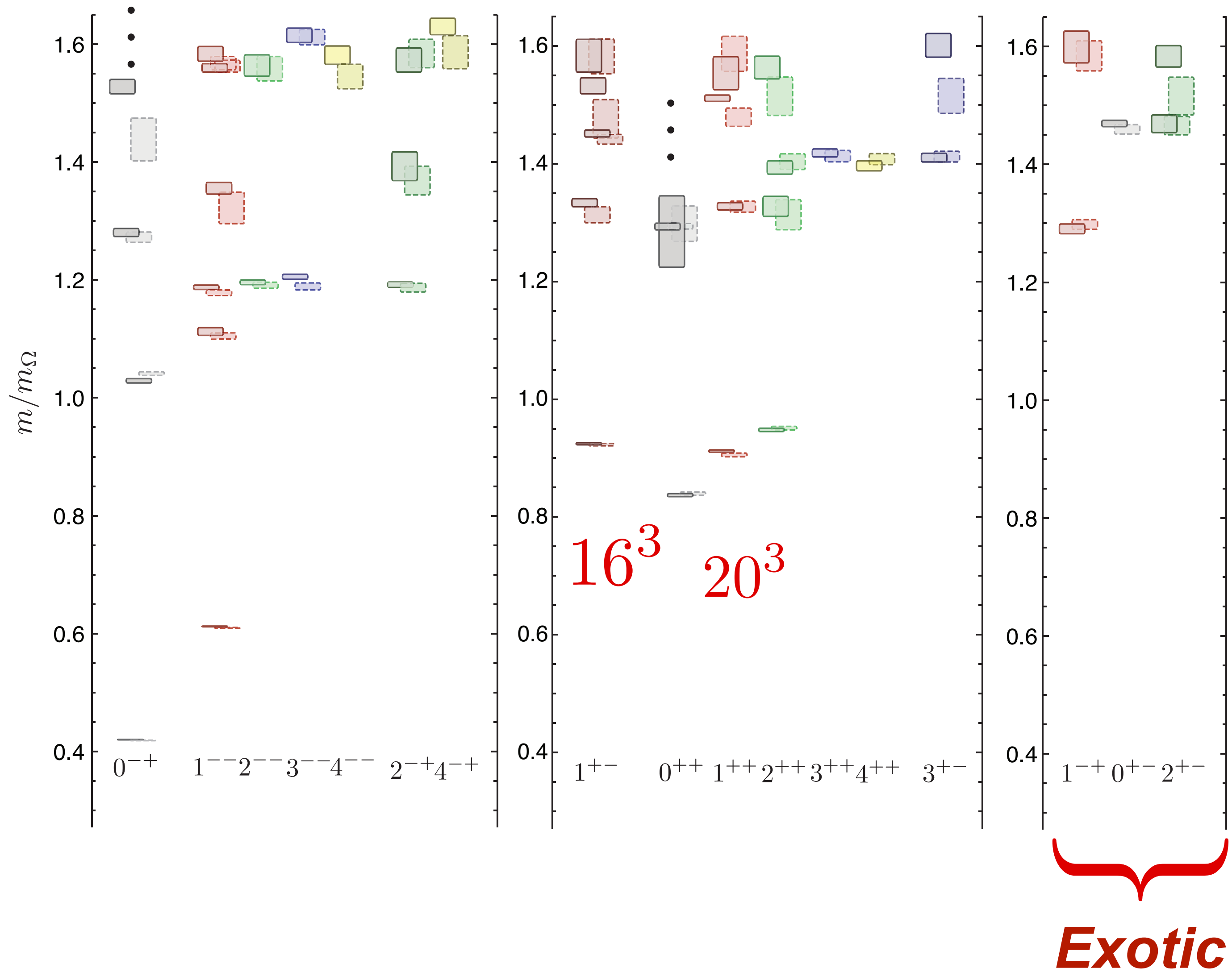
J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

$$N_f = 3$$



Lattice ops. retain memory of their continuum ancestors

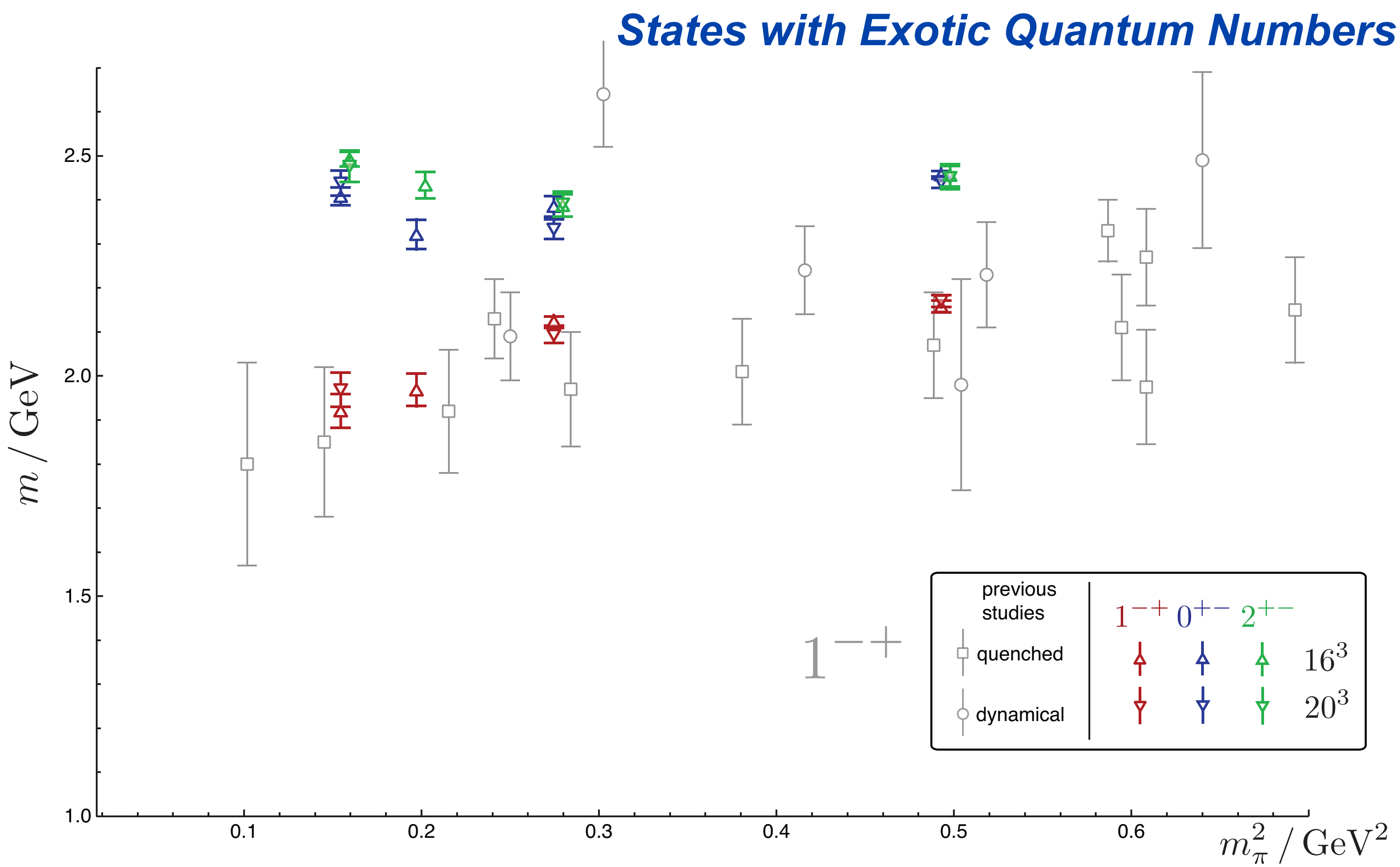
Isovector Meson Spectrum - I



PRL 103:262001,2009

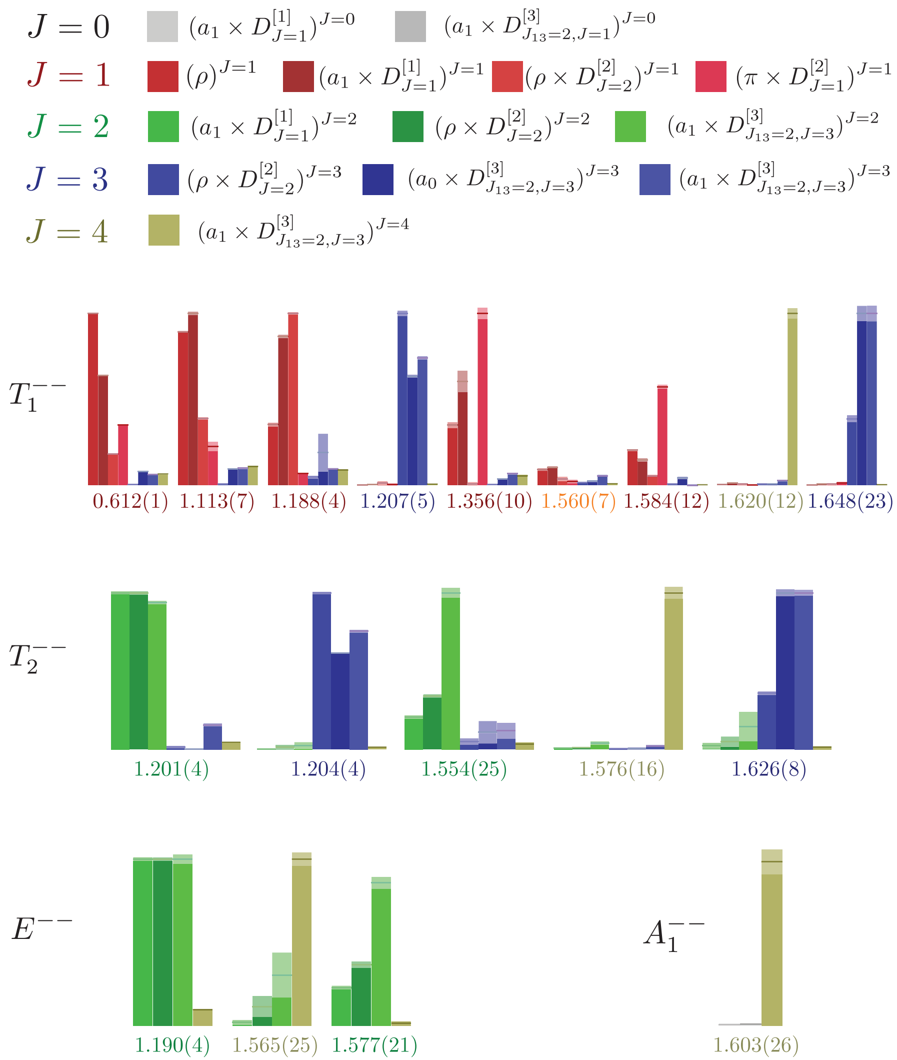
Isovector spectrum with quantum numbers reliably identified

Isovector Meson Spectrum - II



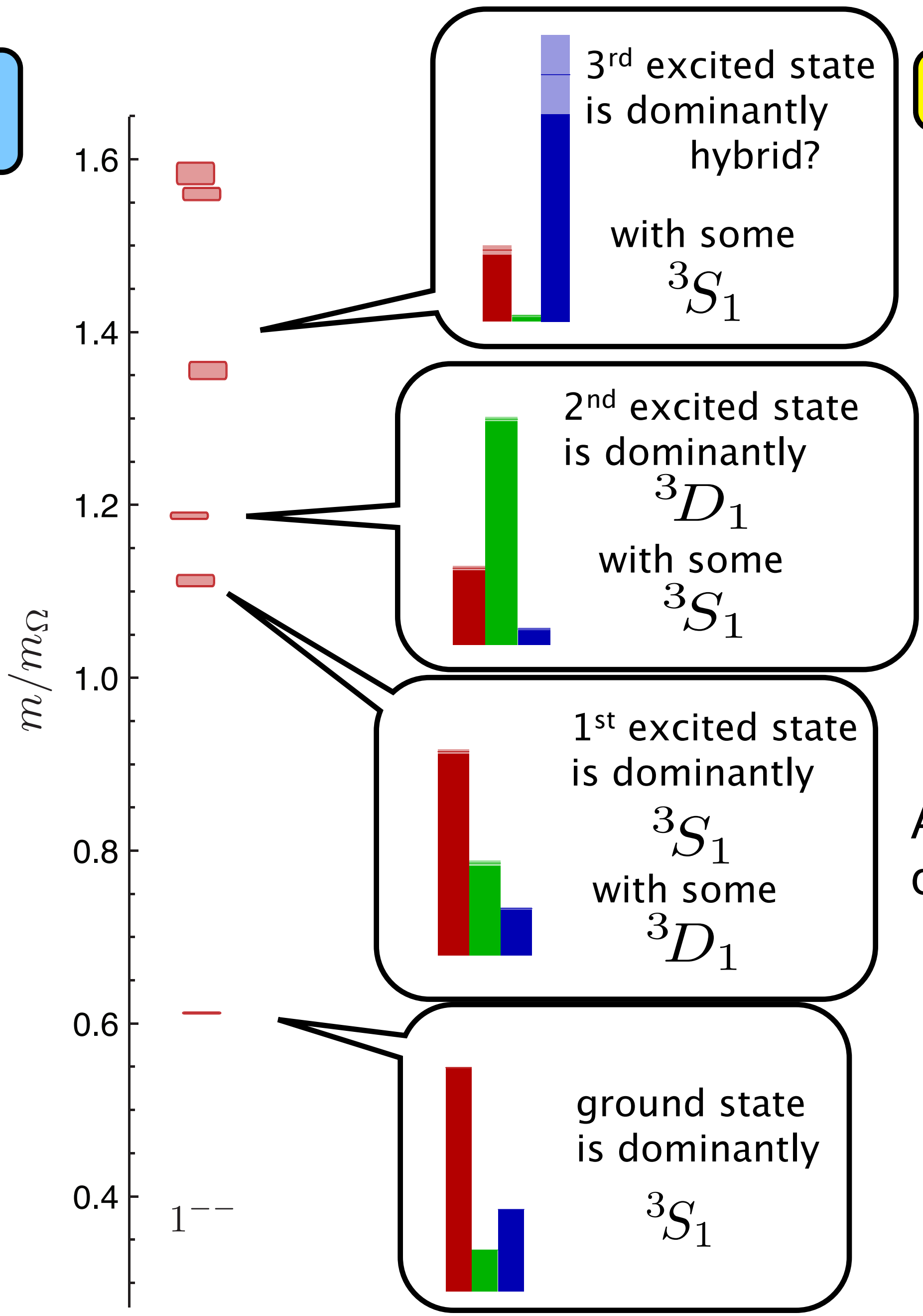
Dudek, Edwards, DGR, Thomas, arXiv:1004.4930

Interpretation of Meson Spectrum



In each Lattice Irrep, state dominated by operators of particular J

1^{--}



look at the 'overlaps' $Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi} \mathbf{\Gamma} \psi | 0 \rangle$

ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

Anti-commutator of covariant derivative: vanishes for unit gauge!

build a **bound state model** phenomenology comparable to the quark model using non-perturbative **QCD** calculations

Dudek, arXiv:1106.5515

Excited Baryon Spectrum - I

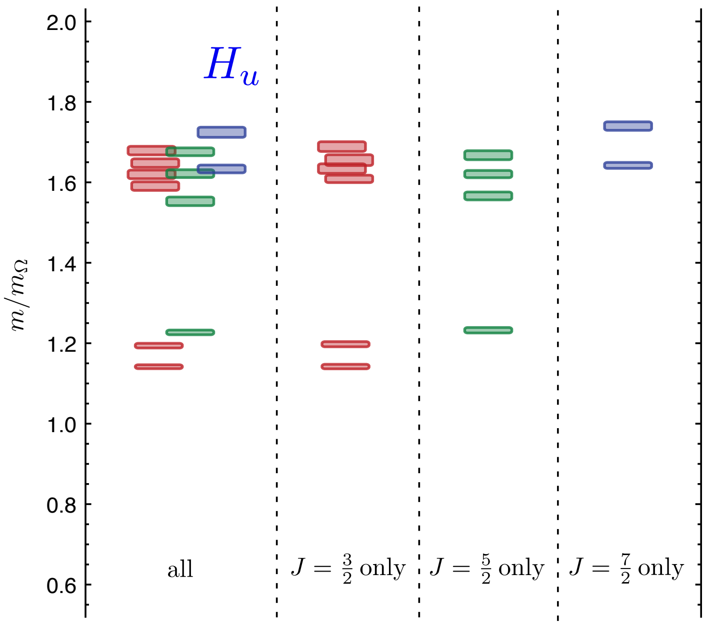
- **Construct basis of 3-quark interpolating operators in the continuum:**

$$\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{7}{2}}$$

"Flavor" x Spin x Orbital

- **Subduce to lattice irreps:**

$$\mathcal{O}^{[J]}_{n\Lambda,r} = \sum_{M} \mathcal{S}^{J,M}_{n\Lambda,r}\mathcal{O}^{[J,M]} \; : \Lambda = G_{1g/u}, H_{g/u}, G_{2g/u}$$



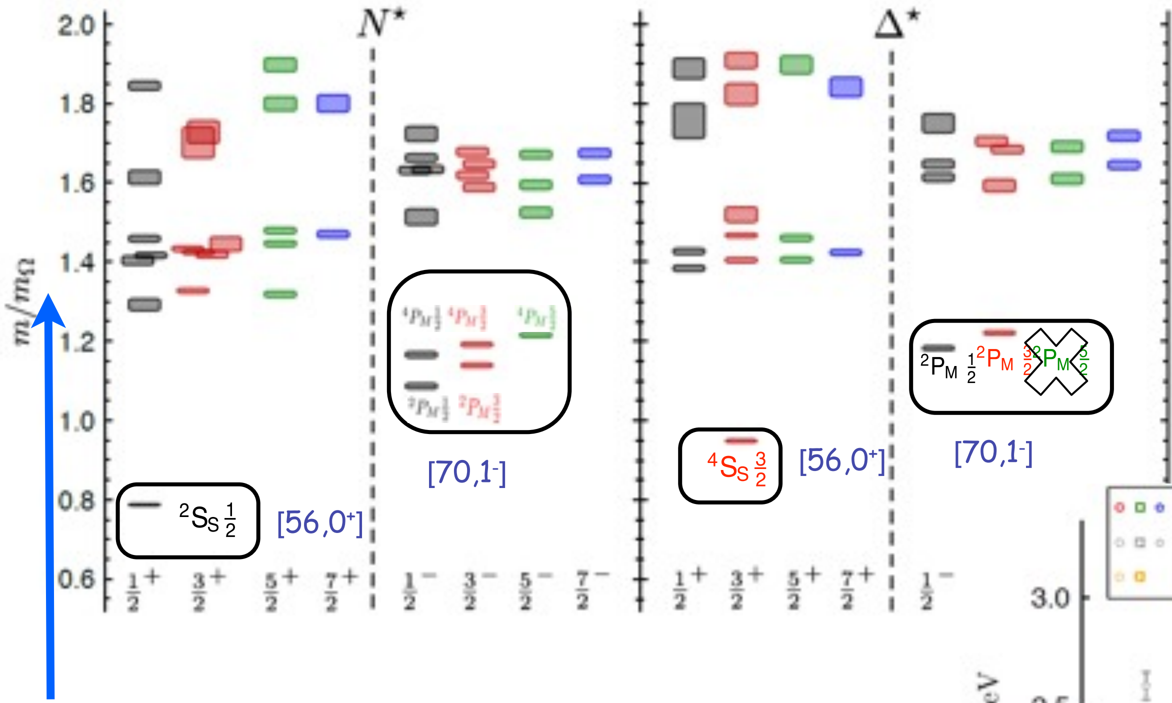
Continuum antecedents

R.G.Edwards et al., arXiv:1104.5152

$16^3 \times 128$ lattices $m_\pi = 524, 444$ and 396 MeV

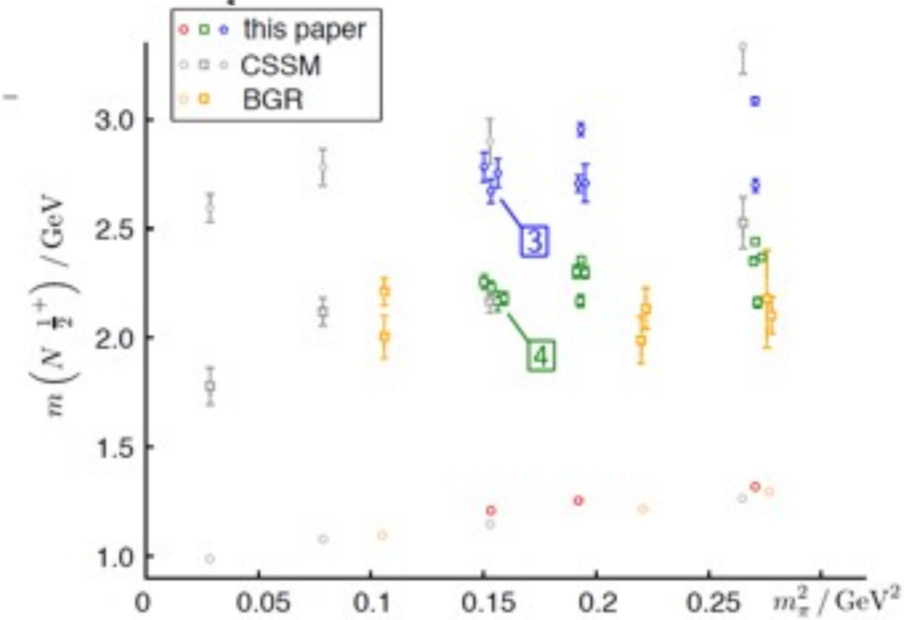
Observe remarkable realization of rotational symmetry at hadronic scale: *reliably determine spins up to 7/2, for the first time in a lattice calculation*

Excited Baryon Spectrum - II



- **Broad features of SU(6)xO(3) symmetry**
- **Counting of states consistent with NR quark model**
- **Inconsistent with quark-diquark picture or parity doubling**

$[70, 0^+], [56, 2^+], [70, 2^+], [20, 1^+]$



N $^{1/2+}$ sector: need for complete basis to faithfully extract states

Isoscalar Meson Spectrum - I

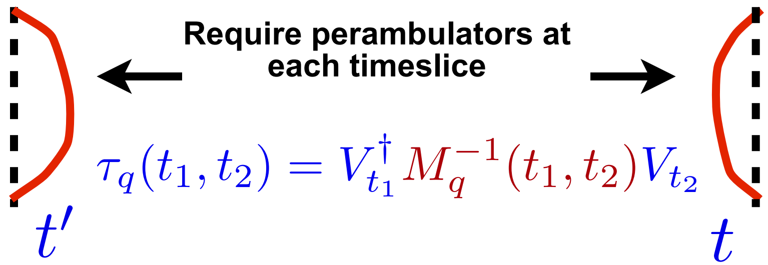
Connected components

$$\mathcal{C}^{q'q}_{AB}(t',t) = \delta_{qq'}\,\mathrm{Tr}\big[\Phi^A(t')\tau_{q'}(t',t)\Phi^B(t)\tau_q(t,t')\big],$$

disconnected components that can mix flavor,

$$\mathcal{D}^{q'q}_{AB}(t',t) = \mathrm{Tr}\big[\Phi^A(t')\tau_{q'}(t',t')\big]\;\mathrm{Tr}\big[\Phi^B(t)\tau_q(t,t)\big].$$

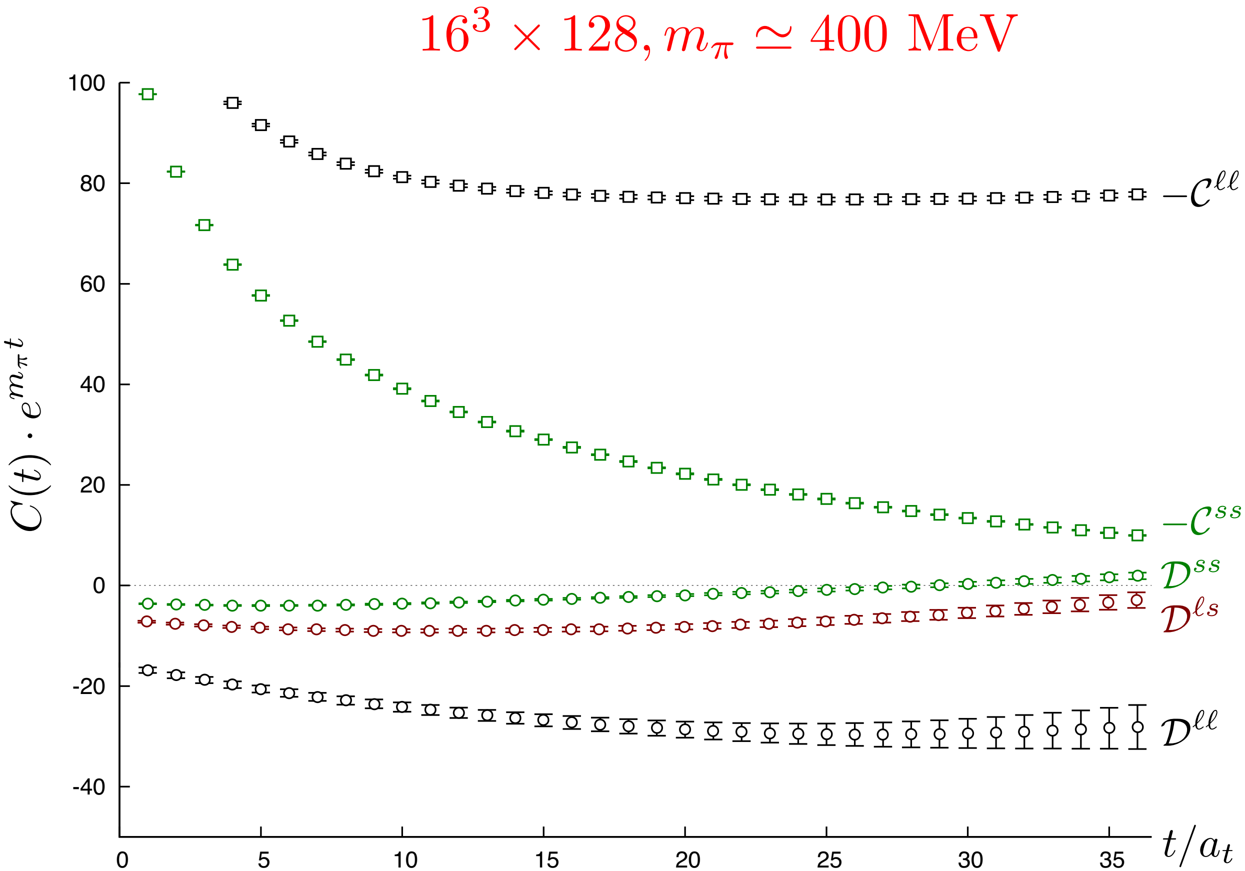
Isoscalar requires disconnected contributions



Dominated by quark-propagator inversions - ENABLED BY GPU

$$C = \begin{pmatrix} -\mathcal{C}^{\ell\ell} + 2\,\mathcal{D}^{\ell\ell} & \sqrt{2}\,\mathcal{D}^{\ell s} \\ \sqrt{2}\,\mathcal{D}^{s\ell} & -\mathcal{C}^{ss} + \mathcal{D}^{ss} \end{pmatrix}.$$

Isoscalar - II



From 32 time sources

From 128 time sources

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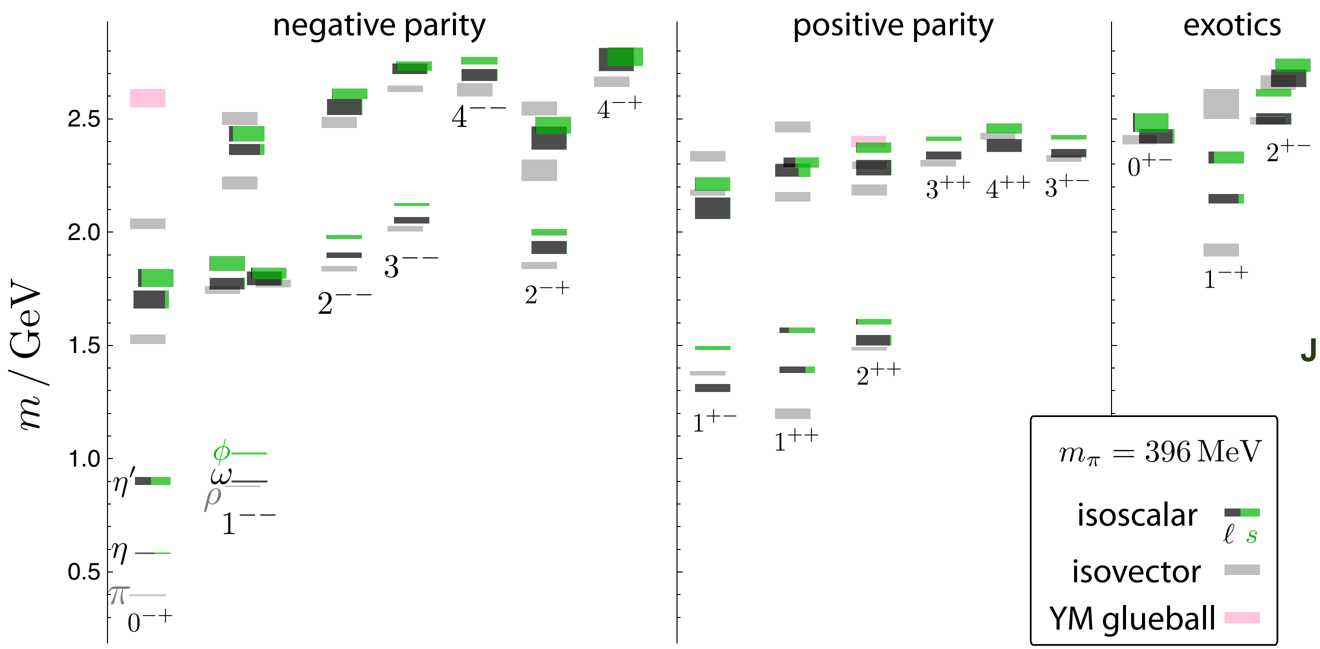
Isoscalar Meson Spectrum - III



J. Dudek *et al.*, PRD73, 11502

Diagonalize in 2x2 *flavor space*

- **Spin-identified single-particle spectrum: states of spin as high as four**
- **Hidden flavor mixing angles extracted - except 0^{-+}, 1^{++} near ideal mixing**
- ***First determination of exotic isoscalar states: comparable in mass to isovector***

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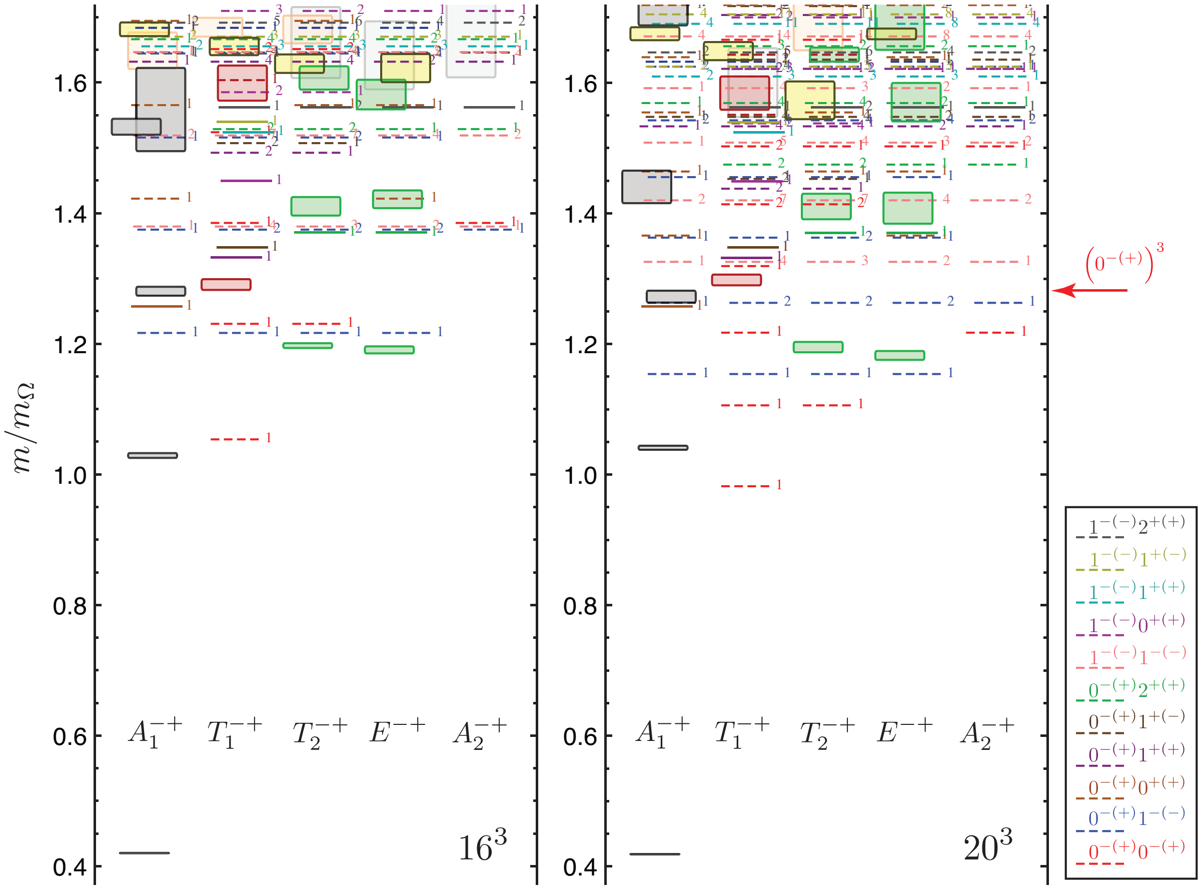
Where are the multi-hadrons?

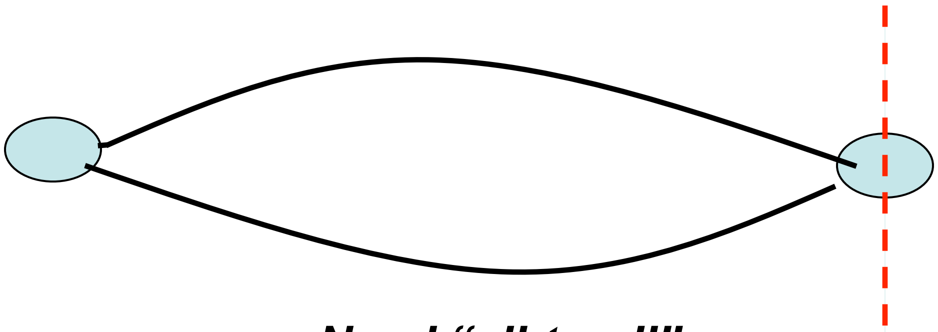


Meson spectrum on two volumes: dashed lines denote expected (non-interacting) multi-particle energies.

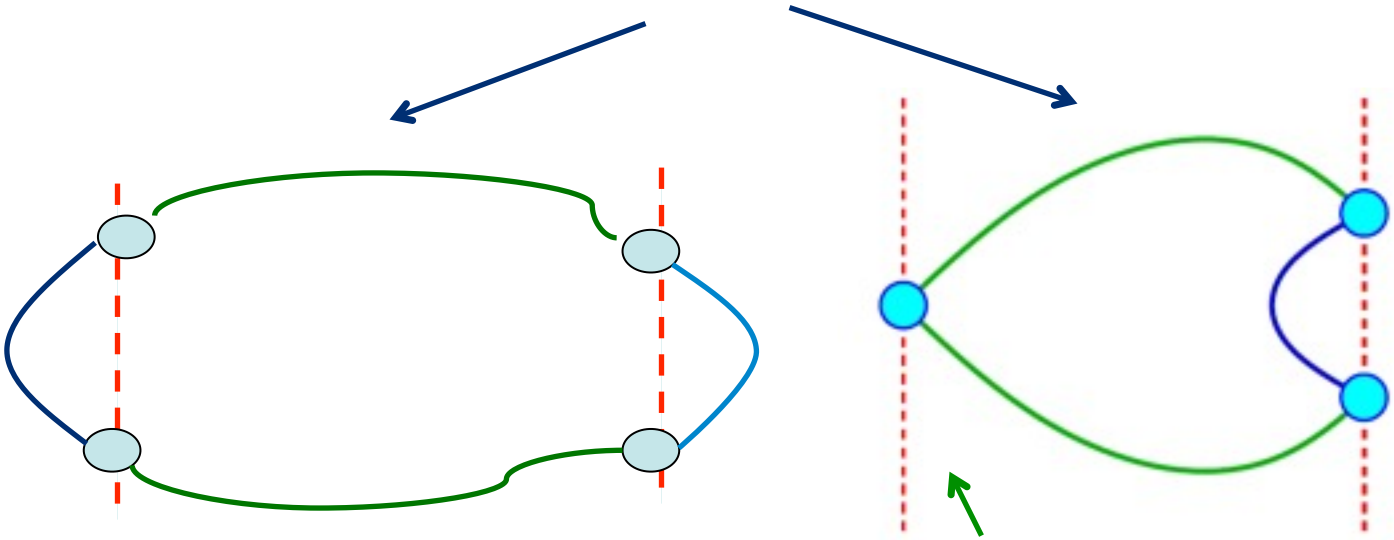
Allowed two-particle contributions governed by cubic symmetry of volume

Calculation is incomplete.

Multi-hadron Operators



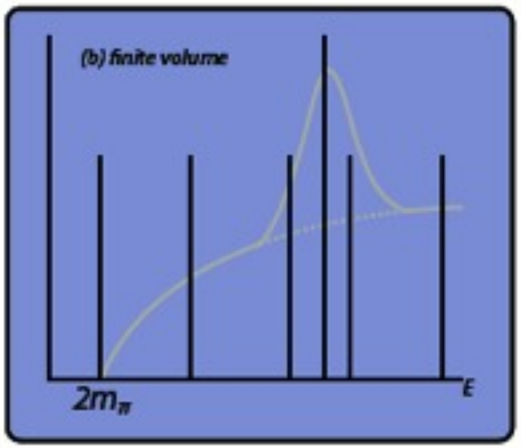
Need "all-to-all"



Usual methods give "point-to-all"

Strong Decays

- In QCD, even ρ is unstable under strong interactions – *resonance in π-π scattering (quenched QCD not a theory – won't discuss).*
- Spectral function continuous; finite volume yields discrete set of energy eigenvalues



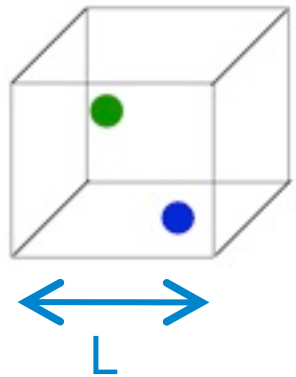
Momenta quantised: known set of free-energy eigenvalues

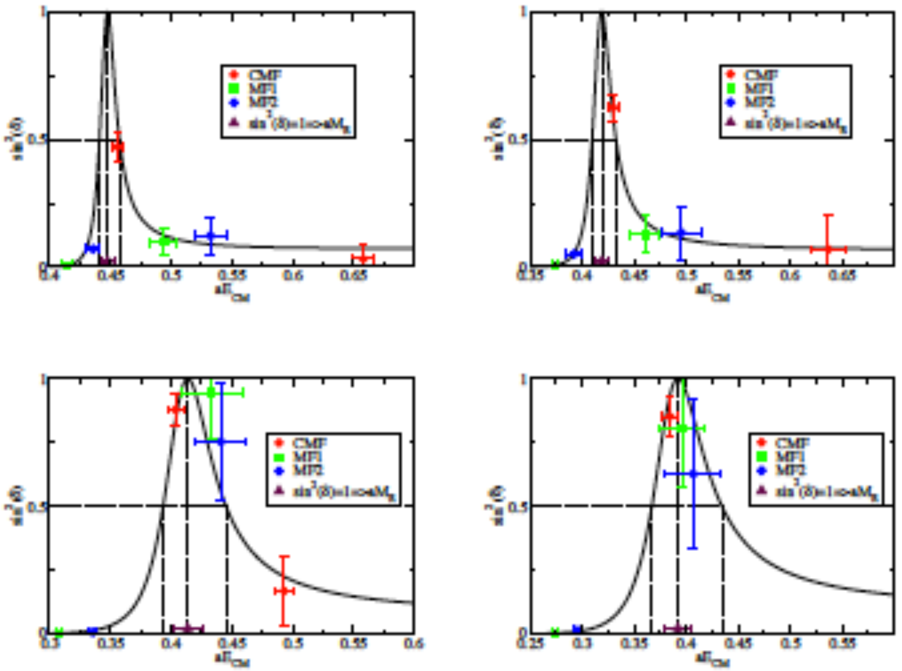
$$E_n = 2\sqrt{m_\pi^2 + (\frac{2n\pi}{L})^2}$$

Strong Decays - II

- For interacting particles, energies are shifted from their free-particle values, by an amount that depends on the energy.
- Luscher: relates shift in the free-particle energy levels to the phase shift at the corresponding E.

$$\delta E(L) \leftrightarrow \delta(E)$$





$$\tan\delta_1 = \frac{{g_{\rho\pi\pi}}^2}{6\pi}\frac{p^3}{E_{\mathrm{CM}}({m_\rho}^2 - E_{\mathrm{CM}}^2)}$$

$$p = \sqrt{E_{\mathrm{CM}}^2/4 - m_\pi^2}$$

Feng, Jansen, Renner, 2010

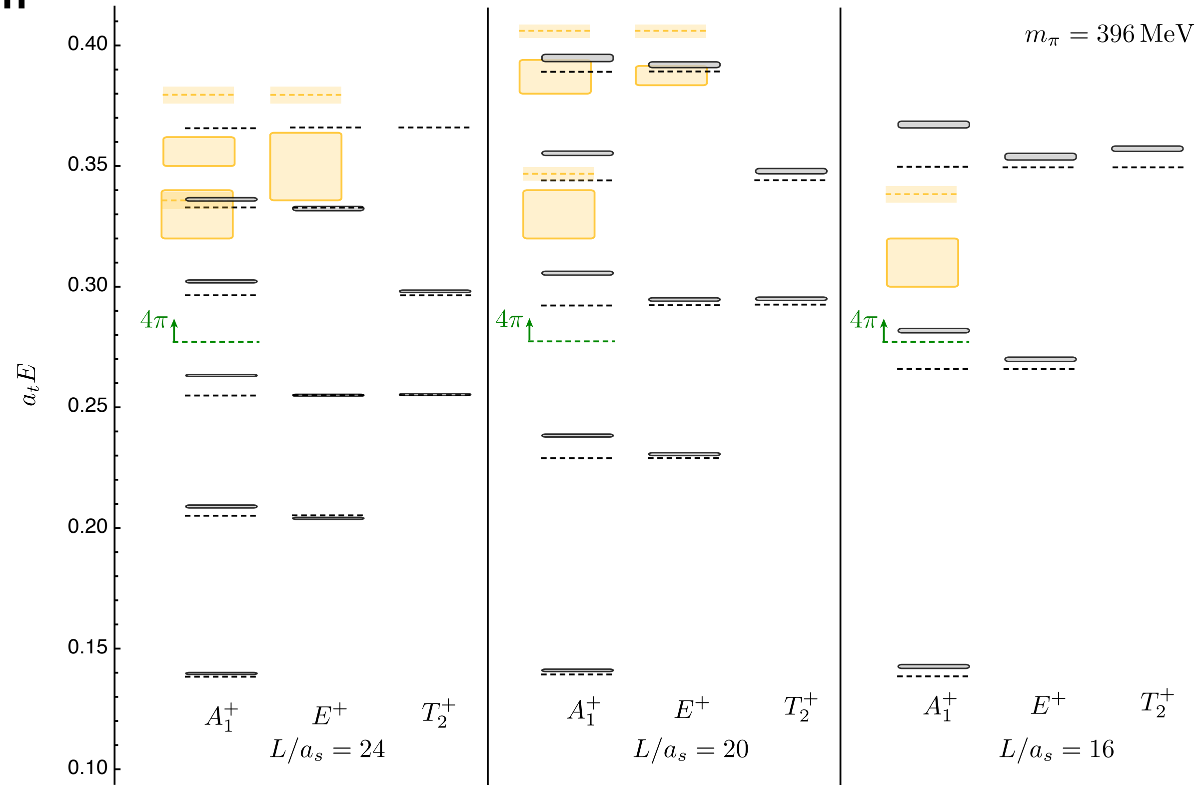
Momentum-dependent I = 2 $\pi\pi$ Phase Shift

Dudek *et al.*, Phys Rev D83, 071504 (2011)

Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

Operator basis $$\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_m \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^m_\ell(\hat{p})\, \mathcal{O}_\pi(\vec{p})\mathcal{O}_\pi(-\vec{p})$$

Total momentum zero - pion momentum ±*p*



Momentum-dependent I = 2 ππ Phase Shift

Dudek *et al.*, Phys Rev D83, 071504 (2011)

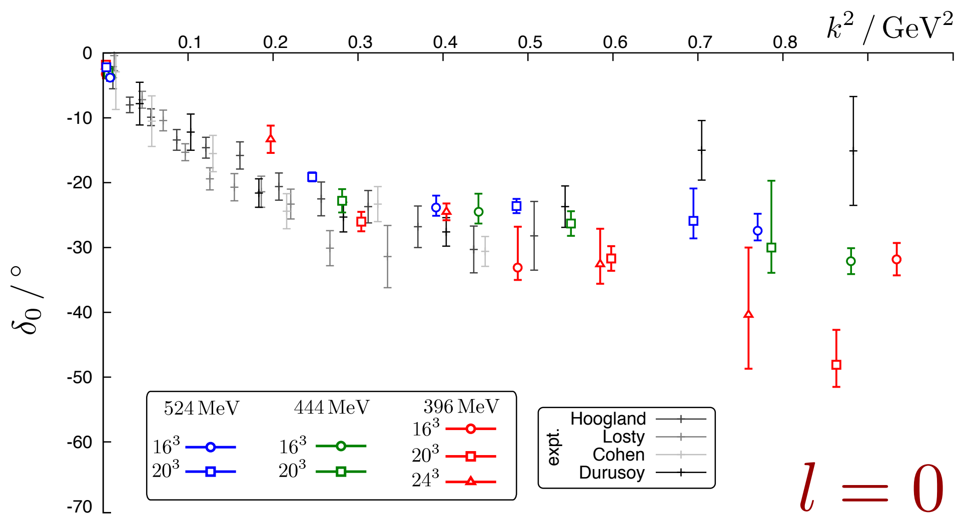
Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

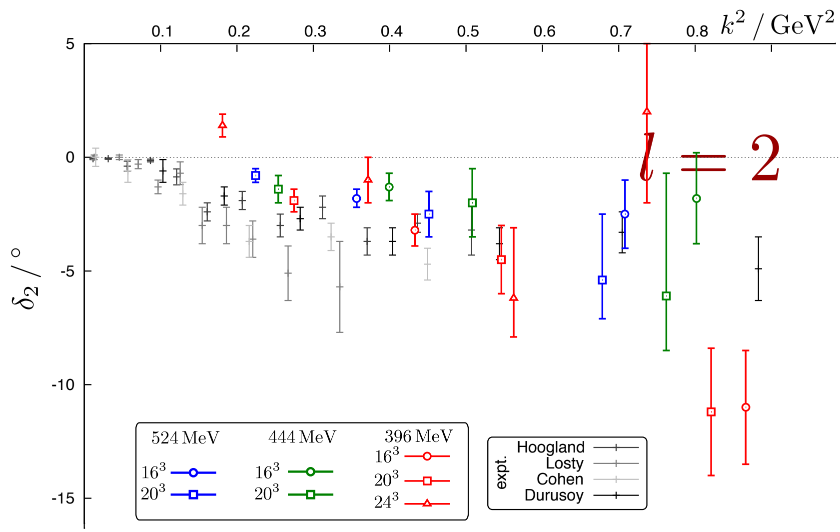
$$\det\left[e^{2i\boldsymbol{\delta}(k)} - \mathbf{U}_\Gamma\left(k\frac{L}{2\pi}\right)\right] = 0$$

Matrix in l

lattice irrep

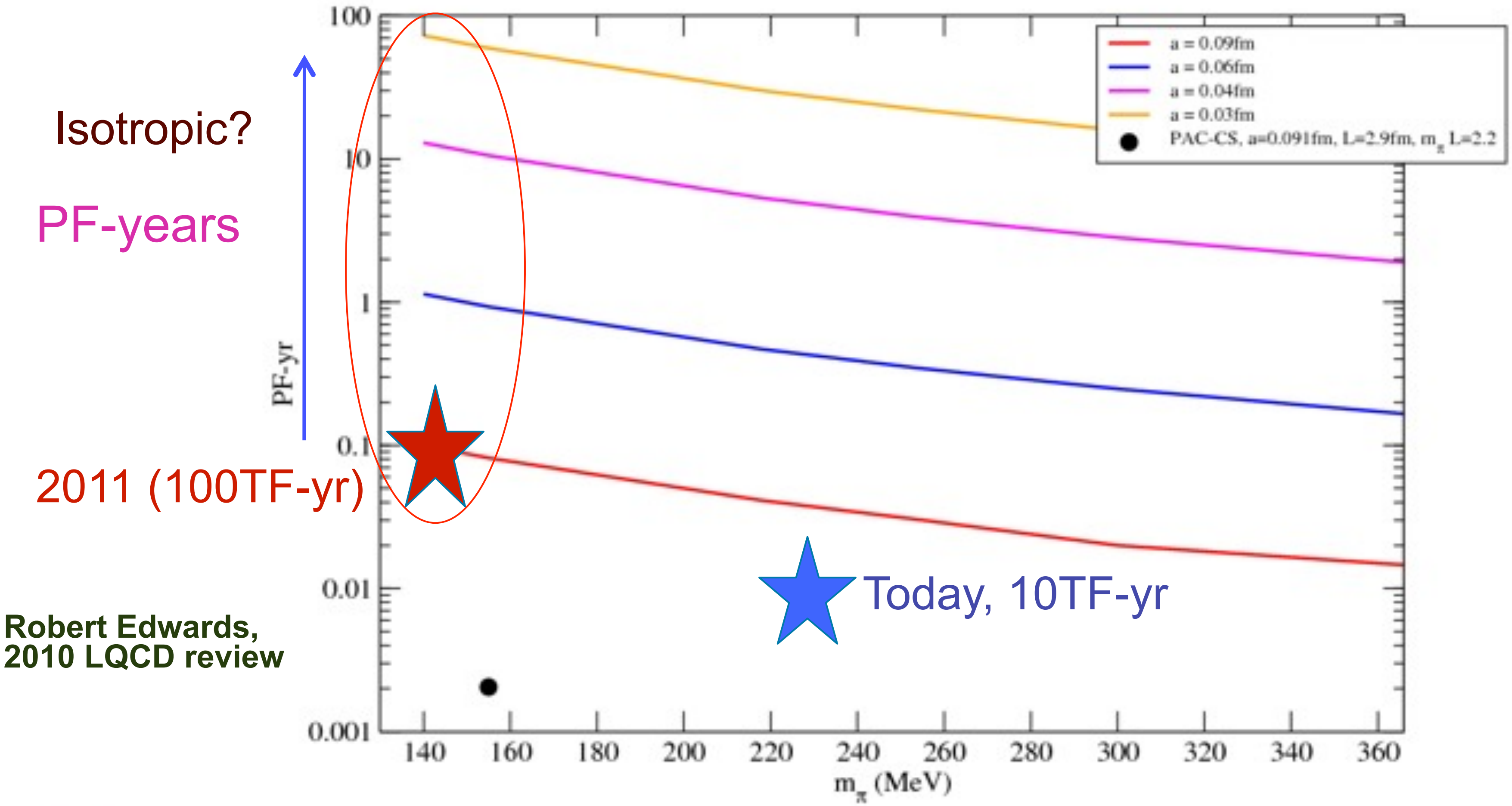
4π at $m_\pi = 396$ MeV





Gauge Generation: Cost Scaling

- Cost: reasonable statistics, box size and "physical" pion mass
- Extrapolate in lattice spacings: 10 ~ 100 PF-yr



Capability vs Capacity: GPUs

- **Gauge generation: (next dataset)**
  - **INCITE: Crays BG/P-s, ~ 16K – 24K cores**
  - **Double precision**
- **Analysis (existing dataset): two-classes**
  - **Propagators (Dirac matrix inversions)**
    - **Few GPU level**
    - **Single + half precision**
    - **No memory error-correction**
  - **Contractions:**
    - **Clusters: few cores**
    - **Double precision + large memory footprint**

~ 5 Tflop-years

Capability

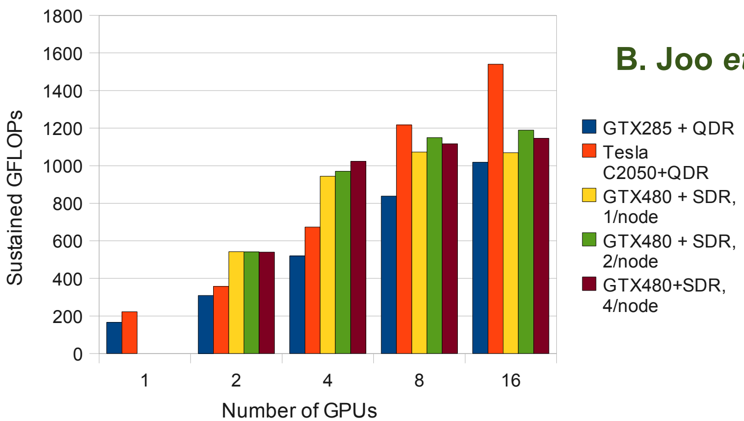
~ 20-30 Tflop-years

Capacity

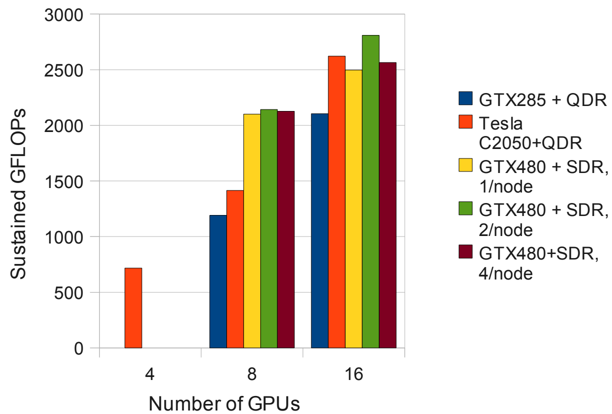
~ 1 Tflops

2010-2011 lattices: $24^3 \times 128$

2011-2012 lattices: $32^3 \times 256$



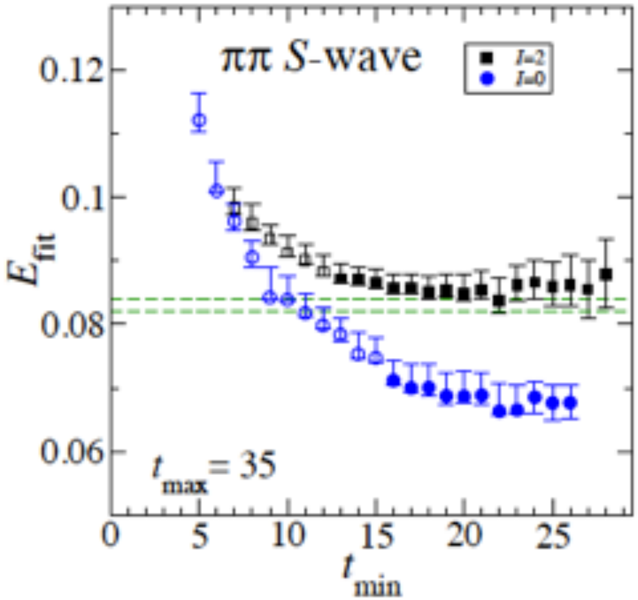
B. Joo *et al*, SciDAC 2010



Capacity Computing

- Calculation of isoscalars and pi-pi scattering enabled by GPUs - for calculation of perambulators
- Contraction costs increasingly dominant

$$\begin{aligned} N_{\mathrm{eigen}} &\simeq \mathrm{Vol} && \text{GPUs} \\ \text{Meson contractions} &\simeq N_{\mathrm{eigen}}^{3} && \\ \text{Baryon contractions} &\simeq N_{\mathrm{eigen}}^{4} && \text{CPUs} \end{aligned}$$



e.g. Stochastic sampling of distillation vectors

Morningstar *et al.*, PRD83, 114505

Summary

- Spectroscopy of excited states affords an excellent theatre in which to study QCD in low-energy regime.
- Major progress at reliable determinations of the *single-particle* spectrum, *with quantum numbers identified*
- Lattice calculations used to construct new "phenomenology" of QCD
- Next step for lattice QCD:
  - Complete the calculation: where are the multi-hadrons and decay channels?
  - Determine the *phase shifts* - model independent
  - extraction of resonance parameters - model dependent
- Lattice calculations: gauge generation ➡ physics measurement