Collective Neutrino Oscillations *in* Supernovae

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Outline

- Neutrino mixing and “self-coupling”
- Why do collective oscillations occur?
- Where do collective oscillations occur?

Neutrino Mixing

WEAK FLAVOR STATES

VACUUM MASS EIGENSTATES

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

$$\delta m_{12}^2 \simeq \delta m_\odot^2 \simeq 7\text{–}8 \times 10^{-5}\mathrm{eV}^2, \quad \theta_{12} \simeq \theta_\odot \simeq 0.6$$

$$|\delta m_{23}^2| \simeq \delta m_{\mathrm{atm}}^2 \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{23} \simeq \theta_{\mathrm{atm}} \simeq \frac{\pi}{4}$$

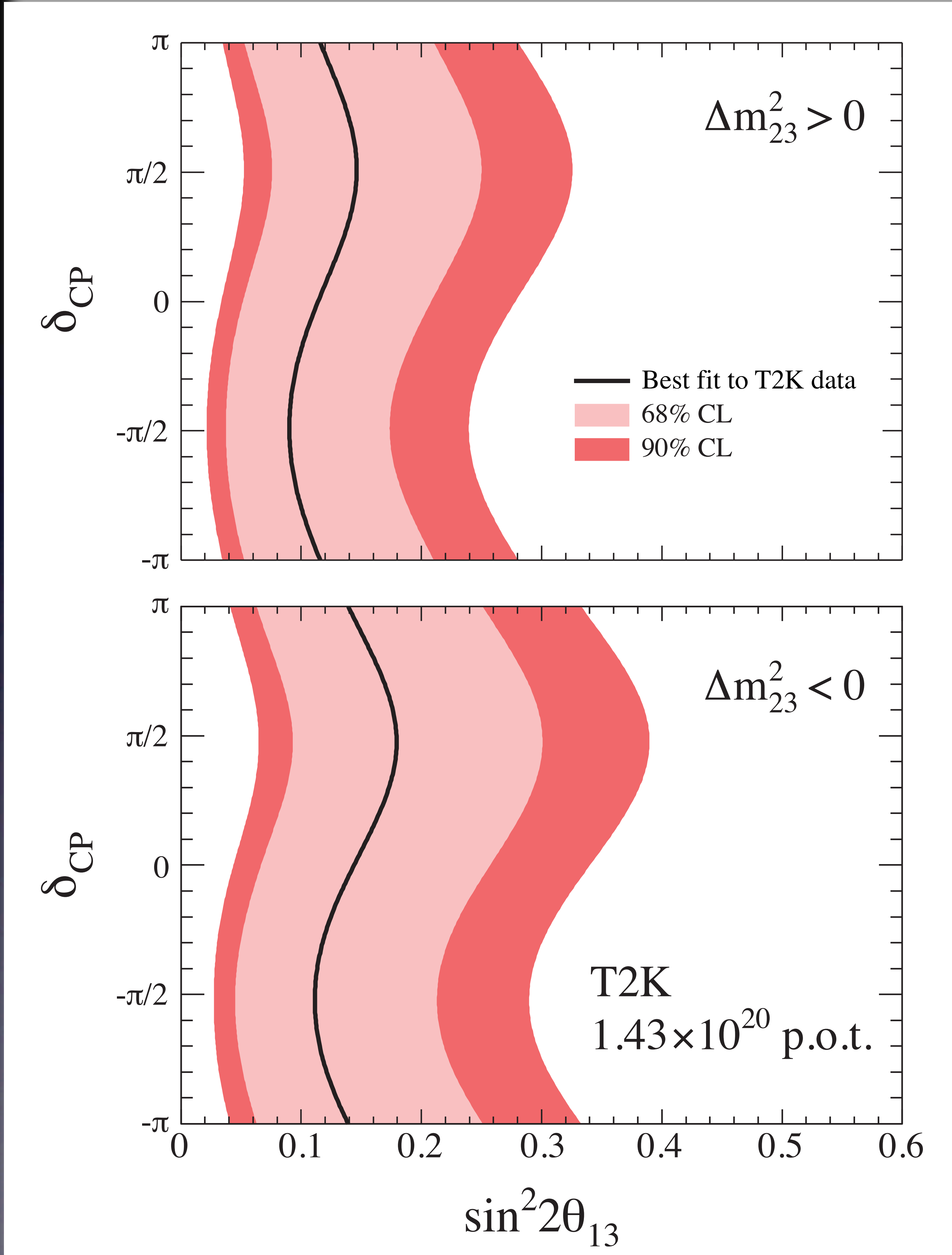
$$|\delta m_{13}^2| \simeq |\delta m_{23}^2| \simeq 2\text{–}3 \times 10^{-3}\mathrm{eV}^2, \quad \theta_{13} \lesssim 0.2$$

ϕ is unknown ← CP VIOLATION PHASE

T2K's New Results

Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam

The T2K experiment observes indications of $\nu_\mu \to \nu_e$ appearance in data accumulated with 1.43×10^{20} protons on target. Six events pass all selection criteria at the far detector. In a three-flavor neutrino oscillation scenario with $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$ and $\sin^2 2\theta_{13} = 0$, the expected number of such events is 1.5±0.3(syst.). Under this hypothesis, the probability to observe six or more candidate events is $7{\times}10^{-3}$, equivalent to 2.5σ significance. At 90% C.L., the data are consistent with $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ for $\delta_{CP} = 0$ and normal (inverted) hierarchy.



T2K's new result

Neutrino Oscillations in SNe

$$i\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

mass matrix

electron density

$$\mathrm{H} = \frac{\mathrm{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$

neutrino energy

ν–ν forward scattering (self-coupling)

Neutrino Oscillations in SNe

$$H_{\alpha\beta}^{\nu\nu} = \sqrt{2} G_{\mathrm{F}} \Big[\sum_{\nu'} (1 - \hat{\mathbf{p}}' \cdot \hat{\mathbf{p}}) n_{\nu'} \langle \nu_\beta | \psi_{\nu'} \rangle \langle \psi_{\nu'} | \nu_\alpha \rangle$$

$$- \sum_{\bar{\nu}'} (1 - \hat{\mathbf{p}}' \cdot \hat{\mathbf{p}}) n_{\bar{\nu}'} \langle \bar{\nu}_\beta | \psi_{\bar{\nu}'} \rangle \langle \psi_{\bar{\nu}'} | \bar{\nu}_\alpha \rangle \Big]$$

Fuller et al, Astrophys. J 322:795, 1987

Nötzold & Raffelt, Nucl. Phys. B307:924, 1988

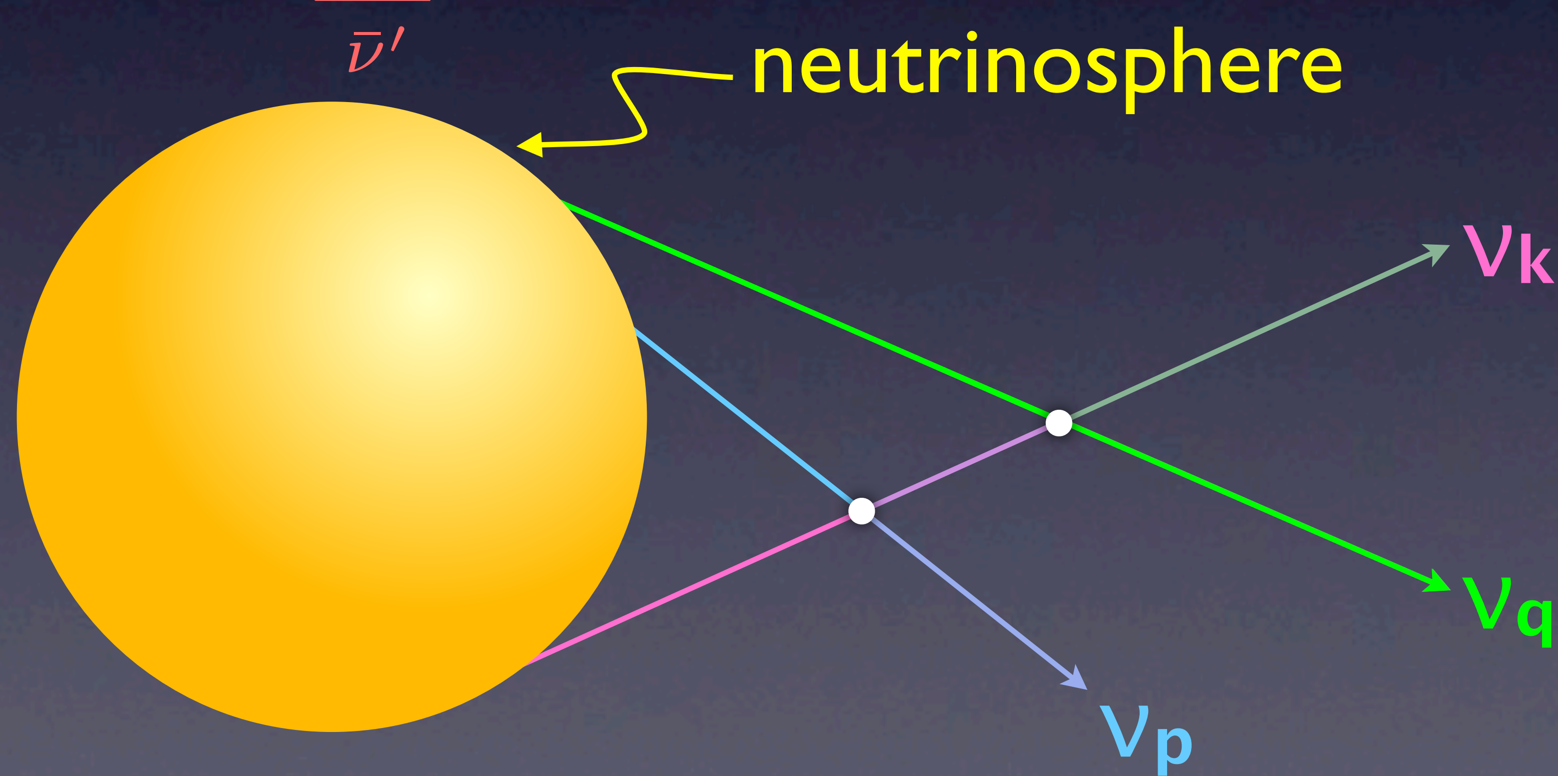
Pantaleone, Phys. Rev. D46:510, 1992

Sigl & Raffelt, Nucl. Phys. B406:423, 1993

Qian & Fuller, Phys. Rev. D51:1479, 1995

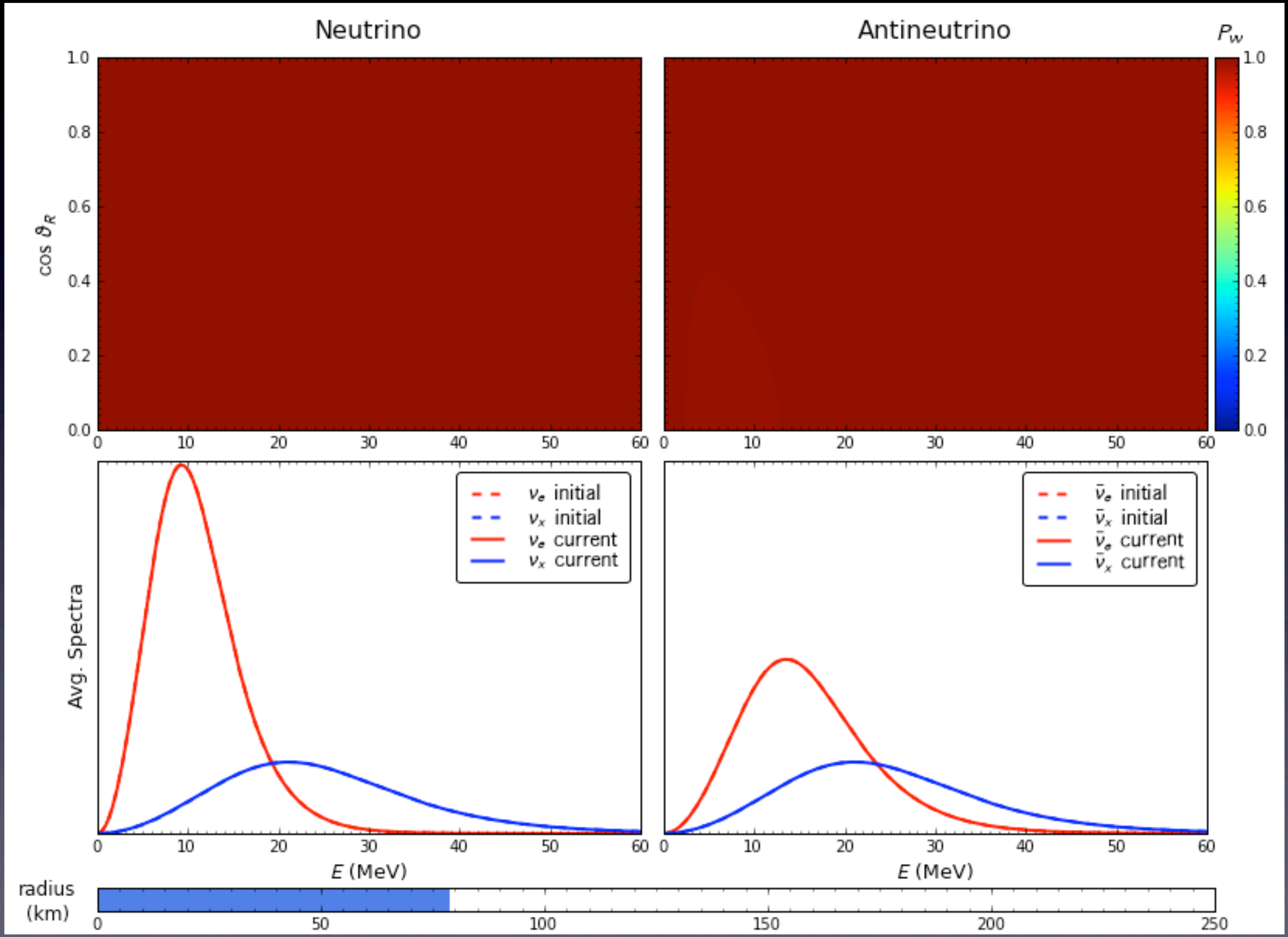
Neutrino Oscillations in SNe

$$H^{\nu\nu}_{\alpha\beta} = \sqrt{2}G_{\mathrm{F}}\Big[\sum_{\nu'}(1-\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}})n_{\nu'}\,\langle\nu_\beta|\psi_{\nu'}\rangle\langle\psi_{\nu'}|\nu_\alpha\rangle$$
$$-\sum_{\bar{\nu}'}(1-\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}})n_{\bar{\nu}'}\,\langle\bar{\nu}_\beta|\psi_{\bar{\nu}'}\rangle\langle\psi_{\bar{\nu}'}|\bar{\nu}_\alpha\rangle\Big]$$



$$\delta m^2 = -3\times 10^{-3}\,\mathrm{eV}^2,\ \theta_\mathrm{v} \ll 1,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$

$$\langle E_{\nu_e}\rangle = 11\,\mathrm{MeV},\ \langle E_{\bar\nu_e}\rangle = 16\,\mathrm{MeV},\ \langle E_{\nu_x,\bar\nu_x}\rangle = 25\mathrm{MeV}$$



Why?

Neutrino Flavor Isospin

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\psi_\nu = \mathsf{H}\psi_\nu$$

$$= -\vec{H}\cdot\frac{\vec{\sigma}}{2}\psi_\nu$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\vec{s} = \vec{s}\times\vec{H}$$

$$\begin{bmatrix} e^- \\ \nu_e \end{bmatrix} \quad \begin{bmatrix} \mu/\tau \\ \nu_\mu/\nu_\tau \end{bmatrix}$$

Weak Isospin

Flavor Isospin

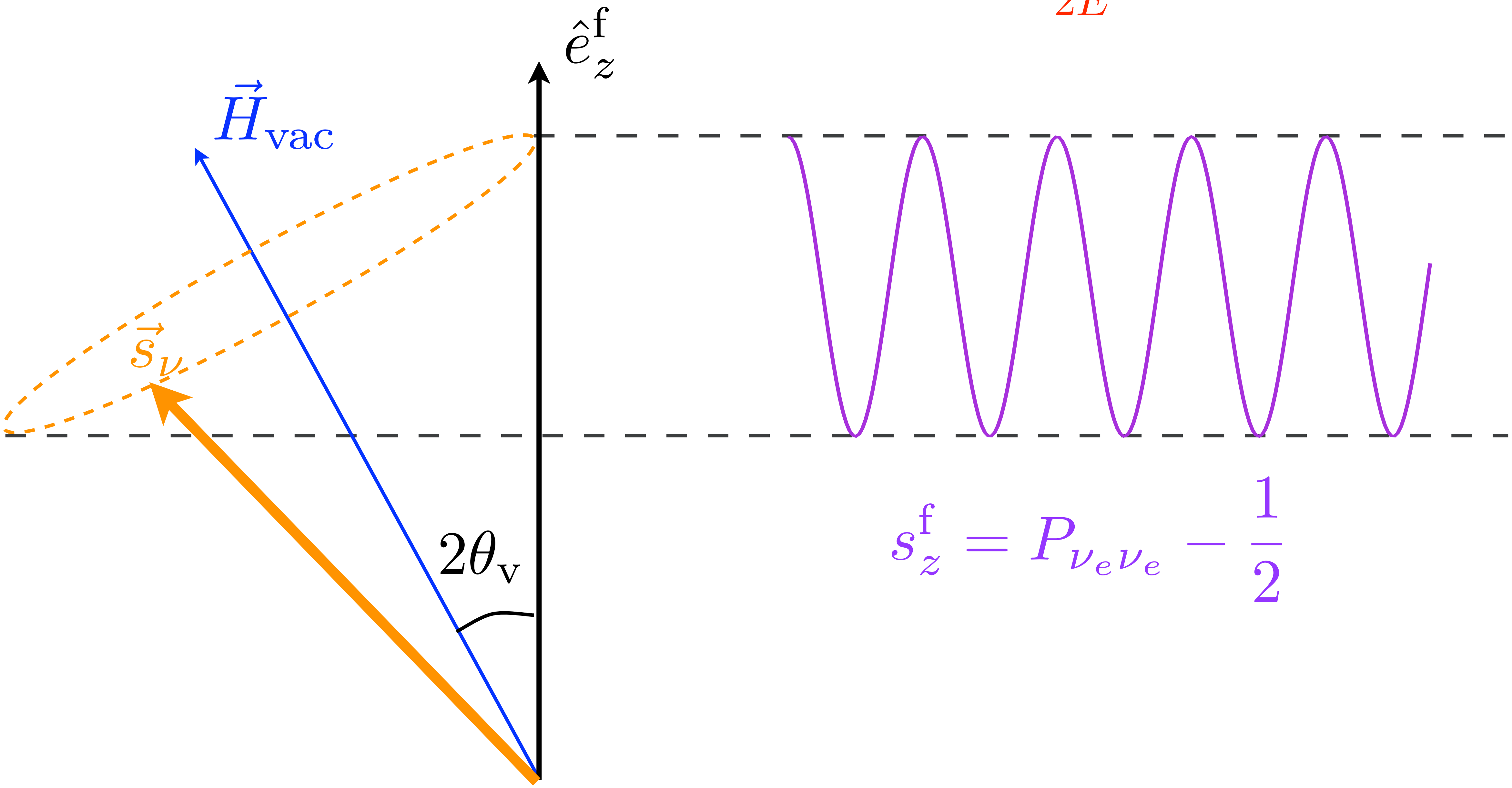
	e-flavor	τ'-flavor	maximally mixed
$\vec{s}_\nu \equiv \psi_\nu^\dagger \frac{\vec{\sigma}}{2}\psi_\nu$	$\uparrow$	$\downarrow$	$\rightarrow$
$\vec{s}_{\bar{\nu}} \equiv (\sigma_y\psi_{\bar{\nu}})^\dagger \frac{\vec{\sigma}}{2}(\sigma_y\psi_{\bar{\nu}})$	$\downarrow$	$\uparrow$	$\rightarrow$

Vacuum Oscillations

$$\vec{H} = \omega \vec{H}_{\mathrm{vac}}$$

$$\vec{H}_{\mathrm{vac}} \equiv -\hat{e}^{\mathrm{f}}_x \sin 2\theta_{\mathrm{v}} + \hat{e}^{\mathrm{f}}_z \cos 2\theta_{\mathrm{v}}$$

$$\omega \equiv \pm \frac{\delta m^2}{2E}$$

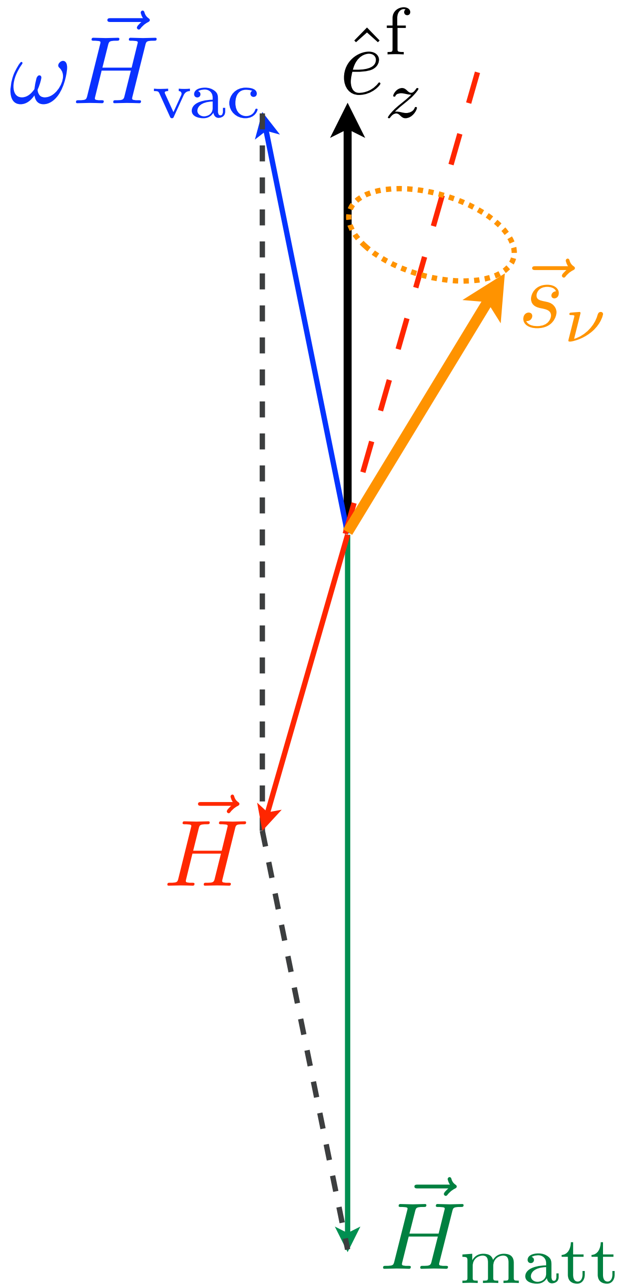


$$s^{\mathrm{f}}_z = P_{\nu_e \nu_e} - \frac{1}{2}$$

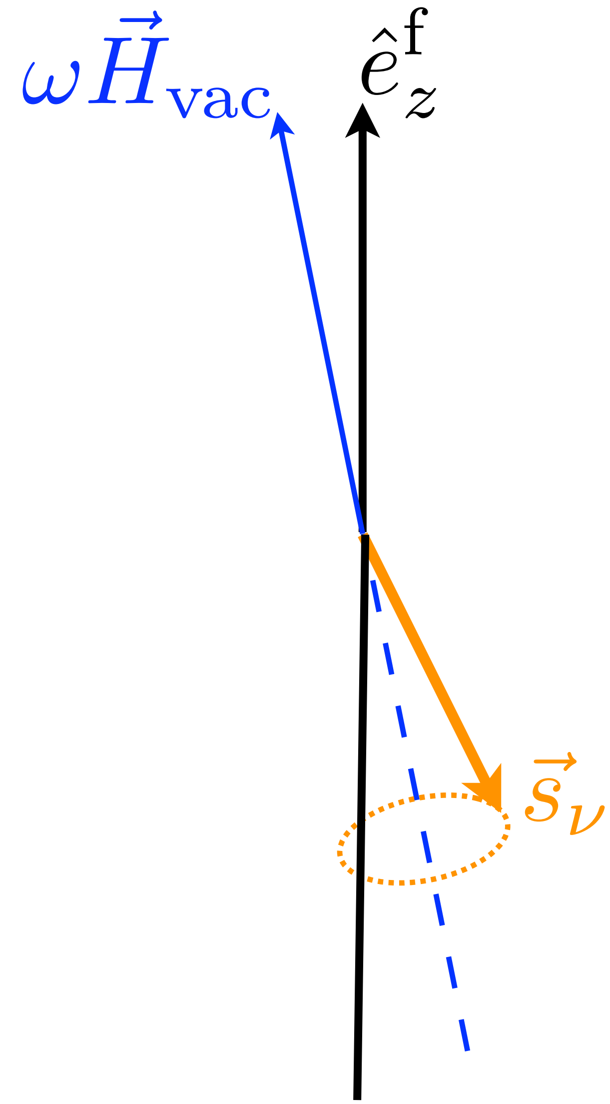
MSW Mechanism

$$\vec{H} = \omega\vec{H}_{\rm vac} + \vec{H}_{\rm matt}$$

$$\vec{H}_{\rm matt} \equiv -\hat{e}_z^{\rm f}\sqrt{2}G_{\rm F}n_e$$



$n_e \to 0$



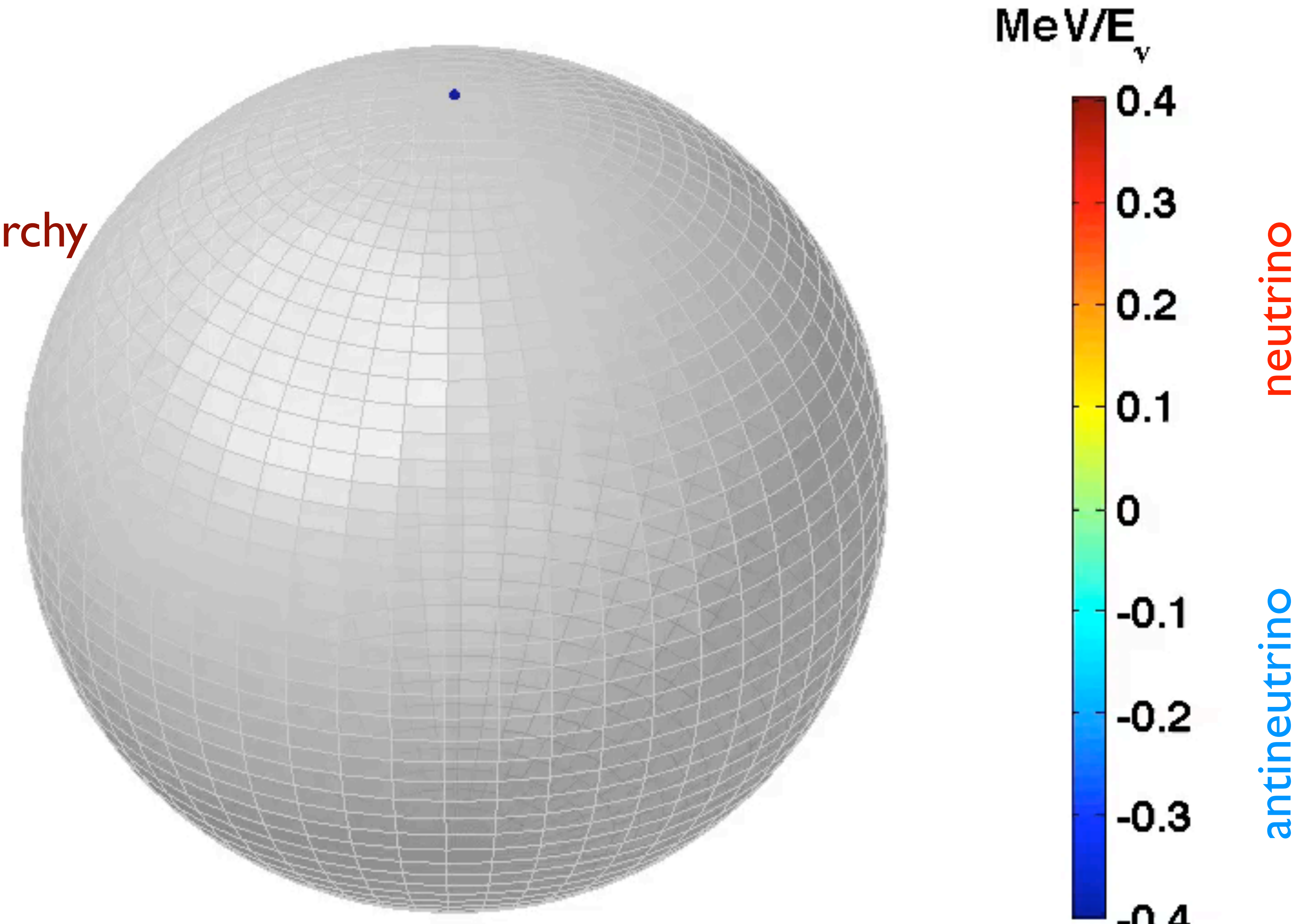
$$\omega = \frac{\delta m^2}{2E},\ \delta m^2 > 0$$

neutrino, normal hierarchy

MSW Mechanism

$\delta m^2 < 0$

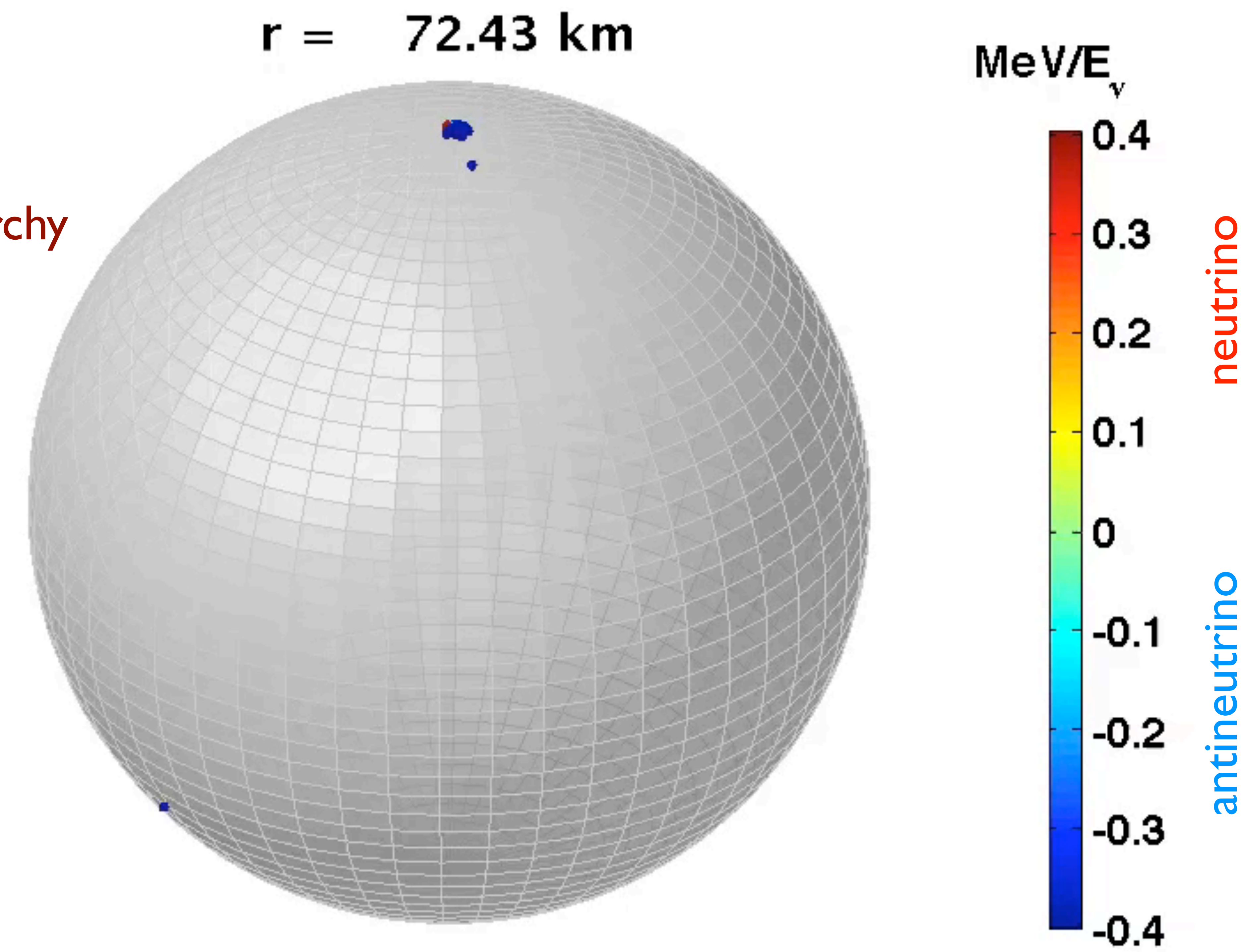
inverted hierarchy



Collective Oscillations

$\delta m^2 < 0$

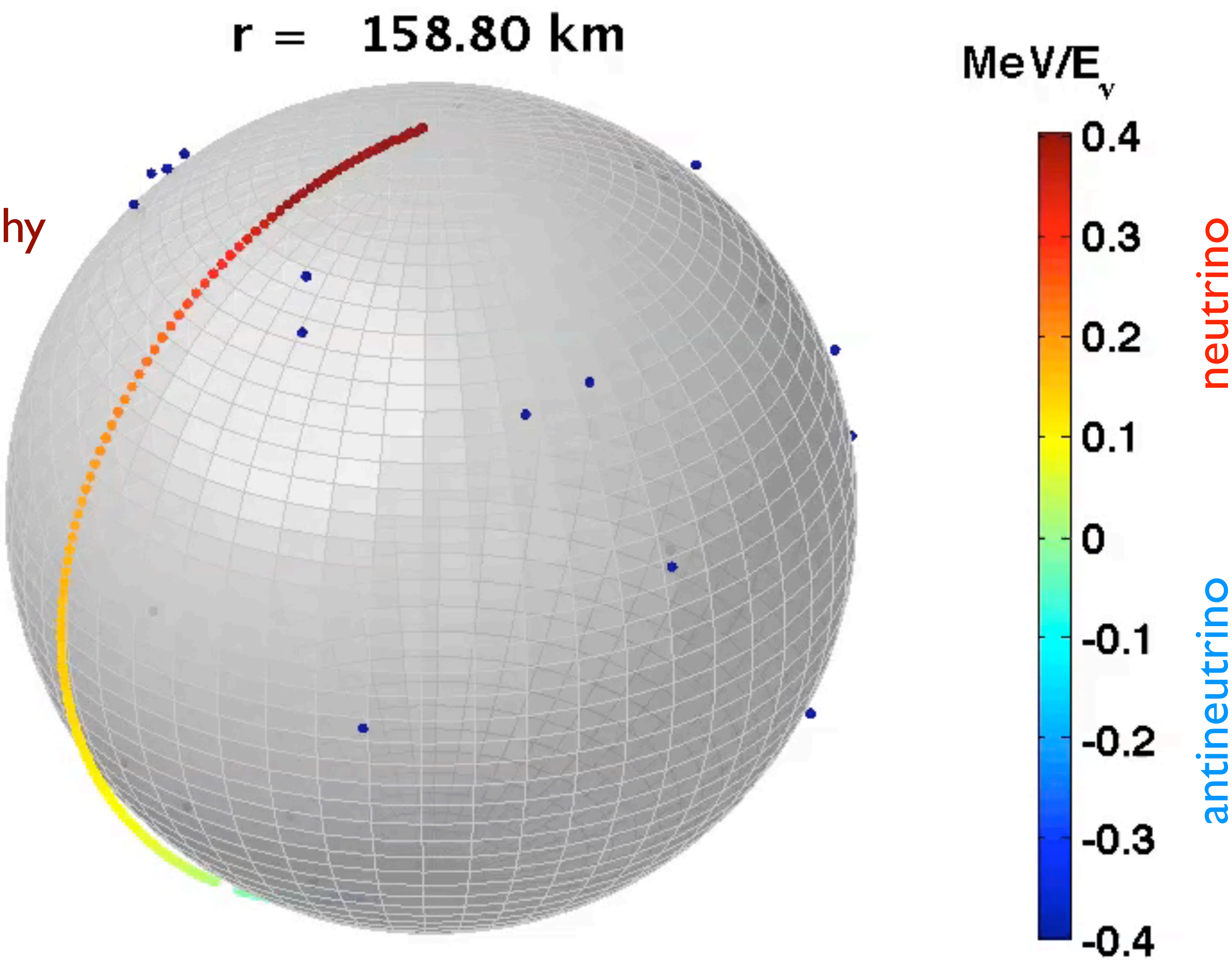
inverted hierarchy



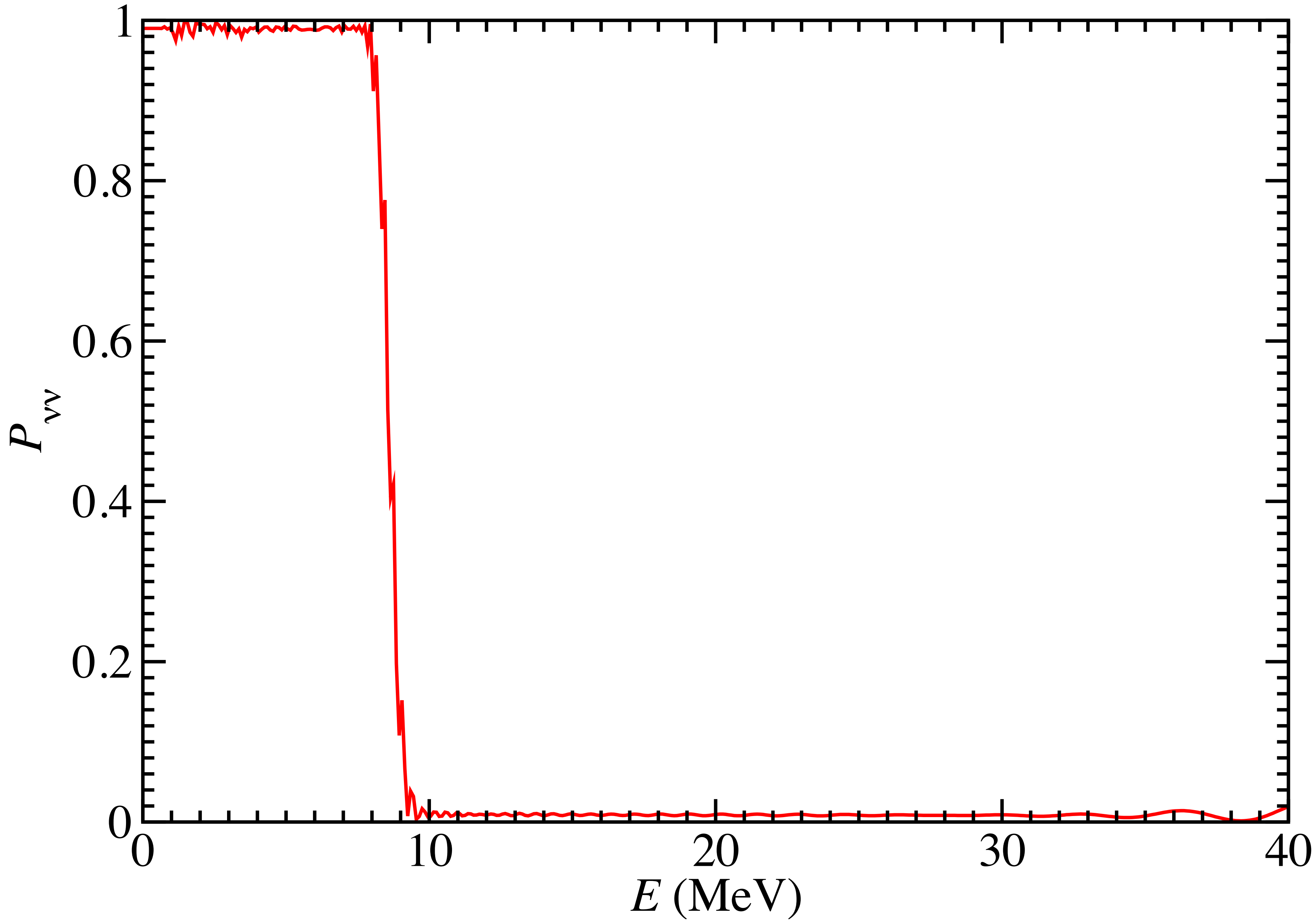
Collective Oscillations

$\delta m^2 < 0$

inverted hierarchy



Collective Oscillations



Collective Oscillations

homogeneous, isotropic neutrino gas

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{s}_\omega = \vec{s}_\omega \times \vec{H}_\omega$$

$$= \vec{s}_\omega \times \left(\omega \vec{H}_{\mathrm{vac}} + \cancel{\vec{H}}_{\mathrm{matt}} + \vec{H}_{\nu\nu}\right)$$

cause collective oscillations

neutral (HD et al, PRD, 2005)

special axis

disrupt collective oscillations

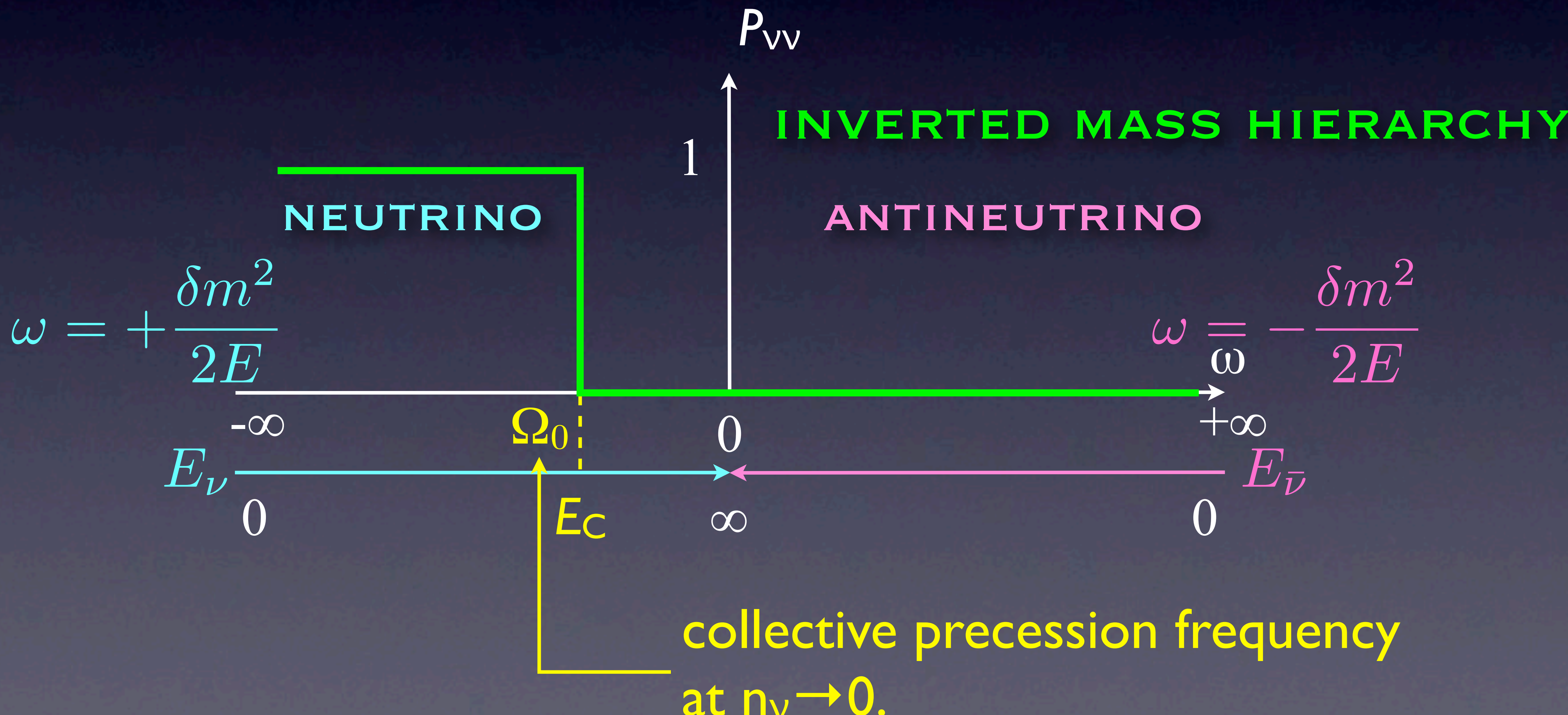
$$\vec{H}_{\nu\nu} = -\mu \int_{-\infty}^{\infty} \mathrm{d}\omega' \, f(\omega') \, \vec{s}_{\omega'}$$

distribution function

coupling strength, $\propto$ neutrino density

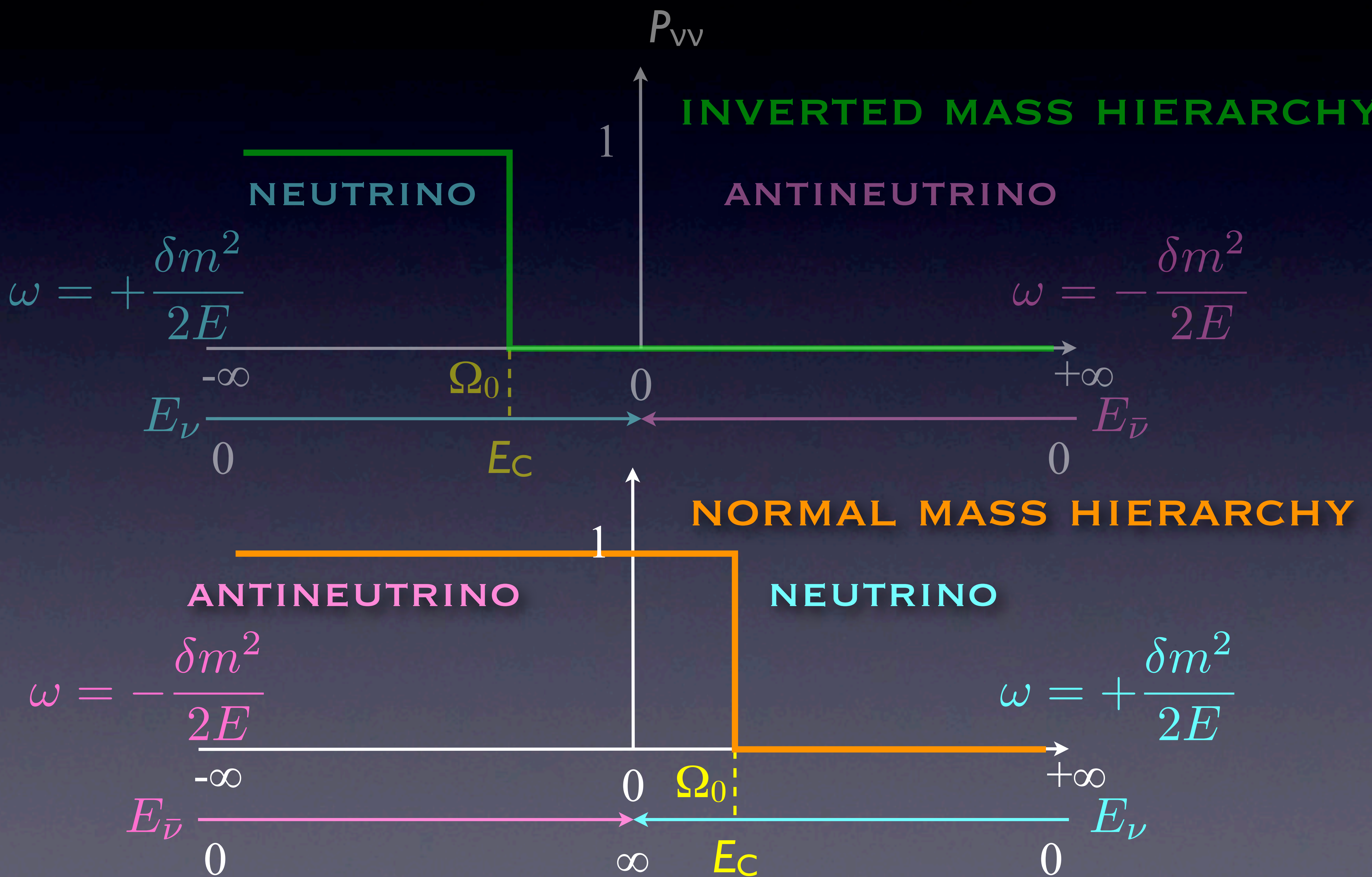
Collective Oscillations

if collective precession exists until $n_\nu \to 0$:



HD et al, PRD (2006)

Collective Oscillations



HD et al, PRD (2006)

Where?

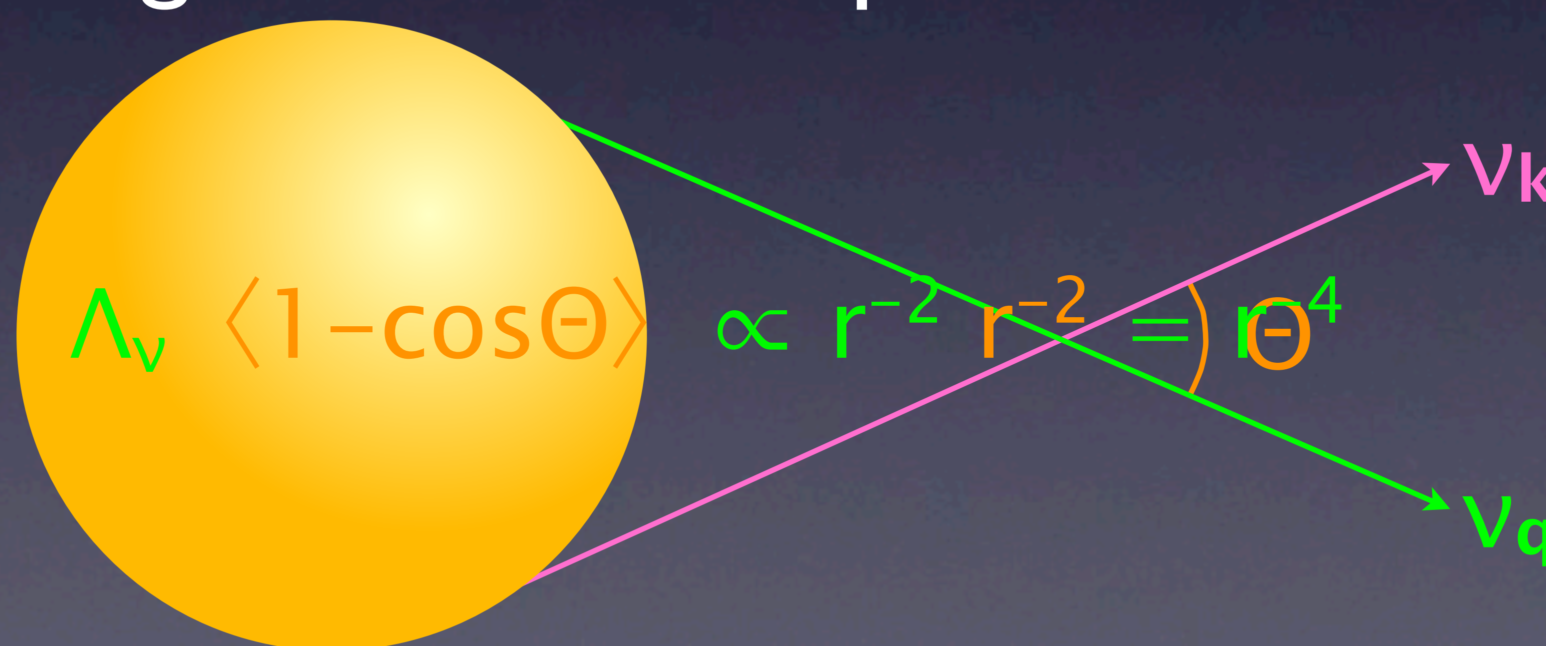
Neutrino Oscillations

$$\mathrm{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\,\mathrm{diag}[n_e, 0, 0] + \mathrm{H}_{\nu\nu}$$

- vacuum term: $\Lambda_{vac}=\Delta m^2/2E$
- matter (electron) density: $\Lambda_{mat}=\sqrt{2}G_F n_e$
- neutrino density: $\Lambda_\nu=\sqrt{2}G_F(n_\nu - n_{\bar{\nu}})$

Collective Oscillations

- Λ_{vac} depends on neutrino energy $\Rightarrow$ $\Delta\Lambda_{vac}$: dispersion in energies
- $\Delta\Lambda_{vac}$ ($\sim \Lambda_{vac}$) $\lesssim$ Λ_ν $\langle 1\text{-}\cos\Theta \rangle$: neutrinos with different energies oscillate in phase



$\Lambda_{vac}=\Delta m^2/2E$ $\qquad$ $\Lambda_\nu=\sqrt{2}G_F(n_\nu-n_{\bar\nu})$

Self-Suppression

- $\Delta\Lambda_{vac}$ ($\sim \Lambda_{vac}$) $\ll \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$: synchronization; no significant oscillations unless experiencing MSW resonance
(Pastor et al 2001, 2002)

- Criterion for significant collective oscillations: $\underline{\Delta\Lambda_{vac} (\sim \Lambda_{vac}) \sim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle}$
(Duan, Fuller & Qian, 2005)

$\Lambda_{vac}=\Delta m^2/2E$ $\quad\quad$ $\Lambda_\nu=\sqrt{2}G_F(n_\nu-n_{\bar{\nu}})$

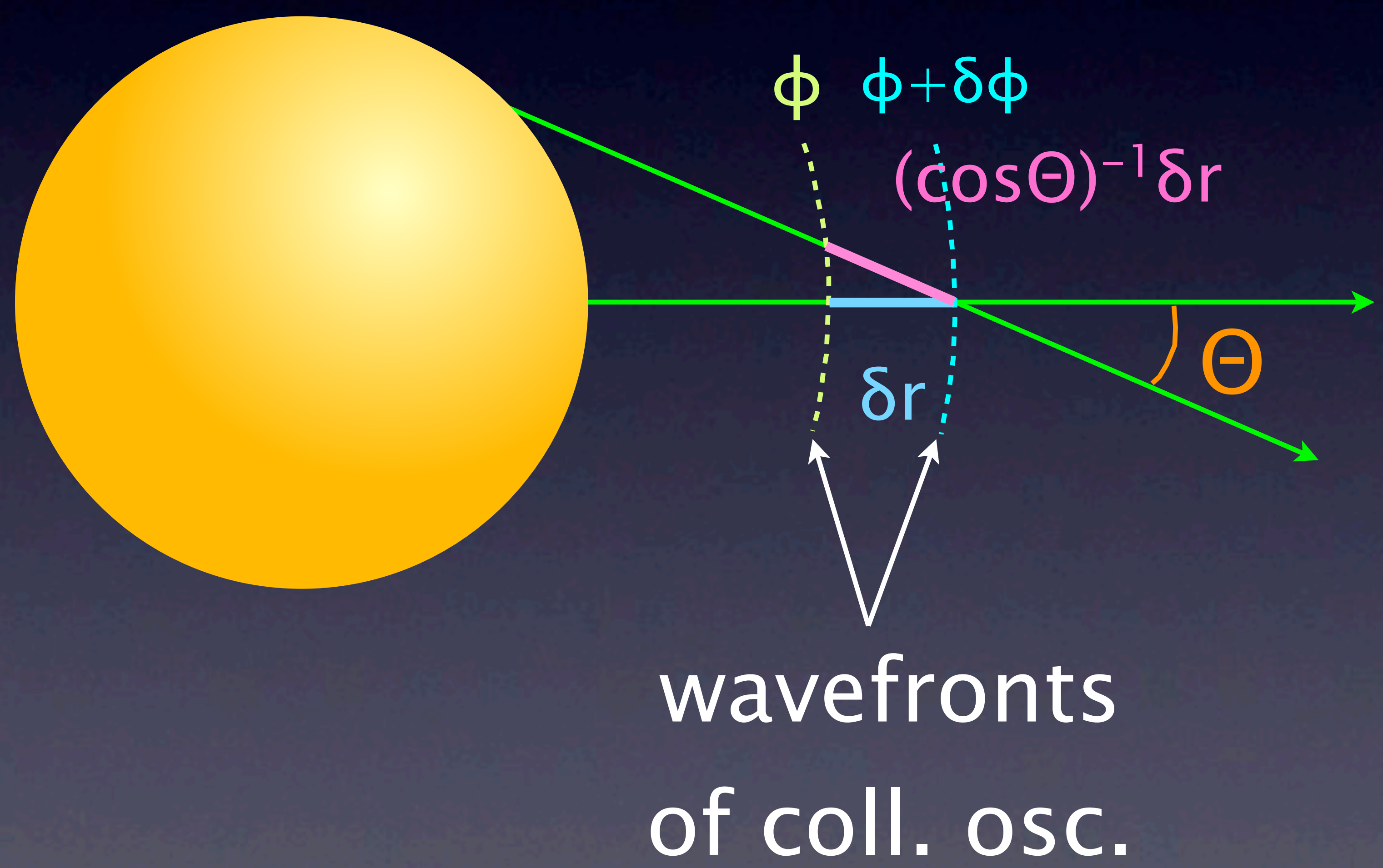
Matter Suppression

- $\Lambda_{mat} \gtrsim \Lambda_\nu \ \langle 1\text{-}\cos\Theta \rangle$: suppression of collective oscillations?
- No. Uniform matter distribution does not suppress collective oscillations in the homogeneous and isotropic neutrino gas.
  (Duan, Fuller & Qian, 2005)

$\Lambda_{mat} = \sqrt{2} G_F n_e$ $\qquad$ $\Lambda_\nu = \sqrt{2} GF (n_\nu - n_{\bar{\nu}})$

Matter Suppression

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$



Matter Suppression

- Dispersion in $\Lambda_{mat} d\lambda/dr$: $\Delta\Lambda_{mat}$
- Criterion for significant collective oscillations: $\underline{\Delta\Lambda_{vac} + \Delta\Lambda_{mat} \sim \Lambda_{\nu} \langle 1\text{-}\cos\Theta \rangle}$
- $\Delta\Lambda_{mat} \gtrsim \Lambda_{\nu} \langle 1\text{-}\cos\Theta \rangle \Rightarrow$ suppression of collective oscillations (Esteban-Pretel et al, 2008)
- $\Delta\Lambda_{mat} \propto r^{-2} \rho(r) \Rightarrow$ suppression only at early-time and/or very close to NS

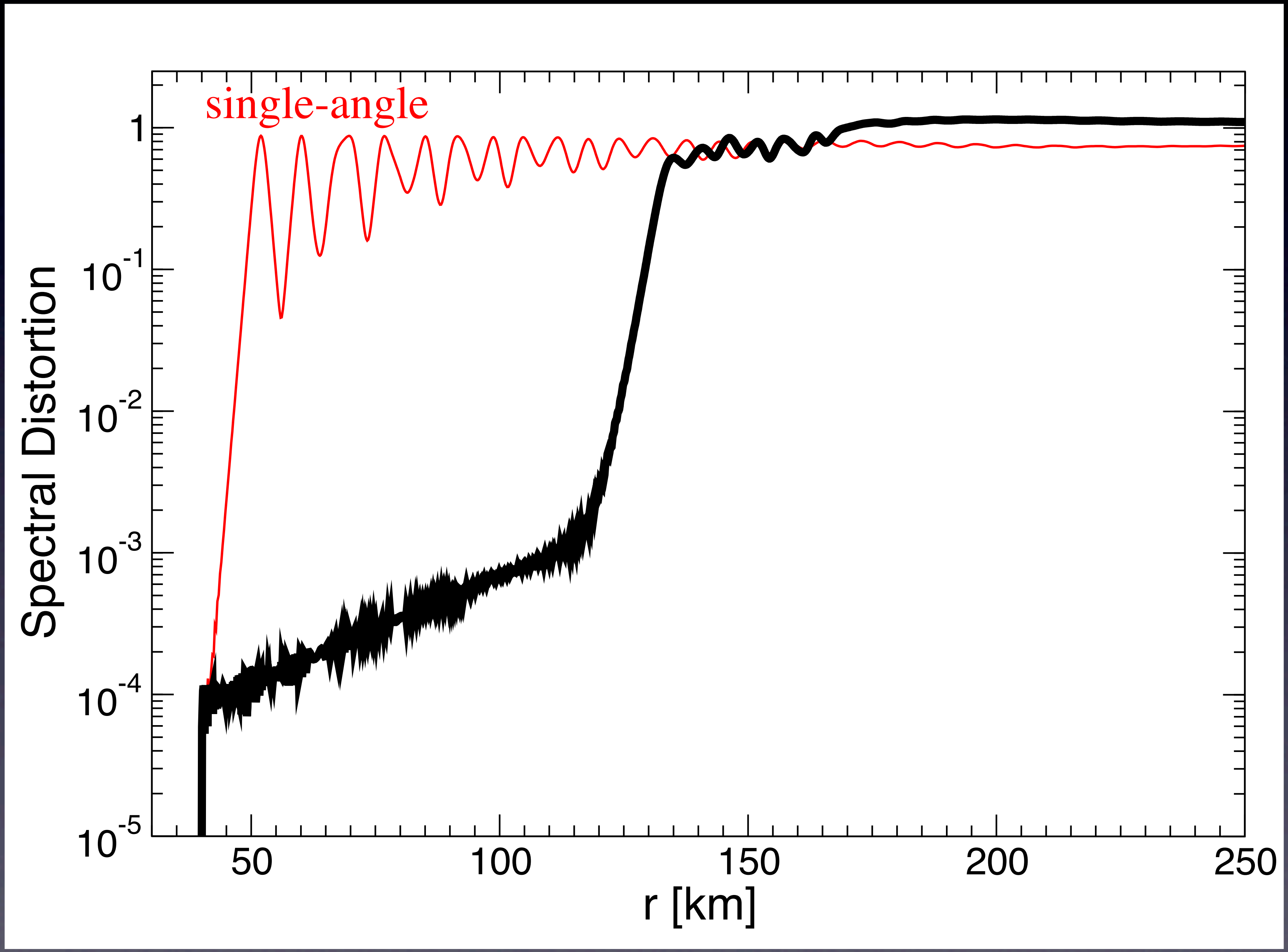
$\Lambda_{vac} = \Delta m^2 / 2E$ $\Lambda_{mat} = \sqrt{2} G_F n_e$ $\Lambda_{\nu} = \sqrt{2} GF(n_{\nu} - n_{\bar{\nu}})$

Multiangle Suppression

- $\Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$ depends on neutrino emission angle ⇒ dispersion in angles $\Delta\Lambda_\nu$ (~$\Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$)
- Criterion for significant collective oscillations: $\underline{\Delta\Lambda_{vac} + \Delta\Lambda_{mat} + \Delta\Lambda_\nu \sim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle}$
or $\Delta\Lambda_{vac} \sim \Delta\Lambda_\nu$ for low matter density
(Duan & Friedland, 2010)

$\Lambda_{vac} = \Delta m^2/2E$ $\quad\quad$ $\Lambda_\nu = \sqrt{2} G_F (n_\nu - n_{\bar{\nu}})$

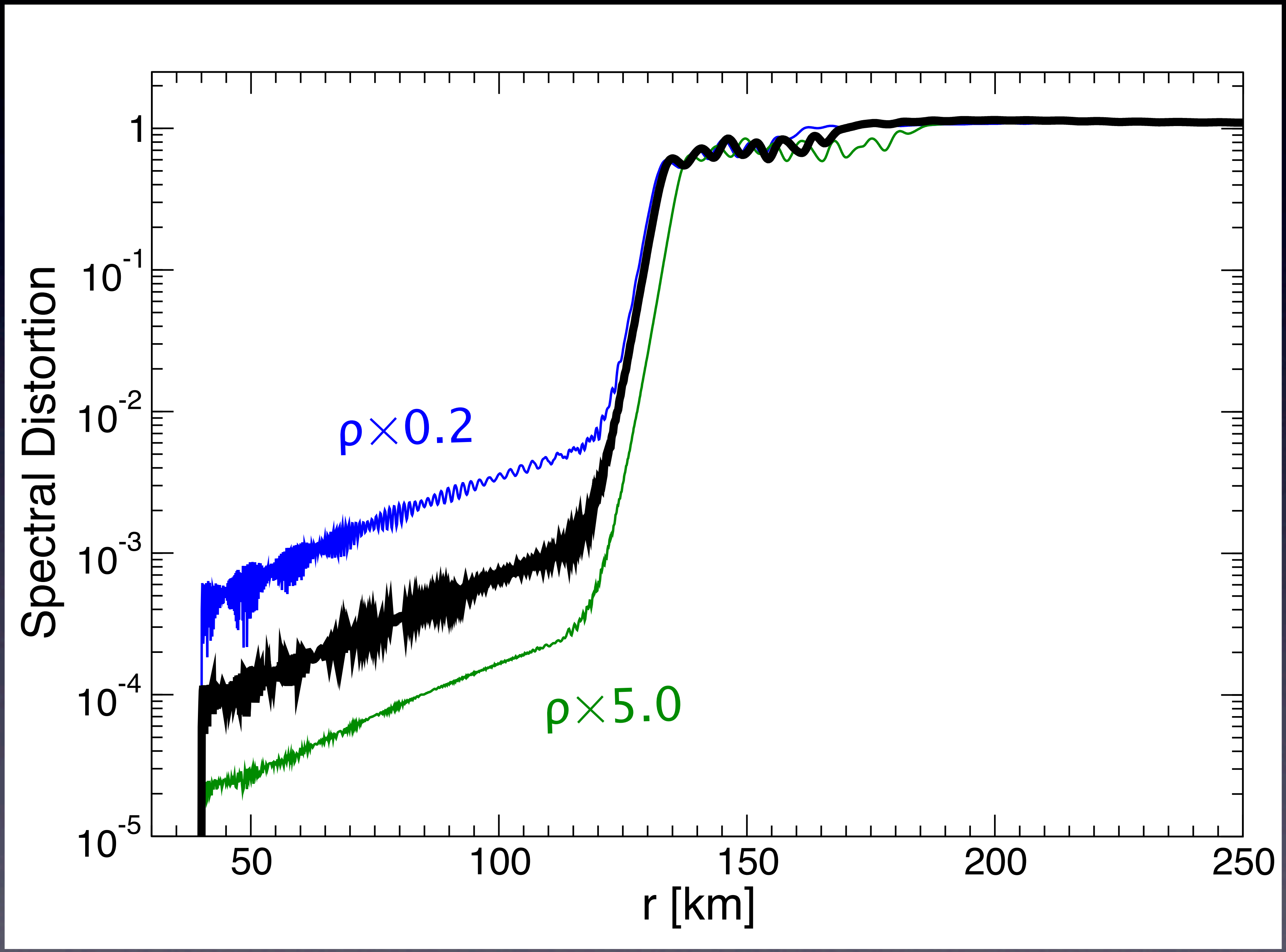
Multiangle Suppression



Using late-time spectra from Monte Carlo simulations (Keil et al 2002)

Duan & Friedland, PRL 106, 091101 (2011)

Multiangle Suppression



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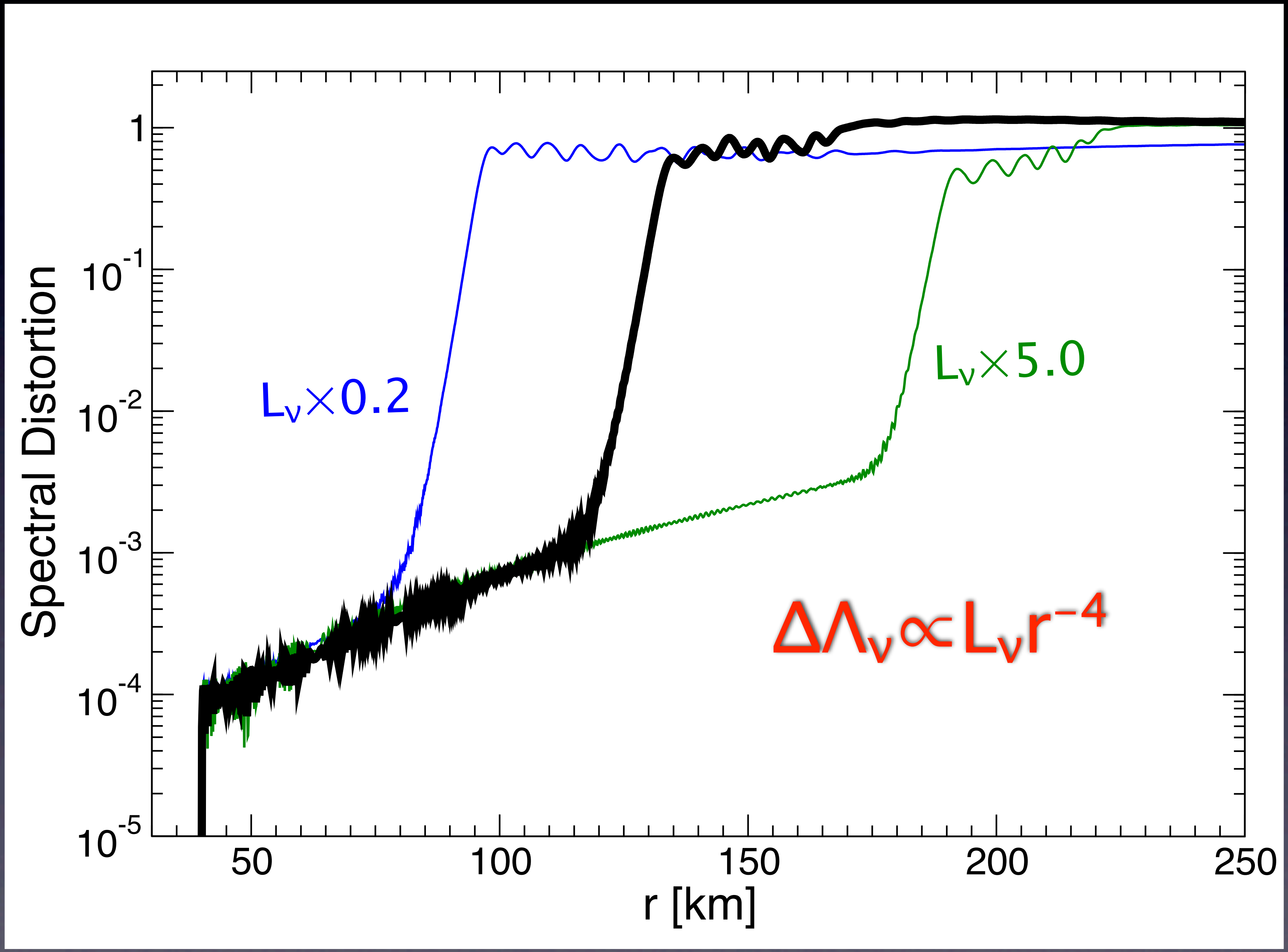

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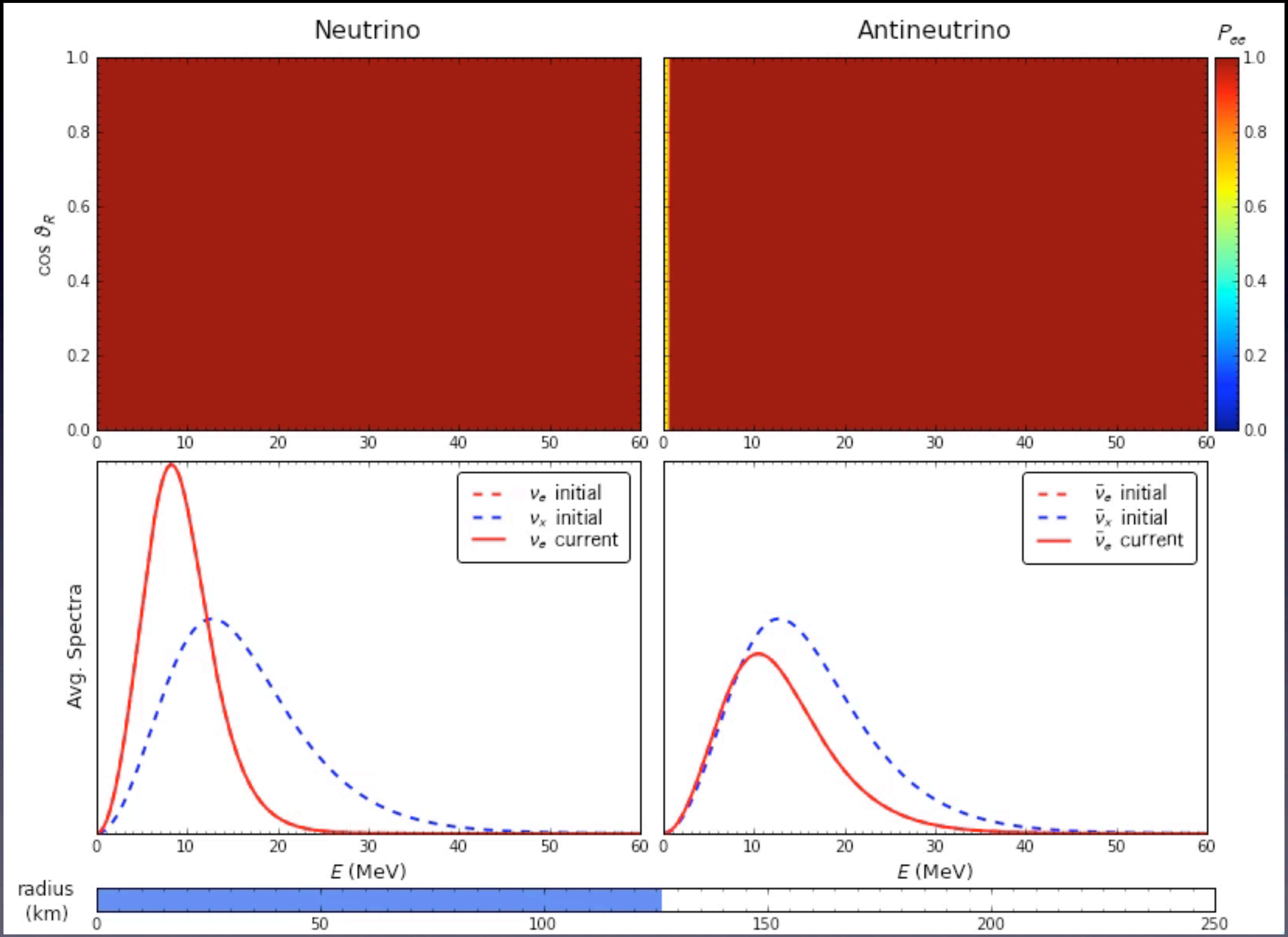
$$\langle L_{\nu_e} \rangle = 4.1\,\mathrm{foe},\ \langle L_{\bar{\nu}_e} \rangle = 4.3\,\mathrm{foe},\ \langle L_{\nu_x,\bar{\nu}_x} \rangle = 7.9\,\mathrm{foe}$$

$$\langle E_{\nu_e} \rangle = 9.4\,\mathrm{MeV},\ \langle E_{\bar{\nu}_e} \rangle = 13.0\,\mathrm{MeV},\ \langle E_{\nu_x,\bar{\nu}_x} \rangle = 15.8\mathrm{MeV}$$



Summaries

- Collective neutrino oscillations can occur in dense neutrino media — This is a result of intrinsic symmetry.
- Significant collective oscillations occur when neutrino densities are "moderate".
- Be careful with the single-angle approximation.
- Stay tuned ...