Macroscopic Quantum Phenomena in Spin-orbit Coupled Bose-Einstein Condensate and Their Implications

Shizhong Zhang

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INTRODUCTION

Solid State Physics:

effects of magnetic field B
(abelian gauge field)

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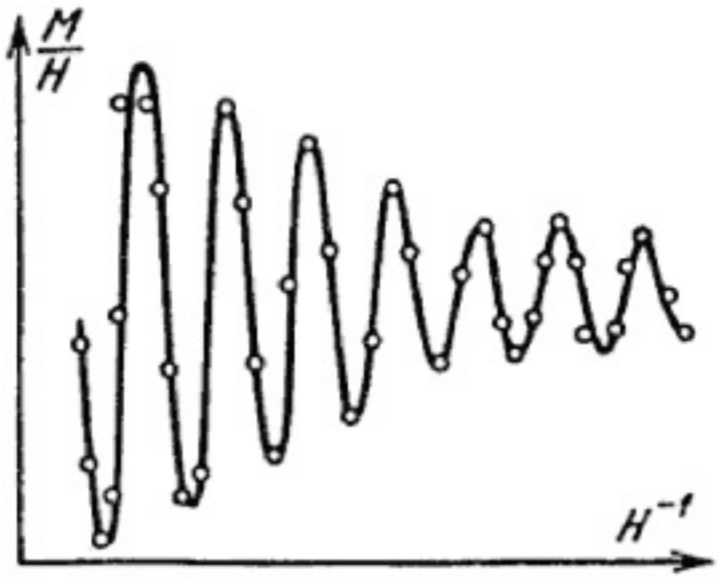

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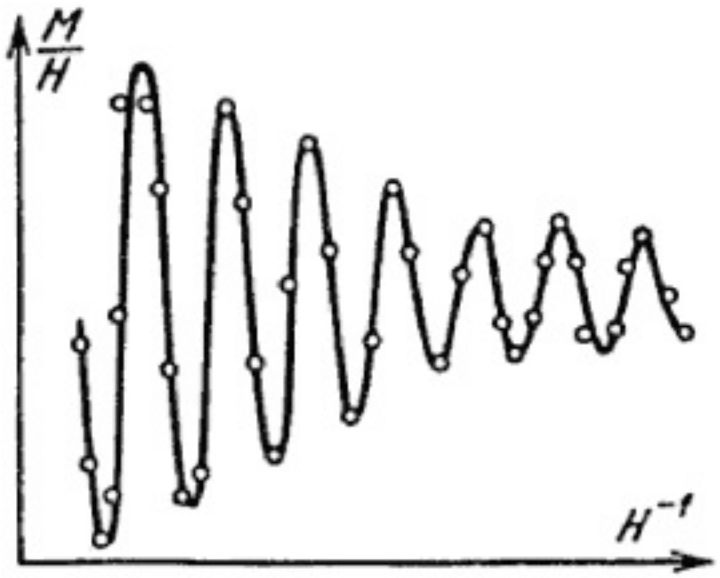

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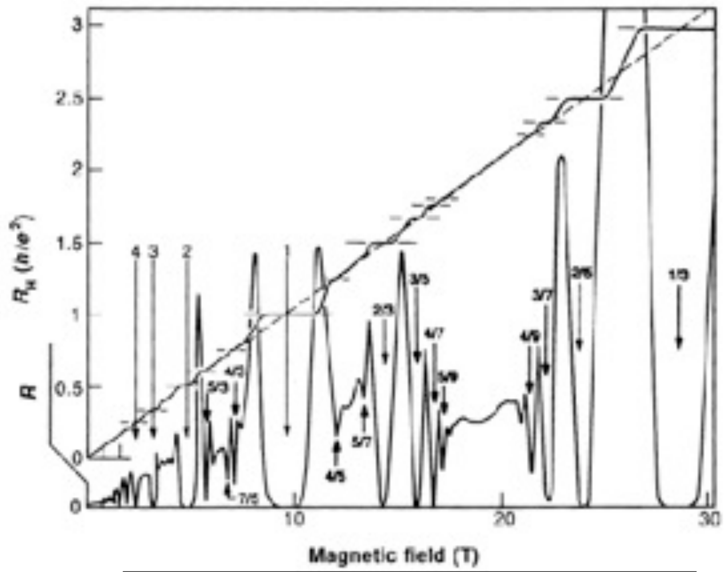

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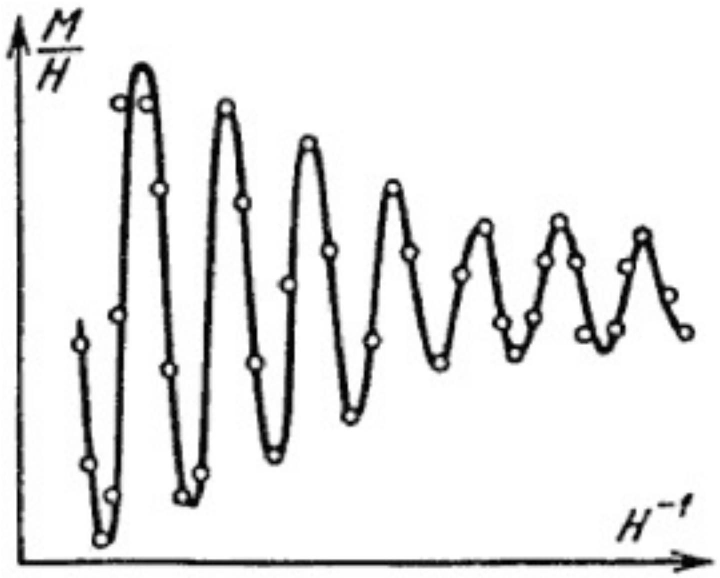

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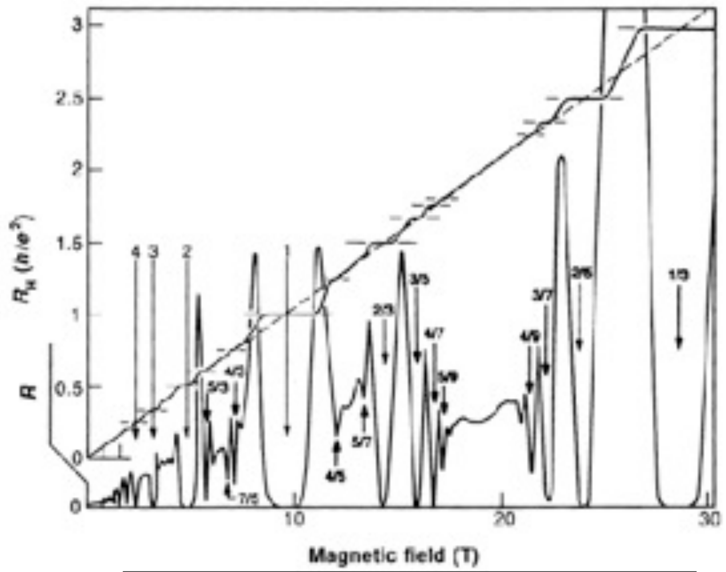

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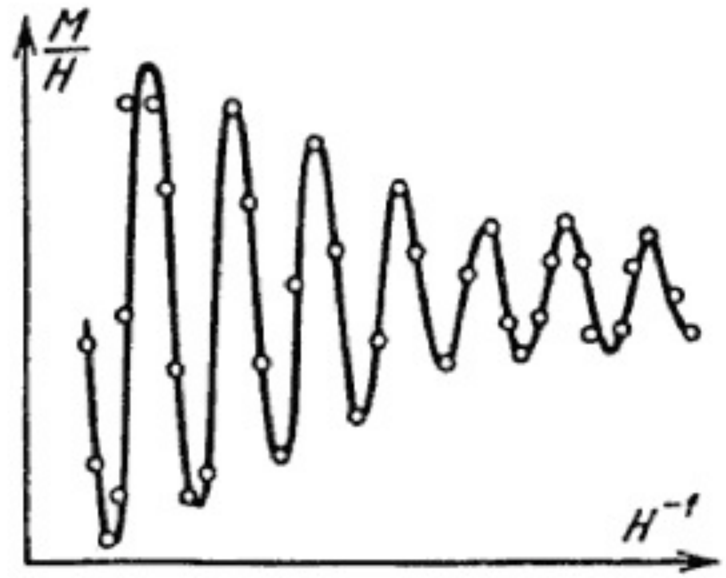

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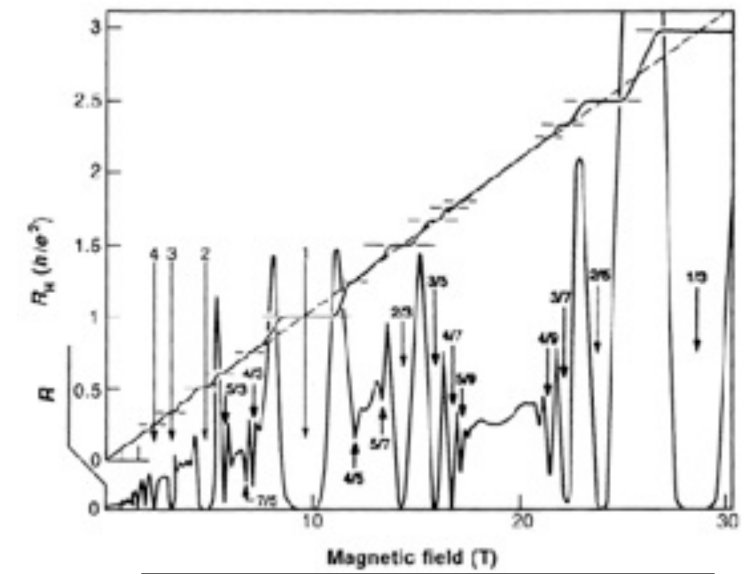

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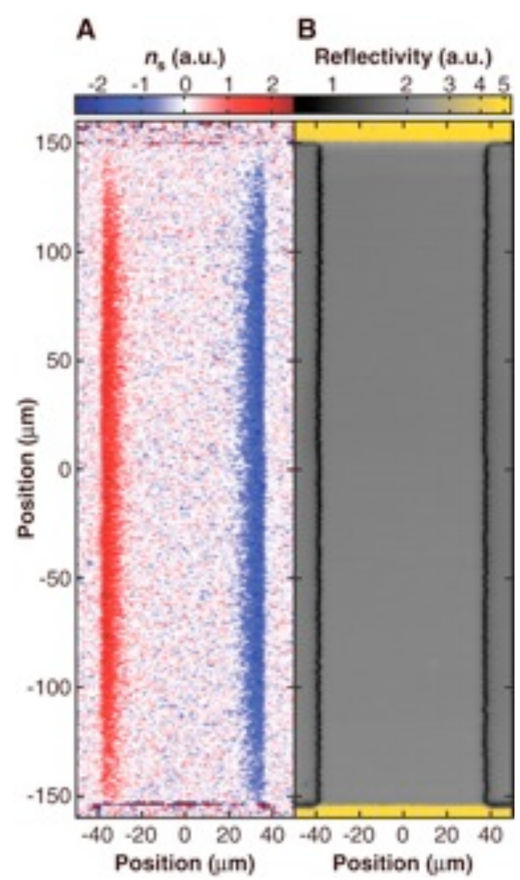

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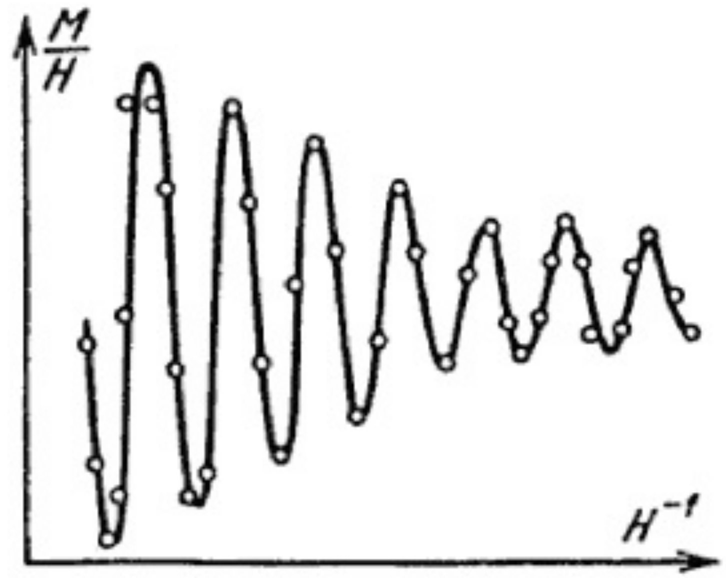

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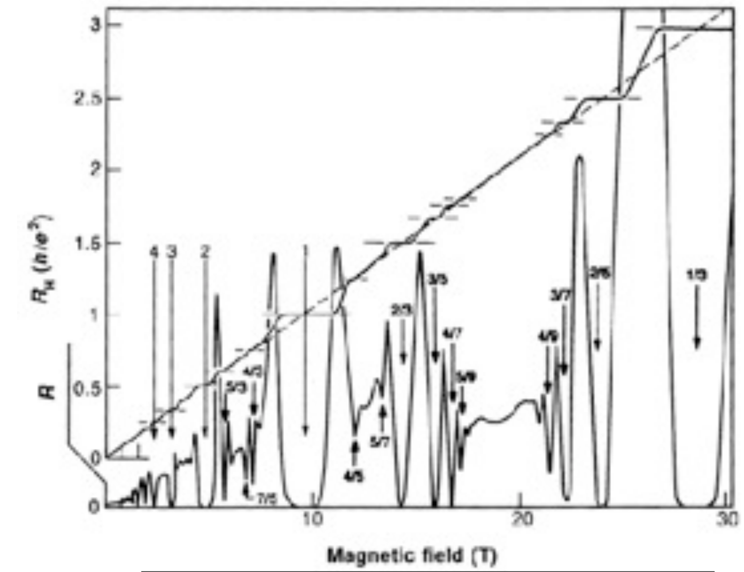

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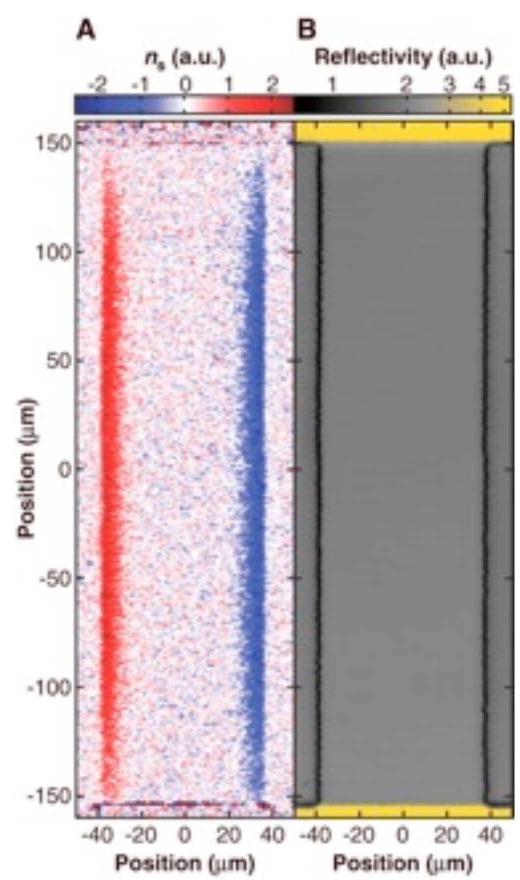

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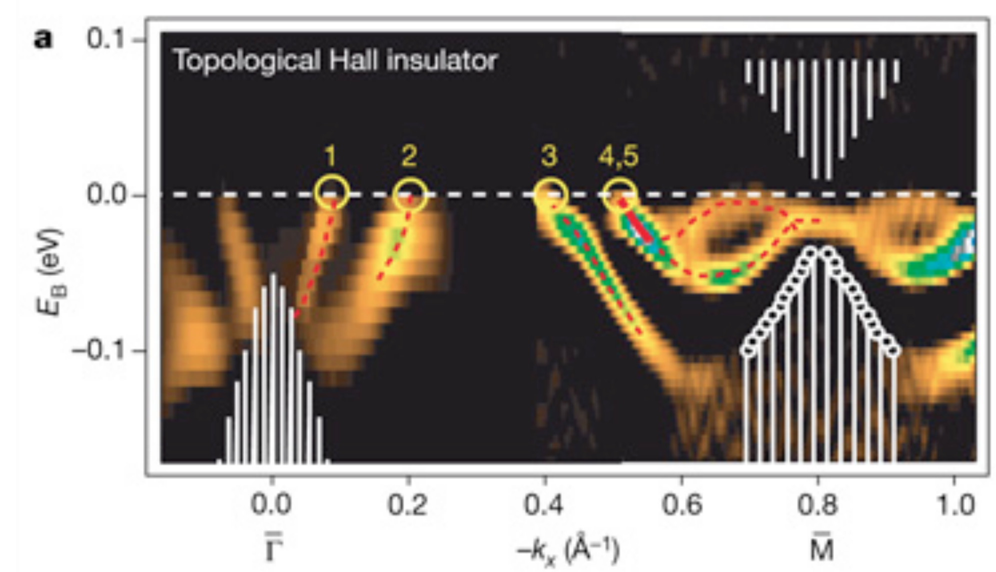

topological insulator

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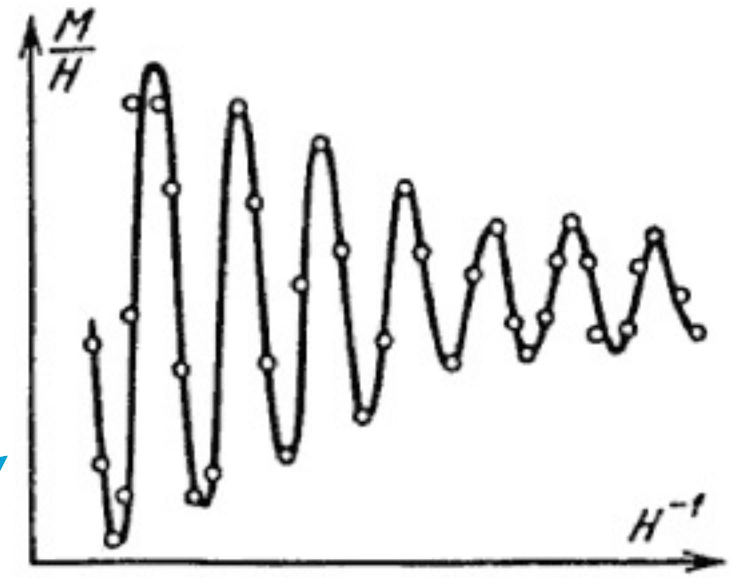

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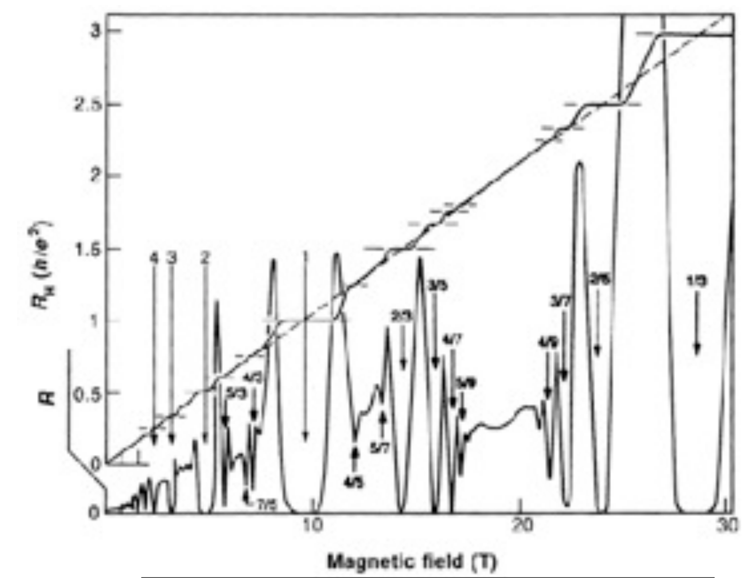

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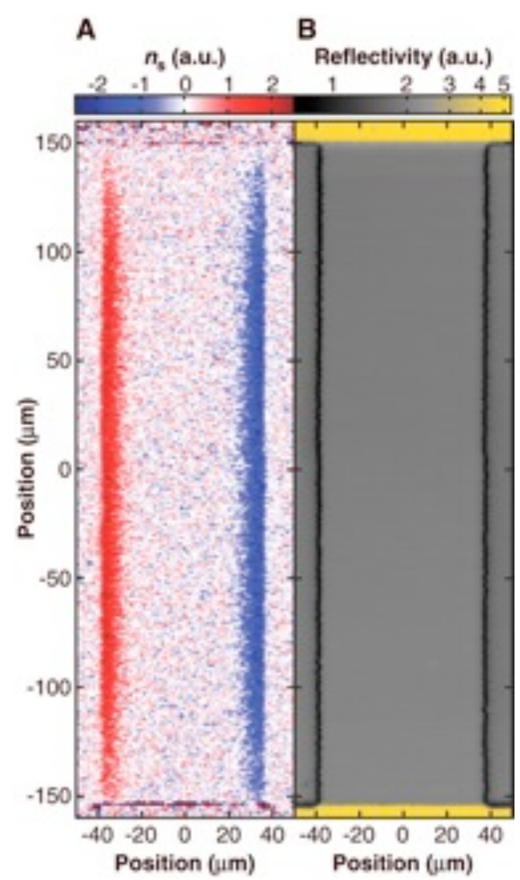

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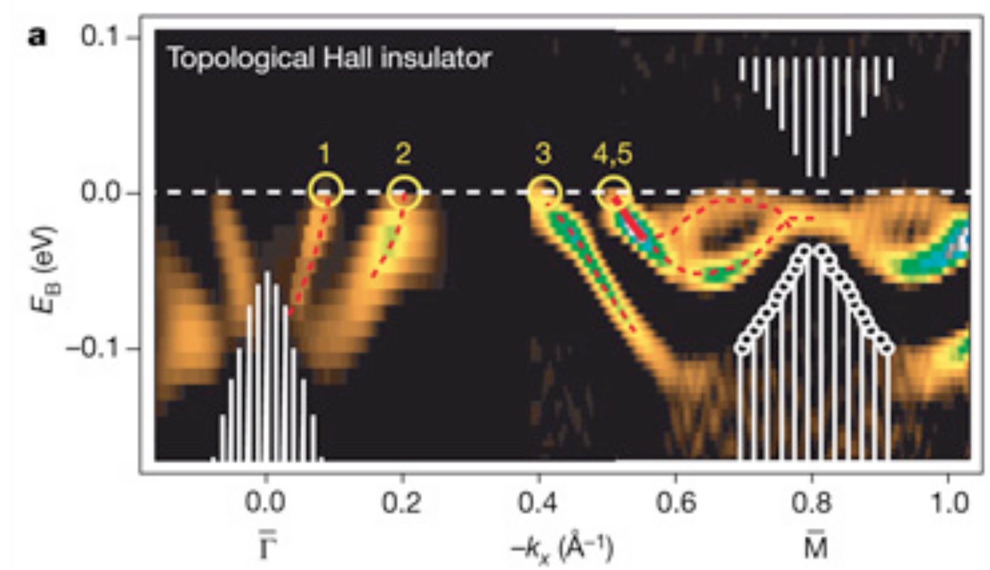

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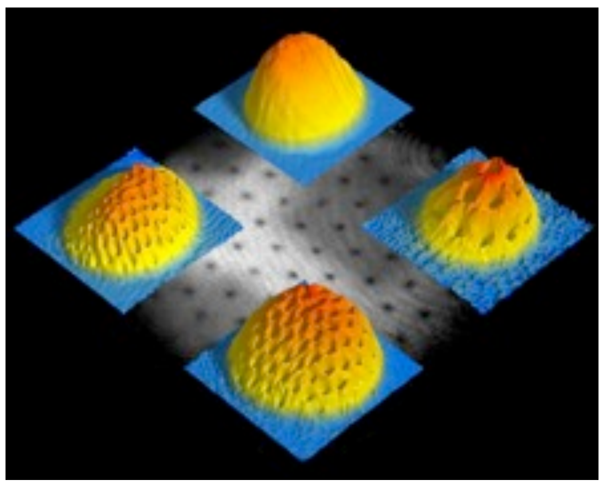

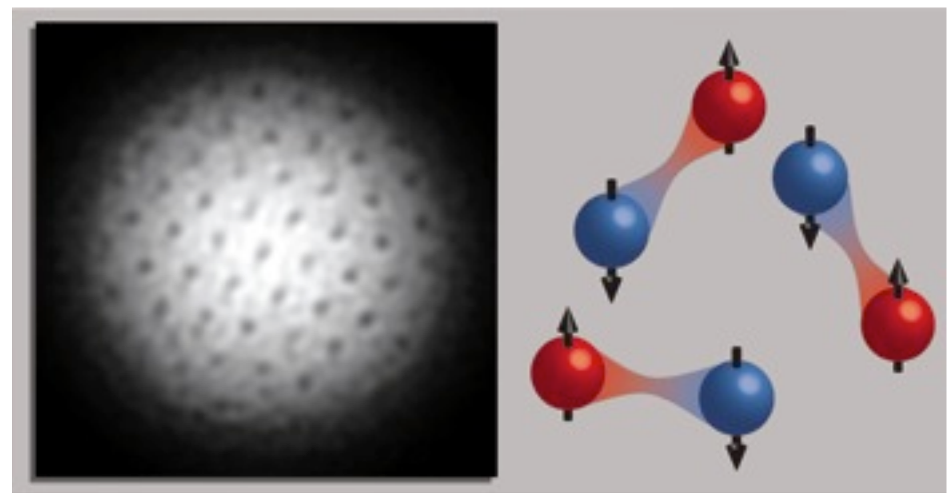

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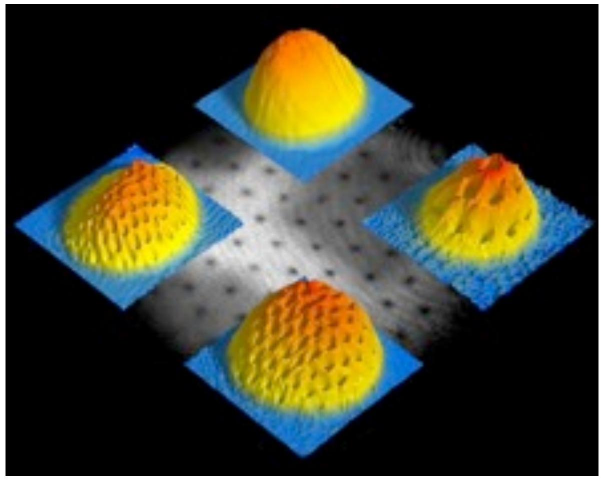

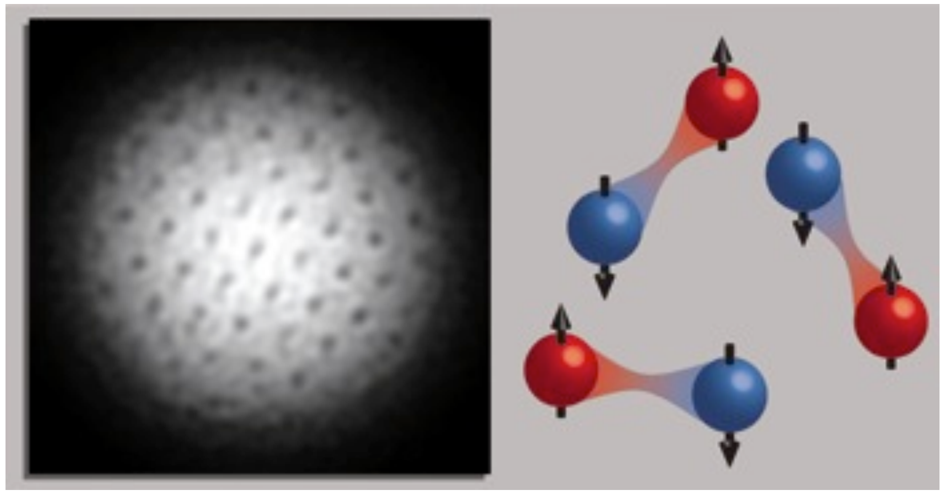

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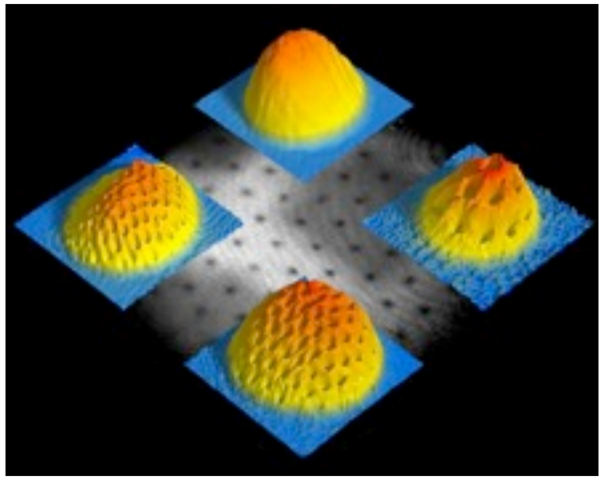

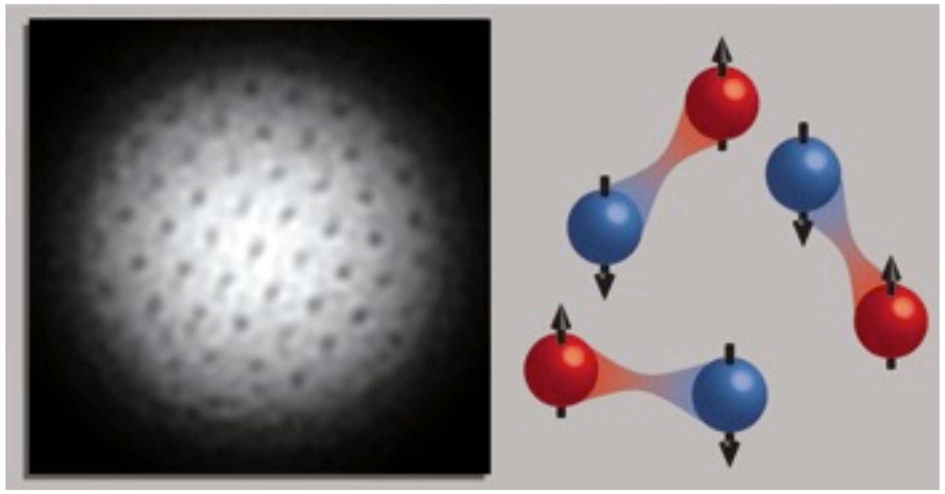

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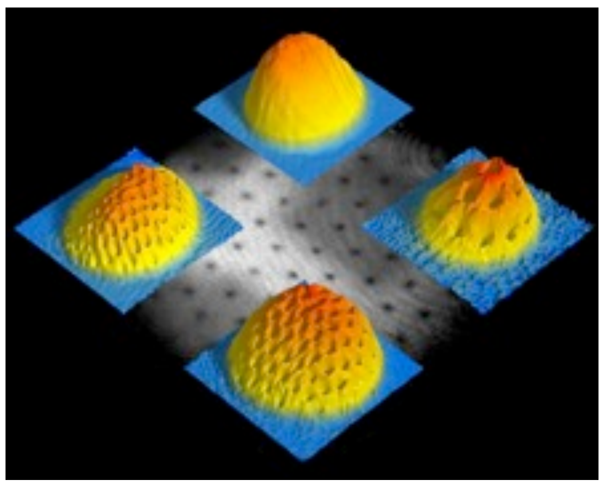

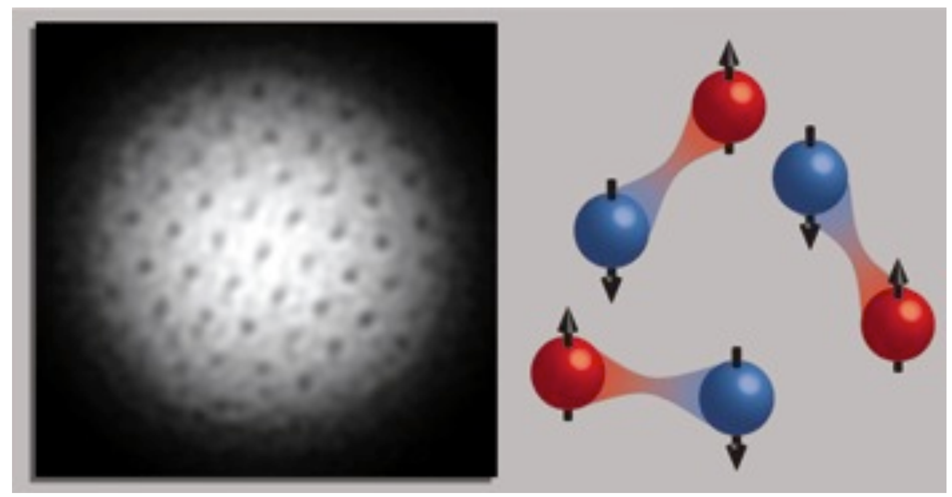

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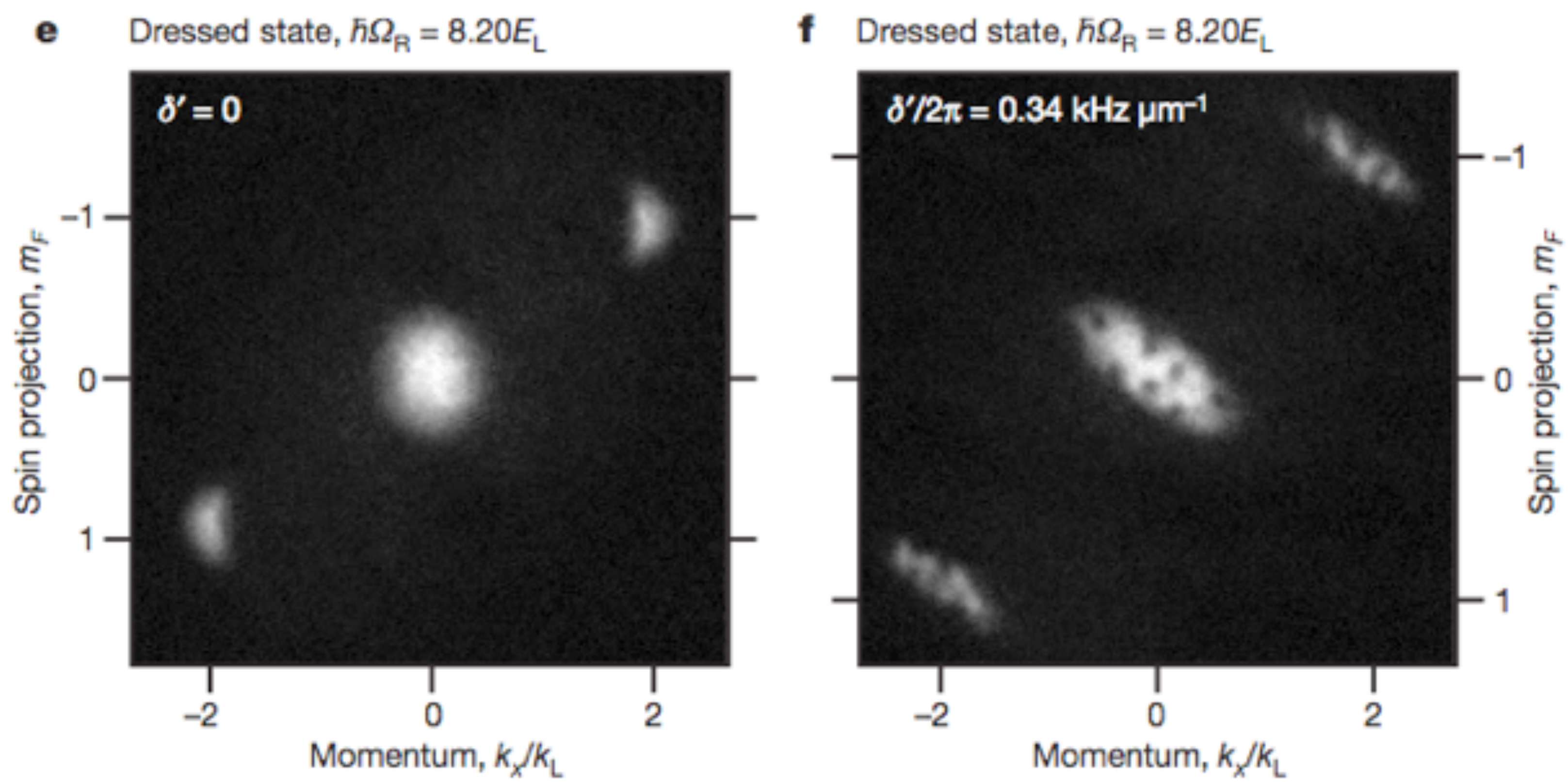

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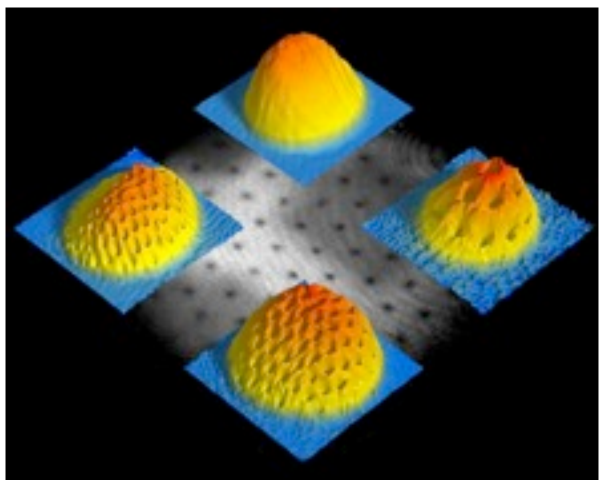

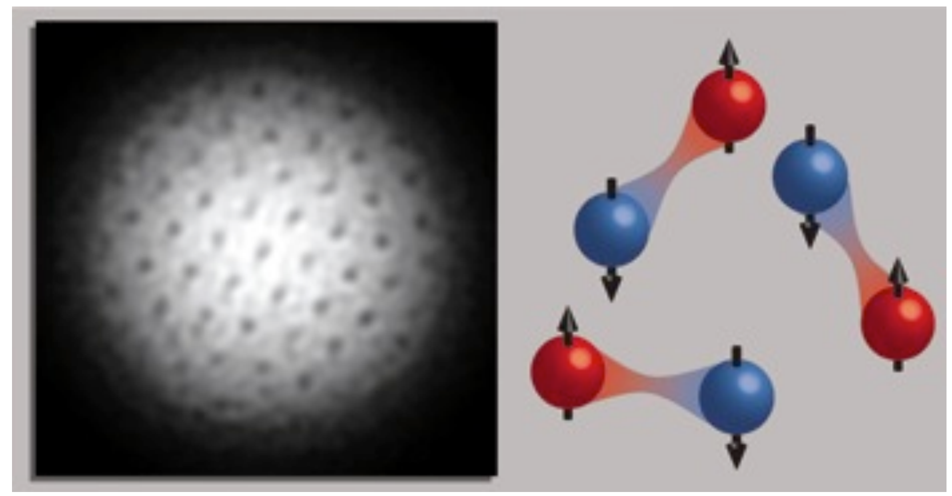

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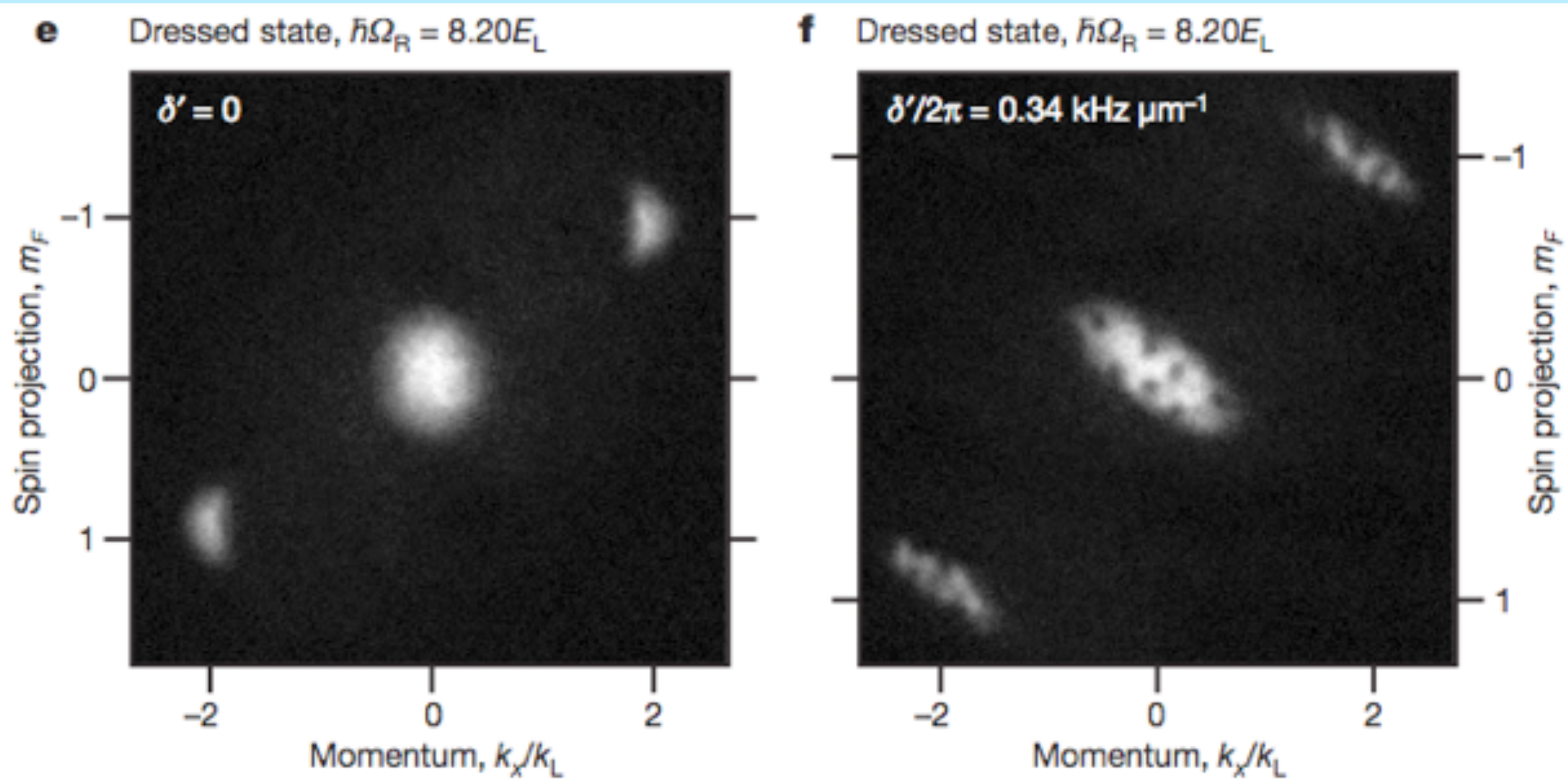

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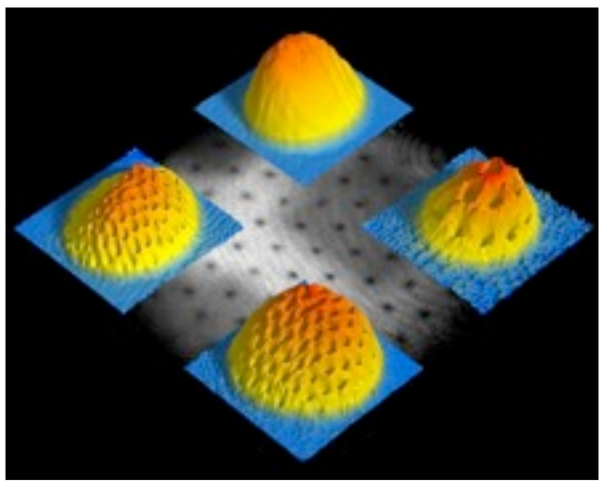

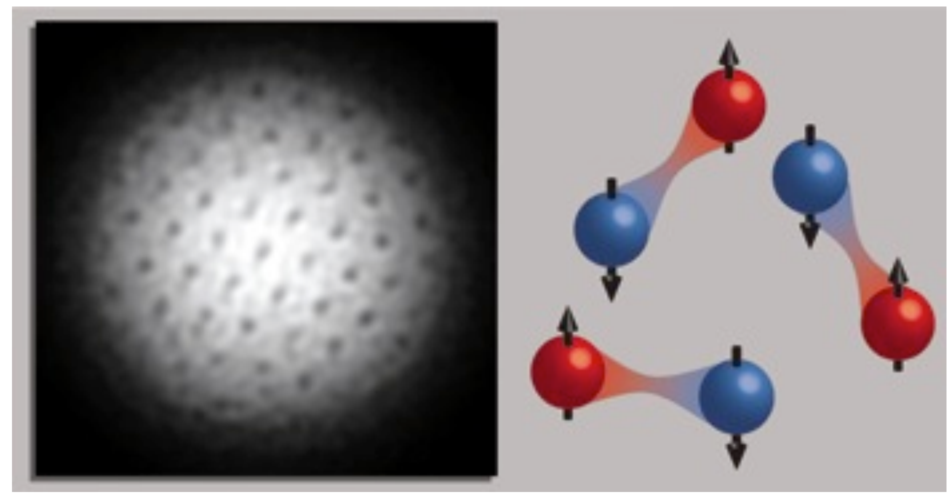

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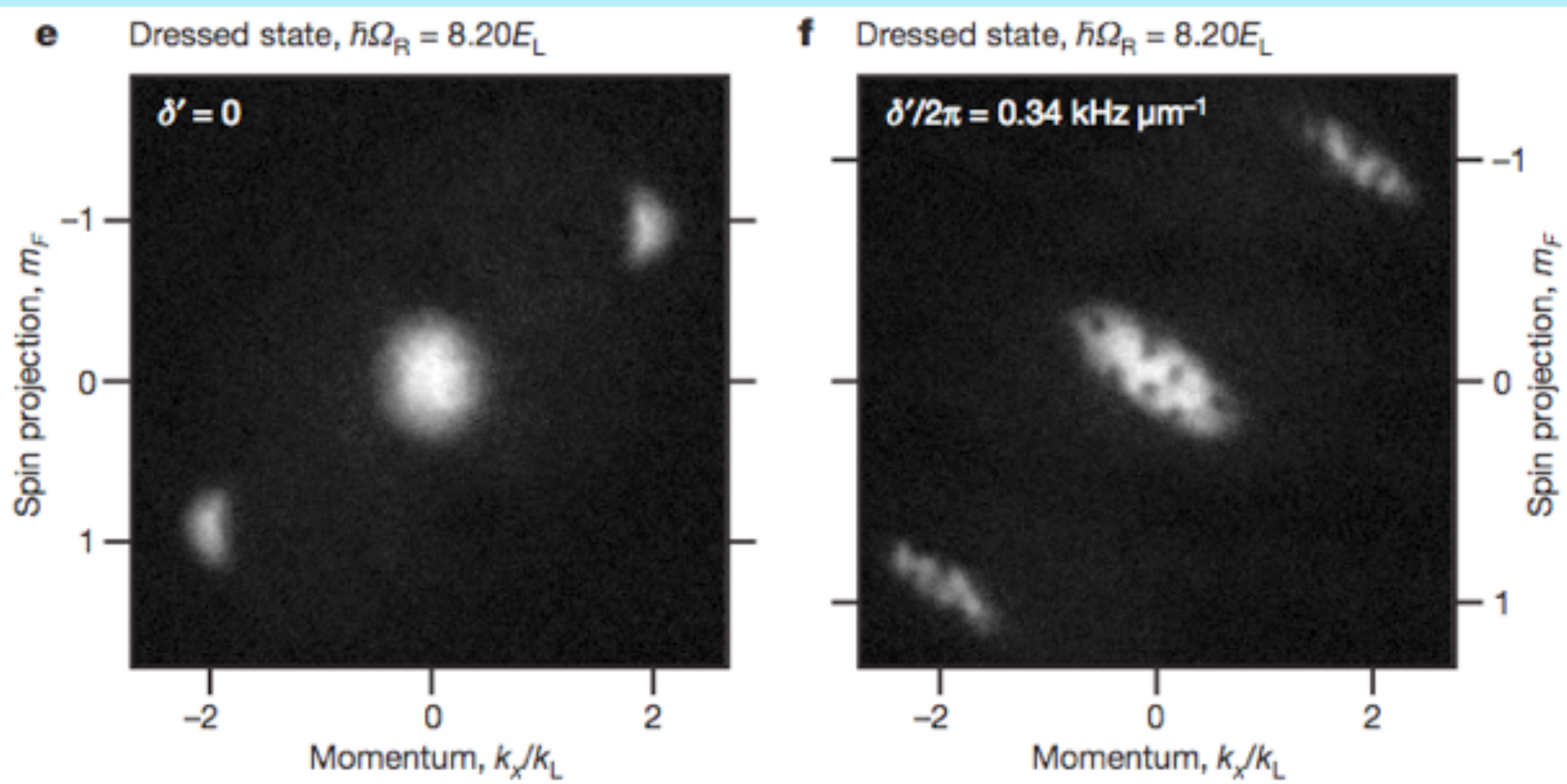

Bosons

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- **Many more possibilities ...**

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IV. Further direction in the field; vortices, fractional charge, transport and some speculations;

References:

Part I and II

Tin-Lun Ho and Shizhong Zhang, arXiv:1007.0650

Part III

Work with Onur Erten, Tin-Lun Ho, to appear.

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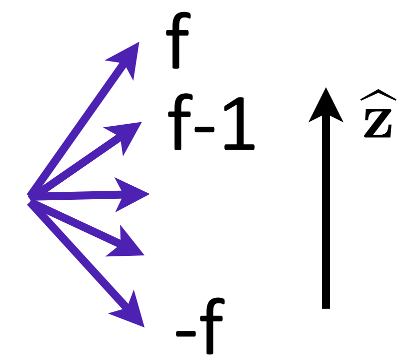
spin-f particles

$\alpha, \beta = 1, 2, \cdots, 2f + 1$



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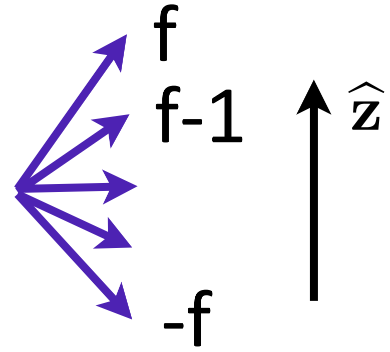

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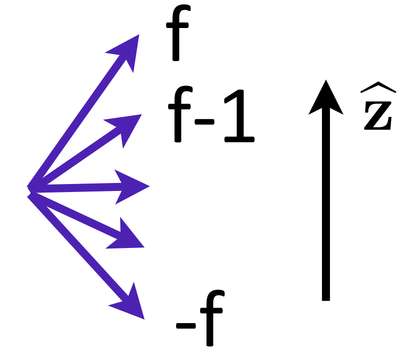

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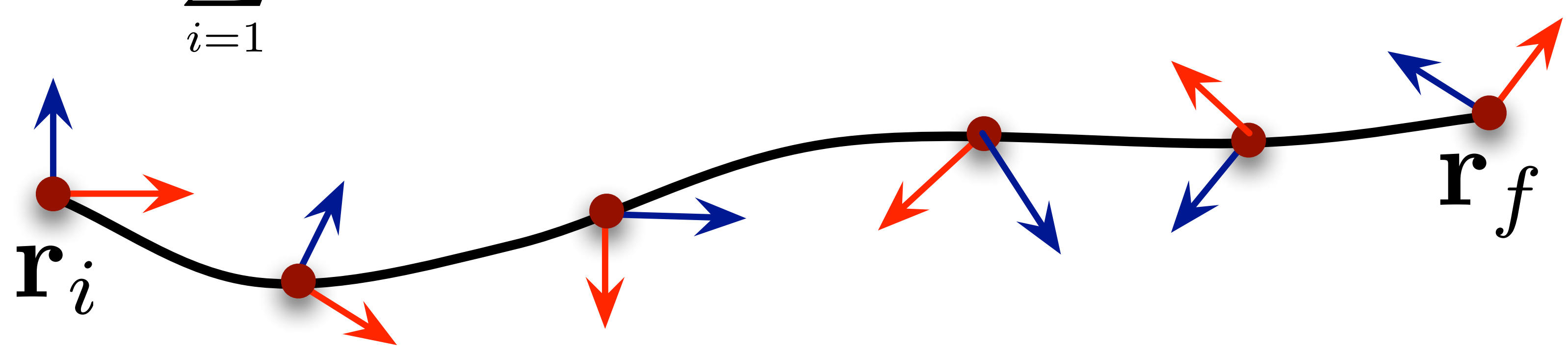

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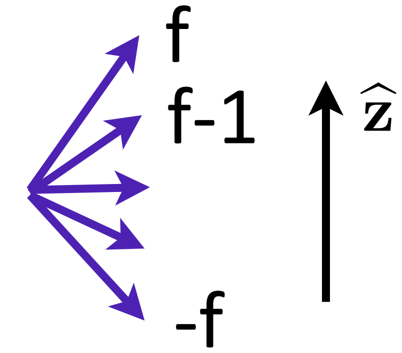

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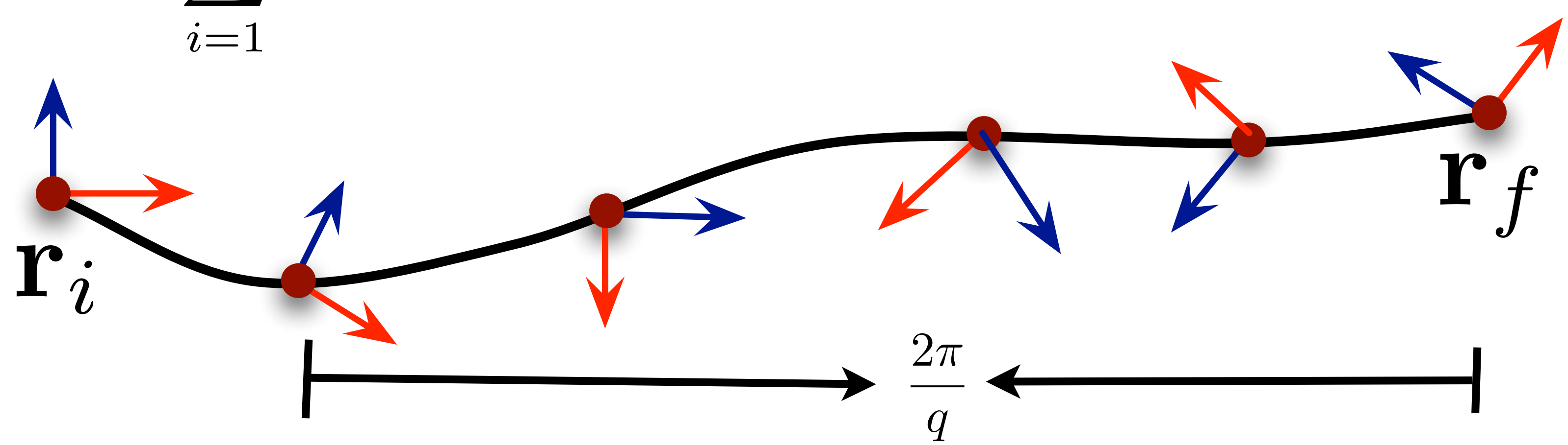

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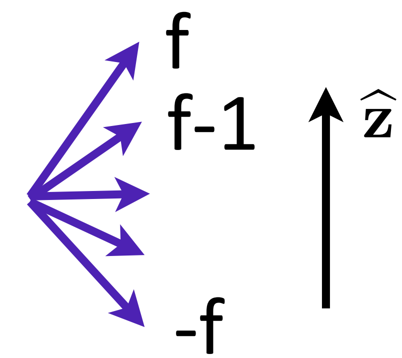

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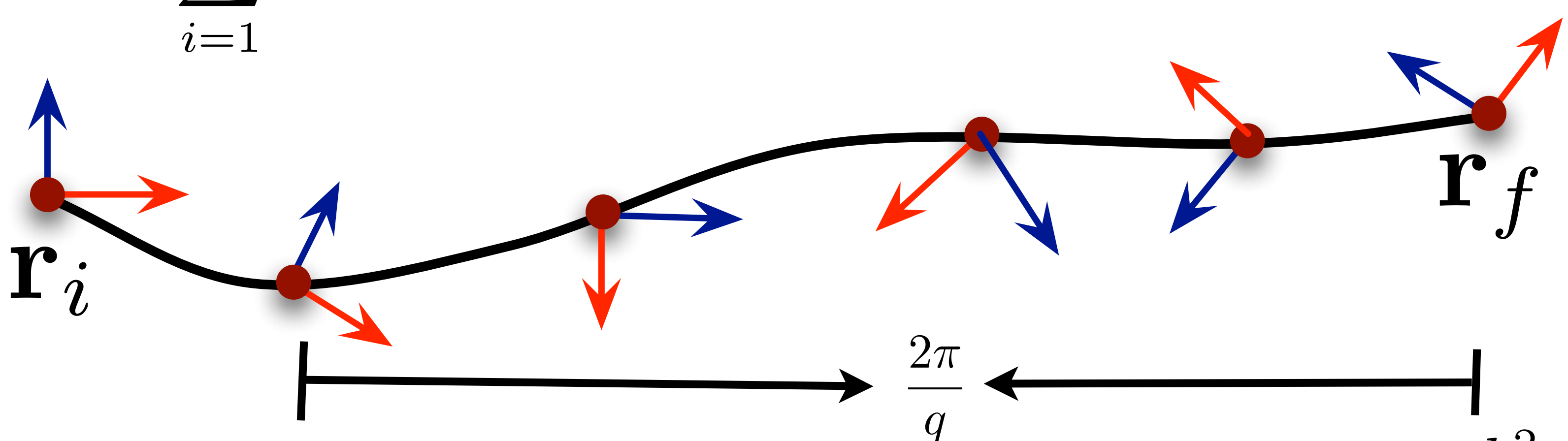

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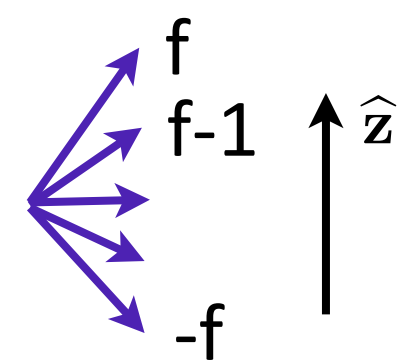

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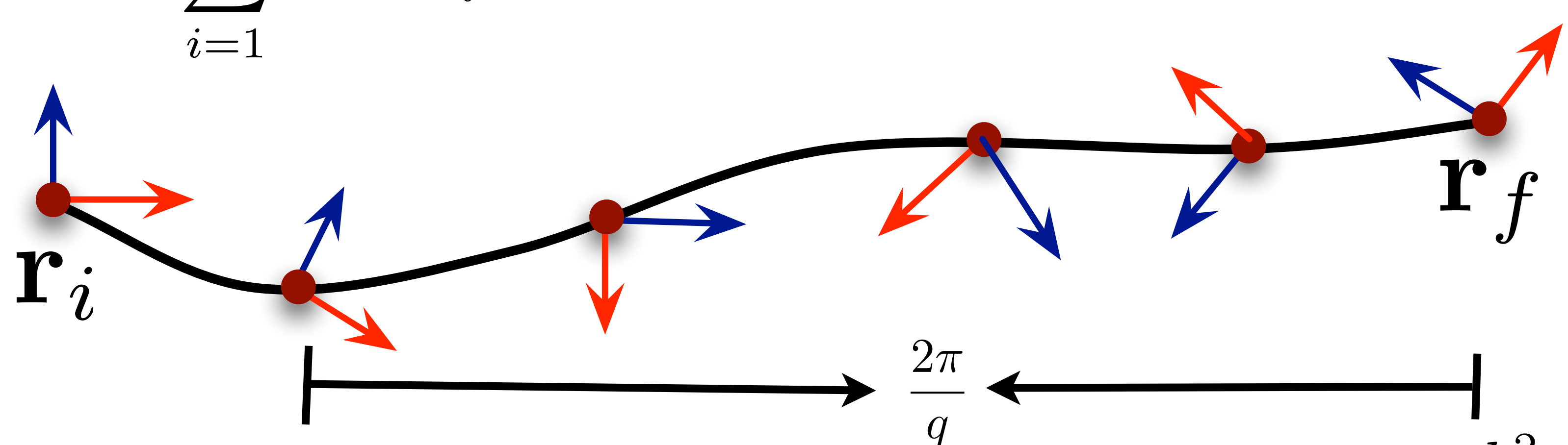

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These L-states will be mixed significantly during the particle motion and we thus obtain an effective low energy manifold, characterized by $\chi_i(r)$,i=1,2,...,L.

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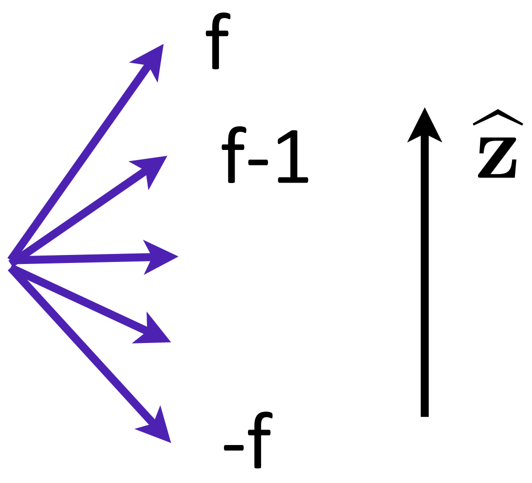

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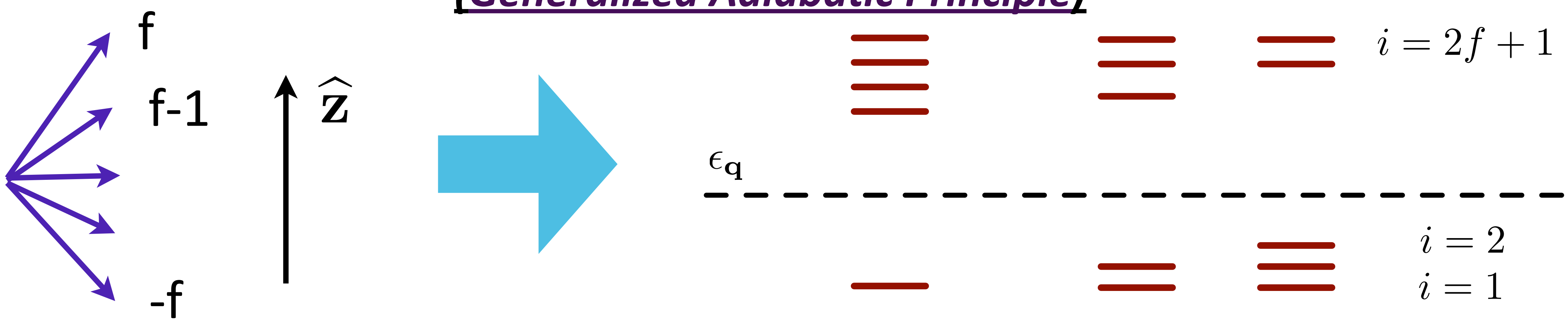

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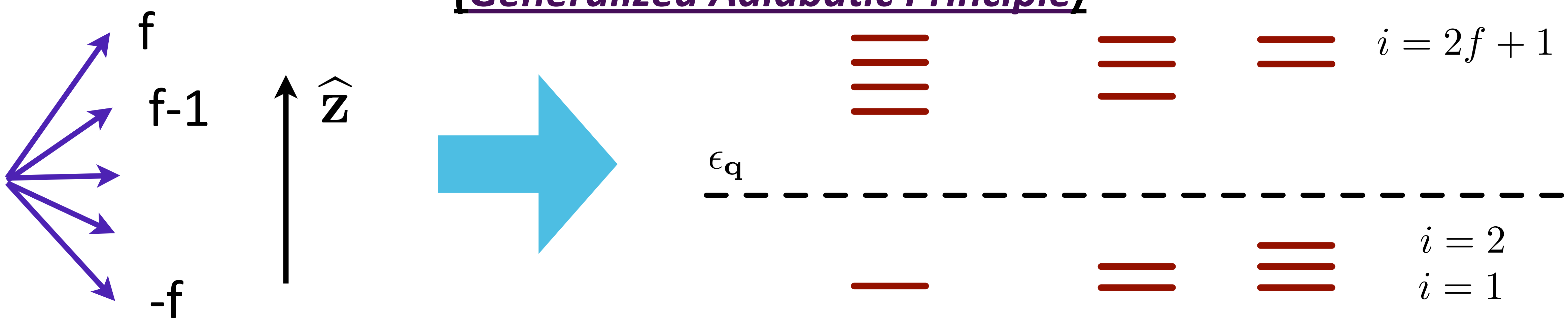

Abelian

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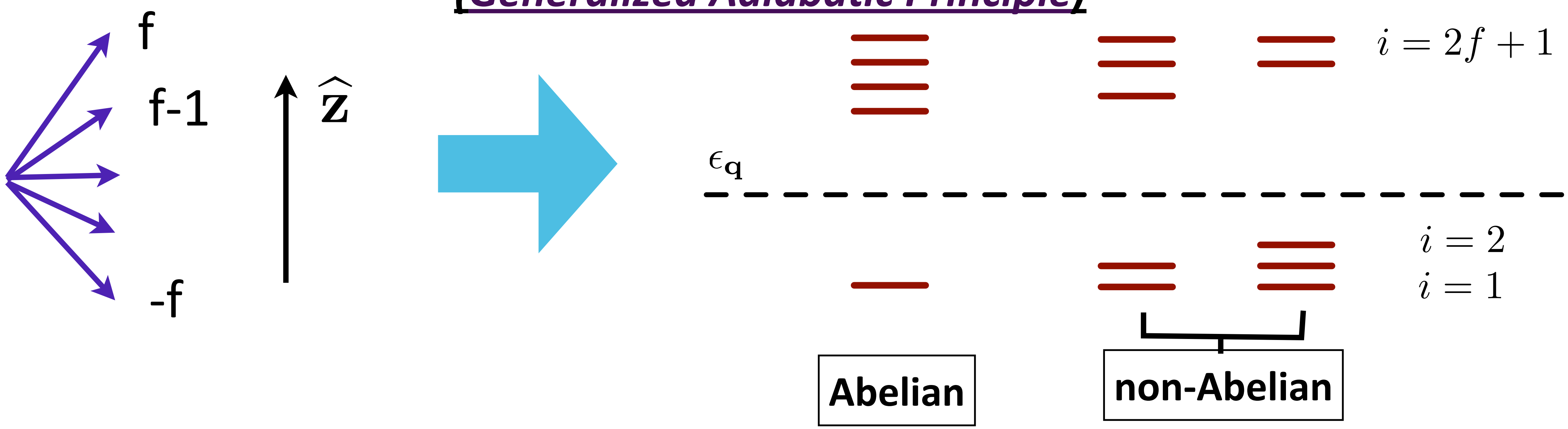

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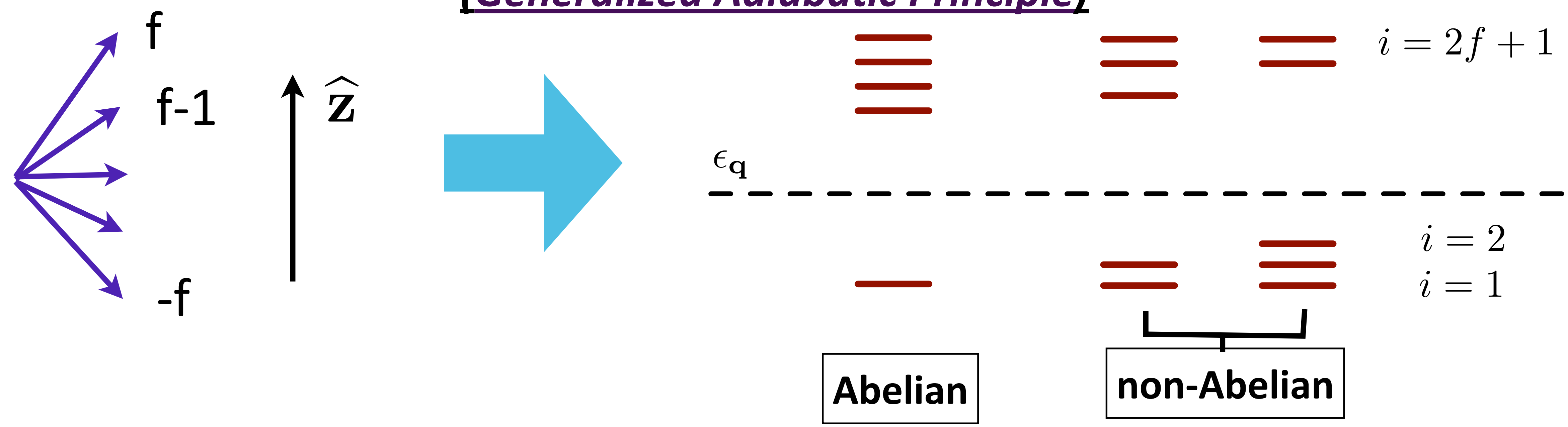

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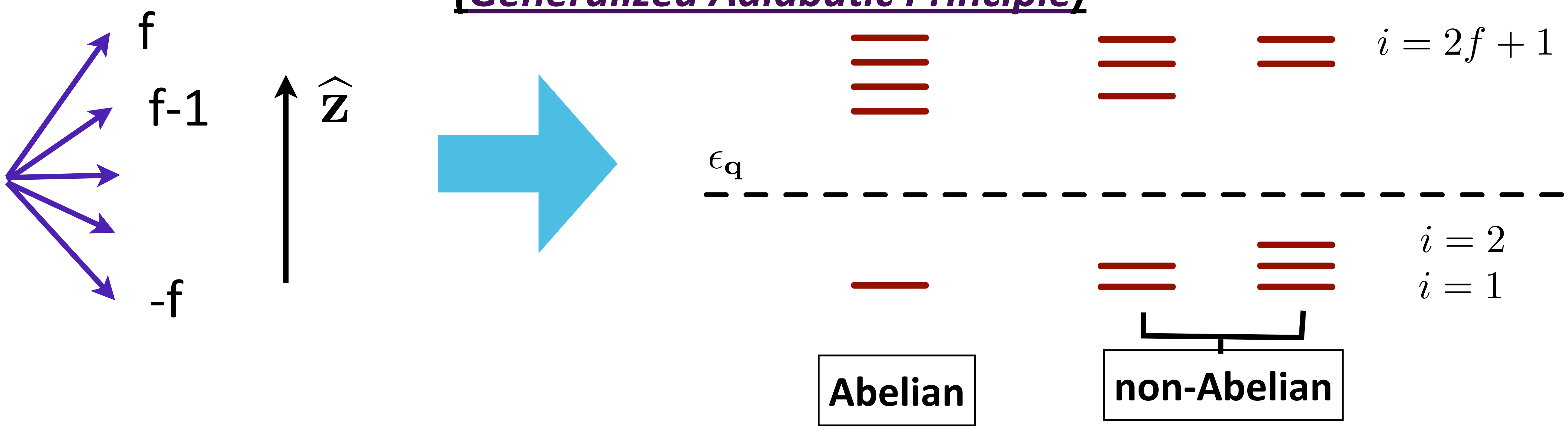

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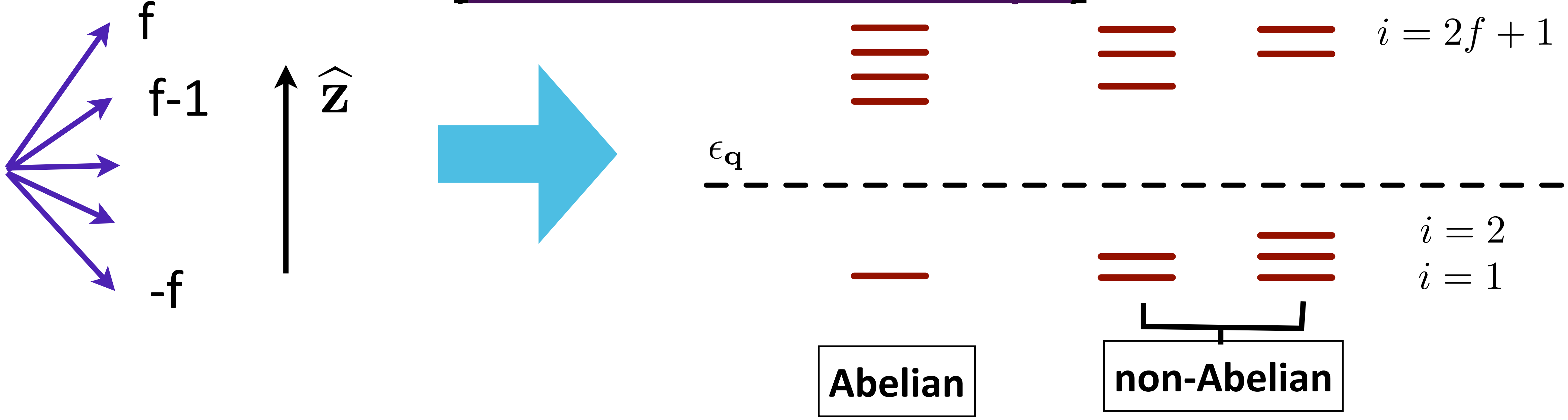

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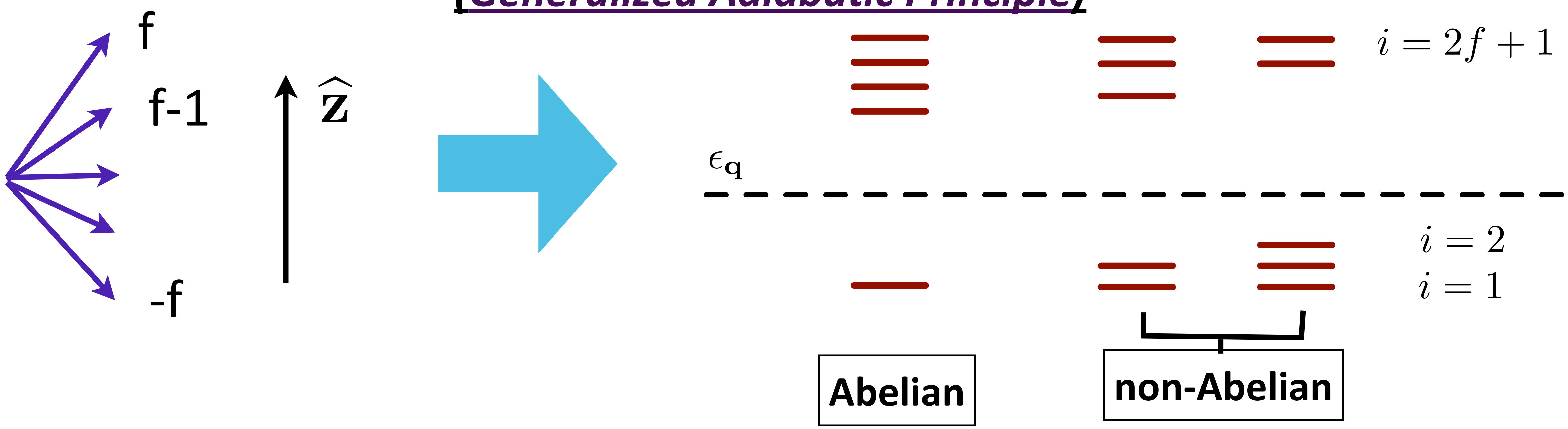

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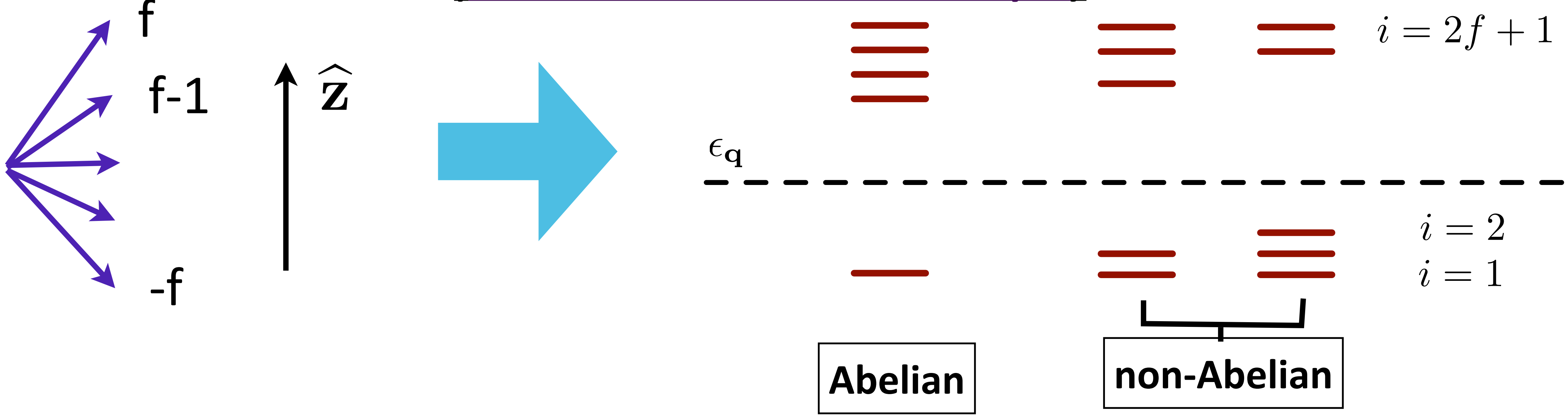

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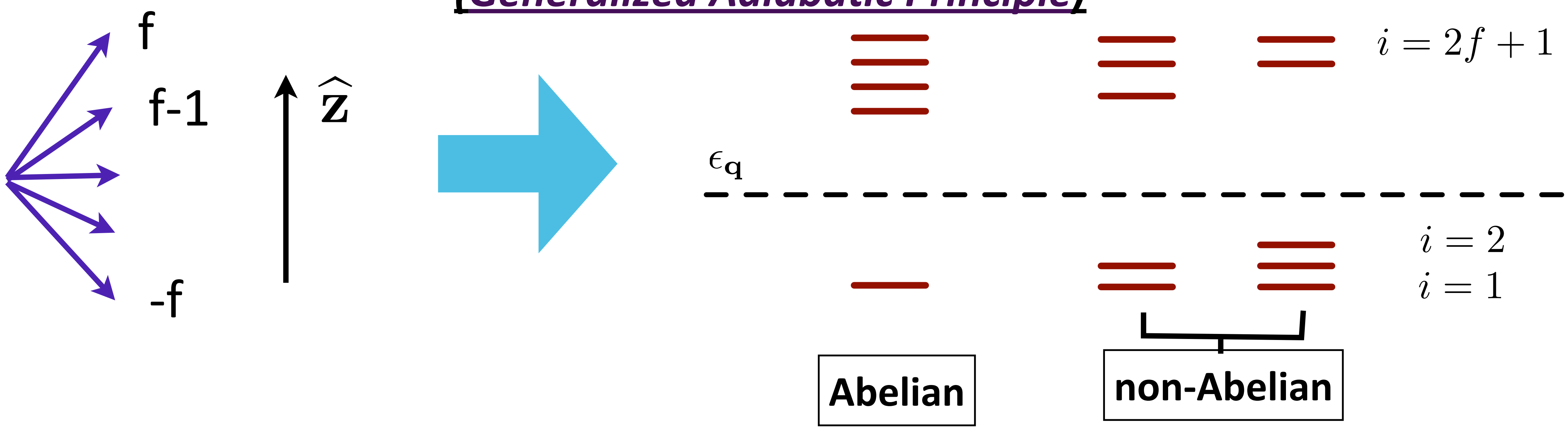

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If different components of $\mathbf{A}_{ij}$ do not commute → Non-abelian !

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A specific realization in NIST of the *__Generalized Adiabatic Principle:__*

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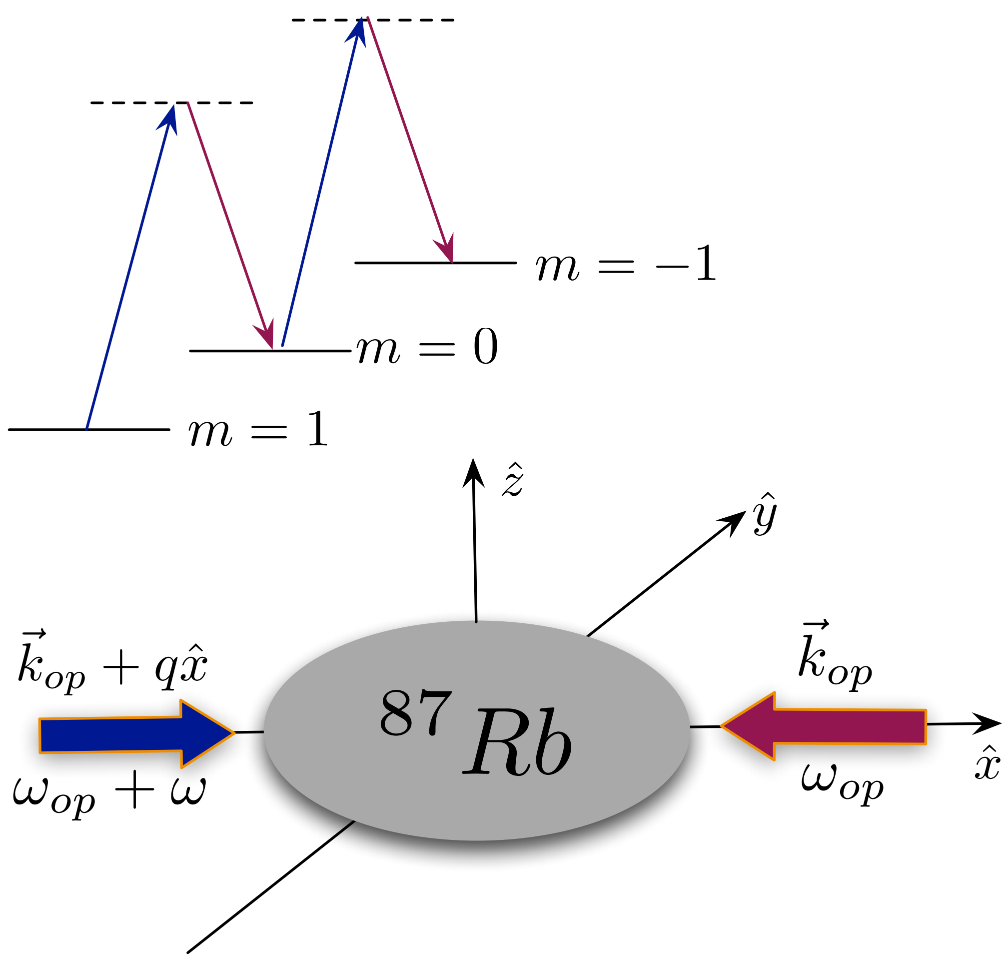

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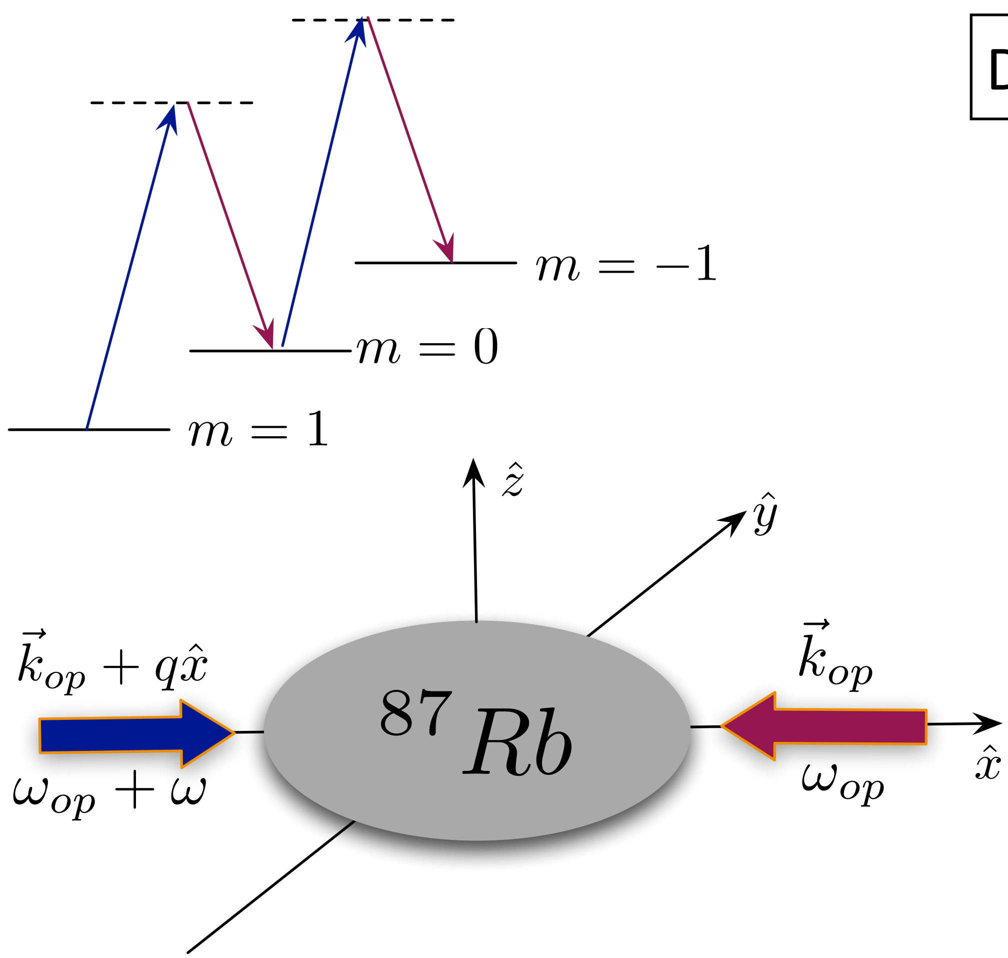

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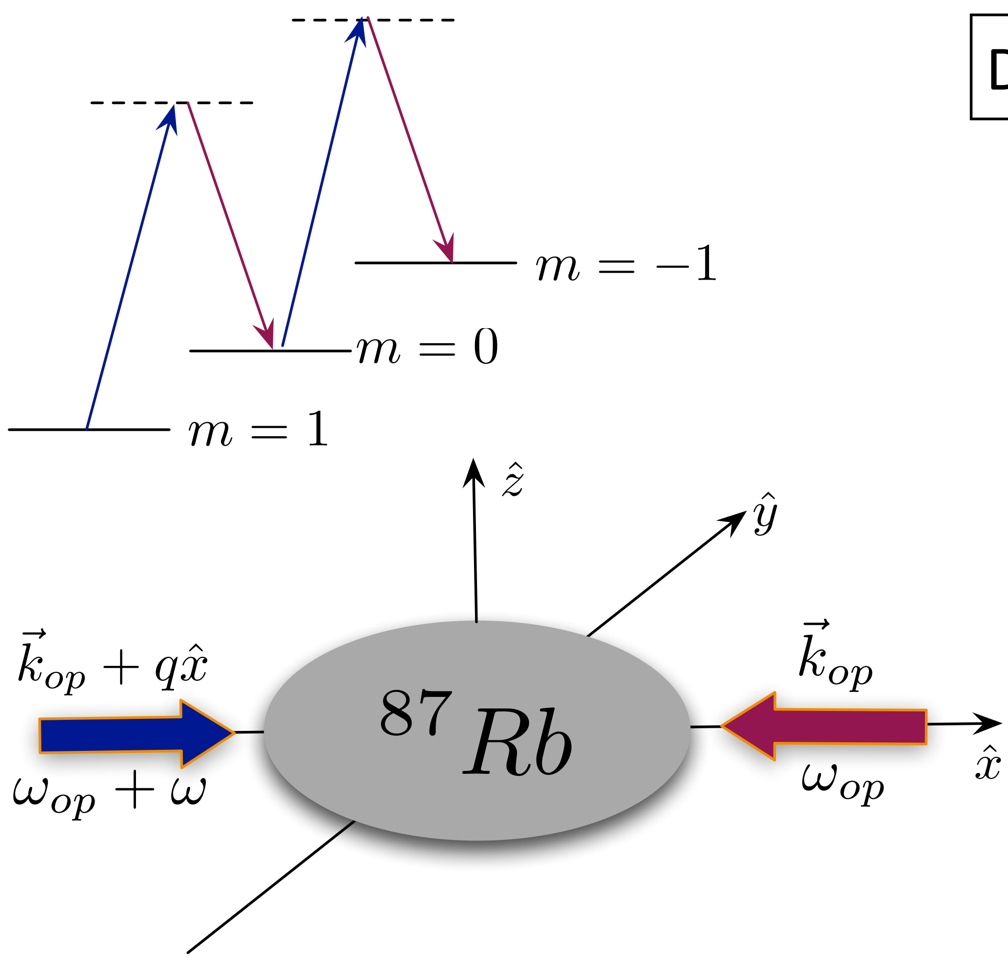

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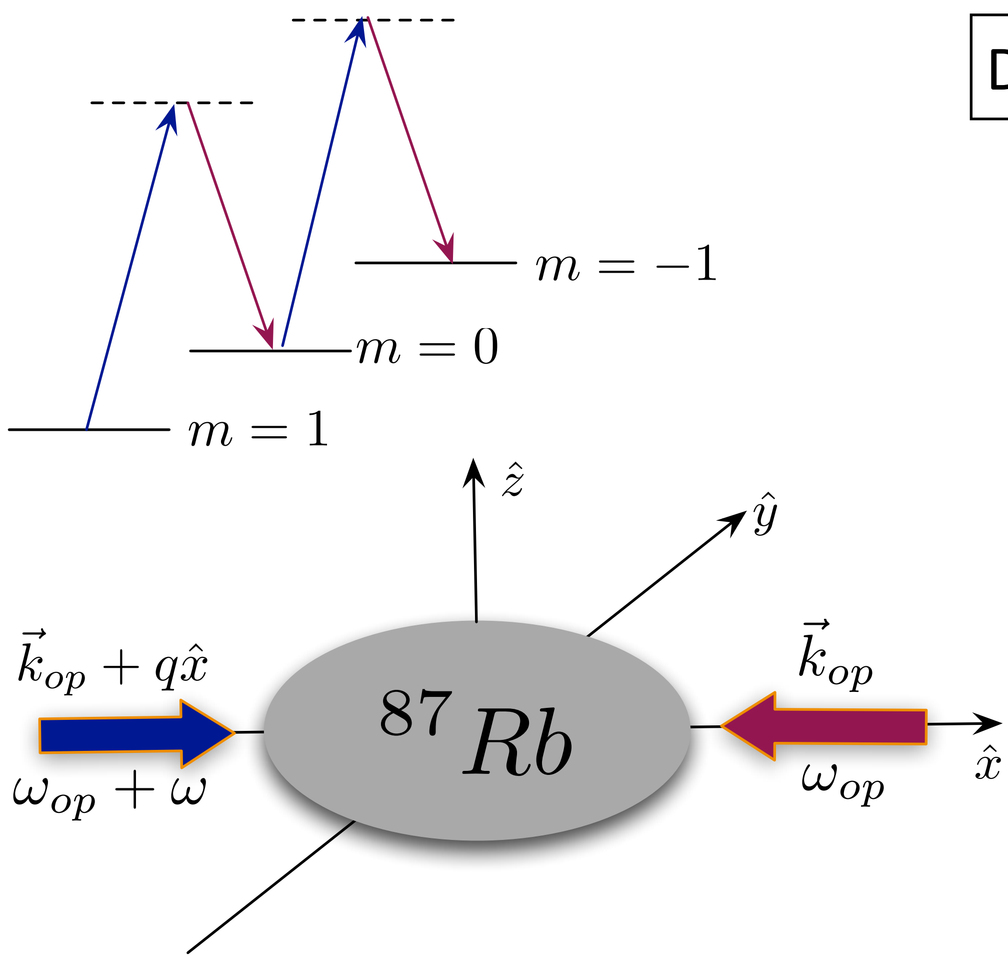

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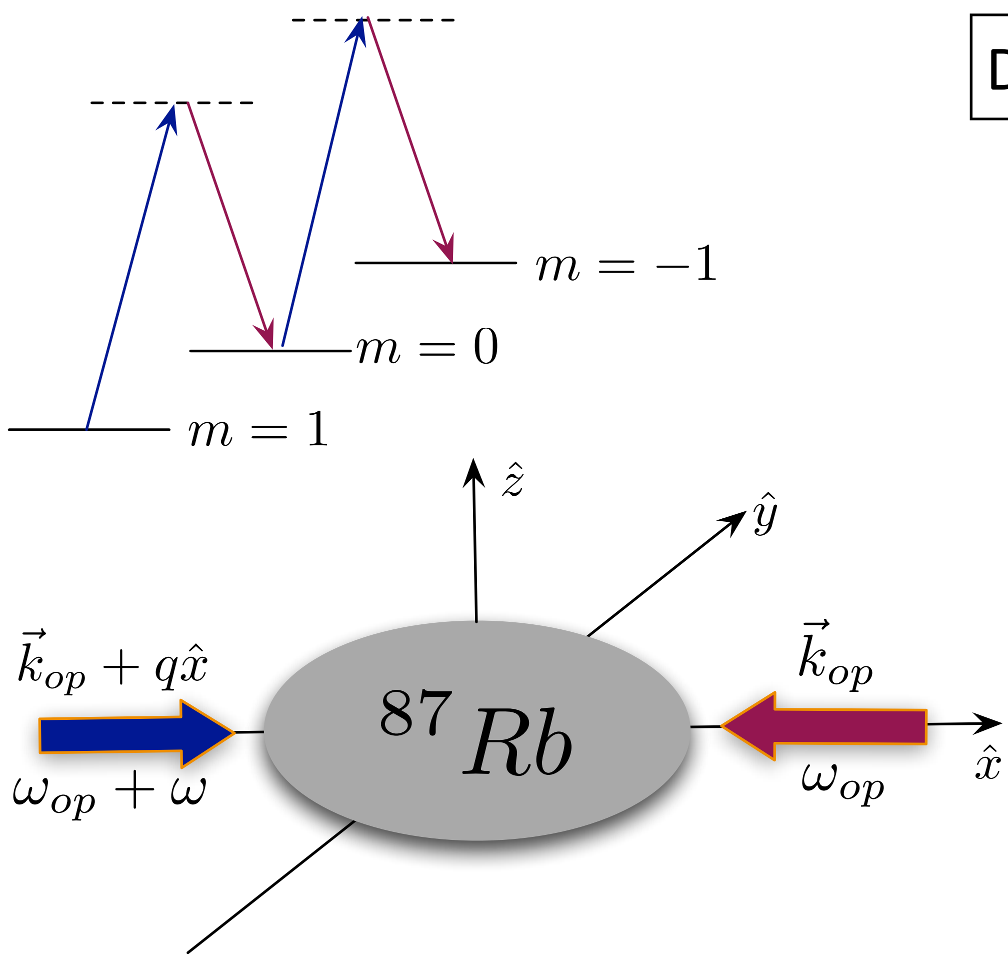

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This Hamiltonian describes <u>both</u> abelian and non-abelian gauge fields
(spin-orbit coupling is a particular case of constant non-abelian gauge field)

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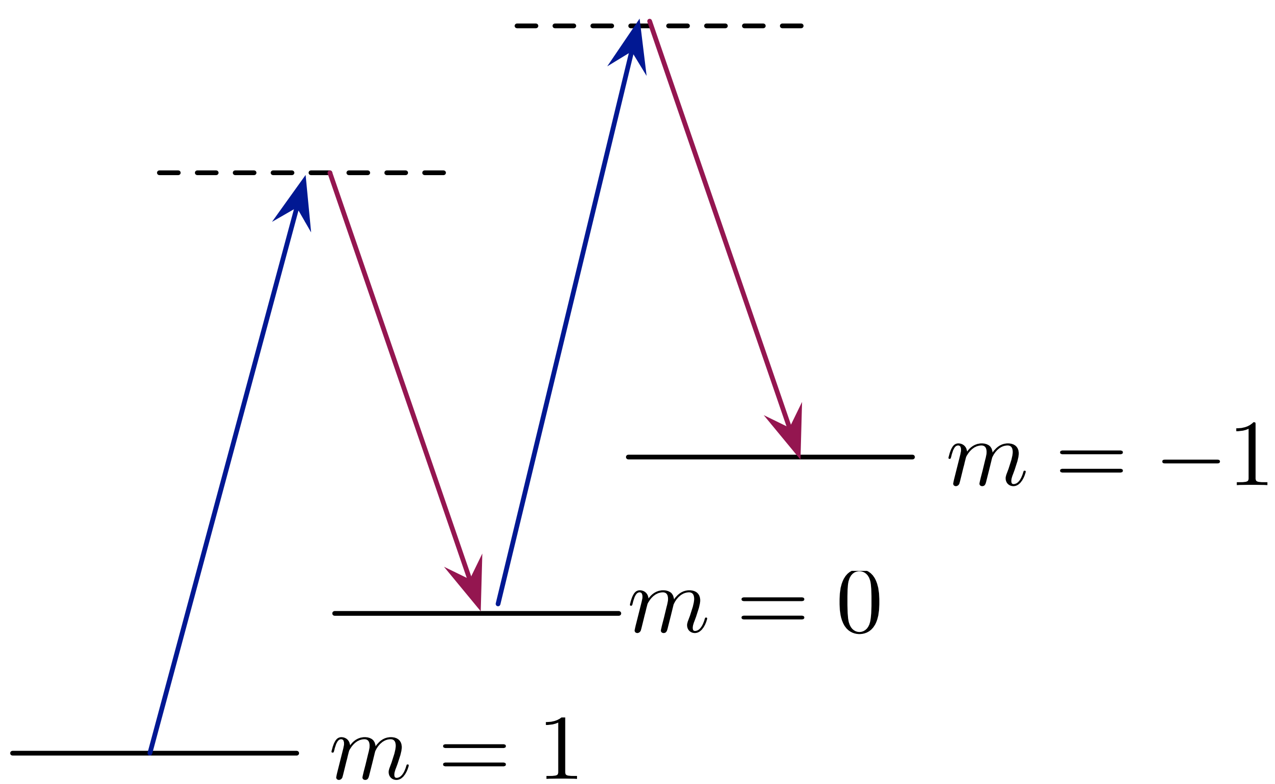

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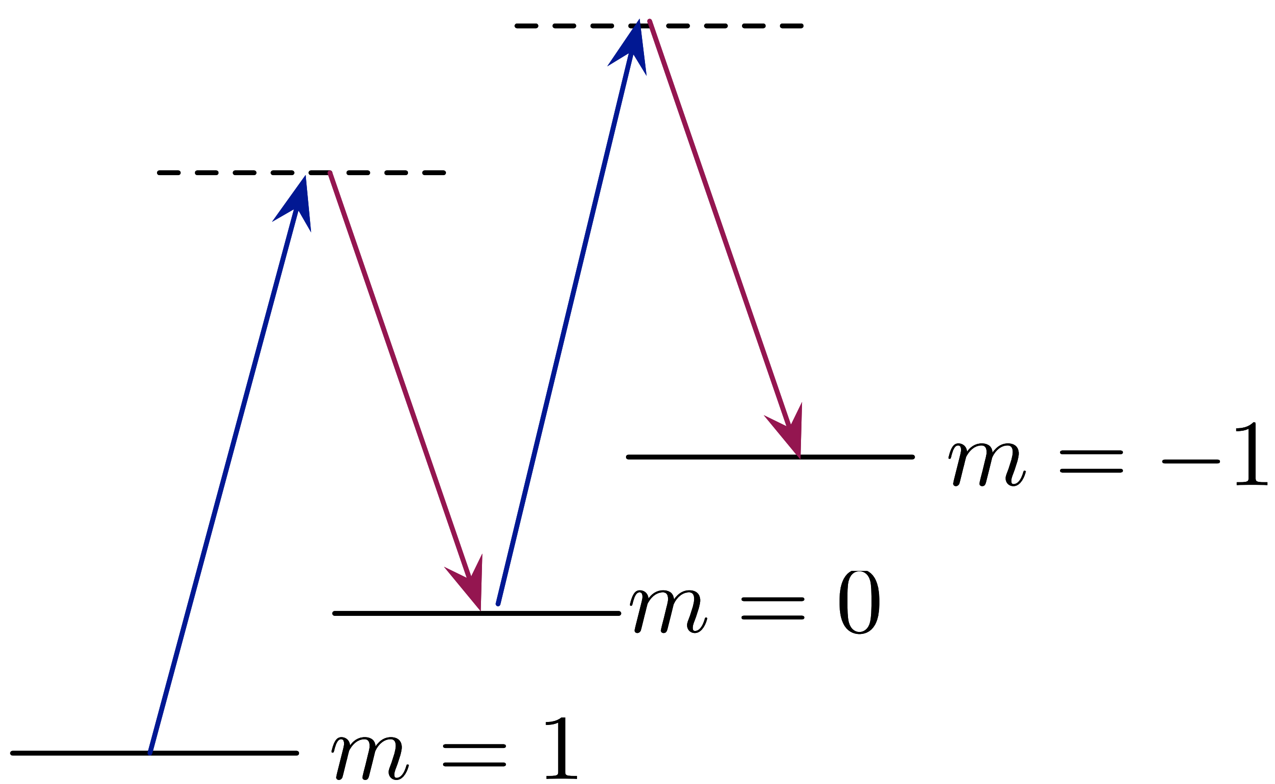

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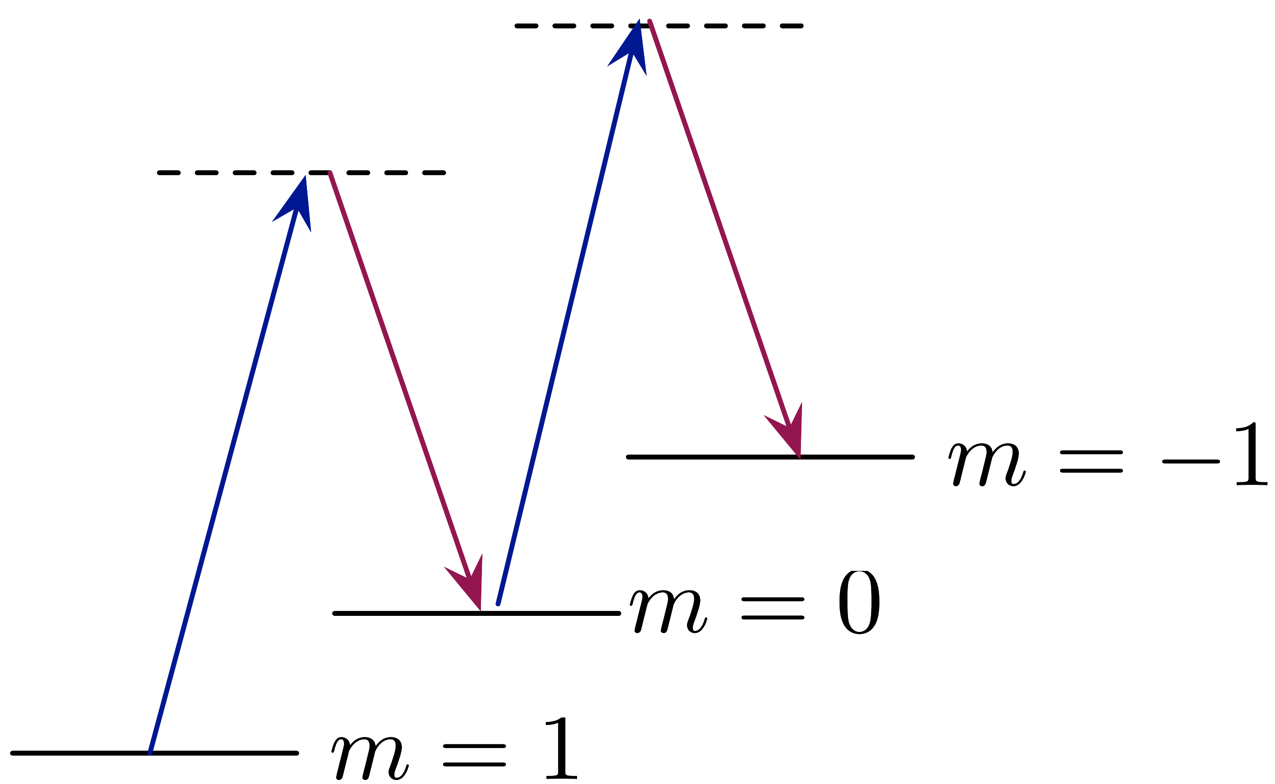

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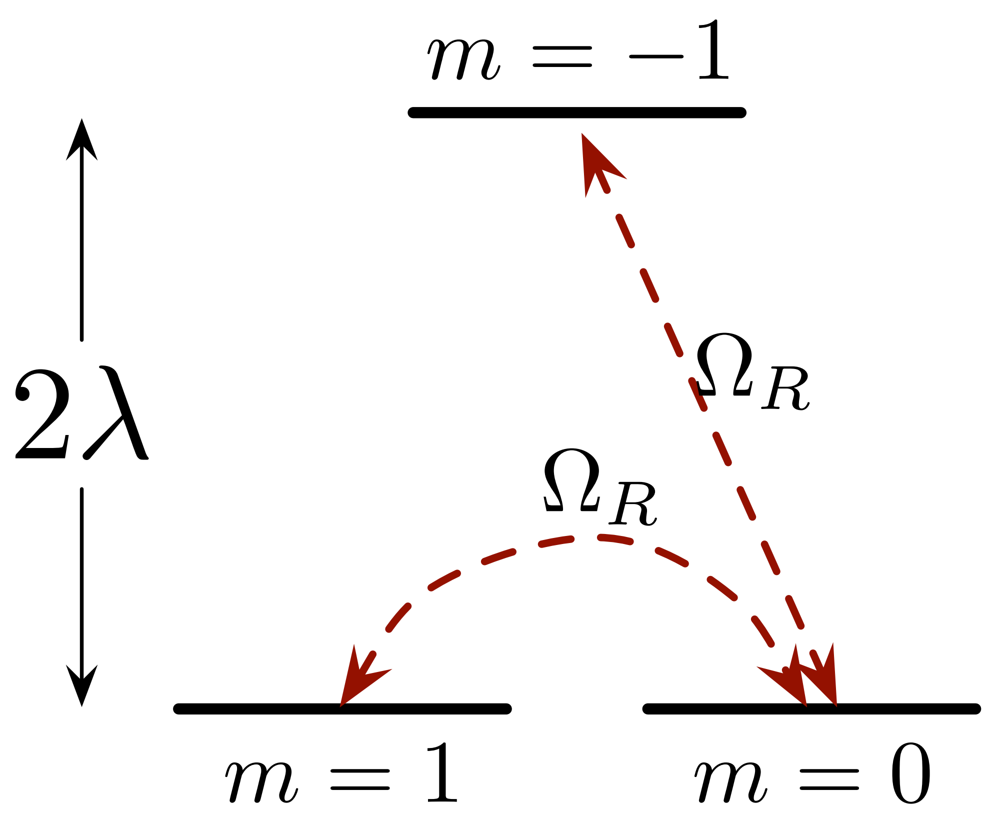

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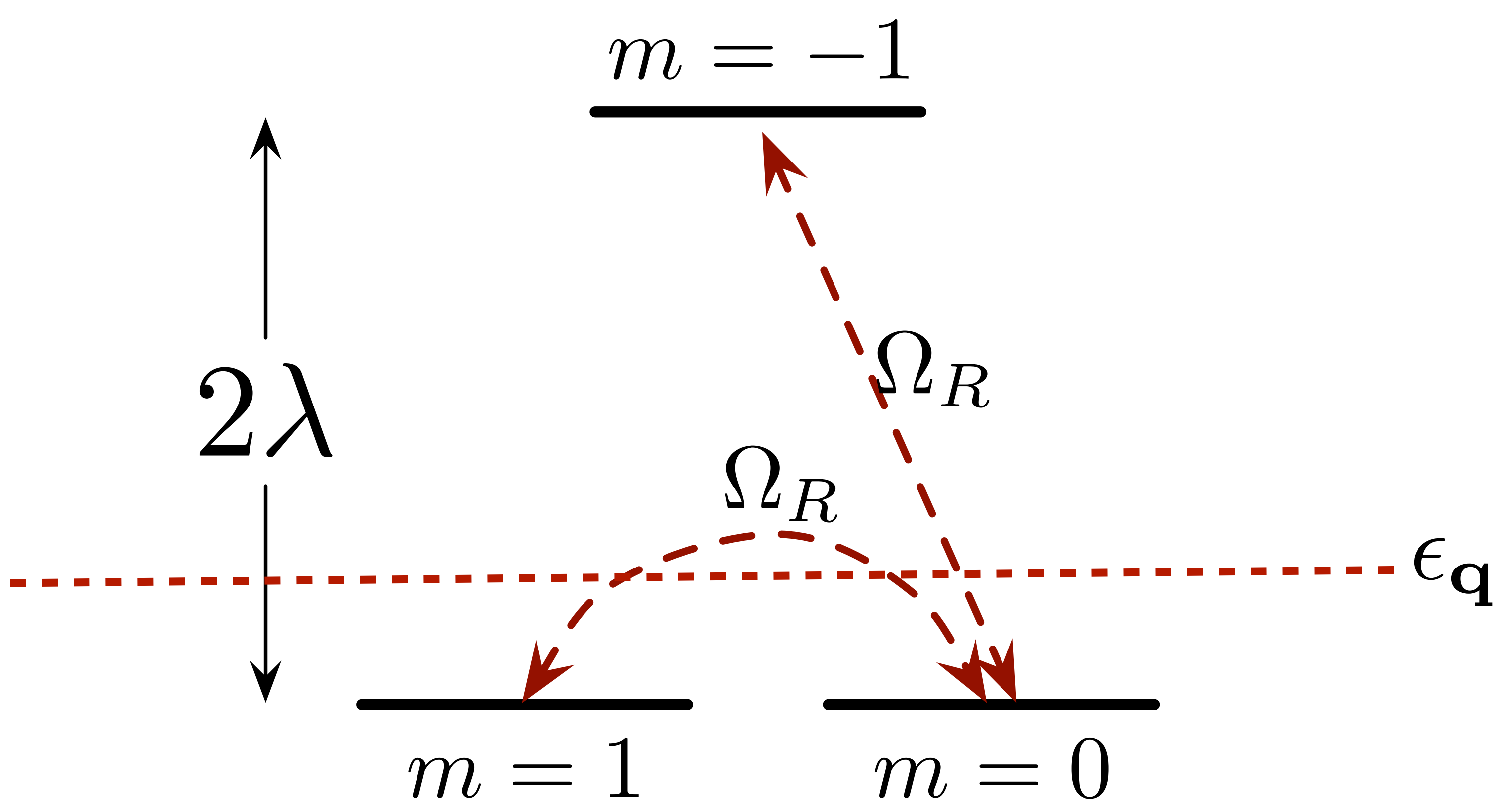

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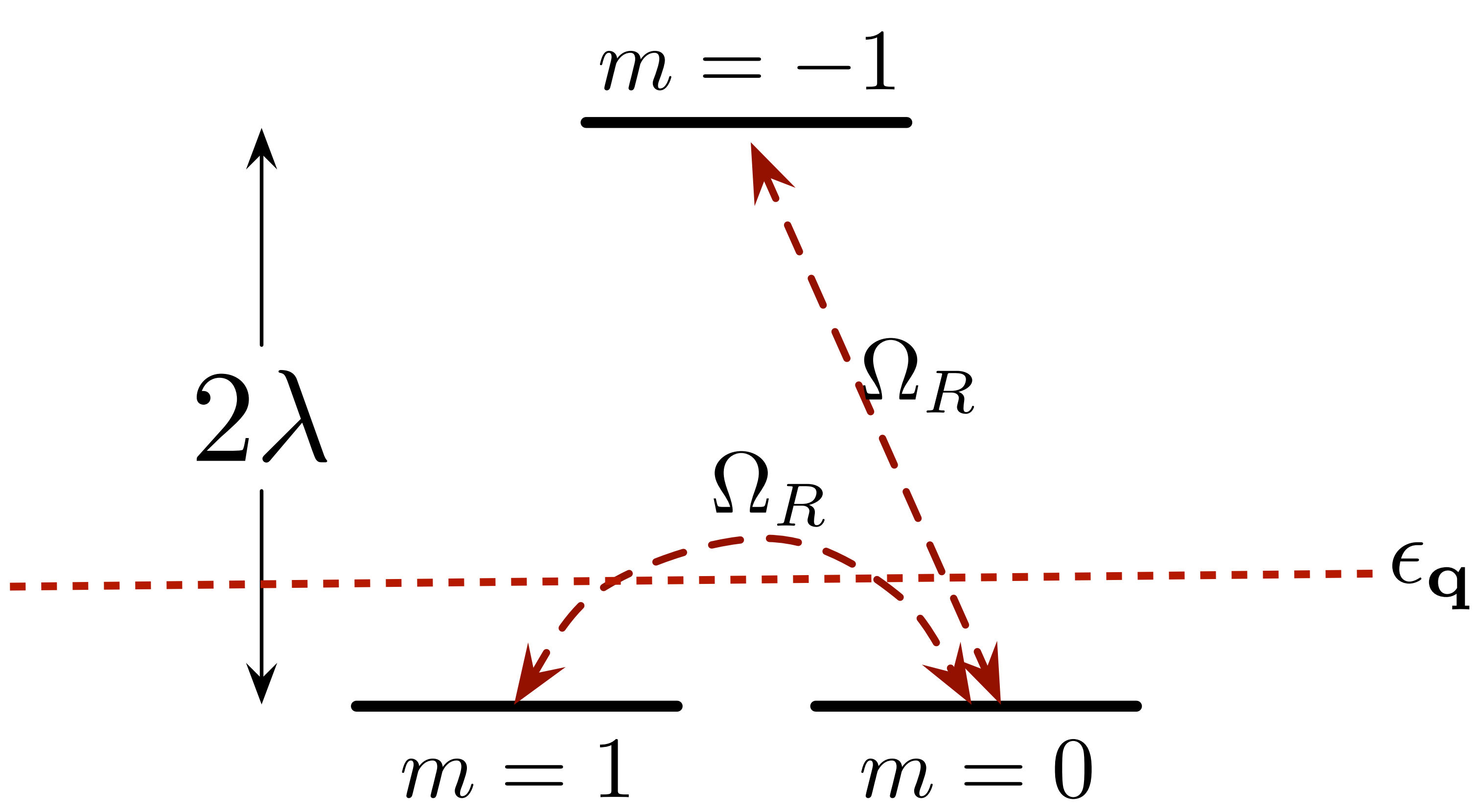

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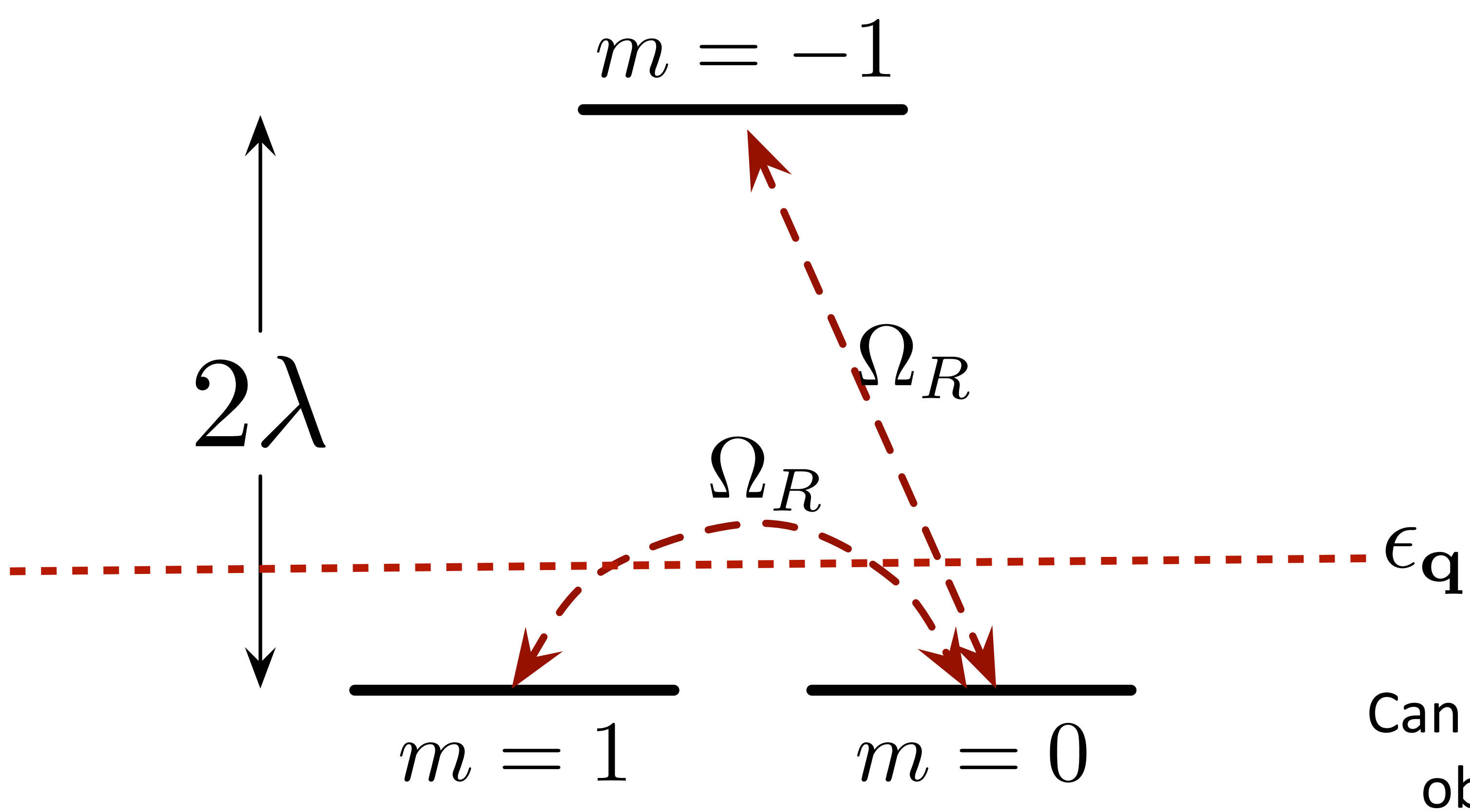

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SU(2) non-abelian gauge field

$$\lambda \gg \Omega_R$$

Can neglect the coupling to m=-1 state and we obtain an system with two internal states!

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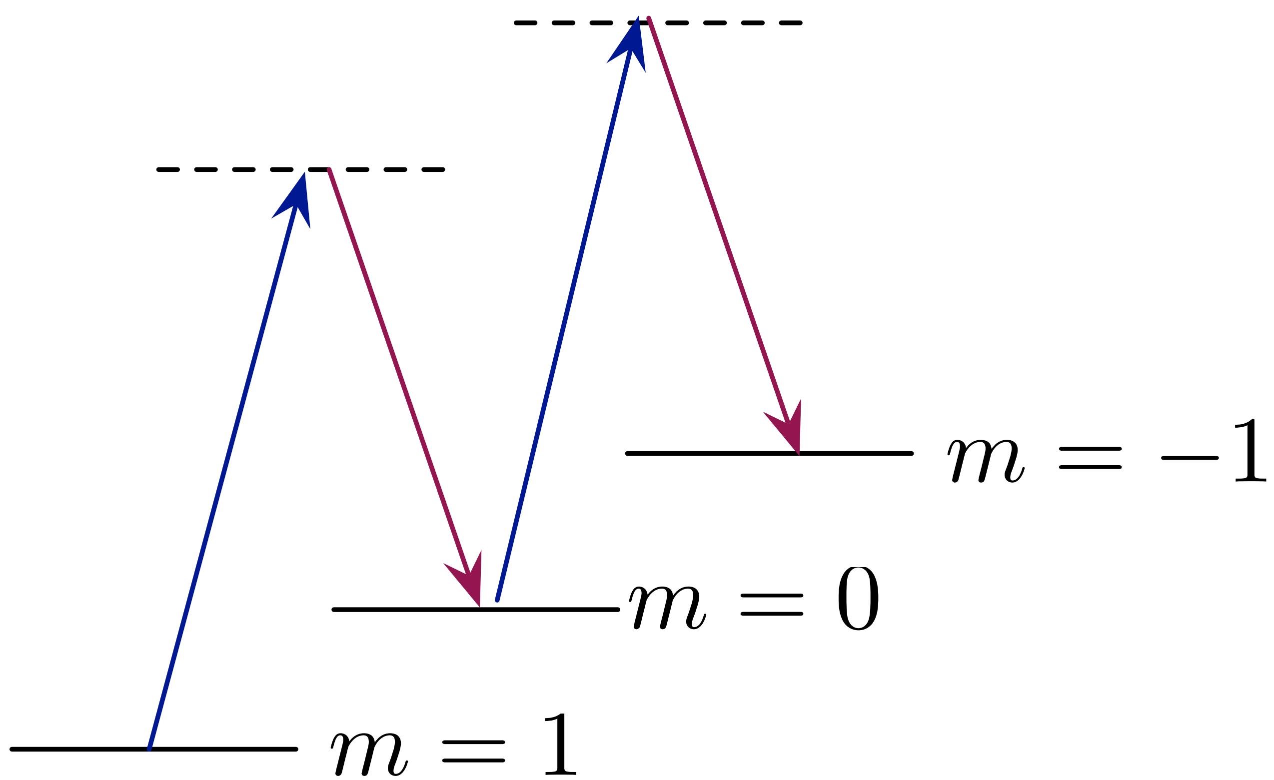

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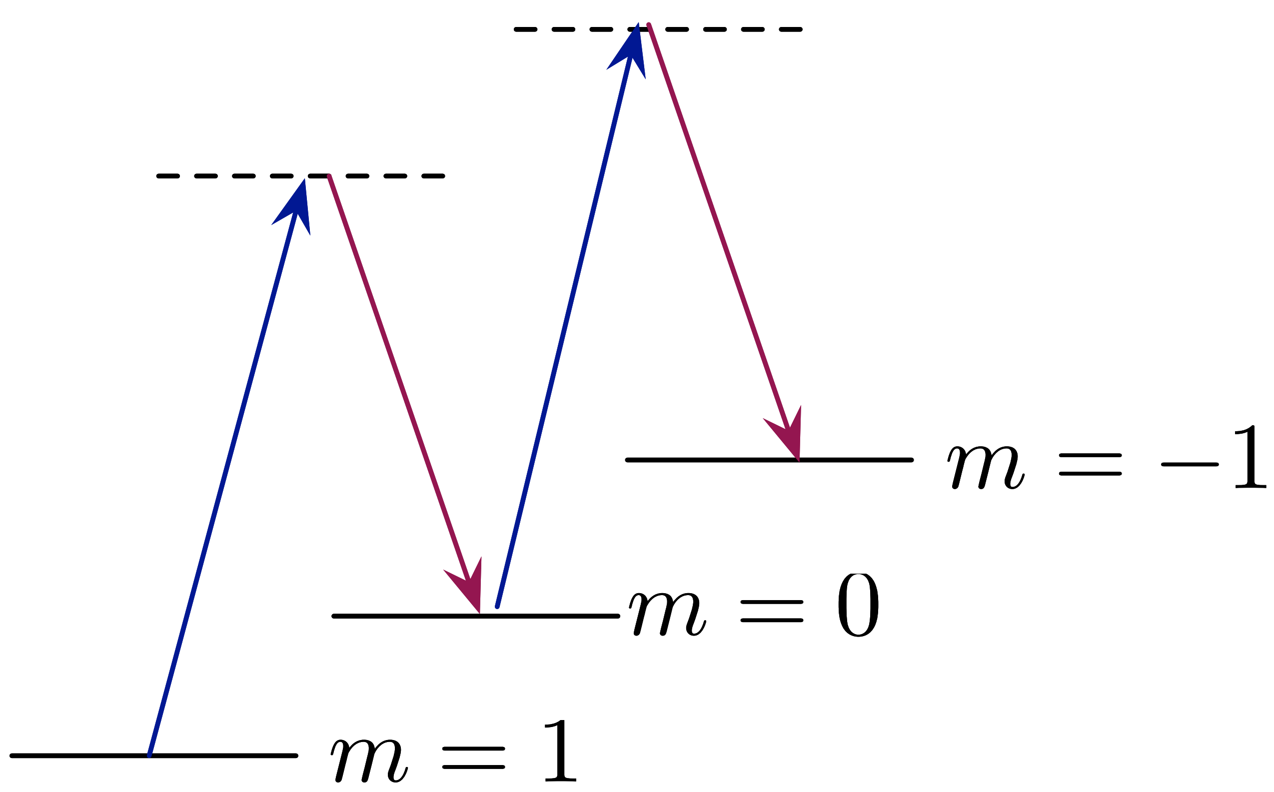

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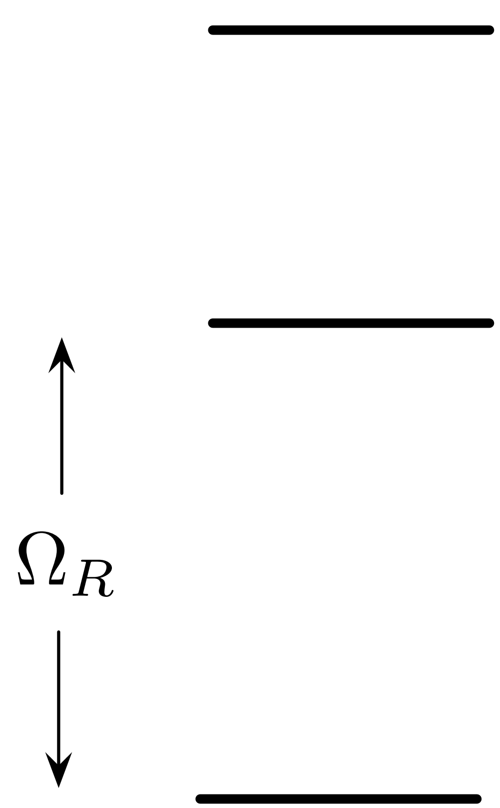

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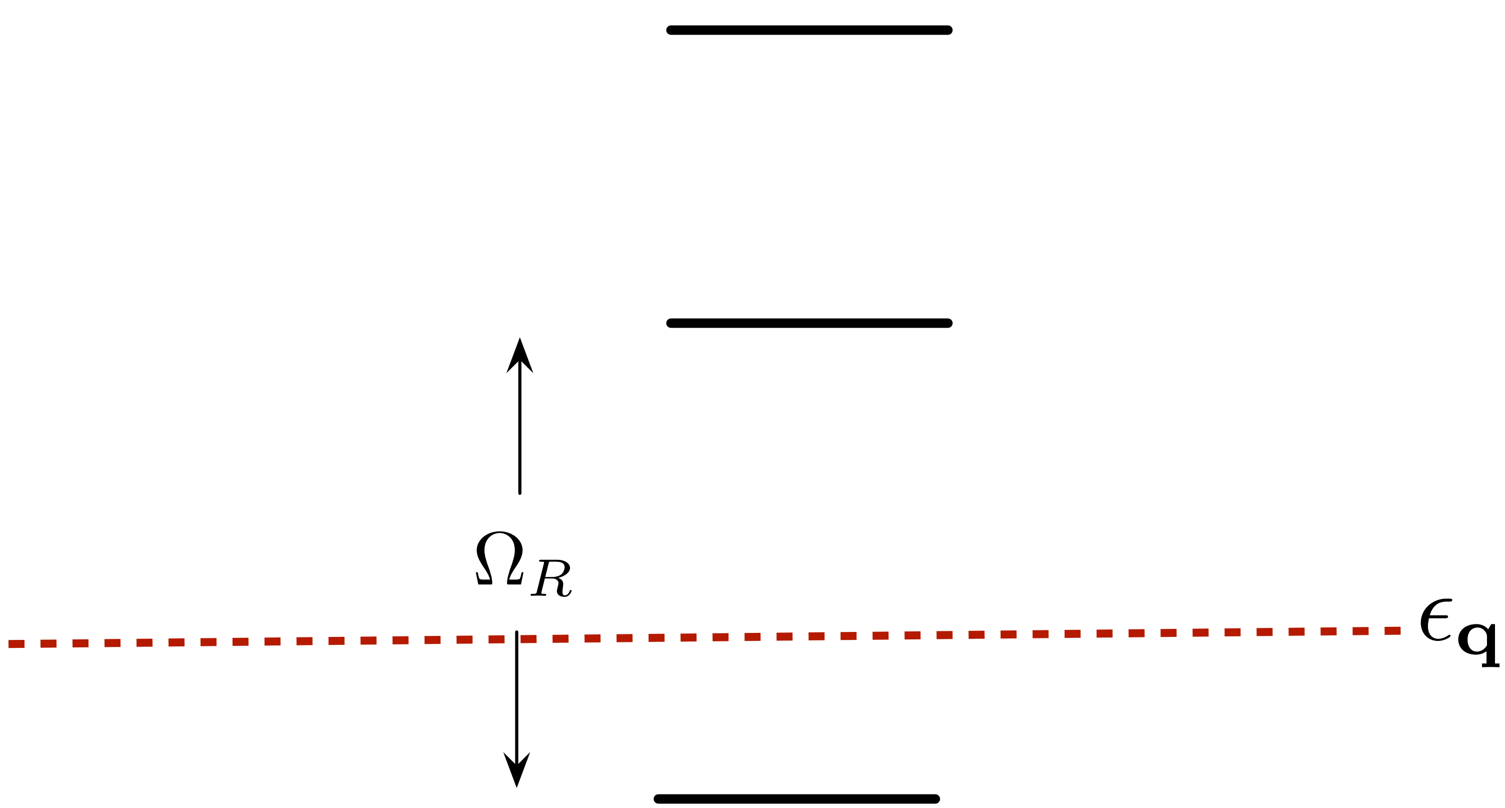

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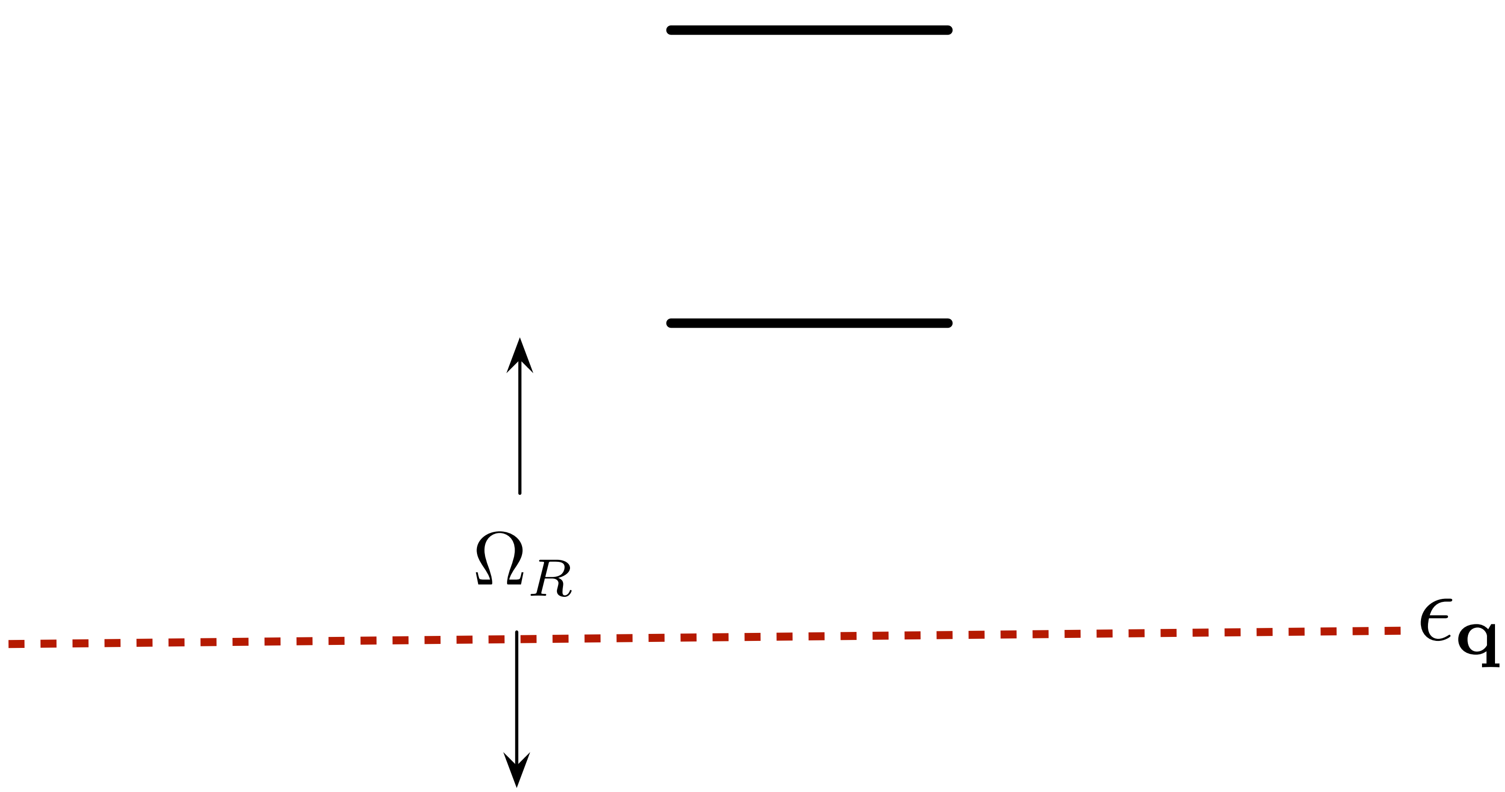

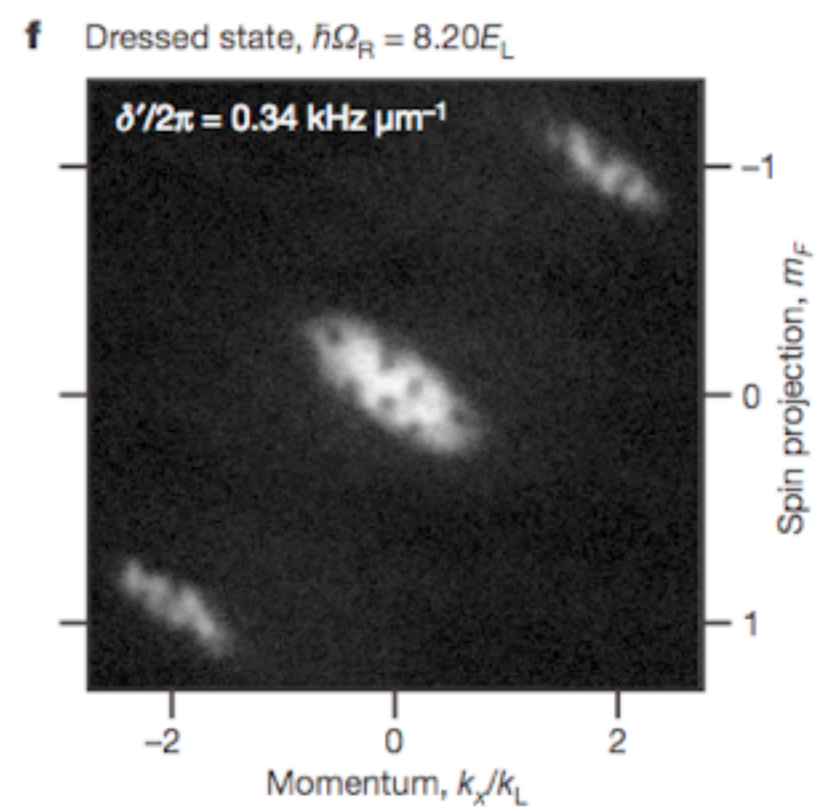

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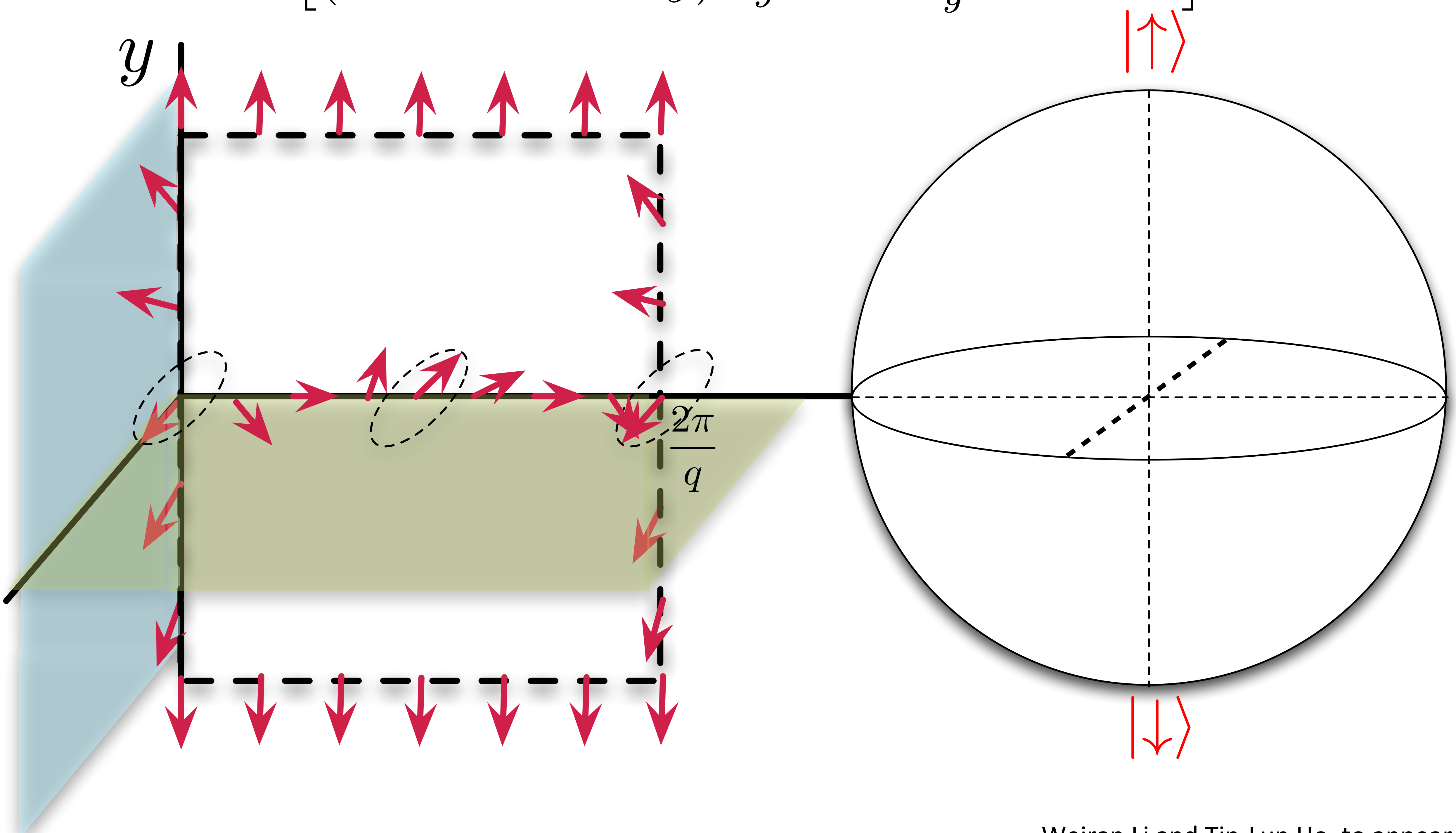

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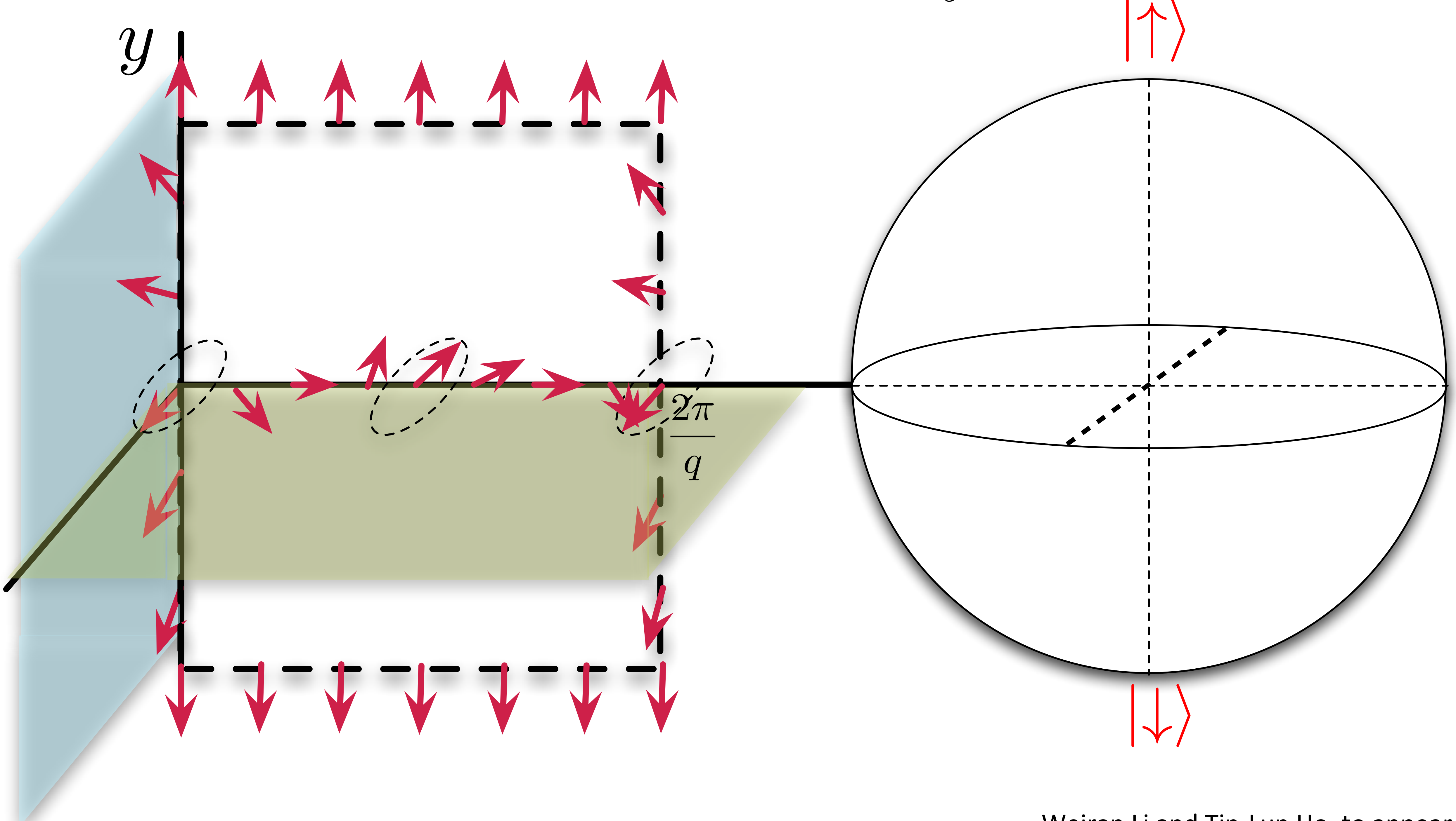

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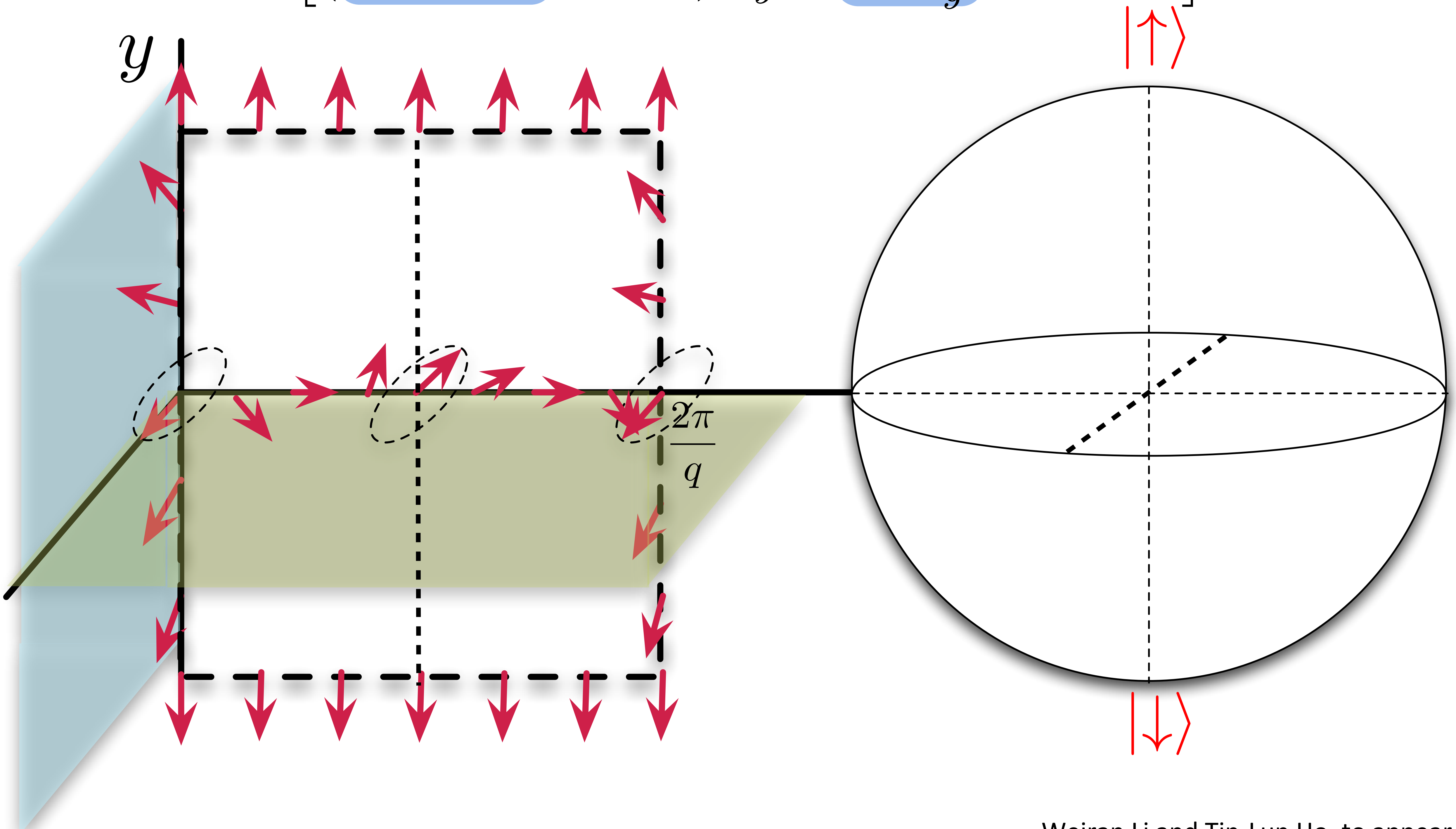

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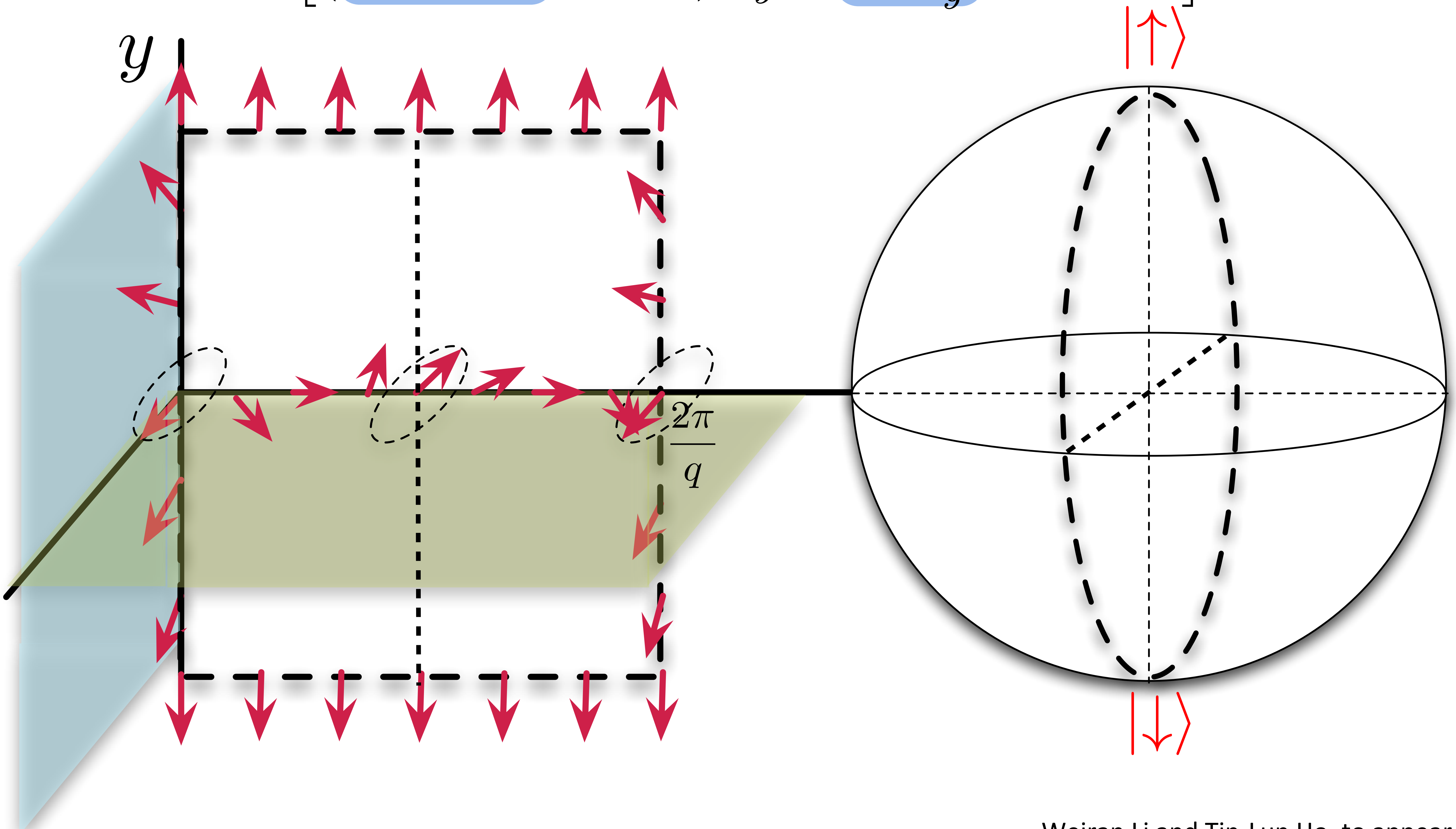

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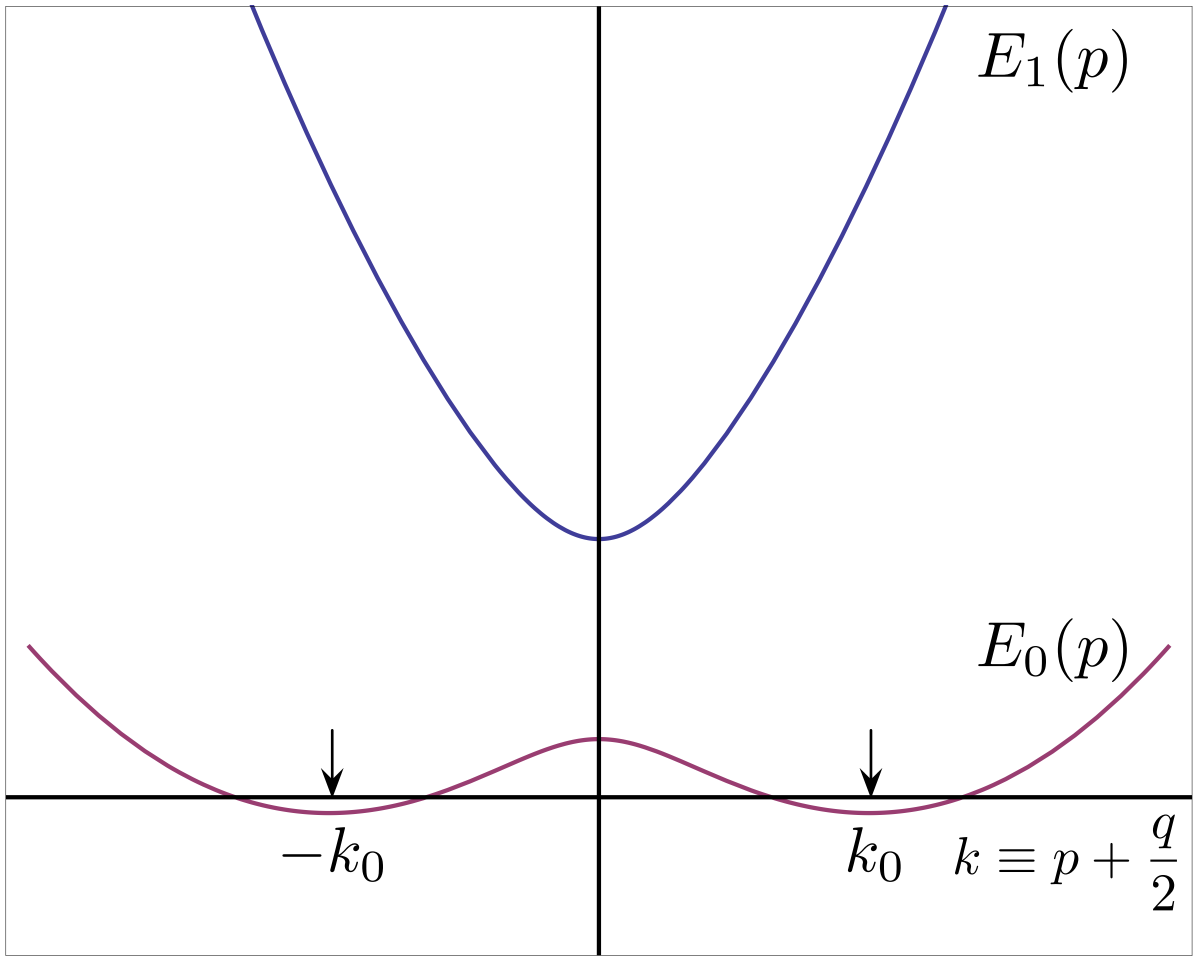

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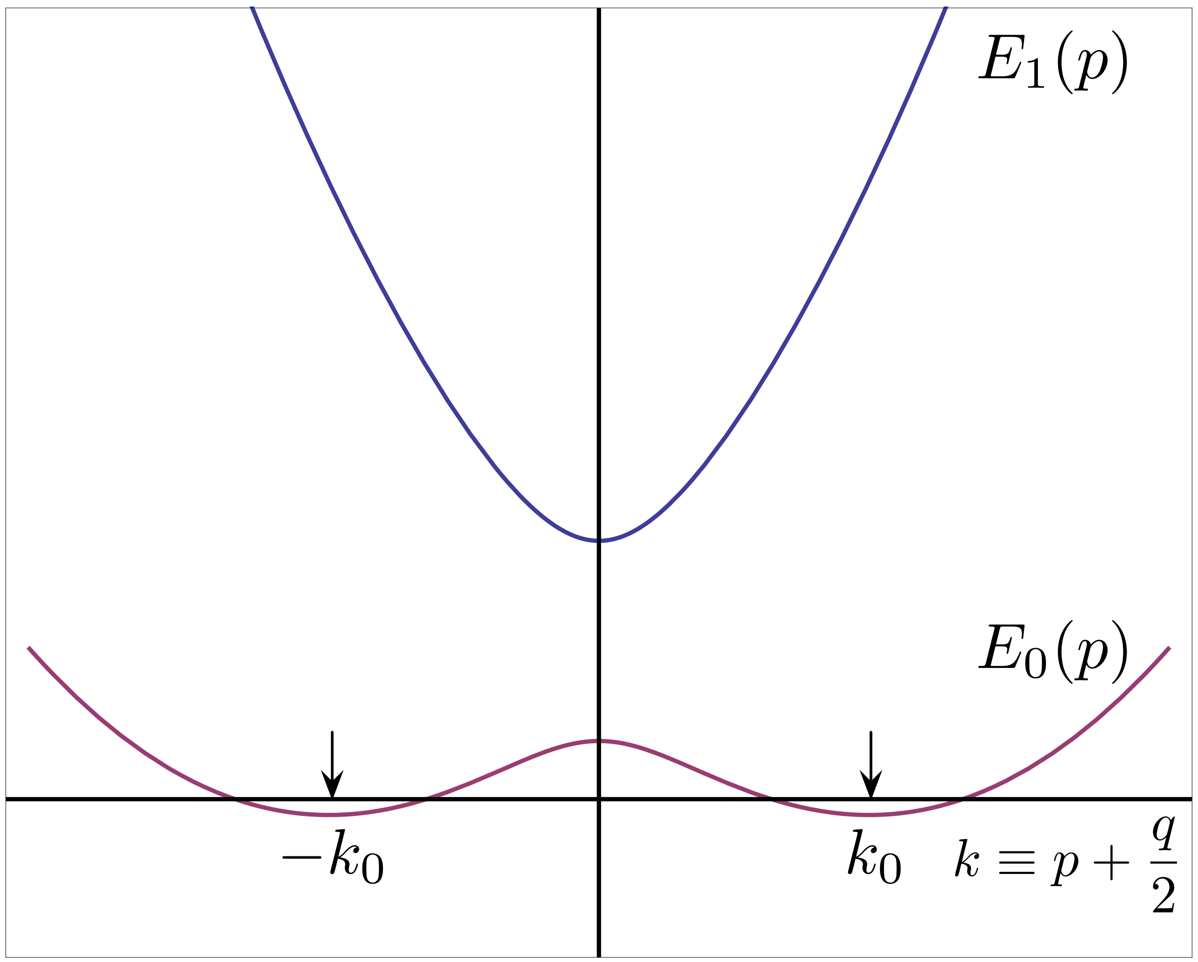

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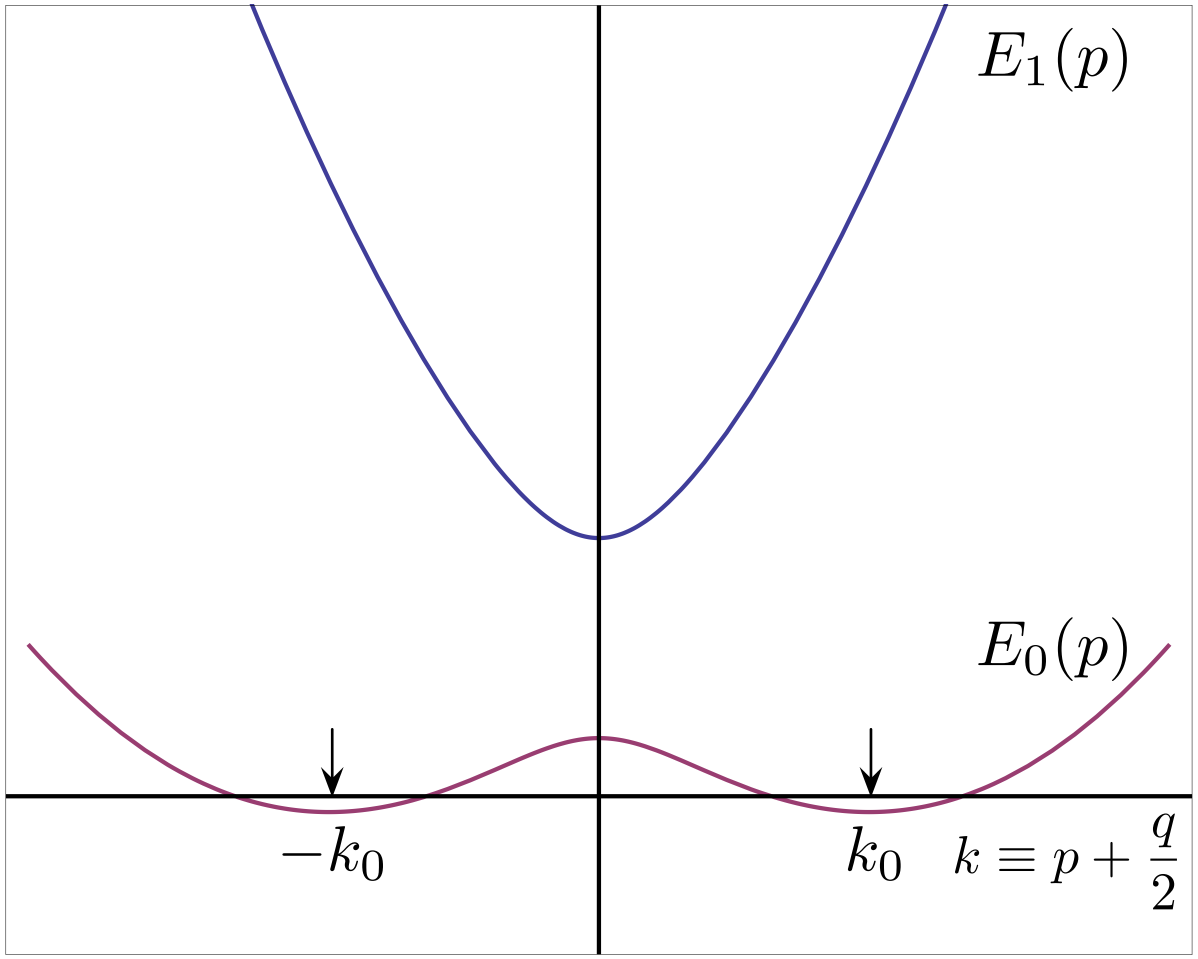

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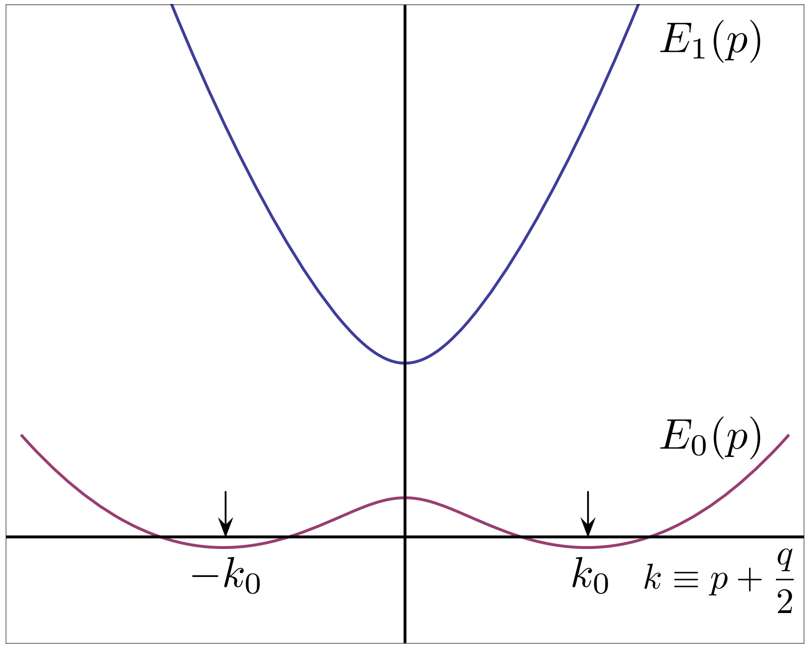
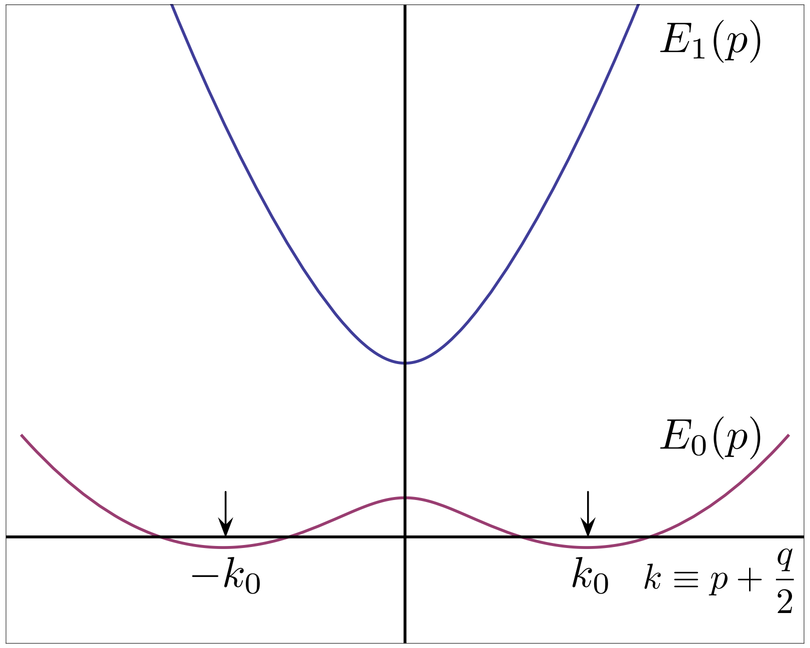


STRUCTURE OF THE CONDENSATE I

What is the Gross-Pitaevskii ansatz?



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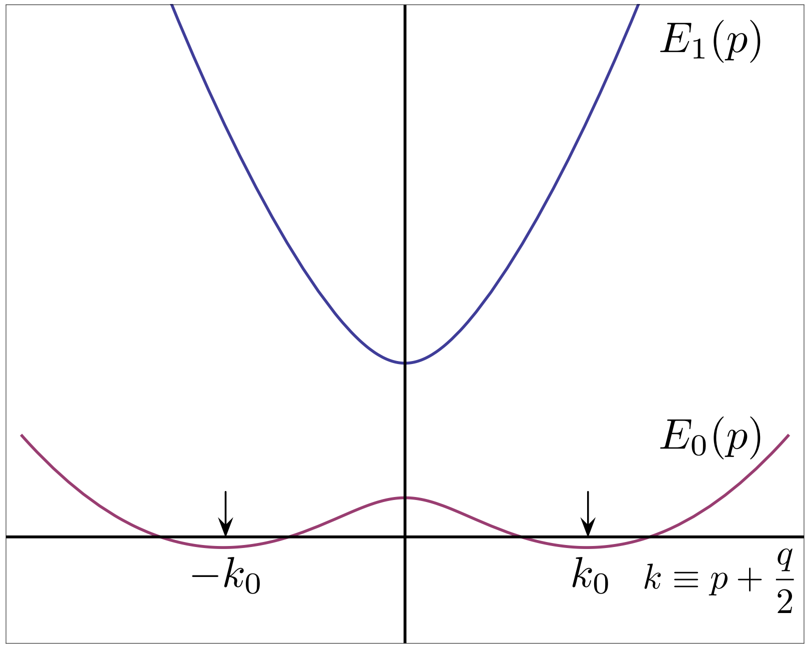

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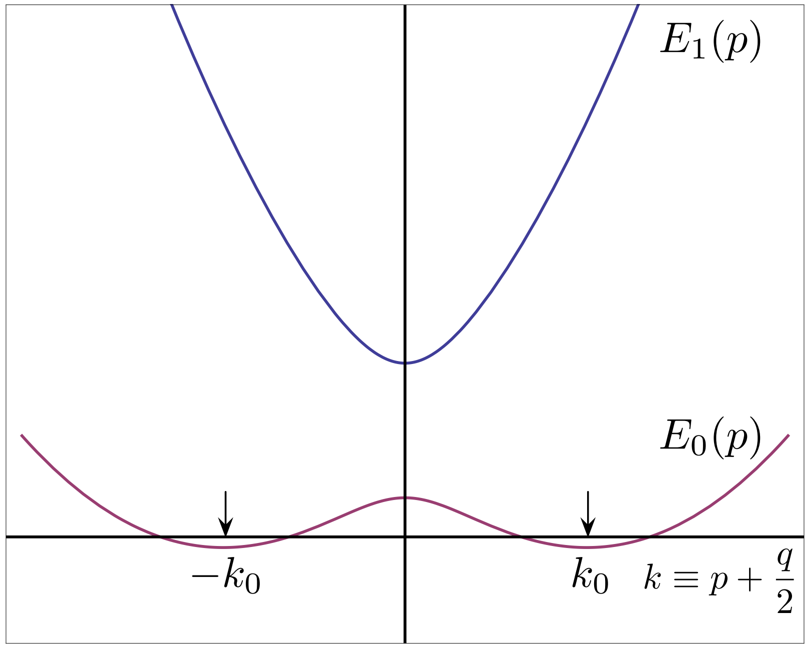

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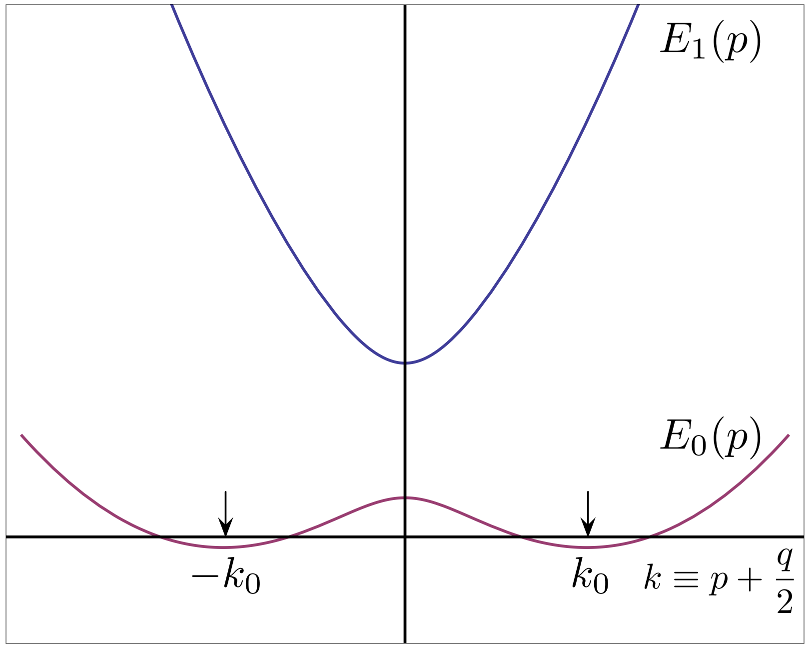

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Note: index m and n correspond to the original spin states m=1 and m=0, rather than the two degenerate states at +(-) k_0

g_{mn} is the interaction matrix between different spin states. For later convenience, we define the following parameters:

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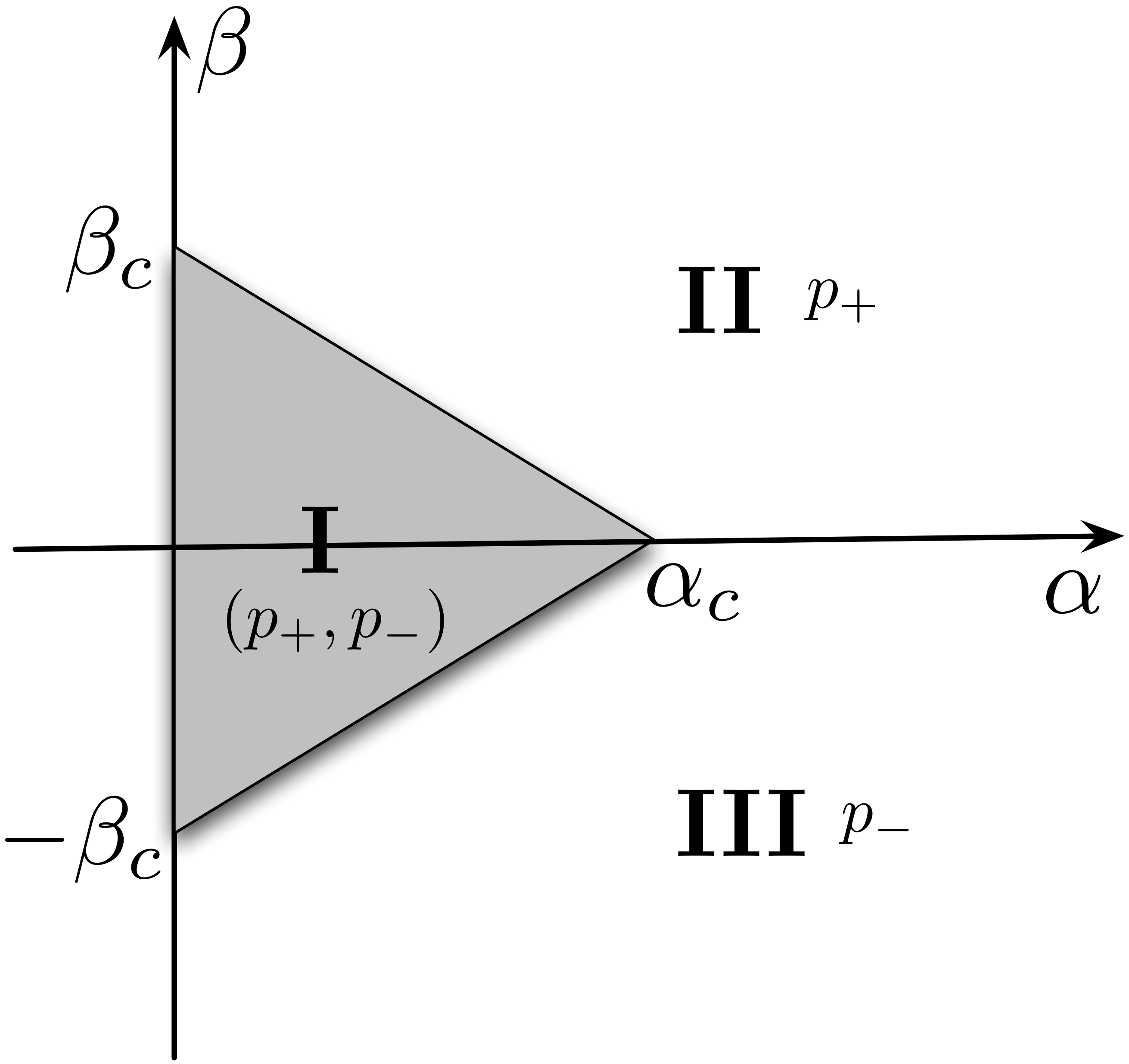

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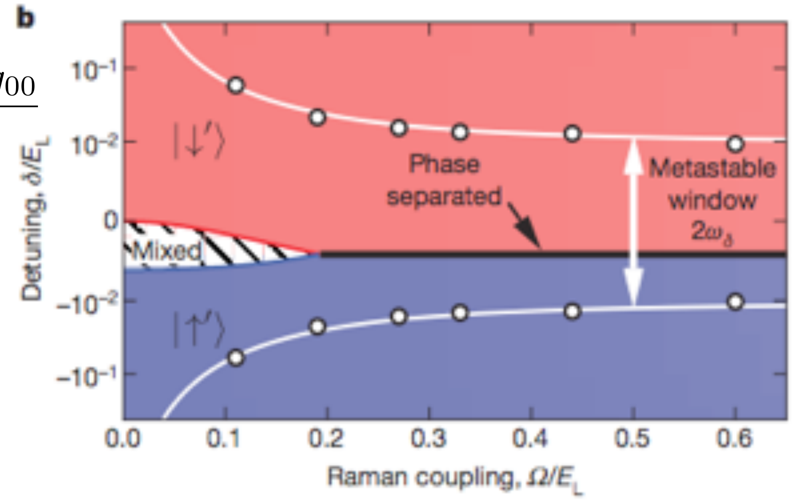

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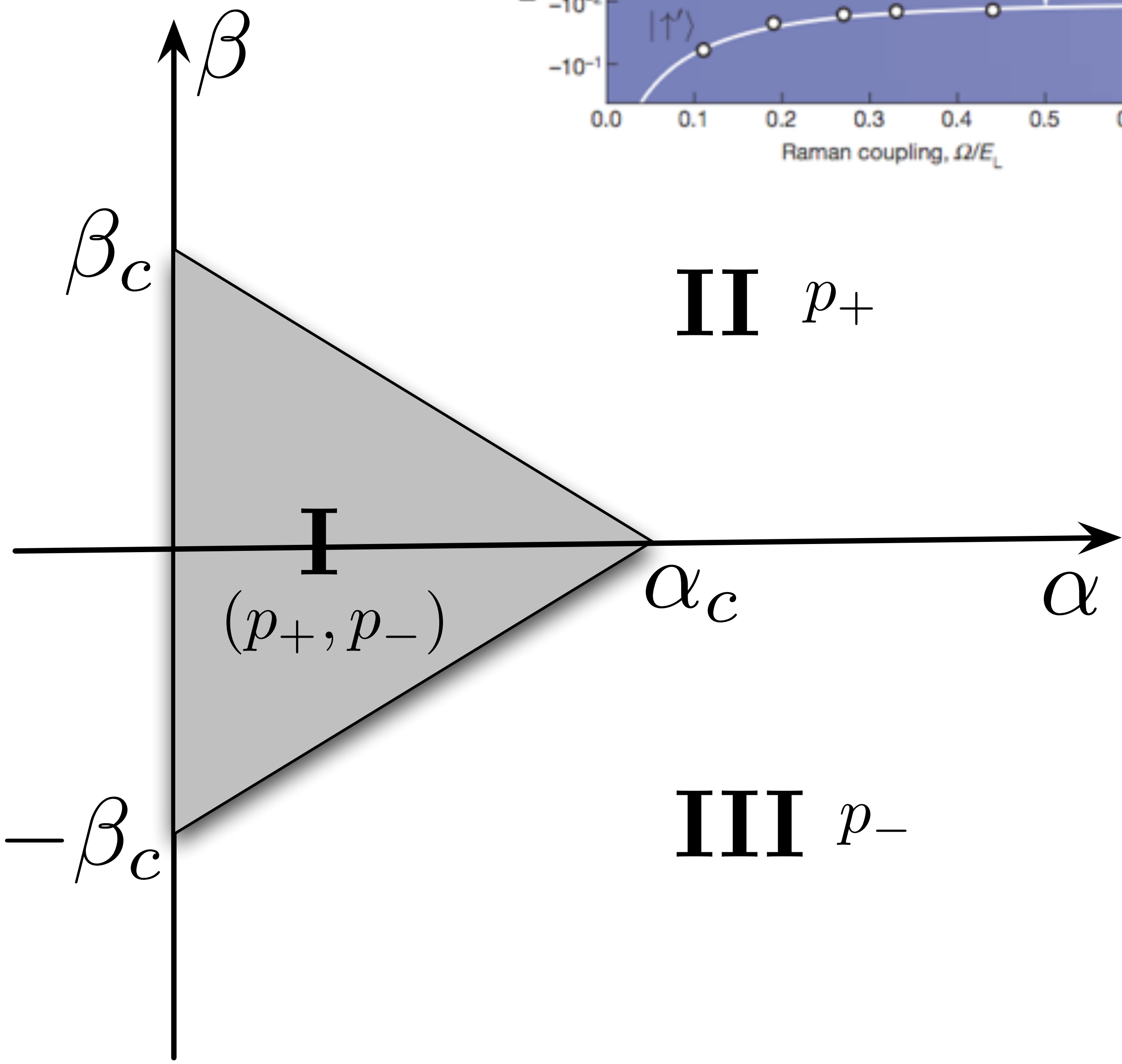

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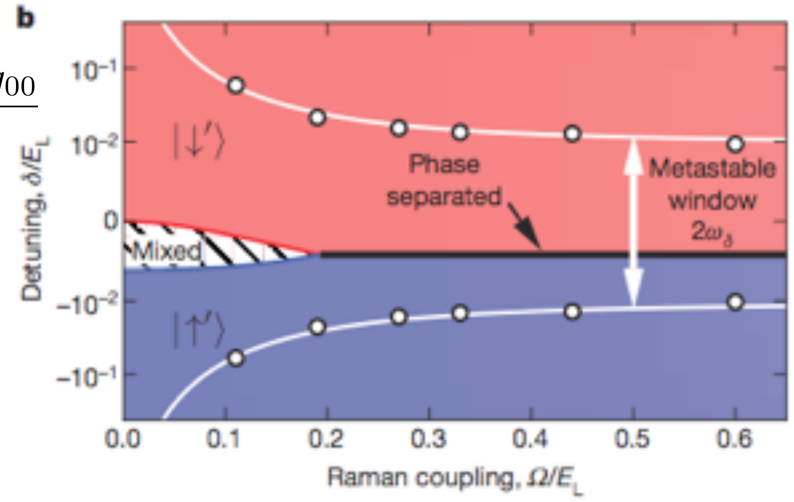

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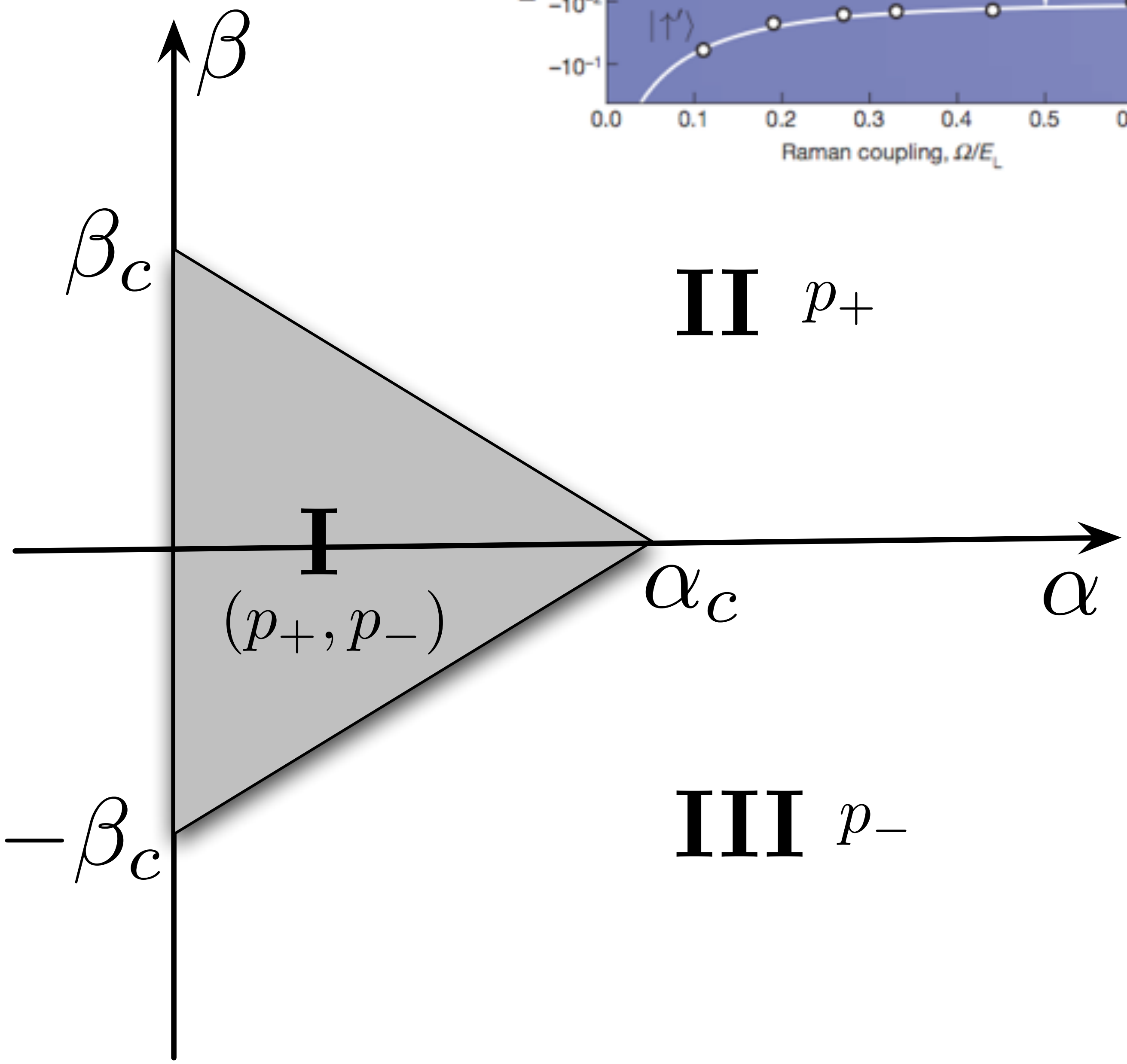

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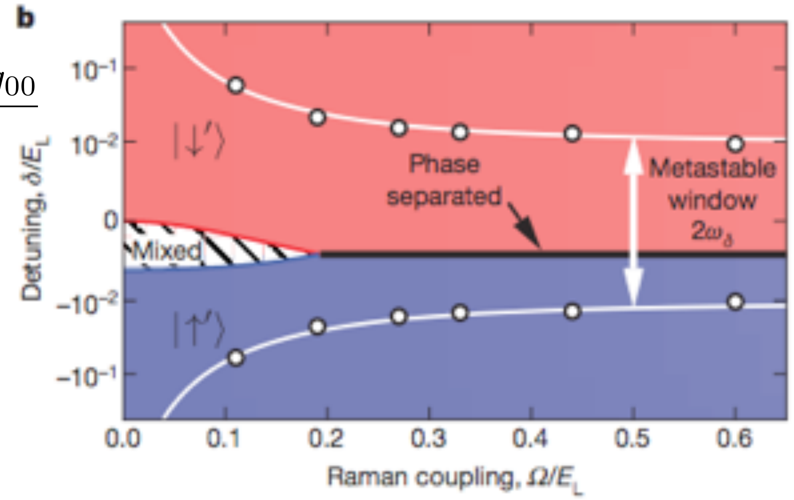

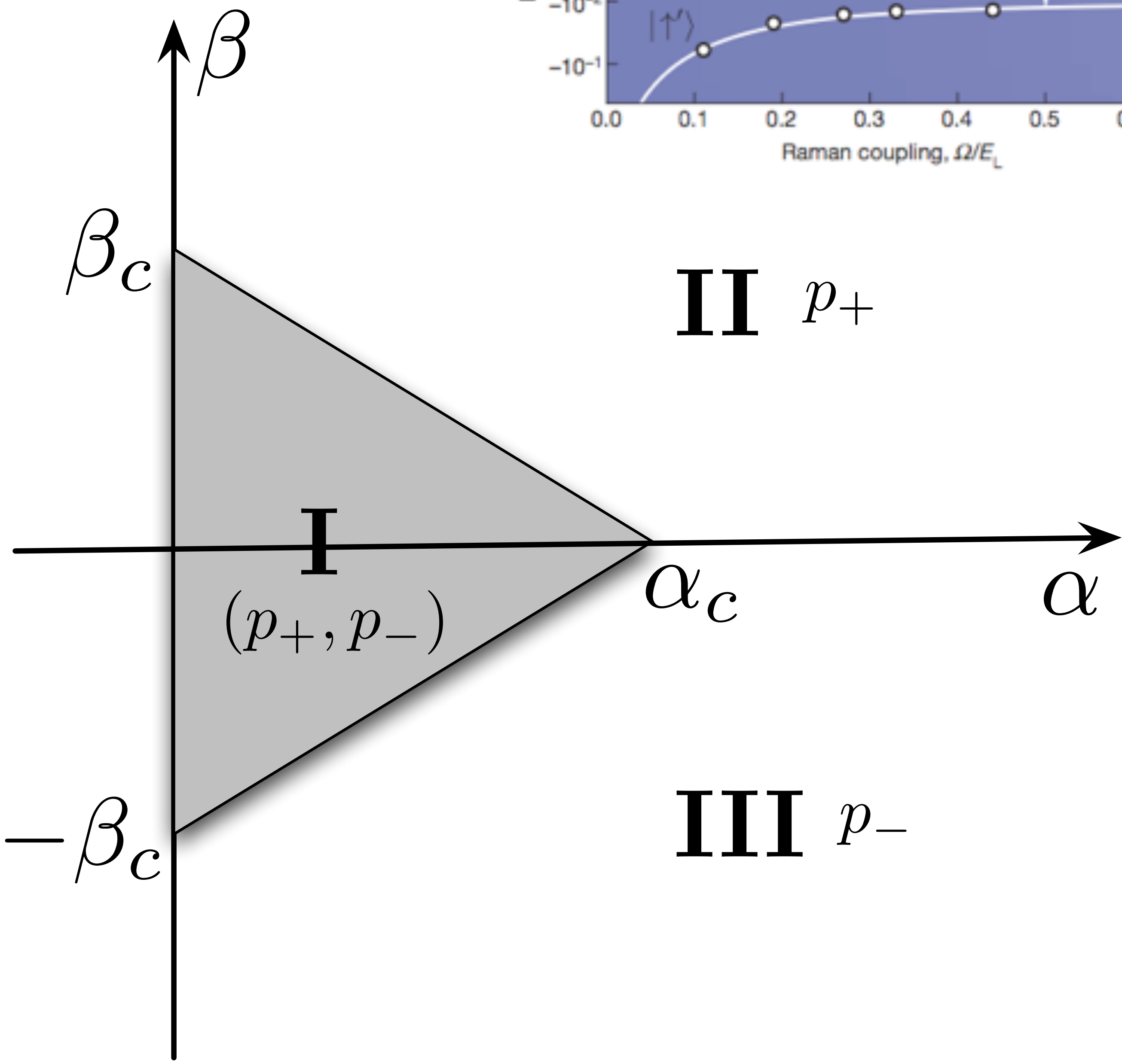

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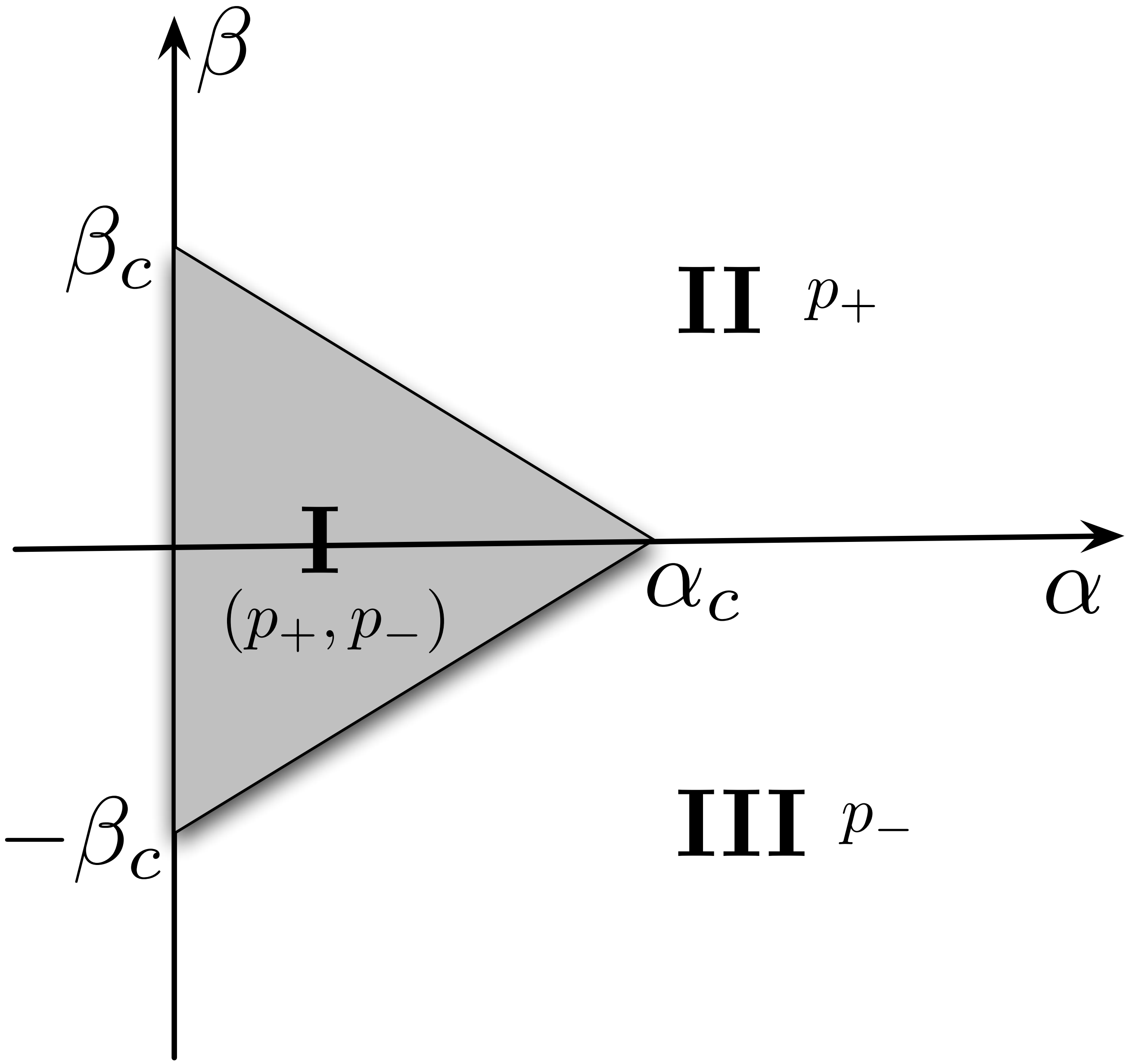

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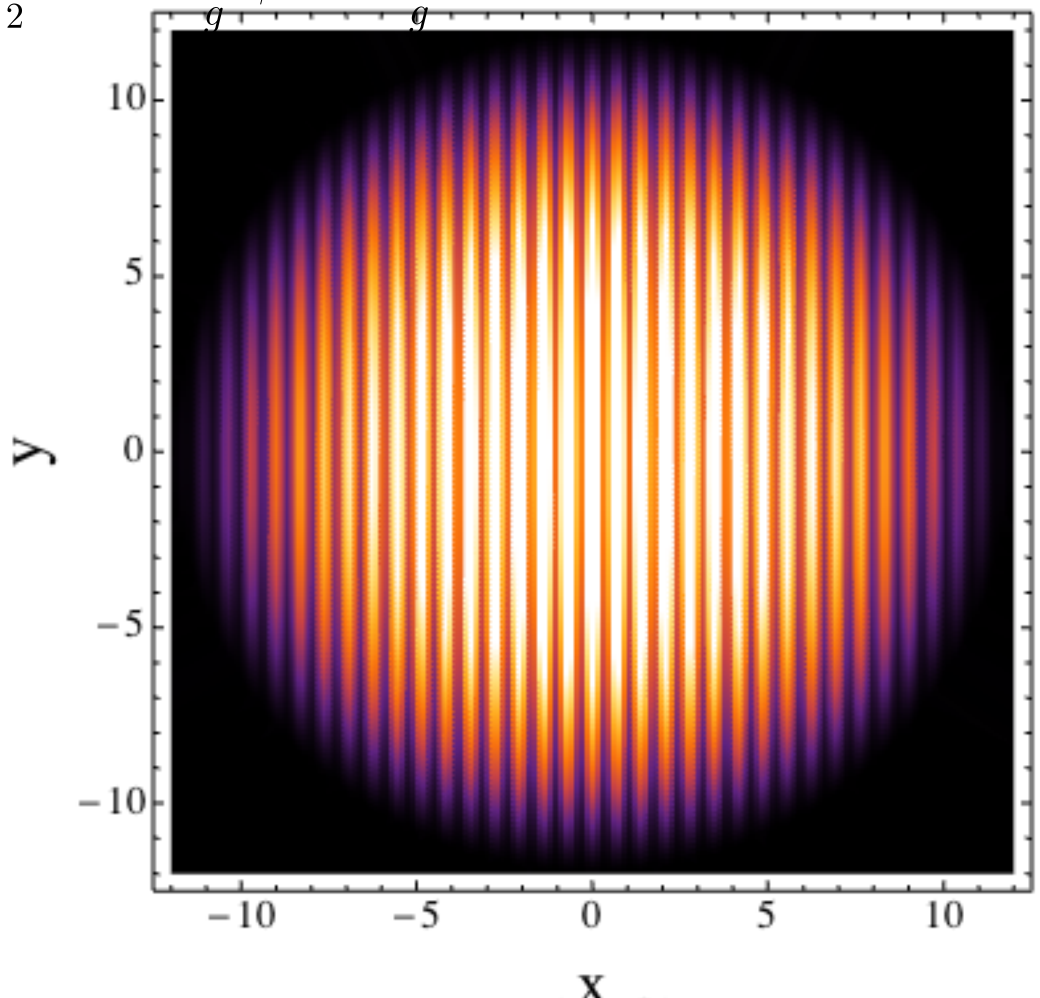

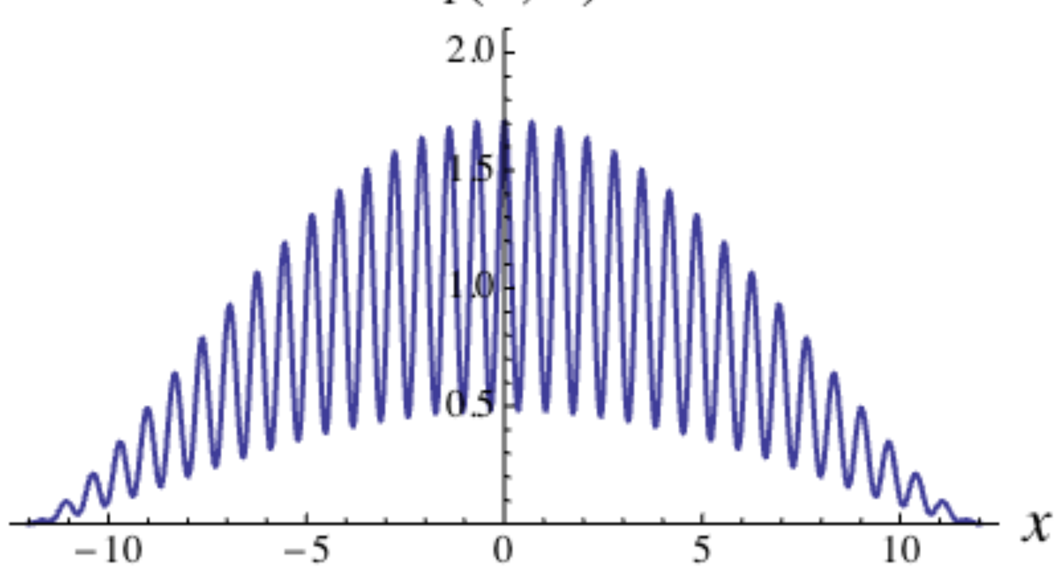

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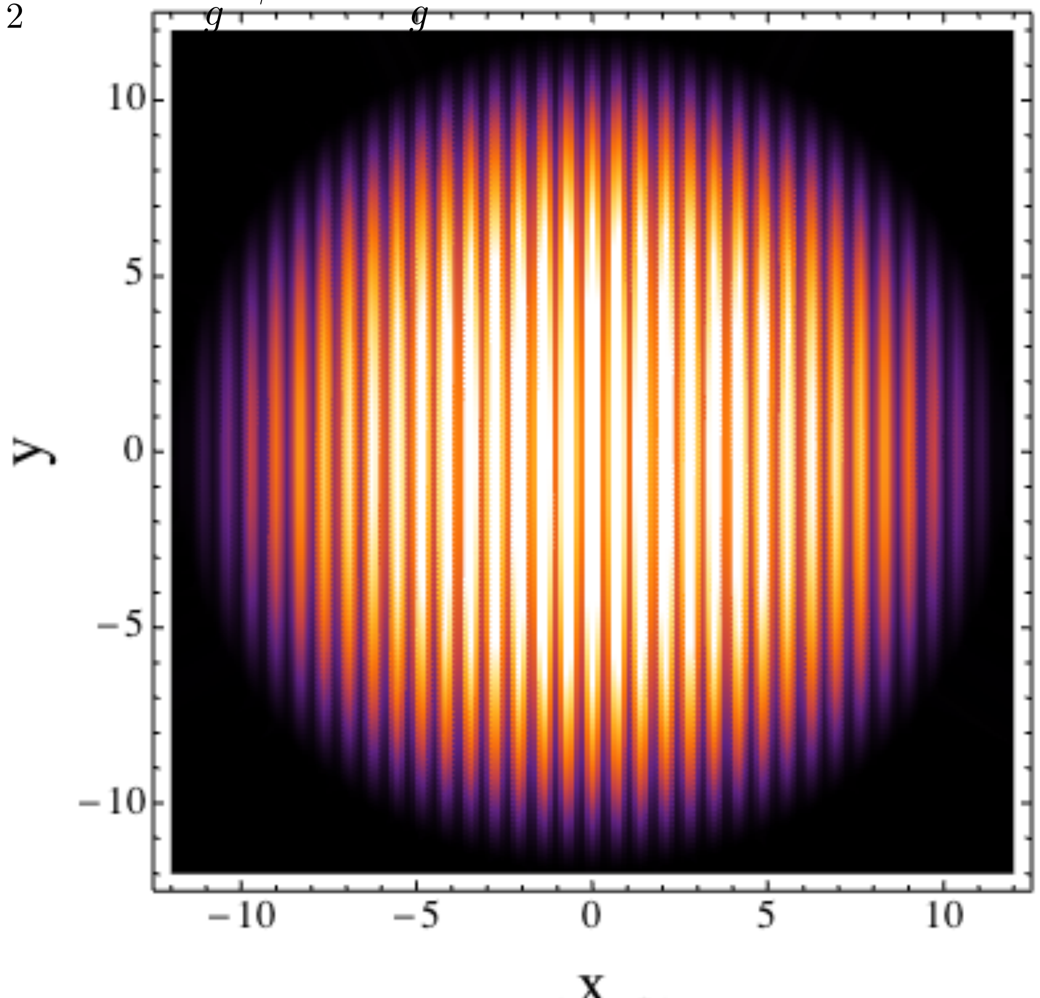

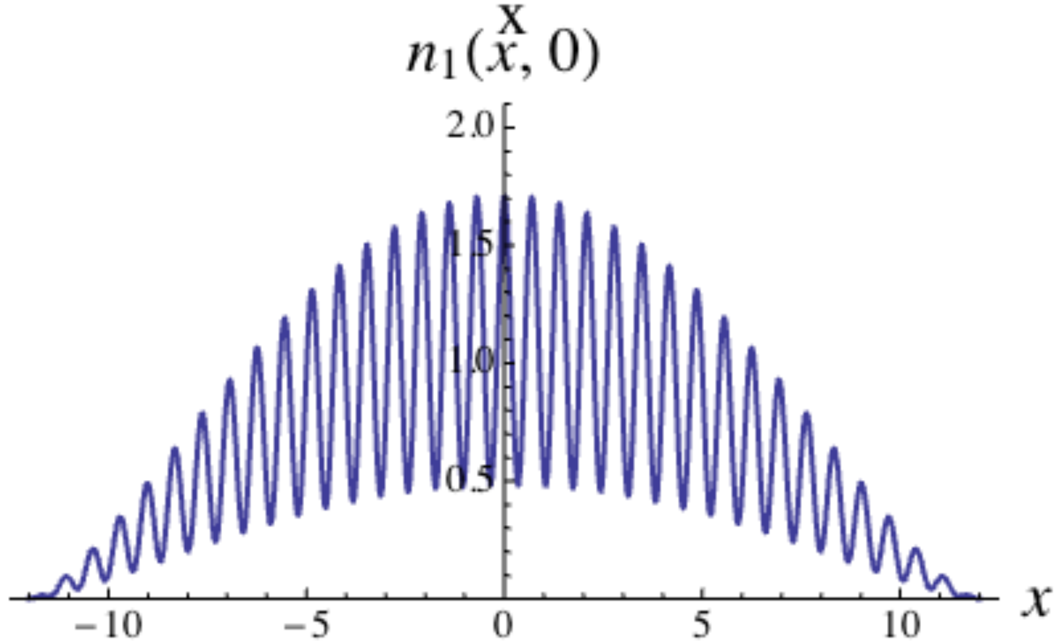

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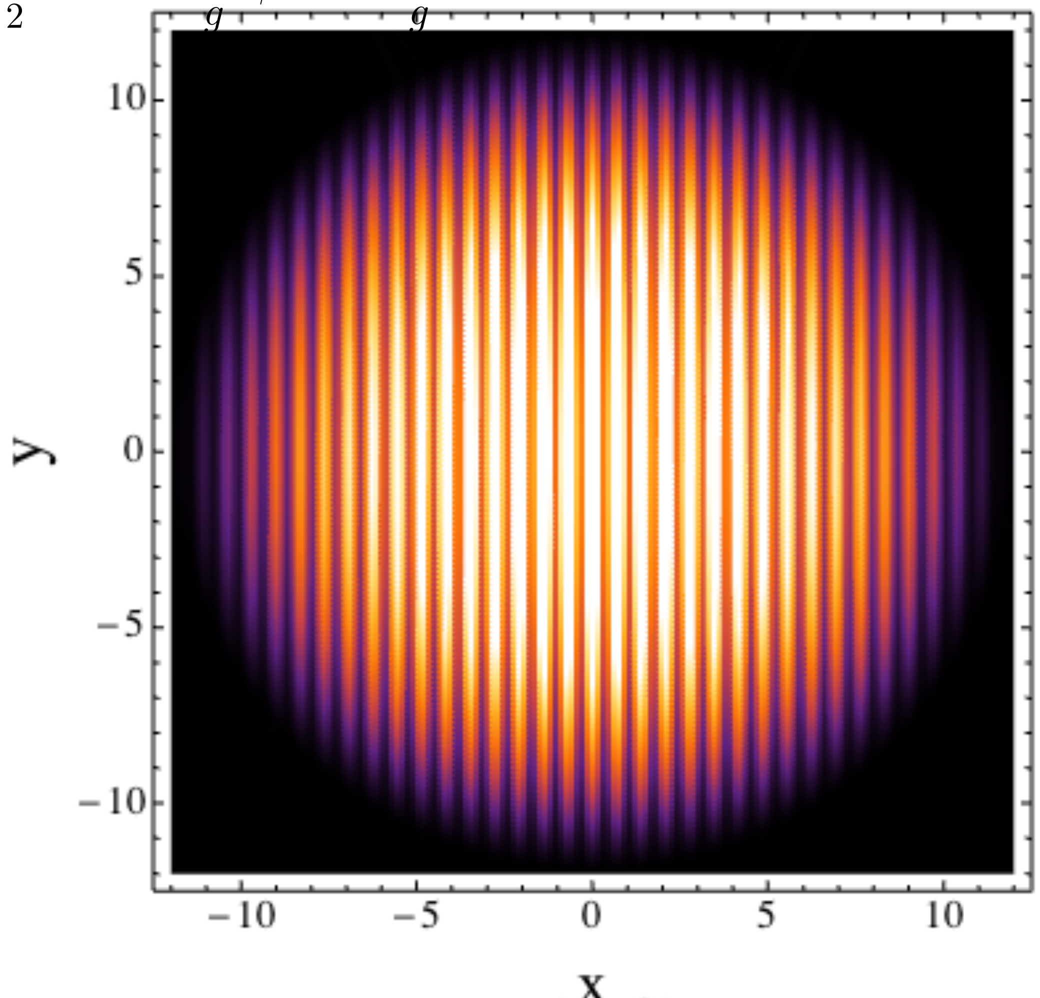

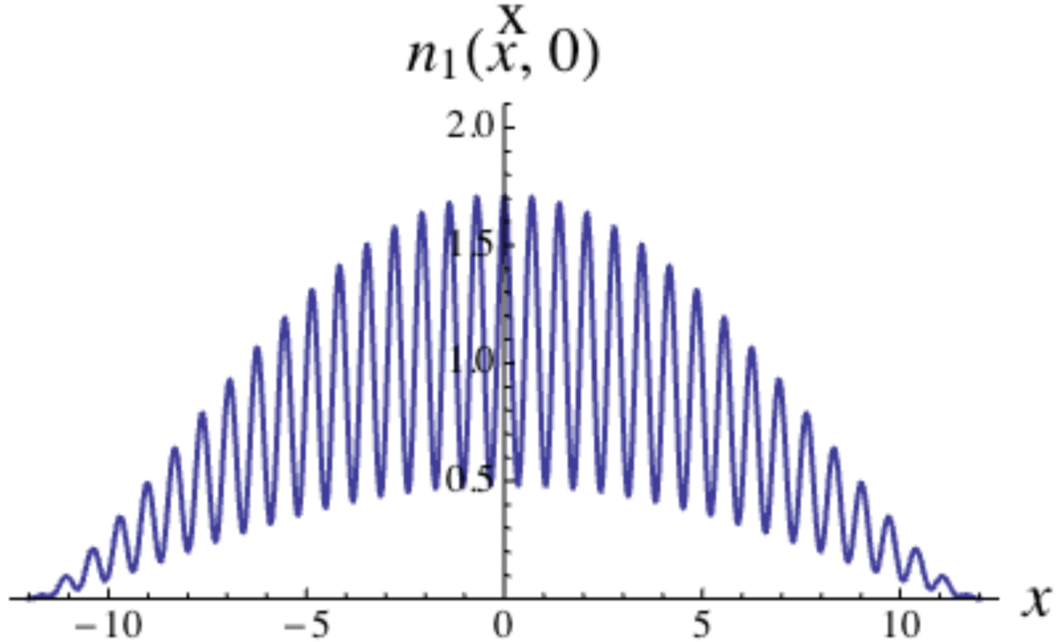

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MORE REFERENCES ON SPIN-ORBIT COUPLE BECS:

Discussions on spin-orbit coupled BEC in the literatures, for example:

T.Stanescu, B.Anderson and V. Galitski, PRA **78**, 023616 (2008)
Jonas Larson and Eric Sjoqvist, PRA **79**, 043627 (2009)
Chunji Wang, Chao Gao,Chao-Ming Jian and Hui Zhai, PRL **105**, 160403 (2010)
Congjun Wu and Ian Mondragon-Shem, arXiv:0809.3532
S.-K. Yip, arXiv:1101.1714

Other effects related to abelian/non-abelian gauge fields:
Jaksch&Zoller, Lewenstein, Reseckas et al., Gerbier&Dalibard ...
......

So, What can we do with it?

Elastic "Matter Lattice"

ELASTIC "MATTER LATTICE"

We have a Bose-Einstein Condensate with density modulations:

$$\Phi_0(x) = A_+ e^{i\theta_+ + ik_0 x} \widetilde{\chi}_+(k_0) + A_- e^{i\theta_- - ik_0 z} \widetilde{\chi}_-(k_0)$$

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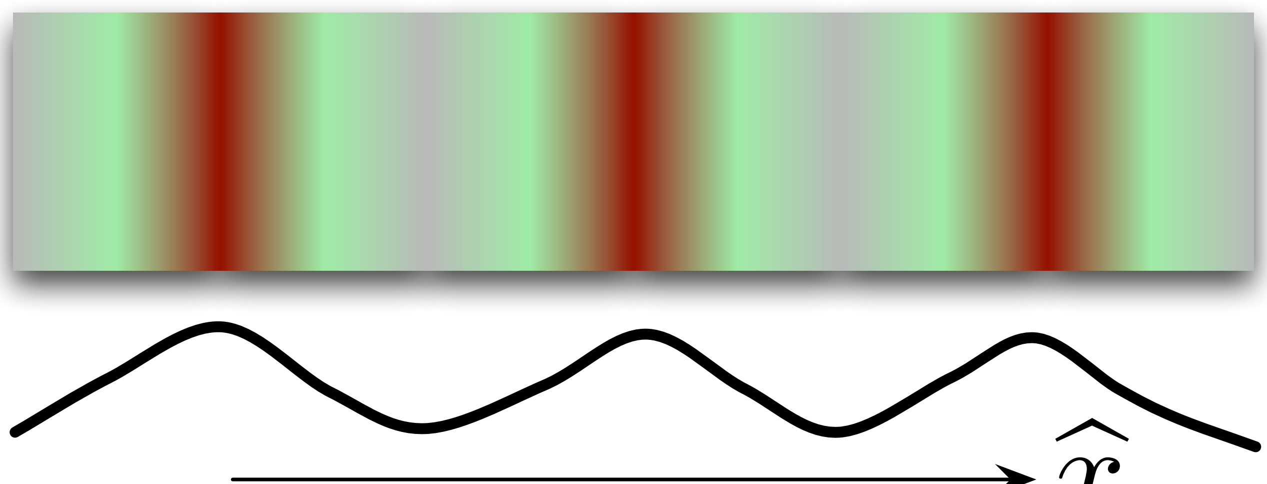

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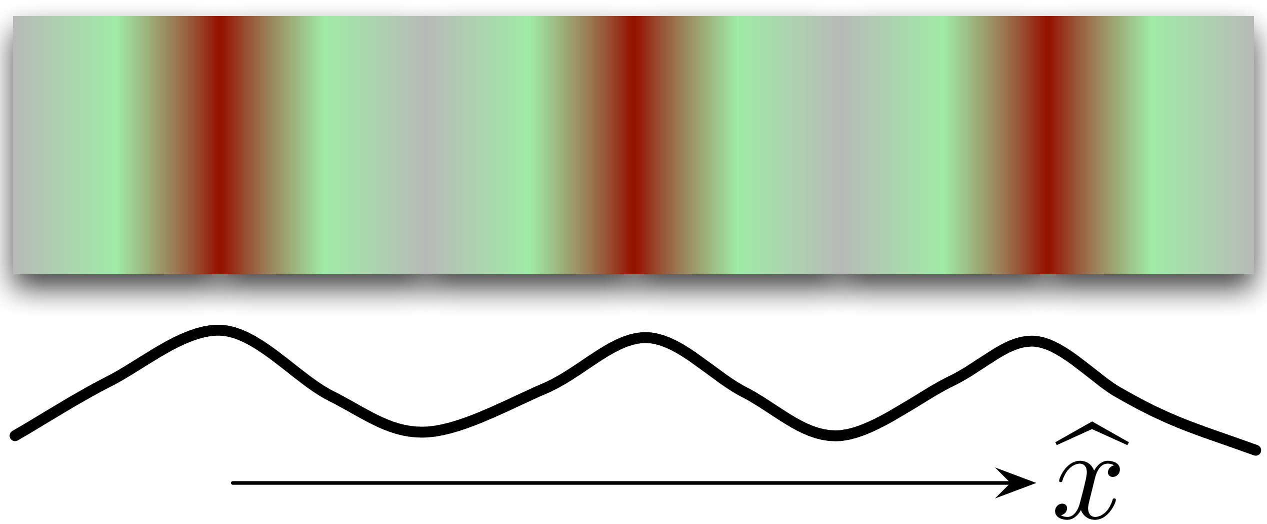


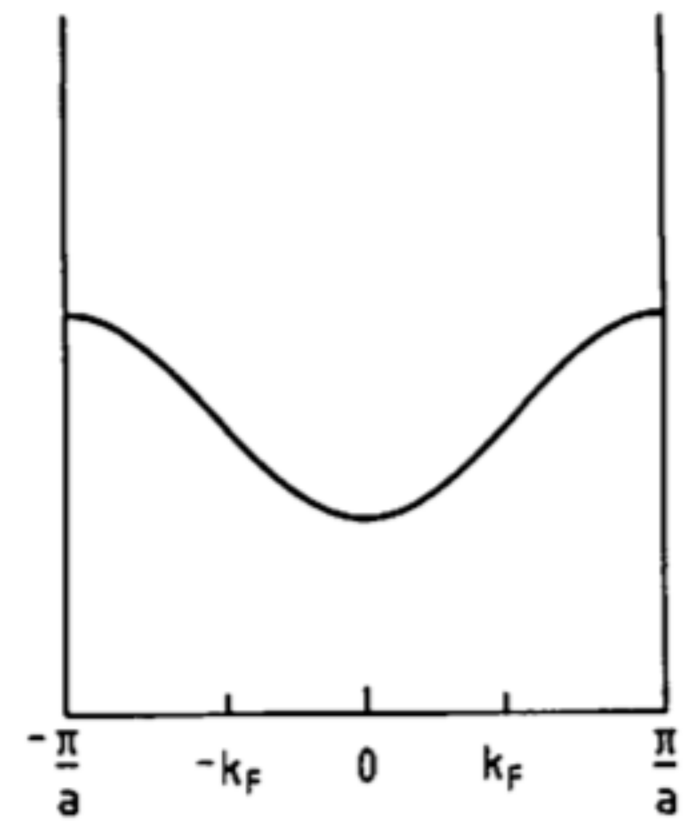
$n_{\mathbf{B}}(x)$

This looks just like the intensity field of light in the optical lattice !

Unlike the usual optical lattice, the potential can respond to external perturbations and support its own dynamics !

Possibility of observing Peierls distortion in the system!

Peierls Distortion I



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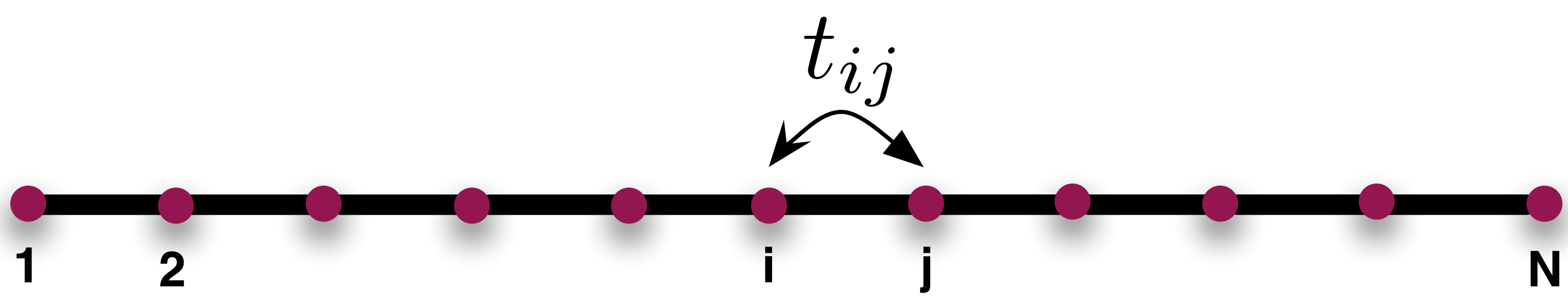

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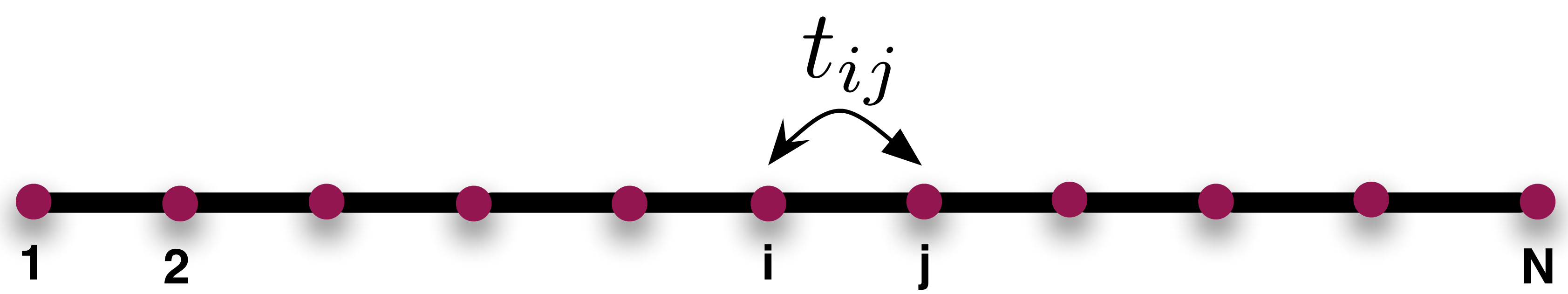

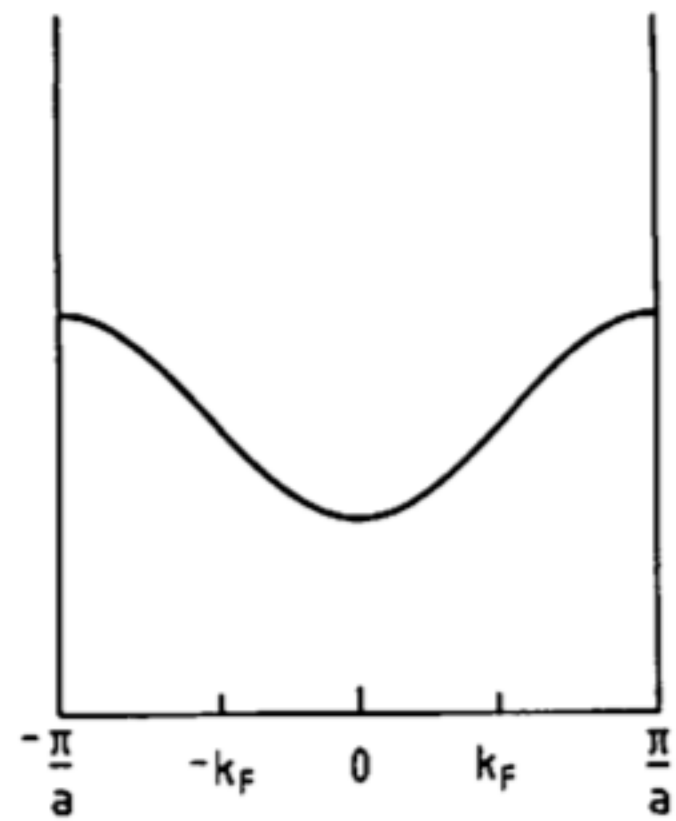

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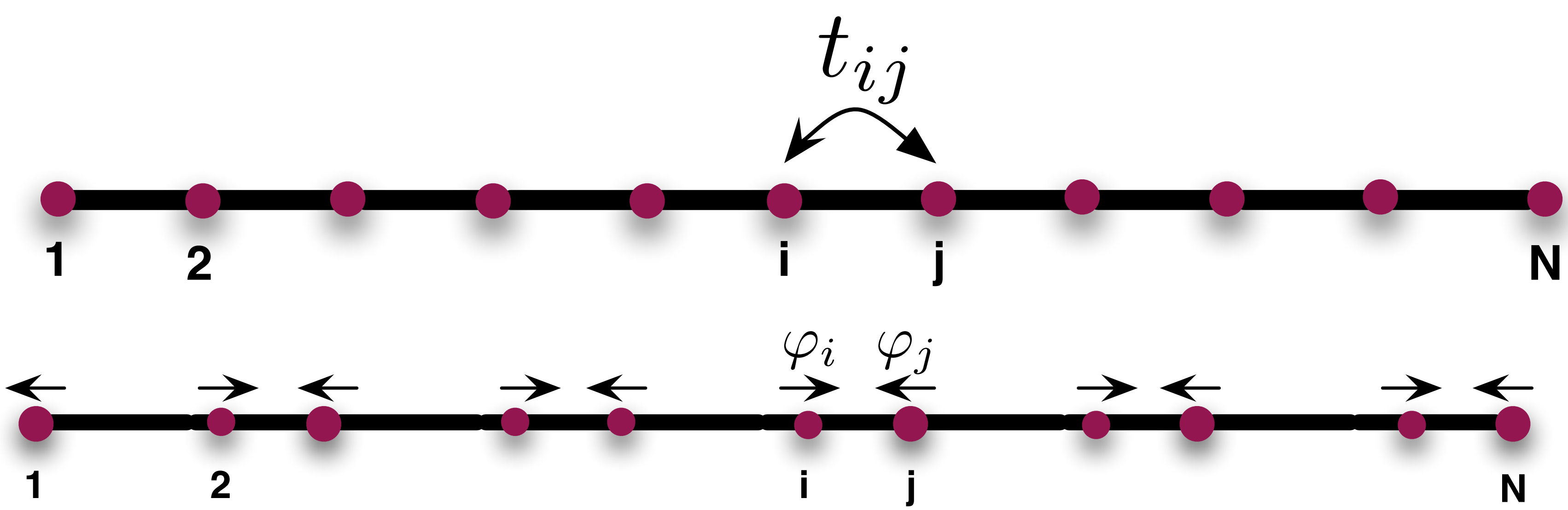

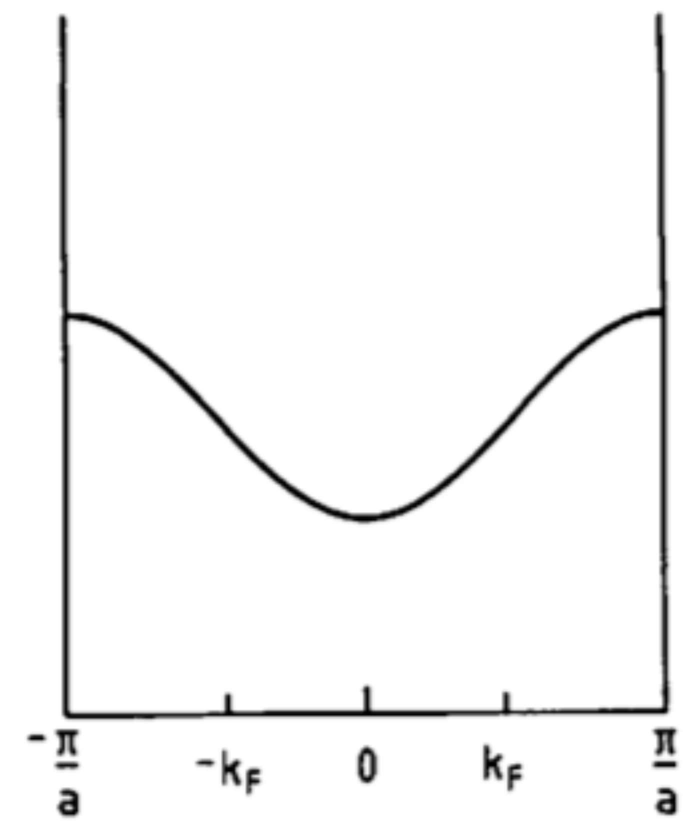

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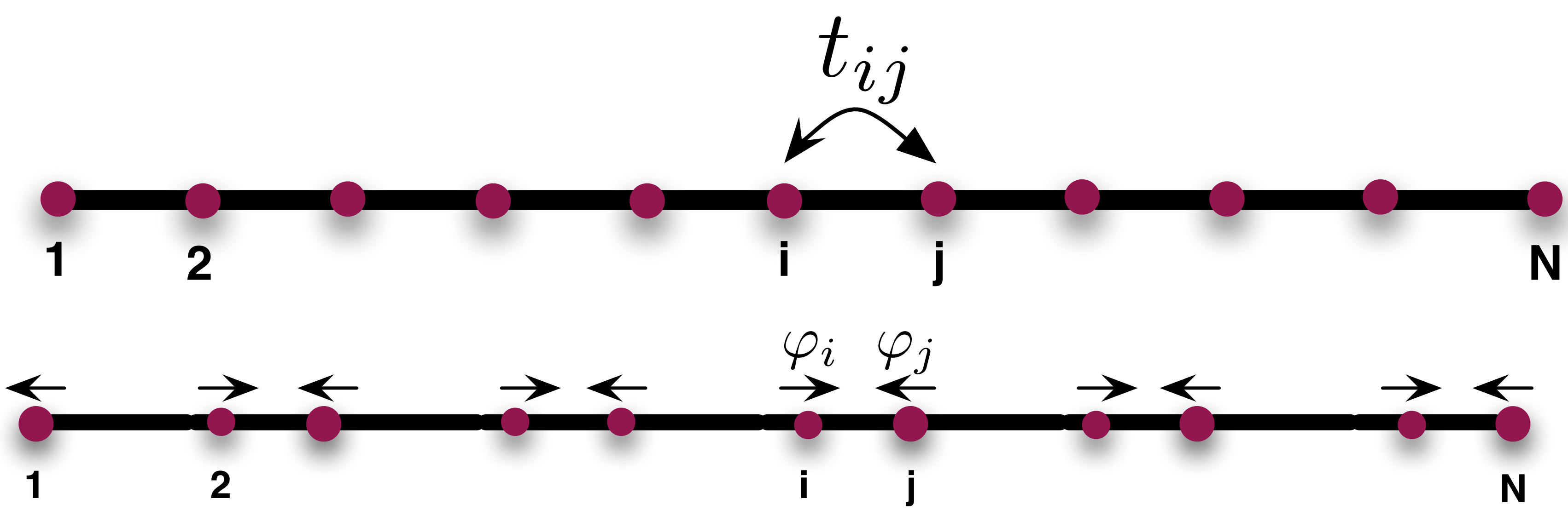

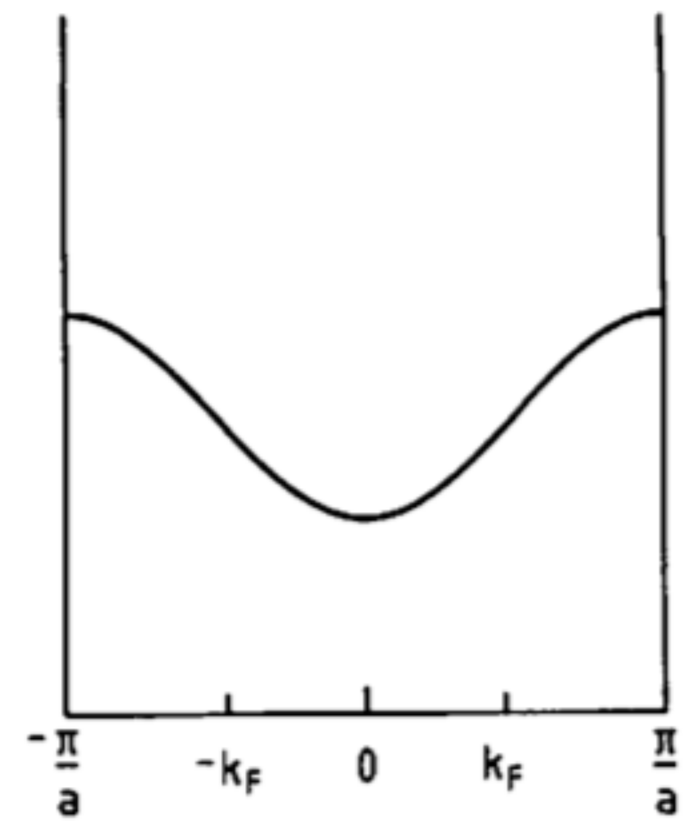

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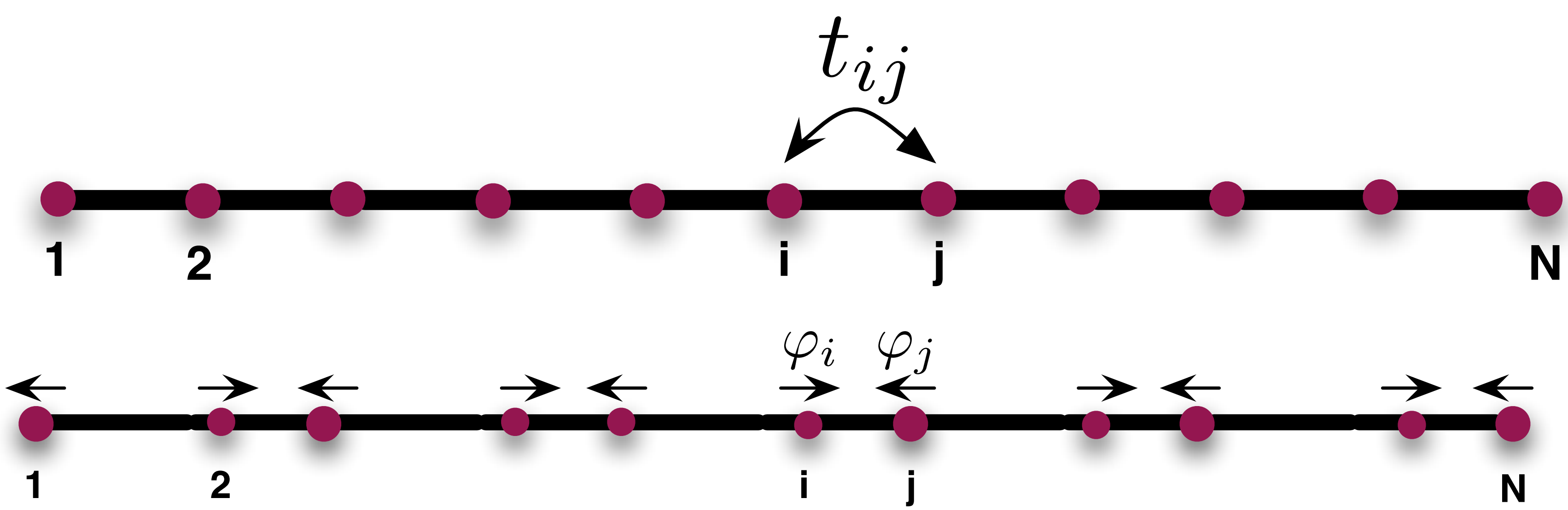

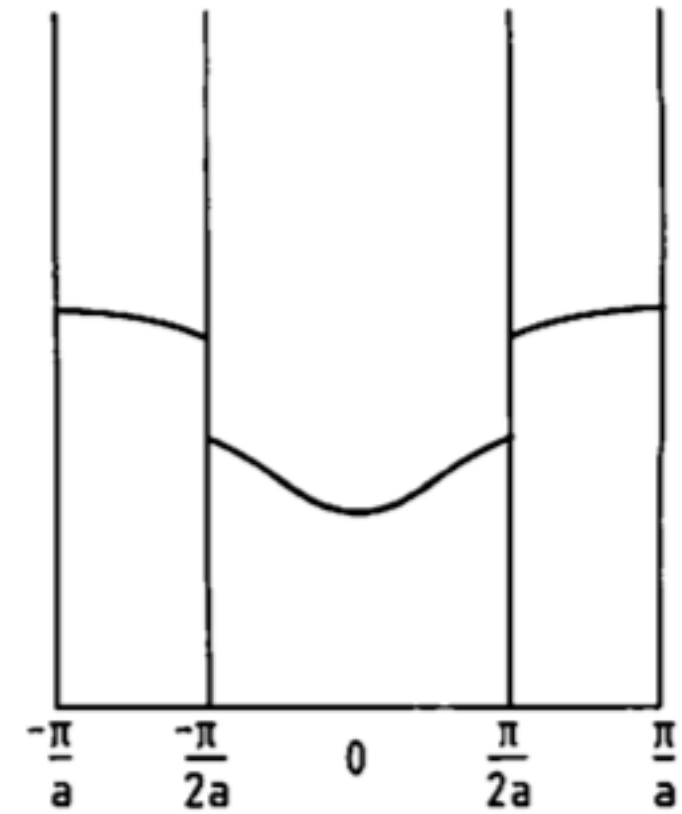

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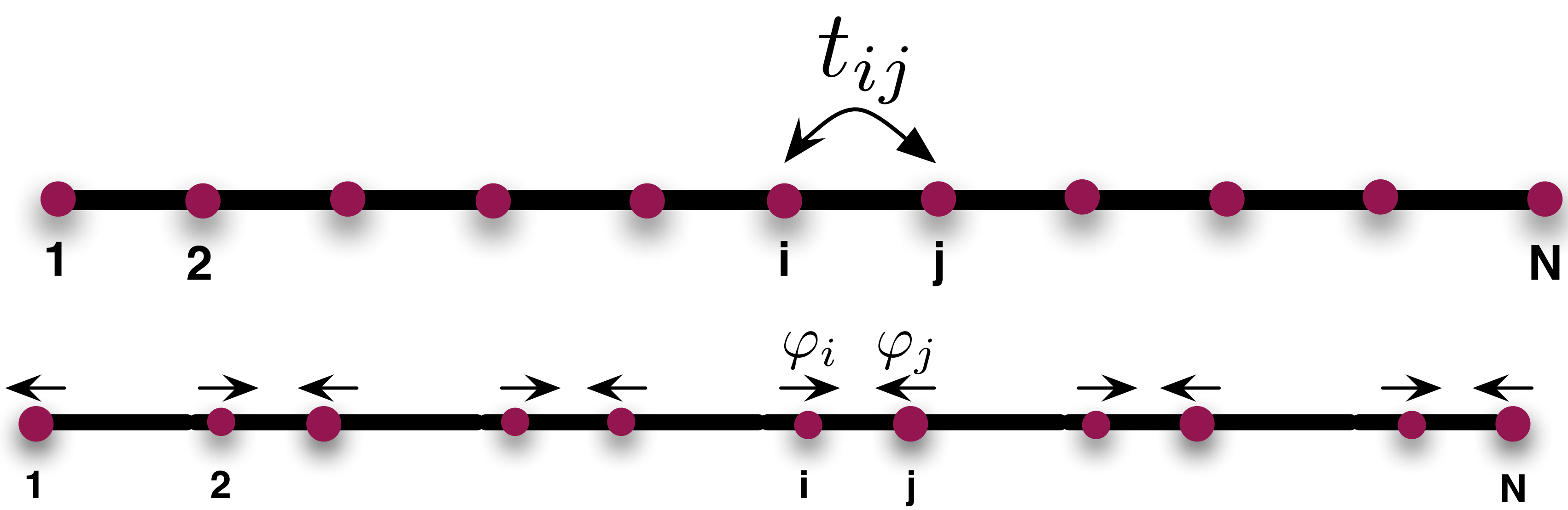

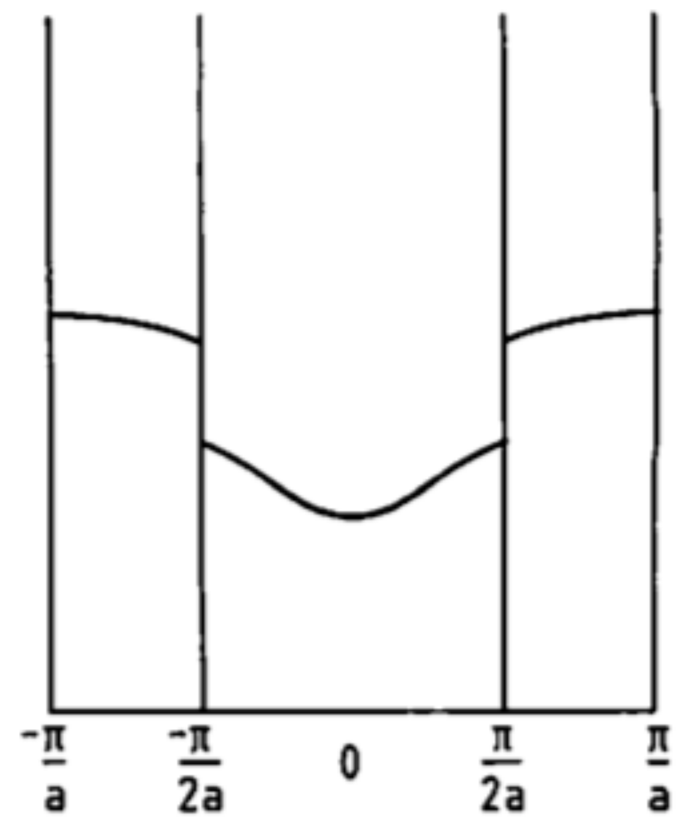

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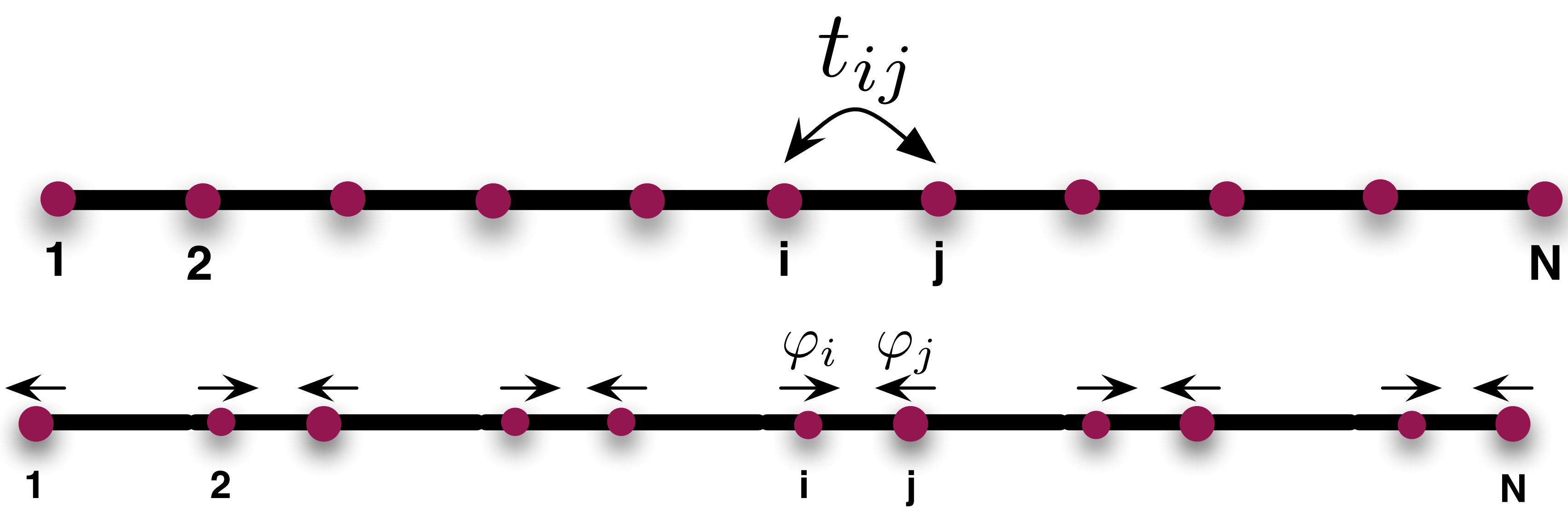

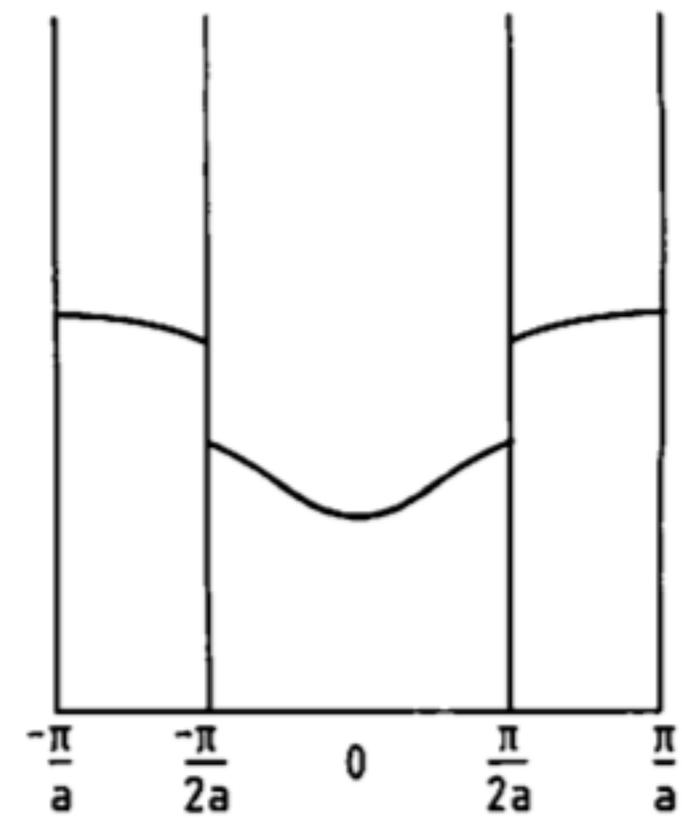

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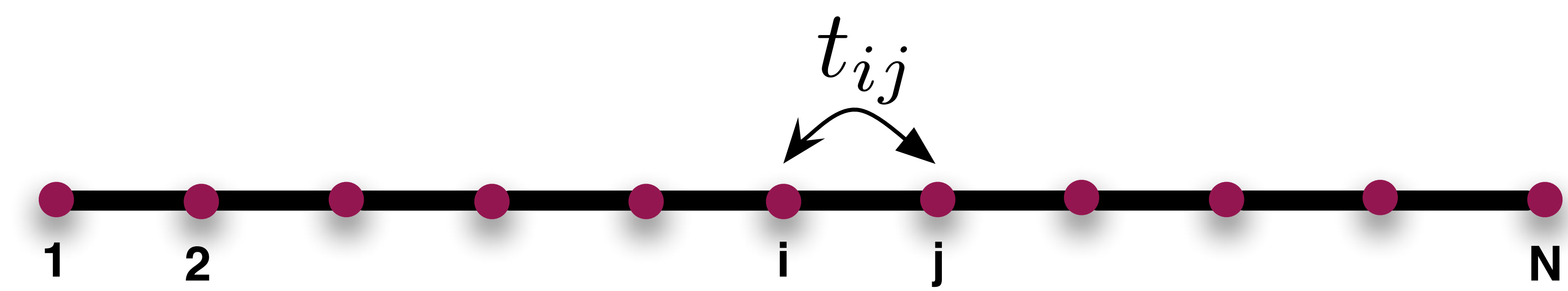

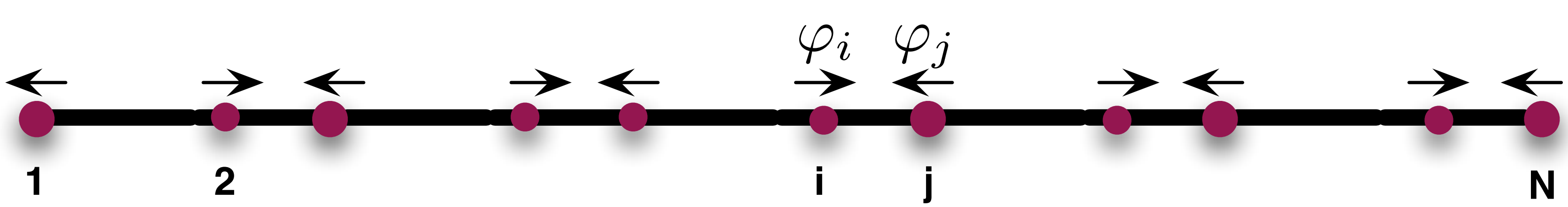

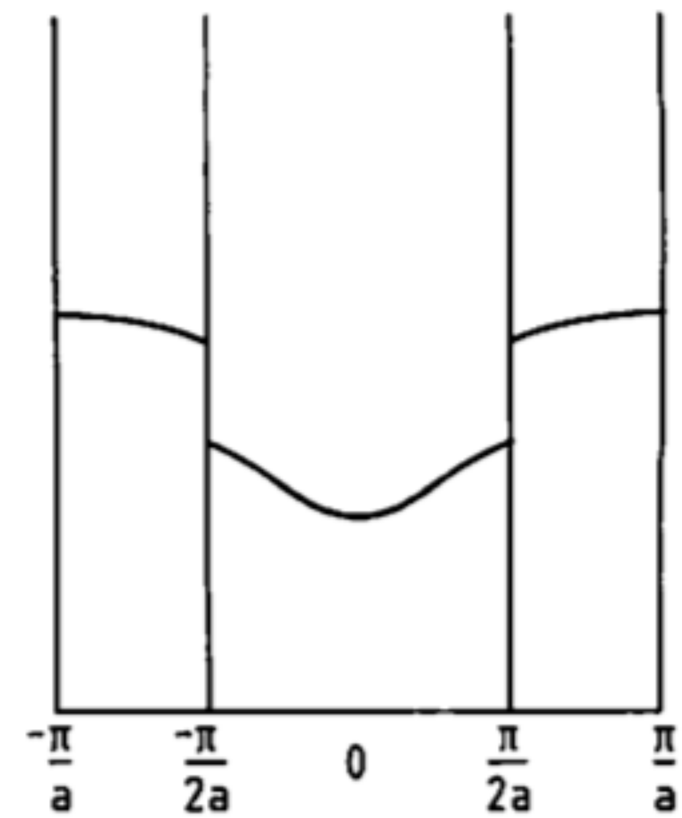

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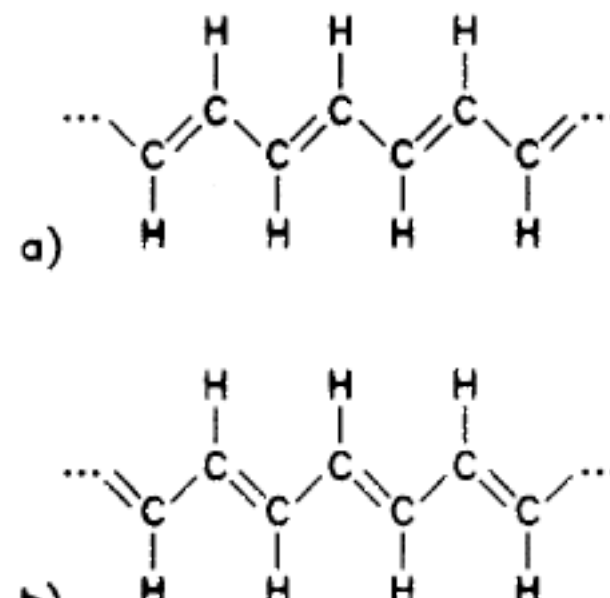

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polyacetylene

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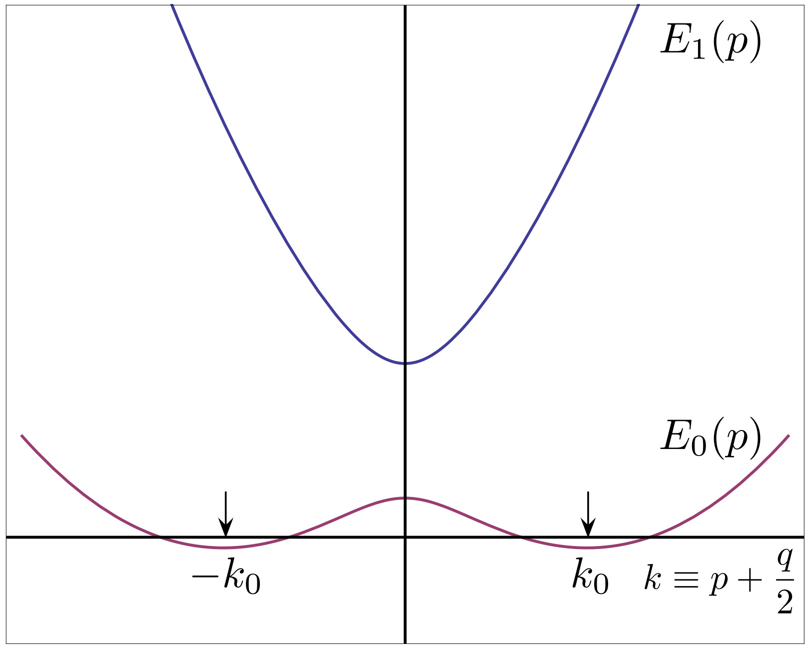

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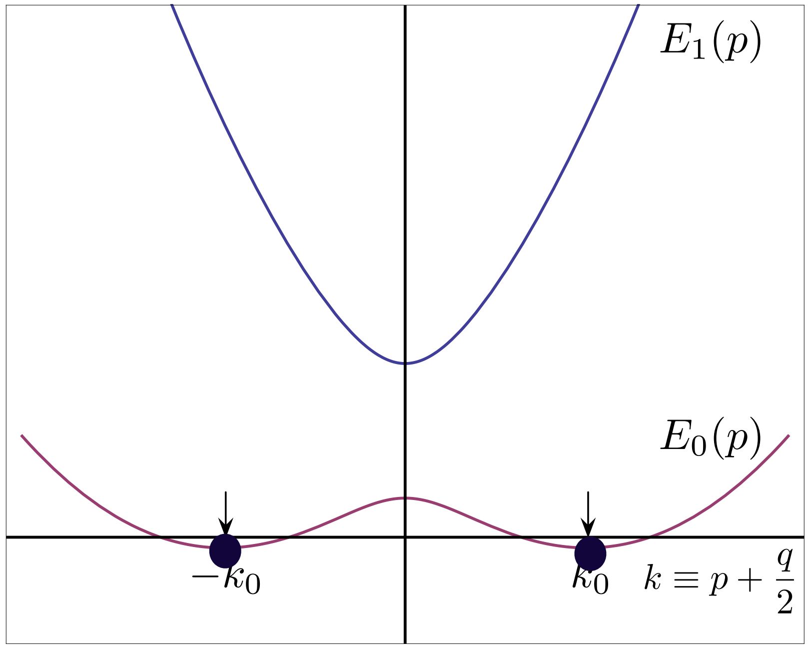

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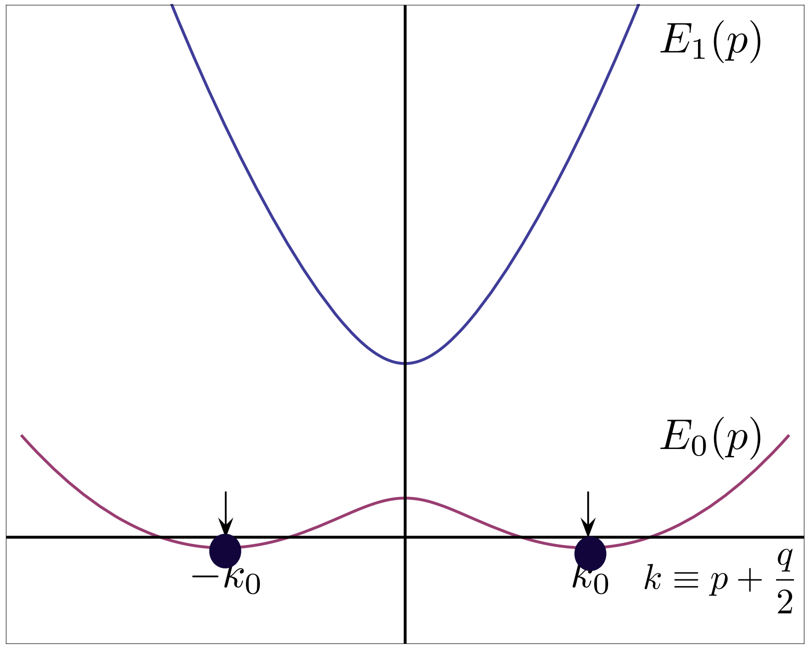

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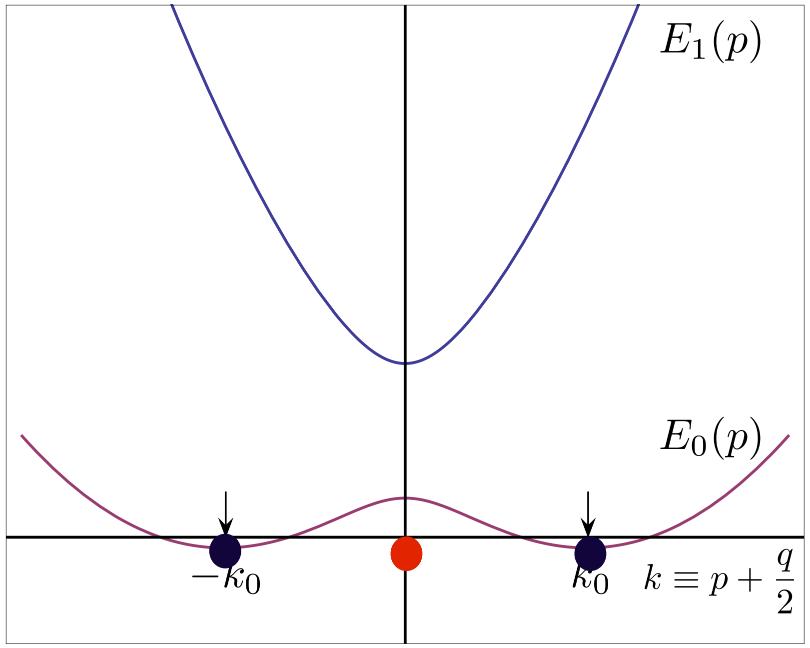

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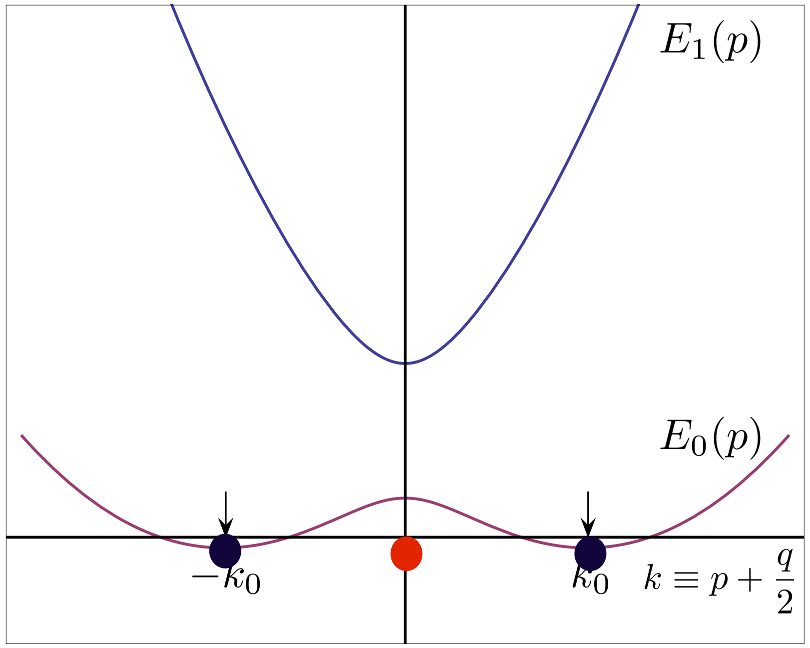

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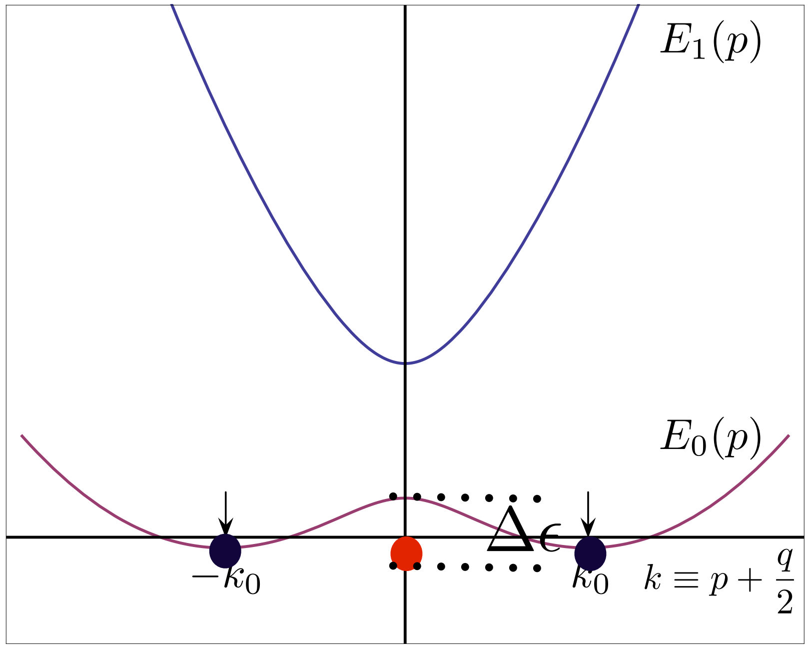
$$n_{\mathbf{B}}^{(0)}(x) = n_0 + 2A_+A_- \sin\theta \cos(2k_0 x + \varphi)$$

Condensate structure:

$$\Phi_1(x) = A_+ e^{i\theta_+ + ik_0 z} \widetilde{\chi}_{k_0}(x) + A_- e^{i\theta_- - ik_0 z} \widetilde{\chi}_{-k_0}(z) + B\widetilde{\chi}_0(x)$$

$$\mathbf{n}_{\mathbf{B}}^{(\mathbf{1})} = n_0 + \text{const.} \times \cos(2k_0 x + \theta_+ - \theta_-) + \text{const.} \times \cos(k_0 x + \varphi')$$

PEIERLS DISTORTION II



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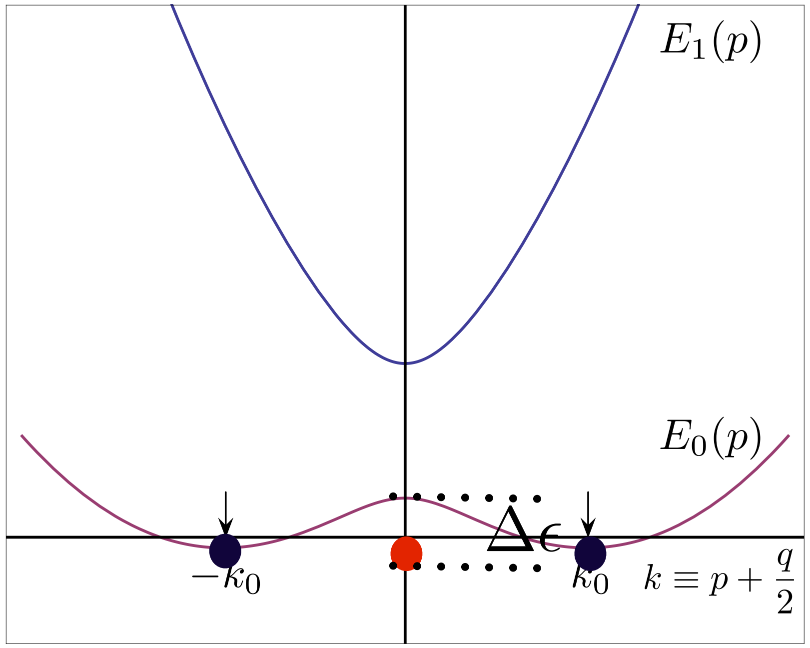

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energy cost for bosons:

$$\Delta\epsilon = E_0(p=0) - E_0(p=k_0)$$

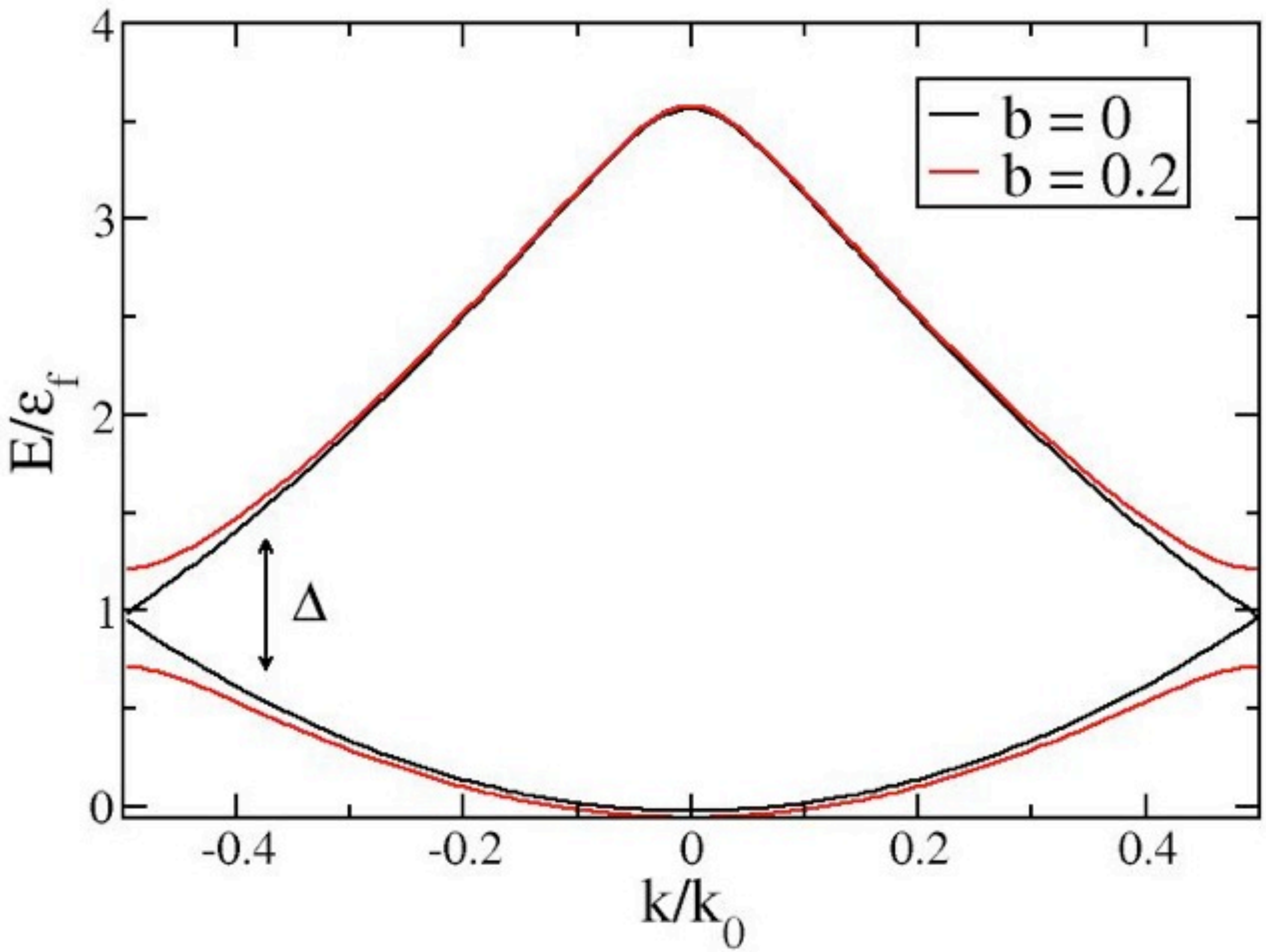
$$\Delta E_B = B^2 \Delta\epsilon$$

PEIERLS DISTORTION III

The fermionic sector: sees an external potential:

within mean field: $$\mathbf{V(x) = g_{bf} n_B^{(1)}(x)}$$ resonant regime?

To determine the size of the effect, we solve the band structure in the potential.



$$\alpha \equiv \frac{\mathbf{g_{bf} w^2}}{\mathbf{2\pi\hbar v_F}}$$

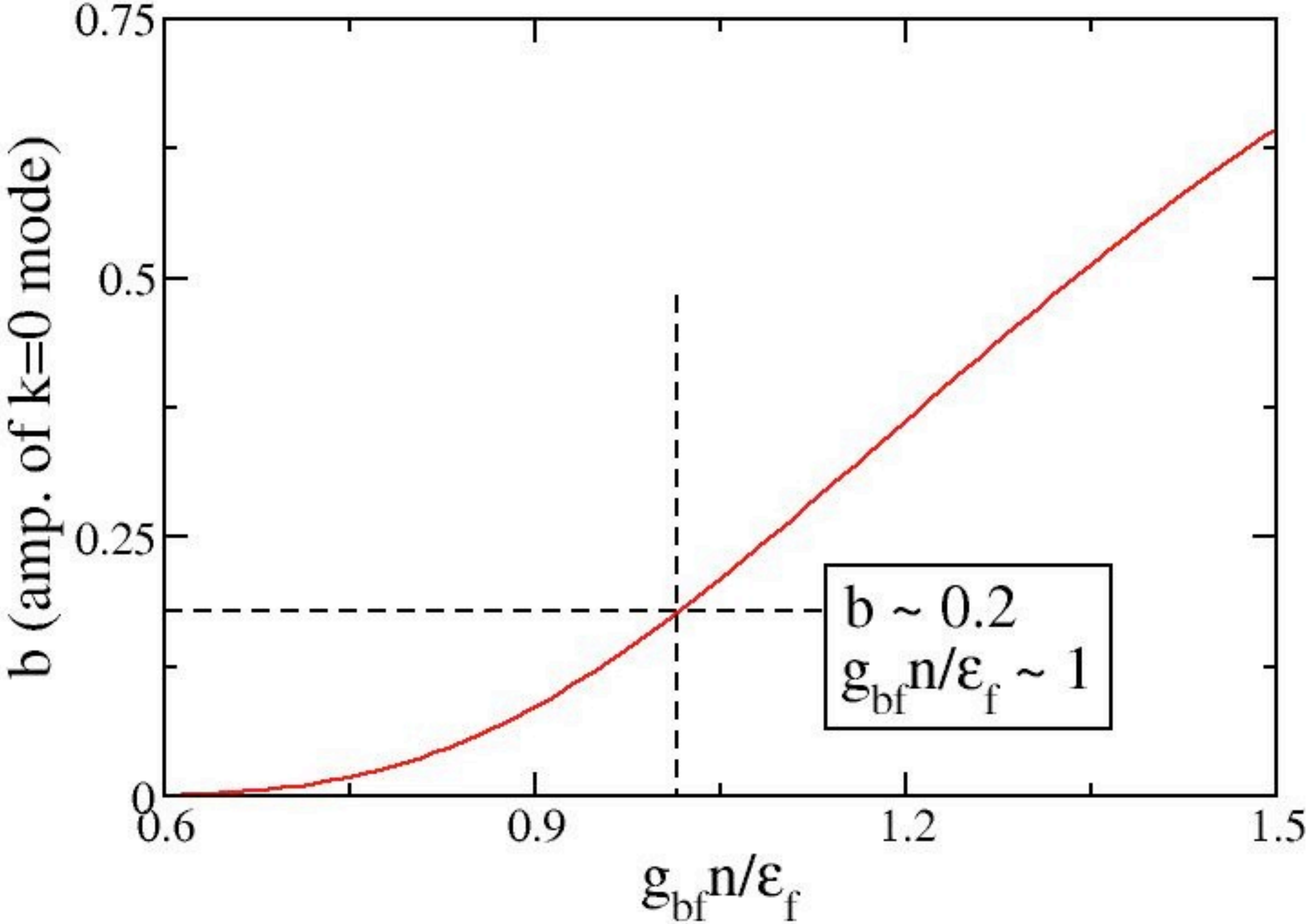
$$\beta \equiv \frac{\mathbf{g_{bf} n_B}}{\delta}$$

The size of the gap can be calculated in the perturbative limit (cf. BCS)

$$\mathbf{\Delta = 2\epsilon_F e^{-\frac{1}{\alpha\beta}}}$$

$$\mathbf{b = \frac{2\epsilon_F}{g_{bf} n_b w} e^{-\frac{1}{\alpha\beta}}}$$

PEIERLS DISTORTION IV

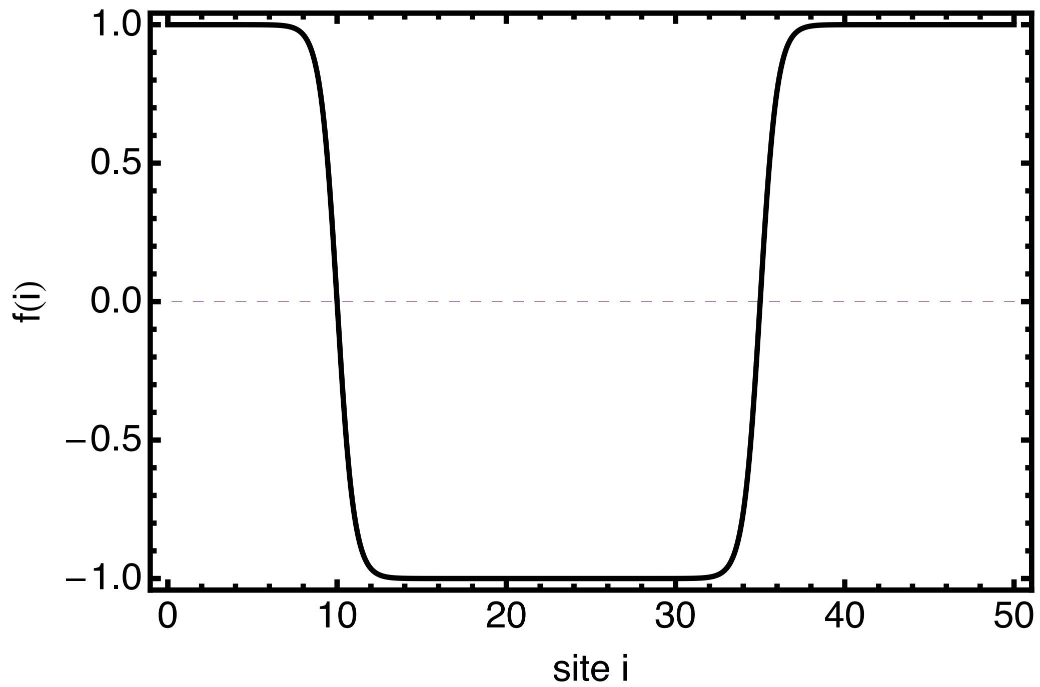


FRACTIONAL CHARGE

As is well-known in the polyacetylene research:

1. Soliton excitations of the lattice: E(-δ)= E(δ)

$$\varphi_i = (-1)^i \delta \times f_i$$



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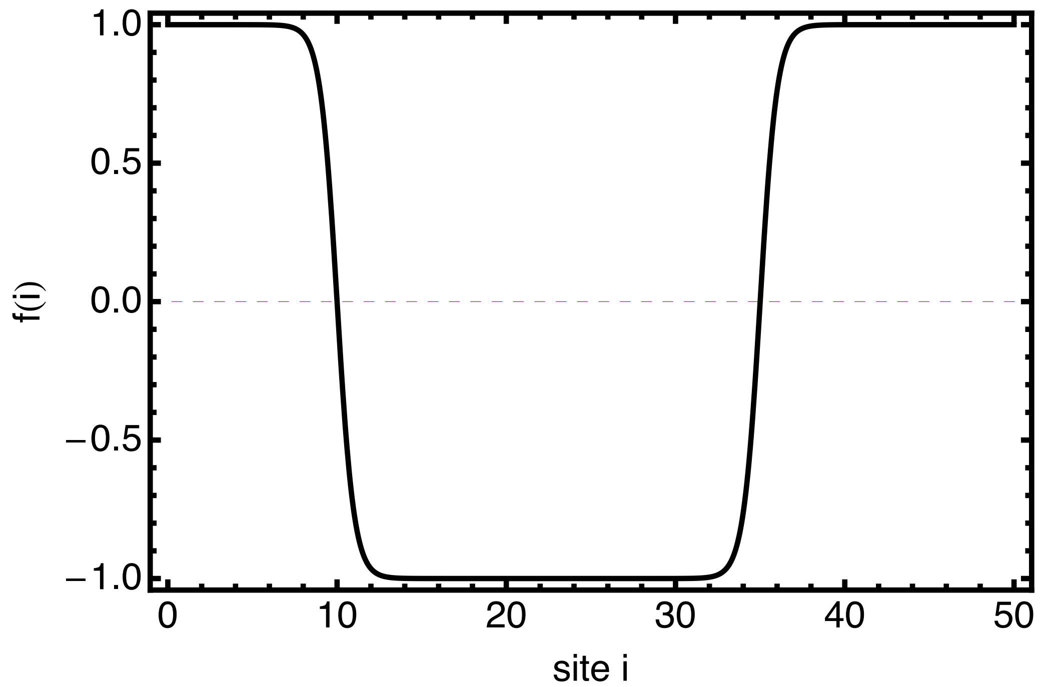

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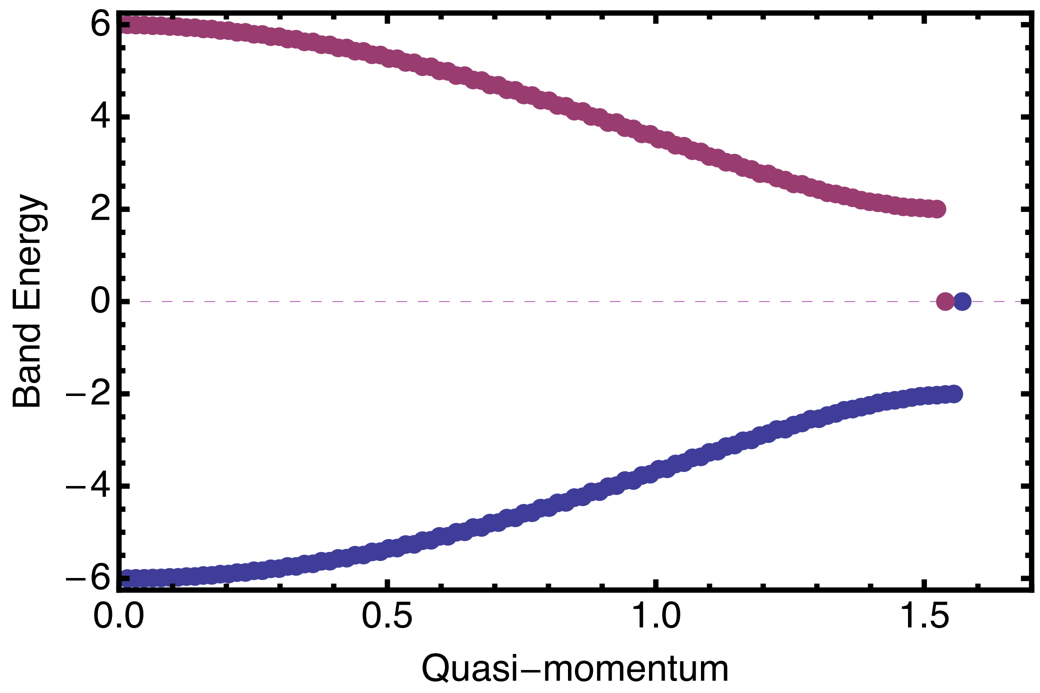

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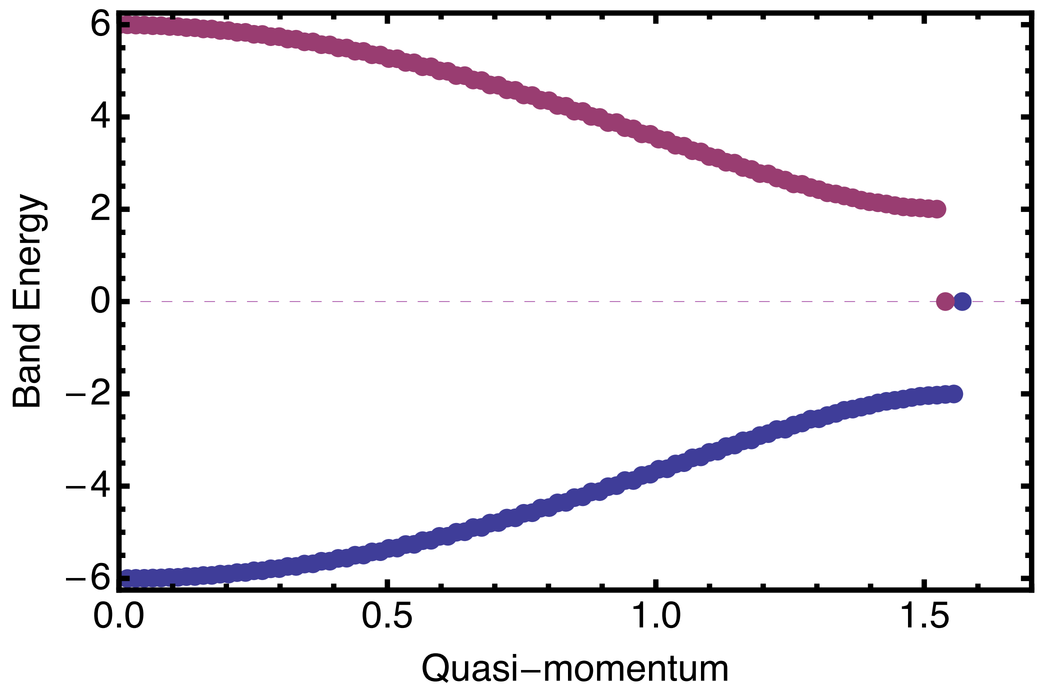

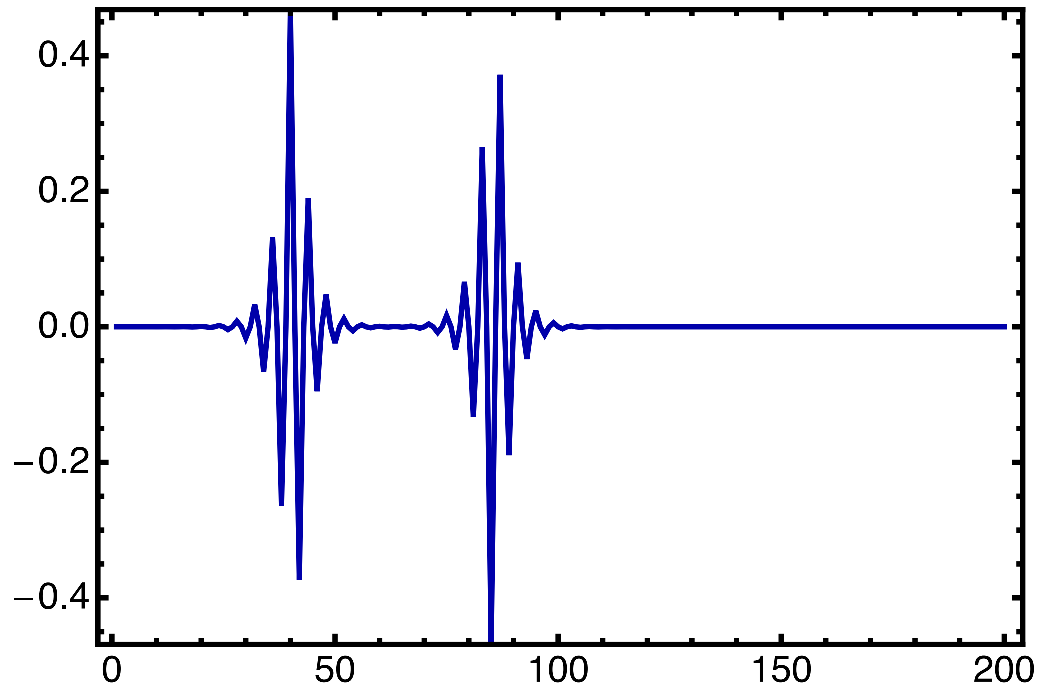

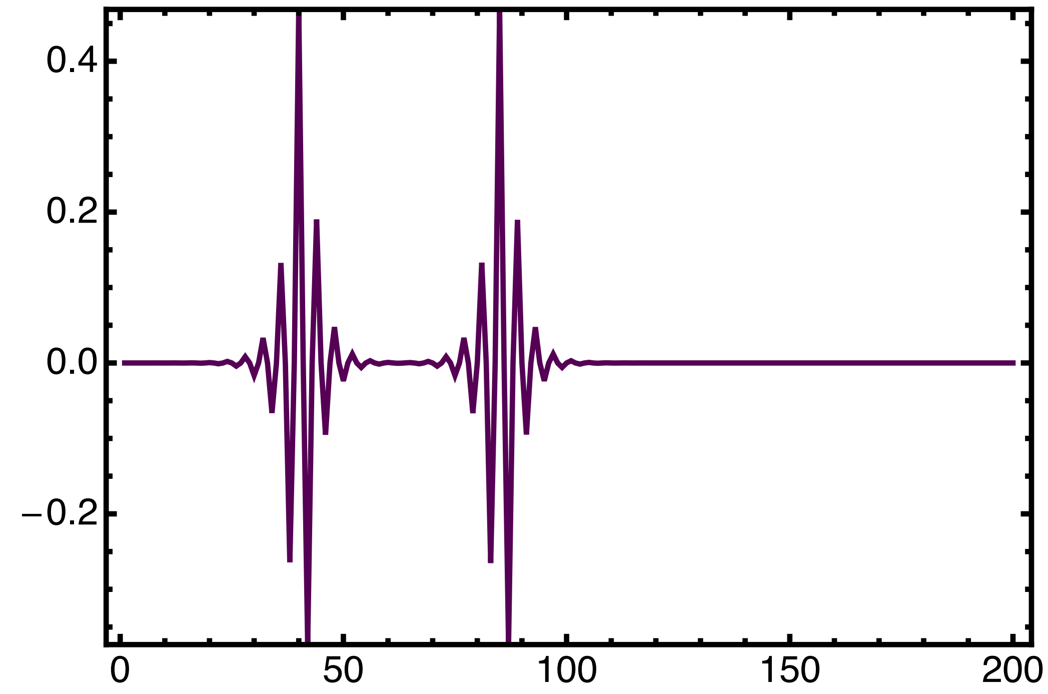

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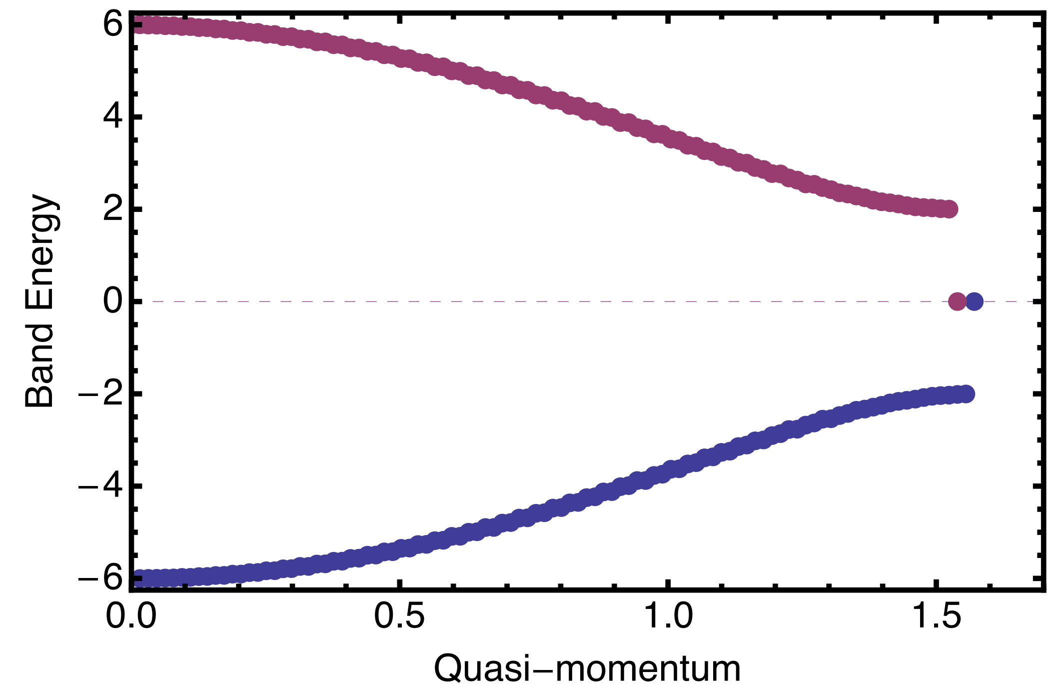

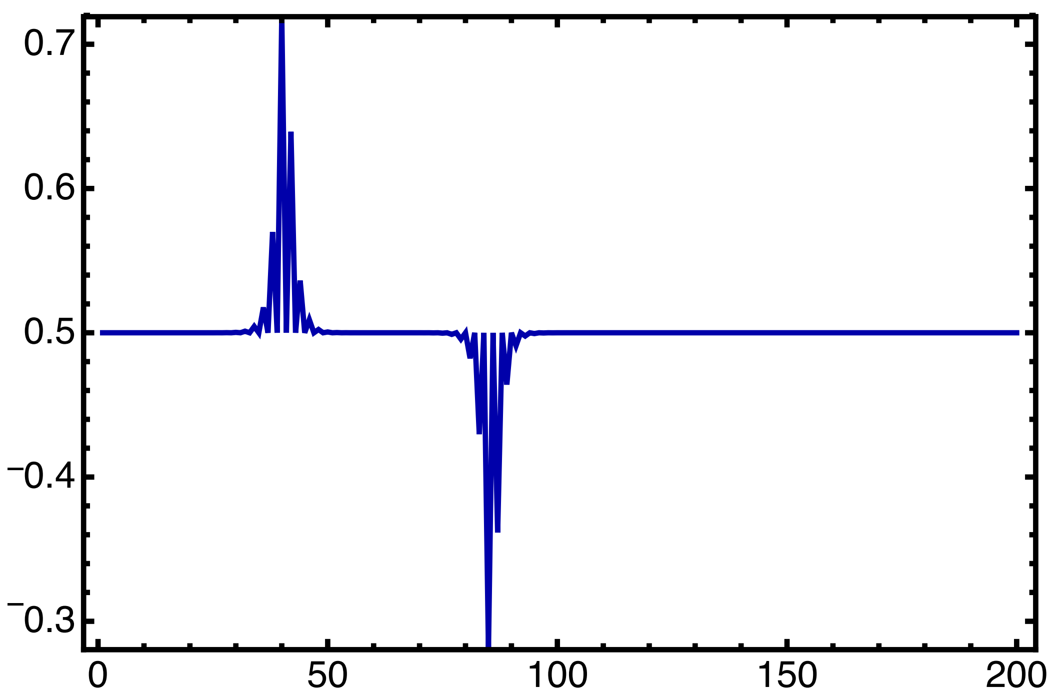

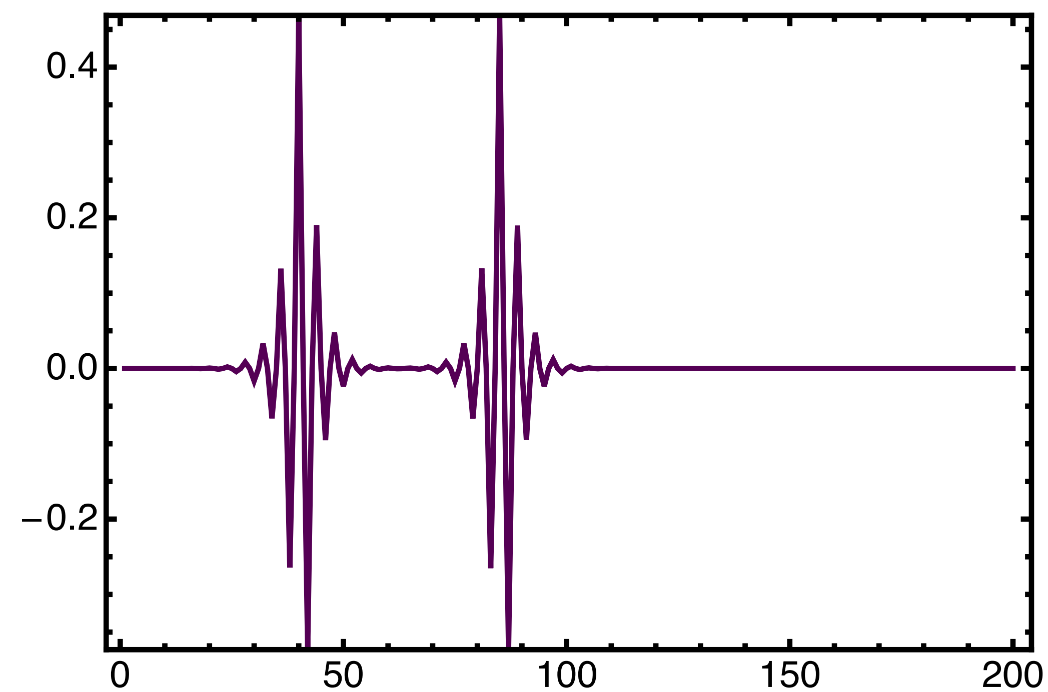

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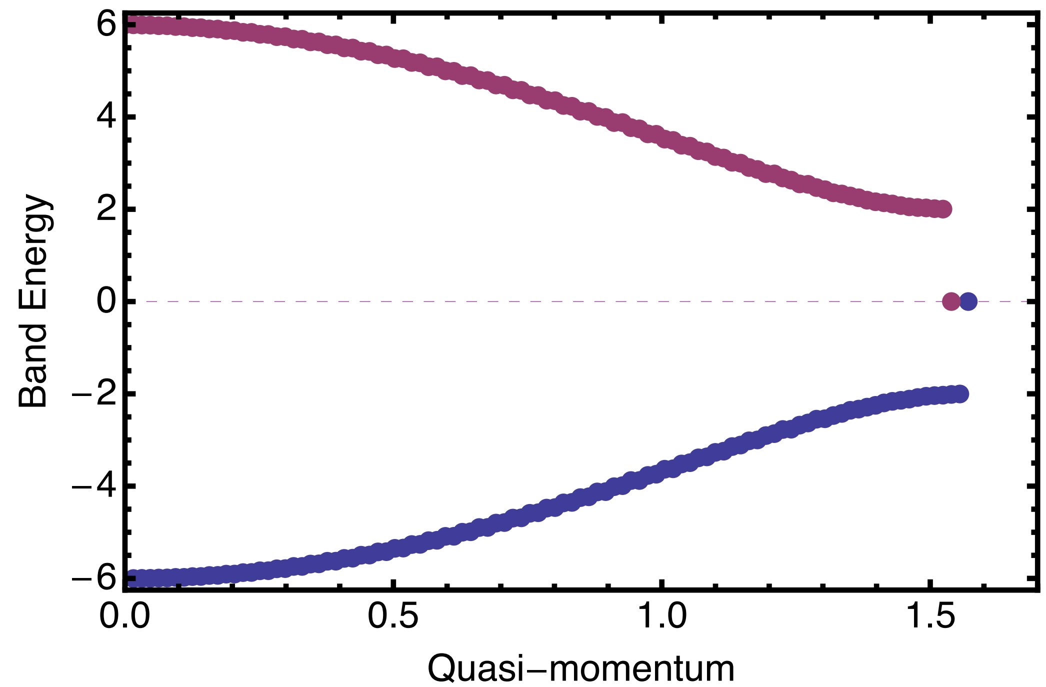

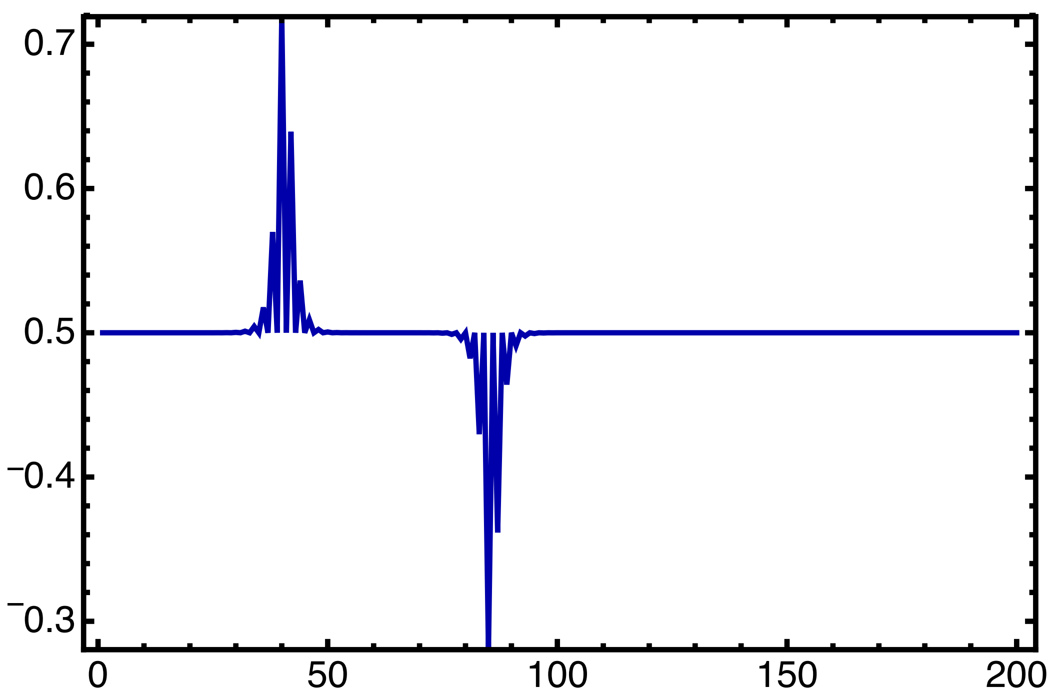

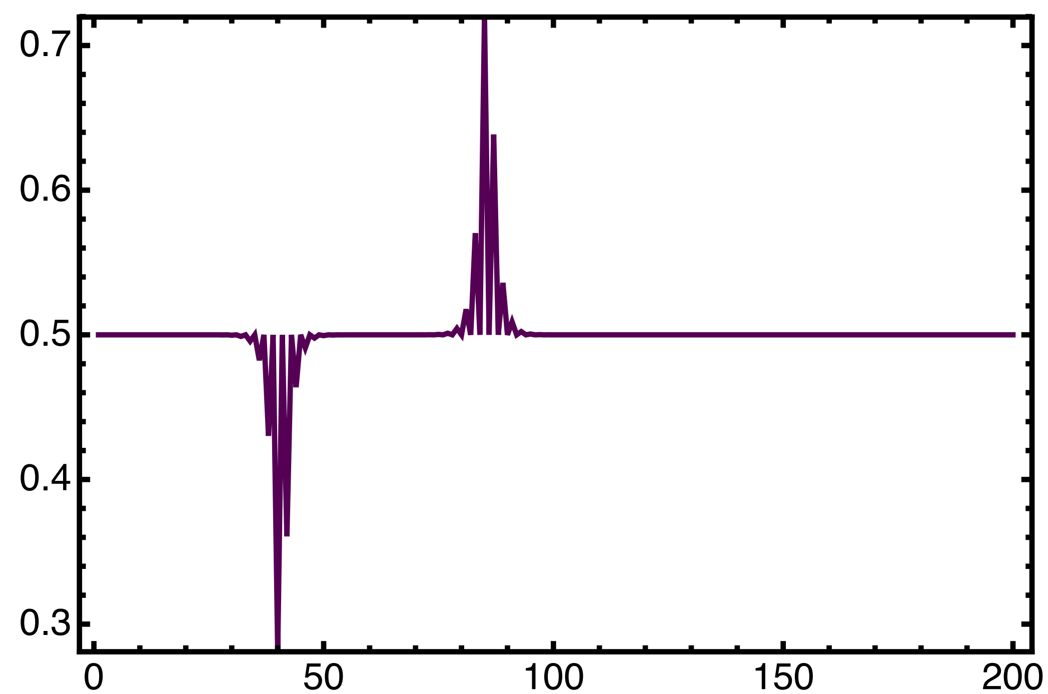

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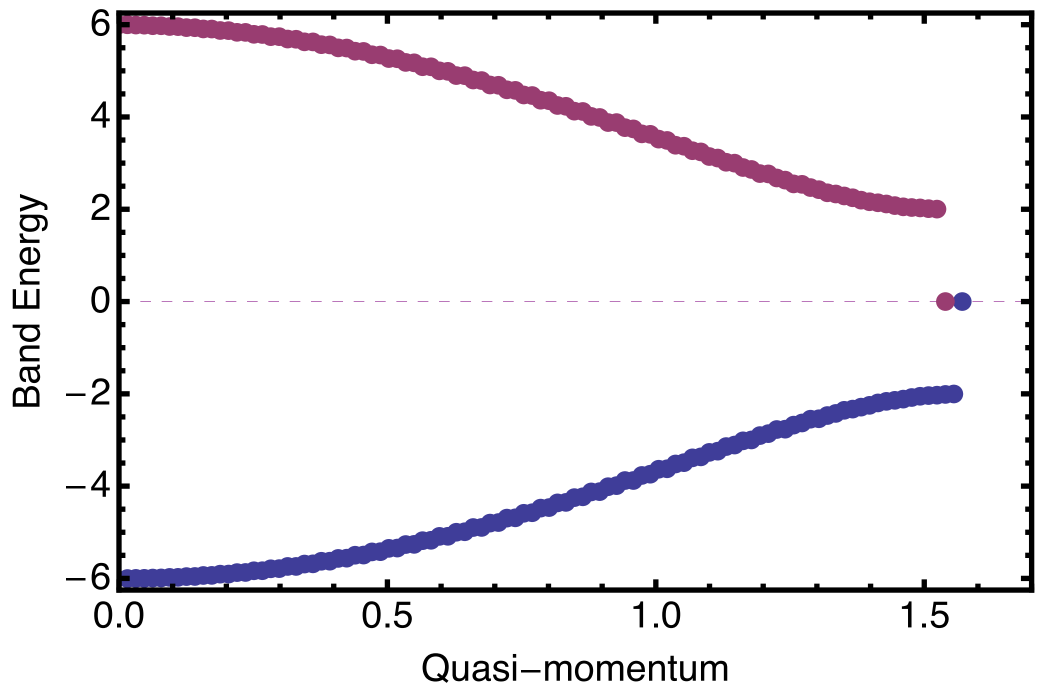
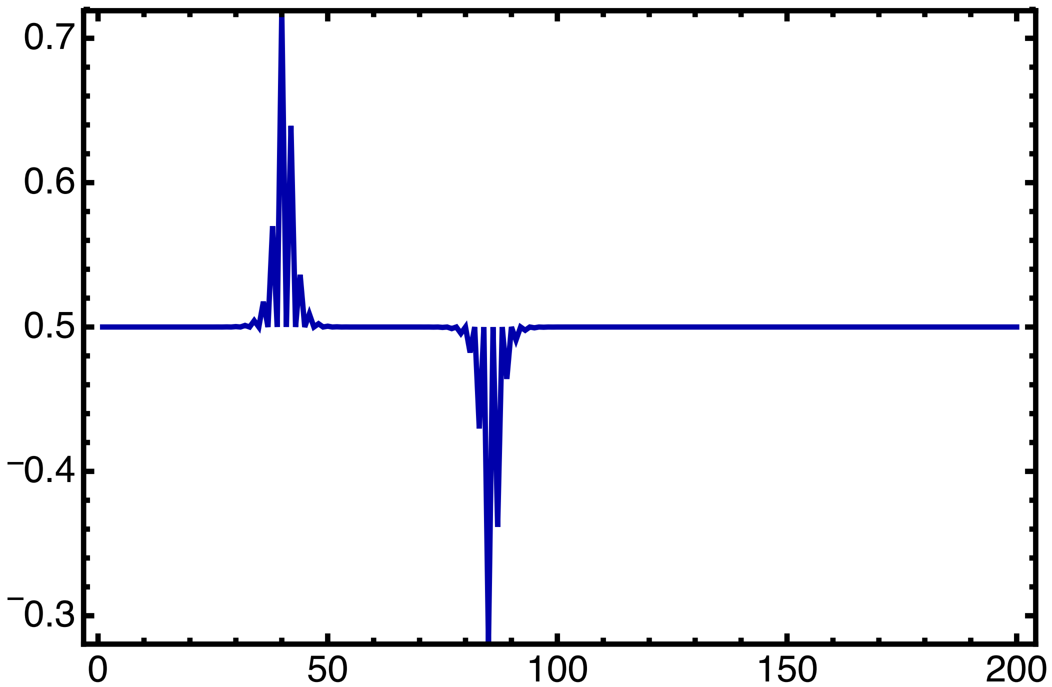
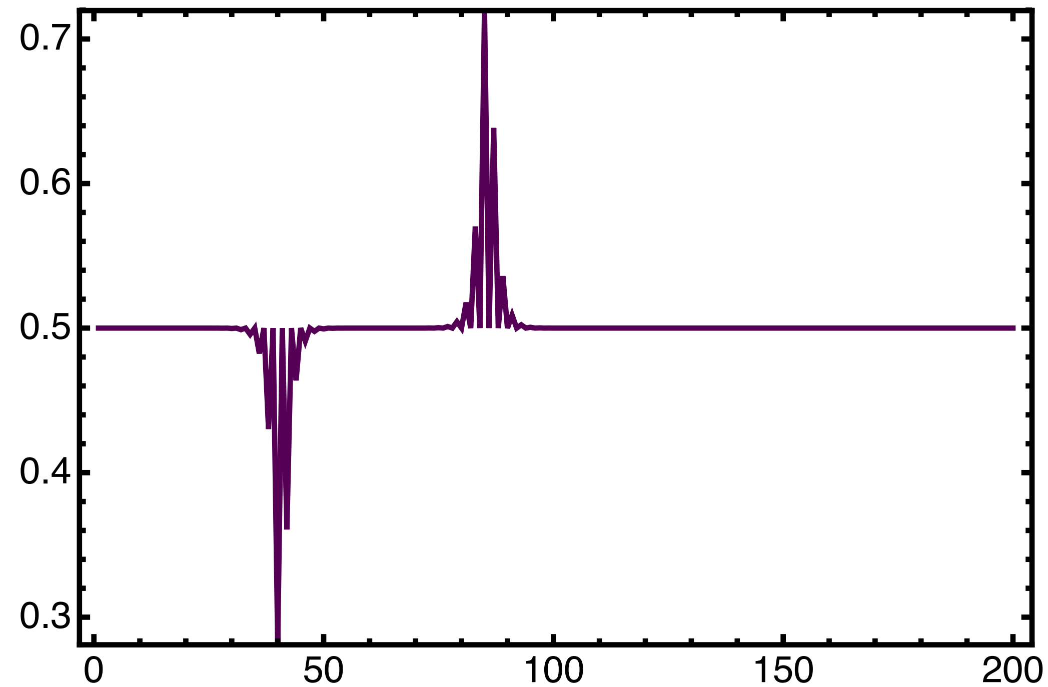




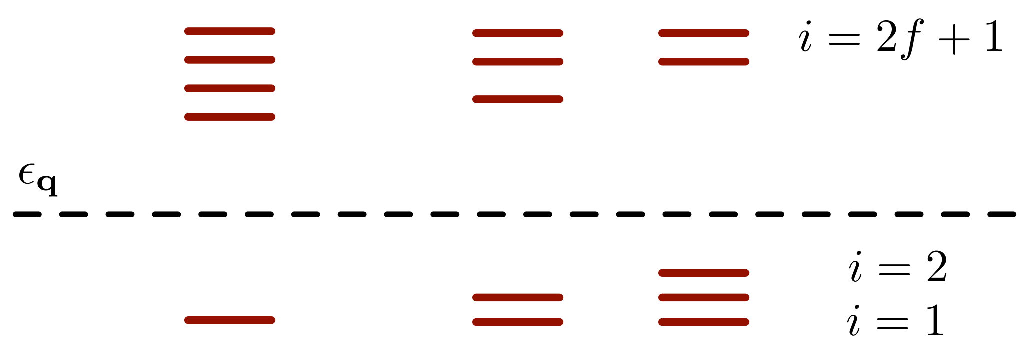


Can be used as a qubit. We have demonstrate all the necessary one qubit operation and CNOT gate !

What have we learned?

SUMMARY

A general way of looking at light induced abelian/non-abelian gauge fields (**N.B.** fermions)



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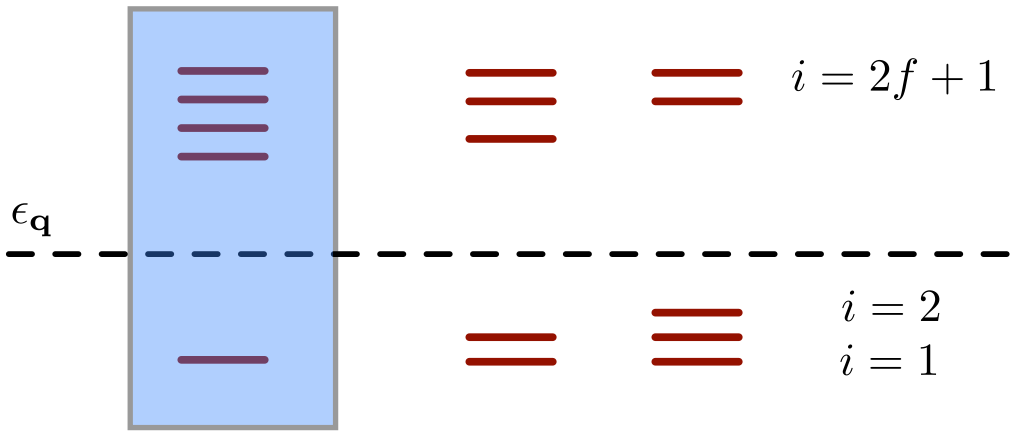

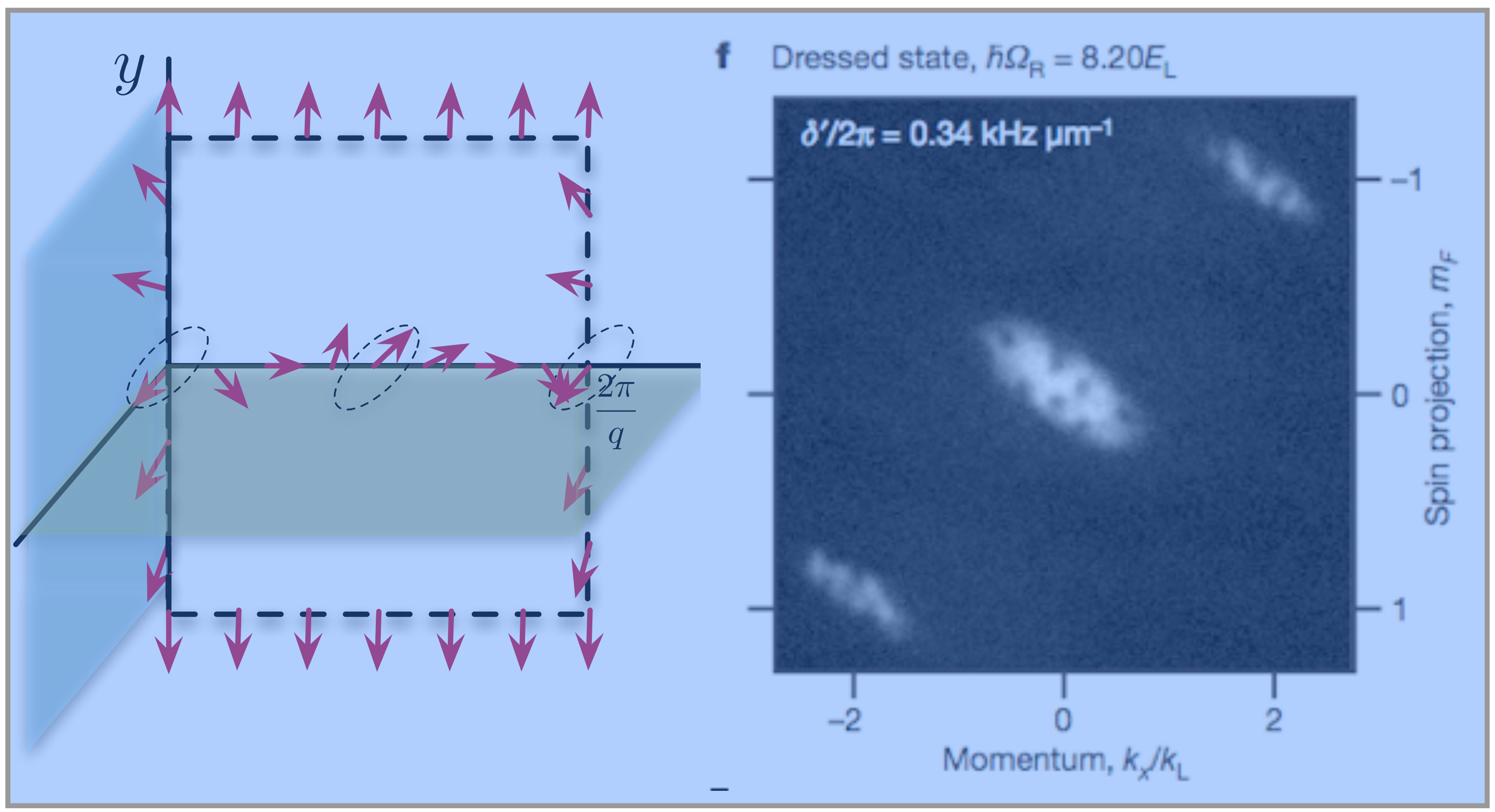

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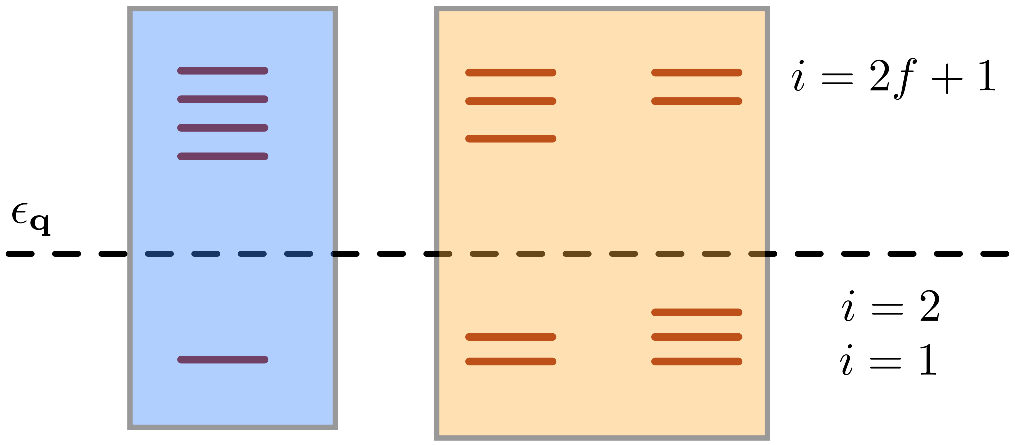

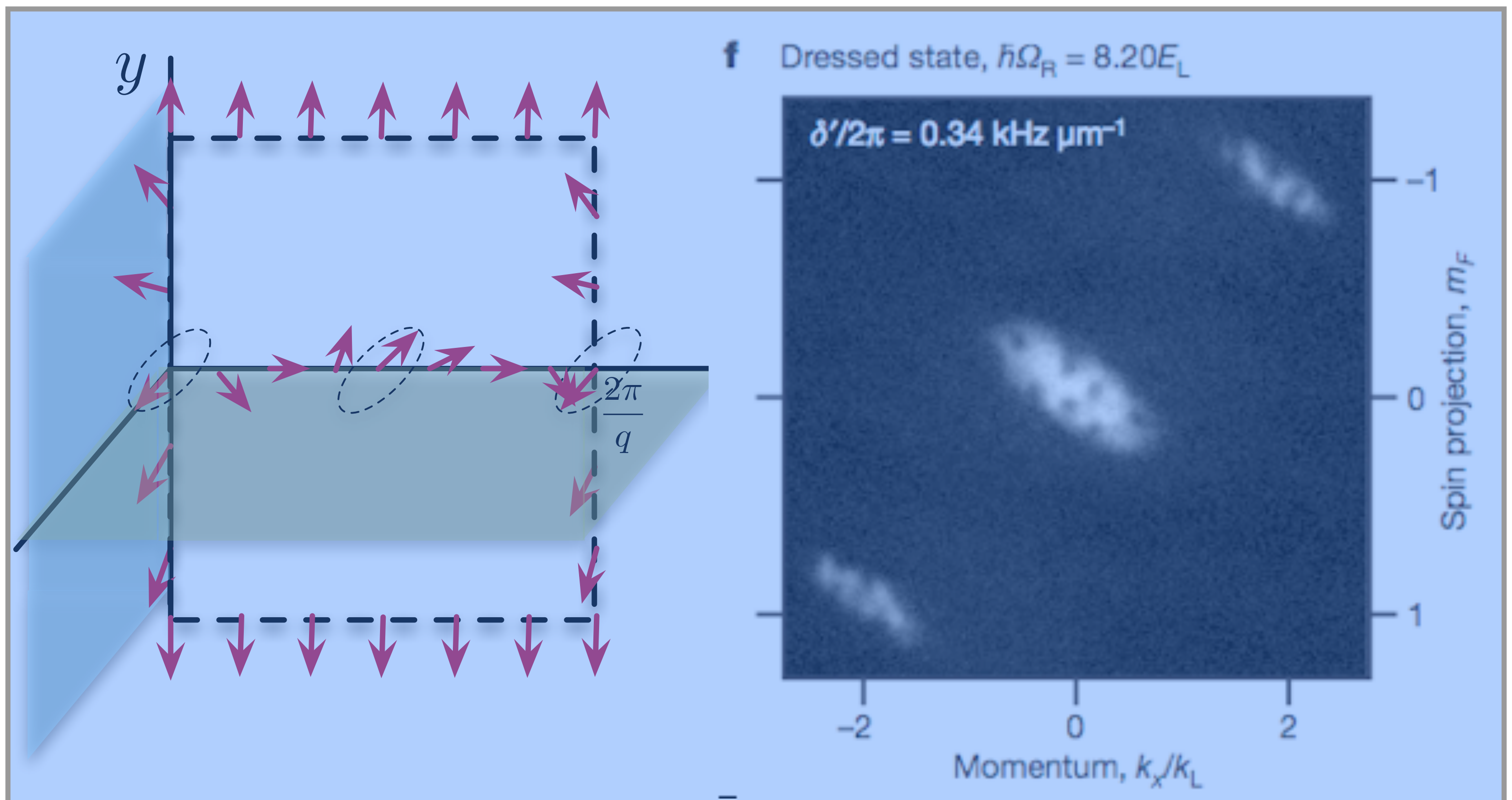

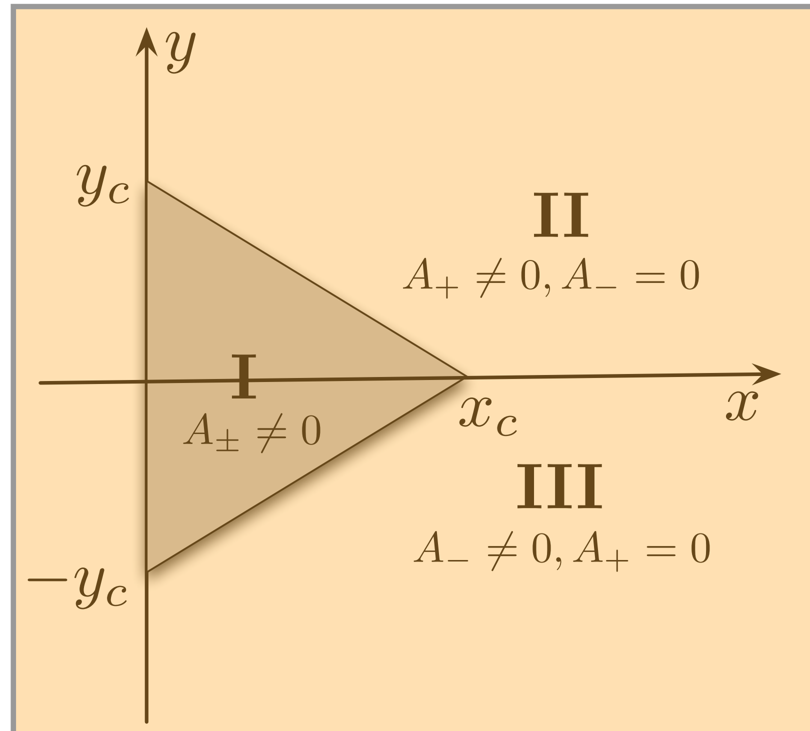

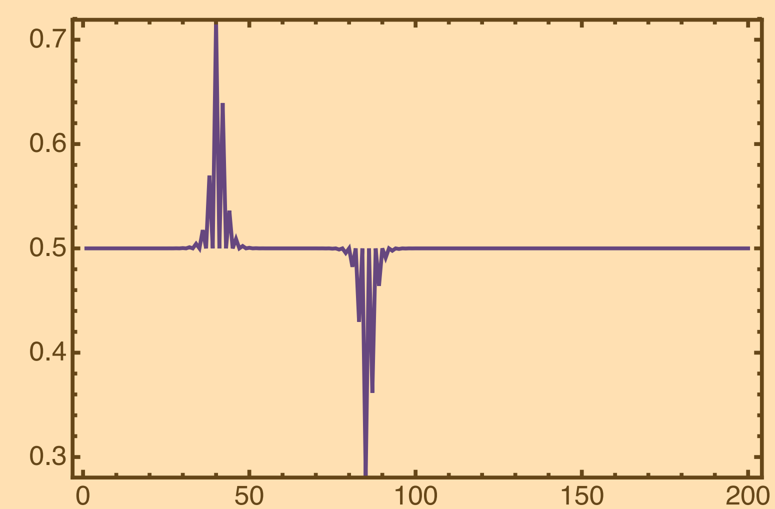

Future Directions

1. Crossover from abelian to non-abelian gauge field; melting of the vortex lattice;

2. Vortices in a spin-orbit coupled Bose-Einstein condensate;

3. Transport properties of bosons with spin-orbit interactions;

4. "Matter lattice" in high dimensions; electron-phonon system (BO approx.)

CONCLUSIONS

1. We discuss the general route to construct abelian/non-abelian gauge field.
Example: NIST experiment;

2. We worked out the phase diagram of the NIST experiment;

3. The condensate will develop appropriate density (spin) modulation in the presence of spin-orbit coupling

4. The possibility of generating Peierls distortion and as a result, fractionalized fermion;