Equation of State of Neutron Matter: Implications for Neutron Stars and Supernova Neutrinos

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Work relating to neutron stars:
Joe Carlson, Stefano Gandolfi

Work relating to supernova neutrinos:
Vincenzo Cirigliano, Jose Pons, Luke Roberts, Gang Shen, Stan Woosley

What can we observe?

- Spin
- Surface Luminosity
- Orbital Characteristics
- Explosions & Flares
- Neutrinos
- Gravitational Waves

What can we infer ?

Hard Physics

- Mass
- Radius
- Crust thickness
- Oscillations frequencies

Ground state EoS

Soft Physics

- Surface and interior temperature
- Neutrino cooling and scattering rates
- Conductivities
- Damping rates

Low energy fluctuations

Neutron Star Masses

Measurement of relativistic effects in compact binaries permits accurate mass measurements

Massive neutron stars provide a very useful constraint on the high density EoS.

Eg. $M > 2\ M_{solar}$

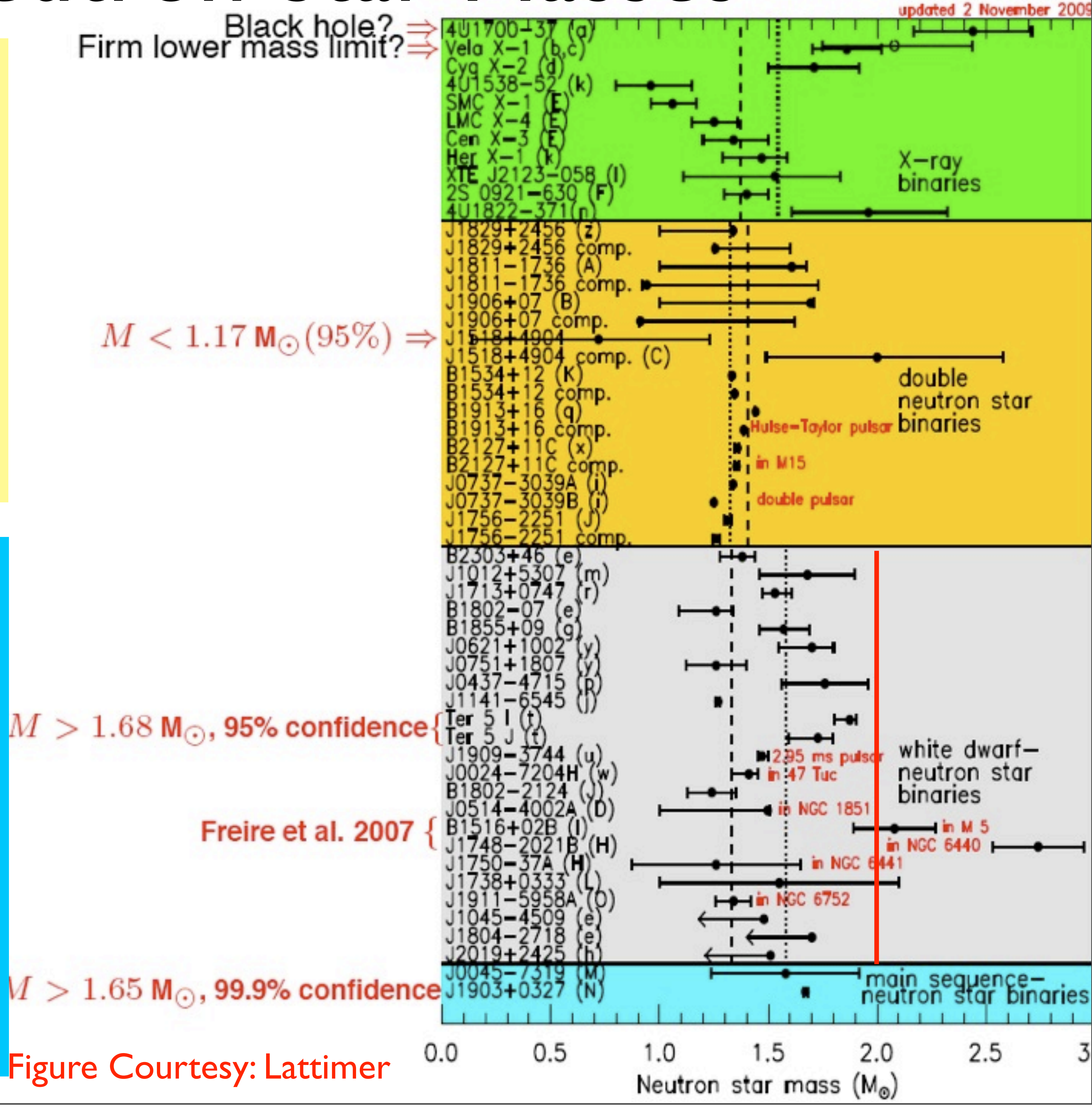
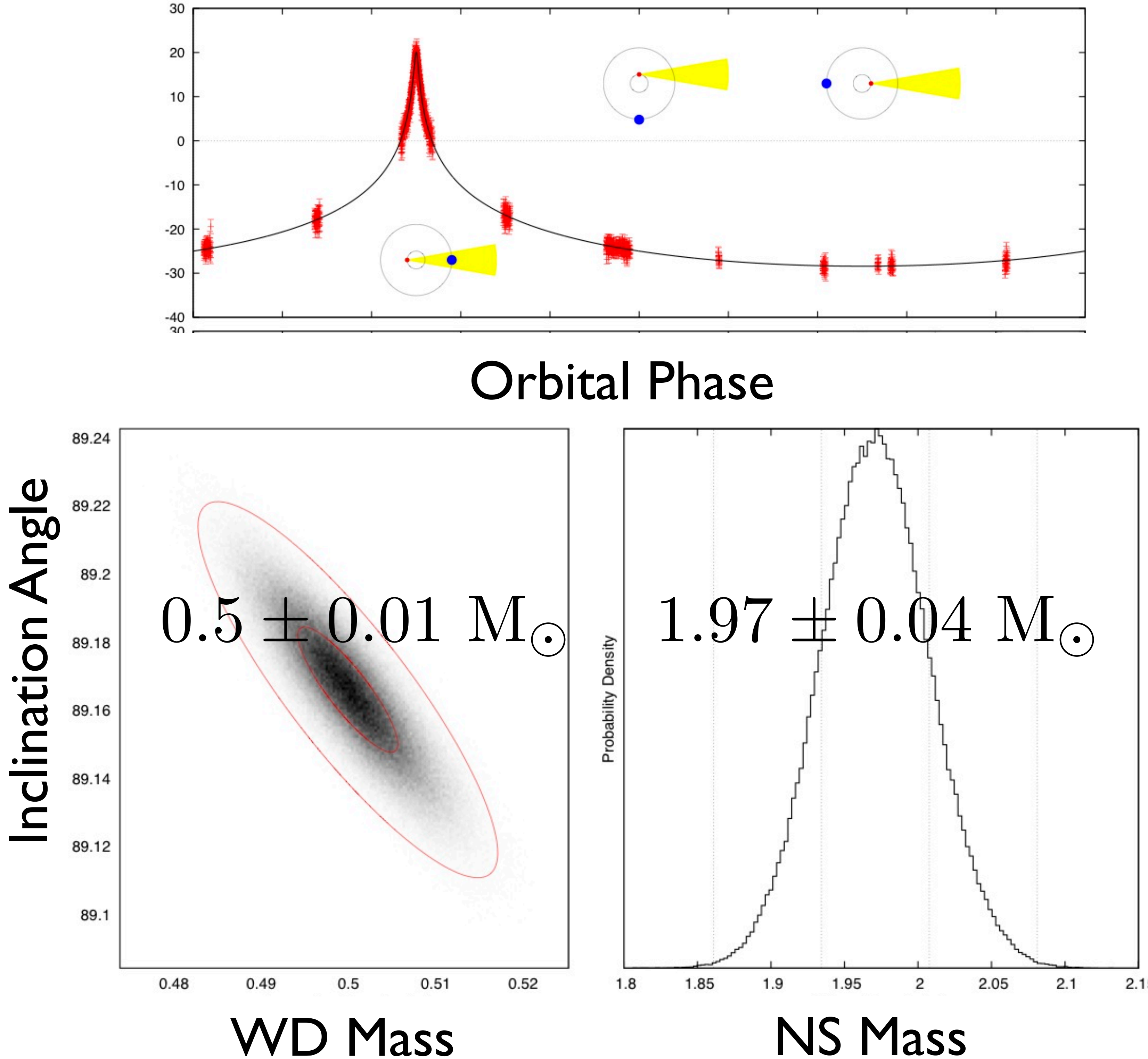


Figure Courtesy: Lattimer

Finally, a 2 $M_\odot$ Neutron Star !

- NS-WD System (J1614-2230)
- Shapiro delay measured in NS + WD system.
- Very accurate measurement of the NS mass: $M = 1.97 \pm 0.04\, M_\odot$

Demorest et al. 2010



Radius

For a black body the observed flux :

$$
\begin{aligned}
F_{\rm BB} &= 4\pi \, \frac{R_\infty^2}{d^2} \, \sigma_{SB} \, T_\infty^4 \\
R_\infty &= \frac{R}{\sqrt{1 - R_S/R}} \\
T_\infty &= \sqrt{1 - R_S/R} \quad T
\end{aligned}
$$

For NSs: Atmosphere and magnetic fields can affect the spectra

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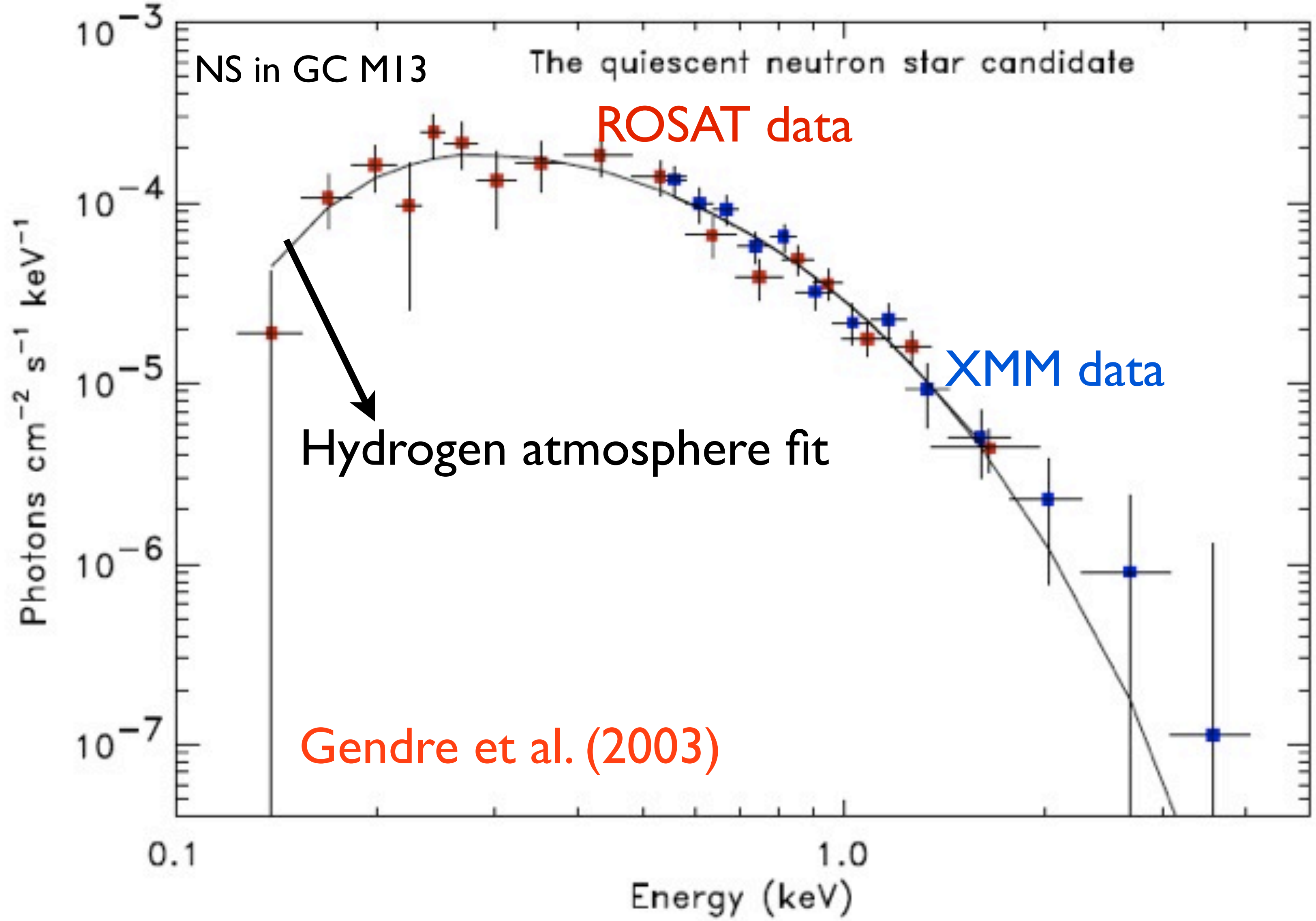
$$R_\infty = \frac{R}{\sqrt{1 - R_S/R}}$$

$$T_\infty = \sqrt{1 - R_S/R} \quad T$$

For a non-magnetized hydrogen atmosphere:

Spectra is easily modeled - Can extract R_∞



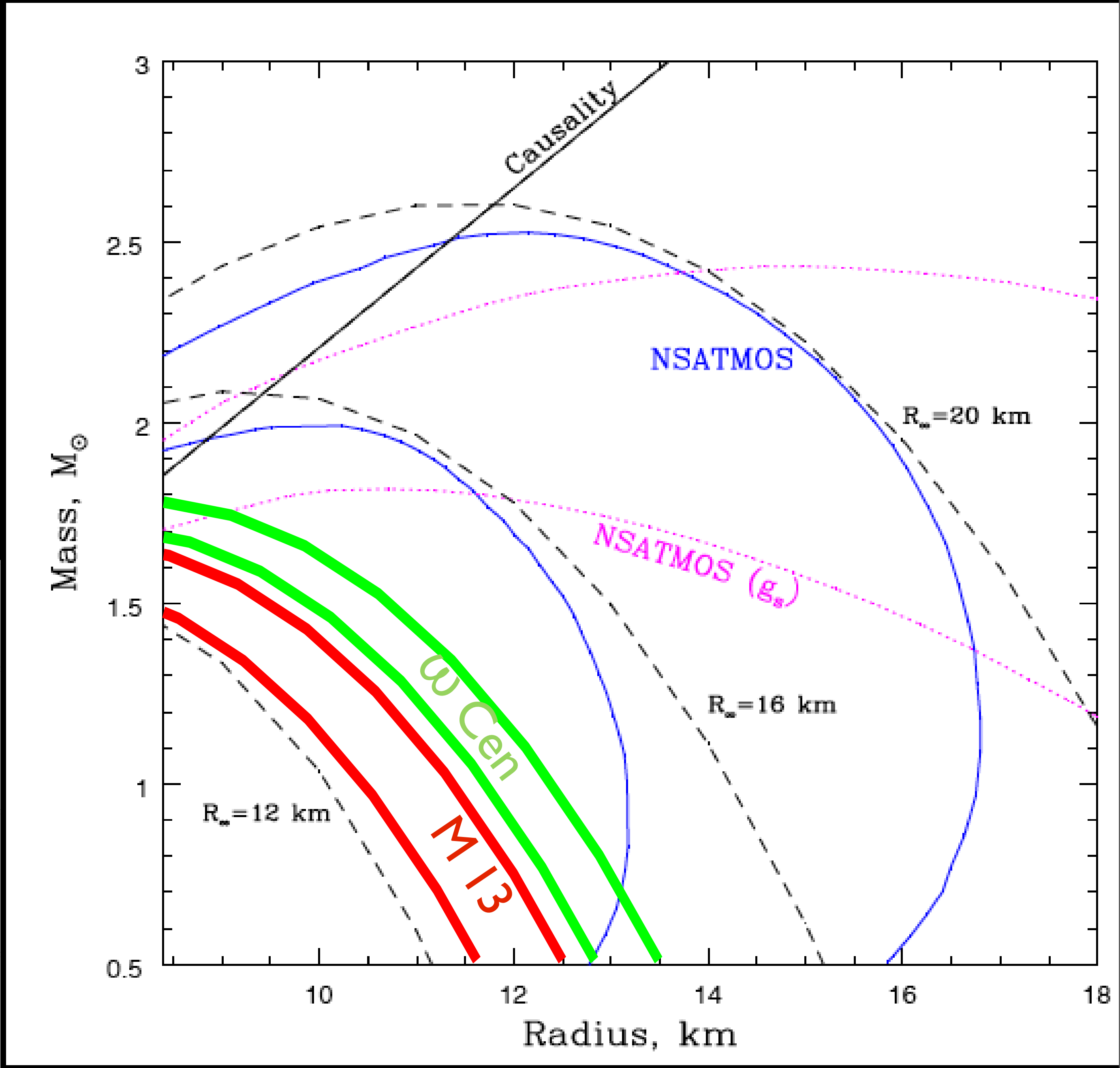
Quiescent NSs in LMXB

Transiently accreting neutron stars in globular clusters:

1. Hydrogen atmosphere
2. Negligible Magnetic Fields
3. Distances are known

Rutledge et al. (2004)

NS	R∞	Ref.
ω Cen	13.6± 0.3	Gendre et al. (2002)
M13	12.6± 0.4	Gendre et al. (2002)
X7*	14.5 + 1.6-1.4	Heinke et al. (2005)
M28	14.5+6.8-3.9	Becker et al. (2003)



3 more found in GC : NGC 6304

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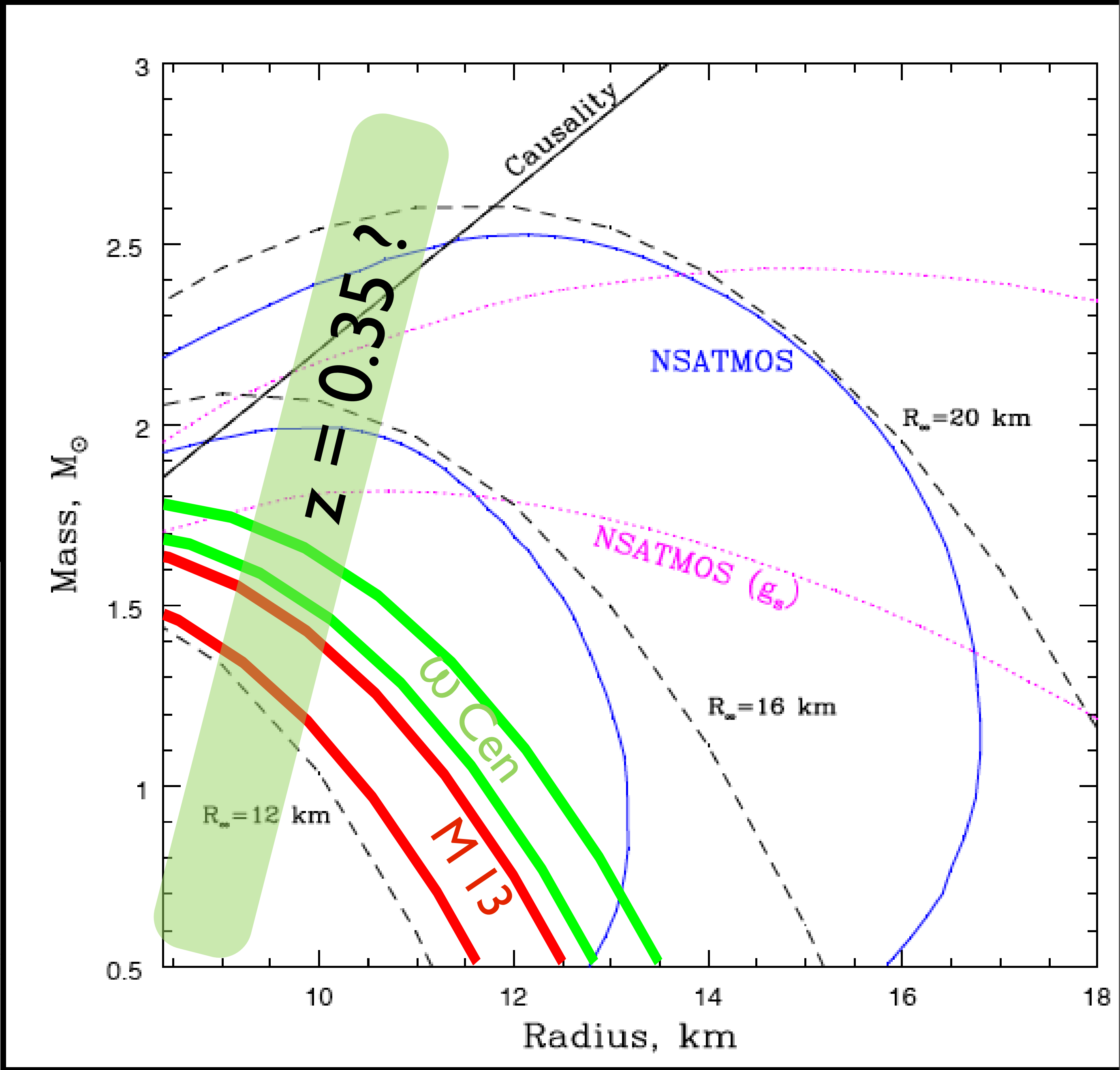

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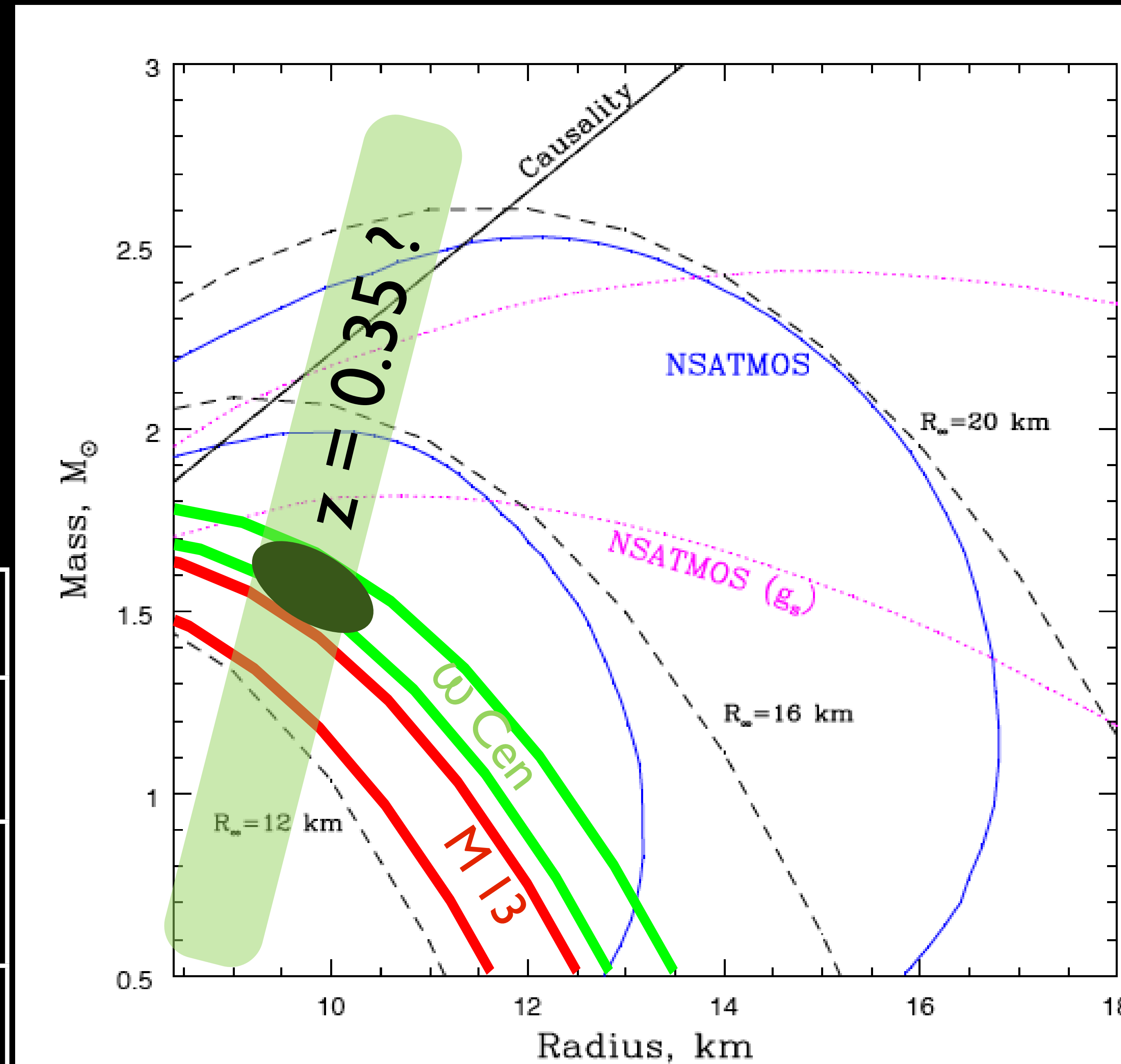

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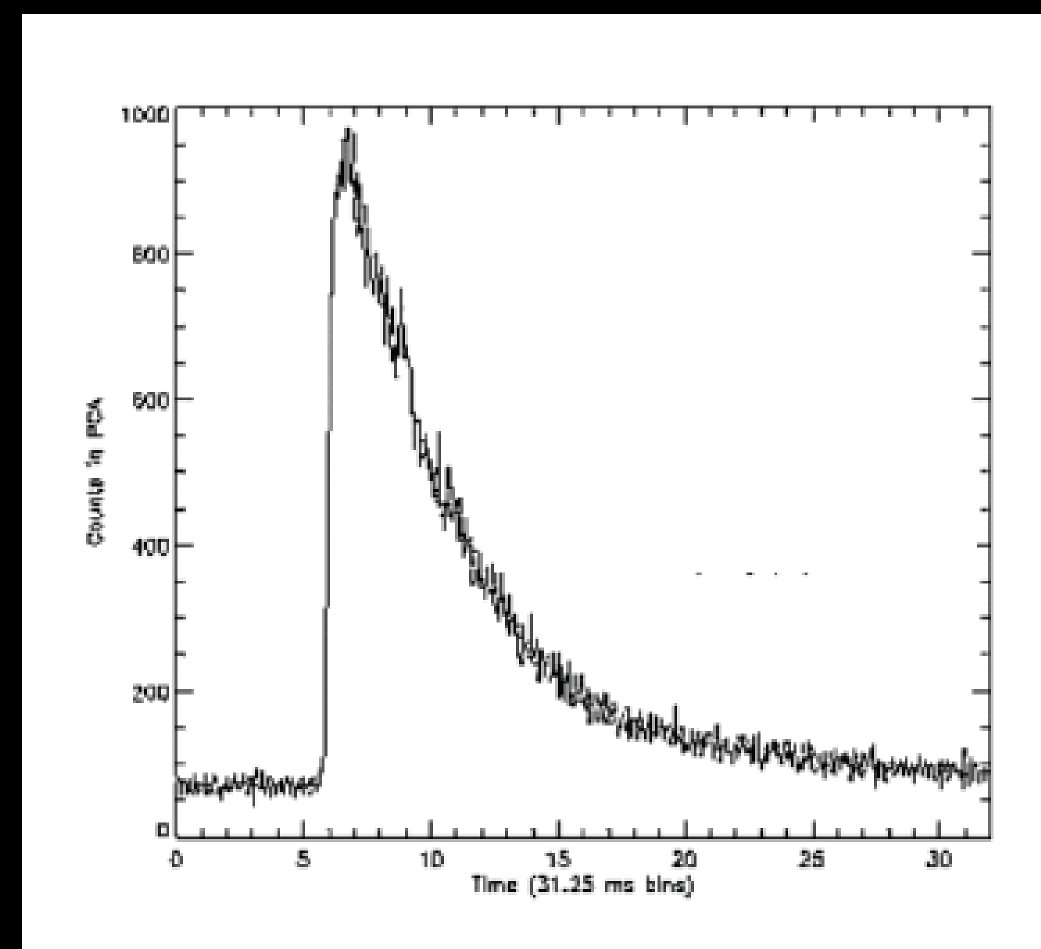
Accreting Neutron Stars: Bursts

Unstable burning of accreted material produces x-ray bursts

- Most common cosmic explosion in the universe.
- Light curve powered by nuclear reactions (rp -process).
- Features in the light curve are be sensitive to Mass and Radius.



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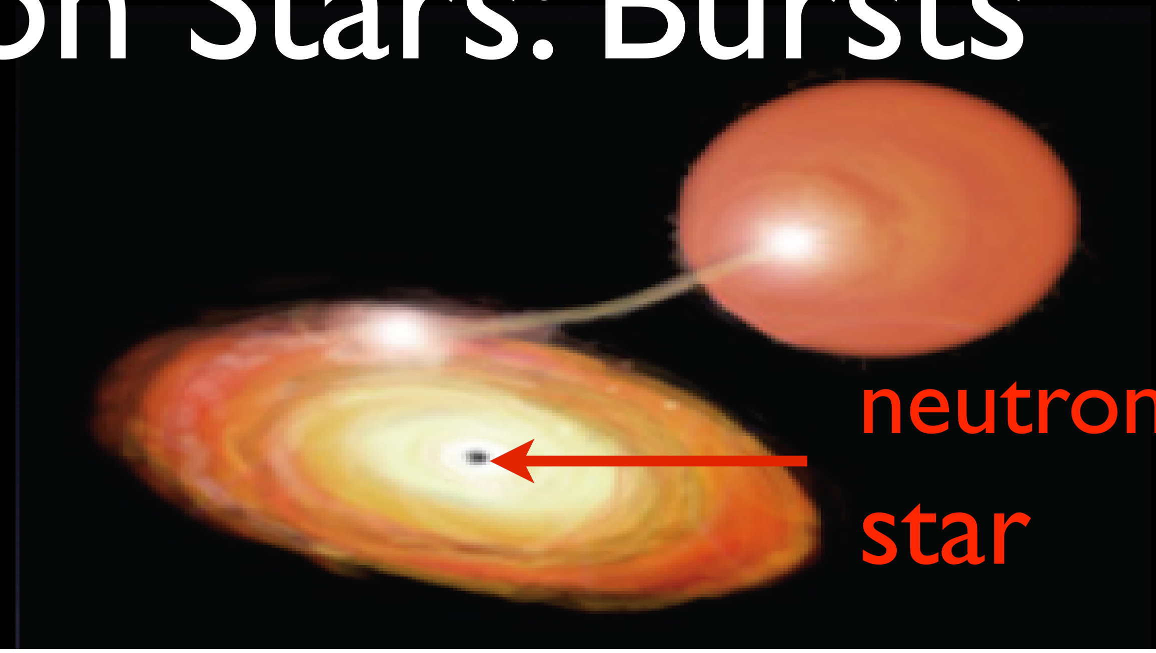

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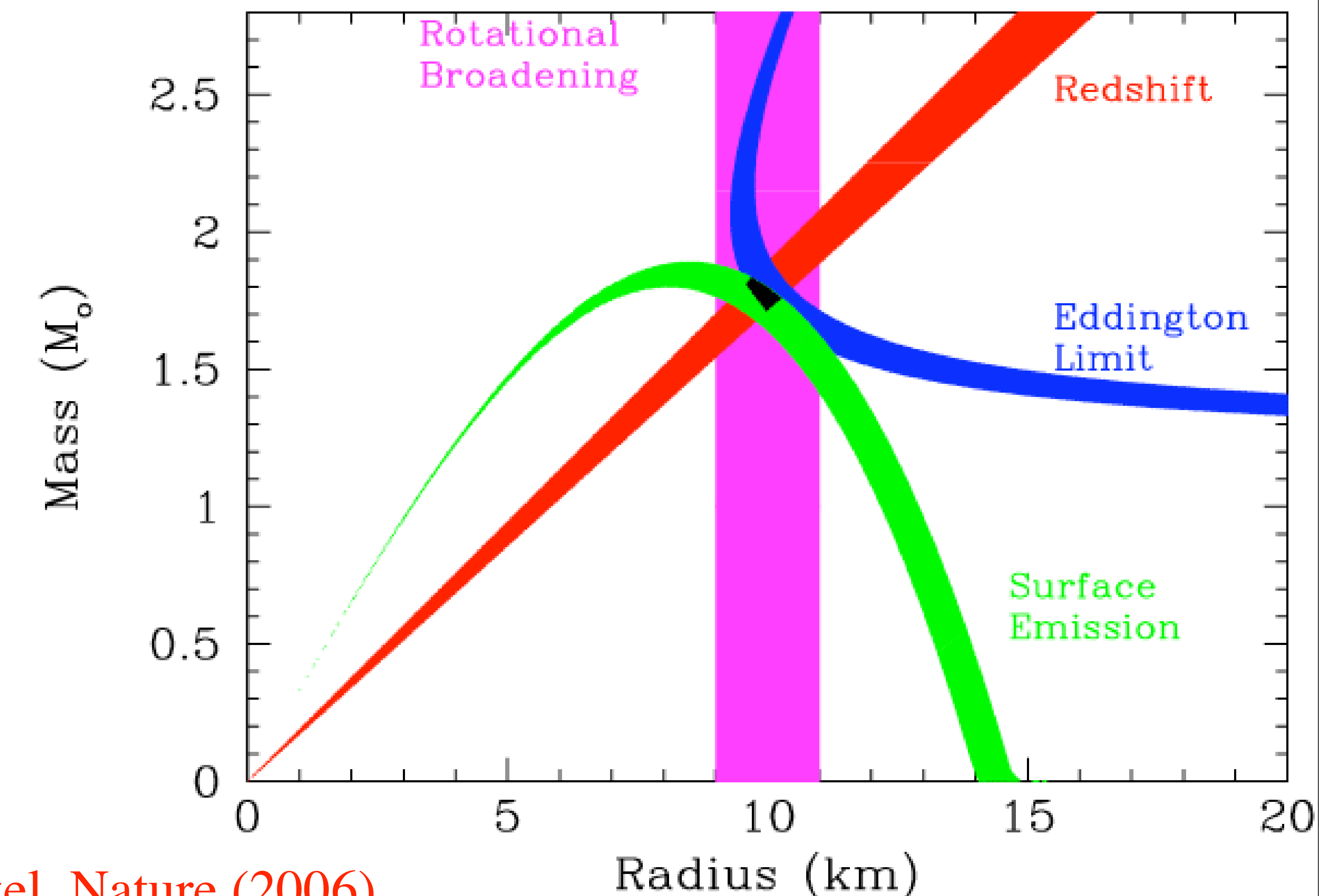
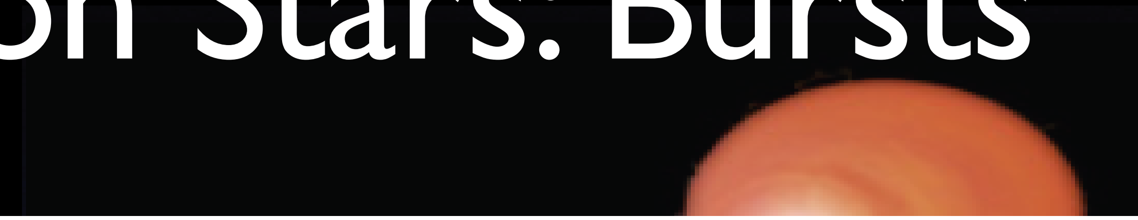


Ozel, Nature (2006)

Accreting Neutron Stars: Bursts



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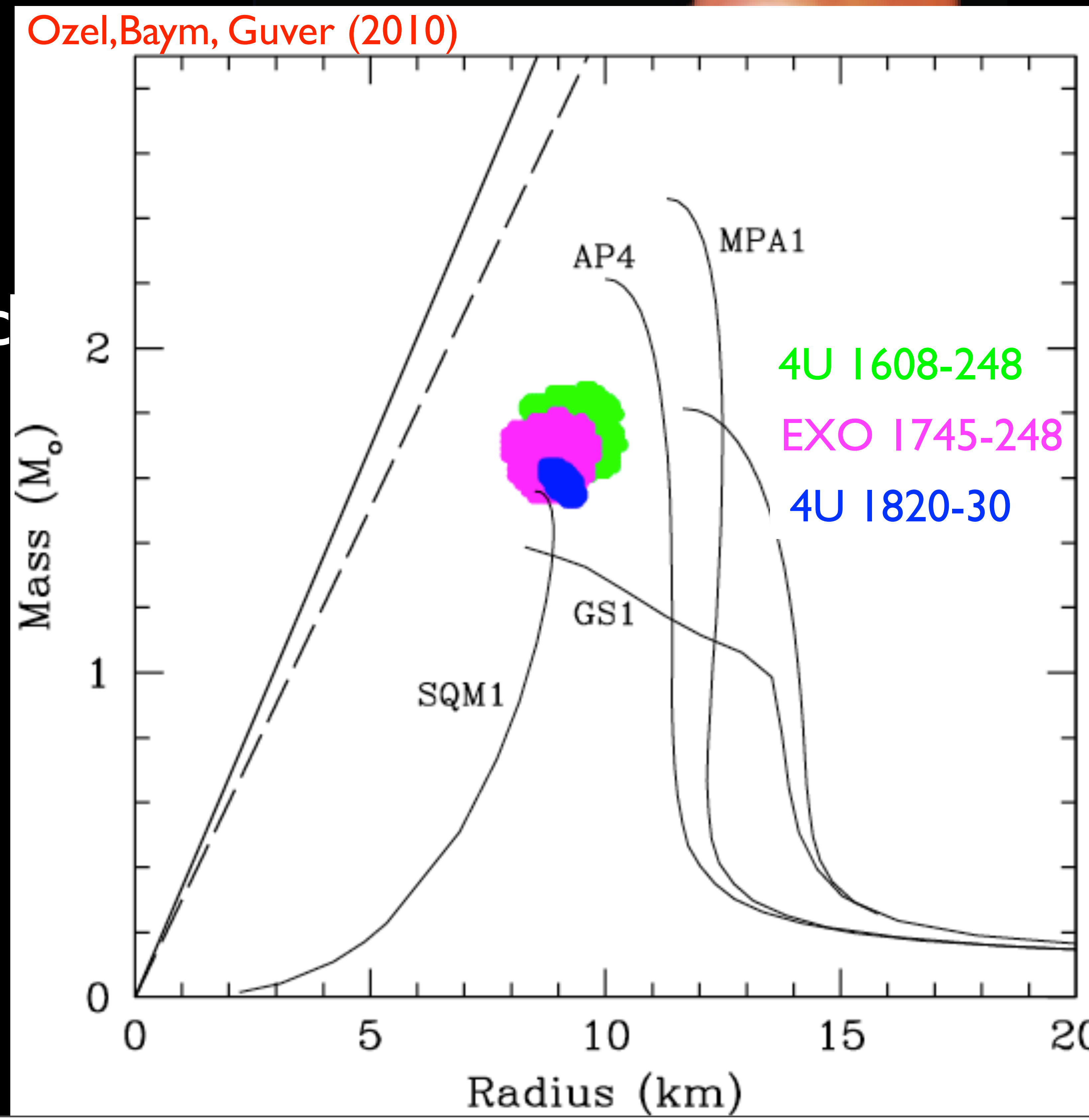

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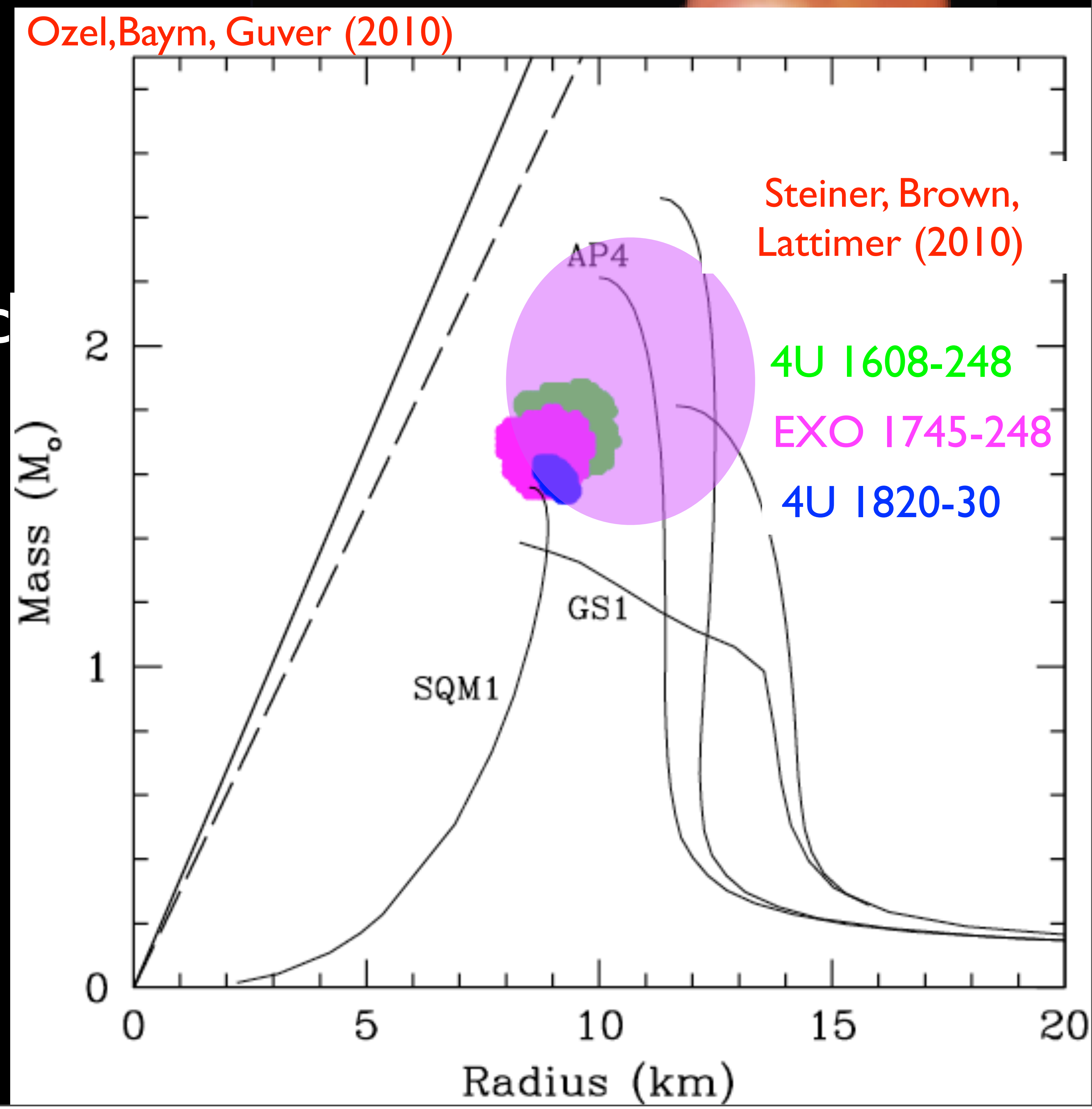

Mass-Radius Constraints from 6 NSs

Object	M ($M_\odot$)	R (km)	M ($M_\odot$)	R (km)
	$r_{ph} = R$		$r_{ph} \gg R$	
4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
EXO 1745–248	$1.55^{+0.12}_{-0.36}$	$10.91^{+0.86}_{-0.65}$	$1.34^{+0.450}_{-0.28}$	$11.82^{+0.47}_{-0.72}$
4U 1820–30	$1.57^{+0.13}_{-0.15}$	$10.91^{+0.39}_{-0.92}$	$1.57^{+0.37}_{-0.31}$	$11.82^{+0.42}_{-0.82}$
M13	$1.48^{+0.21}_{-0.64}$	$11.04^{+1.00}_{-1.28}$	$0.901^{+0.28}_{-0.12}$	$12.21^{+0.18}_{-0.62}$
ω Cen	$1.43^{+0.26}_{-0.61}$	$11.18^{+1.14}_{-1.27}$	$0.994^{+0.51}_{-0.21}$	$12.09^{+0.27}_{-0.66}$
X7	$0.832^{+1.19}_{-0.051}$	$13.25^{+1.37}_{-3.50}$	$1.98^{+0.10}_{-0.36}$	$11.3^{+0.95}_{-1.03}$

Steiner, Lattimer, Brown, ApJ (2010)

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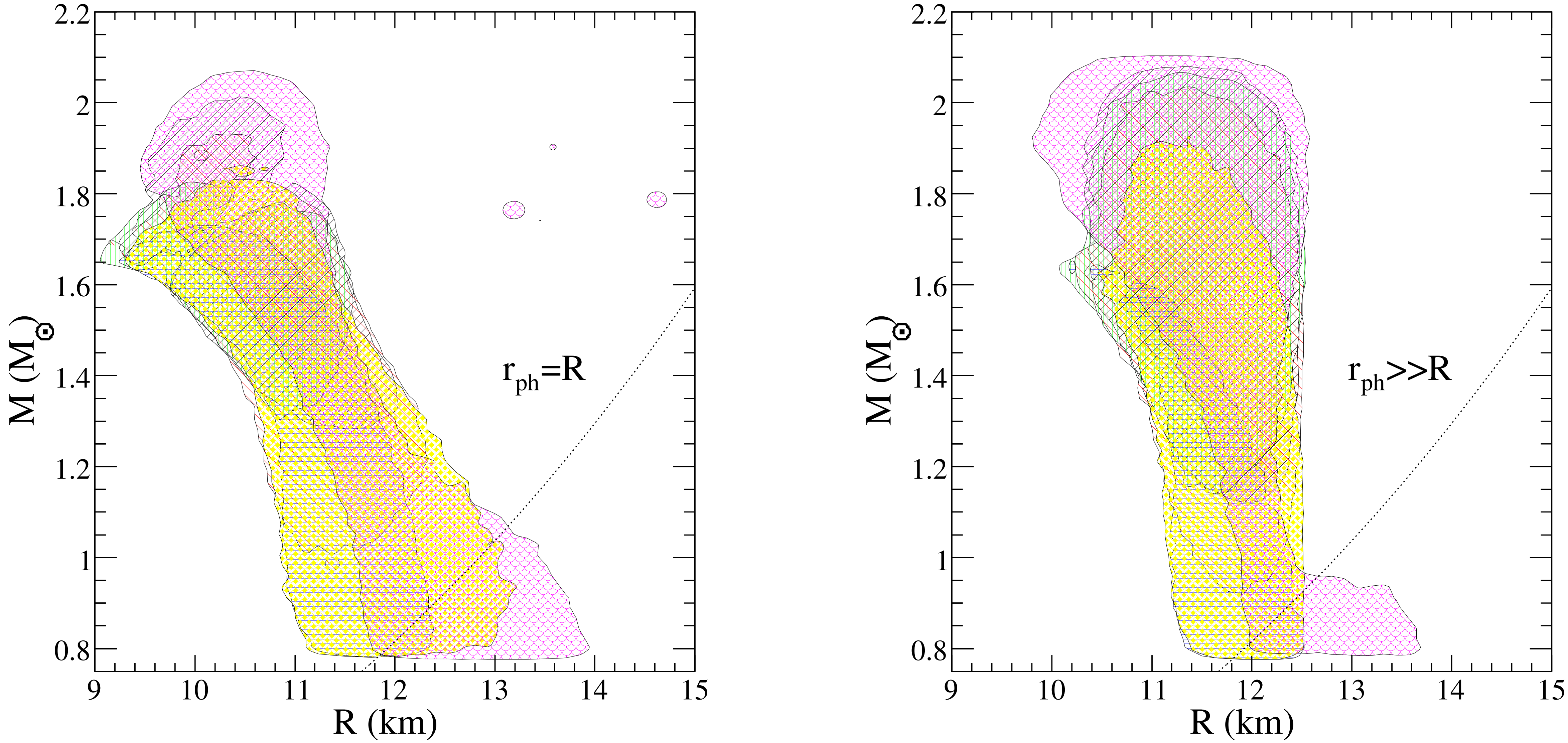

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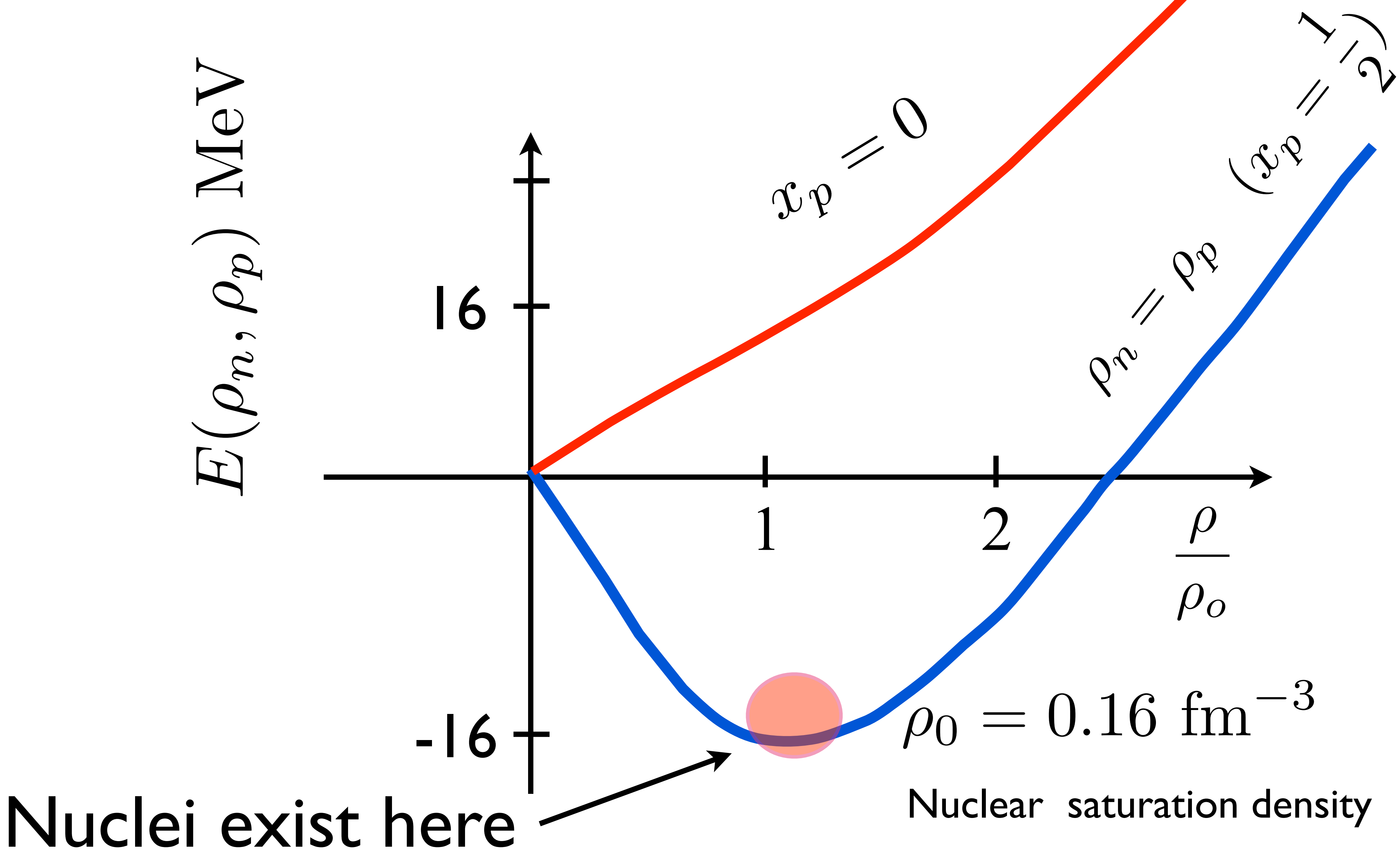
The Nuclear Equation of State

$$\begin{cases} \epsilon = \rho \; E(\rho) \\ P(\epsilon) = \rho^2 \dfrac{\partial E(\rho)}{\partial \rho} \end{cases}$$



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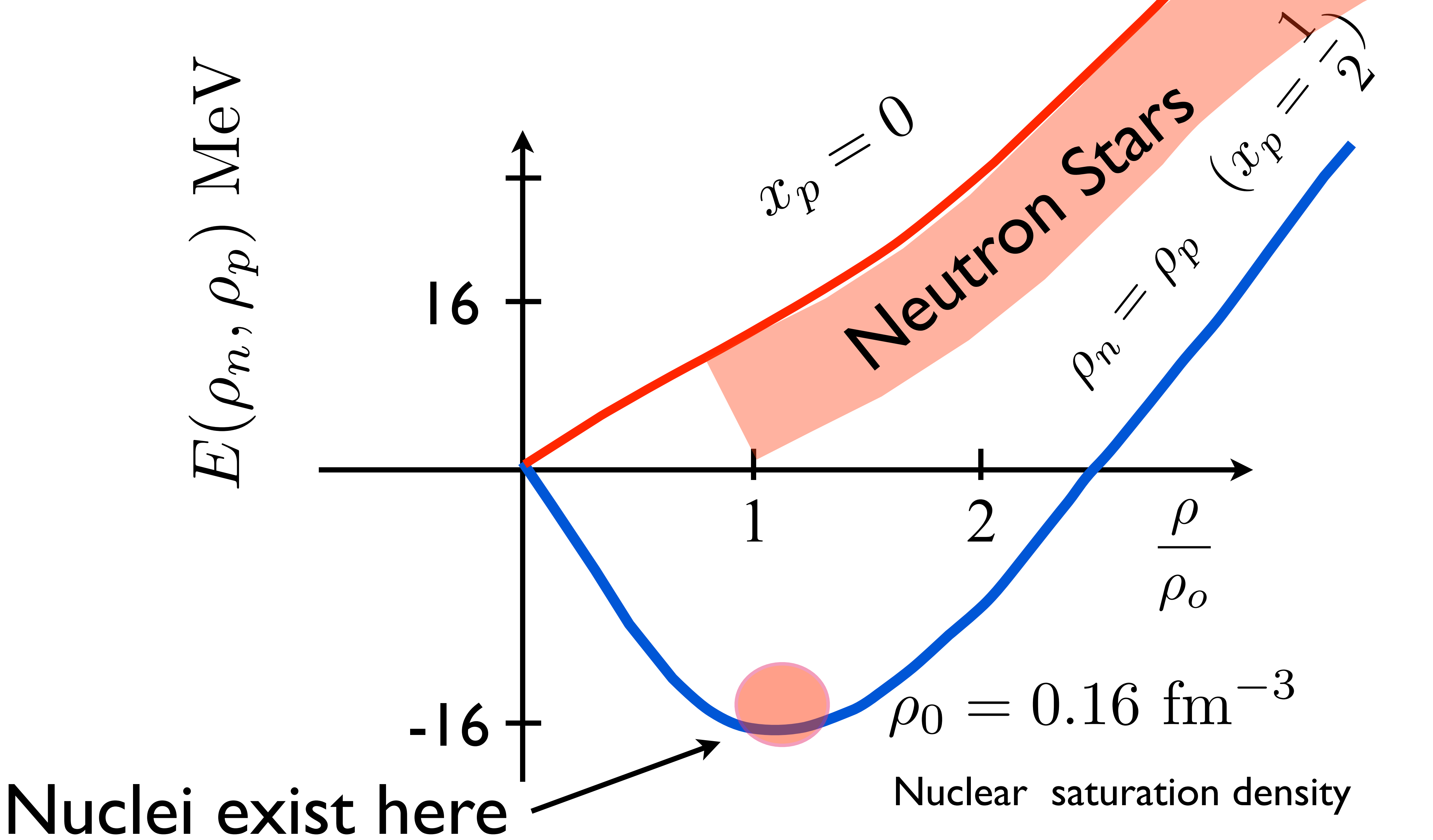

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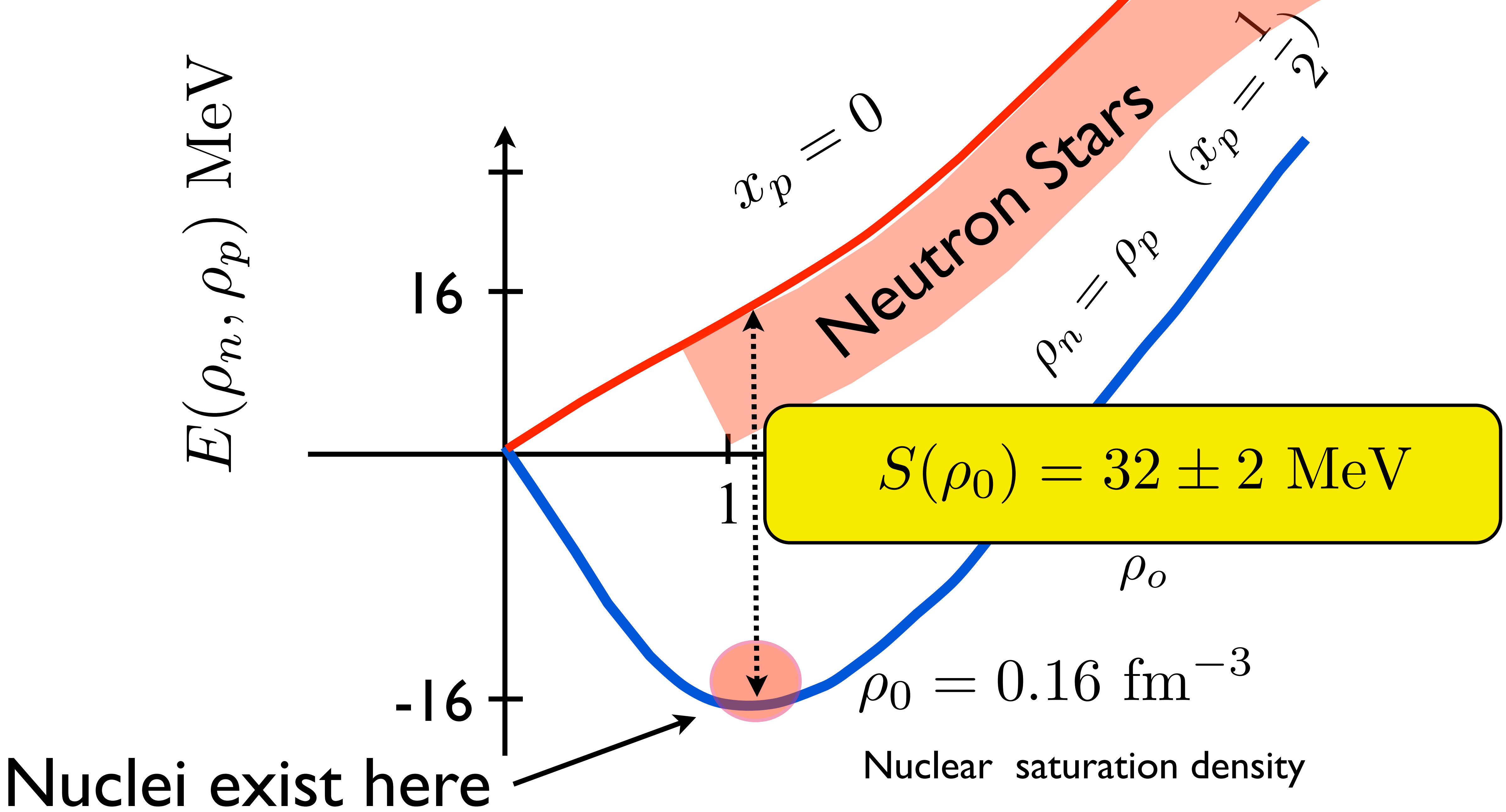

Energy Per Particle in Neutron Matter

$$E(\rho, x_p) = E(\rho, x_p = 0.5) + S_2(\rho)(1-2x_p)^2 + S_4(\rho)(1-2x_p)^4 + \cdots$$

$$E_n(\rho) = E_{np}(\rho_0) + K\,\frac{(\rho-\rho_0)^2}{\rho_o} \quad \textbf{+ ......}$$

$$+S(\rho_0) + \frac{L}{3}\,\frac{(\rho-\rho_0)}{\rho_o} \quad \textbf{+ ......}$$

Empirical Information from Nuclear Structure:

$$E_{np}(\rho_0) = -16 \pm 0.2 \text{ MeV} \quad K = 220 \pm 30 \text{ MeV}$$

$$S(\rho_0) = 32 \pm 2 \text{ MeV} \quad L = 70 \pm 20 \text{ MeV}$$

$$E_n(\rho_0) = 16 \pm 2 \text{ MeV}$$

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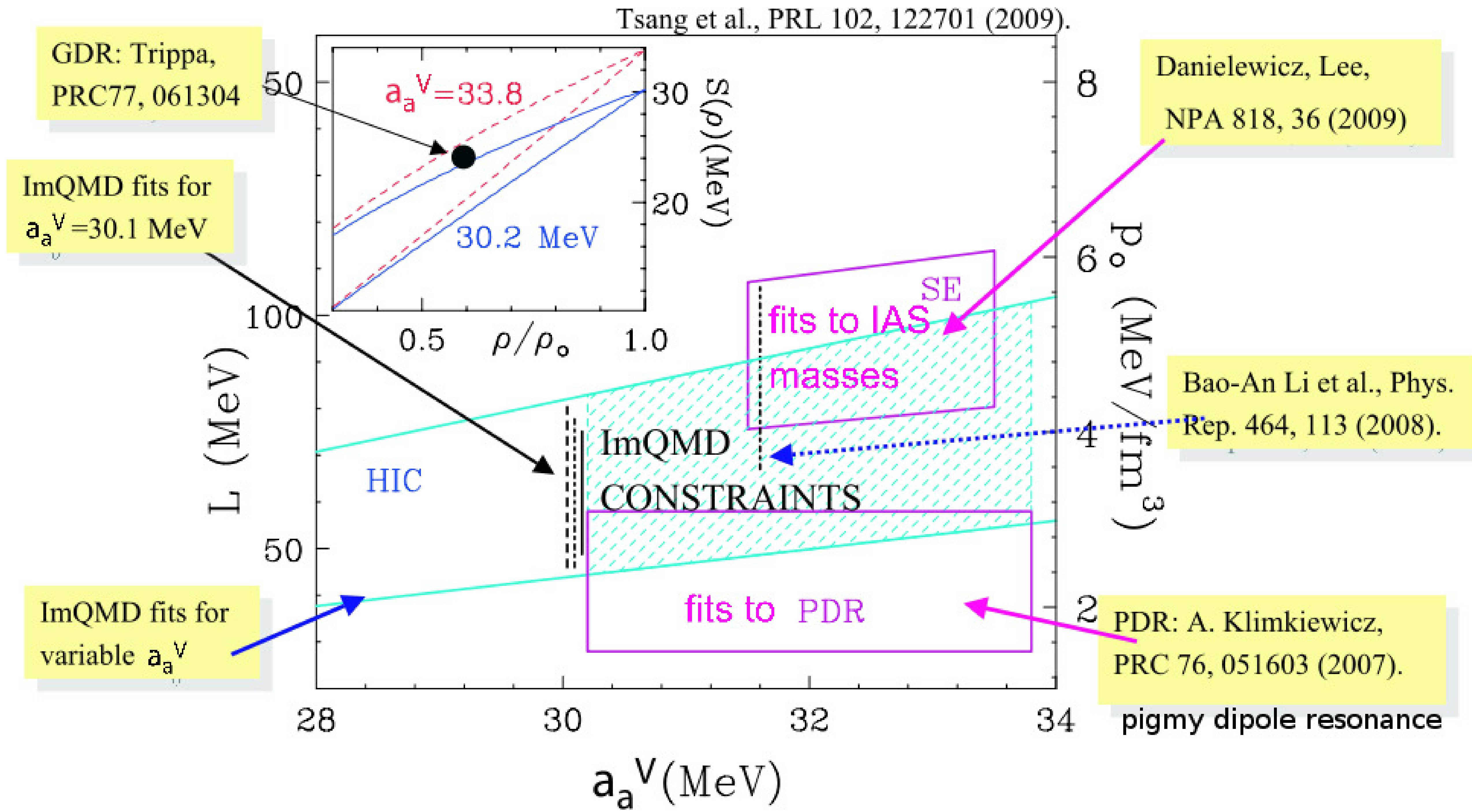
Can we infer S (E_{sym}) and L ($\rho\partial_\rho E_{sym}$) from Experiment ?



Nuclear Many Body Theory

$$H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \cdots$$

Phenomenological potentials (Argonne etc) tuned to fit scattering and light nuclei.

Chiral potentials and softer low energy potentials obtained using RG.

Computational Methods:
Quantum Monte Carlo

Diagrammatic Methods:
Hartree-Fock
Brueckener Hartree-Fock

$E(\rho_n, \rho_p)$: Energy per particle

Model for Nuclear Interactions

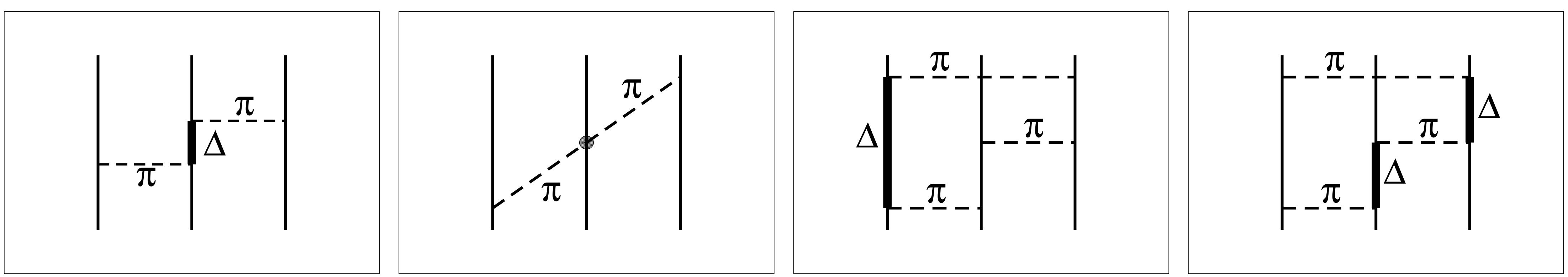
Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} NN (Argonne AV8') fitted on scattering data. Sum of operators:

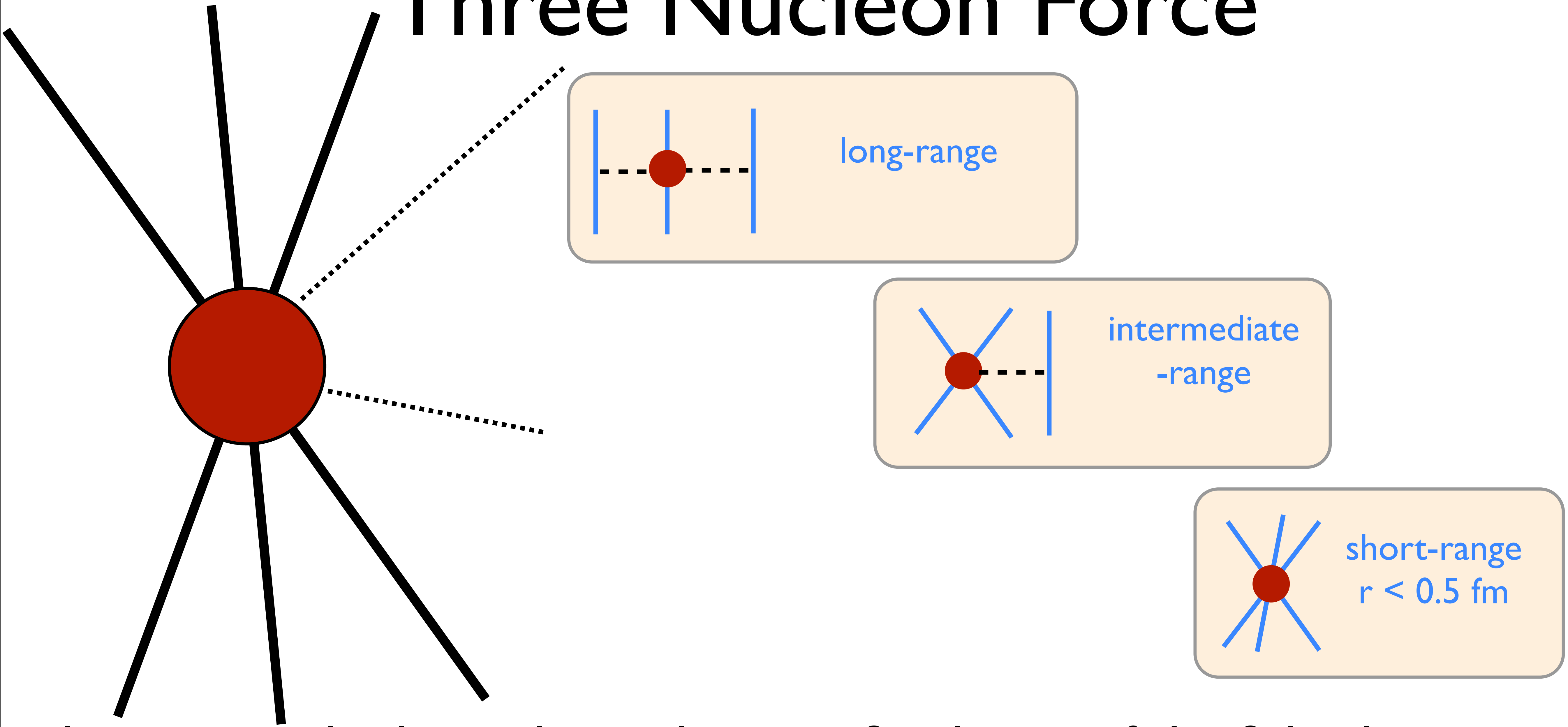
$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij}), \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

V_{ijk} models processes like



$+$ Phenomenological repulsive term.

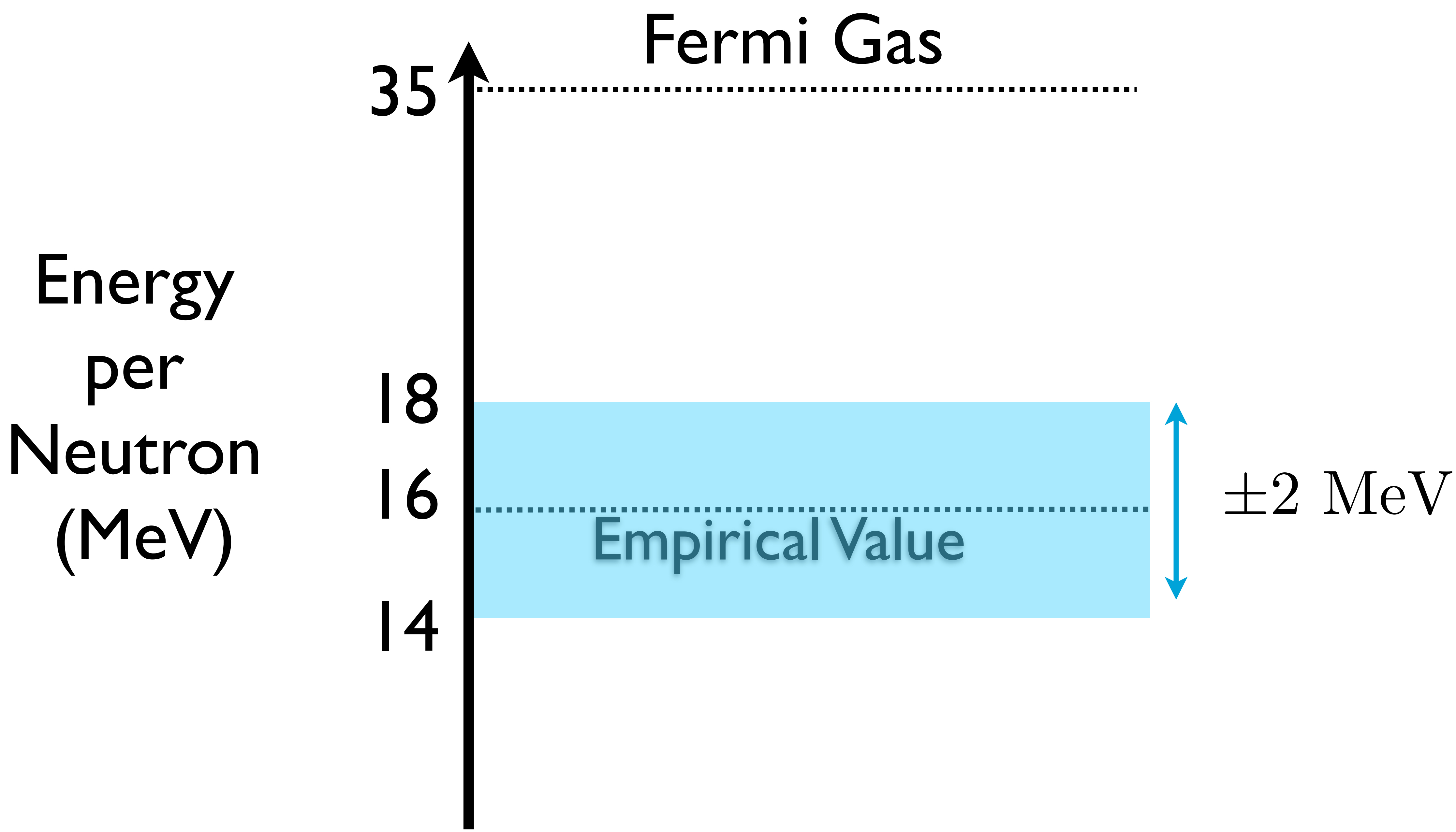
Three Nucleon Force



- Its magnitude depends on the specific choice of the 2-body potential.
- For the Argonne 2-body potentials the long-range 3n contribution is small.
- The short-distanceT=3/2 3-n contribution is not easy to constrain from light nuclei (?) .

3N Forces in Neutron Matter

(at nuclear saturation density)



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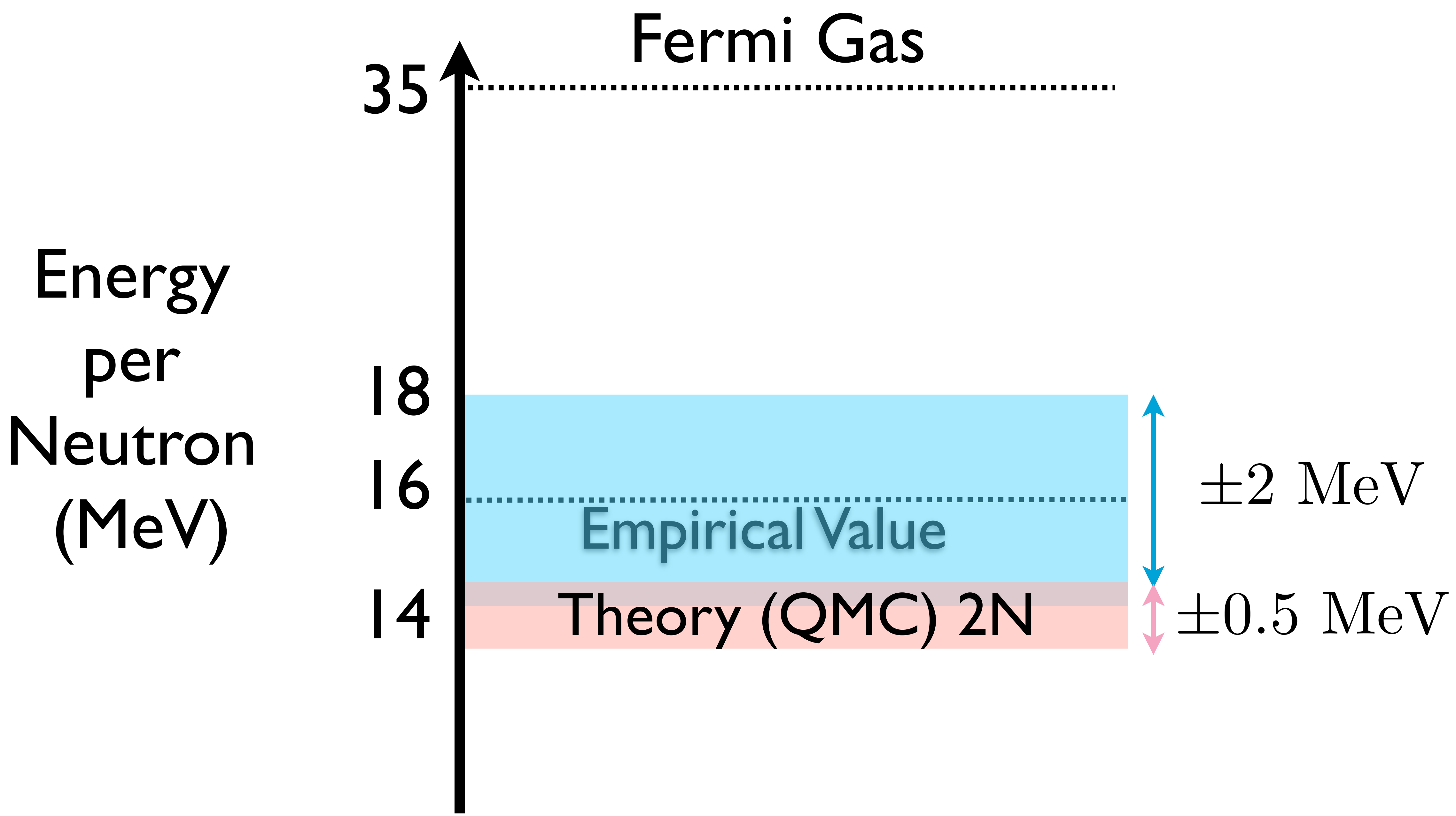

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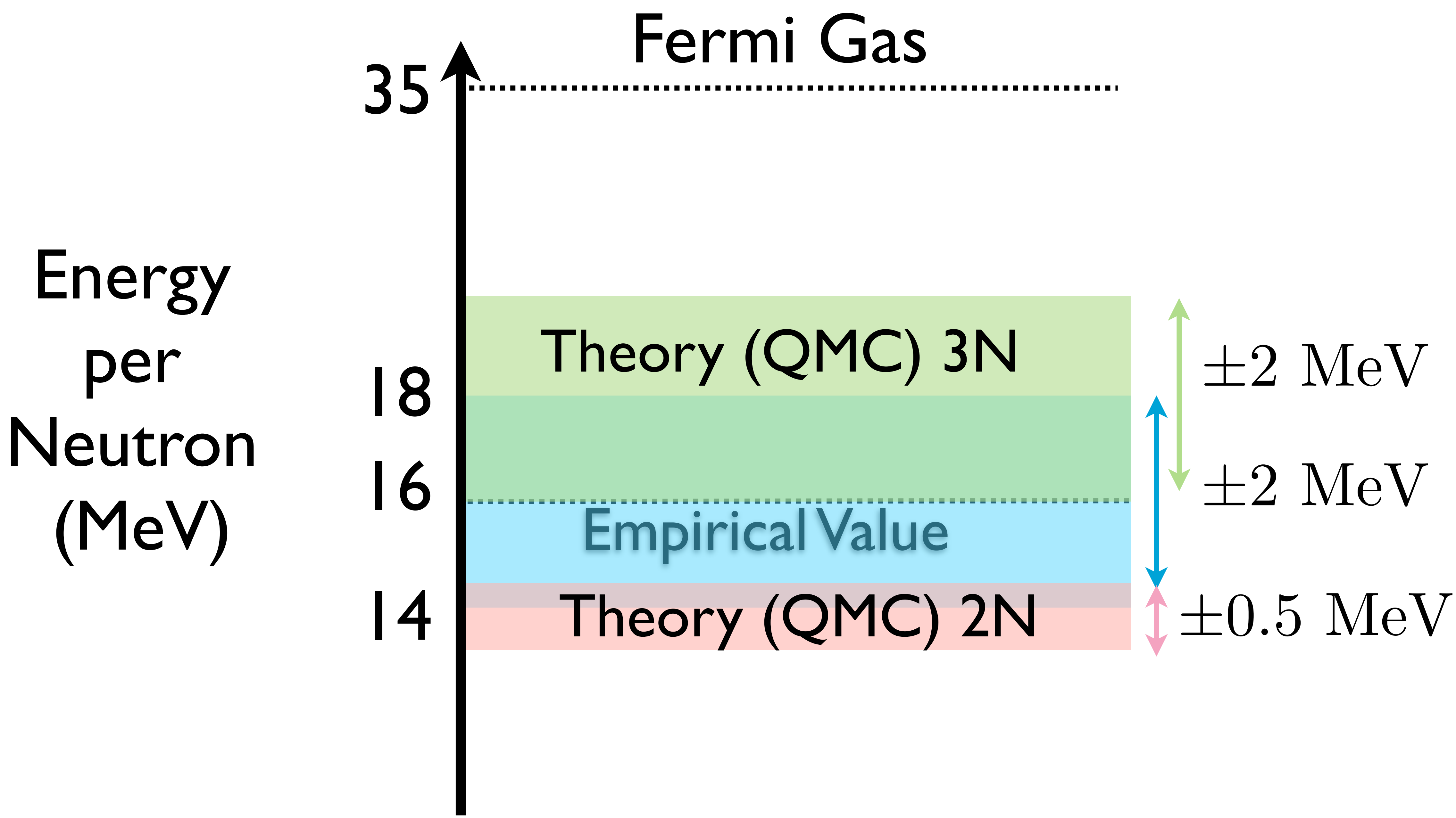
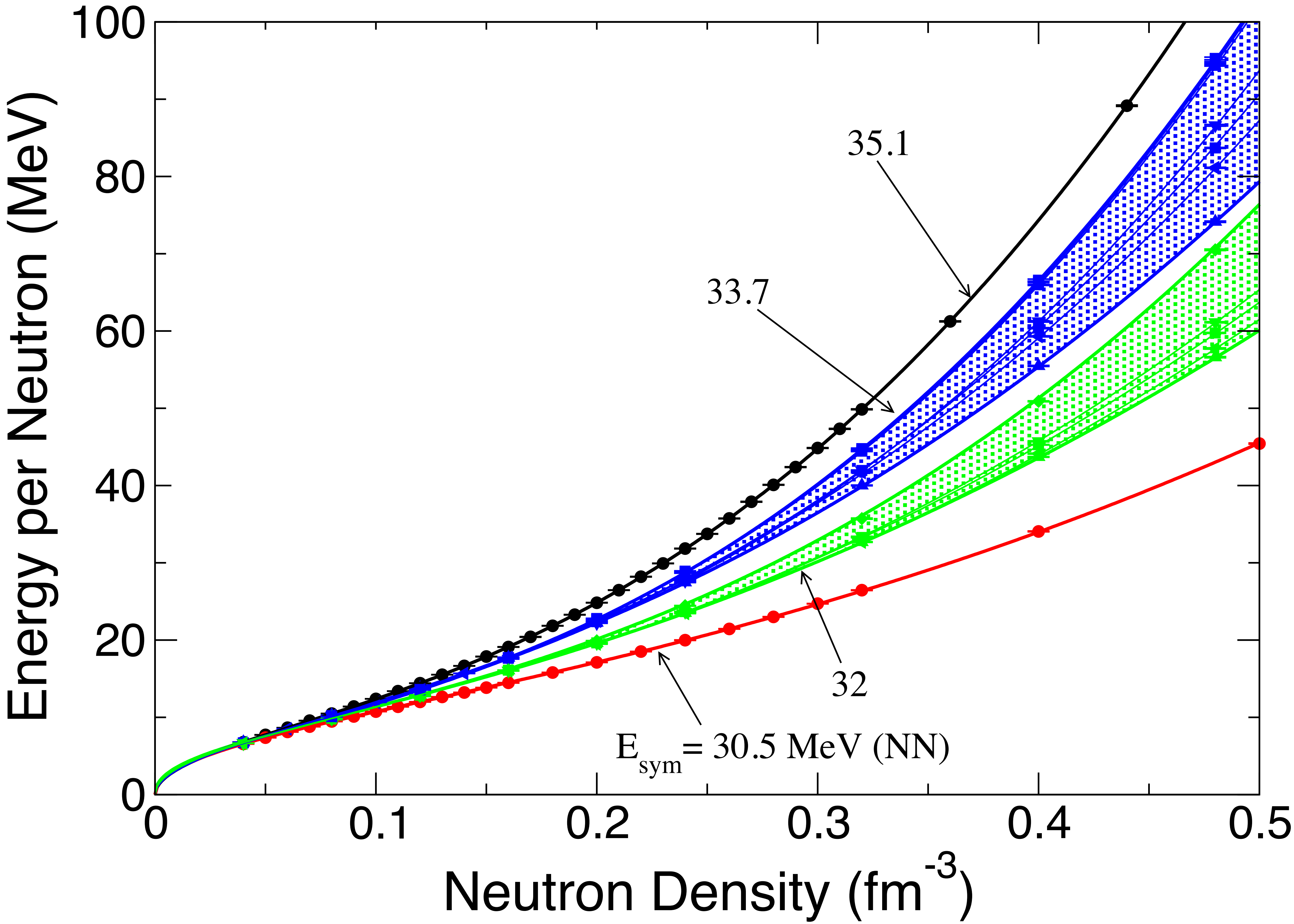


- Phenomenology suggests repulsive 3N forces in neutron matter.

Asymmetry Energy & 3N Forces

Gandolfi, Carlson, Reddy (2010)



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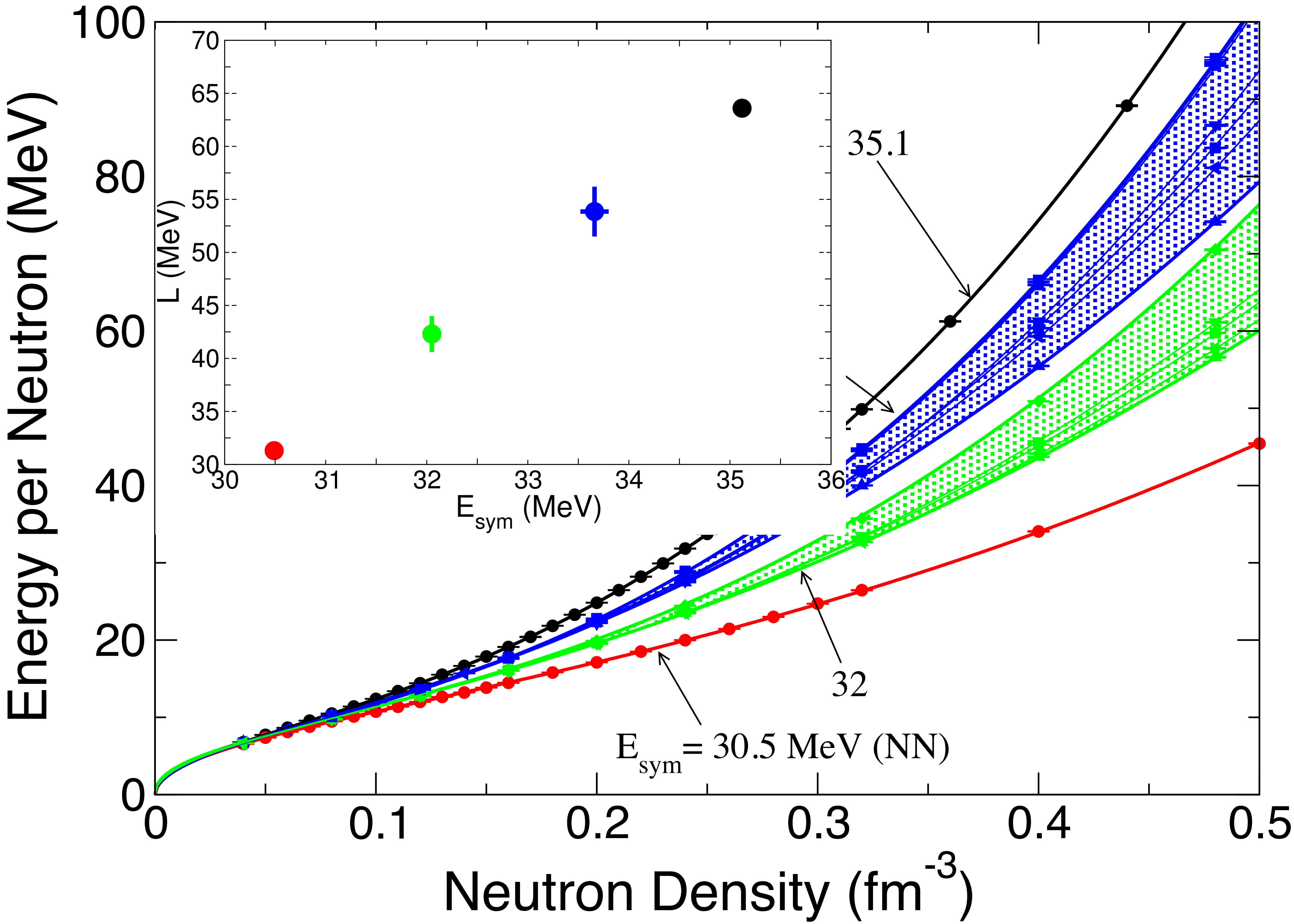

QMC Neutron Matter EoS

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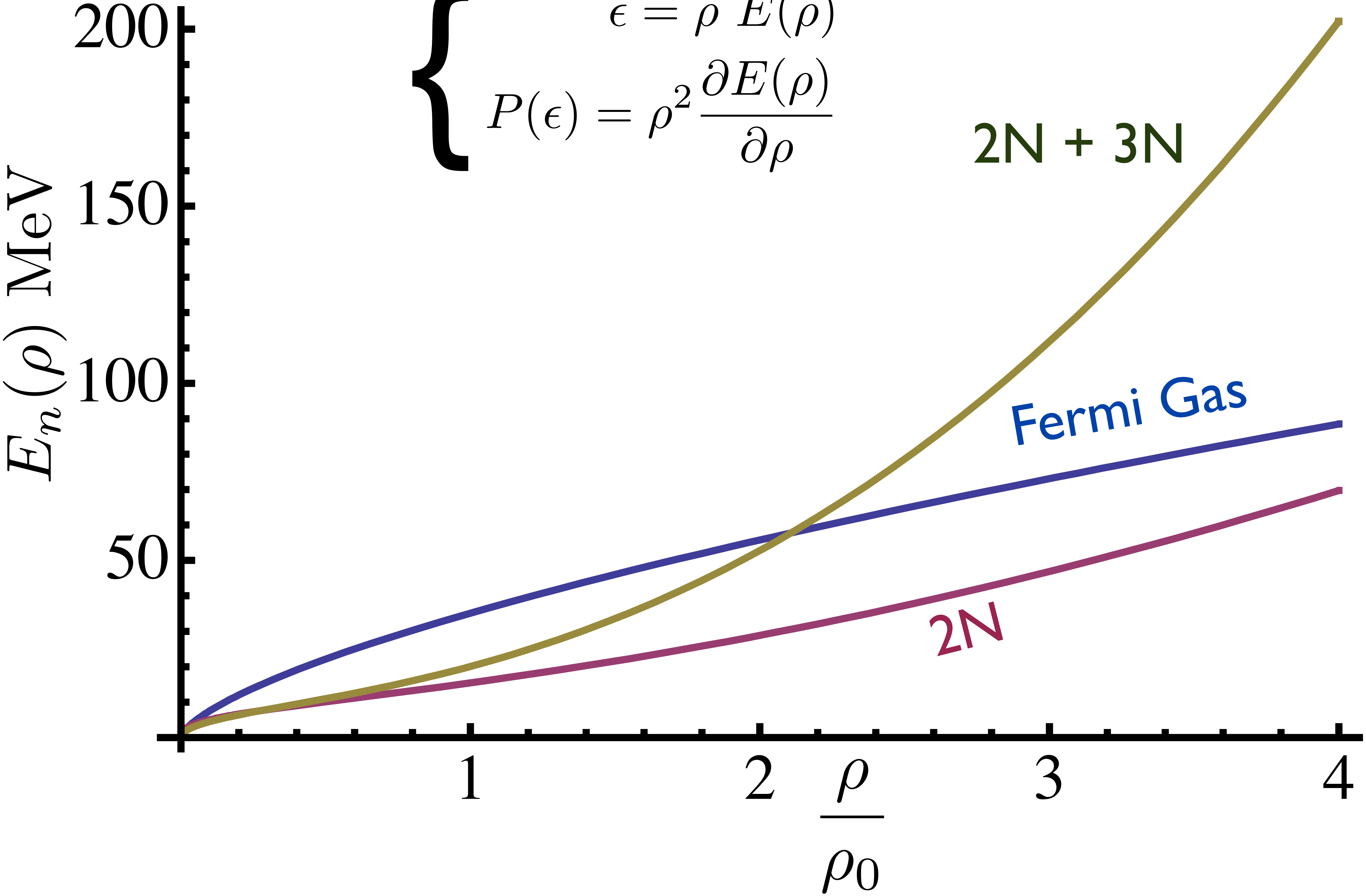
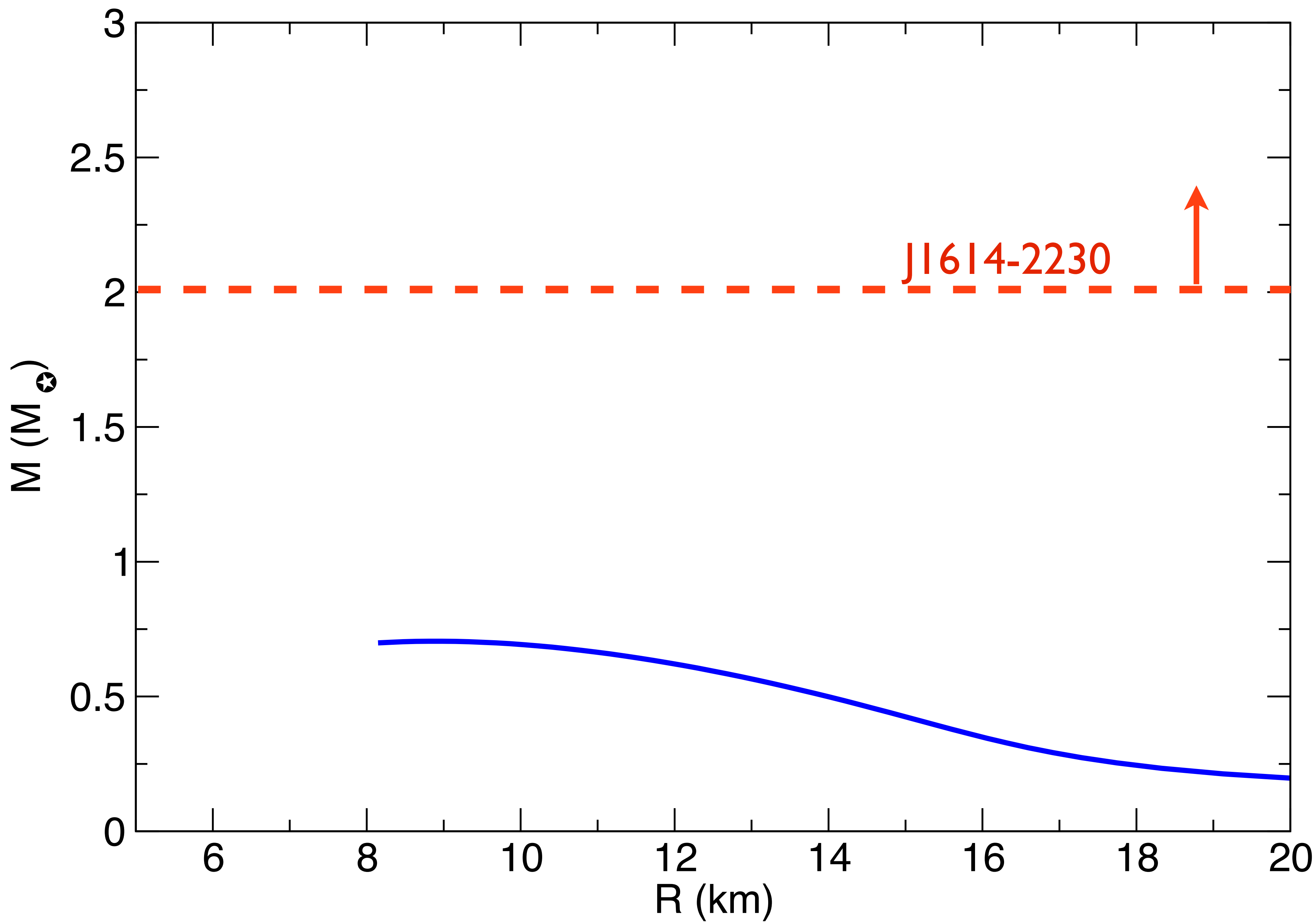


Mass-Radius

$$M(R) \leftrightarrow P(\epsilon)$$



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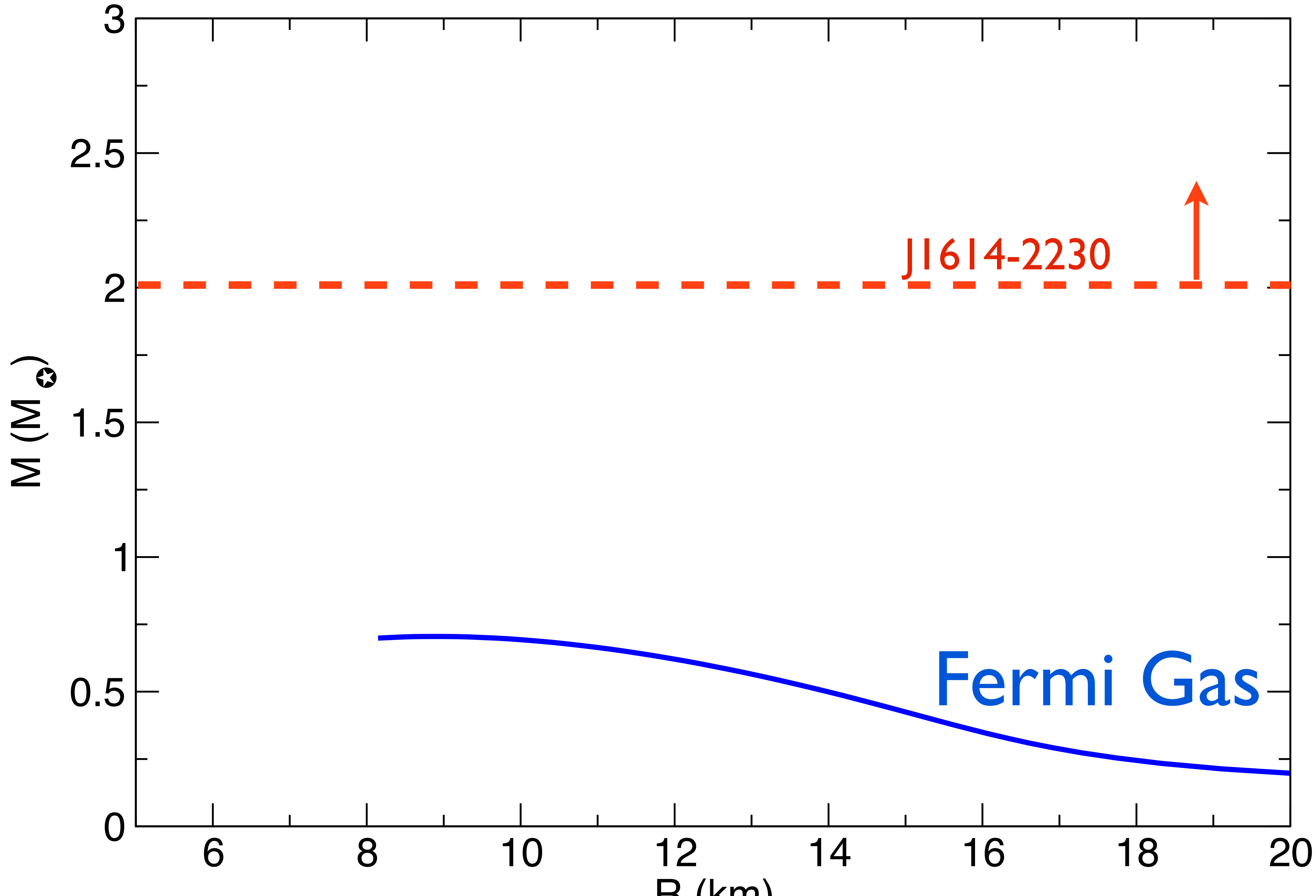

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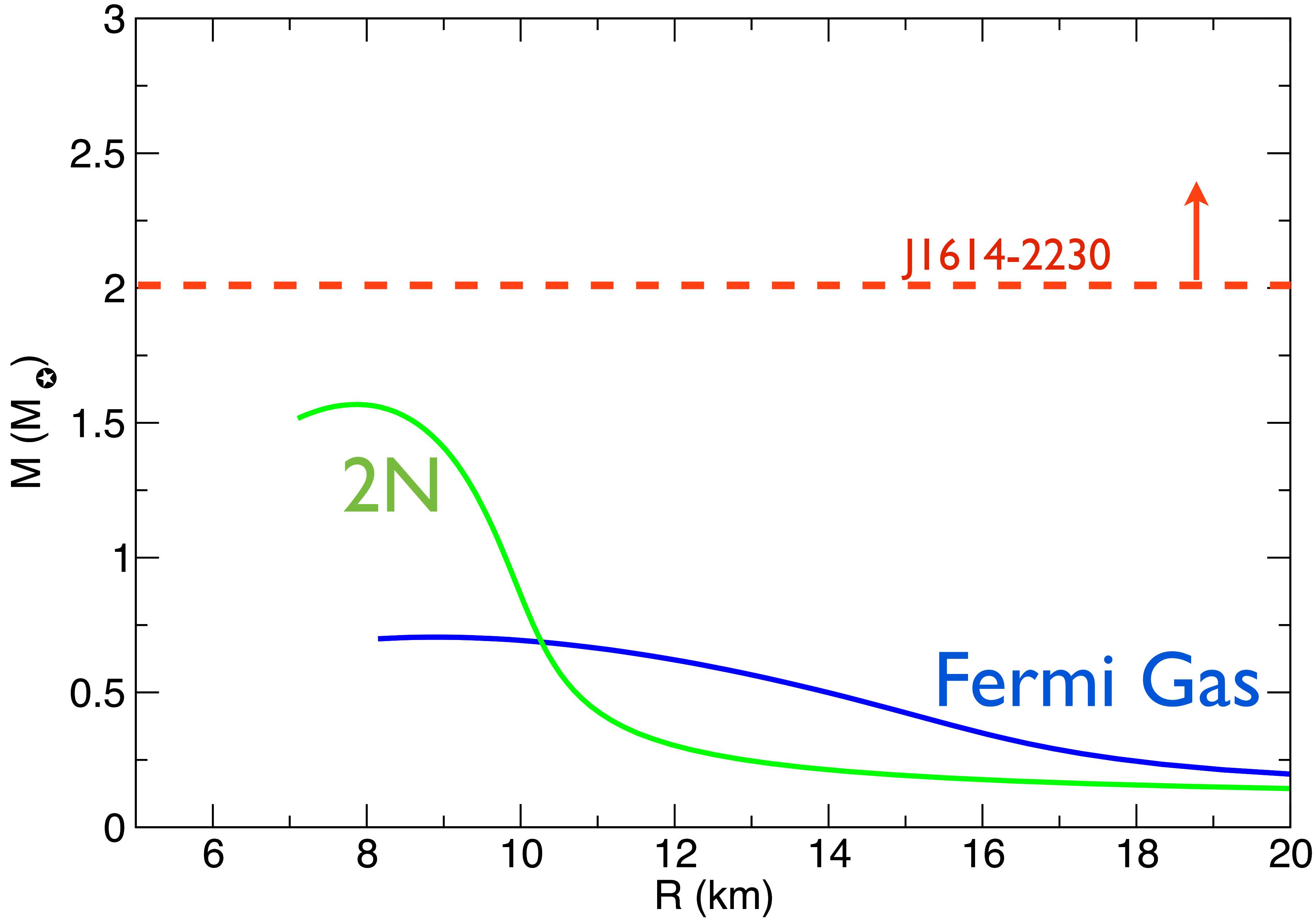

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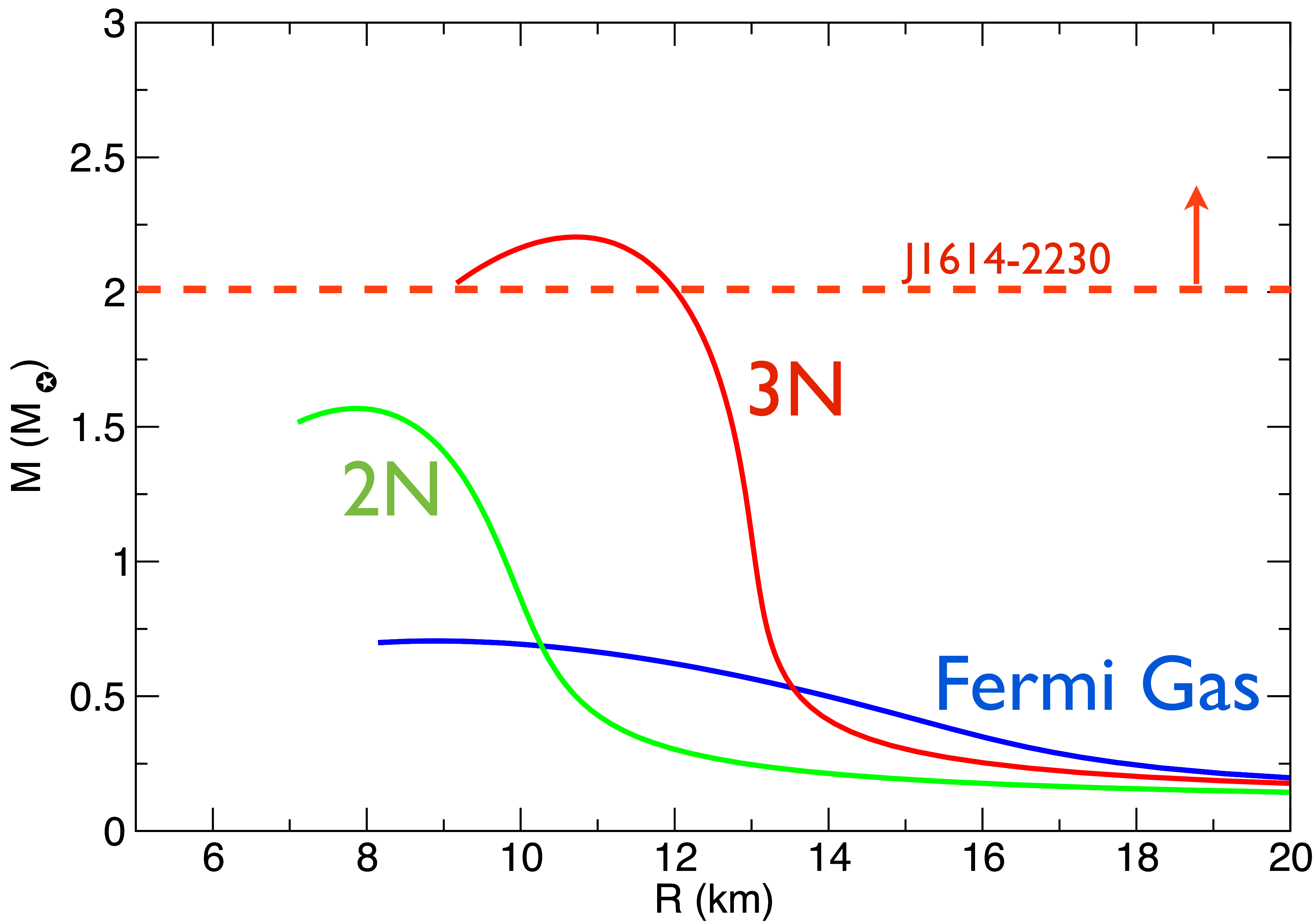

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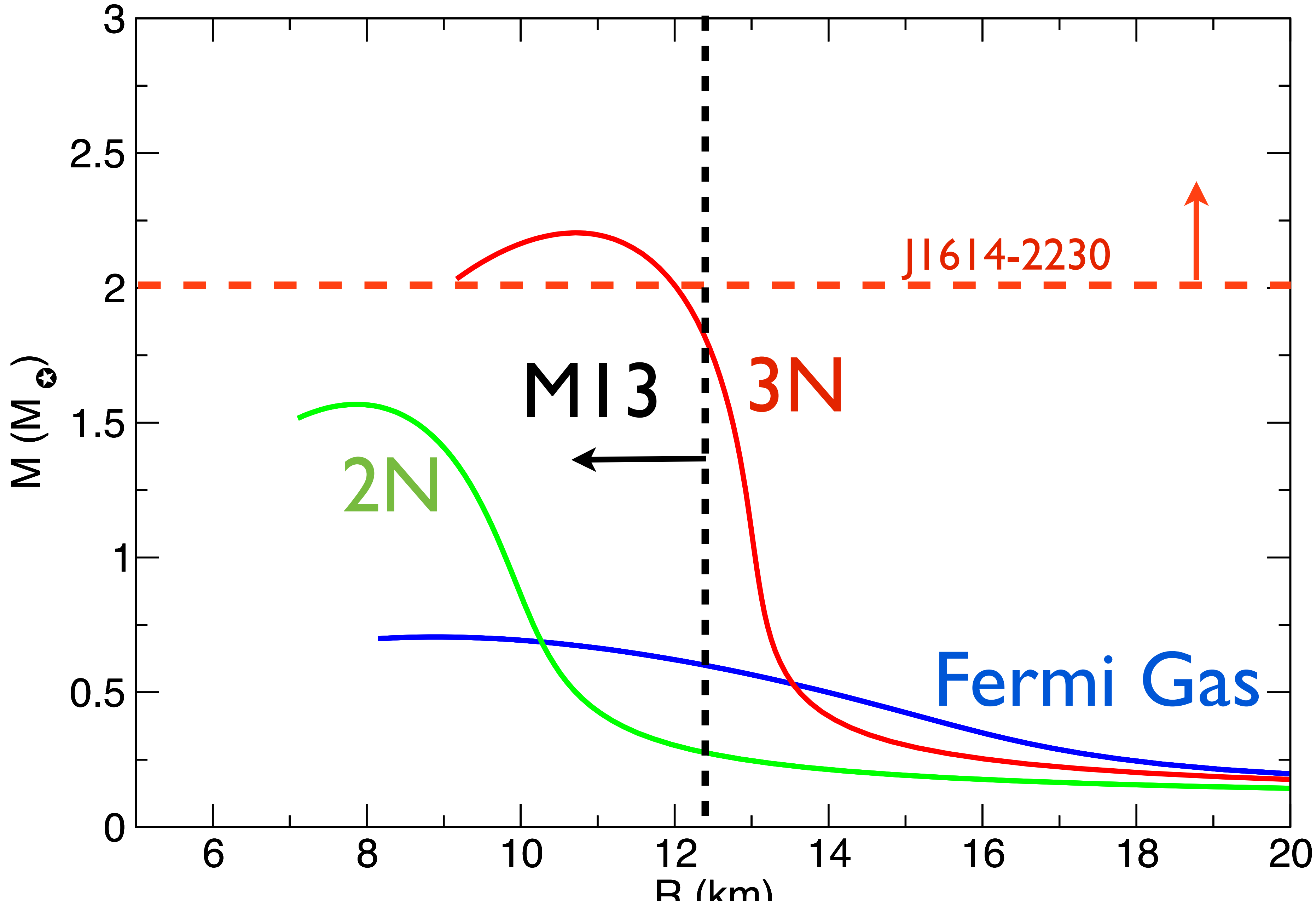
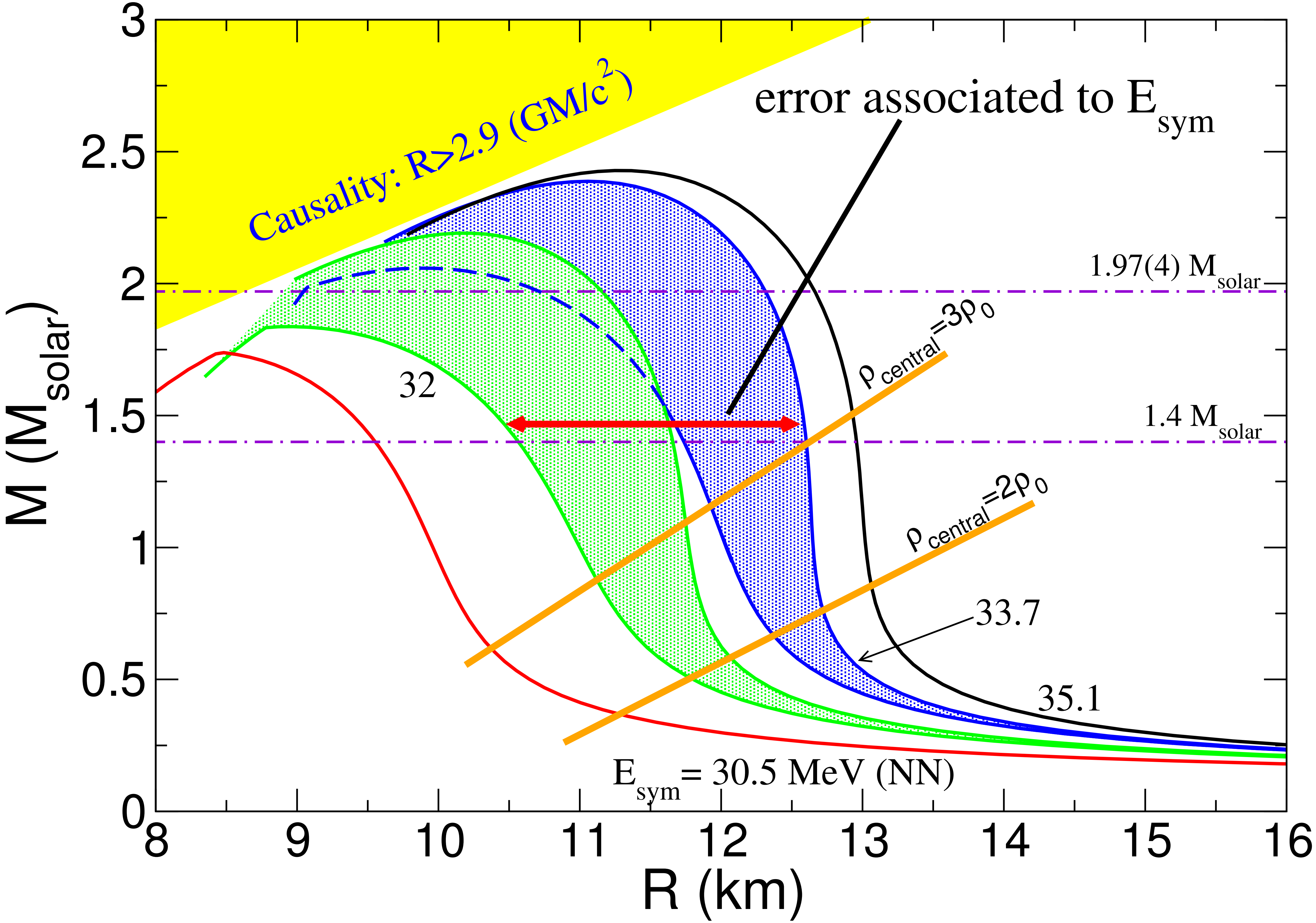


Mass - Radius, E_{sym} and 3n Interactions



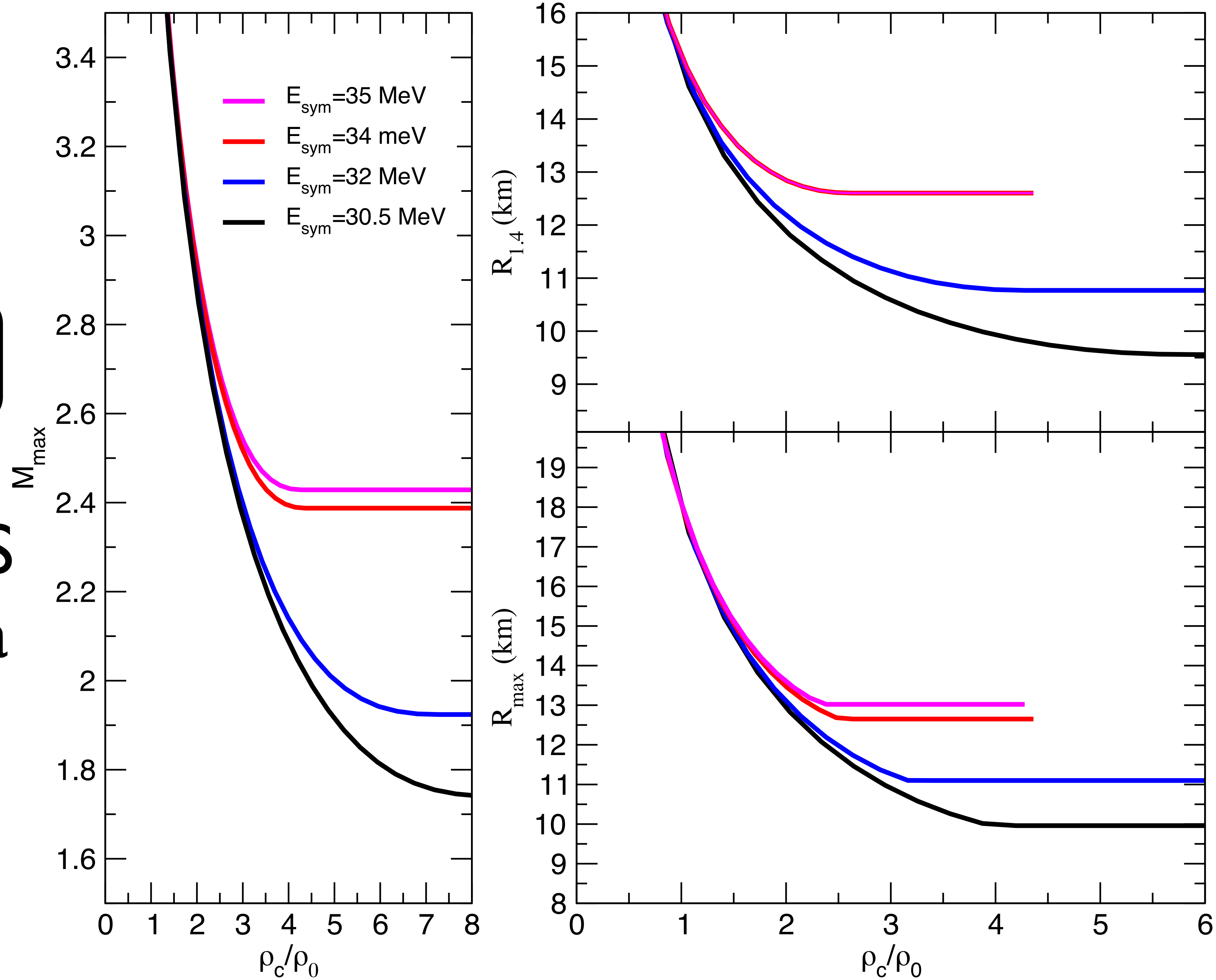
Gandolfi, Carlson, Reddy (2010)

Upper Bounds on M & R

The maximally stiff EoS is the causal EoS:

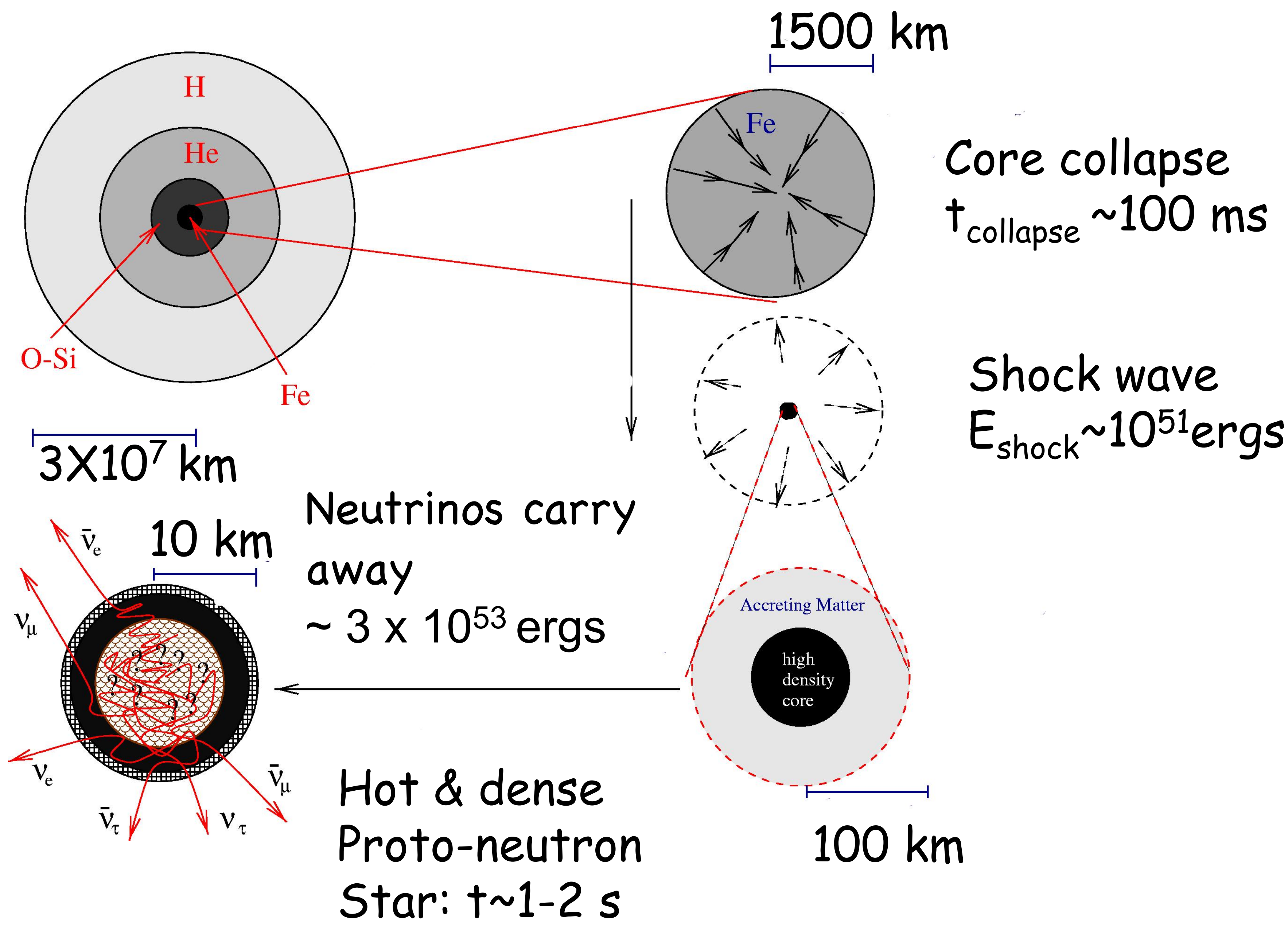
$$P = c\,\epsilon - \epsilon_0$$

Assume that EoS is known up to a critical density and is maximally stiff thereafter.



Supernova Neutrinos and the Protoneutron Star

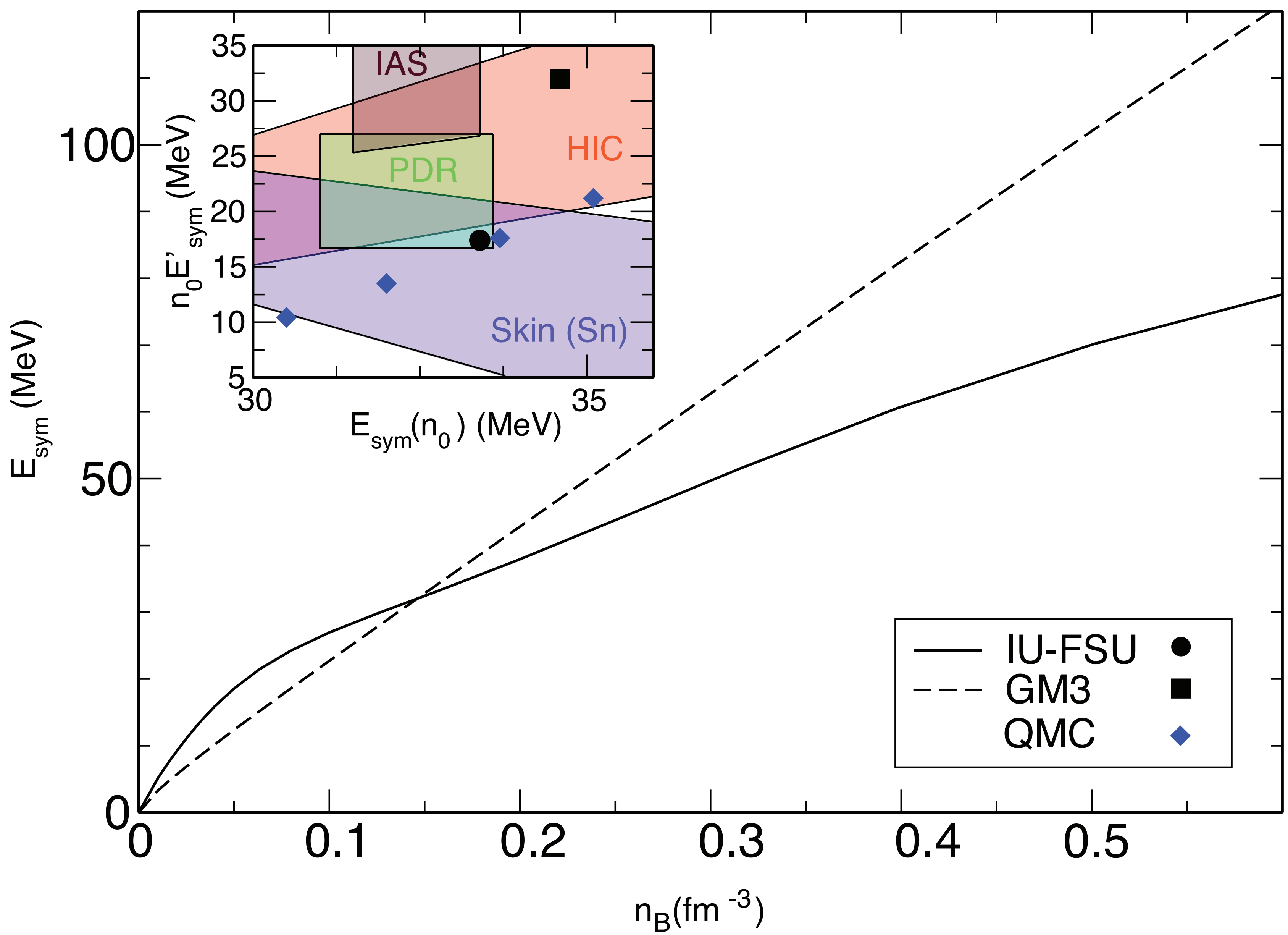
Neutrino flavor and energy distribution can impact explosion mechanism and r-process nucleosynthesis.



Modeling PNS evolution with different EoS.

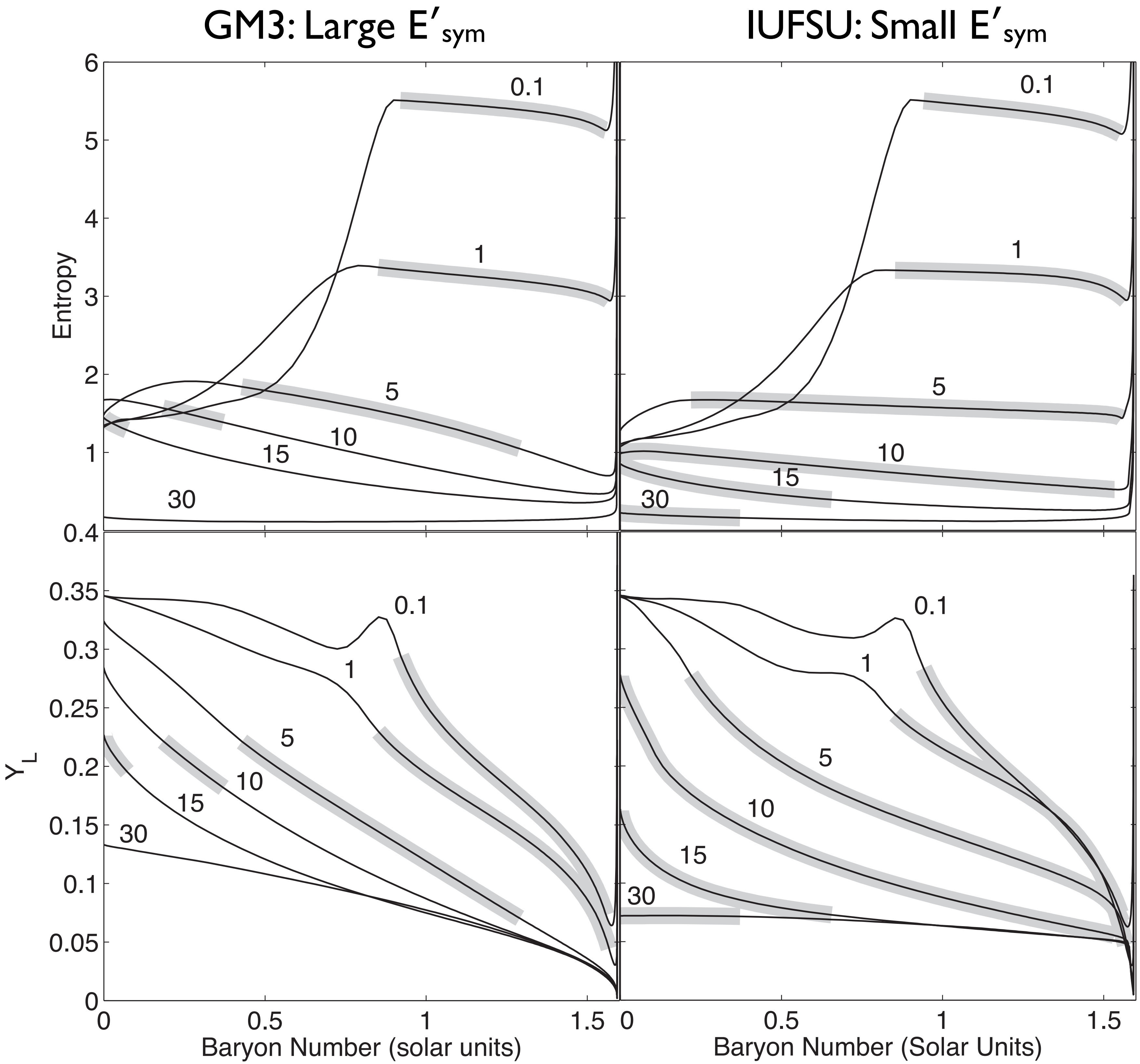
$$\mathscr{L}_{\rm int} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_\mu + \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi$$
$$- \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4 + \frac{\zeta}{4!}g_{\rm v}^4(V_\mu V^\mu)^2 + \Lambda_{\rm v} g_\rho^2\,\mathbf{b}_\mu\cdot\mathbf{b}^\mu g_{\rm v}^2 V_\nu V^\nu$$

- At finite temperature only mean field calculations exist.
- Can tune parameters to mimic different symmetry energies.



Newly born neutron star: An intense neutrino source

- Protoneutron star evolution time scale is set by neutrino diffusion and convection.
- It is imprinted on the temporal structure of the neutrino signal - tomography ?



Roberts, Shen, Cirigliano, Pons, Reddy, Woosley (2011)

Convection is driven by unstable gradients in entropy and lepton number. Convective growth rate:

$$\omega^2 = -\frac{g}{\gamma_{n_B}} \left(\gamma_s \nabla \ln(s) + \gamma_{Y_L} \nabla \ln(Y_L)\right)$$

$$\gamma_{n_B} = \left(\frac{\partial \ln P}{\partial \ln n_B}\right)_{s,Y_L} \gamma_s = \left(\frac{\partial \ln P}{\partial \ln s}\right)_{n_B,Y_L} \gamma_{Y_L} = \left(\frac{\partial \ln P}{\partial \ln Y_L}\right)_{n_B,s}$$

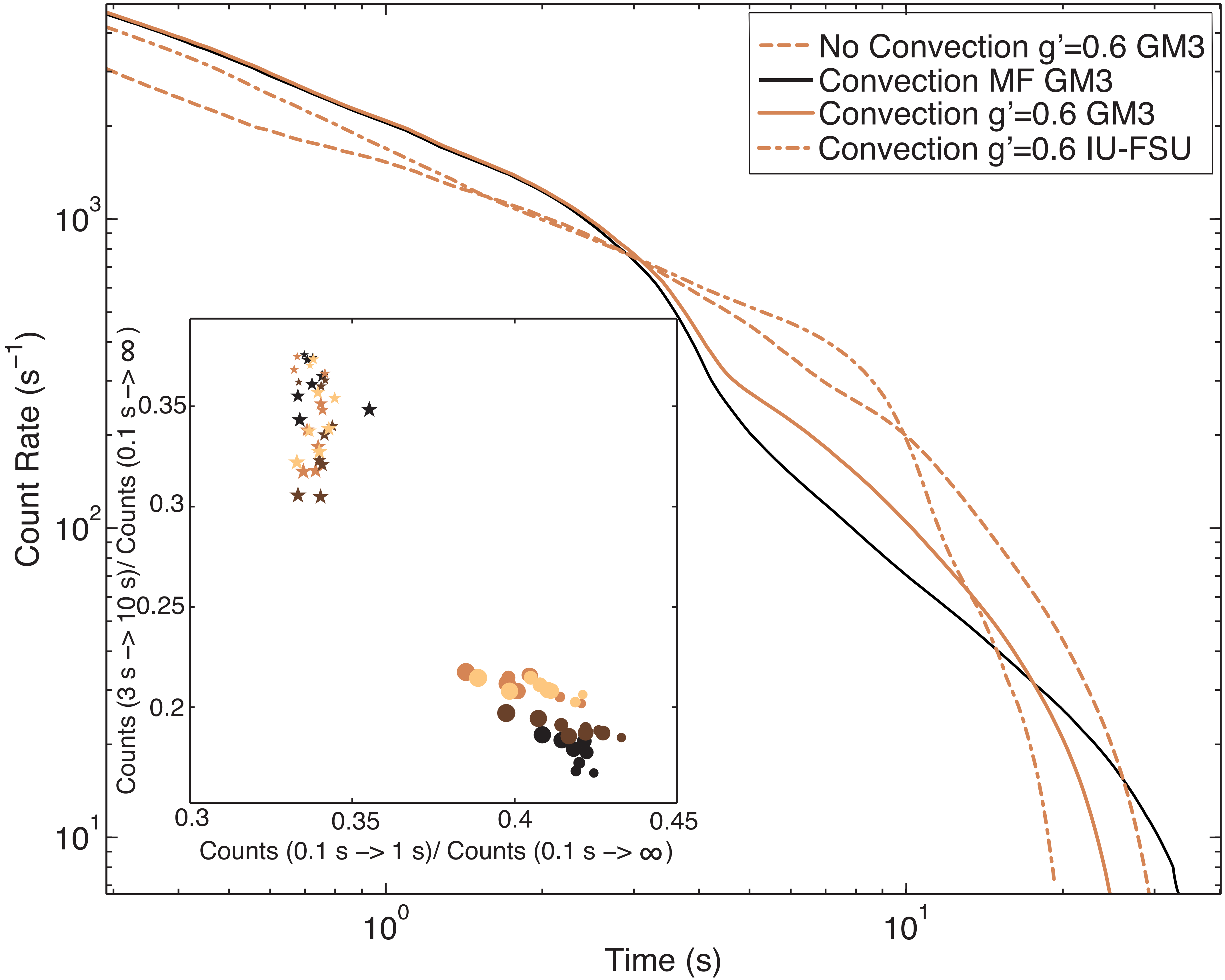
The nuclear symmetry energy is key to understanding composition driven convective instabilities:

$$\left(\frac{\partial P}{\partial Y_L}\right)_{n_B} \simeq n_B^{4/3} Y_e^{1/3} - 4 n_B^2 E'_{\mathrm{sym}} (1 - 2Y_e)$$

Roberts, Shen, Cirigliano, Pons, Reddy, Woosley (2011)

Observable signatures of convective transport

Count rate in Super-Kamiokande for galactic supernova at 10 kpc.



Conclusions & Outlook

- QMC + phenomenological potentials predict a strong correlation between: i) nuclear asymmetry energy; ii) its derivative; (iii) 3n forces; & iv) upper bounds on the NS radius (and mass).
- Need to understand why the long-range 3n interaction is small in neutron matter. Non-perturbative long-range 2 n physics or short-distance assumptions ?
- If small radii are confirmed by other measurements, it demands relatively rapid transition from soft to stiff behavior above nuclear density.
- The behavior of the nuclear symmetry influences convection and the temporal features of the supernova neutrino signal.
- How does the enhanced early time luminosity and the knee-like feature at late time impact other supernova related observables ?