Spatially Modulated Interaction and Dipolar Interaction Induced Resonances

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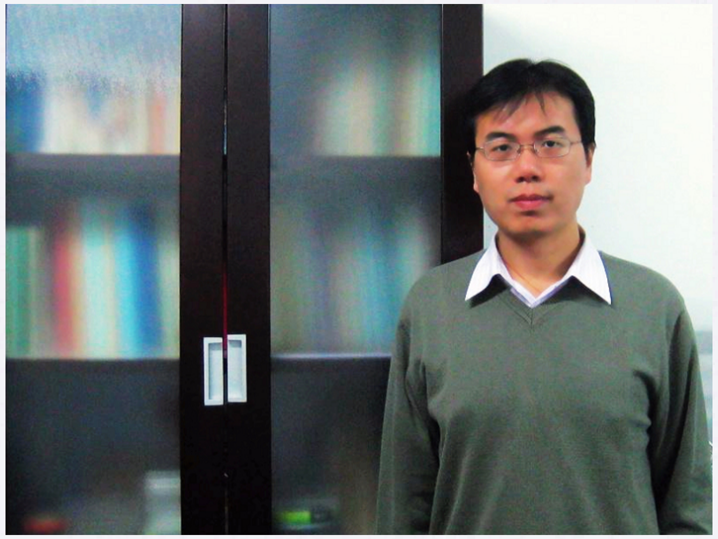
清华大学百年校庆
TSINGHUA UNIVERSITY
CENTENARY CELEBRATION

Three major recent topics:

1. Quantum gases in synthetic gauge field (with spin-orbit coupling)
2. New strongly interacting systems in quantum gases
3. Magnetism in quantum gases

Main Group Members

Ran Qi



Zeng-Qiang Yu

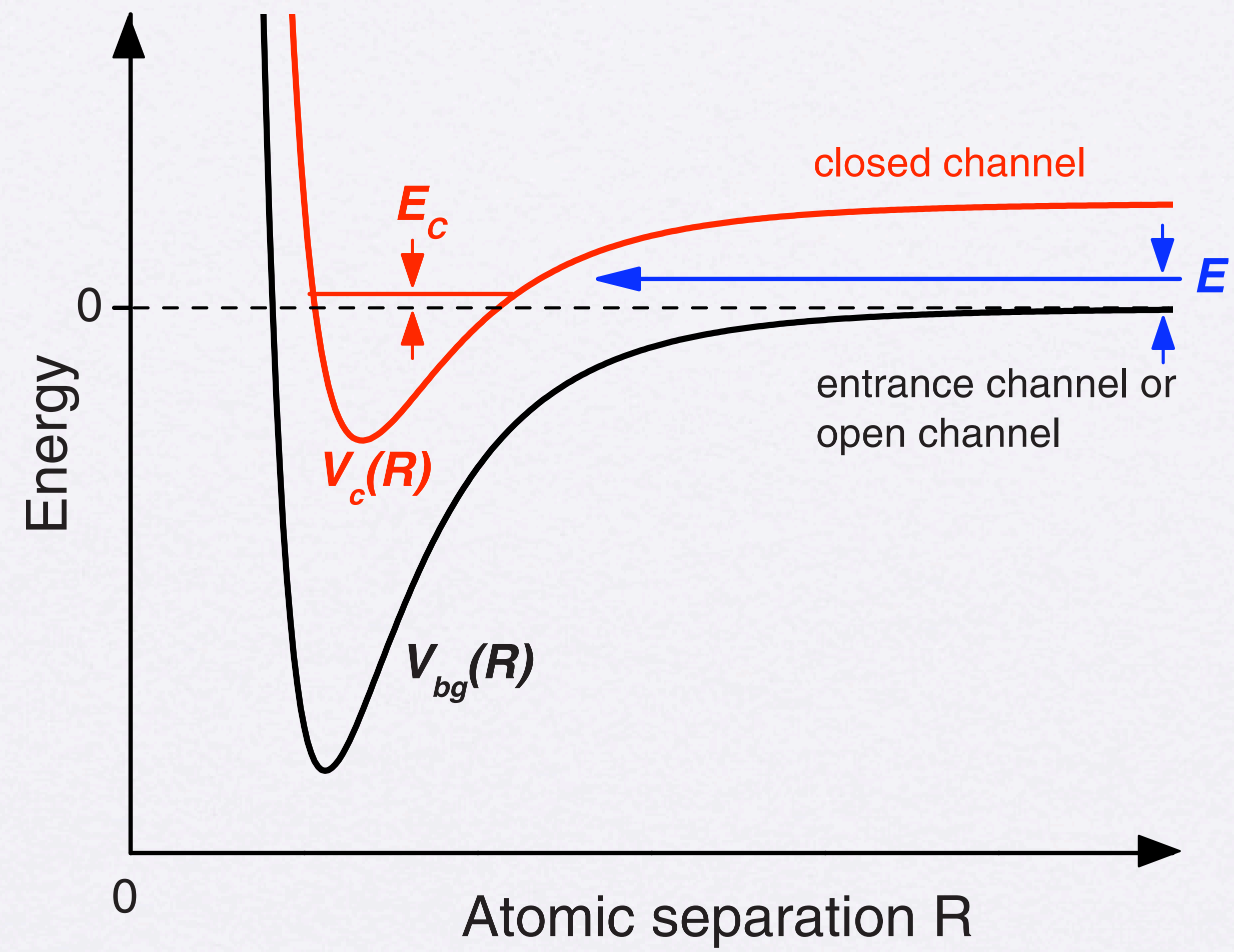
Xiaoling Cui

Chao Gao

Zheyu Shi

Chao-Ming Jian

Feshbach resonance is an important tool to achieve strong interactions in ultracold Fermi gases



magnetic Feshbach resonance; optical Feshbach resonance; confinement induced resonance

New Way to achieve scattering resonance in New cold atom systems with Novel features

A: Spatially modulated interaction induced resonance

Alkali-earth-(like) atomic gases

B: Strong dipolar interaction induced resonace

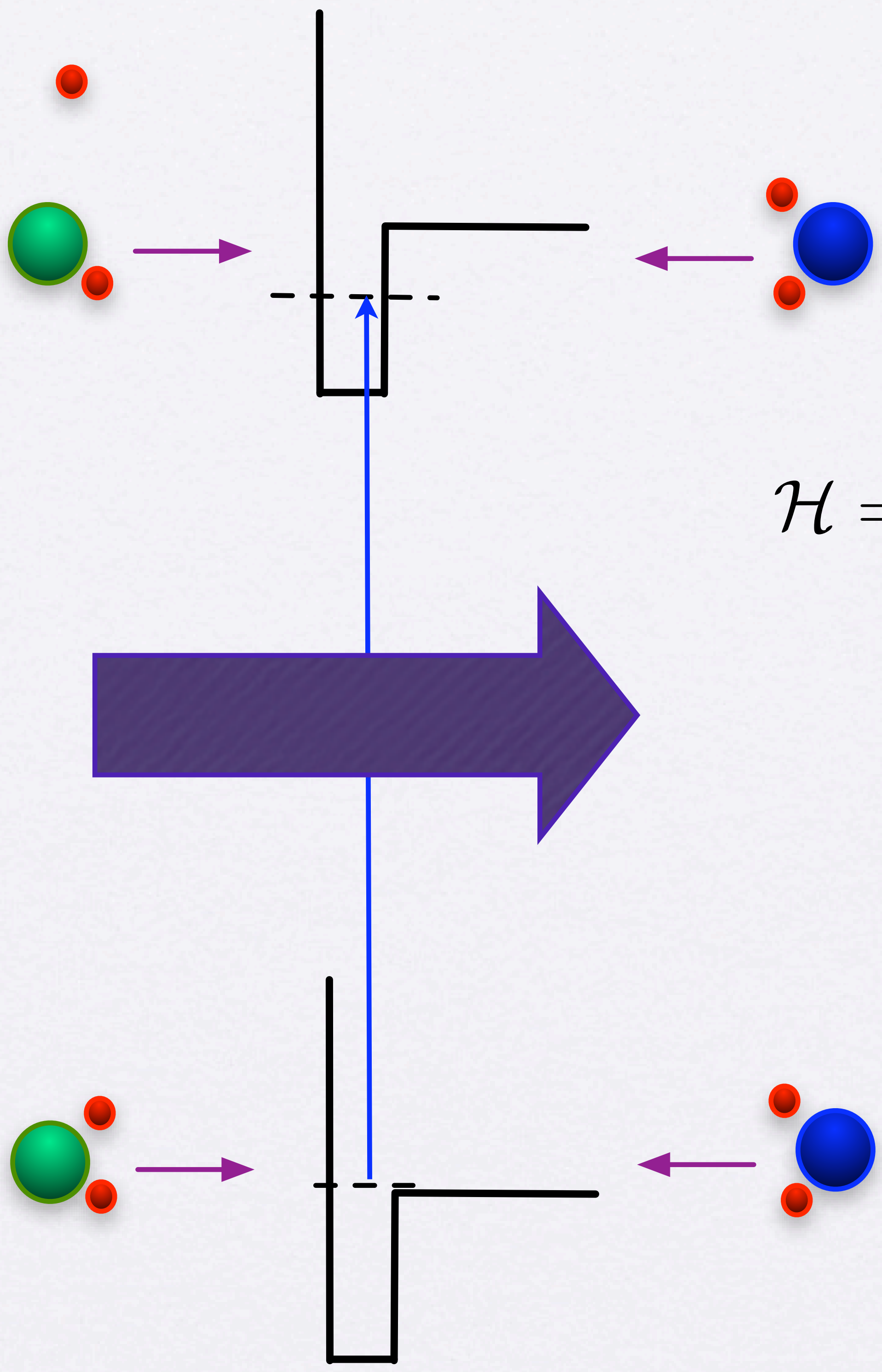
Polar molecular gases

New Way to achieve scattering resonance in New systems with New features

A: Spatially modulated interaction induced resonance

Alkali-earth-(like) atomic gases

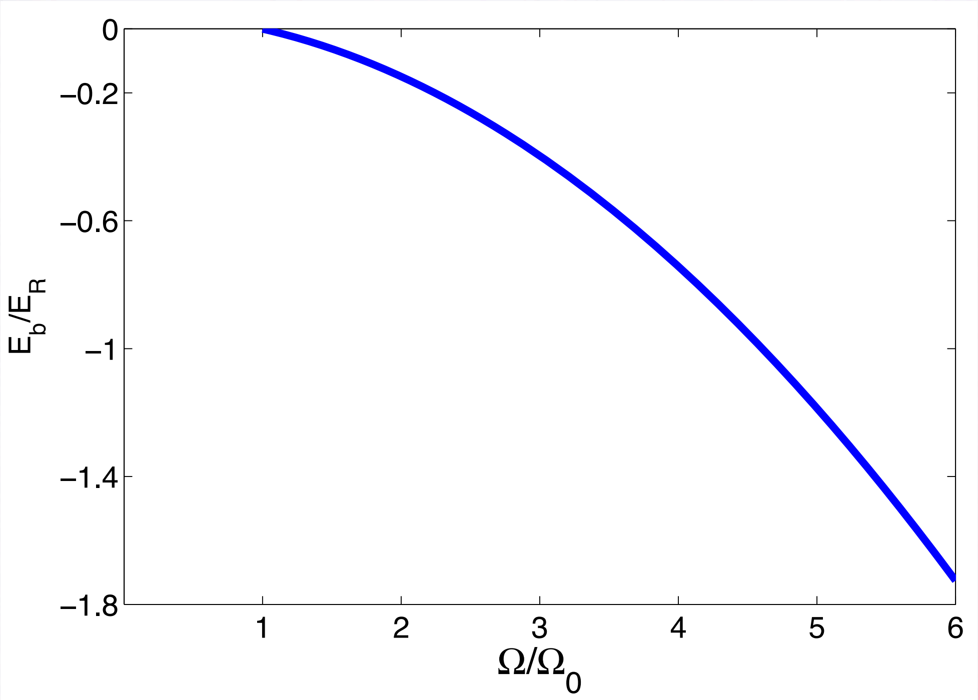
Idea of Optical Feshbach resonance



$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m}\nabla^2_{\mathbf{r}} + v(\mathbf{r})$$

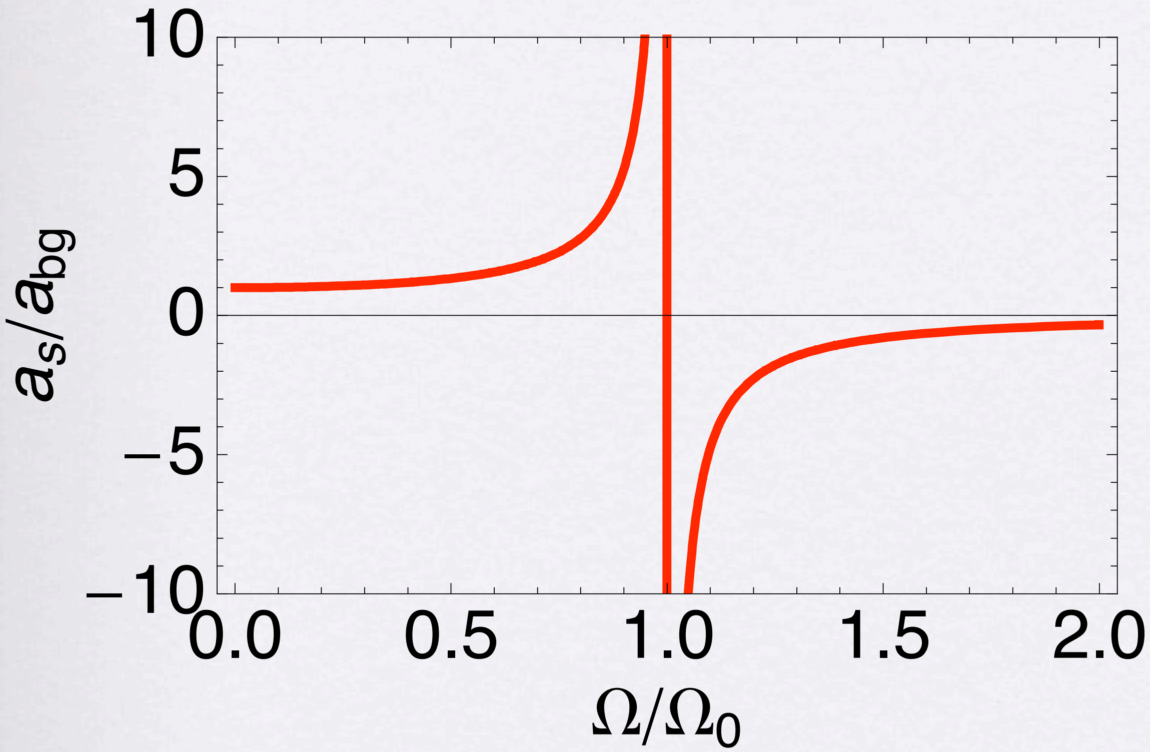
$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega \\ \hbar\Omega & -V_c \end{pmatrix}$$

Optical Feshbach resonance



$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m}\nabla^2_{\mathbf{r}} + v(\mathbf{r})$$

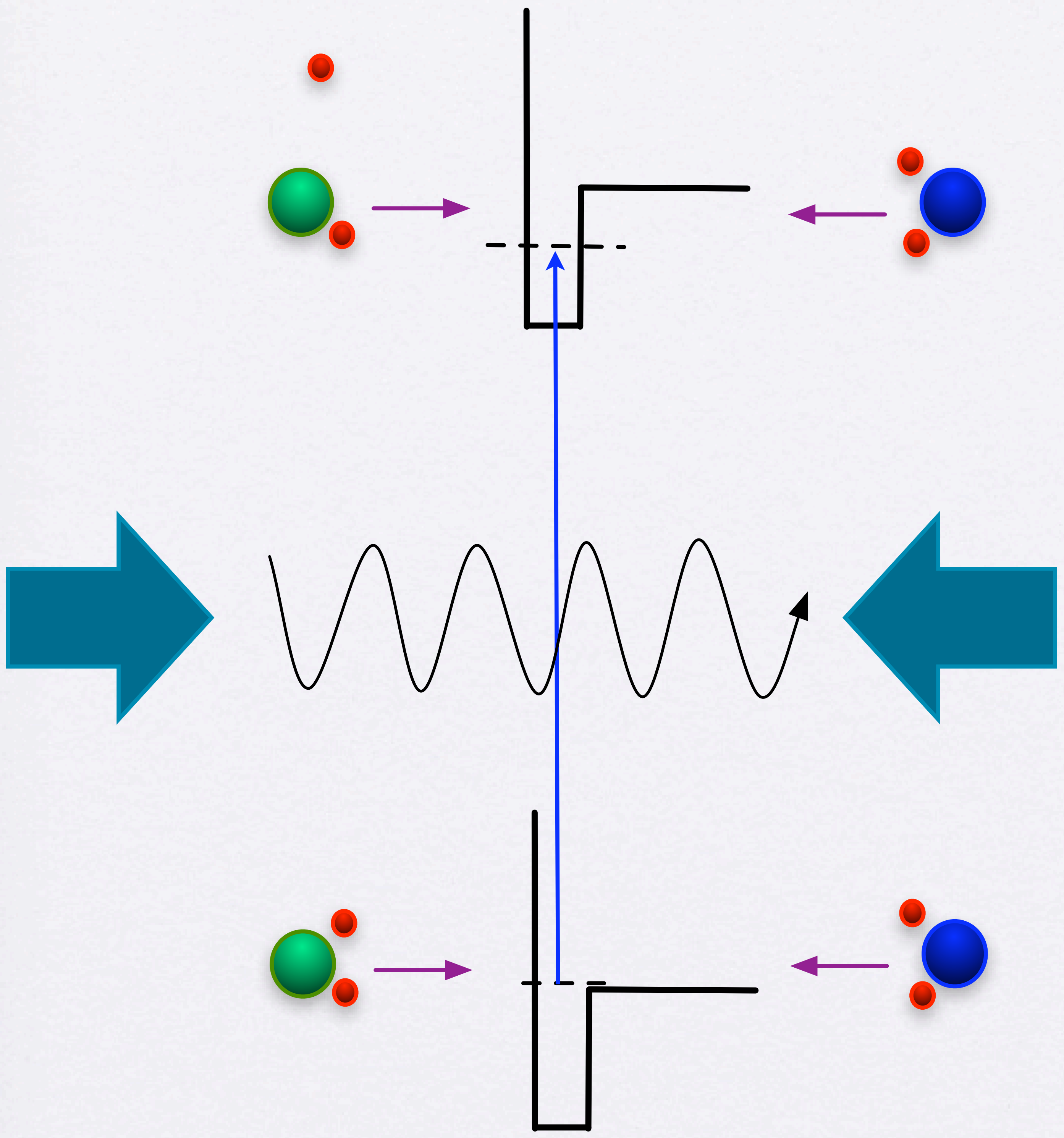
$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega \\ \hbar\Omega & -V_c \end{pmatrix}$$



$$a_{\mathrm{s}} = a_{\mathrm{bg}}\left(1 - \frac{\Omega^2}{\Omega^2 - \Omega_0^2}\right)$$

C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. 82, 1225 (2010).

Optical Feshbach resonance with Standing wave

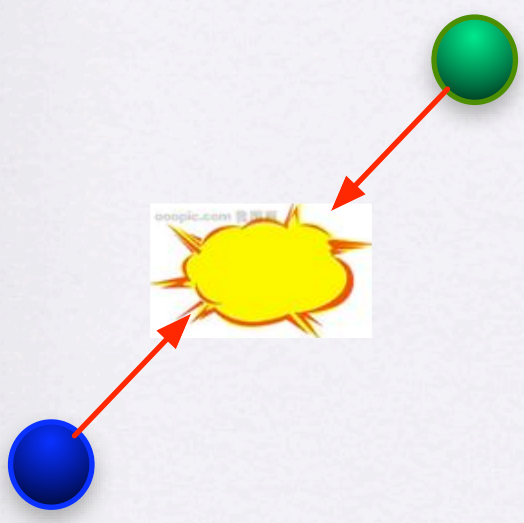


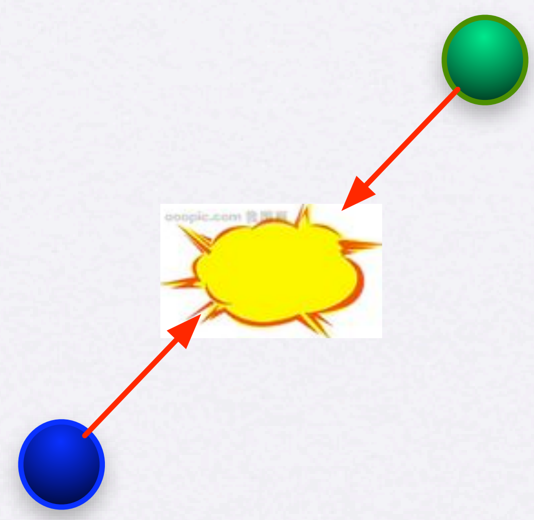
Spatial dependent interaction

Two-body interaction potential:

$$V(\mathbf{r_1}, \mathbf{r_2}) = V(\mathbf{r_1} - \mathbf{r_2})$$

Spatial independent





Spatial dependent interaction

Two-body interaction potential:

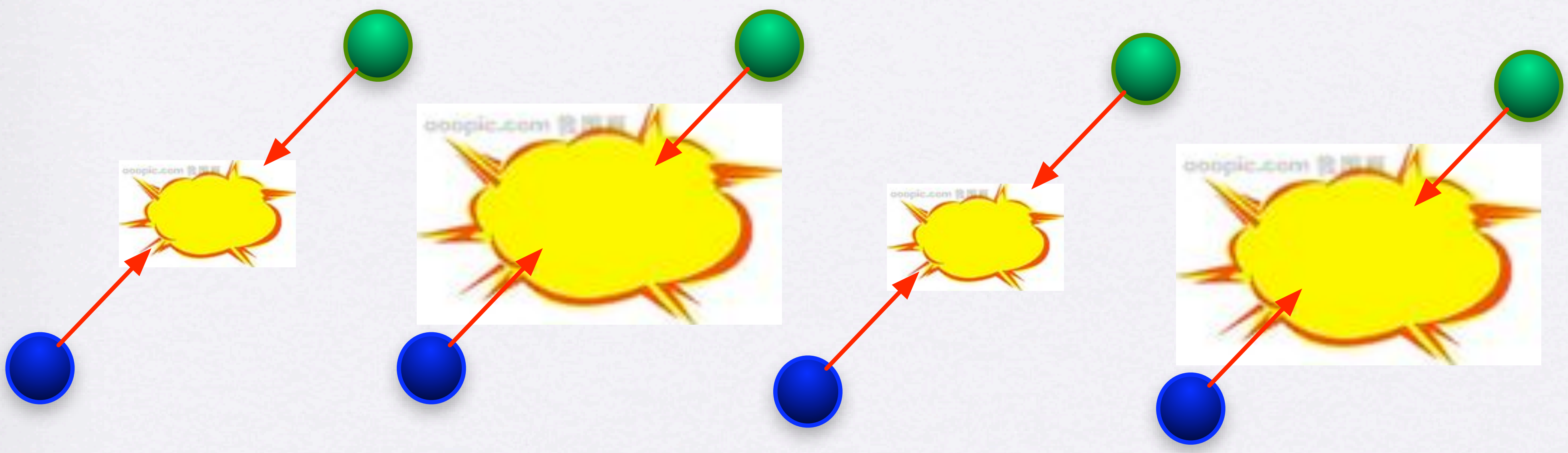
$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m}\nabla^2_{\mathbf{r}} + v(\mathbf{r}, \mathbf{R})$$

$$V(\mathbf{r_1}, \mathbf{r_2}) = V\left(\mathbf{r_1} - \mathbf{r_2}, \frac{\mathbf{r_1} + \mathbf{r_2}}{2}\right)$$

$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega(\mathbf{R}) \\ \hbar\Omega(\mathbf{R}) & -V_c \end{pmatrix}$$

Spatially periodically modulated

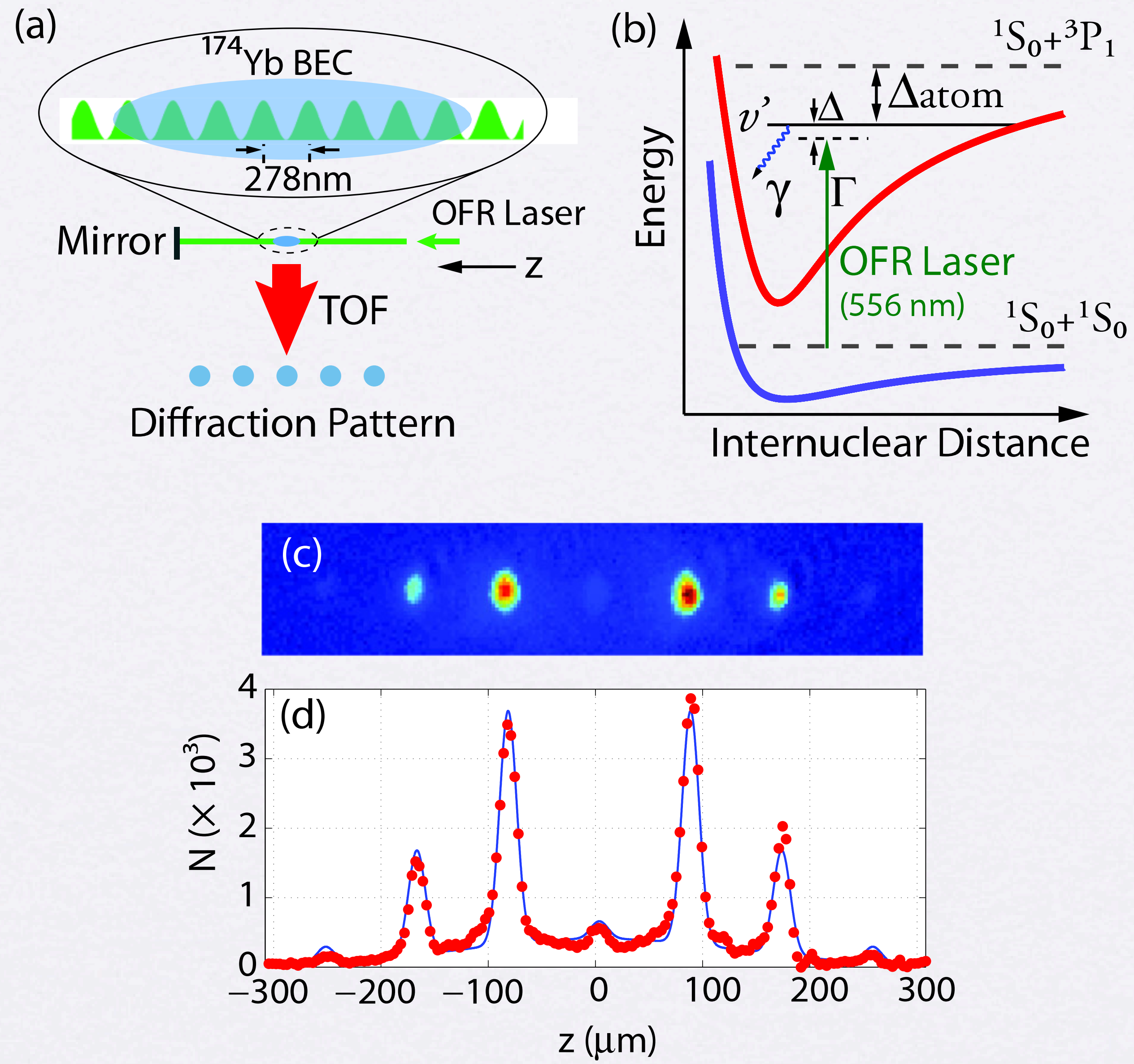
$$\Omega(\mathbf{R}) = \Omega\cos(Kx)$$



$a_{\mathrm{s}}(x)$ **is spatially dependent and modulates periodically in space**

Experimental Realization

Submicron spatial modulation of an interatomic interaction in a Bose-Einstein condensate, PRL, 105, 050405 (2010) Kyoto group



How $a_{\mathrm{s}}(x)$ modulates in space?

$$a_{\mathrm{s}} = a_{\mathrm{bg}}\left(1 - \frac{\Omega^2}{\Omega^2 - \Omega_0^2}\right) \qquad \Omega(\mathbf{R}) = \Omega\cos(Kx)$$

$$a_{\mathrm{s}}(x) = a_{\mathrm{bg}}\left(1 - \frac{\Omega^2\cos^2(Kx)}{\Omega^2\cos^2(Kx) - \Omega_0^2}\right)$$

?

Any other physics effects?

What we have done:
Solve two-body problem of this Hamiltonian

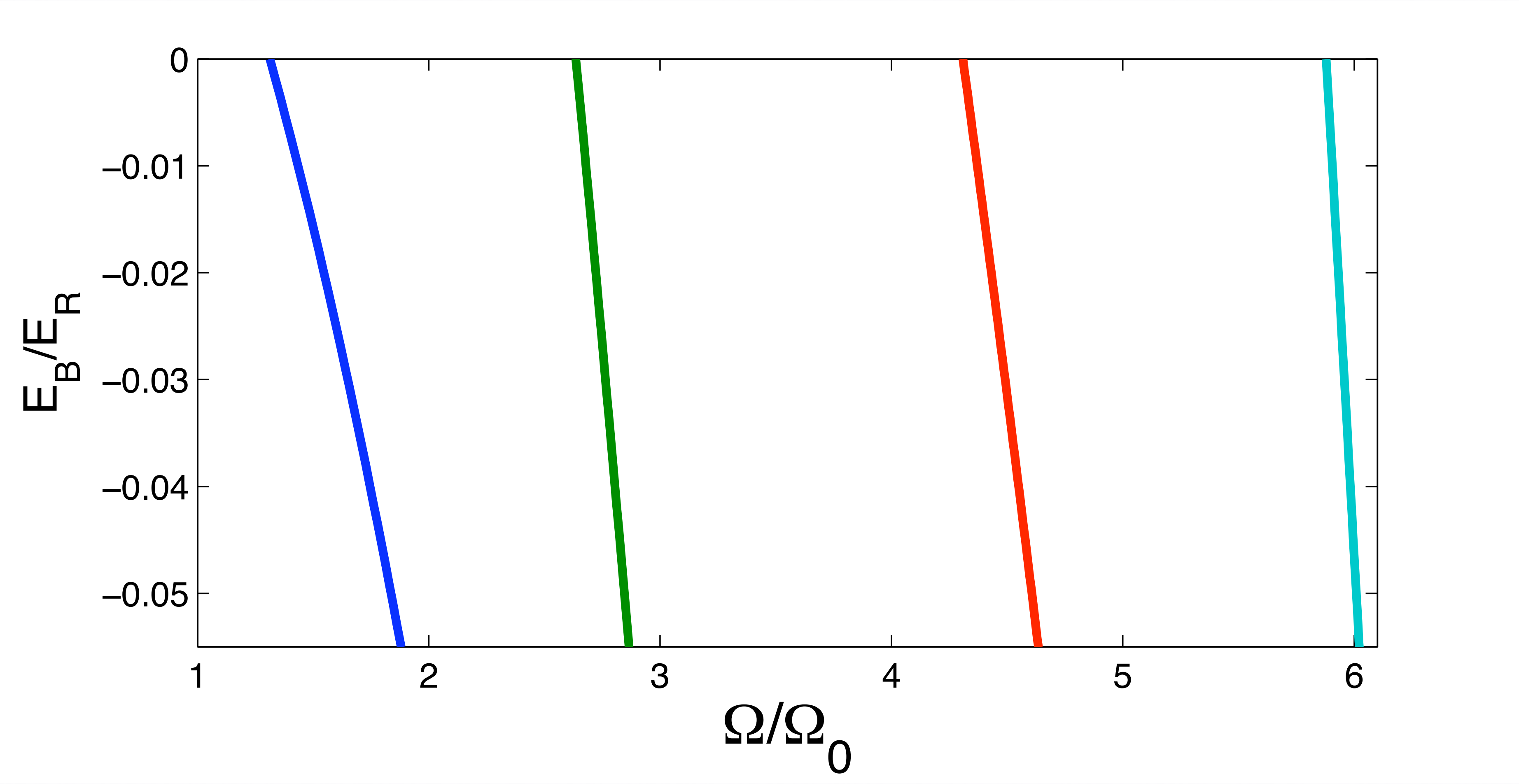
$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m}\nabla^2_{\mathbf{r}} + v(\mathbf{r})$$

$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega(\mathbf{R}) \\ \hbar\Omega(\mathbf{R}) & -V_c \end{pmatrix}$$

$$\Omega(\mathbf{R}) = \Omega\cos(Kx)$$

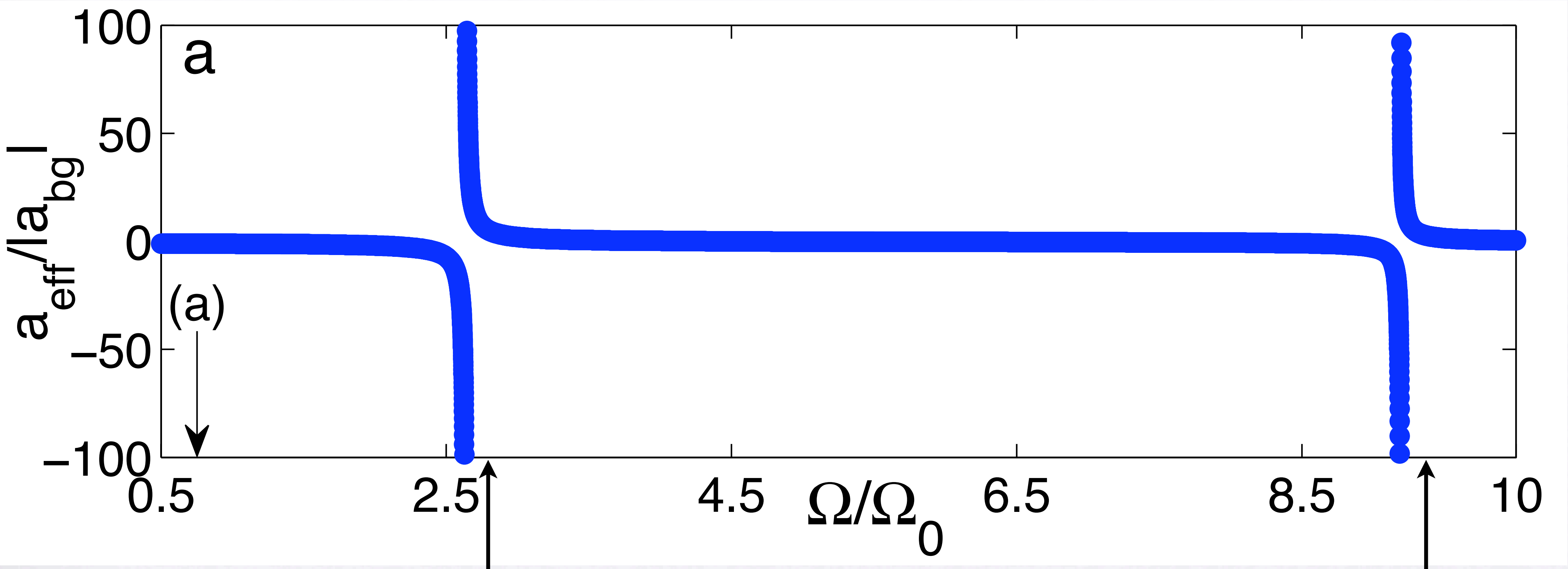
Qi Ran and HZ, arXiv: 1101.4464

Results I: Bound States



Results II: Scattering Resonances

$$a_{\text{eff}} = \lim_{k \to 0} \frac{\tan \delta(k)}{k}$$



Resonances

Resonances

Strongly interacting many-body system !!
Universal Behavior ?

Results III: Local Scattering Length
--- related to local interaction energy

Bethe-Peierls condition:

$$\lim_{r\to 0}\psi(r,x) = \frac{1}{r} - \frac{1}{a_{loc}(x)}$$

Local scattering length

$$a_{\rm loc}(x) = -\lim_{r\to r_0}\frac{r\psi_{\rm o}(x,r)}{\partial_r(r\psi_{\rm o}(x,r))}$$

The mean-field energy for a BEC:

$$\mathcal{E} = \int dx\left[-\frac{\hbar^2}{2m}\varphi^*\nabla^2\varphi + \frac{4\pi\hbar^2}{m}a_{loc}(x)n^2(x)\right]$$

Results III: Local Scattering Length

Exact formula:

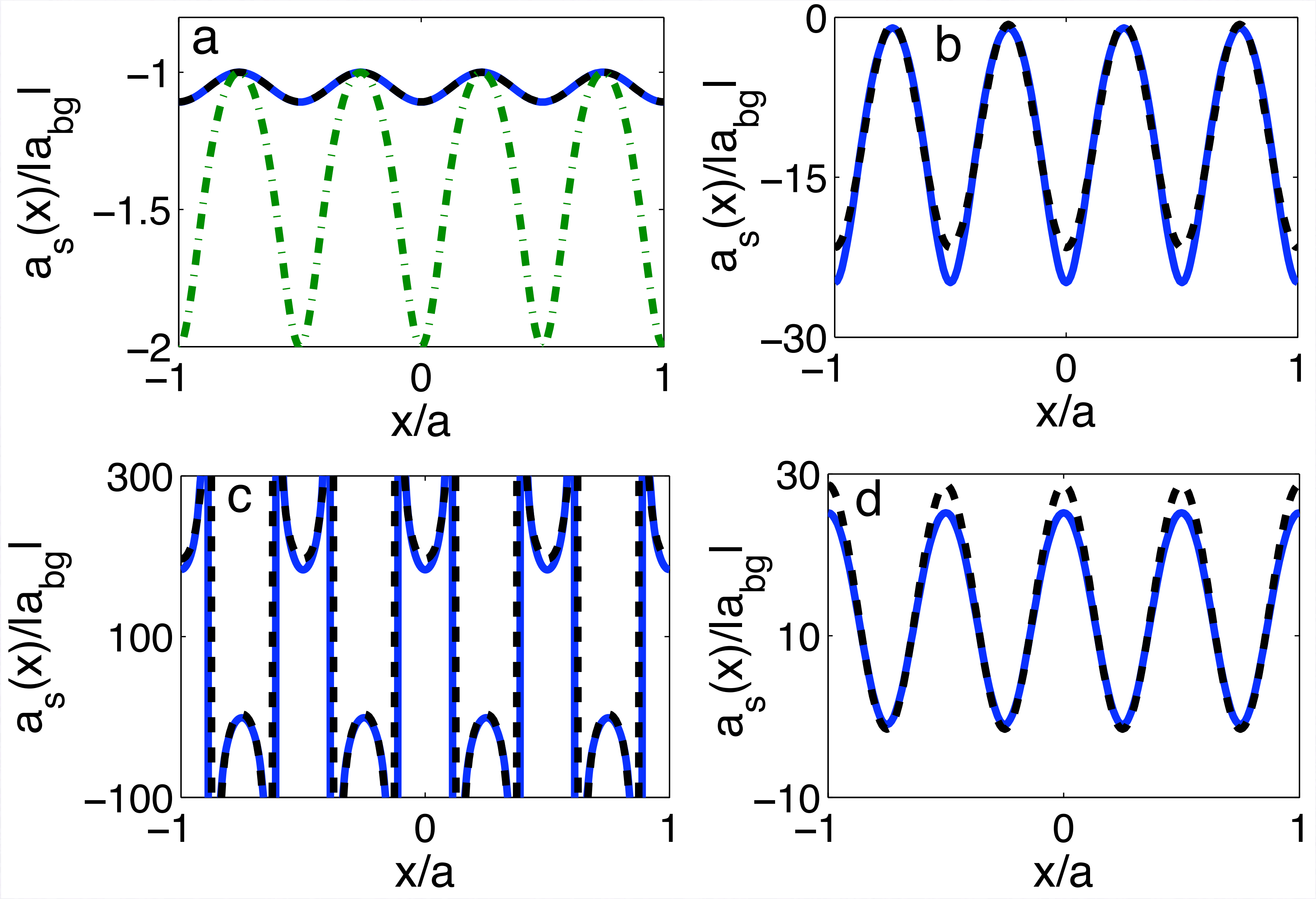
$$a_{\rm loc}(x) = \frac{1 - \sum_{m\neq 0} U_m \cos(mKx)/U_0}{a_{\rm eff}^{-1} - \sum_{m\neq 0} U_m |m| K \cos(mKx)/(2U_0)}$$

Simplified formula

$$K a_{\rm eff} \ll 1 \quad a_{\rm loc}(x) = a_{\rm eff}\left[1 - \frac{2U_2}{U_0}\cos(2Kx)\right]$$

$$K a_{\rm eff} \gg 1 \quad a_{\rm loc}(x) = \frac{1}{K}\left[1 - \frac{U_0}{2U_2\cos(2Kx)}\right]$$

Results III: Local Scattering Length



Summary:
Take Home Message

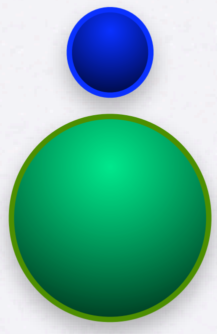
New Mechanism	New System	New Features
Two-body interaction potential has center-of-mass dependence	Alkali-earth-(like) atomic gases: Sr, Ca, Yb	Spatially dependent local scattering length

New Way to achieve scattering resonance
in New systems
with New features

B: Strong dipolar interaction induced resonace

Polar molecular gases

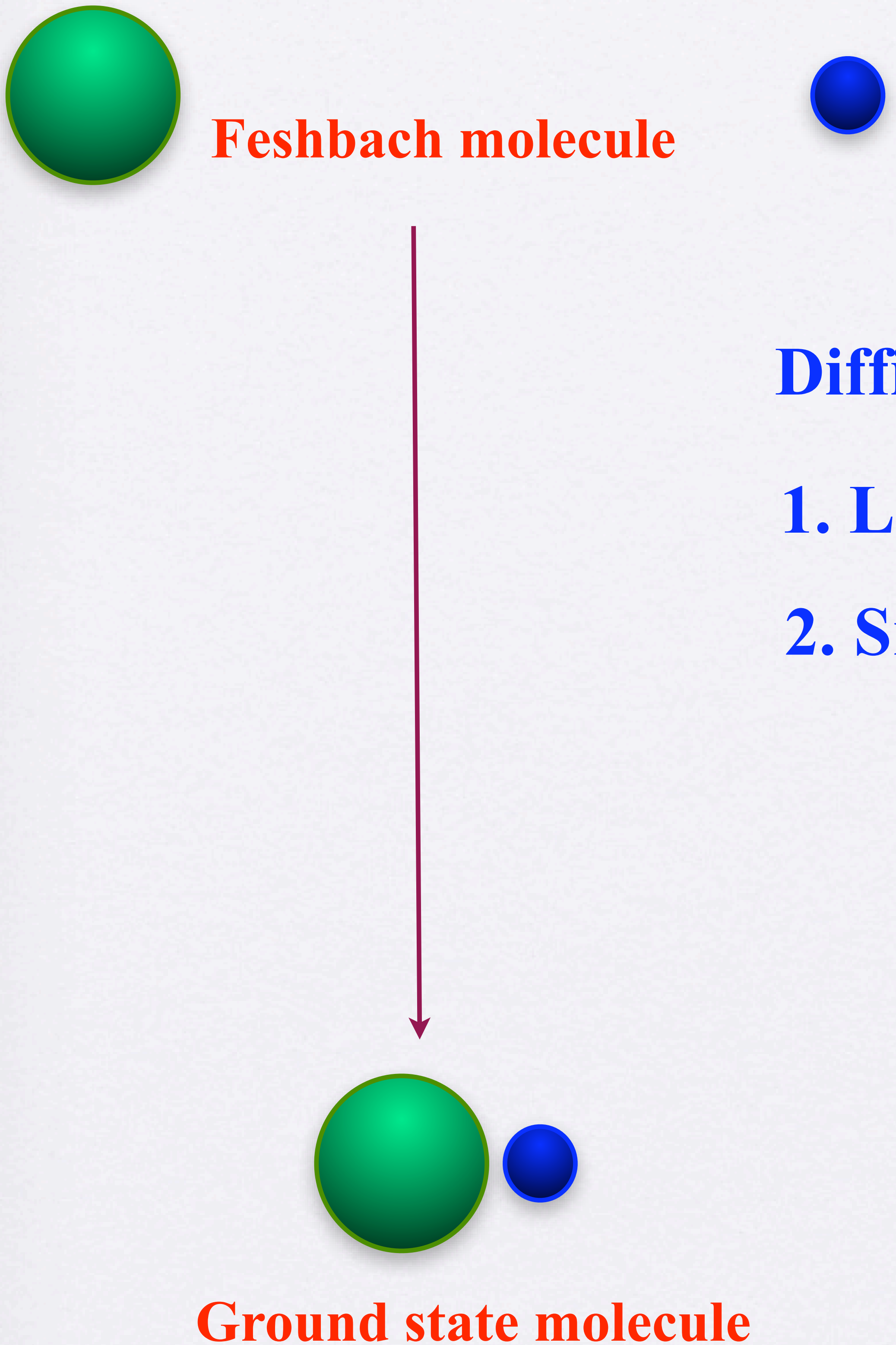
Polar molecular gases



large dipole moment: d

Hard to cool it directly !!

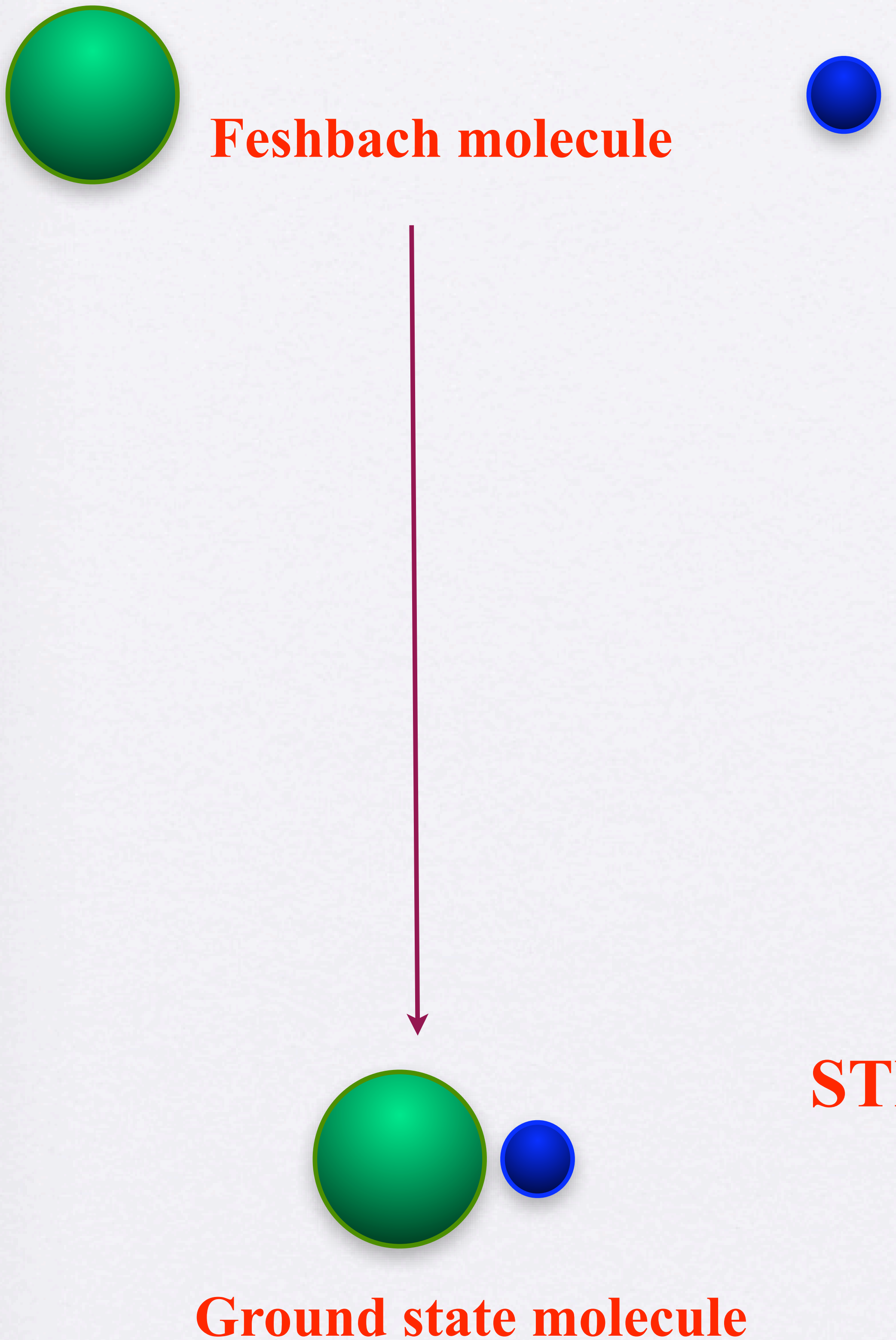
Polar molecular gases

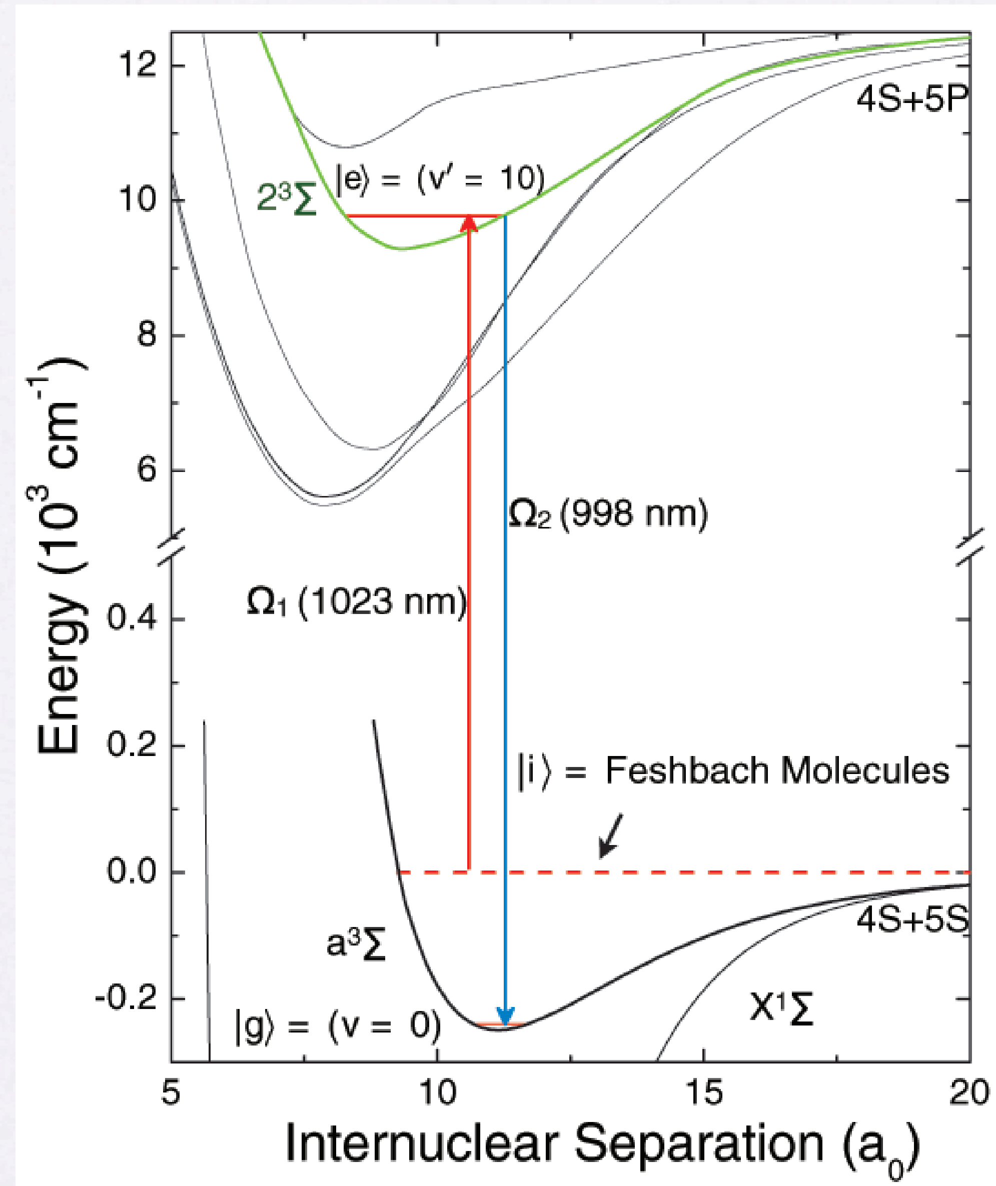


Difficulties:

1. Large energy detuning: 10-100 THz
2. Small transition matrix elememt

Polar molecular gases





STImulated Raman Adiabatic Passage

D. S. Jin and Jun Ye's group

Polar molecular gases

KRb+KRb --> K2+Rb2 **Chemically unstable !!**

	Na	K	Rb	Cs
Li	−328(2)	−533.9(3)	−618(200)	−415.38(2)
Na		74.3(3)	45.5(5)	236.75(20)
K			−8.7(9)	37.81(13)
Rb				29.1(1.5)

Zuchowski and Hutson, PRA (2010)

$$E_{\mathrm{D}} \sim \frac{D}{\langle r \rangle^3} \sim k_{\mathrm{F}}^3 D \qquad E_{\mathrm{F}} \sim k_{\mathrm{F}}^2$$

$$k_{\mathrm{F}} D \sim 1$$

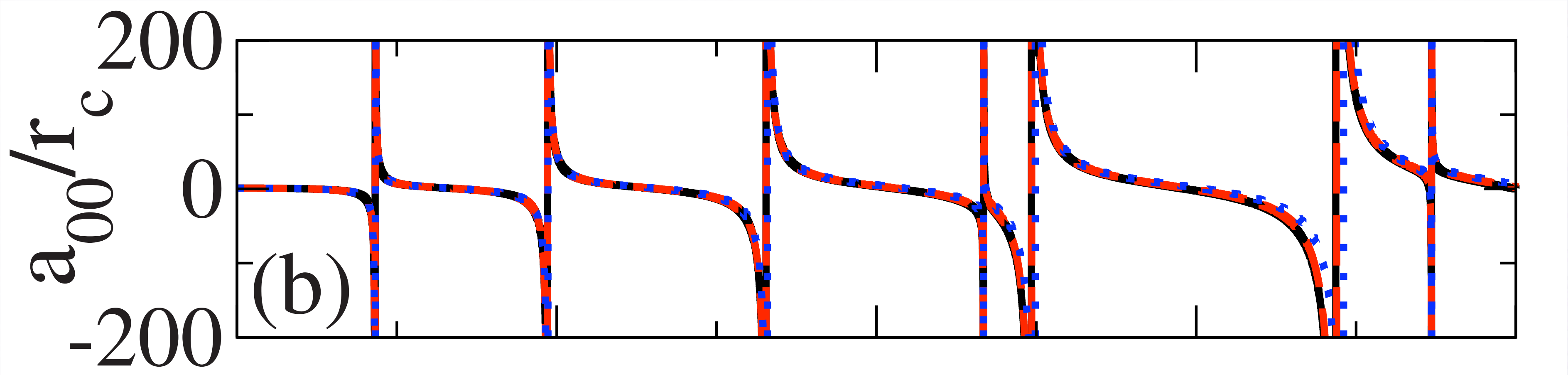
$$E_{\mathrm{D}} \sim E_{\mathrm{F}}$$

Dipole moments of vibrational ground state: υ=0

Mixture	[Debeye]
Li-Na	0.56
Li-K	3.6
Li-Rb	4.2
Li-Cs	5.5
Na-K	2.8 ←
Na-Rb	3.3 ←
Na-Cs	4.6 ←
K-Rb	0.6
K-Cs	1.9 ←
Rb-Cs	1.2 ←

M. Aymar and O. Dulieu, J. Chem.Phys., 122, 204302 (2005)

s-wave resonances with strong dipolar interactions

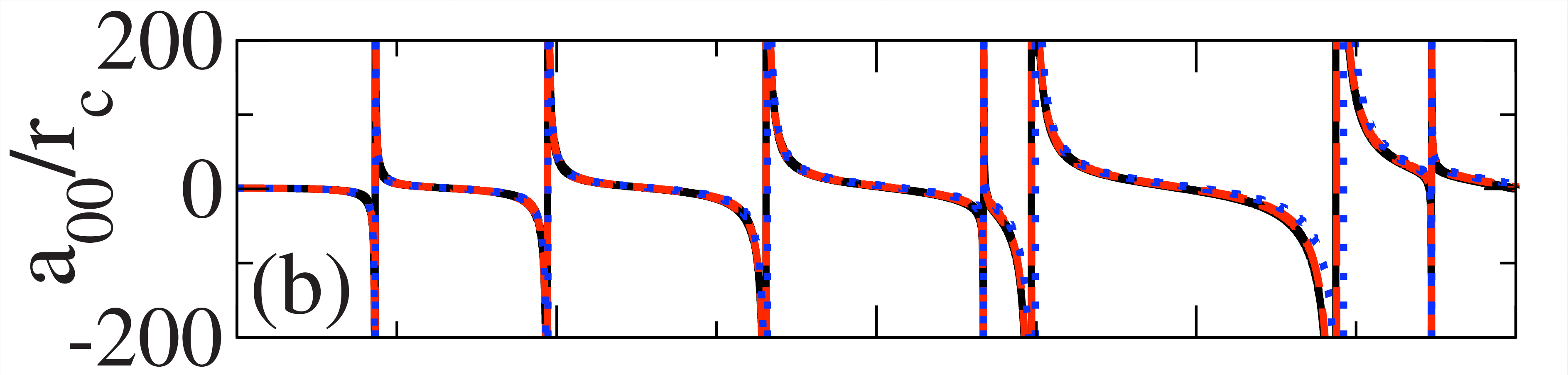


Kanjilal and Blume PRA (2008)

$$V_{\mathrm{D}} = \frac{D(1 - 3\cos^2\theta)}{r^3}$$

$$\langle Y_{00} | V_{\mathrm{D}} | Y_{00} \rangle = 0$$

s-wave resonances with strong dipolar interactions

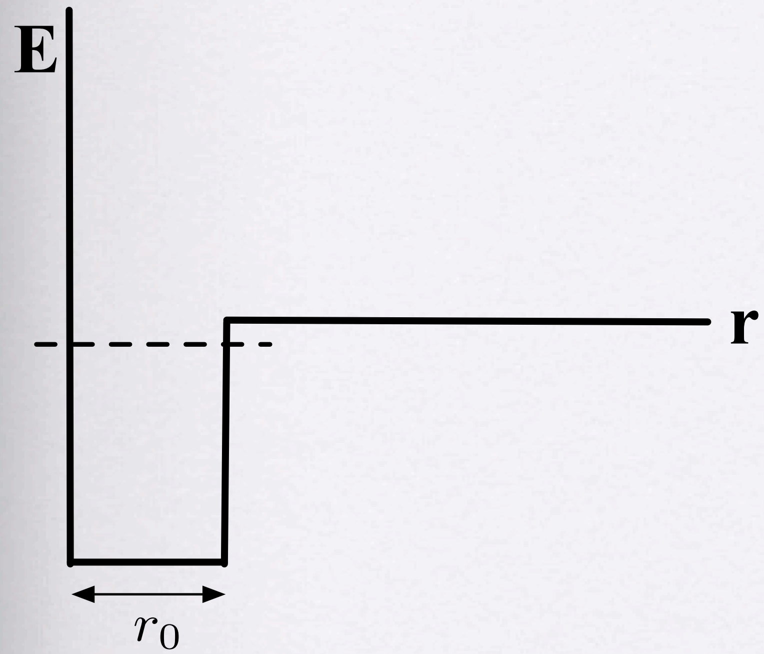


Kanjilal and Blume PRA (2008)

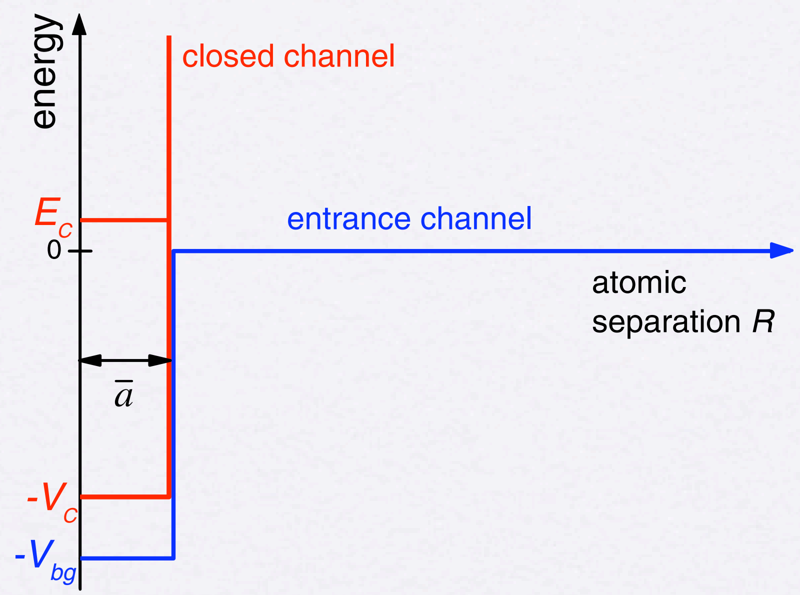
$$\langle Y_{20}|V_{\mathrm{D}}|Y_{00}\rangle \neq 0$$

$$V_{\mathrm{eff}} = -\frac{\langle Y_{00}|V_{\mathrm{D}}|Y_{20}\rangle\langle Y_{20}|V_{\mathrm{D}}|Y_{00}\rangle}{l(l+1)/r^2} \propto -\frac{D^2}{r^4}$$

How well can we tune a (positive) effective range in cold atoms ?
-- from the wiki



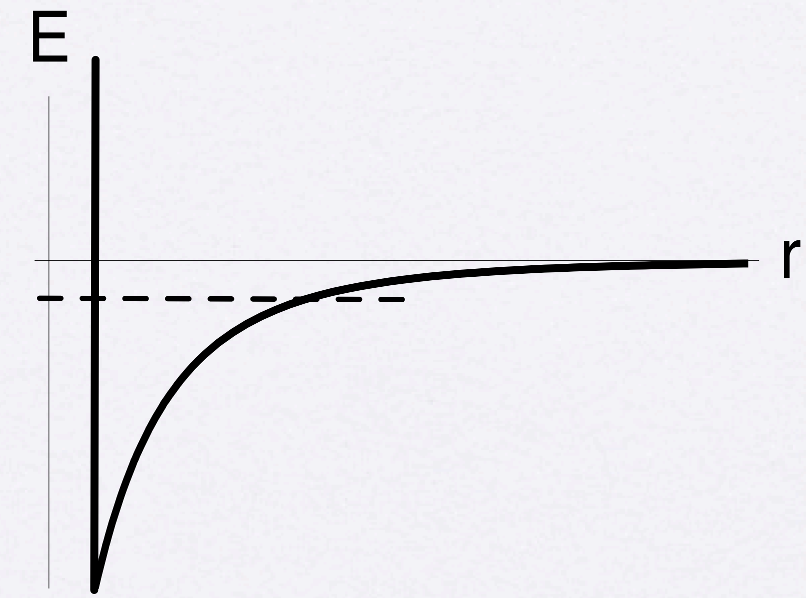
$$k_F r_{\text{eff}} \ll 1$$



$$k_F r_{\text{eff}} \gtrsim 1$$

$$r_{\text{eff}} < 0$$

Narrow resonance !



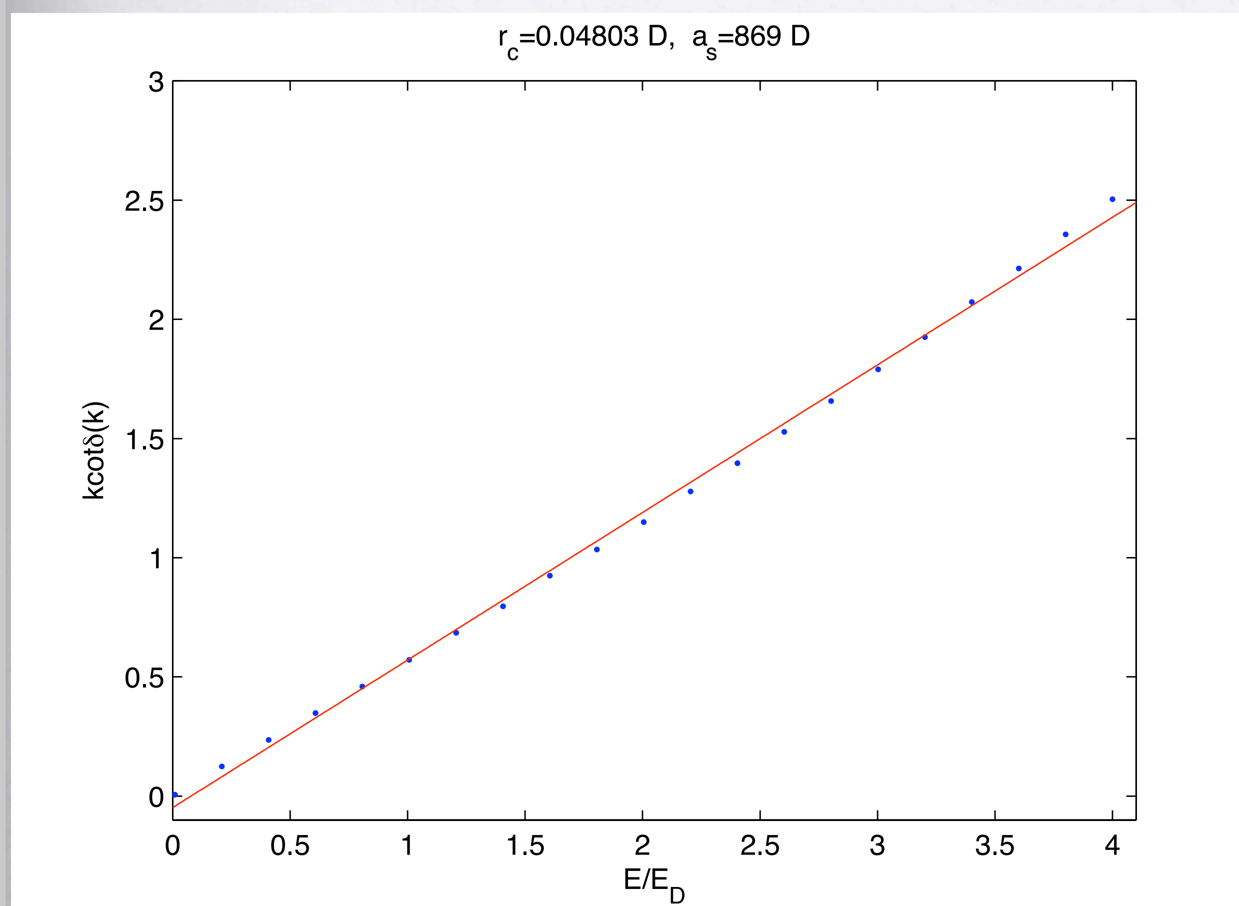
$$k_F r_{\text{eff}} \sim 1$$

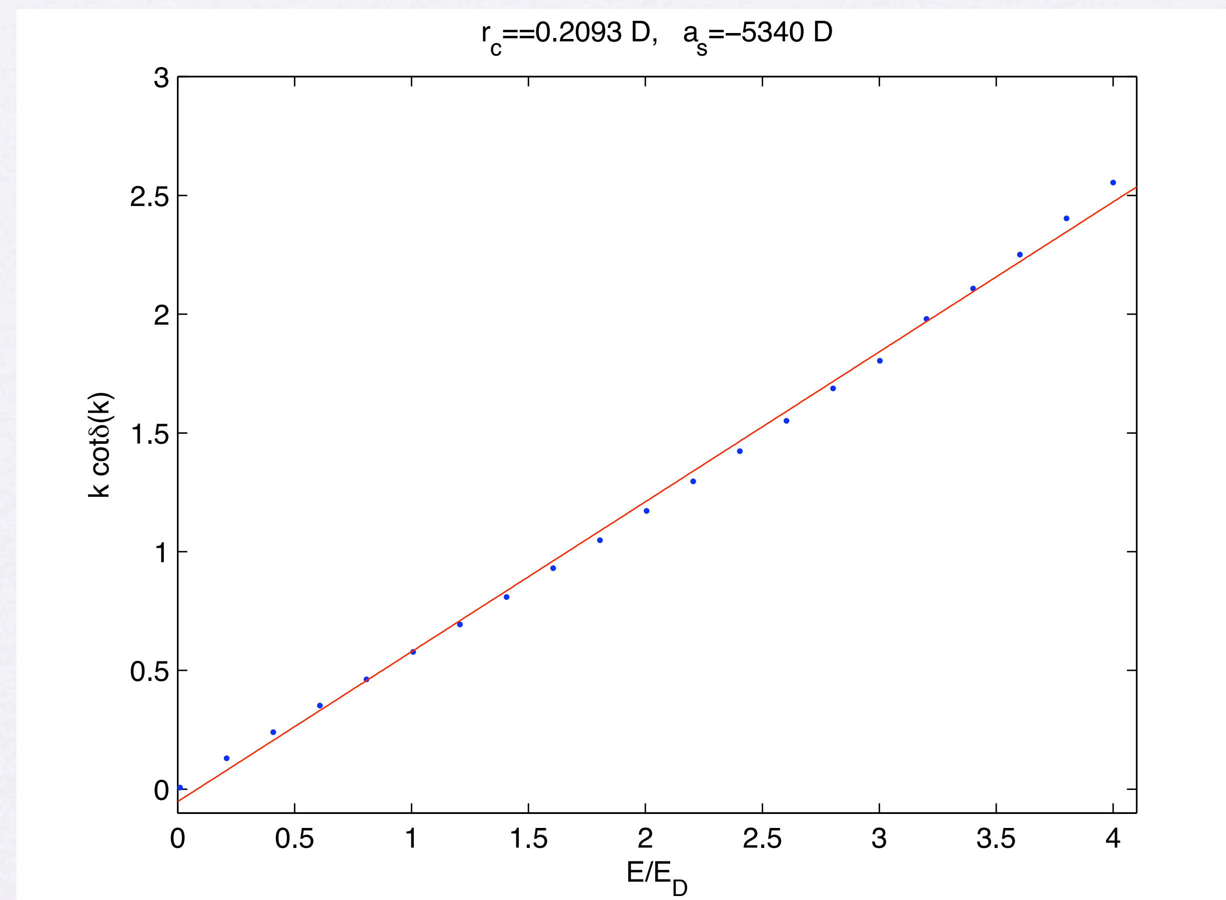
$$r_{\text{eff}} > 0$$

Controlled by D, D can be tuned by electronic field

Positive effective range from a dipole induced resonance

$$\frac{k}{\tan\delta(k)} = -\frac{1}{a_s} + \frac{1}{2} r_{\text{eff}} k^2$$



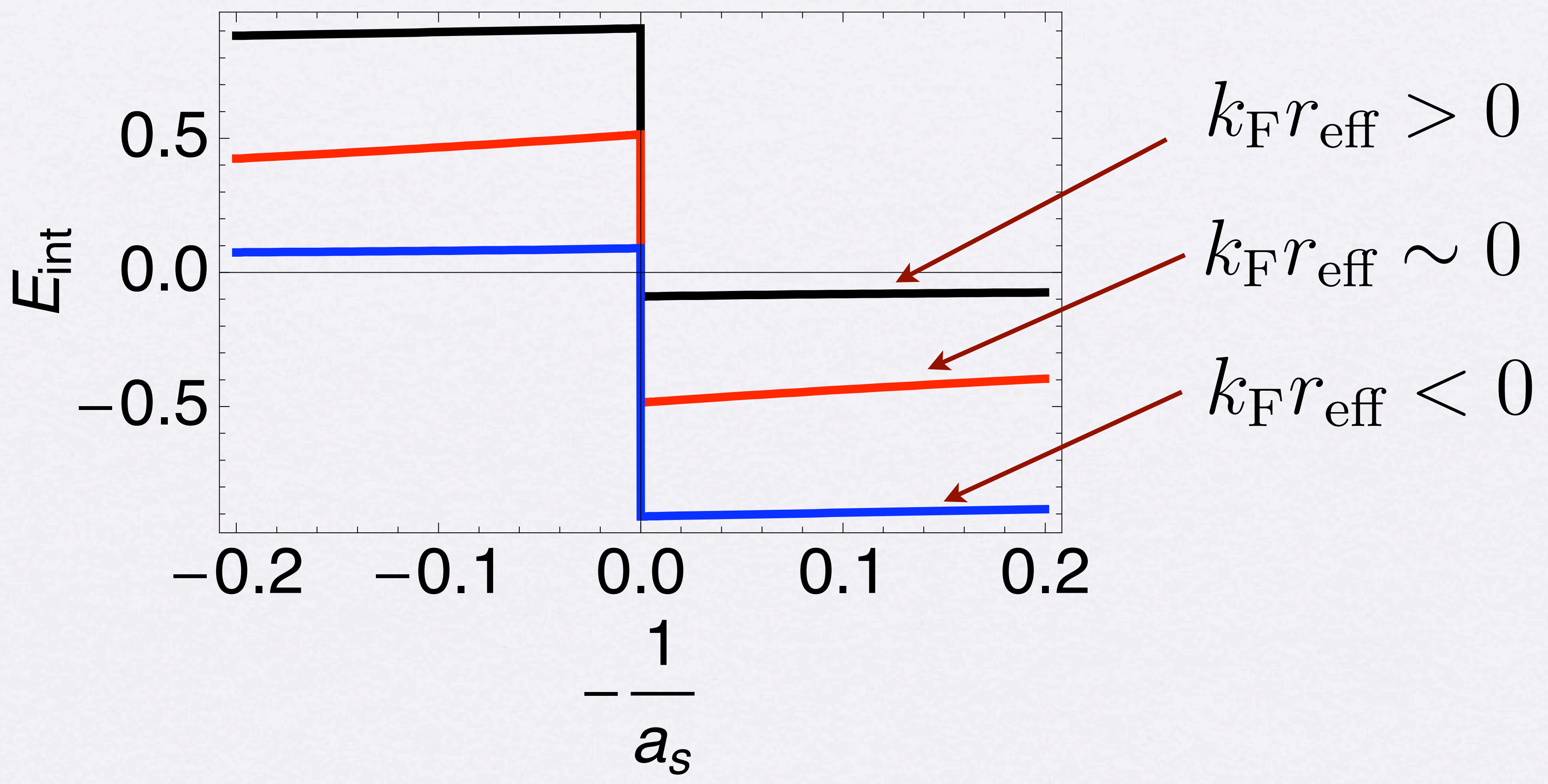


What is the physical effect of effective range being positive or negative

High temperature regime:

$$b_2 = \int_0^{+\infty} \frac{dk}{\pi} \frac{d\delta(k)}{dk} e^{-\lambda^2 k^2/(2\pi)}$$

$$E_{\rm int} = \frac{3k_{\rm B}Tn}{2}(n\lambda^3)\left[-\frac{b_2}{\sqrt{2}} + \frac{\sqrt{2}}{3}T\frac{\partial b_2}{\partial T}\right]$$



What is the physical effect of effective range being positive or negative

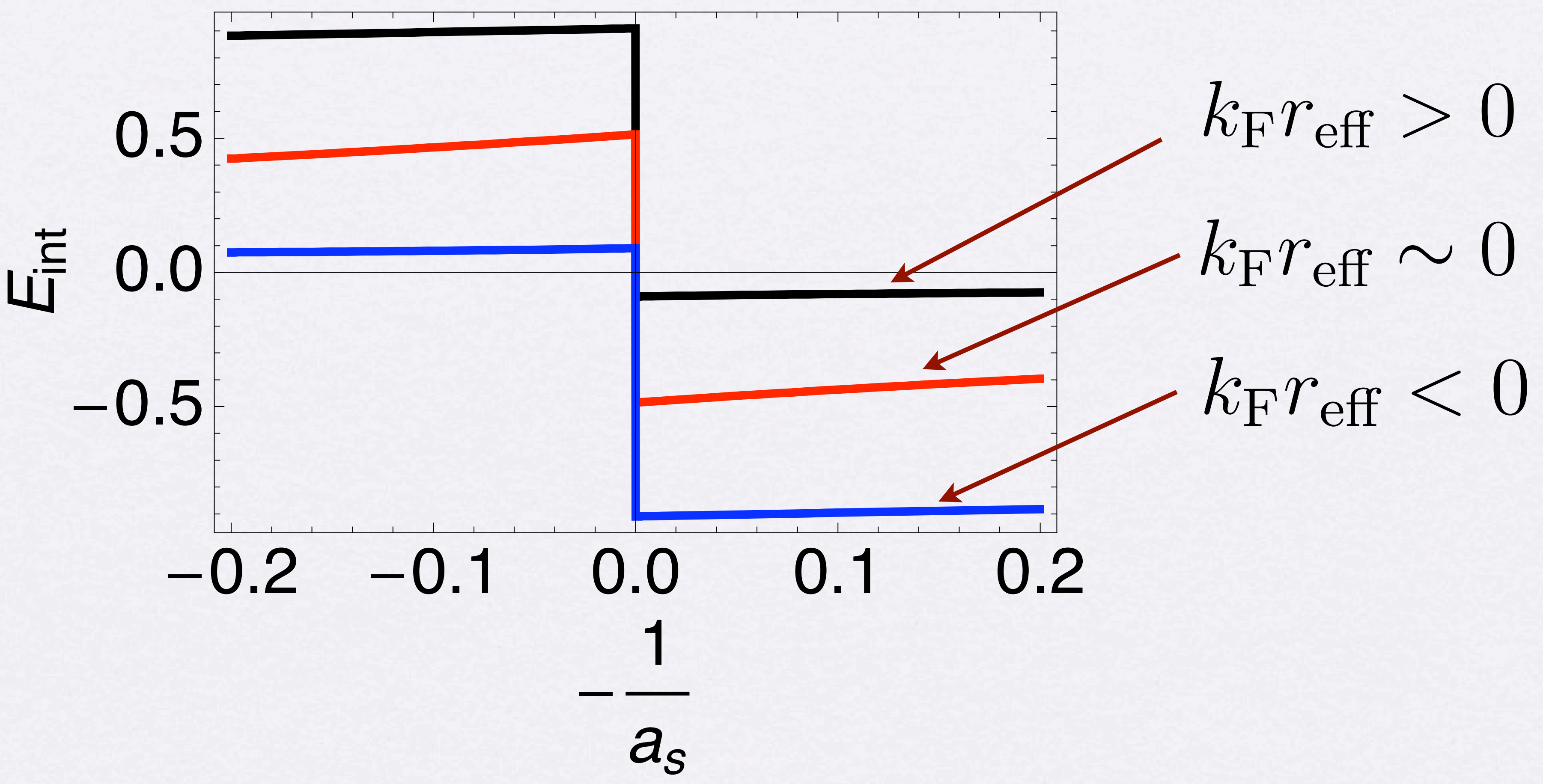
$$-\frac{1}{a_{\mathrm{avg}}} = -\frac{1}{a_{\mathrm{s}}} + \frac{1}{2} r_{\mathrm{eff}} \langle k^2 \rangle$$

$a_{\mathrm{s}} < 0 \quad r_{\mathrm{eff}} < 0 \quad \longrightarrow \quad a_{\mathrm{avg}} < a_{\mathrm{s}}$

$a_{\mathrm{s}} < 0 \quad r_{\mathrm{eff}} > 0 \quad \longrightarrow \quad a_{\mathrm{s}} < a_{\mathrm{avg}}$

$a_{\mathrm{s}} > 0 \quad r_{\mathrm{eff}} < 0 \quad \longrightarrow \quad a_{\mathrm{avg}} < a_{\mathrm{s}}$

$a_{\mathrm{s}} > 0 \quad r_{\mathrm{eff}} > 0 \quad \longrightarrow \quad a_{\mathrm{s}} < a_{\mathrm{avg}}$



Summary:
Take Home Message

New Mechanism	New System	New Features
Two-body interaction potential has center-of-mass dependence	Alkali-earth-(like) atomic gases: Sr, Ca, Yb	Spatially dependent local scattering length
Strong long range dipolar interactions	Polar molecules	positive and sizable effective range

Thank you very much for your attention !