Bosons in a disordered double-well potential: a simple system for understanding the interplay between disorder and interaction

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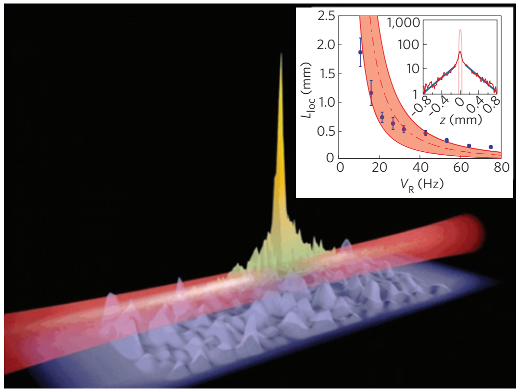
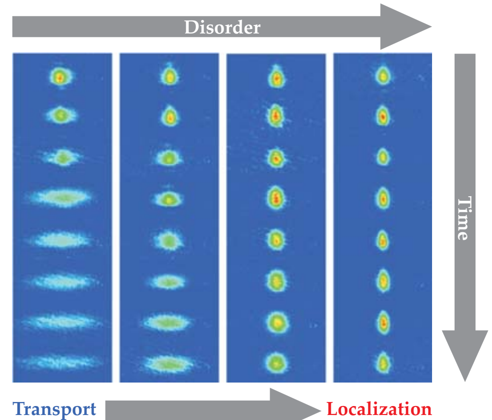
INT, University of Seattle, Washington, 2011

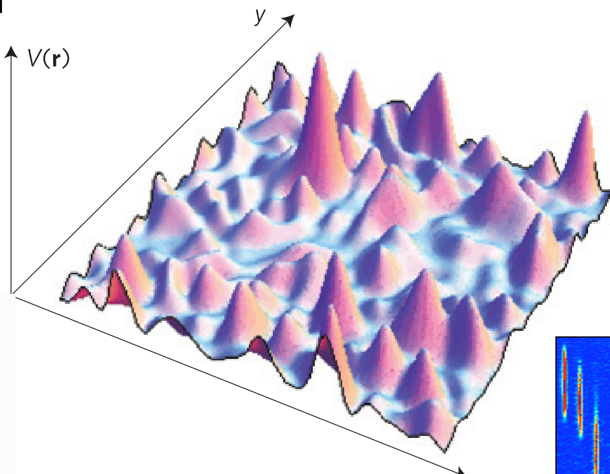
Experiments on cold atoms in disordered potentials

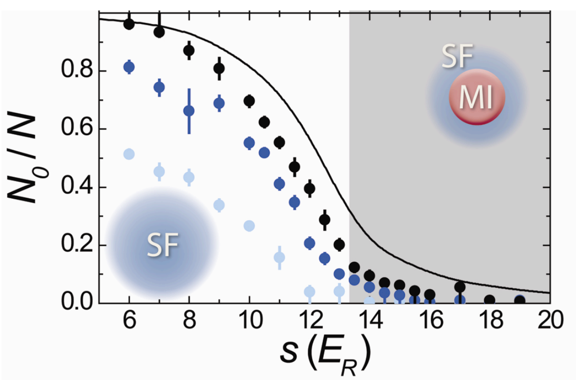


A. Aspect's group
Nature 453, 891 (2008)

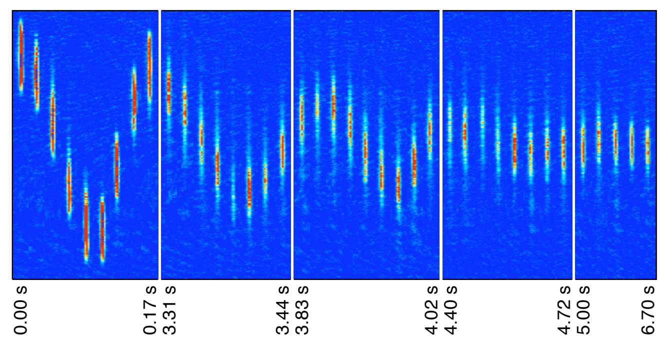


M. Ingusico's group
Nature 453, 895 (2008)





B. Demarco's group
Nature Physics 6, 677 (2010)



R.Hulet's group
Phys. Rev. A 82, 033603 (2010)

Motivation of these experiments:

To understand disorder effects in a many-body system

A particularly interesting question

Interplay between interaction and disorder

Disorder has been known to be important in solids

affects both thermodynamic and transport properties

For non-interacting case: Anderson localization

P. W. Anderson, Phys. Rev. 109, 1492 (1958)

In the presence of interaction: More difficult

The simplest case:

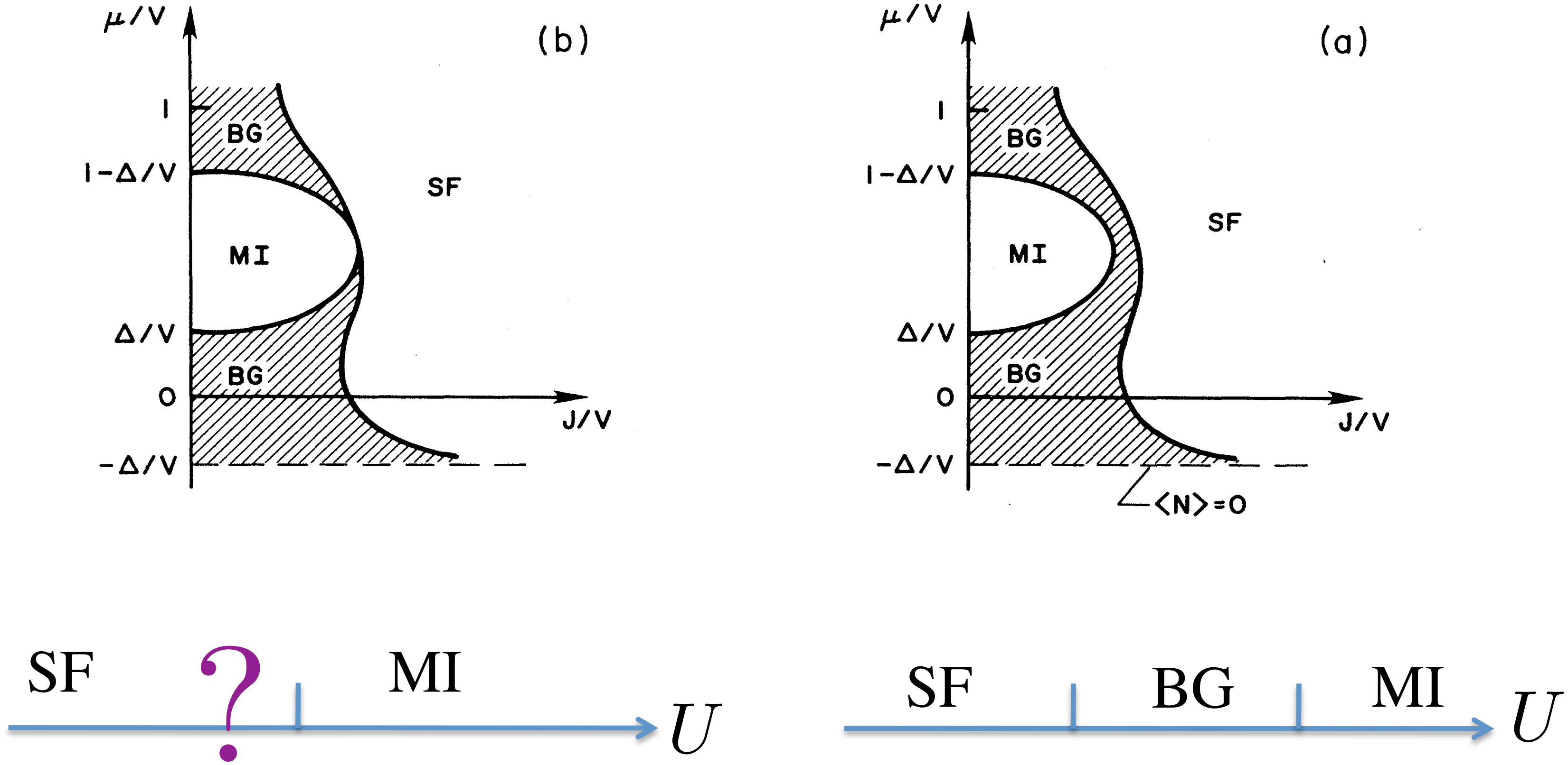
Disordered Bose-Hubbard Model:

$$H = -t\sum_{\langle i,j \rangle}(b_i^\dagger b_j + c.c) + \frac{U}{2}\sum_i n_i(n_i - 1) + \sum_i \epsilon_i n_i \quad \epsilon_i \in [-\Delta, \Delta]$$

M. P. A. Fisher, et al., PRB, 40, 546 (1989)

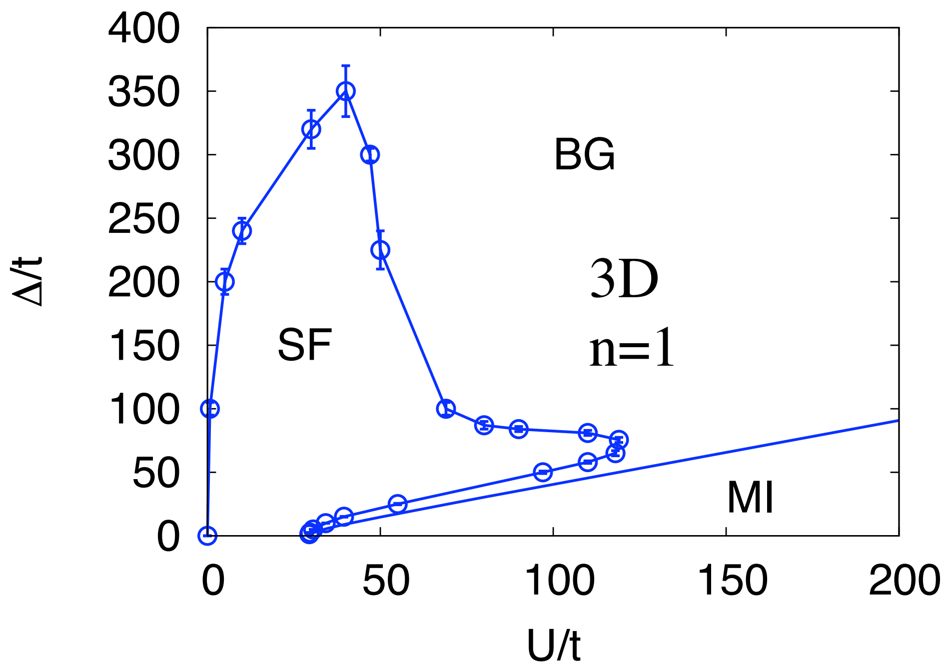
Superfluid(SF)	$\rho_s \neq 0, \kappa \neq 0$
Bose glass(BG)	$\rho_s = 0, \kappa \neq 0$
Mott insulator(MI)	$\rho_s = 0, \kappa = 0$

Debates on the structure of phase diagram over decades



Whether a direct transition between SF and MI is possible?

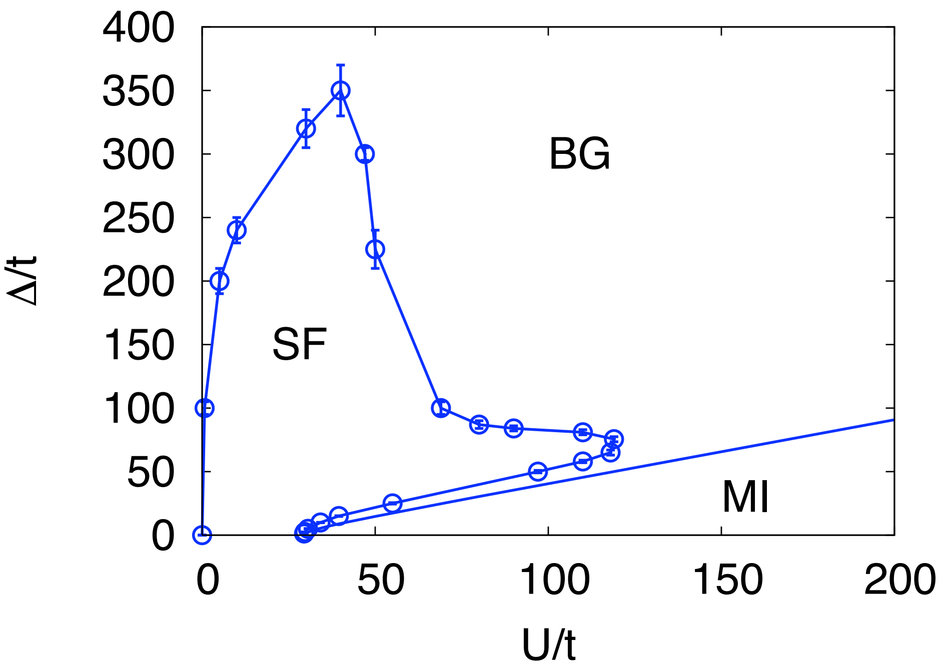
Solution from recent Quantum Monte-Carlo simulations



L. Pollet, et al, PRL 103, 140402 (2009)

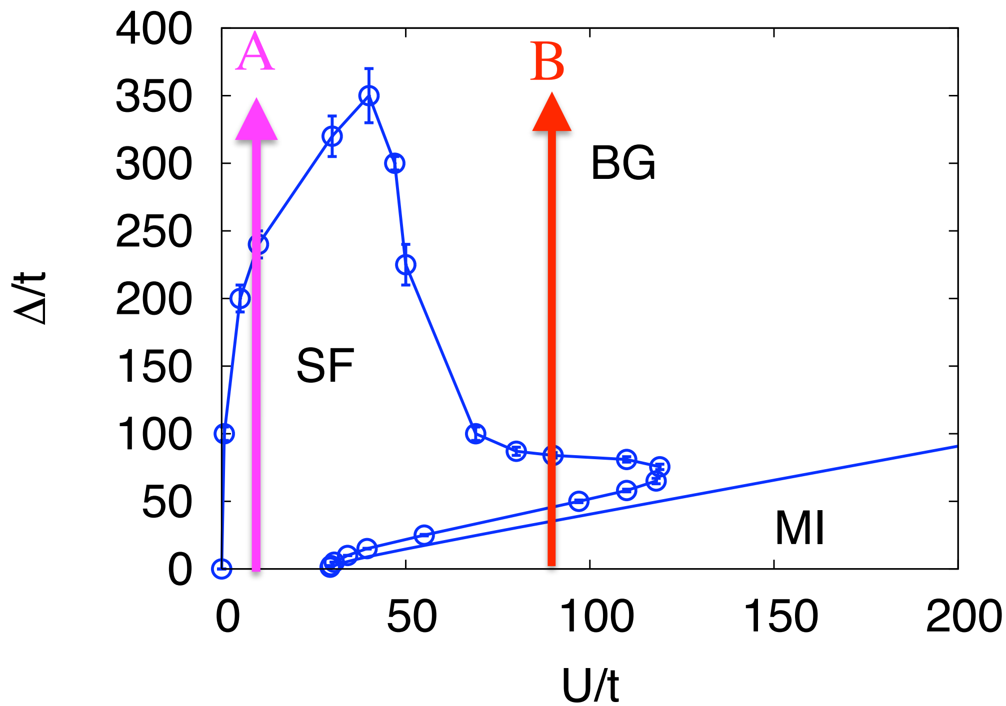
V. Gurarie, et al, PRB 80, 214519 (2009)

Besides the proof

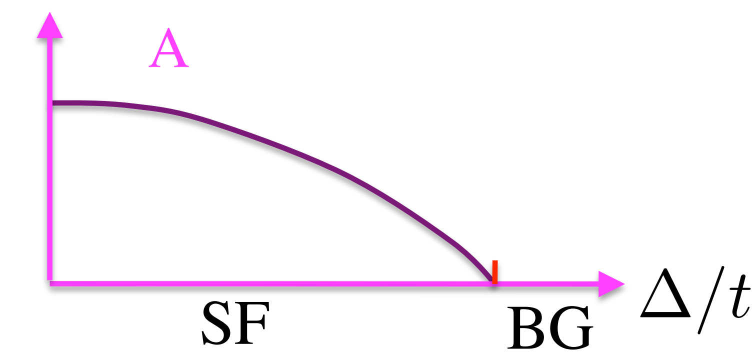


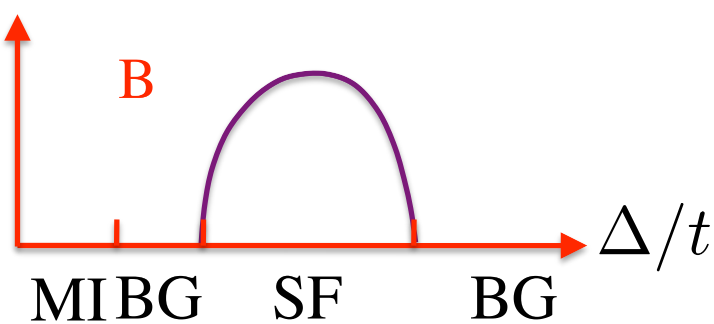
Many striking features on this single phase diagram

Striking feature 1: disorder enhanced phase coherence



V. Gurarie, et al., PRB 80, 214519 (2009)



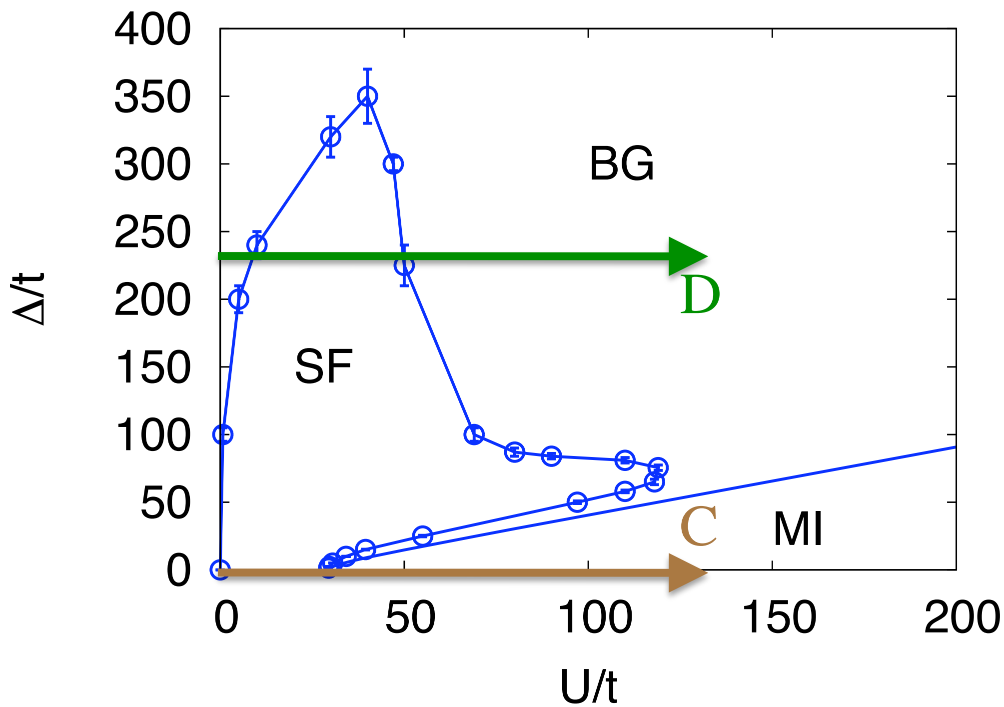


Q1: Why increasing disorder strength can enhance phase coherence at large U?

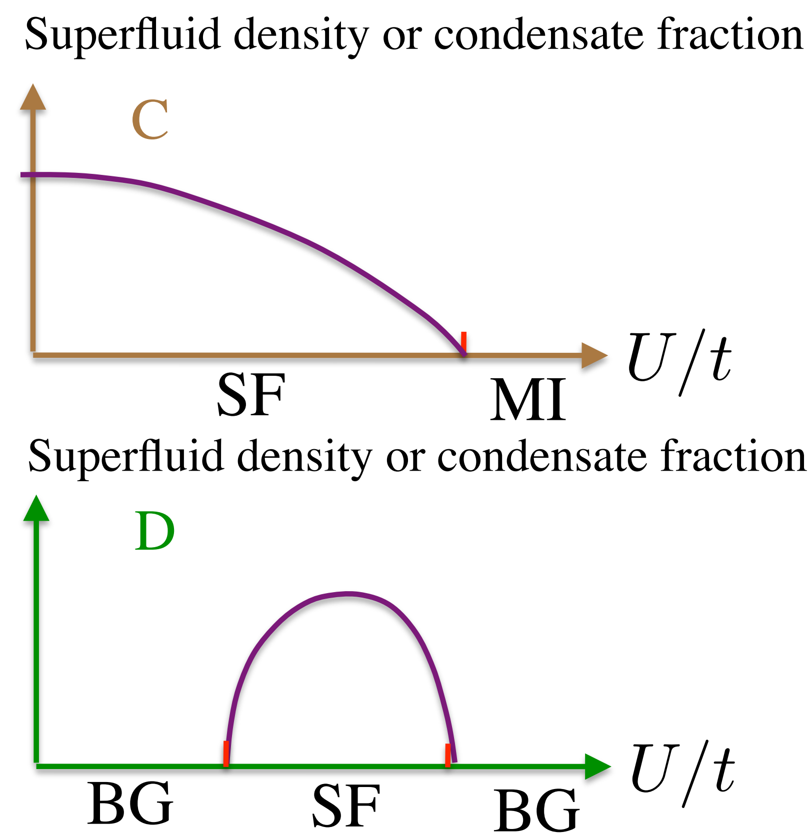
counterintuitive

Why the system behaviors completely differently at small and large U?

Striking feature 2: interaction enhanced phase coherence

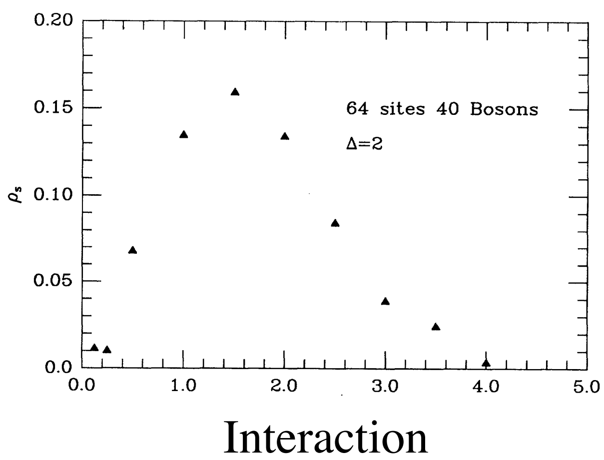


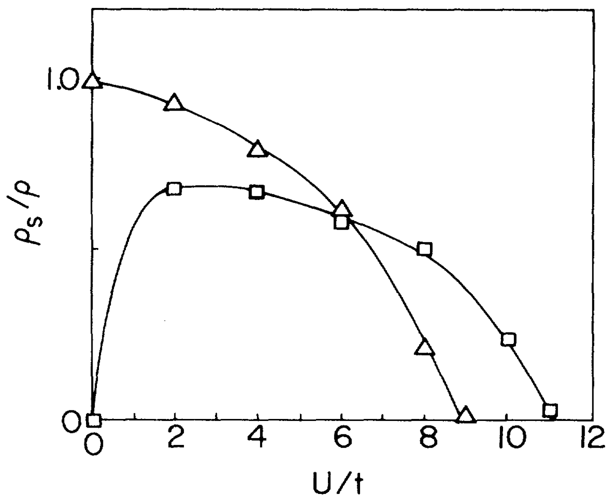
V. Gurarie, et.al.,PRB 80, 214519 (2009)



Q2: Why interaction can enhance the phase coherence in the presence of disorder?

Also counterintuitive





R.T. Scalettar, G. G. Batrouni, G. T. Zimanyi, Phys. Rev. Lett. 66, 3144 (1991)

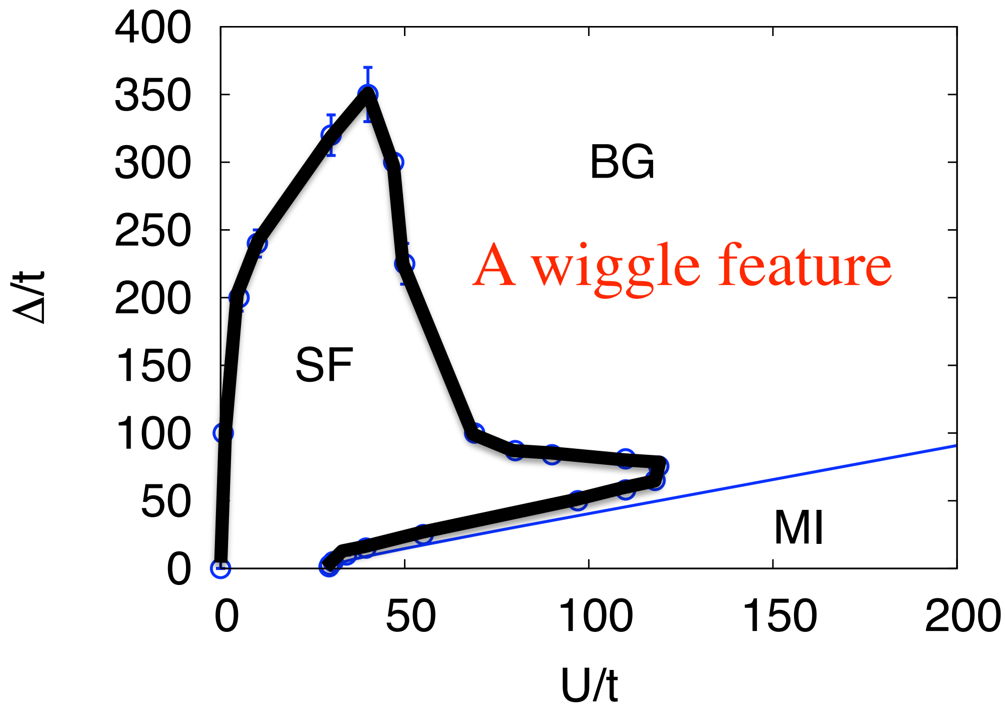
W. Krauth, N. Trivedi, D. Ceperley, Phys. Rev. Lett. 67, 2307 (1991)

Answers to above questions may be known to some experts

A SIMPLE way to understand all these counterintuitive phenomena

TRANSPARENTLY without resorting to numerical simulations?

Striking feature 3: wiggle on the phase diagram

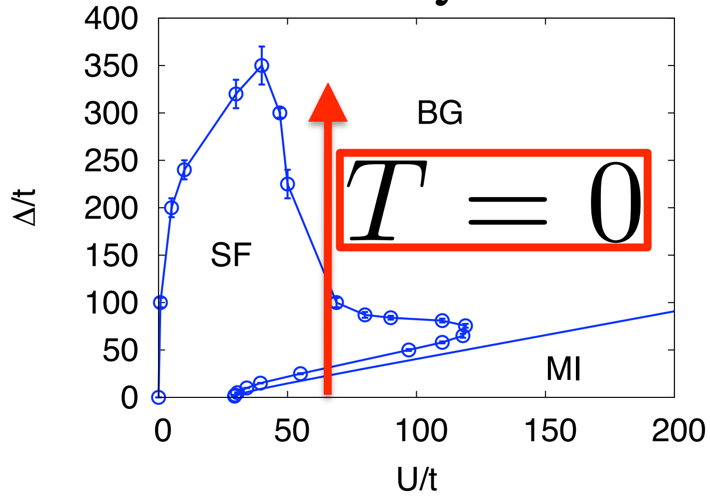


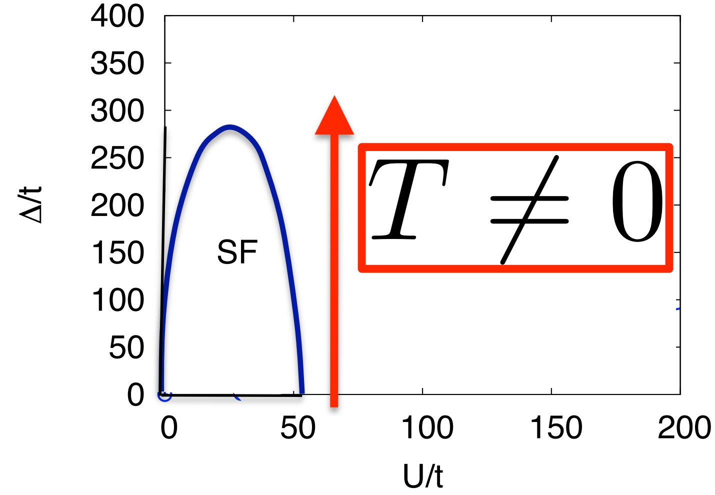
V. Gurarie, et.al., PRB 80, 214519 (2009)

Q3: What is the origin for this non-trivial shape (wiggle) of the phase diagram?

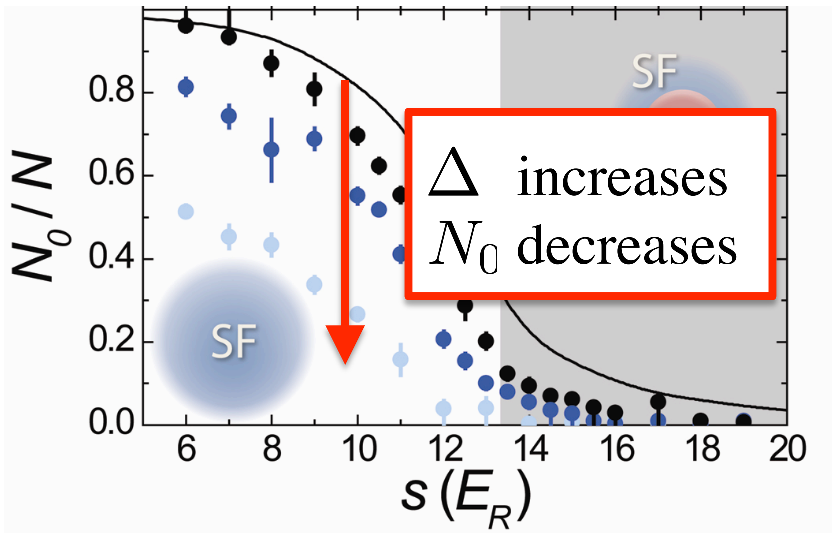
A puzzle

Theory





Experiment: B.Demarco (2009)



Q4: Why the topology of the phase diagram changes as temperature increase?

It is not easy to access the underlying physics for above features from sophisticated numerical simulations

Especially for those who don't know how to do Quantum Monte Carlo simulations, like me

Our approach:

Qualitative understandings from a simpler system

$\boxed{\text{Bosons in a "disordered" double well}}$

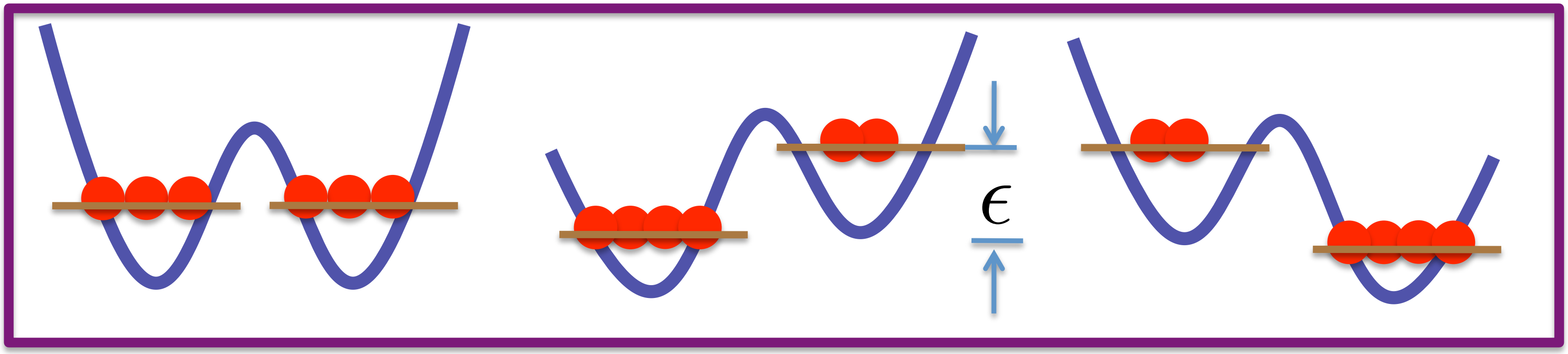
QZ, S. Das Sarma, PRA 82, 041601(R) (2010)

Even though there is no long-range order

- ✧ A simple system capturing all above features
- ✧ A minimal model incorporating interaction & disorder
- ✧ Exactly solvable & Easily computed
- ✧ Reveal underlying qualitative physics transparently

What do we mean by a “disordered” double well?

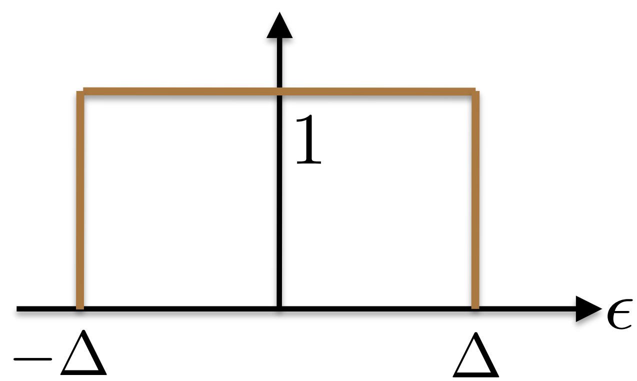
Consider an **ENSEMBLE**



$$-t(b_L^\dagger b_R + c.c) + \frac{U}{2}(n_L(n_L-1) + n_R(n_R-1)) + \epsilon(n_L - n_R)$$

$\epsilon \in [-\Delta, \Delta]$ Randomly distributed: Probability $P(\epsilon)$

Any quantity is an ensemble averaged one



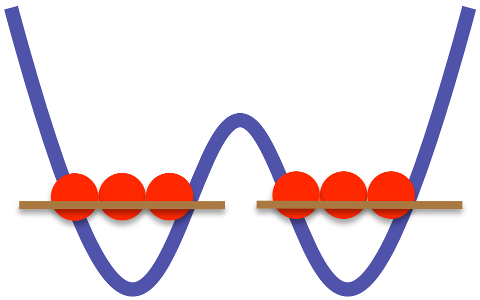
$$\overline{\langle \hat{O} \rangle} = \int_{-\Delta}^{\Delta} d\epsilon \langle \hat{O} \rangle_\epsilon / (2\Delta)$$

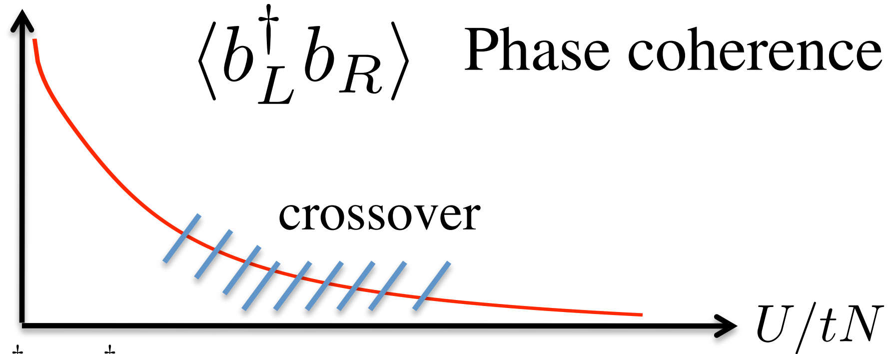
$\langle \hat{O} \rangle_\epsilon$ Expectation value of $\hat{O}$ at fixed ϵ

Bosons in a "clean" double well

$$\epsilon \equiv 0 \quad -t(b_L^\dagger b_R + c.c) + \frac{U}{2}\left(n_L(n_L-1) + n_R(n_R-1)\right)$$

Two sites model

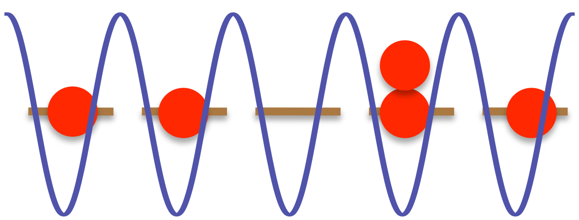




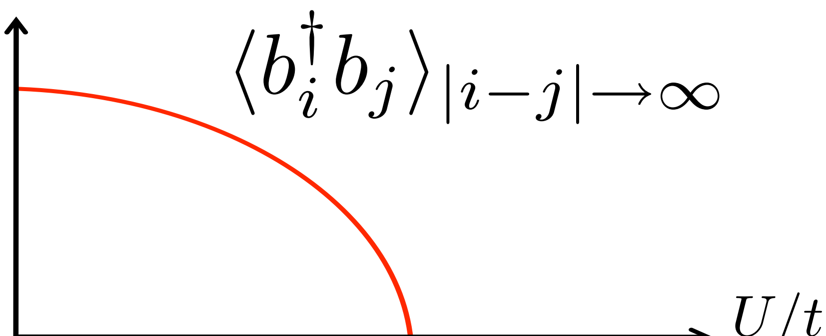
$$|G\rangle = \left(\frac{b_L^\dagger + b_R^\dagger}{\sqrt{2}}\right)^N |0\rangle \qquad |G\rangle = b_L^{\dagger\frac{N}{2}} b_R^{\dagger\frac{N}{2}} |0\rangle$$

$\boxed{\text{Interaction suppresses phase coherence}}$

Lattice model



$$-t\sum_{i,j}(b_i^\dagger b_j + c.c) + \frac{U}{2}\sum_i n_i(n_i-1)$$



$$|G\rangle = \left(\frac{\sum_i b_i^\dagger}{\sqrt{L}}\right)^N |0\rangle \qquad |G\rangle = \prod_i b_i^{\dagger\frac{N}{L}} |0\rangle$$

$$\boxed{-t(b_L^\dagger b_R + c.c) + \frac{U}{2}\left(n_L(n_L-1) + n_R(n_R-1)\right) + \epsilon(n_L - n_R)}$$

At a fixed ϵ , exact diagonalization

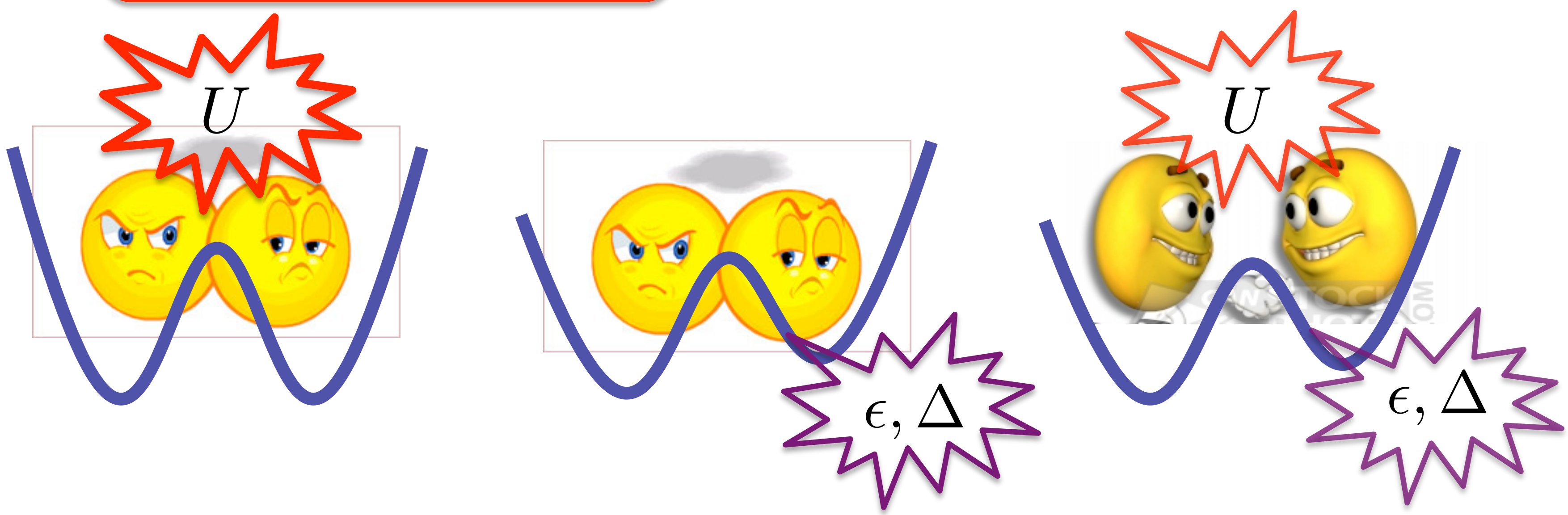
$\epsilon \in [\Delta, \Delta]$

$$H(\epsilon)|\Psi\rangle_n = E_n(\epsilon)|\Psi\rangle_n$$

$$|\Psi\rangle_n = \sum_{l=0}^{N} c_{n,l}|l, N-l\rangle$$

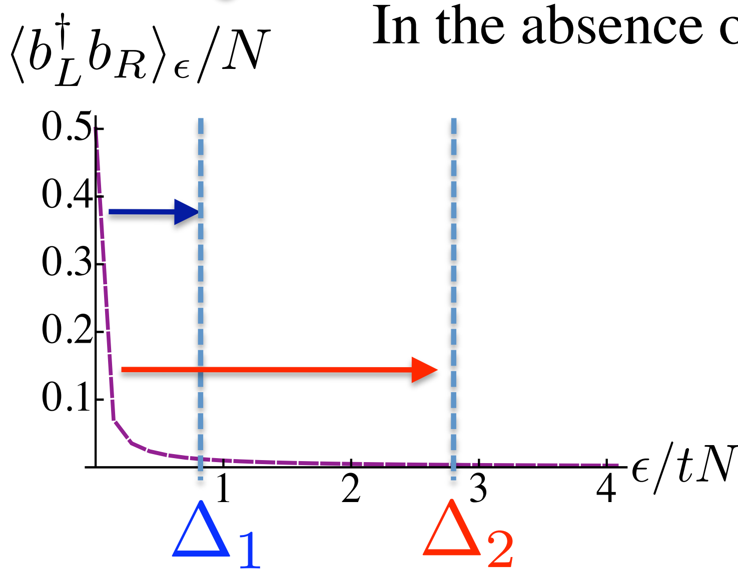
Interesting results on the coherence between left and right well

$$\langle b_L^\dagger b_R\rangle_\epsilon \ \& \ \overline{\langle b_L^\dagger b_R\rangle}$$



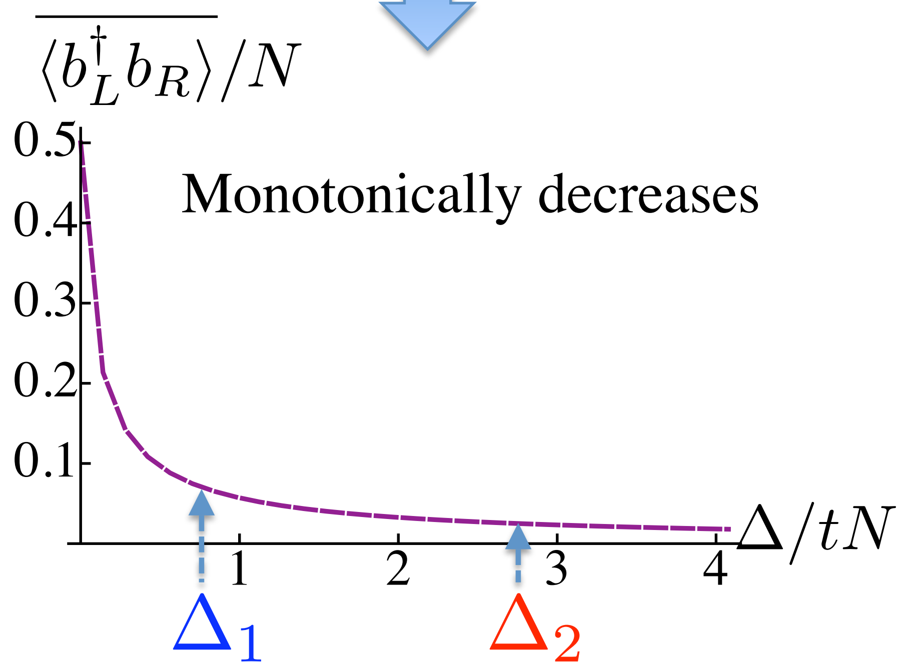
Whether and why disorder can enhance phase coherence?

In the absence of interaction NO!

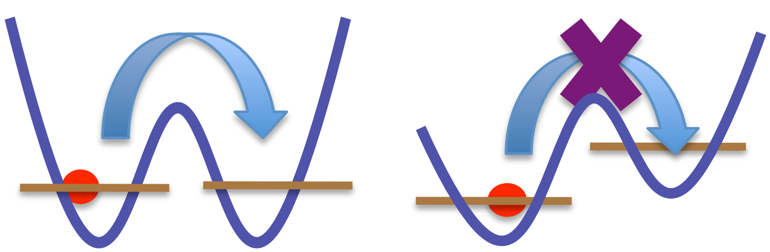


$$\overline{\langle b_L^\dagger b_R\rangle}/N = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} d\epsilon \langle b_L^\dagger b_R\rangle_\epsilon/N$$

Ensemble average



$\epsilon = 0$ $\qquad$ $\epsilon \gg t$

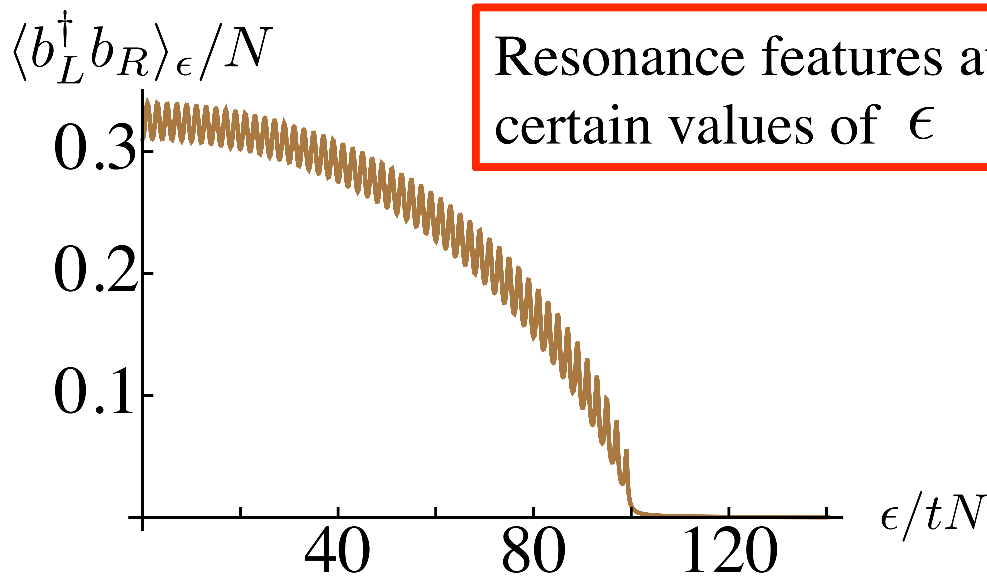


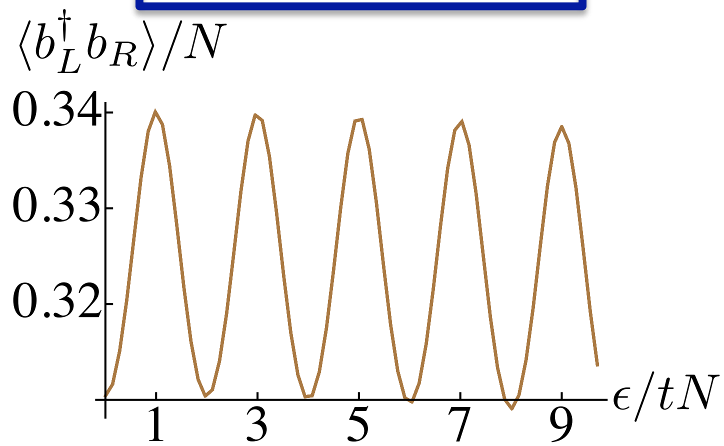
Whether and why disorder can enhance phase coherence?

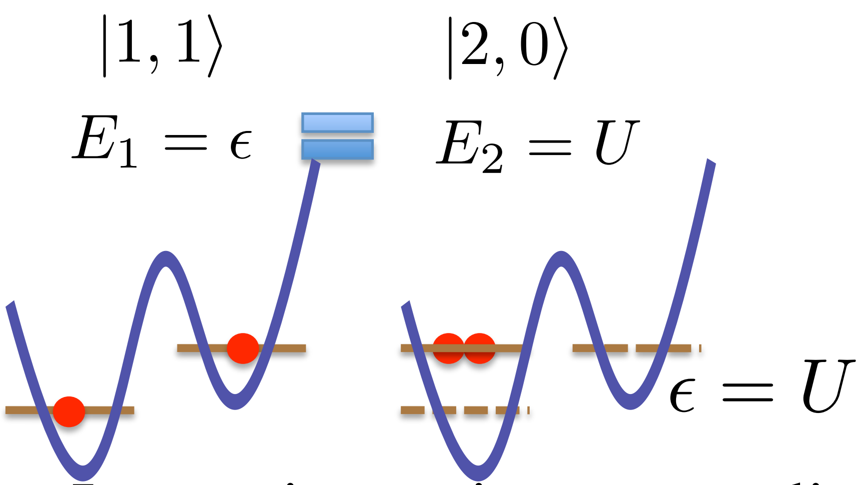
In the presence of interaction **Yes!**

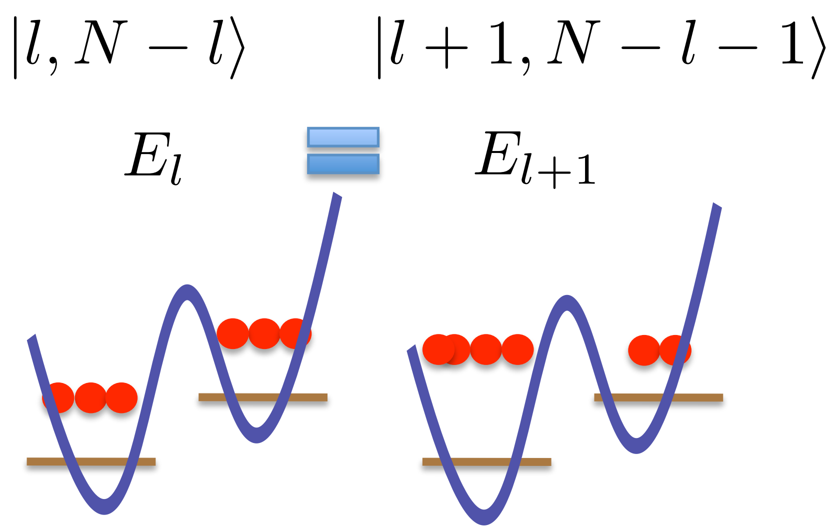
Resonance features at certain values of ϵ

$\epsilon^* = U,\ 3U,\ 5U...$

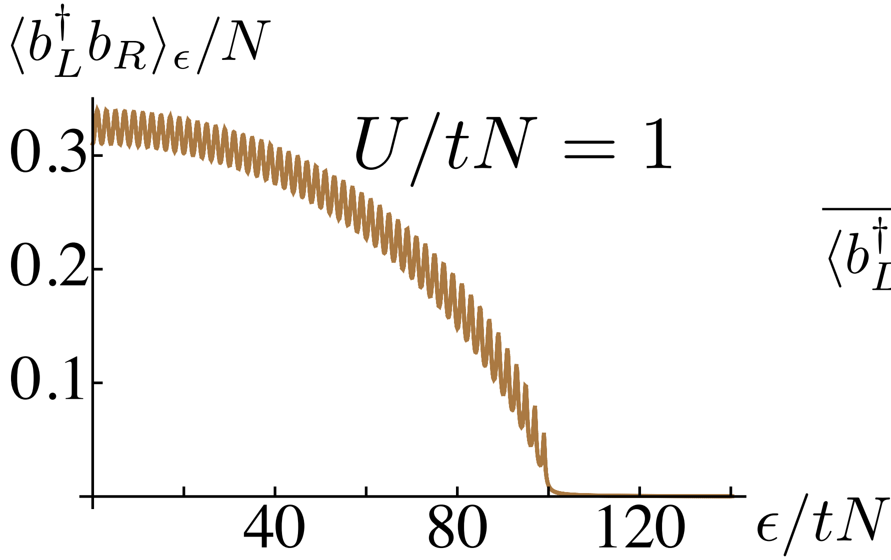






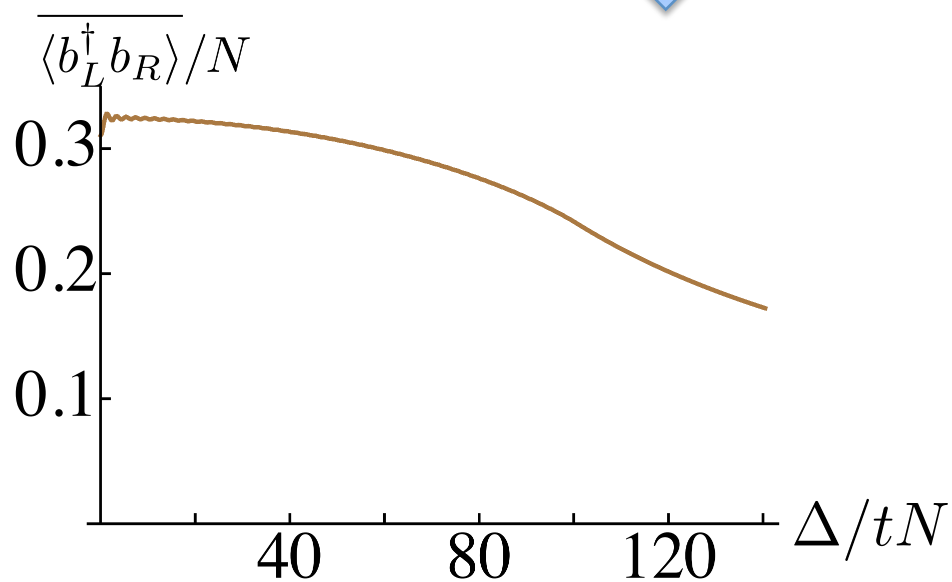


Interaction assistant tunneling



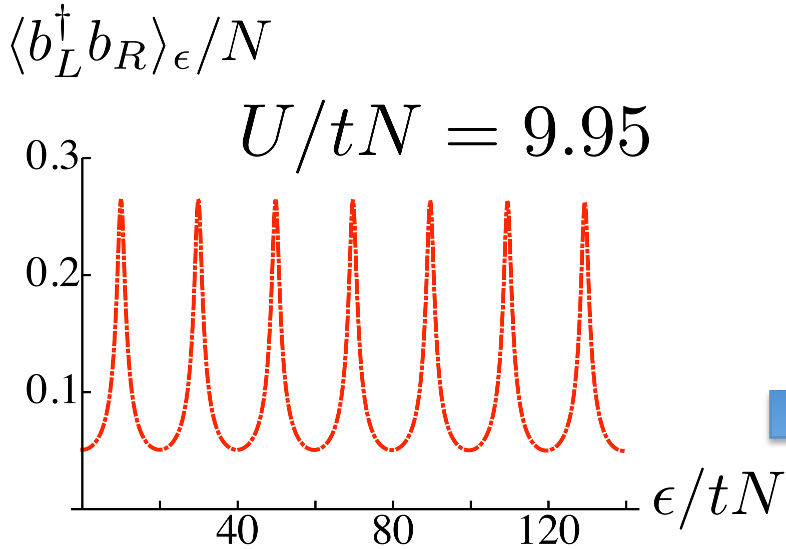
$$\overline{\langle b_L^\dagger b_R\rangle}/N = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} d\epsilon \langle b_L^\dagger b_R\rangle_\epsilon/N$$

Ensemble average



Disorder suppresses the
resonance feature
at small U

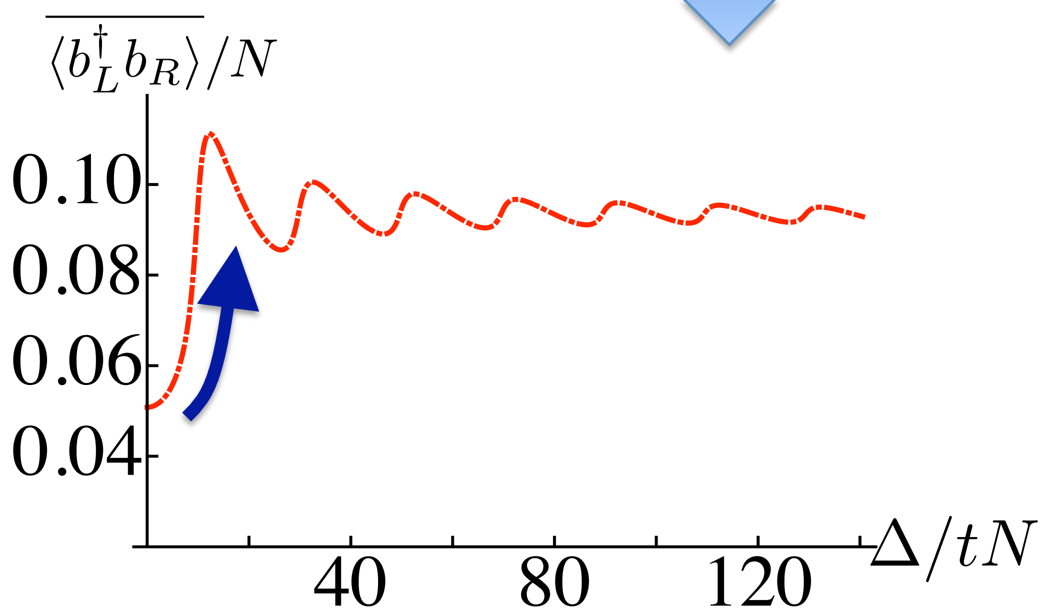
At large U



Resonance feature remains after ensemble averaging

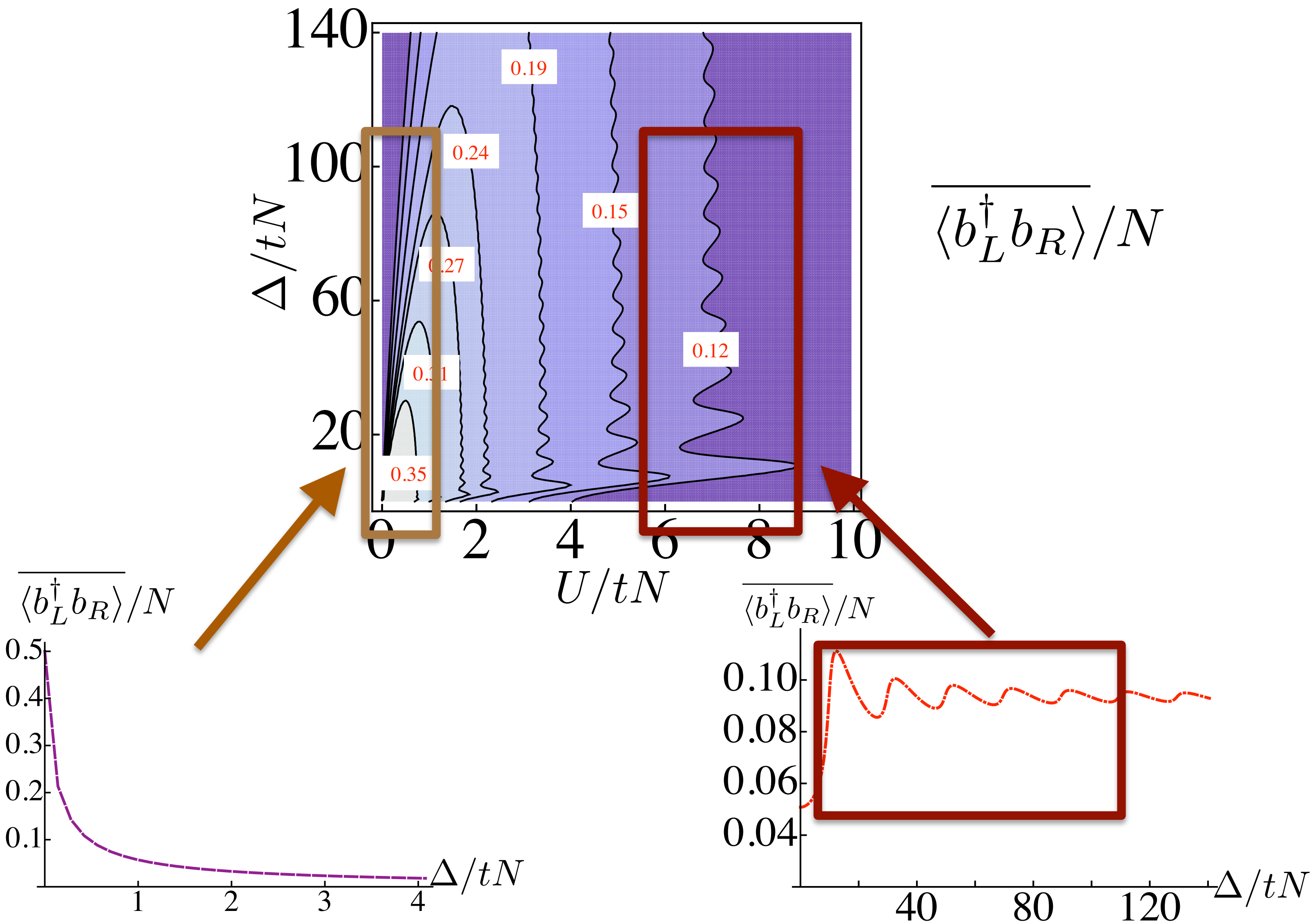
$$\overline{\langle b_L^\dagger b_R \rangle}/N = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} d\epsilon \langle b_L^\dagger b_R \rangle_\epsilon / N$$

Ensemble average

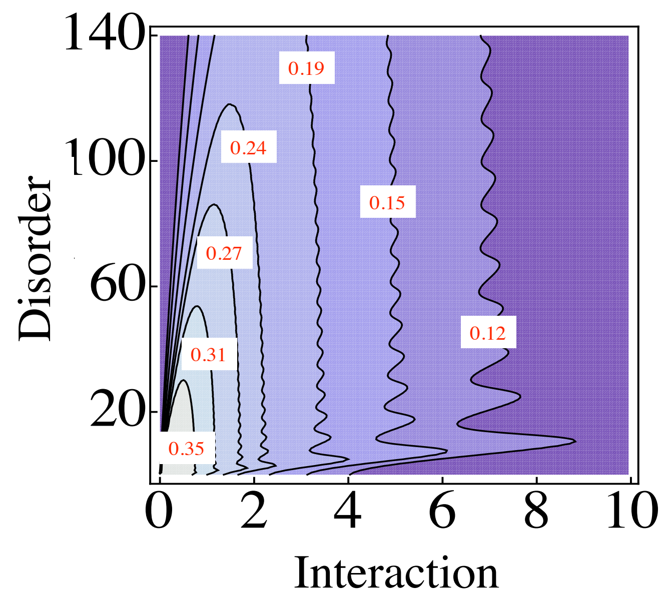


Disorder enhances phase coherence!

the contours of phase coherence



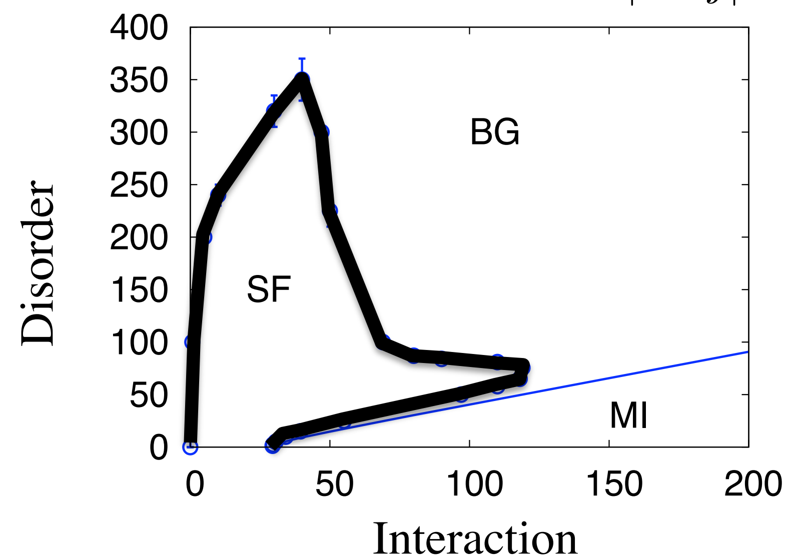
Contours of $\overline{\langle b_L^\dagger b_R \rangle}/N$



QZ, S. Das Sarma,
PRA 82, 041601(R) (2010)

Contours of $\overline{\langle b_i^\dagger b_j \rangle}_{|i-j|} = 0$

$|i-j| \to \infty$

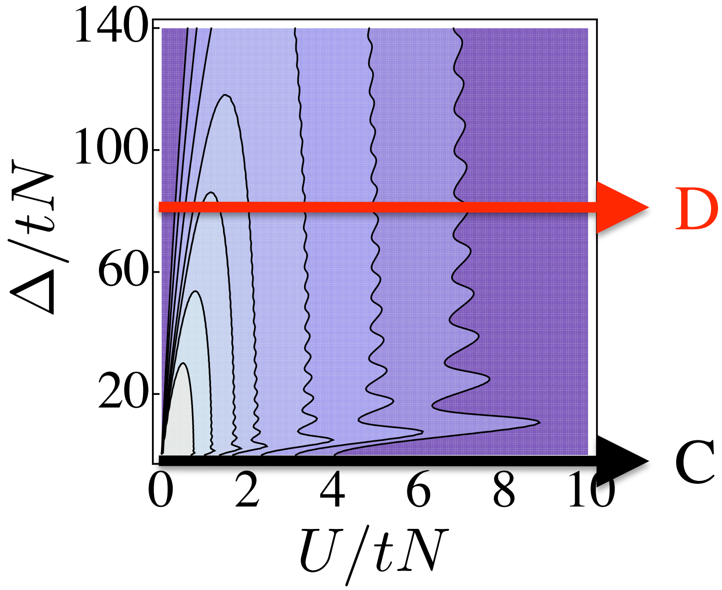


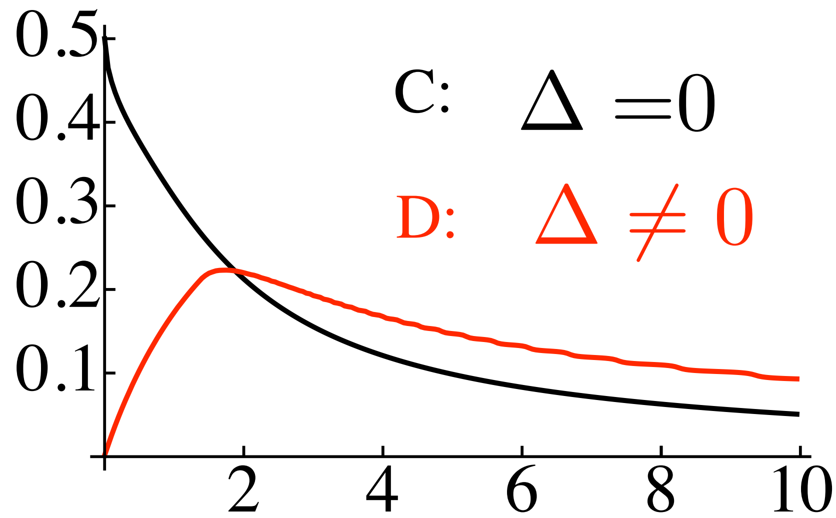
V. Gurarie, et al.,
PRB 80, 214519 (2009)

The only difference: Number of wiggles

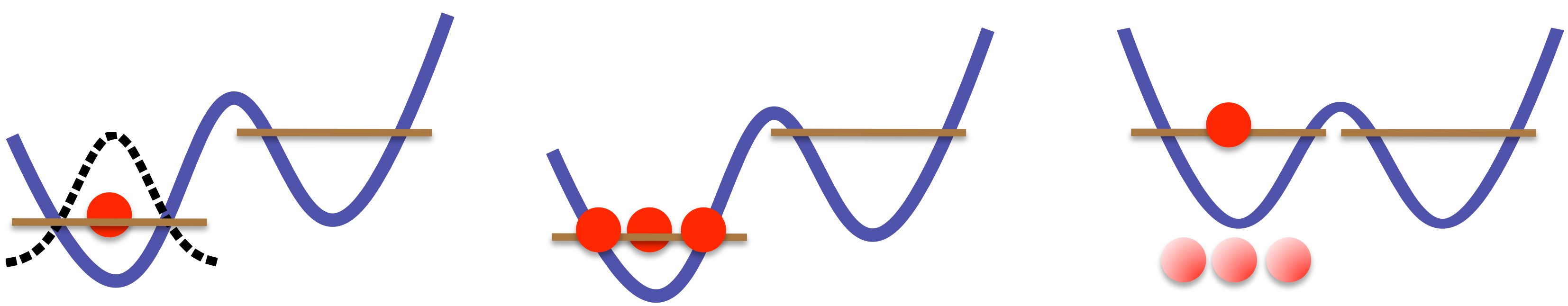
Different particle number per site

Interaction enhanced coherence when $\Delta \neq 0$

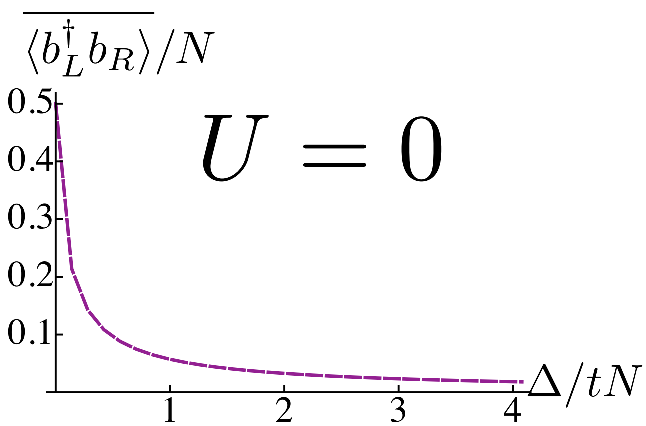


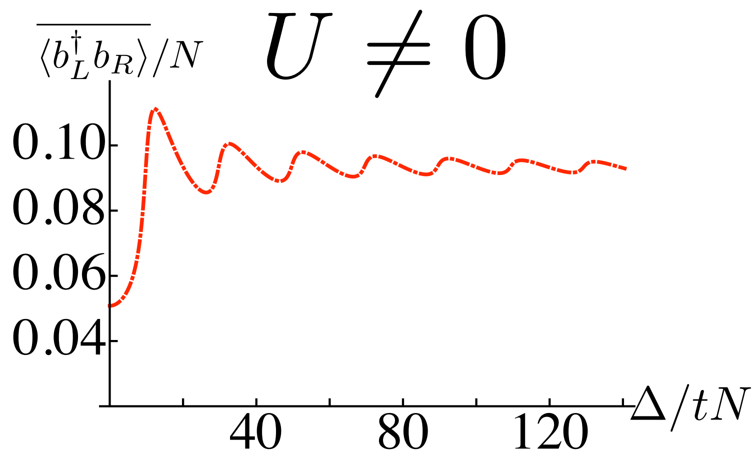


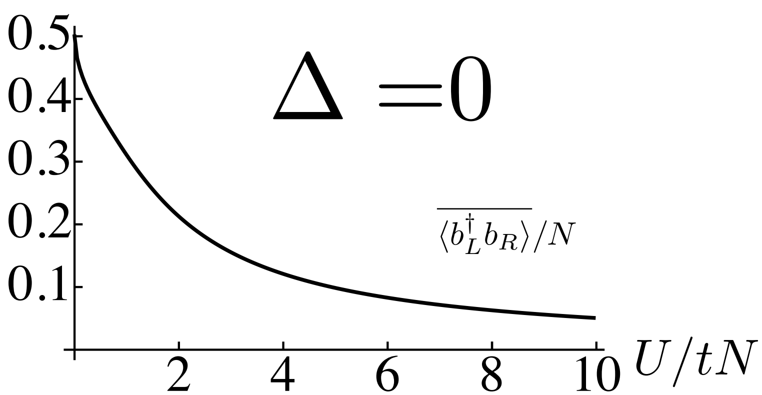
Interaction smoothes the disordered potential

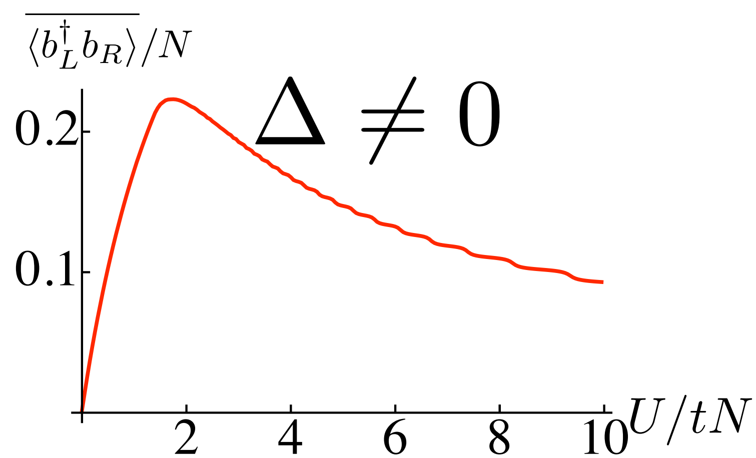


Two negatives make a positive

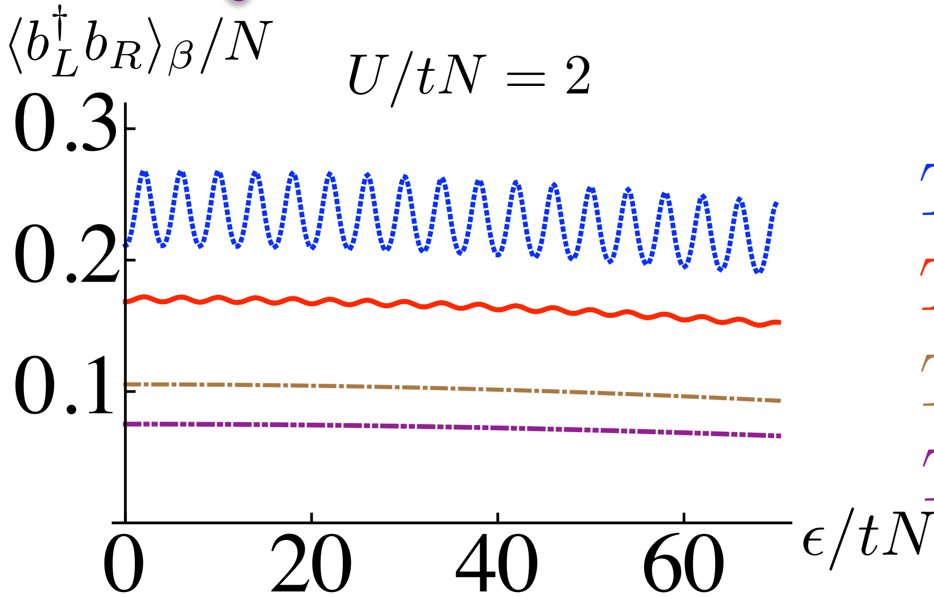








Finite temperature effect



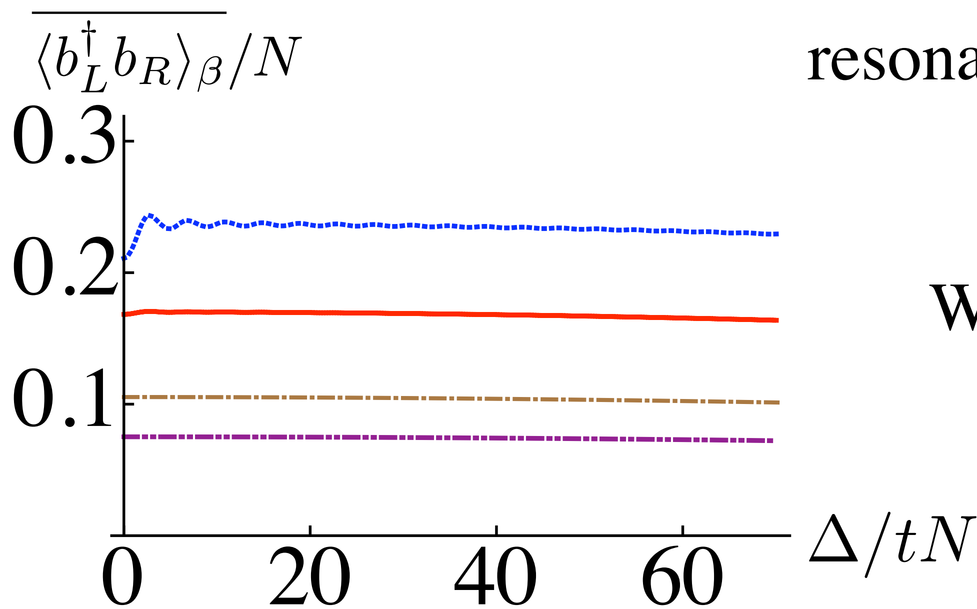
$T/tN = 0.4$

$T/tN = 1$

$T/tN = 2$

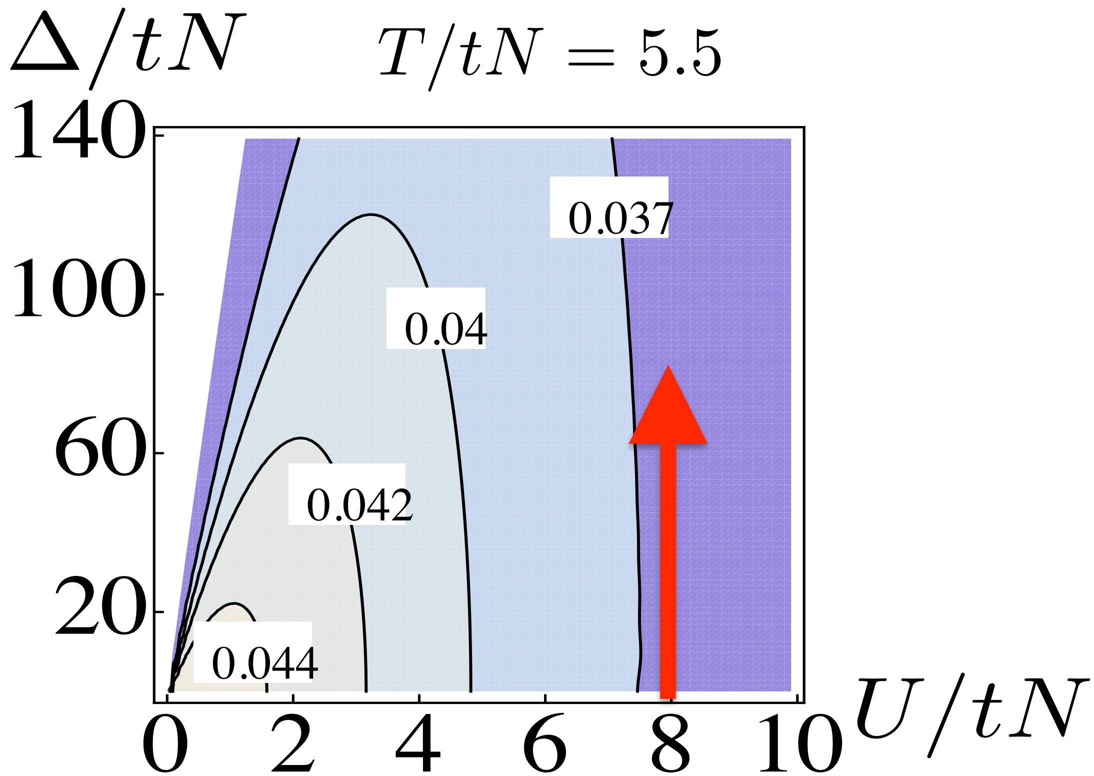
$T/tN = 3$

At finite temperatures, resonance features are suppressed



Wiggles are also suppressed

the contours of phase coherence at $T \neq 0$



No enhancement of coherence by disorder

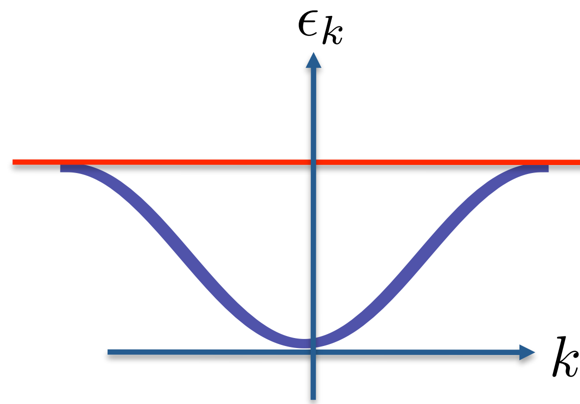
From two-site problem to lattice case

Any exactly solvable lattice mode for helping understand disordered systems?

An exactly solvable model in a clean system

1D hard core bosons in a lattice $U \to \infty$

Map to ideal fermions on a lattice



$$\mu_c = 2t$$

Tomonaga-Tonk Liquid

MI (Fully filled band in fermion model)

$$\langle \hat{b}_i^\dagger \hat{b}_j \rangle \sim \frac{1}{|i-j|^{1/2}}$$

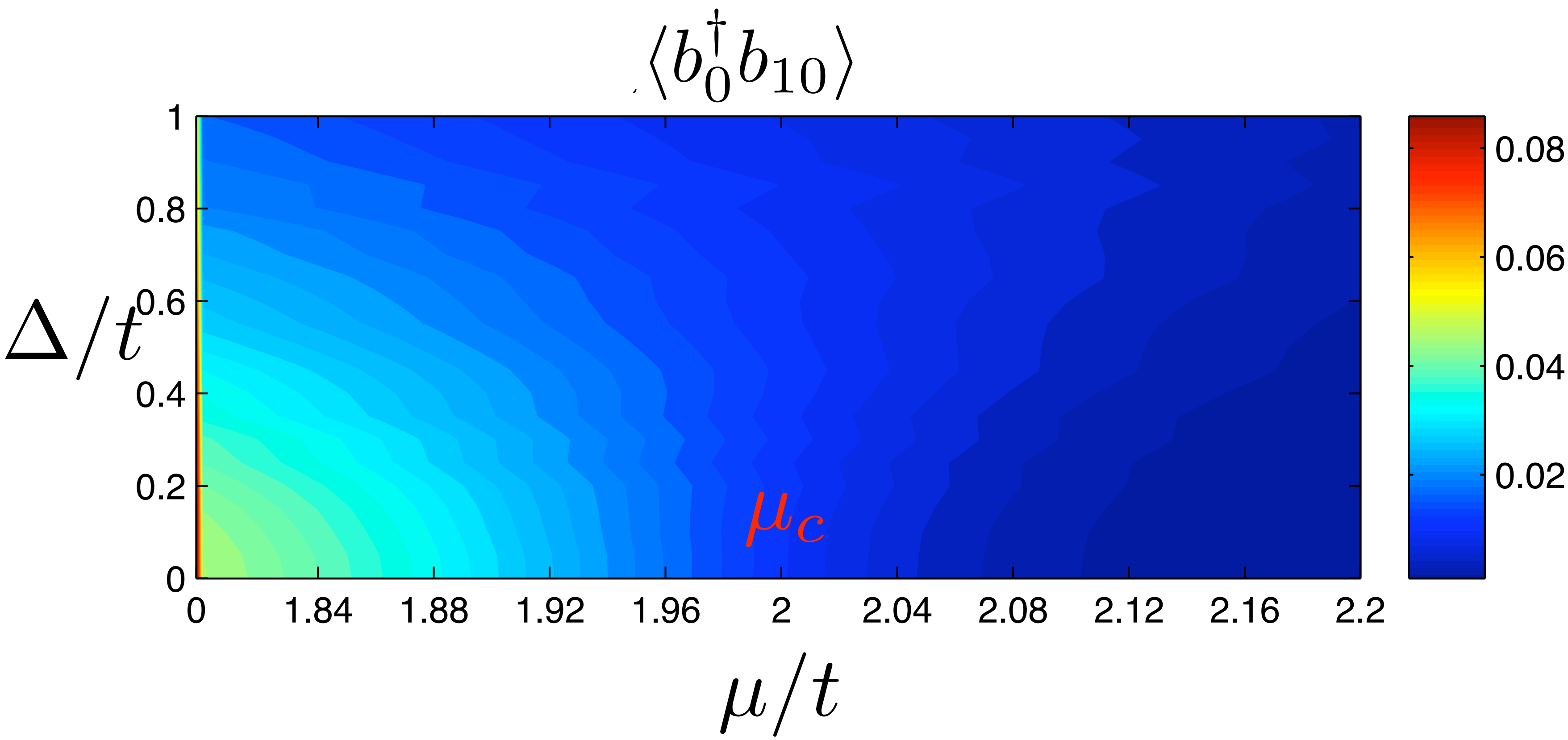
Replace the role of interaction

Disordered case

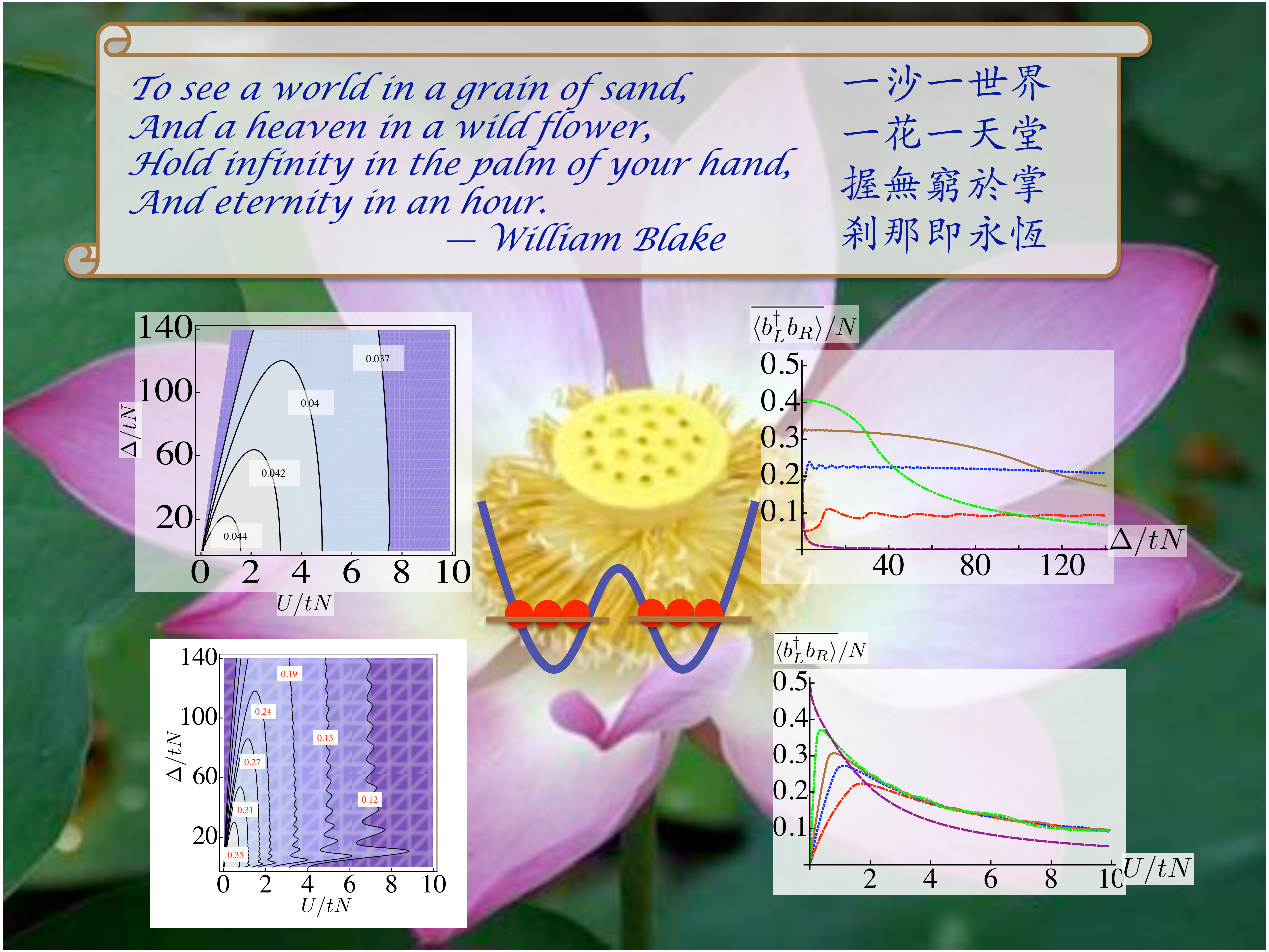
QZ, Q. Wang, W.M. Liu, S. Das Sarma, to be published

$$H = -t\sum_{i,j}(b_i^\dagger b_j + c.c) + \frac{U}{2}\sum_i n_i(n_i-1) + \sum_i \epsilon_i n_i - \mu\sum_i n_i$$

$$U \to \infty \qquad \epsilon_i \in [-\Delta, \Delta]$$



Disorder enhanced coherence is a general feature



To see a world in a grain of sand,
And a heaven in a wild flower,
Hold infinity in the palm of your hand,
And eternity in an hour.
— William Blake
一沙一世界
一花一天堂
握無窮於掌
剎那即永恆
Δ/tN
U/tN
0.037
0.04
0.042
0.044
$\overline{\langle b_L^\dagger b_R \rangle}/N$
Δ/tN
$\overline{\langle b_L^\dagger b_R \rangle}/N$
U/tN
Δ/tN
U/tN
0.19
0.24
0.27
0.31
0.35
0.15
0.12