The search for a perfect fluid: is string theory relevant for ultracold atoms ?

T. Schäfer and D. Teaney, Rep. Prog. Phys. (2009)

J. McGreevy, arXiv:0909.0518 [hep-th]

T. Enss, R. Haussmann, W. Zw., Ann. of Phys. (2011)

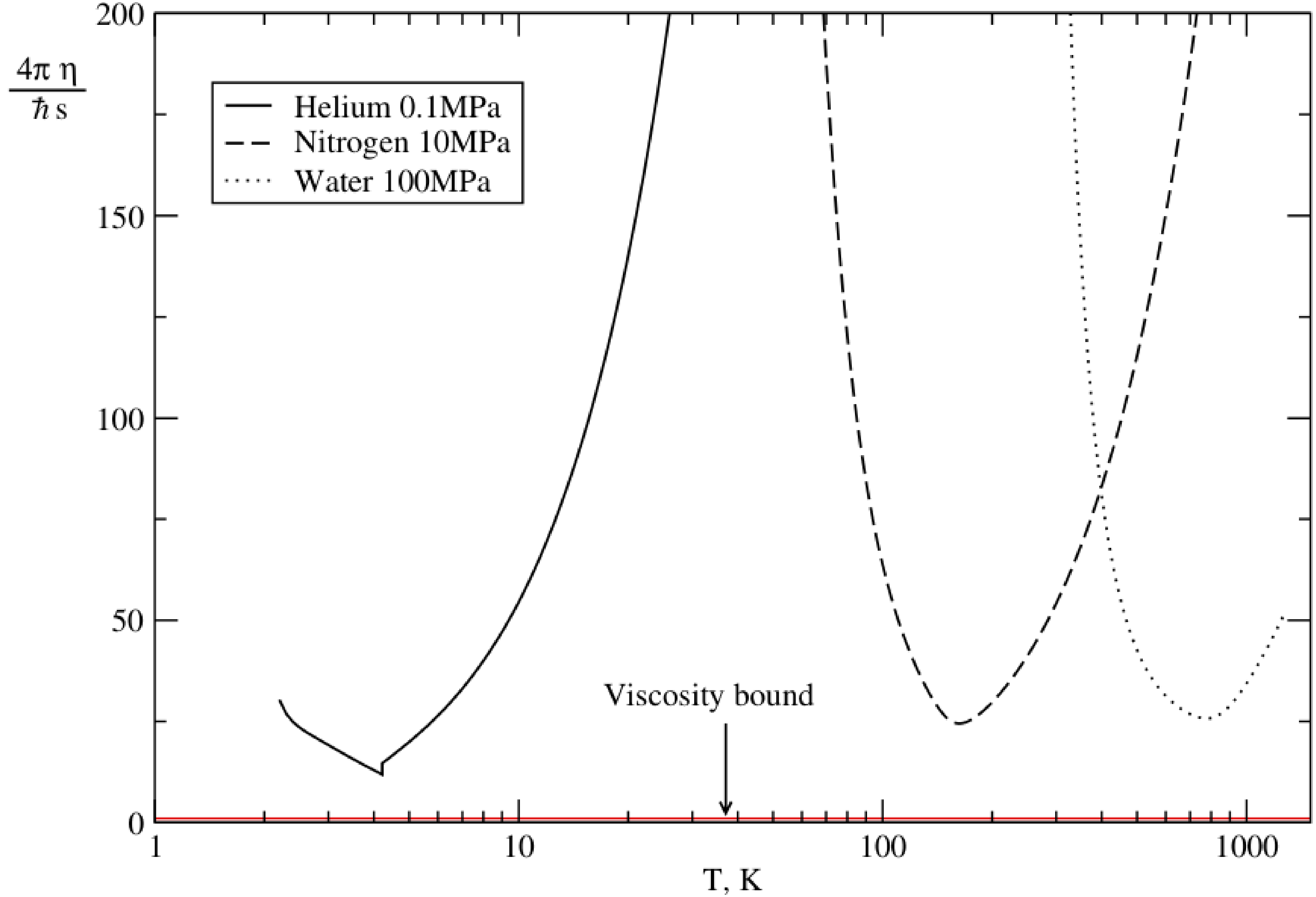

RUFFINO
RISERVA DUCALE
Chianti Classico
Riserva 2001


Wine Spectator
TENUTA
GREPPONE MAZZI
1999
BRUNELLO DI MONTALCINO
DENOMINAZIONE DI ORIGINE CONTROLLATA E GARANTITA
95 points
Ripe and opulent,
with currant, mineral and
fresh herbs.
Layered and stylish
J.S. February, 2010
TENUTA
GREPPONE MAZZI
BRUNELLO DI MONTALCINO
RUFFINO

Definition A fluid is perfect if $\frac{\eta}{s} \equiv \frac{\hbar}{4\pi k_B}$

Conjecture Kovtun/Son/Starinets '05

All (relativistic, scale invariant) fluids have $\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$



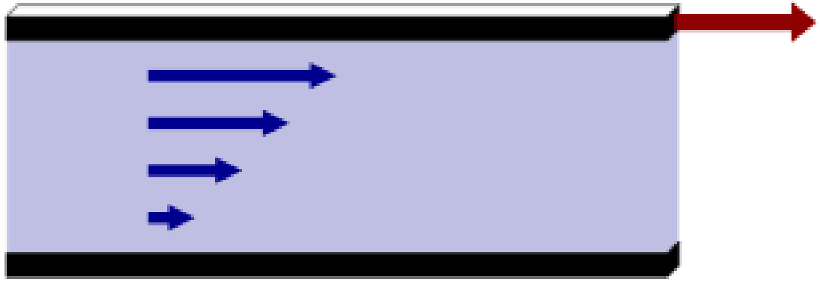
200
150
100
50
0
$\frac{4\pi\,\eta}{\hbar\, s}$
Helium 0.1MPa
Nitrogen 10MPa
Water 100MPa
Viscosity bound
1
10
100
1000
T, K

Outline

Viscosity: ideal, viscous, super- and perfect fluids

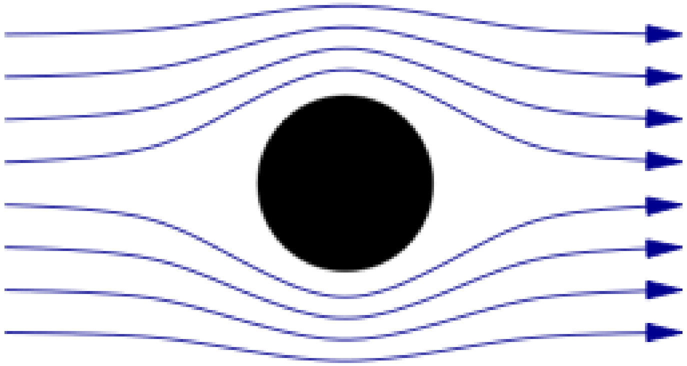
The KSS-bound

Viscosity of the unitary Fermi gas



shear force per area

$$T_{xy} = \eta \cdot \partial_y v_x$$



Reynolds number

$$\mathrm{Re} = \frac{vL}{\nu} \qquad \nu = \frac{\eta}{\rho} \equiv D_\perp$$

momentum balance $\partial_t(\rho v_i) + \partial_j \Pi_{ij} = 0$

$$\Pi_{ij} = p\delta_{ij} + \rho v_i v_j - \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\cdot\partial_k v_k\right) - \zeta\delta_{ij}\cdot\partial_k v_k$$

positivity: $\eta \geq 0$ and $\zeta \geq 0$ due to $dS/dt \geq 0$

fluids are (approx.) **ideal** $\eta_{\rm id} \equiv 0$ if $\eta|\nabla_\perp v| \ll p \;\rightarrow\; L \gg \ell$

shear viscosity: liquids versus gases

liquids: thermally activated $\rightarrow \eta(T)$ grows as $T\downarrow$

gases $\eta = \frac{1}{3}mn\langle v\rangle\ell \simeq \sqrt{mk_BT}/\sigma(T)$ grows as $T\uparrow$

A lower bound on the viscosity ?

mean free path $\ell \gtrsim n^{-1/3}$ average velocity $\langle v \rangle \gtrsim \frac{\hbar}{m} n^{1/3}$

gives $\boxed{\eta \geq \alpha_\eta \cdot \hbar n}$ ($\alpha_\eta \simeq 0.5$ for ^{4}He at 2K)

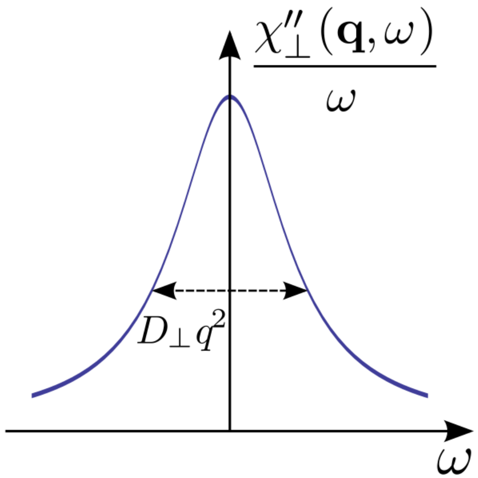
superfluids have $\eta_{\mathsf{SF}} \equiv 0$ but at any finite T there is

a normal comp. $\rho_n(T) \neq 0$; relaxation of shear due to

phonon-phonon collisions $\eta \sim T^{-5}$ Landau/Khal. '49

shear diffusion transverse currents relax diffusively

$$\frac{\chi''_{\perp}(\mathbf{q},\omega)}{\omega} \longrightarrow \frac{\eta q^2}{\omega^2 + \left(D_{\perp} q^2\right)^2}$$



sum rule $\int \frac{\chi''_{\perp}(\mathbf{q},\omega)}{\omega} = \chi_{\perp}(\mathbf{q}) \rightarrow \rho_n$ normal fluid density

Einstein relation $\boxed{\eta \equiv D_{\perp} \cdot \rho_n}$ Hohenberg/Martin '65

$\eta = \alpha_\eta \cdot \hbar n$ implies $D_{\perp} = \alpha_\eta \cdot \frac{\hbar}{m}$ (exp. Zwierlein '10)

The KSS bound extend Yang-Mills Theory

$\mathcal{L}_{YM} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu a}$ to a $\mathcal{N}=4$ supersymmetric one

$\beta(g) \equiv 0 \;\rightarrow$ no confinement or asymptotic freedom!

AdS/CFT $\mathcal{N}=4$ SSYM-Theory in the **t'Hooft limit**

$\lambda = g^2 N \rightarrow \infty$ is equiv. to a **classical** theory of gravity

entropy/viscosity gas of massless bosons with

interactions $\mathcal{O}(1)$ gives $s(T) \simeq k_B\,(k_B T/\hbar c)^3$ and

$\eta(T) \simeq \hbar\,(k_B T/\hbar c)^3$ with $\eta/s = \mathcal{O}(\hbar/k_B)$ at all T!

The unitary Fermi gas

two-component Fermigas with zero-range interactions

$$\mathcal{L}_E = \sum_{\sigma=\uparrow,\downarrow} \Psi_\sigma^\dagger \left(\hbar\partial_\tau - \frac{\hbar^2}{2m}\nabla^2 \right) \Psi_\sigma + \frac{g(\Lambda)}{2} \Psi_\sigma^\dagger \Psi_{-\sigma}^\dagger \Psi_{-\sigma} \Psi_\sigma$$

renormalized coupling $g(\Lambda) \to g = 4\pi\hbar^2 a/m$

Hubbard-Stratonovich transformation

$$\mathcal{L}[\Psi] \to \mathcal{L}[\Psi, \Phi] = \mathcal{L}_0 + \left(\Psi_\uparrow \Psi_\downarrow \bar{\Phi} + \text{h.c.} \right) - \frac{1}{g} \Phi \bar{\Phi}$$

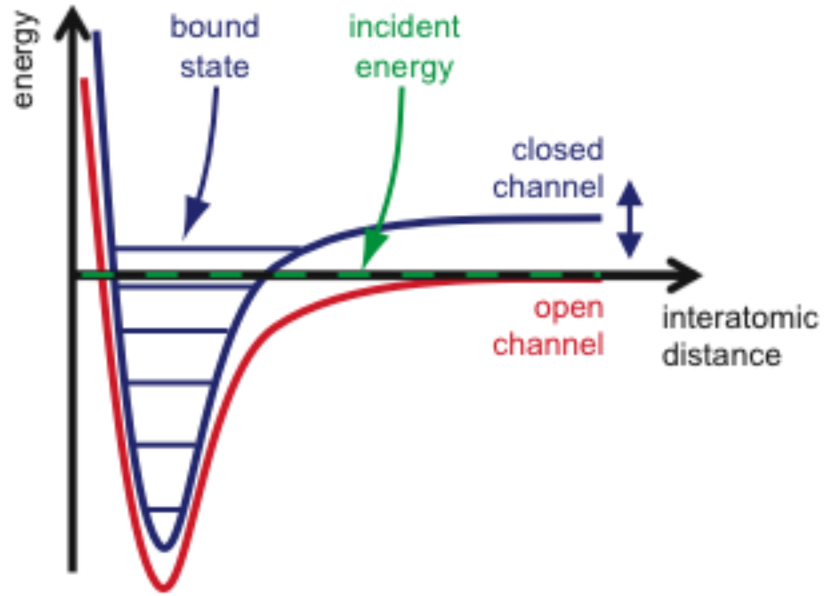
Φ is massless at infinite coupling $g = \infty$

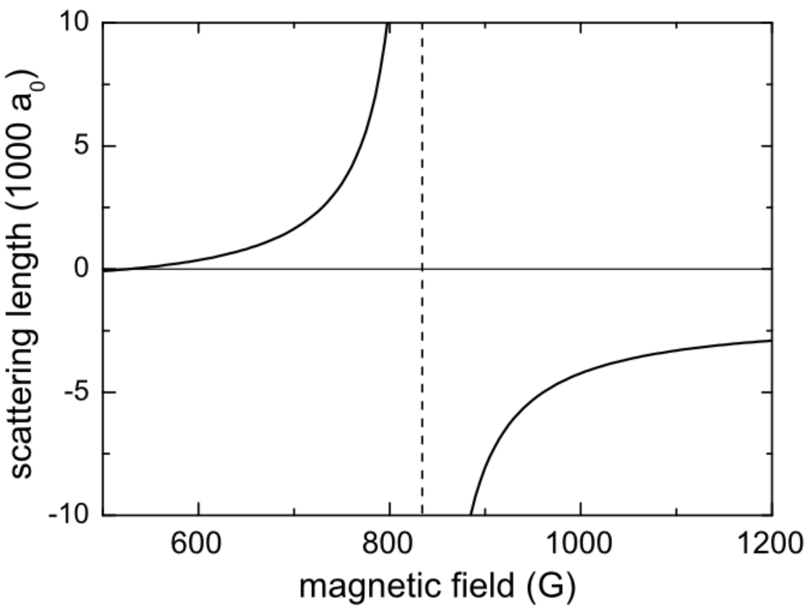
Feshbach-resonances

closed channel bound state

couples resonantly

$$a_s = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$





scattering length in ^{6}Li

(two lowest hyperfine states)

Scale invariance at infinite scattering length

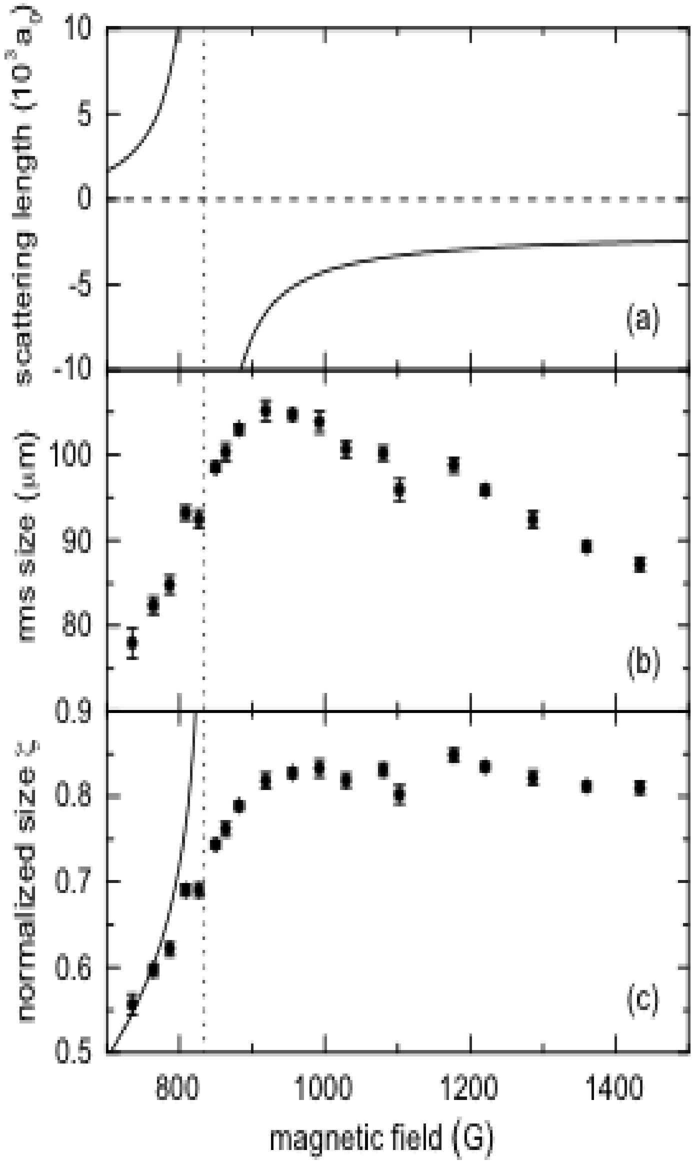
$\mathbf{x} \to \lambda \mathbf{x}$ gives $H \to H/\lambda^2 \;\to\; \partial_\nu D^\nu = 0 \;\to$ **Tr T** $= 0$

pressure $p = 2u/3$ Ho '04 bulk viscosity $\zeta = 0$ Son '07

$p(\infty) = \xi \cdot p_F^{(0)}$ **Bertsch-parameter** $\xi < 1$

determines cloud size in a trap $R_{TF} = R_{TF}^{(0)} \cdot \xi^{1/4}$

universal numbers $\xi = 0.36,\ \Delta_0 = 0.46\,\epsilon_F,\ T_c = 0.16\,T_F$



Bertsch parameter from

cloud size $\xi_{exp} = 0.32 \pm 0.1$ 2004

$\xi = 0.41$ variational MC 2004

$\xi = 0.36$ Luttinger-Ward 2007

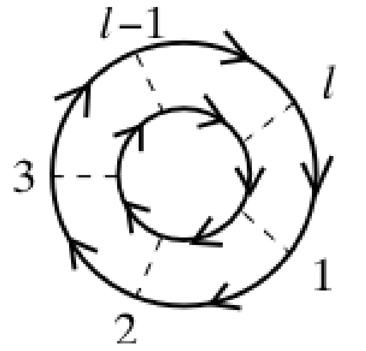
$\xi = 0.36 \pm 0.02$ field theory 2009

$\xi_{exp} = 0.36 \pm 0.01$ MIT 2011

Many-body theory pseudopotential $V_{\uparrow\downarrow}(\mathbf{x}) = \bar{g}\,\delta(\mathbf{x})$

Luttinger/Ward '60 $\Omega = -T \ln Z = \Omega[\widehat{G}]$

$$\Omega[\widehat{G}] = \beta^{-1}\Big(-\tfrac{1}{2}\mathrm{Tr}\{-\ln \widehat{G} + [\widehat{G}_0^{-1}\widehat{G} - 1]\} - \Phi[\widehat{G}]\Big)$$

Ladder-approximation $\Phi[\widehat{G}] = \sum_{\ell=0}^{\infty}$ 

$\delta\Omega[\widehat{G}]/\delta\widehat{G} = 0$ variational principle for functions !

why does Luttinger-Ward work well ?

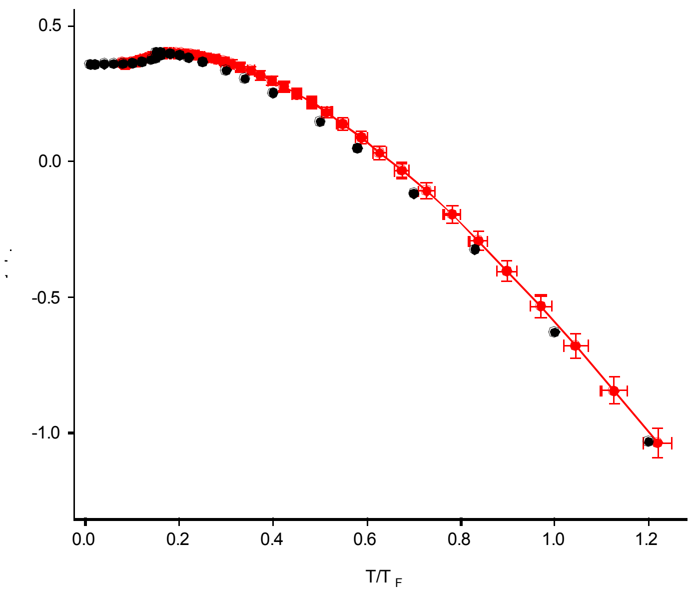
a) it is conserving $\rightarrow$ all th. dyn. relations are obeyed

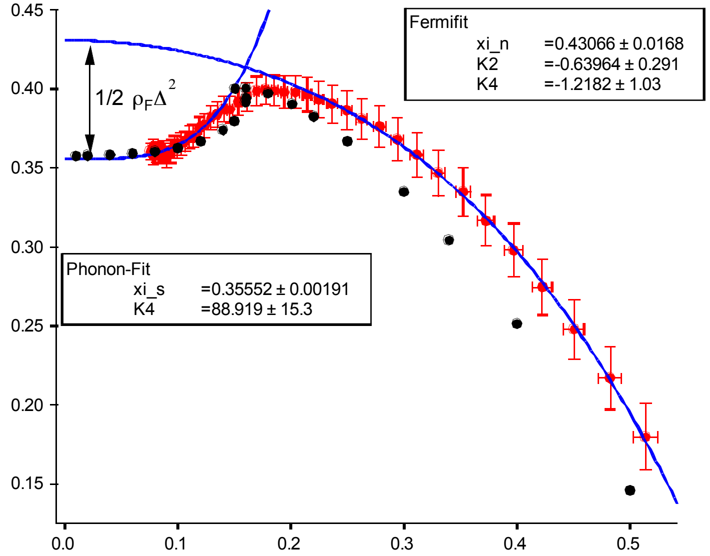
b) it obeys the **Tan relations**

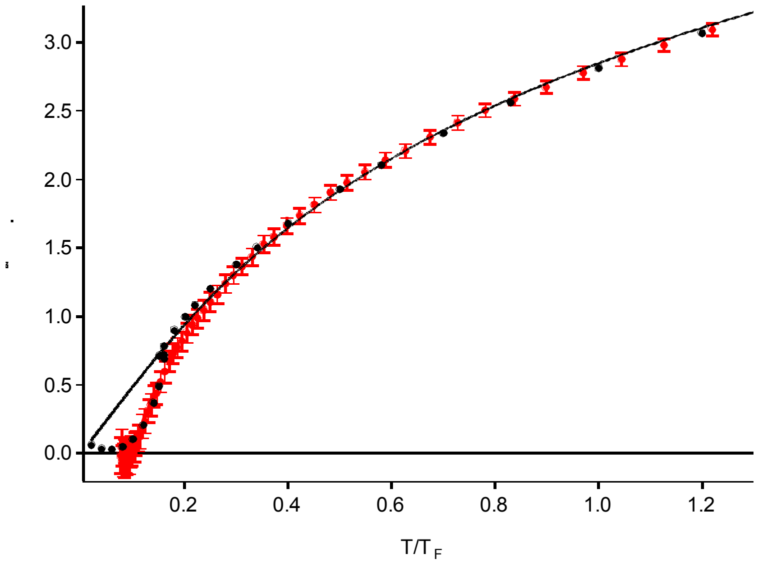
$$\mathcal{L}_E = \mathcal{L}[\psi_\sigma] + \mathcal{L}[\Phi] + \tilde{g}\Big(\bar{\Phi}_B \psi_\uparrow \psi_\downarrow + \text{h.c.}\Big)$$

change of Ω with scattering length $\dfrac{\partial \Omega}{\partial(-1/a)} =$

$$= \text{Tr}\left[G_B \frac{\partial G_{B,0}^{-1}}{\partial(-1/a)}\right] = \sum_{X,X'} G_B(X,X')\tilde{g}^2 \frac{m}{4\pi\hbar^2}\delta_{X,X'} = \frac{\hbar^2 C}{4\pi m}$$







chemical potential

and entropy of the

unitary gas

MIT 2011

viscosity of the unitary gas from Luttinger-Ward

Kubo formula $\quad Re\,\eta(\omega) = \dfrac{Im\,\chi_{xy}^{ret}(\omega)}{\omega}$

perturbation $\quad \hat{H}' = h_\ell(t) \cdot \hat{\Pi}_\ell \quad (\ell = 0, 2 \rightarrow$ bulk, shear)

euclidean time $\tau \rightarrow \quad \chi_\ell(\tau) = \int d^3x \left\langle \tilde{T}\, \hat{\Pi}_\ell(\mathbf{x}, \tau) \hat{\Pi}_\ell(\mathbf{0}, 0) \right\rangle$

from $\quad \chi_\ell(\tau) = -\dfrac{\delta^2 \Omega}{\delta h_\ell(\tau) \delta h_\ell(0)}\Big|_{h=0} \quad \rightarrow \quad \chi_{xy}(i\omega_m)$

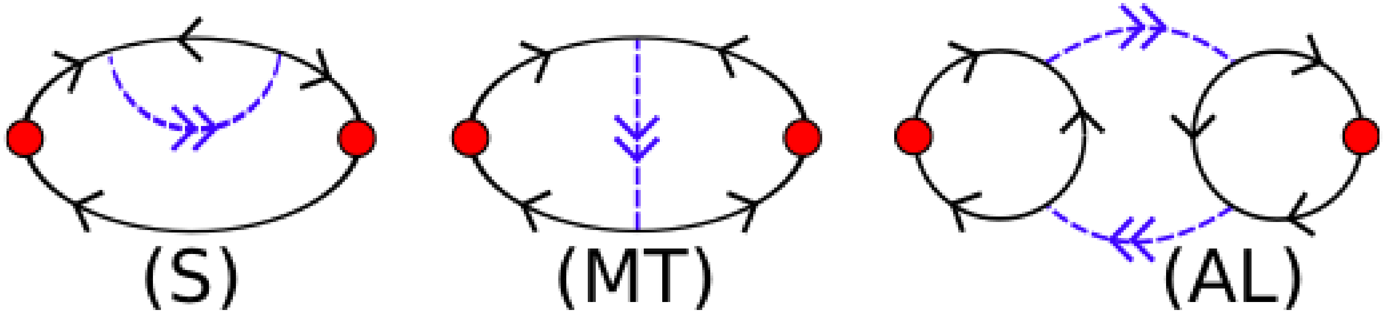
requires contin. to real frequencies $\omega \quad$ (Pade, Ansatz)

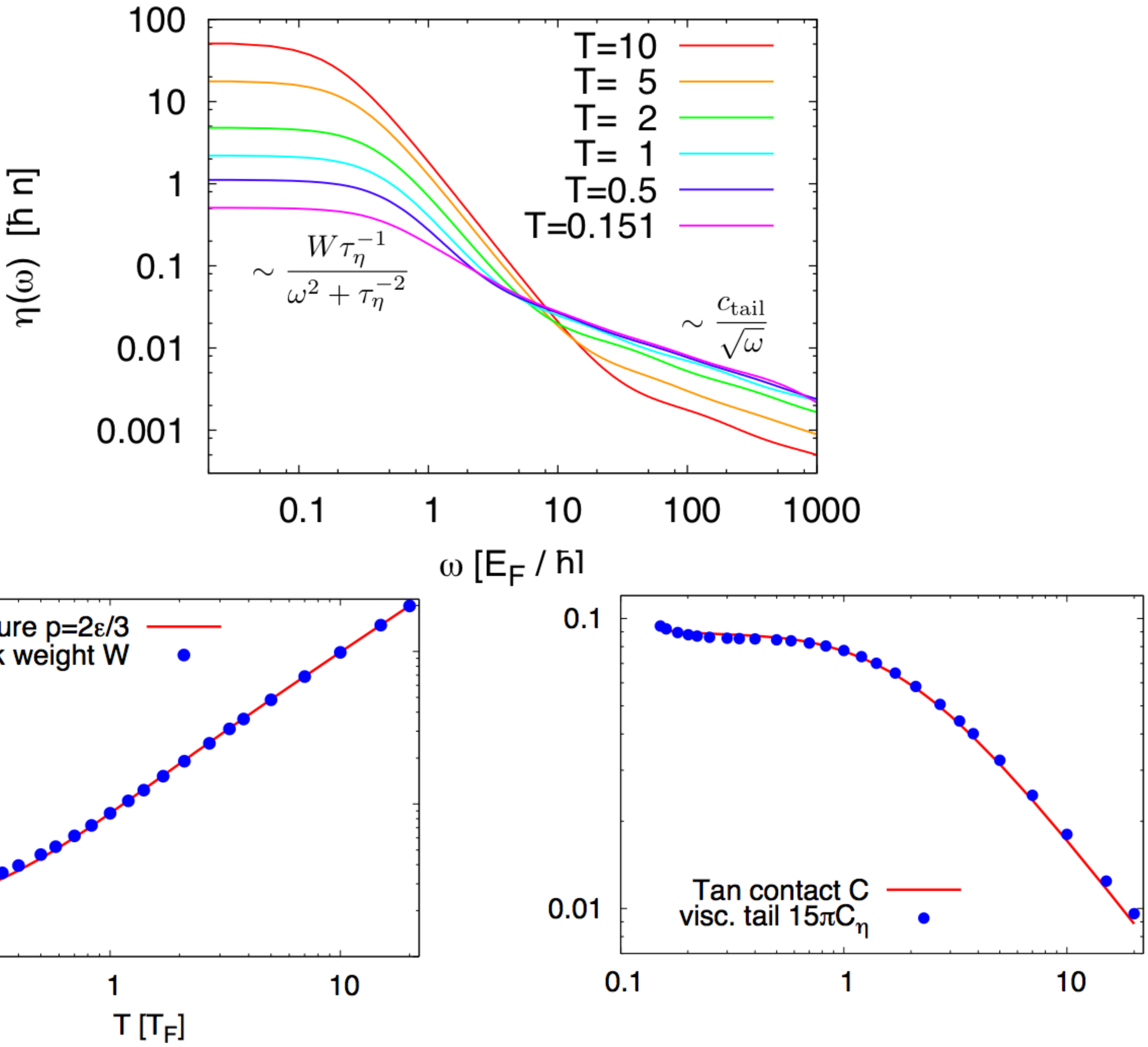
Ward-identities due to scale and translation inv.

a) guarantee that $\zeta(\omega) \equiv 0$

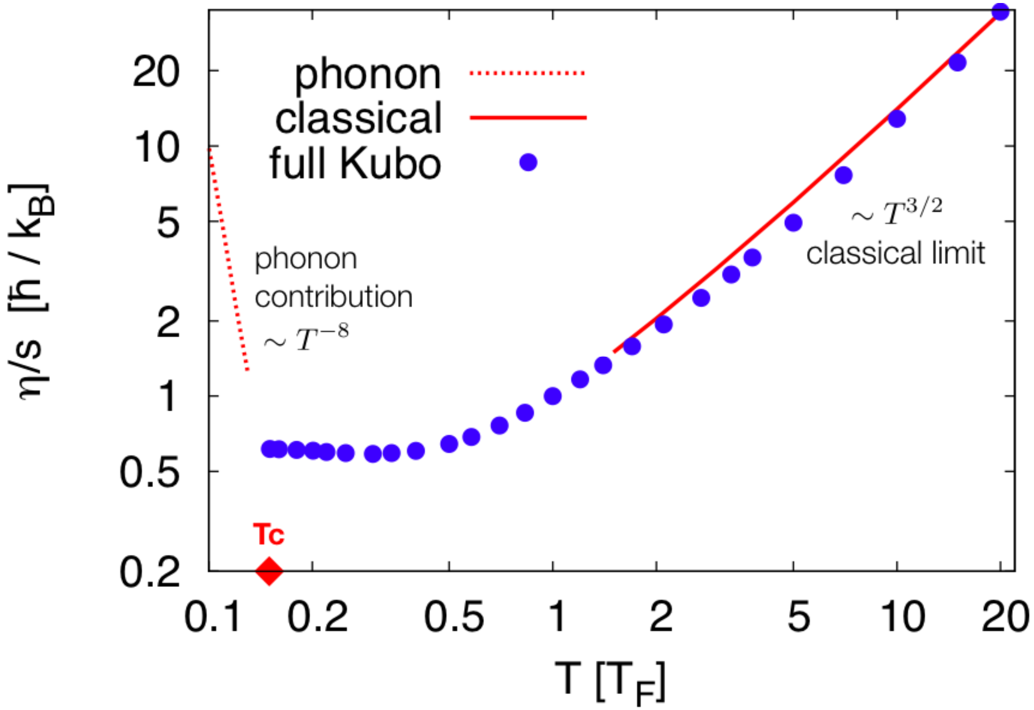
b) sum rule $\frac{2}{\pi}\int_0^\infty d\omega \left[Re\,\eta(\omega) - \frac{\hbar^{3/2}C}{15\pi\sqrt{m\omega}} \right] \equiv p$

c) Boltzmann-limit $\eta(T \gg T_F) = 4.2\,\frac{\hbar}{\lambda_T^3} \sim T^{3/2}$





T=10
T= 5
T= 2
T= 1
T=0.5
T=0.151
$\sim \frac{W\tau_\eta^{-1}}{\omega^2+\tau_\eta^{-2}}$
$\sim \frac{c_{\text{tail}}}{\sqrt{\omega}}$
η(ω) [ħ n]
ω [E_F / ħ]
pressure p=2ε/3
peak weight W
T [T_F]
Tan contact C
visc. tail 15πC_η



Enss, Haussmann, Zw.

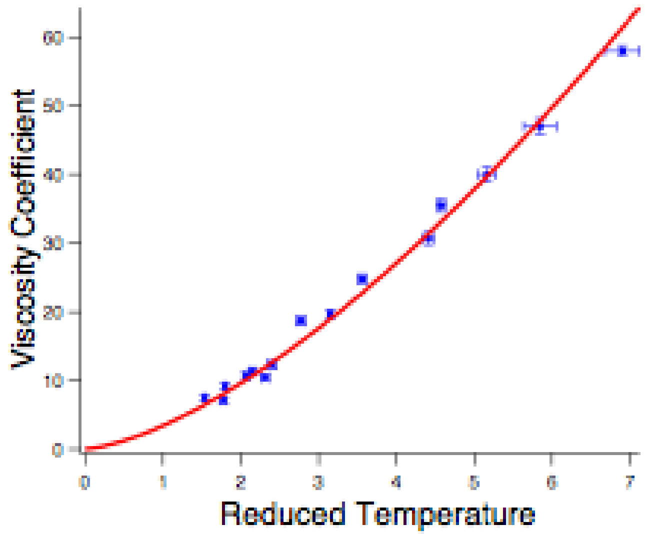
Ann. Phys. **326**, 770 (2011)

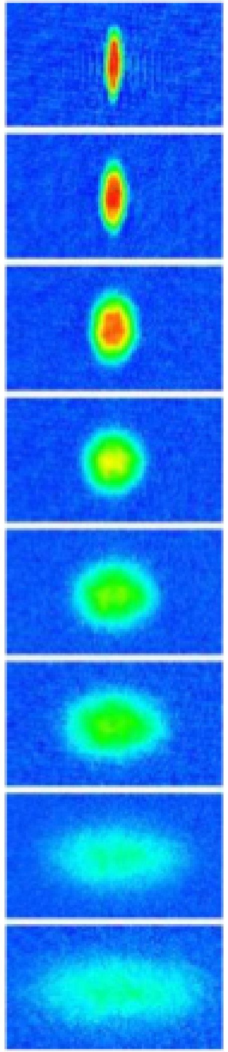
$(\eta/s)_{\text{min}} \simeq 7\,\hbar/4\pi k_B$

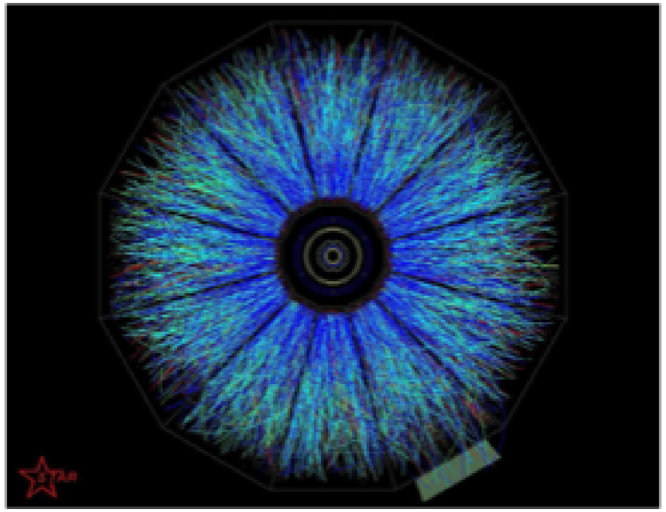
Cao, ... Thomas

Science **331**, 58 (2011)

$\eta(T \gtrsim T_F) = 4.2\,\hbar/\lambda_T^3$



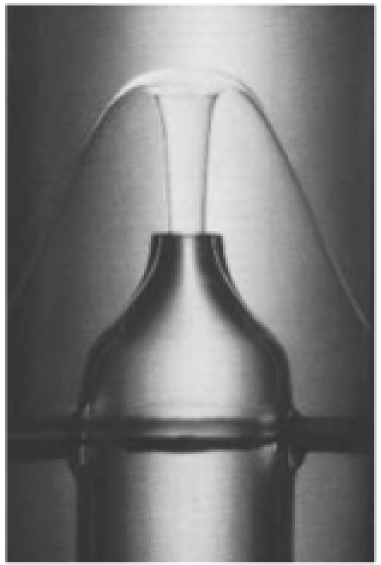




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η / s

$\gtrsim$ 0.4; 0.5; 1 for QGP, ^{6}Li, ^{4}He

Conclusions

1) All known fluids obey the KSS bound on η/s. The quark-gluon-plasma and the unitary Fermi gas come closest to saturating it.

2) Ideas from string theory provide motivation for theory and experiments in the area of ultracold atoms.

Lennard-Jones fluid $V(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$

dim. analysis gives $\eta_{\mathrm{LJ}} = \frac{\varepsilon\tau}{\sigma^3}\eta^\star(n^\star, T^\star)$

reduced density $n^\star = n\sigma^3$ and temp. $T^\star = k_B T/\varepsilon$

critical point at $n_c^\star = 0.36$ and $T_c^\star = 1.36$

time scale for classical dynamics $\tau = \sqrt{m\sigma^2/\varepsilon} \rightarrow$

$\eta_{\mathrm{LJ}}^{\mathrm{min}} = \mathrm{const}\,\frac{\sqrt{m\varepsilon}}{\sigma^2} = \alpha_\eta \hbar n$ with $\alpha_\eta = \mathrm{const}/\Lambda = \mathcal{O}(1)$

because de Boer par. $\Lambda = \hbar/\sigma\sqrt{m\varepsilon}$ cannot be $\gg 1$!

Scale invariant many-body problems

pseudopotential $g_2\delta(\mathbf{x})$ in 2d Pitaevskii/Rosch '97

unitary gas $(a = \infty)$ in 3d Son/Wingate '06

gases in mixed dimensions Nishida/Tan ' 09

electrons with $1/r$-interaction in Graphene Son '07

relativistic fluids: replace ρ by $s\,T/c^2 \;\rightarrow\; D_\perp^{\text{rel.}} = \dfrac{\eta\, c^2}{s\,T}$

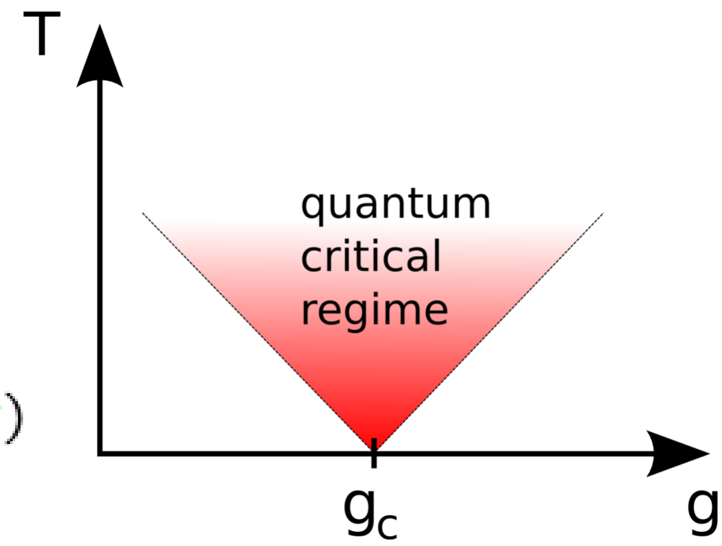
KSS-bound $\boxed{\dfrac{D_\perp}{c^2} = \tau_\perp \geq \dfrac{\hbar}{4\pi k_B T}}$ applies to QFT's

with no well defined quasi-particles $(\hbar/\tau \ll \epsilon_{qp} \simeq k_B T)$

quantum critical regime

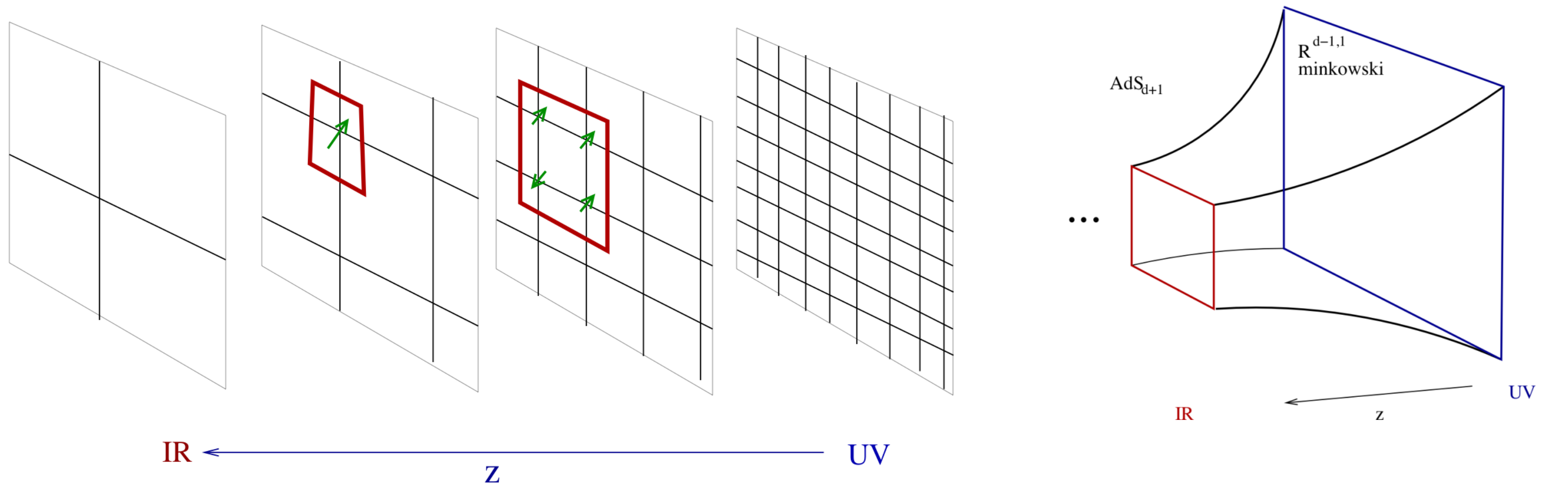
above a QPT $\boxed{\tau_\Psi = \mathcal{C}\dfrac{\hbar}{k_B T}}$

$\mathcal{C}$ is a universal number (Sachdev)



AdS/CFT $\mathcal{N}=4$ SSYM-Theory in the t'Hooft limit

$\lambda = g^2 N \to \infty$ is equiv. to a classical theory of gravity



$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\mathbf{x}^2 + dz^2\right)$$

$$\frac{L}{\ell_P} = \lambda^{1/4} \to \infty$$

radial coord. z is effectively an RG-scale McGreevy '09

Unitary gas entropy in a trap   Thomas '07+'09