The finite temperature QCD phase transition from domain wall fermions

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Outline

1. QCD at Finite Temperature
2. Chiral Symmetry restoration and the anomalous $U(1)_A$ symmetry
3. Domain Wall Fermions
4. Screening correlators and susceptibilities
5. Dirac spectrum

The HotQCD collaboration

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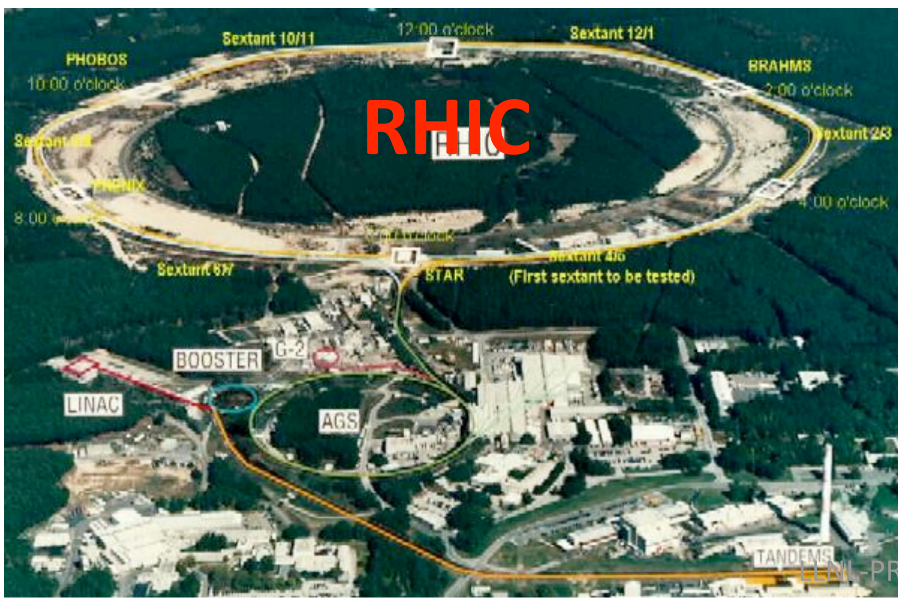
Hot QCD

- Matter of sufficient energy density existed in the first 10^{-5} sec. after Big Bang.
- Interiors of neutron stars?
- Probed experimentally in heavy ion collisions.

RHIC: Au+Au collisions at 200 GeV/nucleon

LHC: Pb+Pb collisions at 2.76 TeV/nucleon



QCD

QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - g\psi A^{a}_{\mu}\bar{\psi}T^{a}\psi - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a}$$

$$G^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - gf^{abc}A^{b}_{\mu}A^{c}_{\nu}$$

- Asymptotic Freedom: Weak coupling at high energies
- Strong coupling at low energies
- Confinement: Only color singlets
- Chiral symmetry: Classical $SU(N_f)_L$ x $SU(N_f)_R$ x $U(1)_V$ x $U(1)_A$ symmetry
- Spontaneous χSB breaks $SU(N_f)_L$ x $SU(N_f)_R$ -> $SU(N_f)_V$
- Anomalous $U(1)_A$ symmetry

Finite Temperature QCD

- Hagedorn (1965) noticed that apparent exponential rise in density of hadronic states could imply a limiting temperature.
- In reality, hadrons are not point particles. At sufficiently high temperature T_c, thermal energy is sufficient to "melt" hadron states.
- Quark-gluon plasma (QGP)
- Deconfinement: Colored gluons, quarks are no longer bound in hadrons – free to propagate.
- Chiral symmetry restoration: Quarks propagate in QGP with current quark masses.
- Is the QGP truly a different phase? What is the nature of the phase transition?

Phase Diagram

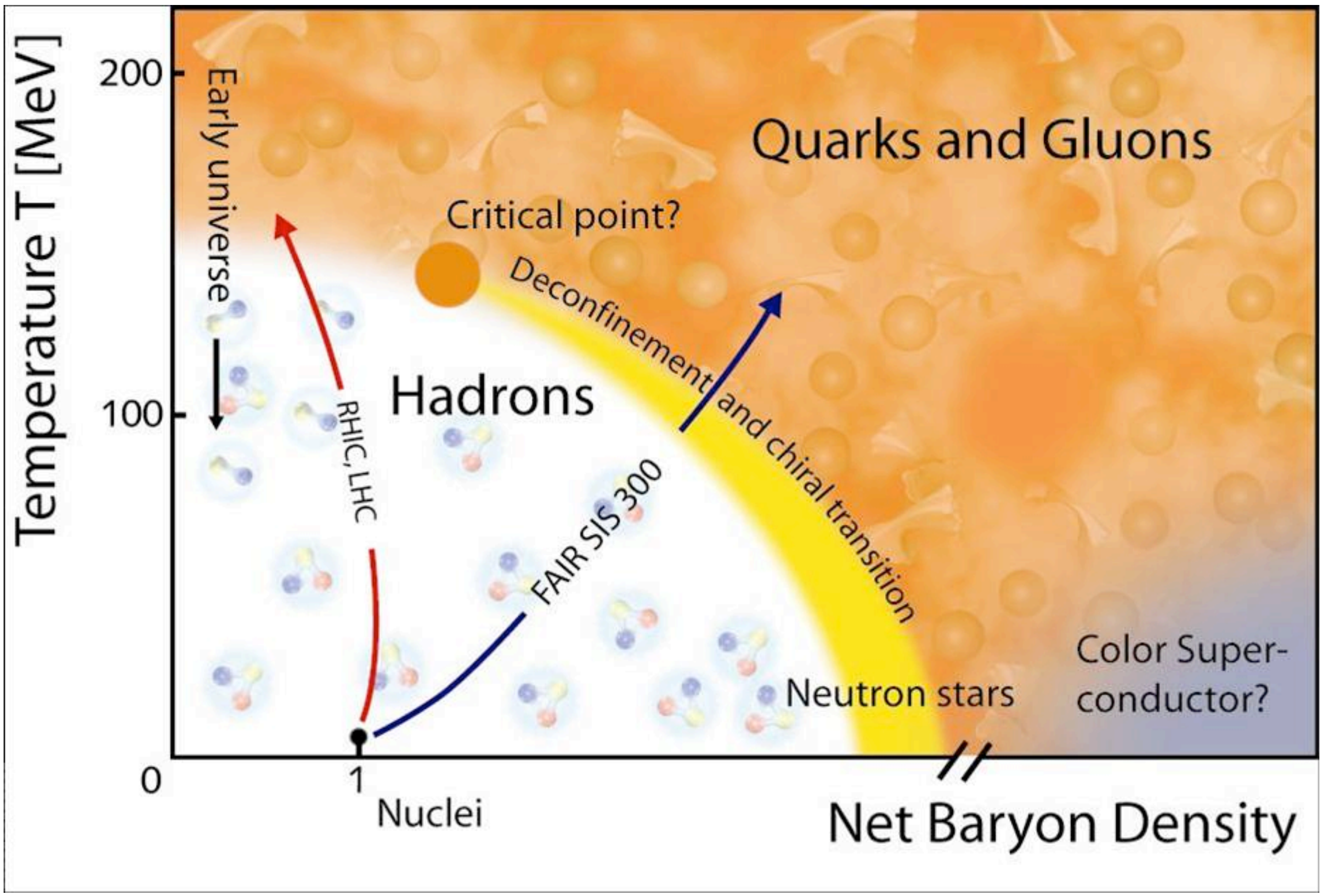


Image from GSI

Lattice QCD

- Normal field theoretic tools, i.e. perturbative expansion, does not work very well.
- Coupling is too strong for perturbation series to converge.
- In addition, at finite temperature, there is added infared divergence that signals true non-perturbative behavior.
- Consider instead (Euclidean) path integral:

$$\int [D\bar{\psi}][D\psi][DA_\mu] \exp\left(-\int \left[\bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right]\right)$$

- Discretize on a 4-dimensional space-time lattice.
- $V = L^3 = (N_s a)^3 \quad t = N_t a$
- a is lattice spacing; a^{-1} sets UV cut-off scale in calculation.

Lattice QCD at Finite Temperature

- Wish to calculate *thermal* expectation values:

$$\langle \mathcal{O} \rangle = \sum_n \langle n | \mathcal{O} e^{-H_{QCD}/T} | n \rangle$$

- Thermal partition function is just Euclidean path integral with T = $1/N_t a$ and (anti-)periodic BCs for (fermion) gauge fields.

$$Z = \int [\mathcal{D}A_\mu] \det(M) \, e^{-\int_0^{1/T} dt \int d^3x S_G(x,t)}$$

- Work at fixed N_t. Change temperature by varying the lattice spacing.

Chiral Symmetry Restoration

- $SU(N_f)_L$ x $SU(N_f)_R$ spontaneously broken to $SU(N_f)_V$
- Chiral condensate develops – analog of magnetization in Ising model.
- Spontaneous ordering of QCD vacuum
- For $T > T_c$ chiral symmetry is restored – chiral condensate vanishes.
- Chiral condensate is an order parameter for chiral symmetry restoration.

Banks-Casher

- Banks-Casher relation (1980):

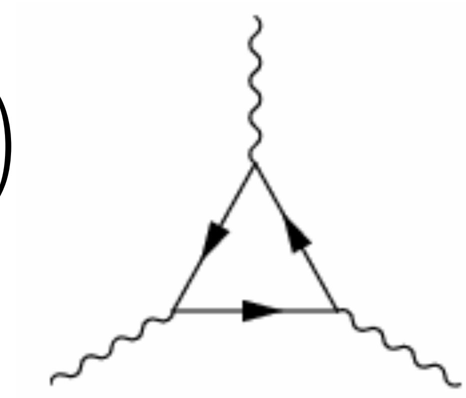
$$\begin{aligned}\langle \bar{q}q \rangle &= \lim_{m_q \to 0} \lim_{V \to \infty} \int d\lambda \rho(\lambda) \frac{-2m_q}{\lambda^2 + m_q^2} \\ &= -\pi\rho(\lambda = 0)\end{aligned}$$

- Remarkable result: Chiral condensate can be reconstructed using low-lying modes of the 4-d Dirac operator.
- Furthermore: Condensation of modes at λ=0 in the chirally broken phase.
- Order of limits important: no χSB in finite volume.

$U(1)_A$

- Classical symmetry of QCD Lagrangian.
- (Non-)conservation of flavor singlet axial current.
- Violated by Adler-Bell-Jackiw anomaly.

$$\partial_\mu J_5^\mu = \frac{g^2 N_f}{16\pi^2} \mathrm{tr}\left(\tilde{F}_{\mu\nu} F^{\mu\nu}\right)$$
$$J_5^\mu = \bar{q}\gamma^\mu\gamma^5 q$$



Gauge field topology

- RHS of anomaly equation corresponds to density of topological modes.

$$Q(x) = \frac{g^2}{16\pi}\mathrm{tr}\left(\tilde{F_{\mu\nu}} F^{\mu\nu}\right)$$

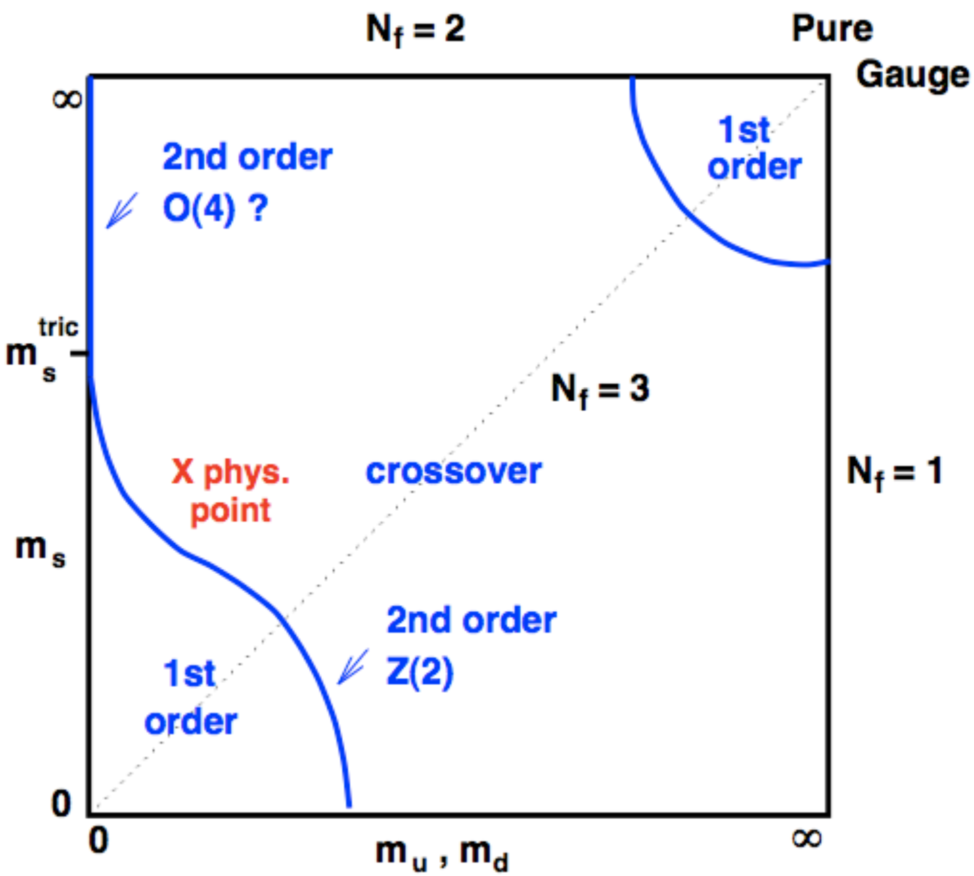
- Global topological charge – gauge field winding number
- Winding number equal to index of Dirac operator, *i.e.* difference between number of positive and negative chiral zero modes.

$$\nu = \int d^4x Q(x) = \mathrm{index}(D) = n_+ - n_-$$

$U(1)_A$ at finite temperature

- What happens to $U(1)_A$ at finite temperature?
- Naively, ABJ anomaly independent of temperature. $U(1)_A$ breaking at finite T?
- $U(1)_A$ problem solved by ABJ anomaly *and* presence of non-trivial topology.
- At high temperature, configurations with non-zero topology may be sufficiently suppressed so that $U(1)_A$ is *effectively* restored.

Phase Diagram



- $N_f \geq 3$ expected to be first order
- $N_f = 2$ is second order
- 2+1f QCD with quarks at physical mass is likely crossover.
- $U(1)_A$ restoration at T_c may cause $N_f = 2$ case to be first order.

Literature Review

- Pisarski Wilczek (1984)
- LQCD calculations in mid-late 1990s (Columbia, MILC, Kogut+Sinclair+Lagae, Karsch *et. al.*, …)
- Connection to low-lying Dirac modes: (Cohen, …)
- Many calculations rely on staggered fermions – difficult because of reduced chiral symmetry at finite lattice spacing – need to be close to continuum…

Domain Wall Fermions

- Chiral fermions on the lattice (Kaplan 1992).
- Left and right-handed chiral modes are bound to 4-d walls of 5-d theory.
- An exact chiral symmetry even at finite lattice spacing.

Residual Mass

- Residual chiral symmetry breaking from finite L_s. Additive renormalization to quark masses.

$$m_{\rm res}(L_s) = \frac{c_0}{L_s} + \frac{c_1}{L_s} \exp\left(-\alpha L_s\right)$$

- First term corresponds to localized lattice dislocations that mix states on the boundaries.
- Zero modes of 4-d Hermitian Wilson operator:

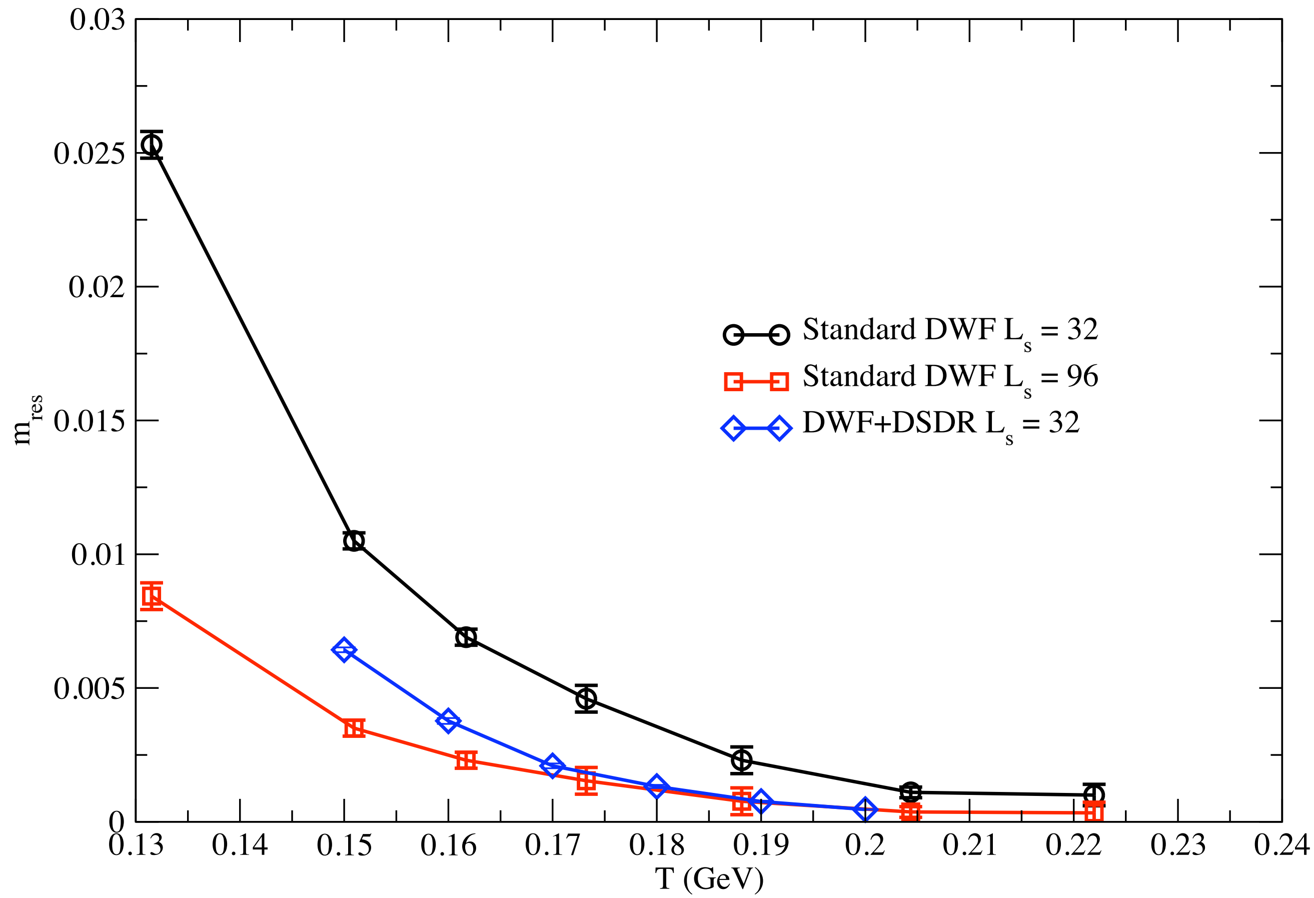
$$\gamma_5 D_W(-M_0)$$

DSDR

- Dislocation Suppressing Determinant Ratio(DSDR).
- Introduce det(D_W(-M_0)) into fermion action (Vranas 2006, Fukaya *et. al.* 2006)
- Suppresses lattice dislocations that dominate m_{res} at coarse couplings.
- However, same modes responsible for topological change.
- Add extra weight factor:

$$
\begin{aligned}
\mathcal{W}(M_0, \epsilon_b, \epsilon_f) &= \frac{\det\left[D_W^\dagger(-M_0 + \imath\epsilon_b\gamma^5) D_W(-M_0 + \imath\epsilon_b\gamma^5)\right]}{\det\left[D_W^\dagger(-M_0 + \imath\epsilon_f\gamma^5) D_W(-M_0 + \imath\epsilon_f\gamma^5)\right]} \\
&= \frac{\det\left[D_W^\dagger(-M_0) D_W(-M_0) + \epsilon_b^2\right]}{\det\left[D_W^\dagger(-M_0) D_W(-M_0) + \epsilon_f^2\right]}
\end{aligned}
$$

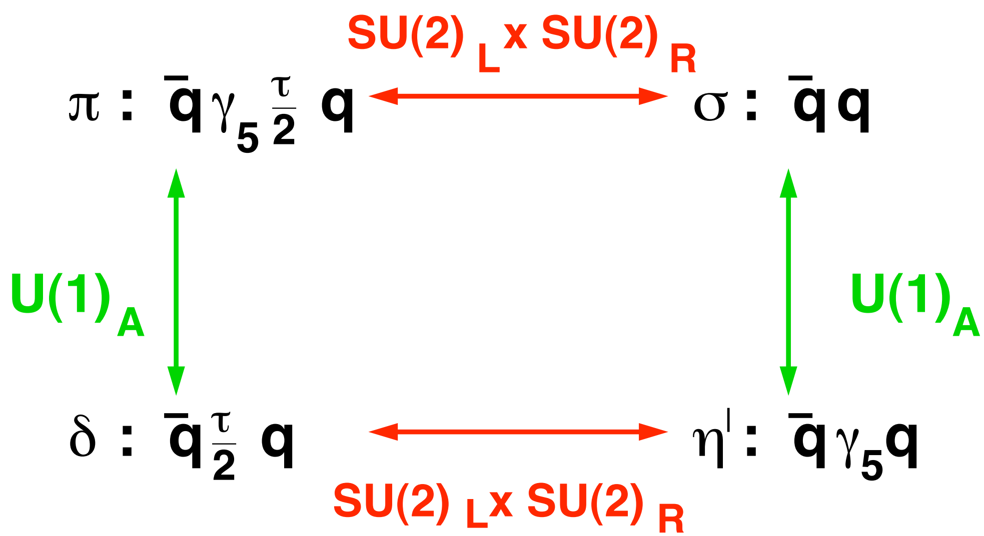
Residual Mass with DSDR



Calculation Details

- 16^3x8 lattice volumes
- 7 temperature in the range T = 140 – 200 MeV
- 3000-6000 HMC trajectories per temperature
- Temperatures fixed by zero-temperature calculations at three different bare couplings
- Adjust m_q so that m_π = 200 MeV
- m_s physical

Screening Correlators



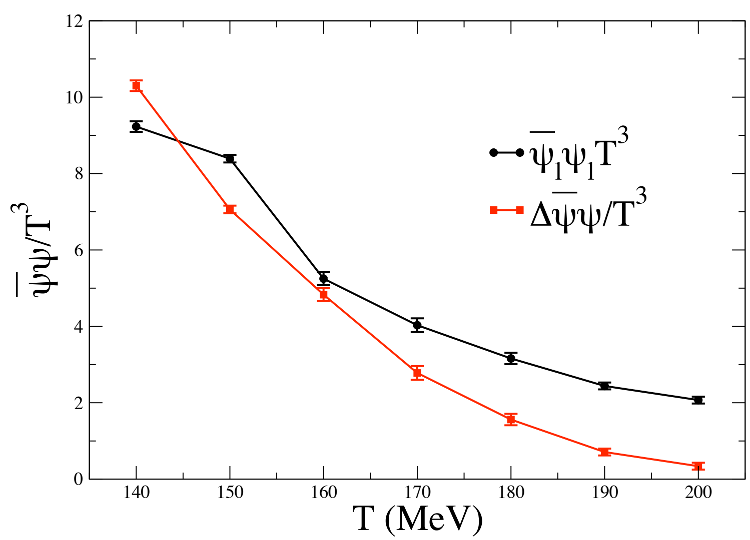
- Correlation functions in mesonic channels with propagation along spatial directions.
- Can be used to extract screening masses.
- Different channels related by chiral symmetry
- Can be used to examine $SU(2)_L$ x $SU(2)_R$ or $U(1)_A$ restoration in high temperature phase.

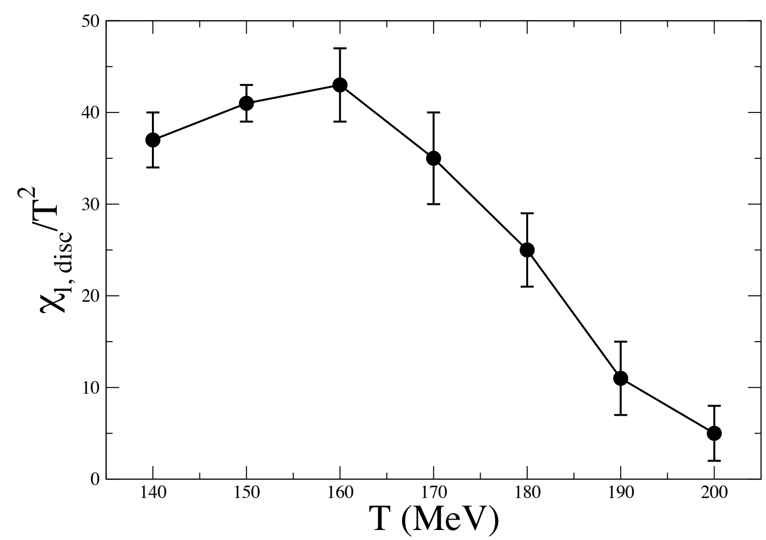
Chiral Susceptibility and Chiral Condensate

$$\frac{\langle\bar{\psi}_q\psi_q\rangle}{T^3} = \frac{1}{VT^2}\frac{\partial \ln Z}{\partial m_q} = \frac{N_\tau^2}{N_\sigma^3}\langle \mathrm{Tr} M_q^{-1}\rangle\ ,\ q=l,\ s\ .$$

$$\Delta_{l,s} = \langle\bar{\psi}_l\psi_l\rangle - m_l/m_s\langle\bar{\psi}_s\psi_s\rangle$$

$$\chi_{l,disc} = \frac{1}{N_\sigma^3 N_\tau}\left\{\langle\left(\mathrm{Tr}M_l^{-1}\right)^2\rangle - \langle \mathrm{Tr}M_l^{-1}\rangle^2\right\}$$

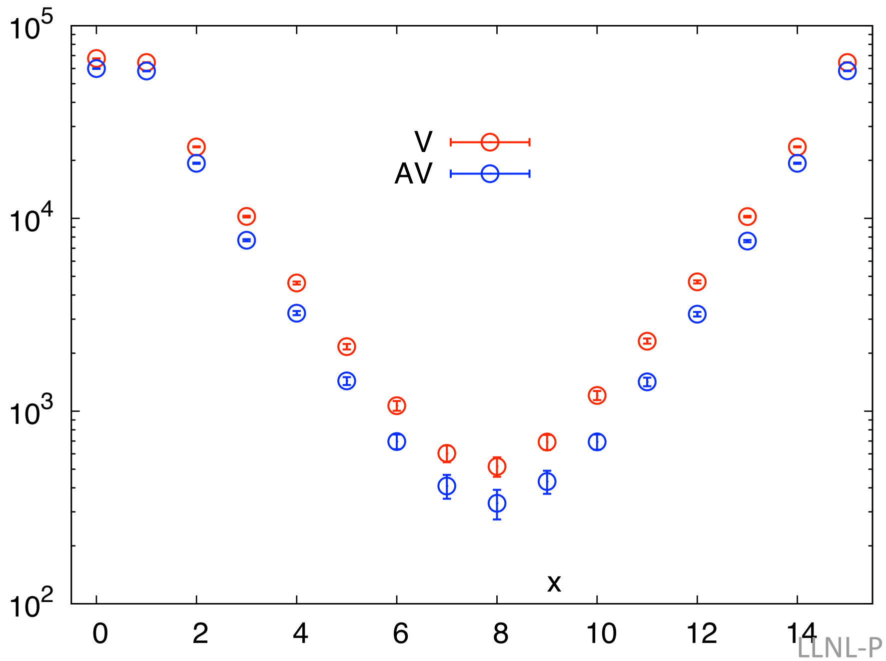


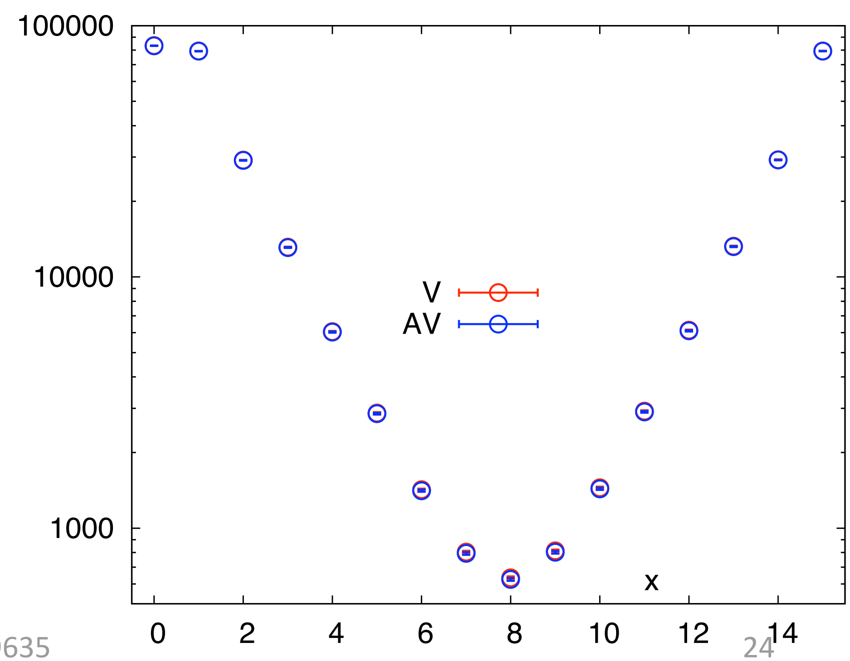


Vector/Axial Vector Correlators

$$\text{Vector:} \qquad \bar{q}\gamma^{\mu}q$$

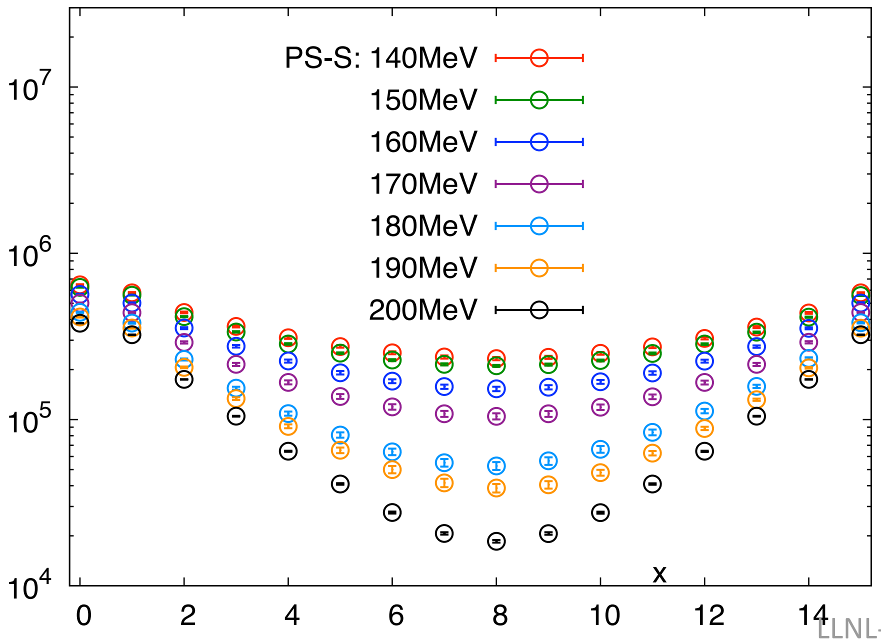
$$\text{Axial Vector:} \qquad \bar{q}\gamma^{\mu}\gamma^{5}q$$

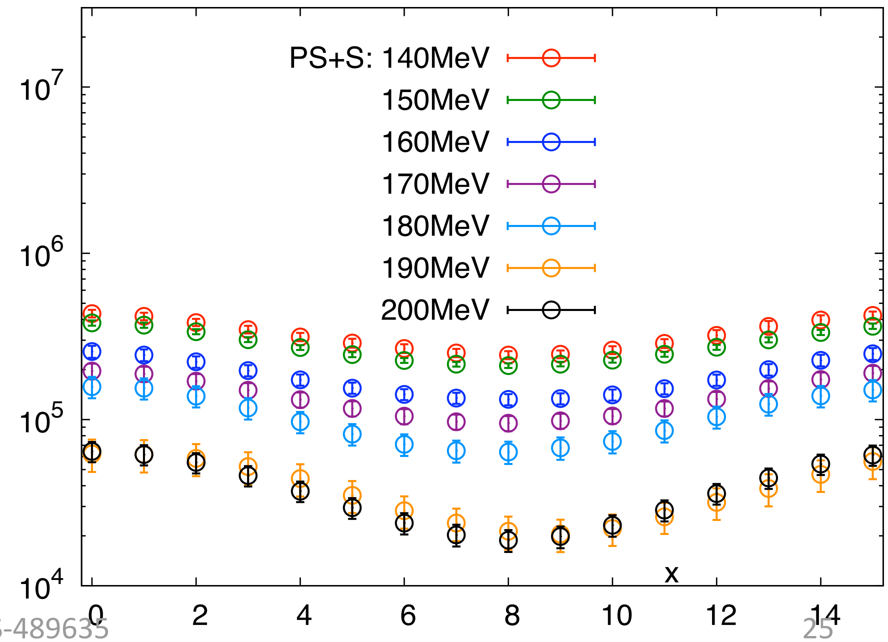




Scalar/Pseudoscalar Correlators

$$C_{SC(PS)}(x) = \langle \bar{u}_L d_R(x) \bar{d}_L u_R(0) + \bar{u}_L d_L(x) \bar{d}_L u_L(0) \rangle \pm \langle \bar{u}_L d_R(x) \bar{d}_R u_L(0) + \bar{u}_R d_L(x) \bar{d}_L u_R(0) \rangle.$$





Susceptibilities

$$\frac{\chi_\sigma}{T^2} = \sum_{\vec{x},\tau} G_\sigma(\tau,\vec{x}) = \frac{\chi_{l,con}}{T^2} + \frac{\chi_{l,disc}}{T^2}$$

$$\frac{\chi_\delta}{T^2} = \sum_{\vec{x},\tau} G_\delta(\tau,\vec{x}) = \frac{\chi_{l,con}}{T^2}$$

$$\frac{\chi_\eta}{T^2} = \sum_{\vec{x},\tau} G_\eta(\tau,\vec{x}) = \frac{\chi_{5,con}}{T^2} + \frac{\chi_{5,disc}}{T^2}$$

$$\frac{\chi_\pi}{T^2} = \sum_{\vec{x},\tau} G_\delta(\tau,\vec{x}) = \frac{\chi_{5,con}}{T^2}$$

$U(1)_A$ restoration:

$$\Delta_{\pi-\delta} = \frac{\chi_\pi - \chi_\delta}{T^2} = \frac{\chi_{5,conn} - \chi_{l,conn}}{T^2}$$

$$\lim_{m_q\to 0} \rightarrow \Delta_{\pi-\delta} = \frac{\chi_{l,disc}}{T^2} = \frac{\chi_{5,disc}}{T^2}$$

$SU(2)_L$ x $SU(2)_R$ restoration:

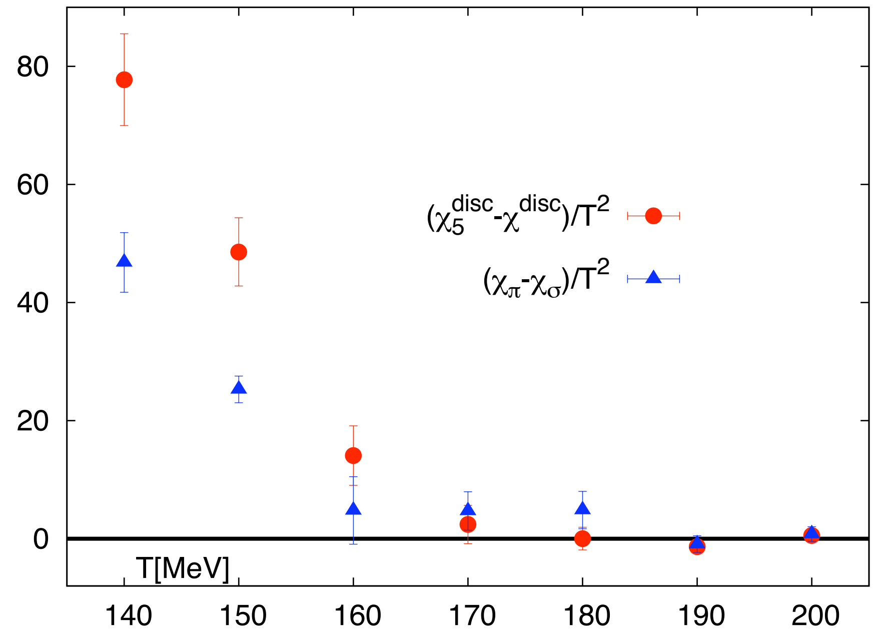
$$\chi_\pi = \chi_\sigma; \quad \chi_\eta = \chi_\delta$$

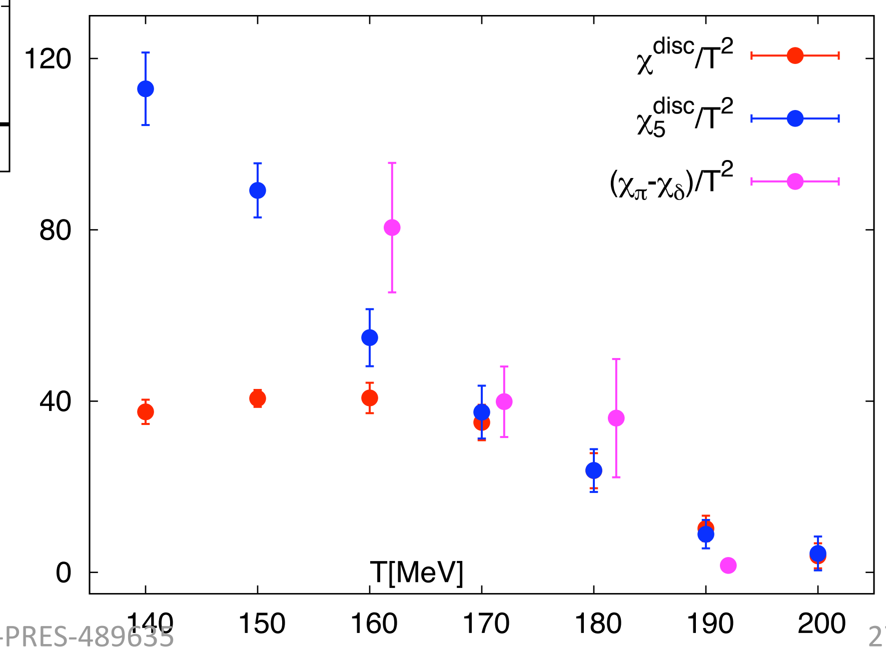
$$\Delta_{\pi-\sigma} = \frac{\chi_\pi - \chi_\sigma}{T^2} = \frac{\chi_{5,conn} - \chi_{l,disc} - \chi_{l,conn}}{T^2}$$

$$\Delta_{\text{disc}} = \frac{\chi_{5,disc} - \chi_{l,disc}}{T^2}$$

$$\lim_{m_q\to 0} \rightarrow \Delta_{\pi-\sigma} = \Delta_{\text{disc}} = 0$$

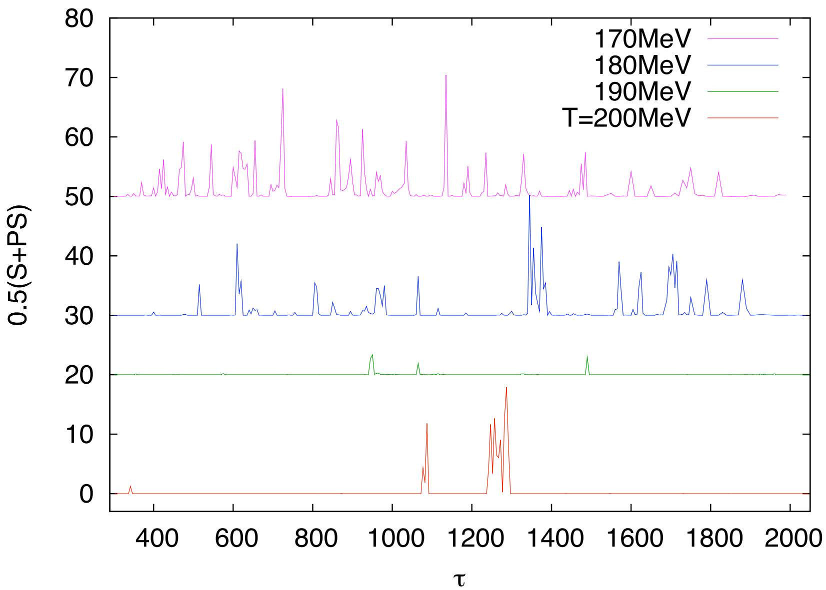
Scalar/Pseudoscalar Susceptibilities

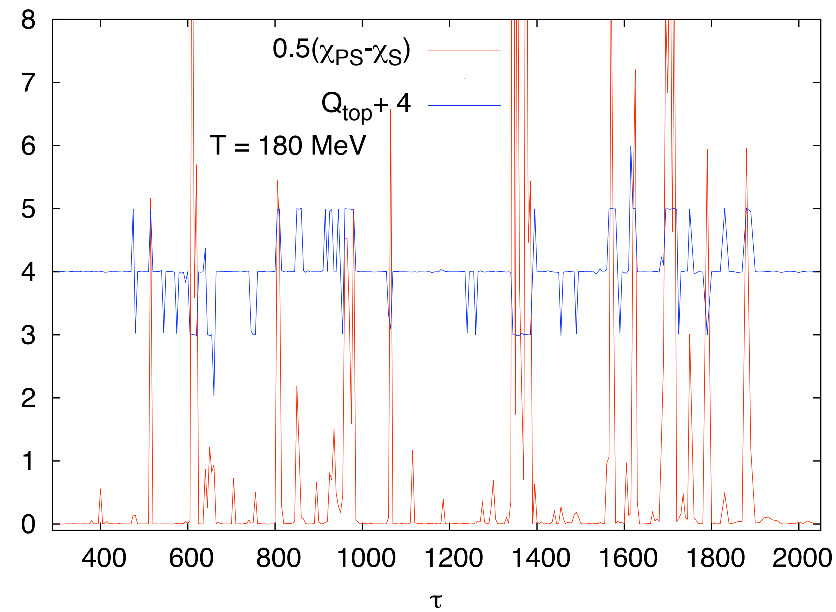




Correlation to Topological Charge

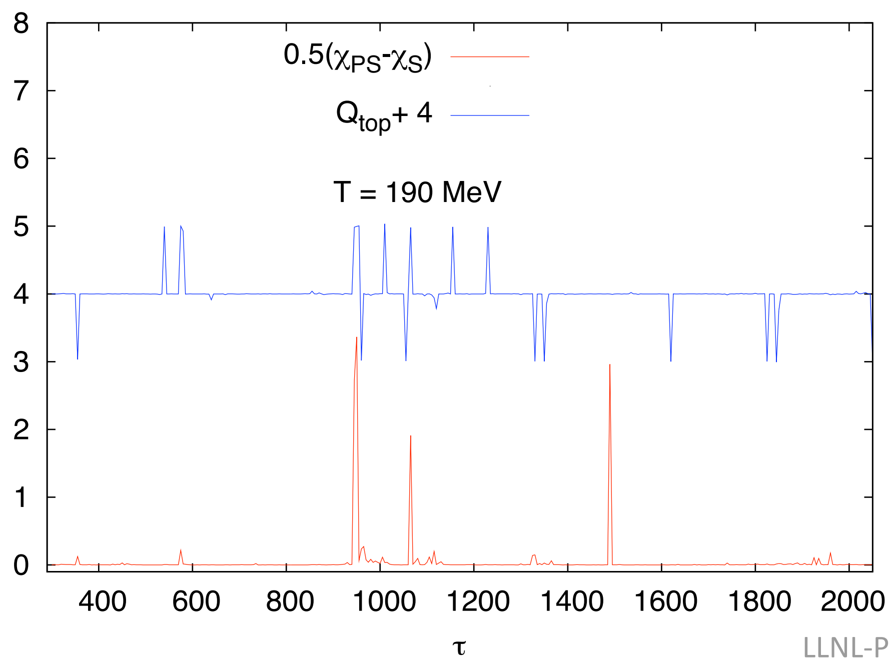
- Contributions to $U(1)_A$ breaking come seem to be highly correlated w/topological charge, *i.e.* $U(1)_A$ breaking comes from objects with non-zero topology.

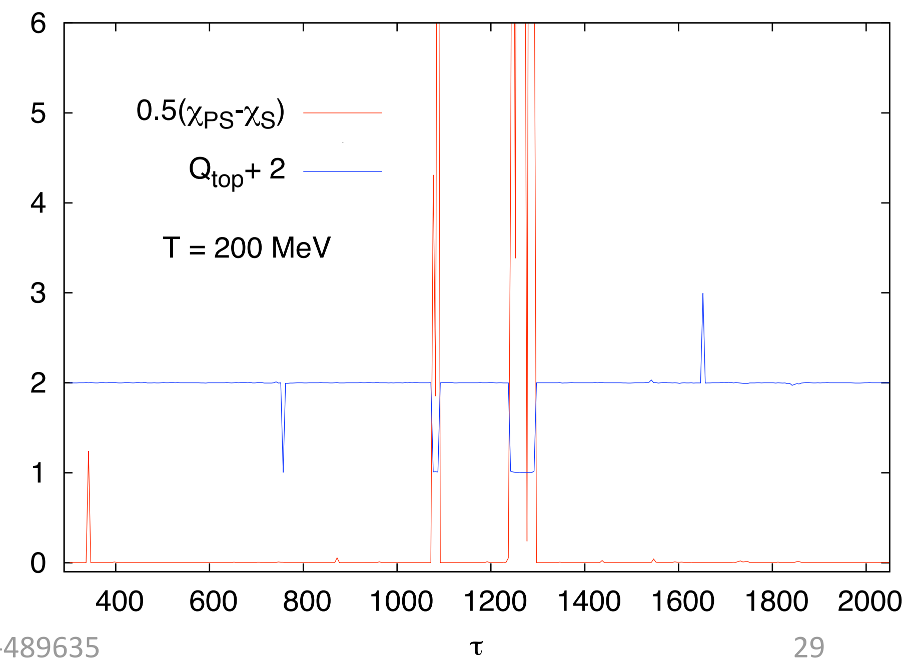




Correlation to Topological Charge

- Topological susceptibility becomes smaller in the high temperature phase, *i.e.* object with non-zero topology are suppressed -> contributions to $U(1)_A$ are smaller.





Dirac Spectrum

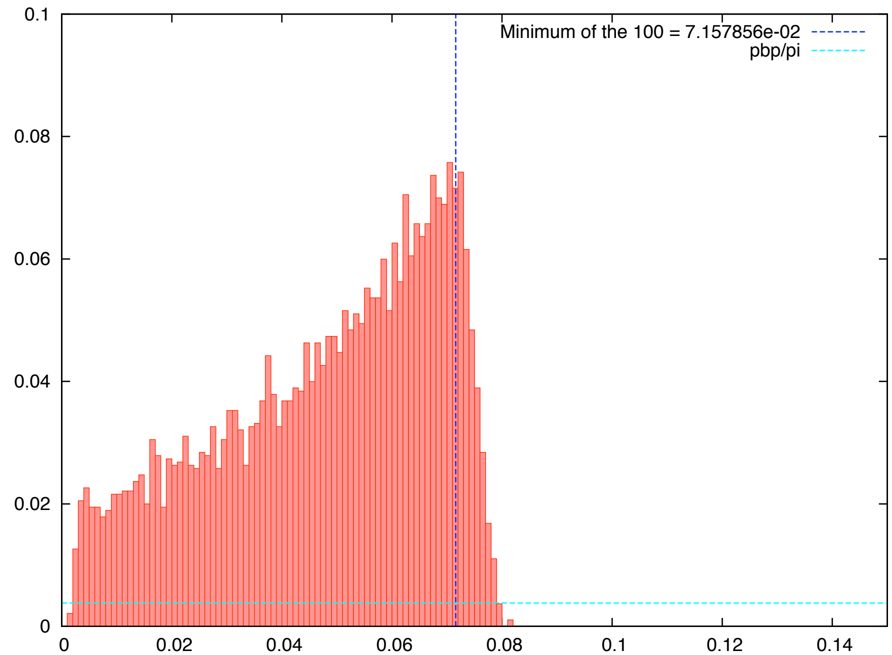
- Calculate low-lying eigenvalues of 5-D DWF operator.
- Lowest-lying modes correspond to 4-d modes bound to the wall.
- Connection between low modes of 5-d operator and 4-d low modes (needed for Casher-Banks) can be made via NPR.
- Modes with $\lambda \sim m_q$ become zero modes relevant in the chiral limit.
- Can derive relation for $\Delta_{\pi\text{-}\delta}$ in terms of $\rho(\lambda)$ as well:

$$\Delta_{\pi-\delta} \quad = \quad \int d\lambda \rho(\lambda) \frac{4m_q^2}{(m_q^2 + \lambda^2)^2}$$

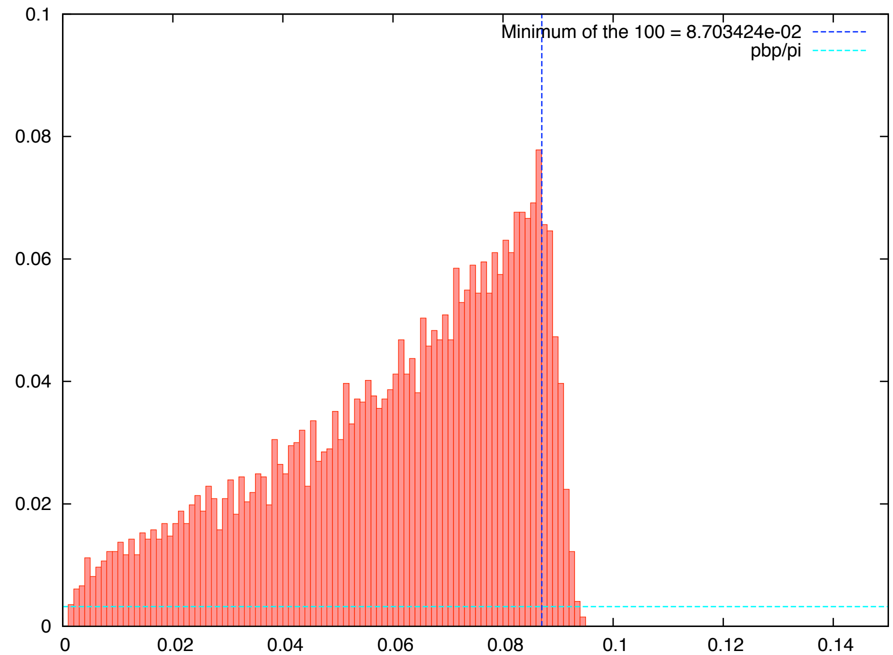
Dirac Spectrum

Pileup of near-zero modes for $T \leq T_c$

T = 150 MeV

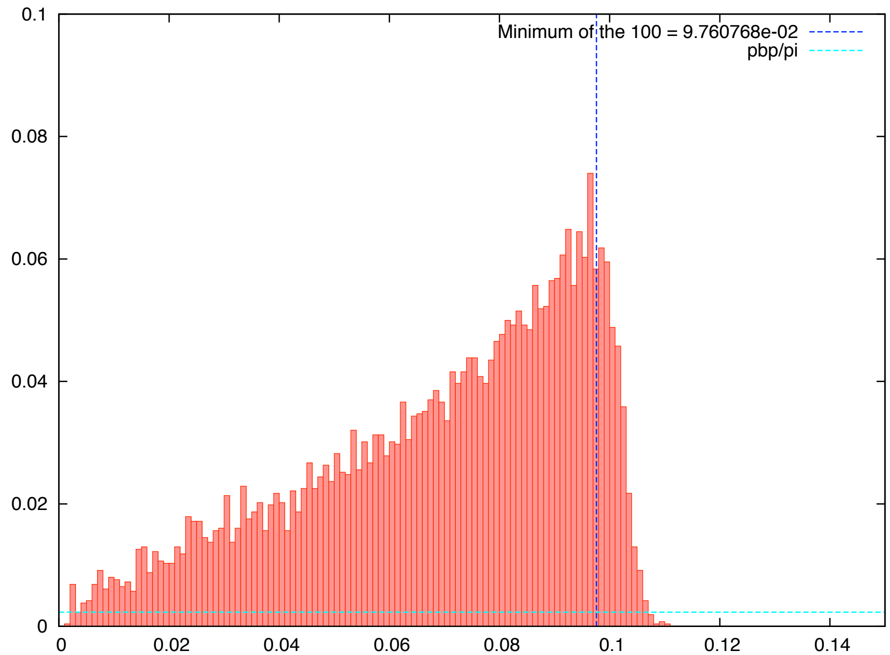


T = 160 MeV

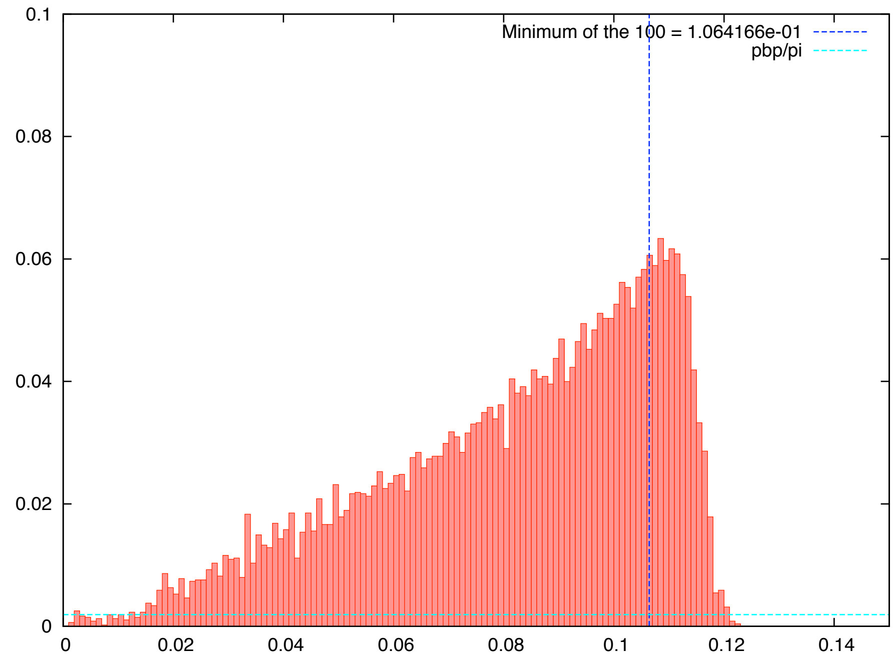


Dirac Spectrum

T = 170 MeV



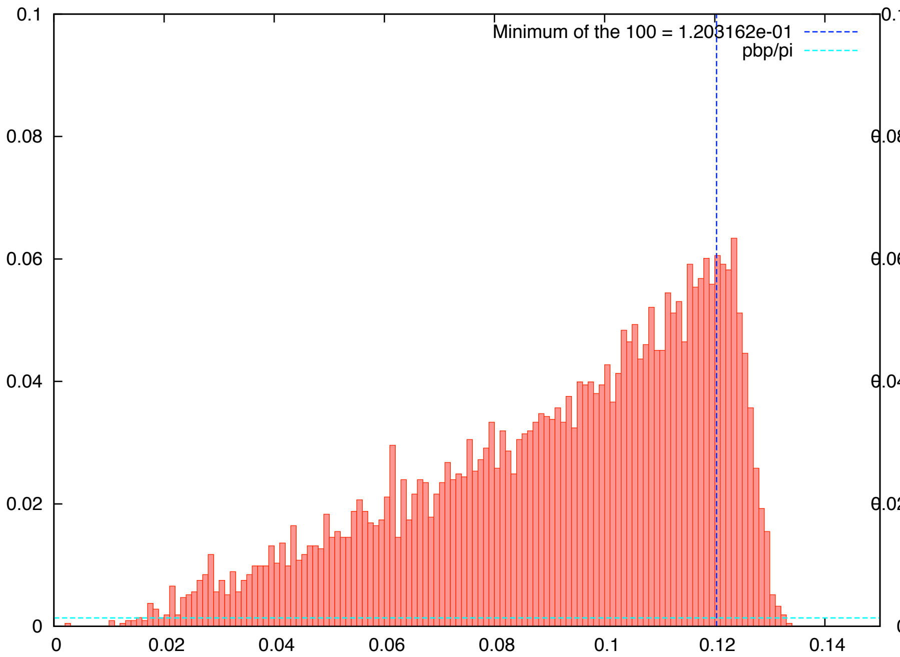
T = 180 MeV



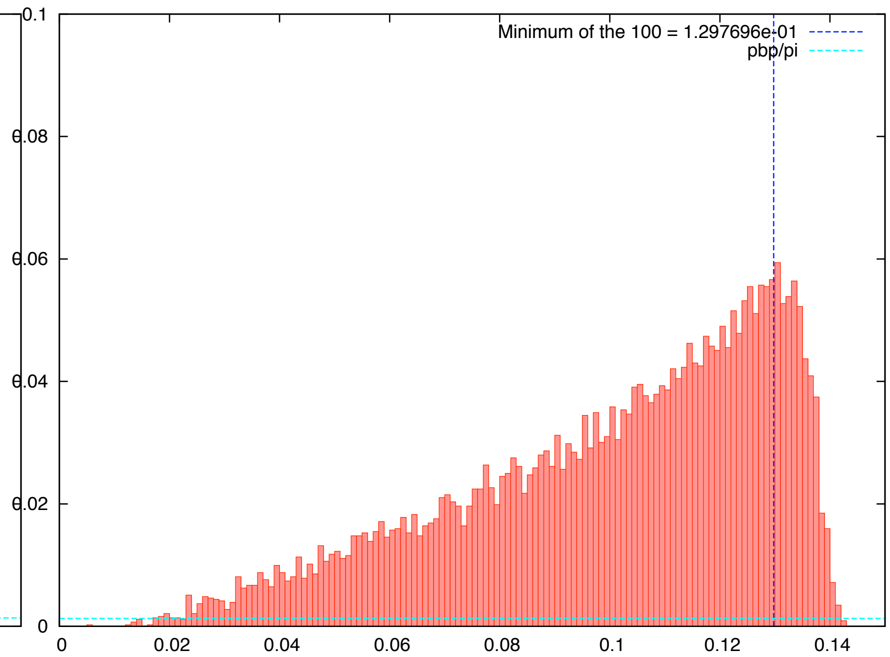
Dirac Spectrum

Development of a gap near λ ≈ 0

T = 190 MeV

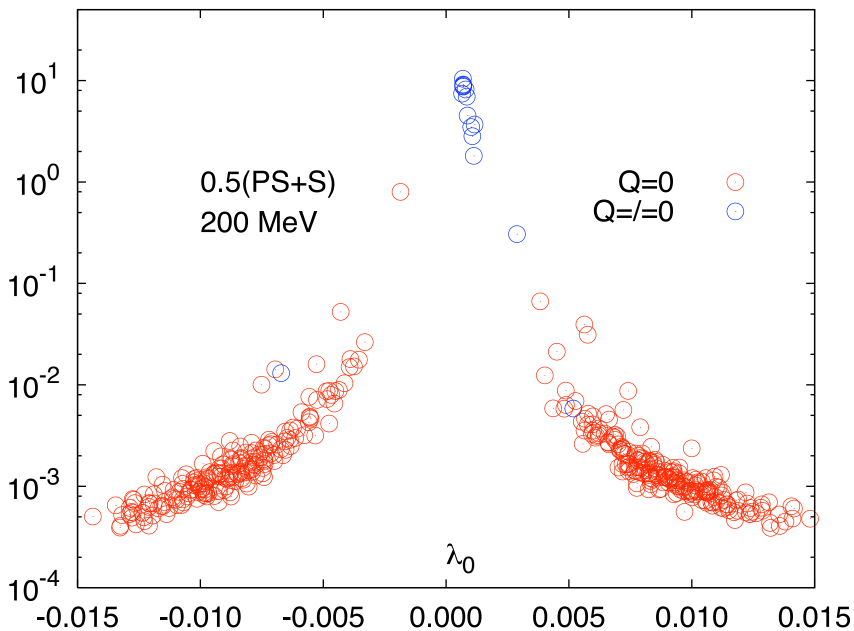


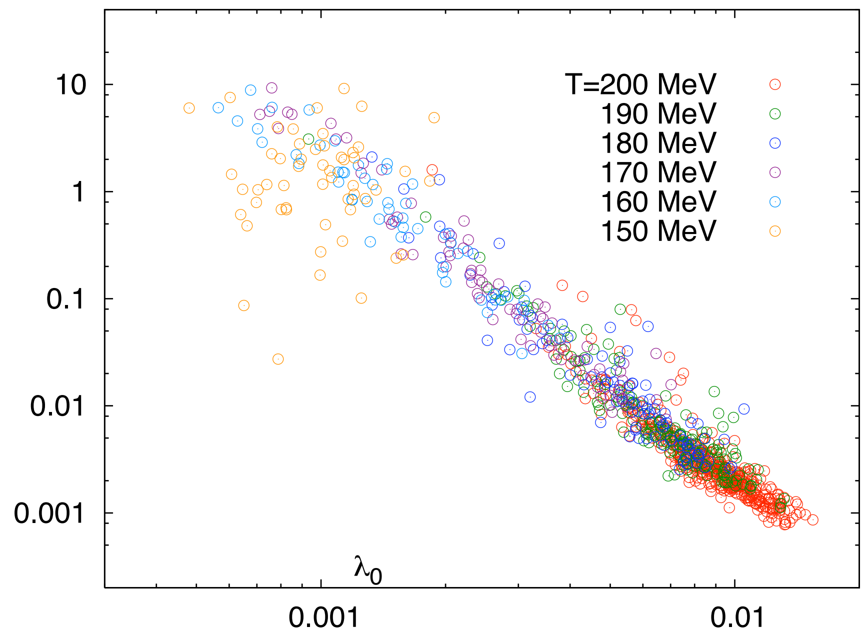
T = 200 MeV



Dirac Specrum and $U(1)_A$

- Contributions to $U(1)_A$ breaking come from configurations with small eigenvalues.
- Development of gap in eigenvalue spectrum: these contributions should disappear.





Conclusions

- The restoration of $SU(2)_L$ x $SU(2)_R$ has been heavily studied because of its direct relevance to heavy ion collisions.
- $T_c \approx 160 – 170$ MeV
- $U(1)_A$ for $T \geq T_c$ less well understood.
- Calculation with chiral fermion formulation needed to clearly see connection between $U(1)_A$ restoration, gauge field topology, low-lying eigenmodes of Dirac operator.
- $U(1)_A$ breaking terms in scalar/pseudoscalar correlators are still non-zero at $T = T_c$
- However, development of gap in near-zero modes of eigenvalue spectrum suggest that $U(1)_A$ may be restored in chiral limit.
- Larger volumes (ongoing), lighter mass?