Higher Spin Fermions

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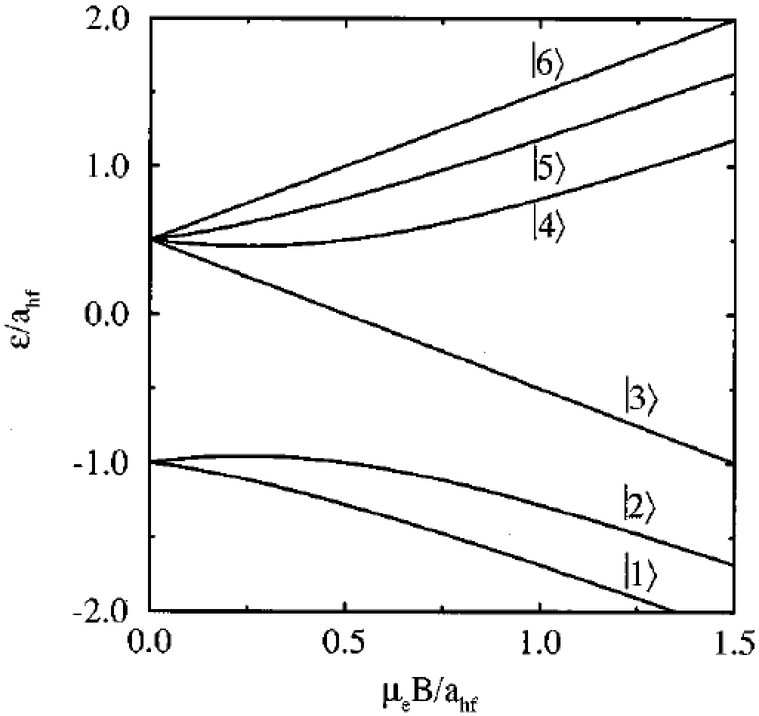
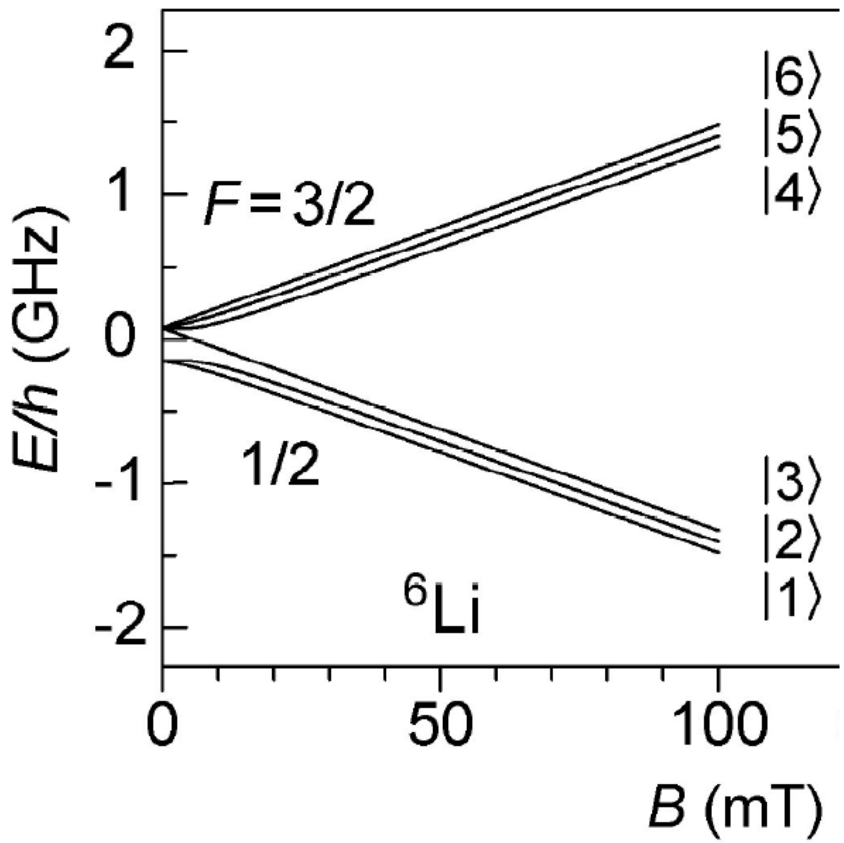




F = S + I

S = ½, I = 1 ; F = 1/2 or 3/2

Most experiments:

$|1\rangle + |2\rangle \longrightarrow |\uparrow\rangle \; |\downarrow\rangle$

Make use of Feshbach resonances for pairing

Full spin degrees of freedom not exploited

Systems with higher spins readily available:

Bosons with hyperfine spins

trapped optically

BEC / superfluids have already been studied:

^{23}Na I = 3/2 F=1 (MIT)
^{87}Rb I = 3/2 F=1 (Georgia Tech, Berkeley ..)

^{87}Rb I = 3/2 F=2 (Hannover, Tokyo, Mainz(Bloch))

Long-lived alkali fermions with f > ½:

^{22}Na, ^{40}K, ^{86}Rb (18 days), What can be interesting ?

dilute gas → s-wave interaction

but interaction still spin dependent

$\frac{1}{2} + \frac{1}{2} \rightarrow 0 \qquad \not{1}$

$3/2 + 3/2 \rightarrow 0, \quad \not{1}, \quad 2, \quad \not{3}$

$5/2 + 5/2 \rightarrow 0, \quad 2, \quad 4$

.....

Can have finite spin Cooper pairs;
internal structures of order parameter ....

Pairing occurs for the most negative a_J

Even given J, still internal structures since spin-J order parameter

e.g. J=2 direct analogy with L=2 (d-wave) Cooper pairs

five component order parameter Δ_{2M} , $-2 \leq M \leq 2$

possible phases:

(1,0,0,0,0) ferro/axial

(1,0,0,0,1) / (0,0,1,0,0) polar / quadrupolar

$(1,0,i\sqrt{2},0,1) \sim (1,0,0,\sqrt{2},0)$ Cylic / tetrahedral

[c.f spin-2 Bosonic condensates]

Pairing between two spin f fermions, assuming a_2 most negative:

$$\Delta^{(J=2)}_{m1,m2} = \sum_M \Delta_{2M} < 2M \mid ff m_1 m_2 >$$

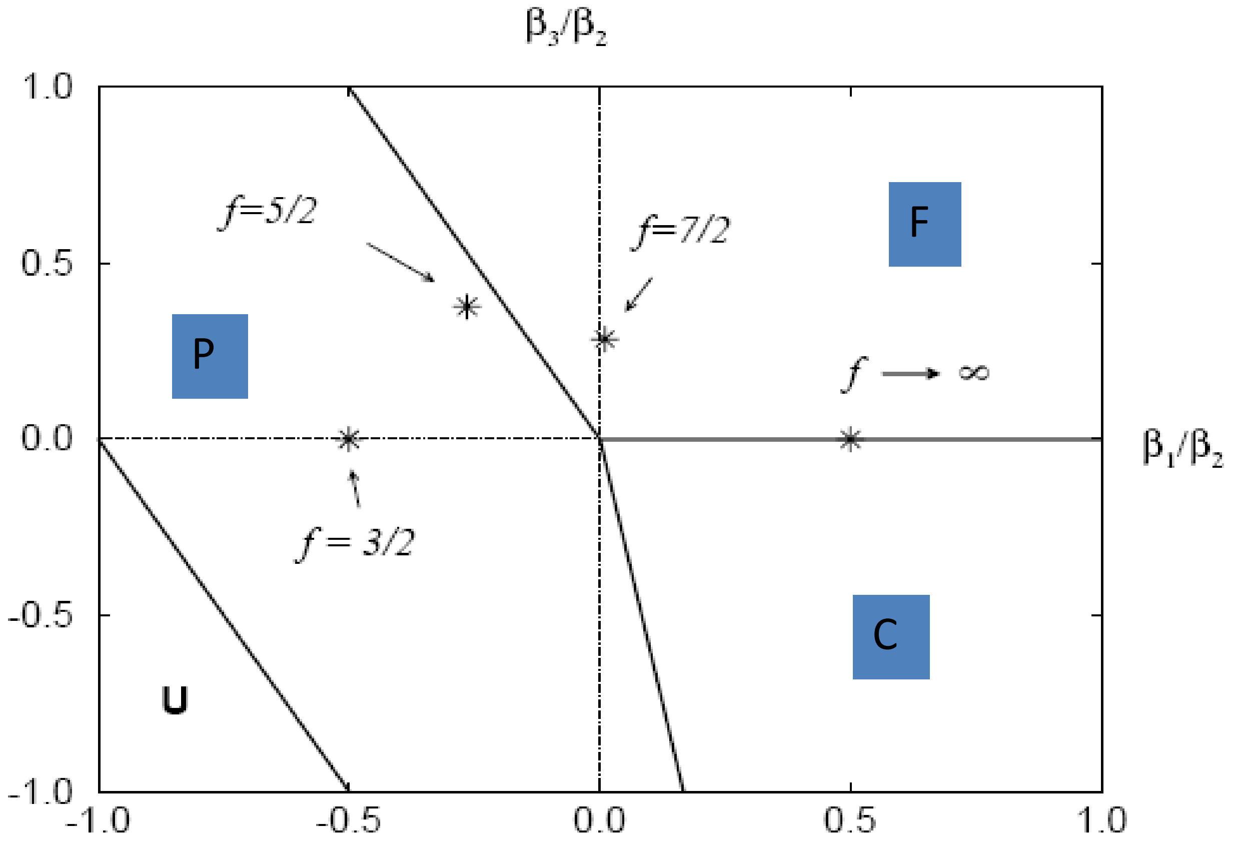
(2f+1) x (2f+1) matrix

Weak-coupling, Ginzburg-Landau theory:

$$\mathcal{F} = -\frac{1}{2}\alpha \, \mathrm{Tr}\, \Delta^{(J)+}\Delta^{(J)} + \frac{1}{4}\beta \, \mathrm{Tr}(\Delta^{(J)}\Delta^{(J)+})^2$$

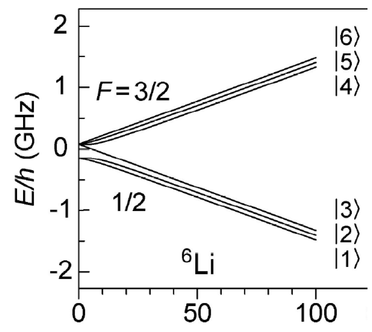
J= 2; but free energy still f dependent

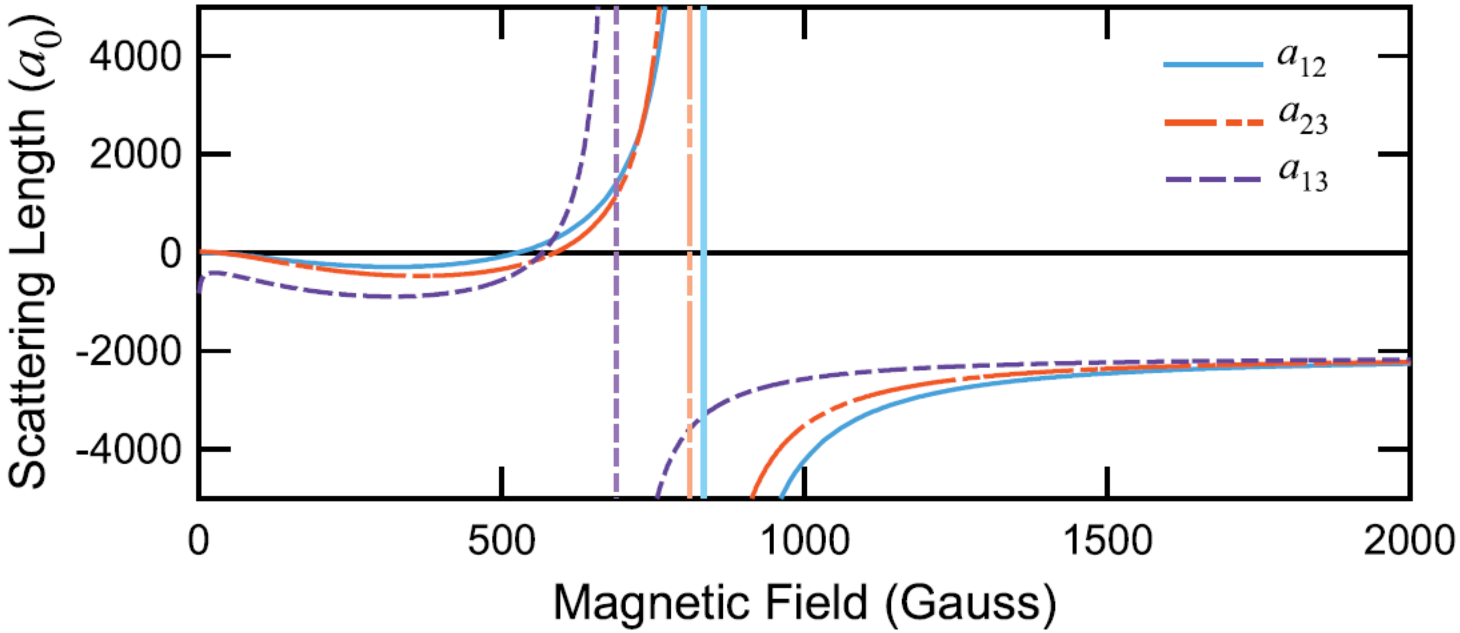
Ho + Yip, 1999



("quantum isotope" effect!)

Other systems with higher symmetry/internal structure:





~ SU(3)

Modawi and Leggett; Hofstetter et al

Also: 40K (I =4) ~ SU(4)

Alkaline

two electrons per atom, closed s-shell -- no net electronic spin

nuclear spin I only

interaction roughly independent of F = I

~ SU(N) N = 2F+ 1

87Sr I = 9/2 Innsbruck , Rice

(40Ca I = 0 Brauncshweig; 84Sr I = 0; Innsbruck, Rice)
[Mg 24, 26; Ca 40,42,44,46,48; Sr 84, 86,88; all I = 0]

Yb isotopes -- Kyoto

Yb: rare-earth; closed electronic shell $4f^{14}$

Kyoto

Yb 168, 170, 172, 174, 176 $I = 0$

Yb 171 $I = ½$ 173 $I = 5/2$

obtained degenerate mixture of 171 and 173

$a_{171-171}$ = -0.15 nm

$a_{173-173}$ = 10.55nm

$a_{171-173}$ = -30.6 nm

$a_{171\text{-}173}$ = -30.6 nm

small, but may be enhanced by lattice

Interspecies Cooper pairing ?

(c.f Hofstetter et al, originally for 6Li)

$$T_c^{\max} \sim 0.3 E_{F,free} k_l |a|$$

171 I = ½ 173 I = 5/2

SU(2) X SU(6)

equal chemical potentials

order parameter

degeneracies

Nambu-Goldstone modes

two-types $\omega \propto q$ $\omega \propto q^2$

$a_{\vec{k},\lambda}$ $\lambda = \pm\frac{1}{2}$ $^{171}\mathrm{Yb}$

$c_{\vec{k},\nu}$ $\nu = -f, ..., f$ $^{173}\mathrm{Yb}$

$$H_K = \sum_{\vec{k},\lambda} \xi_k a^\dagger_{\vec{k},\lambda} a_{\vec{k},\lambda} + \sum_{\vec{k},\nu} \xi_k c^\dagger_{\vec{k},\nu} c_{\vec{k},\nu}$$

$$\xi_k \equiv \frac{k^2}{2m} - \mu$$

$$H_{int} = g \sum_{\vec{k},\vec{k}',\vec{q},\lambda,\nu} a^\dagger_{\vec{k}_+,\lambda} c^\dagger_{-\vec{k}_-,\nu} c_{-\vec{k}'_-,\nu} a_{\vec{k}'_+,\lambda}$$

Superfluid order parameter

$2 \times (2f+1)$ matrix

Minimize:

$$\Omega = \alpha \mathrm{Tr}\left[\mathbf{\Delta}\mathbf{\Delta}^\dagger\right] + \frac{\beta_2}{2}\mathrm{Tr}\left[(\mathbf{\Delta}\mathbf{\Delta}^\dagger)^2\right]$$

$\alpha > 0; T > T_c$

$\alpha < 0; T < T_c$

$$\mathbf{\Delta}\mathbf{\Delta}^\dagger = \begin{pmatrix} D_{1/2}^2 & \sum_\nu \Delta_{1/2,\nu}\Delta^*_{-1/2,\nu} \\ \sum_\nu \Delta^*_{1/2,\nu}\Delta_{-1/2,\nu} & D_{-1/2}^2 \end{pmatrix}$$

$$D_{1/2} \equiv [\sum_\nu \Delta_{1/2,\nu}\Delta^*_{1/2,\nu}]^{1/2} \qquad D_{-1/2} \equiv [\sum_\nu \Delta_{-1/2,\nu}\Delta^*_{-1/2,\nu}]^{1/2}$$

$$\mathbf{\Delta}\mathbf{\Delta}^\dagger = \frac{D_{1/2}^2 + D_{-1/2}^2}{2} + \mathrm{M} \qquad \mathrm{Tr}[\mathrm{M}] = 0$$

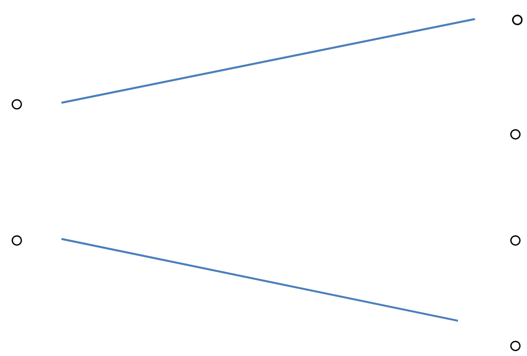
Best choice: M = 0 $\quad \sum_\nu \Delta_{\frac{1}{2},\nu}\Delta^*_{-\frac{1}{2},\nu} = 0 \quad D_{1/2} = D_{-1/2}$

$$\Delta_{1/2} \equiv (\Delta_{1/2,-f},\dots,\Delta_{1/2,f}) \qquad \Delta_{-1/2} \equiv (\Delta_{-1/2,-f},\dots,\Delta_{-1/2,f})$$

Orthogonal vectors of equal magnitude

Pairing $$\Delta_{\lambda,\nu} a^{\dagger}_{\vec{k},\lambda} c^{\dagger}_{-\vec{k},\nu}$$

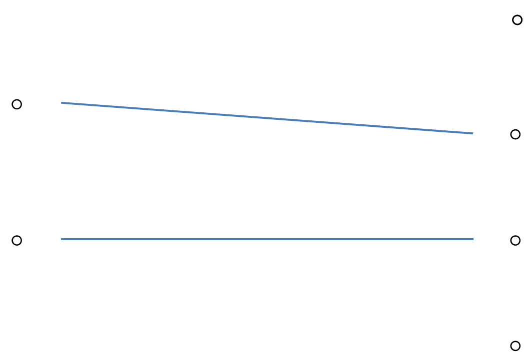
a $_{½}$ and a $_{-1/2}$ pair with orthogonal combinations of c's



Pairing

$$\Delta_{\lambda,\nu} a^{\dagger}_{\vec{k},\lambda} c^{\dagger}_{-\vec{k},\nu}$$

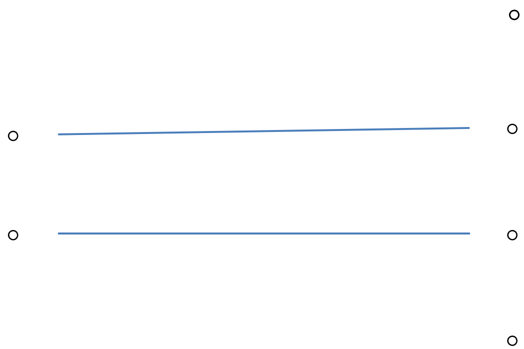
$a_{1/2}$ and $a_{-1/2}$ pair with orthogonal combinations of c's



(left over c's: (2f +1) – 2 = 2f -1 normal components)

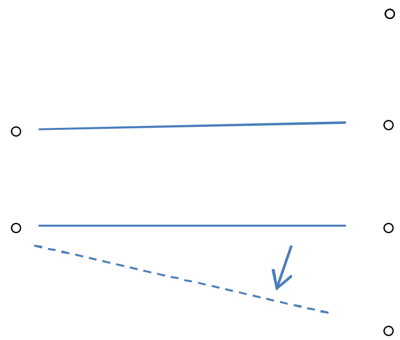
Collective Modes: $\delta\Delta_{\lambda,\nu}$ $\delta\Delta^{*}_{\lambda,\nu}$

Take equilibrium state : non-vanishing $\Delta_{1/2,1/2}$ $\Delta_{-1/2,-1/2}$



Different types

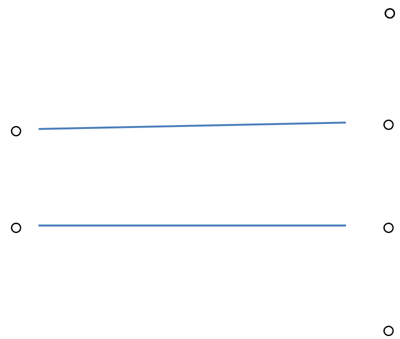
(1)



e.g. $\delta\Delta_{1/2,3/2}(\vec{q})$

2 x (2 f – 1)

(2)

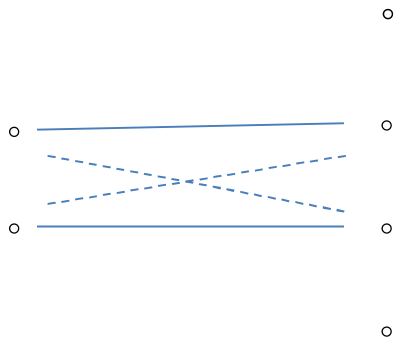


$\delta\Delta_{1/2,1/2}$ $\delta\Delta^{*}_{1/2,1/2}$

$\delta\Delta_{-1/2,-1/2}$ $\delta\Delta^{*}_{-1/2,-1/2}$

2

(3)



$\delta\Delta_{1/2,-1/2}$ $\delta\Delta^{*}_{1/2,-1/2}$

$\delta\Delta_{-1/2,1/2}$ $\delta\Delta^{*}_{-1/2,1/2}$

2

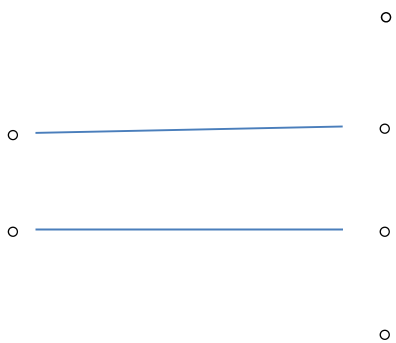
Recall: two-component

$\delta\Delta$ $\delta\Delta^*$ Couple → phase mode (gapless Goldstone)
(+"amplitude mode")

$\omega \propto q$ weak-coupling: $\omega = \frac{v_F}{\sqrt{3}} q$

applies to:

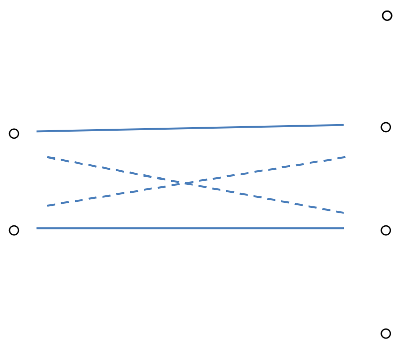
(2)



$\delta\Delta_{1/2,1/2}$ $\delta\Delta^*_{1/2,1/2}$

$\delta\Delta_{-1/2,-1/2}$ $\delta\Delta^*_{-1/2,-1/2}$

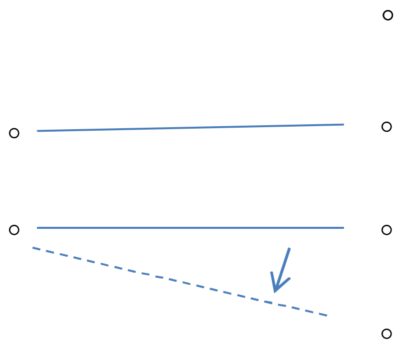
(3)



$\delta\Delta_{1/2,-1/2}$ $\delta\Delta^*_{1/2,-1/2}$

$\delta\Delta_{-1/2,1/2}$ $\delta\Delta^*_{-1/2,1/2}$

(1)



e.g. $\delta\Delta_{1/2,3/2}(\vec{q})$

Do not couple to others (gauge invariance)

$$a_\lambda \longrightarrow a_\lambda e^{i\theta_\lambda} \qquad c_\nu \longrightarrow c_\nu e^{i\phi_\nu}$$

quadratic Nambu-Goldstone modes $\omega \propto q^2$

General theorem on linear vs quadratic Goldstone modes
[Nielsen and Chadha, Nucl. Phys. B, 105, 445 (1976)]

Present problem: two related arguments:

1. Response function

$$< a_{\vec{k}_+,1/2} c_{-\vec{k}_-,3/2} >^{(1)}$$ in response to

$$\delta H = \sum_{\vec{k}} \delta\Delta_{1/2,3/2}(\vec{q}) a^{\dagger}_{\vec{k}_+,1/2} c^{\dagger}_{-\vec{k}_-,3/2}$$

$$\delta\Delta_{1/2,3/2}(\vec{q}) = (-g) \sum_{\vec{k}} < a_{\vec{k}_+,1/2} c_{-\vec{k}_-,3/2} >^{(1)}$$

Consistency → dispersion relation $A_1\omega + Bq^2 = 0$

Quadratic if A_1 not forbidden by symmetry

Weak-coupling:

$$0 = \sum_{\vec{k}} \left\{ \left[\frac{\frac{1}{2}\left(1 - \frac{\xi_{k_+}}{E_{k_+}}\right) f(\xi_{k_-})}{\omega - \xi_{k_-} + E_{k_+}} - \frac{\frac{1}{2}\left(1 + \frac{\xi_{k_+}}{E_{k_+}}\right)(1 - f(\xi_{k_-}))}{\omega - \xi_{k_-} - E_{k_+}} \right] - \frac{1}{2E_k} \right\}$$

response

1/g

$$\vec{k}_\pm \equiv \vec{k} \pm \vec{q}/2$$

$$A_1 = \sum_{\vec{k}} \frac{1}{2E_k} \left[\frac{1 - f(\xi_k)}{(E_k + \xi_k)} - \frac{f(\xi_k)}{(E_k - \xi_k)} \right]$$

2. Continuity equation:

mode couple to spin density $< c^{\dagger}_{-\vec{k}_+,1/2} c_{-\vec{k}_-,3/2} >^{(1)}$

$$\Delta_{1/2,1/2} \times < c^{\dagger}_{-\vec{k}_+,1/2} c_{-\vec{k}_-,3/2} >^{(1)} \longleftrightarrow < a_{\vec{k}_+,1/2} c_{-\vec{k}_-,3/2} >^{(1)}$$

$$\omega \delta n_s - \underbrace{\vec{q} \cdot \vec{J}_s}_{\rightarrow Bq^2} = 0$$

$$\downarrow$$

$$[A_1 \ + \ A_2 \omega \ + \ ...]\Delta^*_{1/2,1/2} \delta \Delta_{1/2,3/2}(\vec{q})$$

$\downarrow$

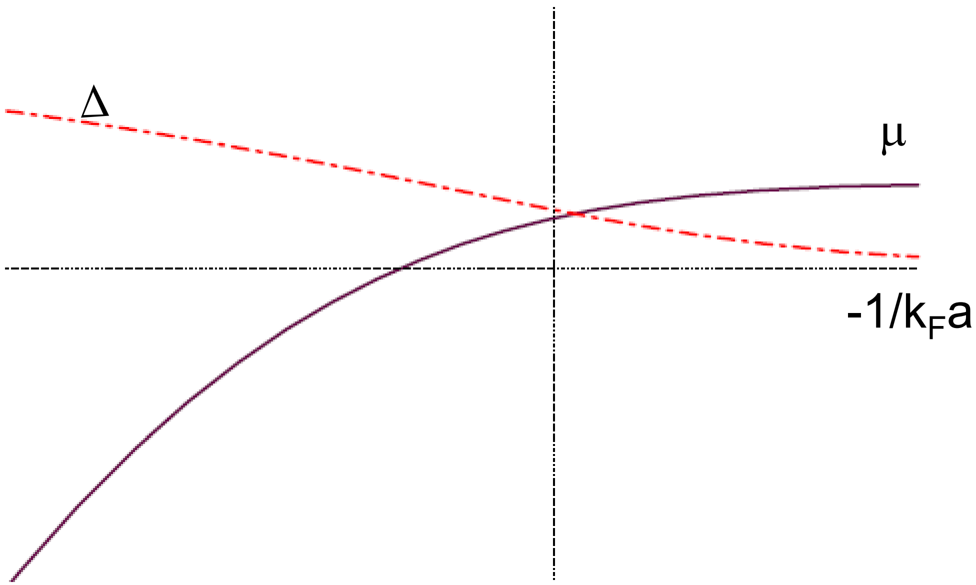
Zero frequency spin-density response

Started with equal chemical potentials

but if

$$\Delta_{1/2,1/2} \neq 0 \qquad\qquad n^c_{1/2} \neq n^c_{3/2}$$

Changes in $n^c_{1/2} \propto \Delta^*_{1/2,1/2}\Delta_{1/2,1/2}$



$$\delta n^{c}_{1/2} \propto \delta[\Delta^{*}_{1/2,1/2}\Delta_{1/2,1/2}]$$

“rotate” in c Hilbert space:

$$< c^{\dagger}_{-\vec{k}_+,1/2} c_{-\vec{k}_-,3/2} >^{(1)} \quad \propto \Delta^{*}_{1/2,1/2}\delta\Delta_{1/2,3/2}$$

coefficient small for weak-coupling limit (particle-hole asymmetry)

$$A_1\omega + A_2\omega^2 + Bq^2 = 0$$

Weak-coupling:

small freq:

$$\omega = \frac{q^2}{2m}\frac{\frac{4}{3}\frac{\mu}{\Delta}}{\left(\frac{\Delta}{2\mu}\ln\frac{\mu}{\Delta}\right)}$$

$$\omega = (q\xi_0)^2\Delta\left[\frac{\frac{2}{3}\pi^2}{\frac{\Delta}{\mu}\ln\frac{\mu}{\Delta}}\right]$$

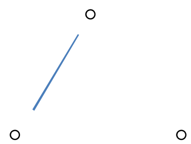
$$\xi_0 \equiv \frac{v_F}{\pi\Delta}$$

eventually:

$$\omega = \frac{v_F}{\sqrt{3}}q$$

Comparison with SU(N):

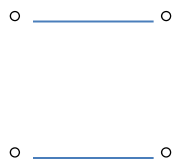
SU(3): (Modawi, Leggett)



$$\Delta_{12} = -\Delta_{21}$$

All others zero

SU(4):



Purely mathematical result (not connected with minimization of energy)

Antisymmetric Δ

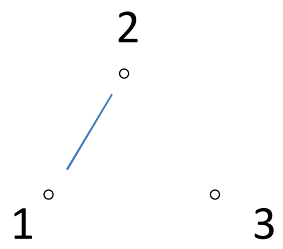
$$\boldsymbol{\Delta} \rightarrow \mathbf{U}\boldsymbol{\Delta}\mathbf{U}^{\mathrm{t}}$$

$$\begin{pmatrix} 0 & \Delta_{12} & 0 & 0 & \dots \\ -\Delta_{12} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \Delta_{34} & \dots \\ 0 & 0 & -\Delta_{34} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots \end{pmatrix}$$

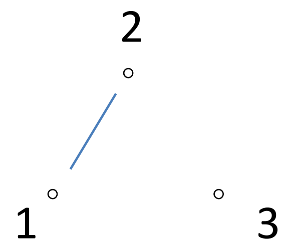
true for any state

SU(3): one normal fermi surface

$$\begin{pmatrix} 0 & \Delta_{12} & 0 \\ -\Delta_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Collective modes:



$\delta\Delta_{12}$ $\delta\Delta^{*}_{12}$ → Phase mode; linear

$\delta\Delta_{13}$

$\delta\Delta_{23}$

} Quadratic, becoming linear at higher freq

[$\delta\Delta^{*}_{13}$ $\delta\Delta^{*}_{23}$ just hermitian conjugates / annihilation vs creation]

Honerkamp,Hofstetter did not distinguish linear vs quadratic

He,Jin,Zhuang pointed out quadratic* but incorrect dispersion

Catelani and Yuzbashyan pointed out linear at higher freq

$$* \Delta_{12} \neq 0 \Rightarrow n_3 \neq n_1 = n_2$$

Quadratic modes from finite zero frequency density response

c.f. Ferromagnet,
sometimes attributed to broken time-reversal

Higher Spins

More “complex” order parameter

SU(2) x SU(6): two pairs, 6-2=4 normal Fermi surfaces

linear and quadratic modes

similarities and differences with SU(N)

Nambu-Goldstone modes couple to densities/ spin densities

in principle measureable by Bragg scattering
through structure factor