Bragg spectroscopic studies of the contact

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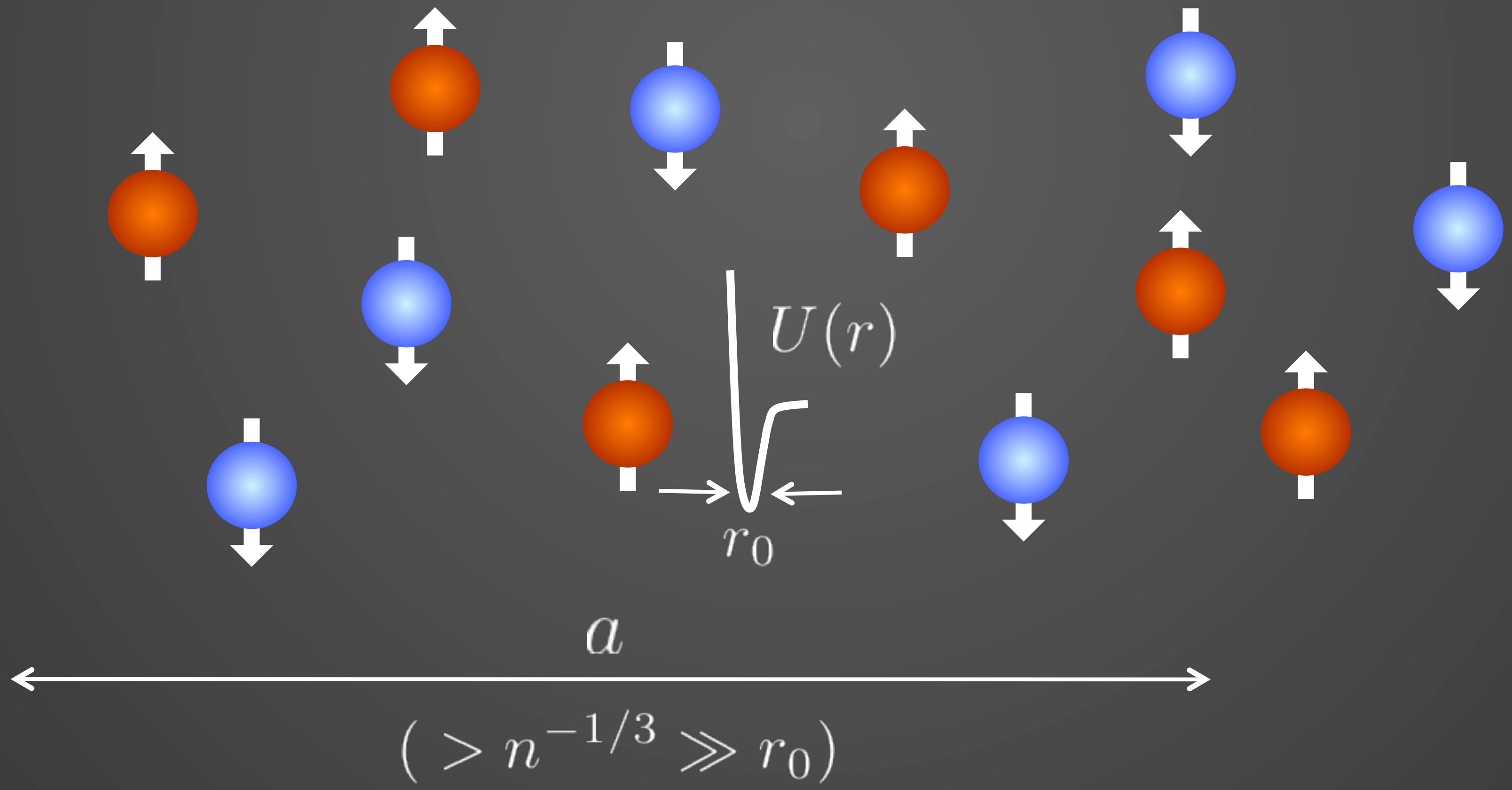
Overview

- Universality in interacting Fermi gases
- Bragg spectroscopy
- Measurements of the contact
- Progress towards spatially resolved Bragg spectroscopy
- Outlook

Universality in Fermi Gases

Fermionic Universality

Universality: All dilute Fermi systems with sufficiently strong interactions behave identically on a scale given by the mean interparticle spacing, independent of *U*(*r*)



Tan's Universal Relations

- In 2005 Shina Tan derived several exact relations linking macroscopic and microscopic system properties through a single parameter, the contact – $\mathcal{C}$

Tan, Ann Phys **323**, 2952; 2971; 2987 (2008).

- Two examples of Tan relations are:

$$\frac{dE}{d(1/a)} = \frac{-\hbar^2 \mathcal{C}}{4\pi m} \qquad E - 2V = -\frac{\mathcal{C}}{4\pi k_F a}$$

now verified experimentally

Stewart *et al.*, PRL **104**, 235301 (2010)

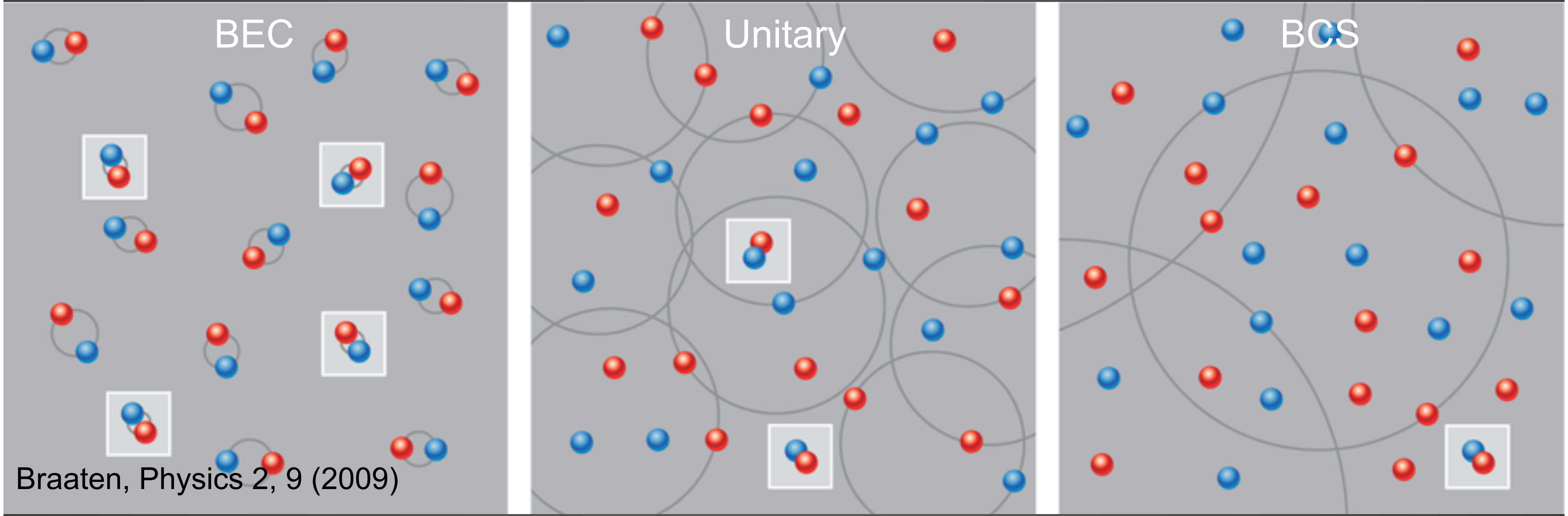
- These apply to:
  - Superfluid / normal phases (0 or finite *T*)
  - Few-body / many-body systems

Punk and Zwerger, PRL **99**, 170404 (2007)
Braaten and Platter, PRL **100**, 205301 (2008)
Zhang and Leggett, PRA **79**, 023601 (2009)
Palestini *et al.*, PRA **82**, 021605(R) (2010)

Partridge *et al.*, PRL **95**, 020404 (2005)
Werner, Tarruell and Castin, EPJ B **68**, 401 (2009)
Son and Thompson, PRA **81**, 063634 (2010)
Gandolfi *et al.*, PRA **83**, 041601(R) (2011)

Tan's Universal Relations

- Contact is defined as: $\mathcal{C} = \lim_{k \to \infty} k^4 n(k)$



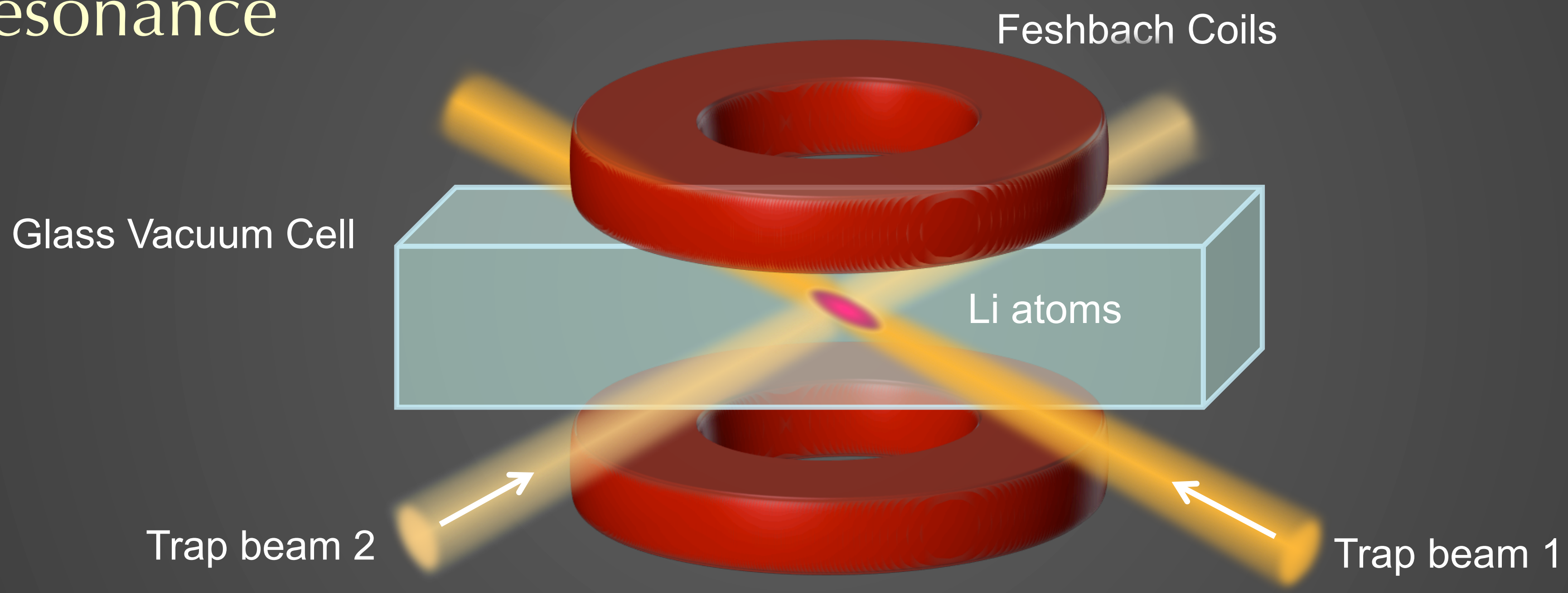
$\mathcal{C}$ quantifies the number of closely spaced pairs!

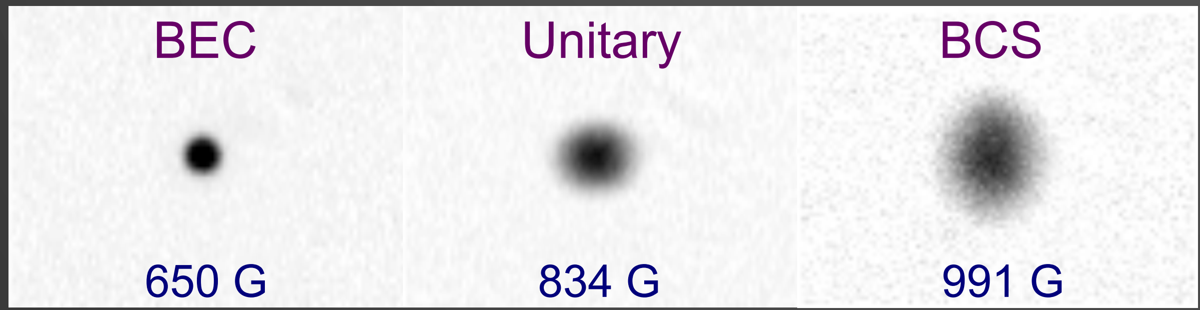
- $\mathcal{C}$ depends upon :- $\frac{T}{T_F}$ and $\frac{1}{k_F a}$

Bragg Spectroscopy of Fermi gases

Ultracold Fermi Gases

- We cool a 50/50 mixture of ^{6}Li $|F = \frac{1}{2}, m_F = \pm\frac{1}{2}\rangle$ atoms in an optical trap near the 834 G Feshbach resonance



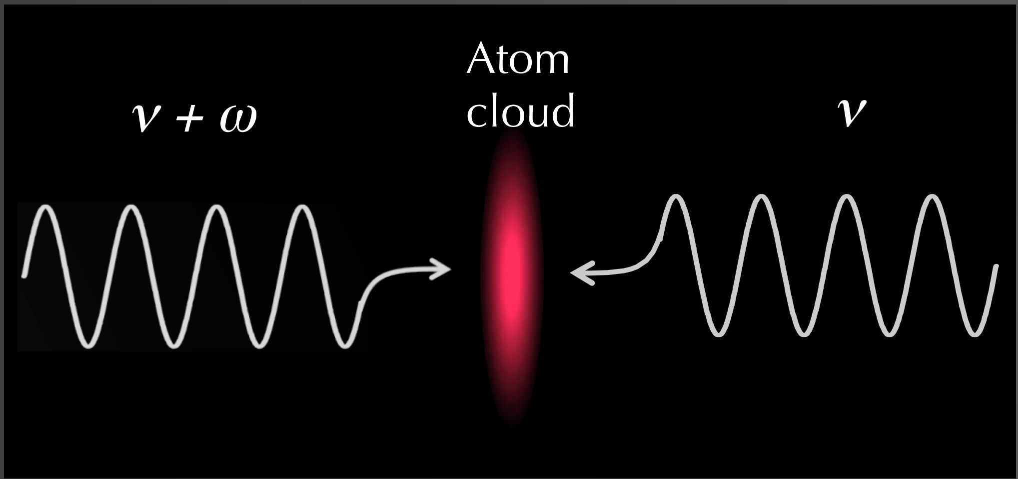


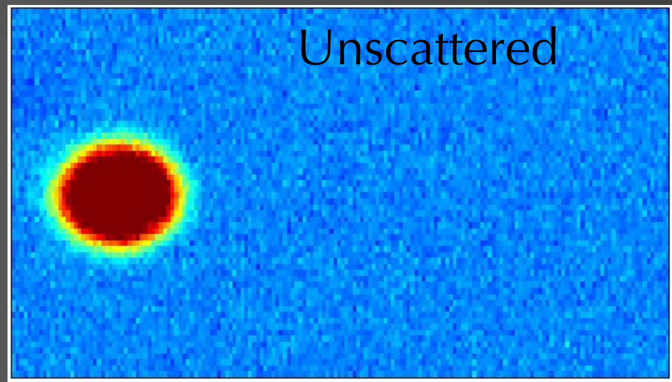
$$N_\sigma = 1.5 \times 10^5$$

$$\frac{T}{T_F} \lesssim 0.1 \quad \text{(at Unitarity)}$$

Bragg Scattering

- Illuminate a cloud with a "moving" standing wave
- Can scatter molecules (pairs) / atoms by selecting ω



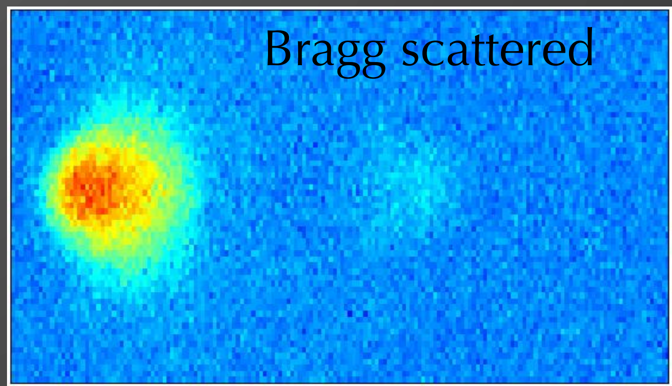


Bragg condition

$$\omega_{Br} = \frac{2\hbar k_l^2}{m}$$

$$\frac{\omega_{at}}{2\pi} = 294\,\mathrm{kHz}$$

$$\frac{\omega_{mol}}{2\pi} = 147\,\mathrm{kHz}$$



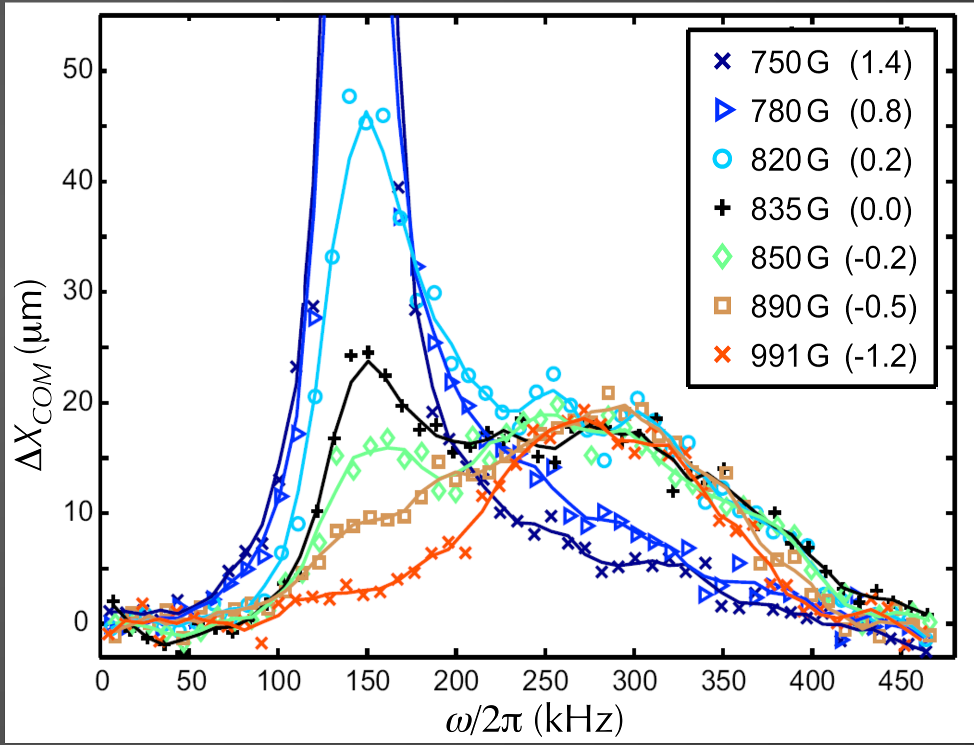
Bragg Spectroscopy

- We obtain a Bragg spectra by:

(i) Measuring either

- ΔP vs. ω
- ΔE vs. ω

These provide the dynamic structure factor – $S(k,\omega)$



(ii) Integrating these give $S(k)$

Veeravalli *et al.*, PRL **101**, 250403 (2008)

Structure Factors

- The Static Structure Factor measures the integrated response at a particular momentum

$$S(k) = \frac{\hbar}{N} \int S(k,\omega) d\omega$$

$$\propto \int \Delta X_{COM}(k,\omega) d\omega$$

where the proportionality depends the 2-photon Rabi frequency which is difficult to measure accurately

We overcome this by invoking the *f*-sum rule

f-Sum Rule

- Particle conservation dictates $S(k,\omega)$ obeys:

$$NE_r = \hbar^2 \int S(k,\omega)\omega d\omega \quad \propto \int \Delta X_{COM}(k,\omega)\omega d\omega$$

where the proportionality is the same as for $S(k)$

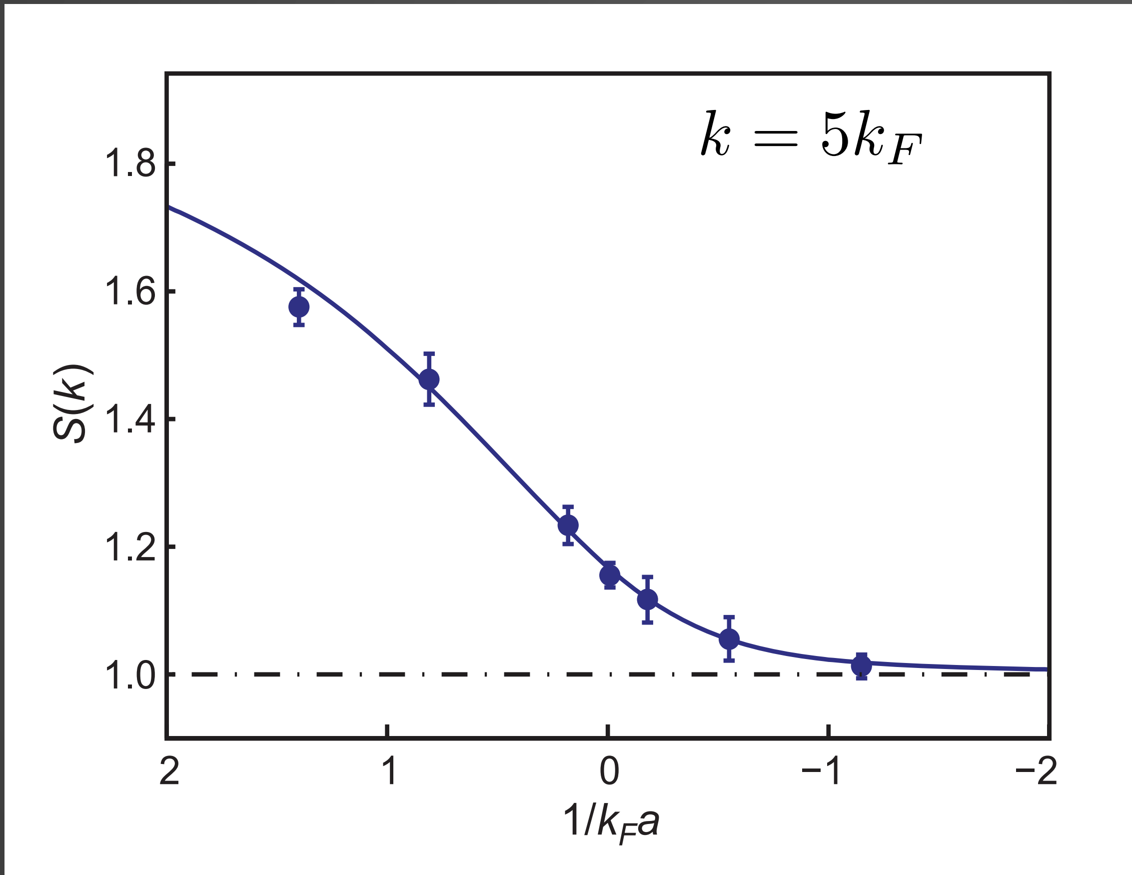
$$S(k) = \frac{\hbar}{N} \int S(k,\omega) d\omega \quad \propto \int \Delta X_{COM}(k,\omega) d\omega$$

Combining these provides an **absolute** measure of $S(k)$ requiring only the recoil energy, E_r

$$\boxed{S(k) = \frac{E_r}{\hbar} \frac{\int \Delta X_{COM}(k,\omega) d\omega}{\int \Delta X_{COM}(k,\omega)\omega d\omega}}$$

Static Structure Factor

- This greatly improves the measurement accuracy of $S(k)$ through the BEC-BCS crossover



- $S(k)$ decays from 2 – 1 through the BEC-BCS crossover due to the decay of $g_{\uparrow\downarrow}^{(2)}(r)$, in good agreement with theory

Veeravalli *et al.*, PRL **101**, 250403 (2008)

Kuhnle *et al.*, PRL **105**, 070402 (2010)

$$S(k) = 1 + n\int [g^{(2)}(r) - 1]e^{-ikr}dr$$

Universal Structure Factor

Universal Pair-Correlations

- Tan showed that the spin-up / spin-down density-density correlation function is given by

$$g^{(2)}_{\uparrow\downarrow}(r) = \frac{\mathcal{C}}{16\pi^2}\left(\frac{1}{r^2} - \frac{2}{ar}\right) \qquad (a \gg r)$$

- Correlation functions are generally hard to measure
  - BUT, we can consider the Fourier transform

$$S(k) = 1 + n\int [g^{(2)}(r) - 1]e^{-ikr}dr$$

Universal Pairing

- The Fourier transform of this expression gives a new universal relation for the static structure factor

$$S_{\uparrow\downarrow}(k) = \frac{\mathcal{I}}{4Nk_F}\left(\frac{k_F}{k}\right)\left[1 - \frac{4}{\pi k_F a}\left(\frac{k_F}{k}\right)\right]$$

- $S_{\uparrow\downarrow}(k)$ has a simple analytic dependence on (k/k_F)
- $S(k)$ can be measured experimentally using inelastic Bragg spectroscopy

Stamper-Kurn *et al.*, PRL **83**, 2876 (1999)
Steinhauer *et al.*, PRL **88**, 120407 (2002)

$$S(k) = S_{\uparrow\uparrow}(k) + S_{\uparrow\downarrow}(k) \cong 1 + S_{\uparrow\downarrow}(k) \qquad (k \gg k_F)$$

Combescot *et al.* EPL **75**, 695 (2006)
Veeravalli *et al.*, PRL **101**, 250403 (2008)

Universal $S(k)$

- To verify the universal relation we need to measure the dependence of $S(k)$ on k/k_F and $1/(k_F a)$

$$S_{\uparrow\downarrow}(k) = \frac{\mathcal{I}}{4Nk_F}\left(\frac{k_F}{k}\right)\left[1 - \frac{4}{\pi k_F a}\left(\frac{k_F}{k}\right)\right]$$

- Rather than change k we vary k/k_F through the density

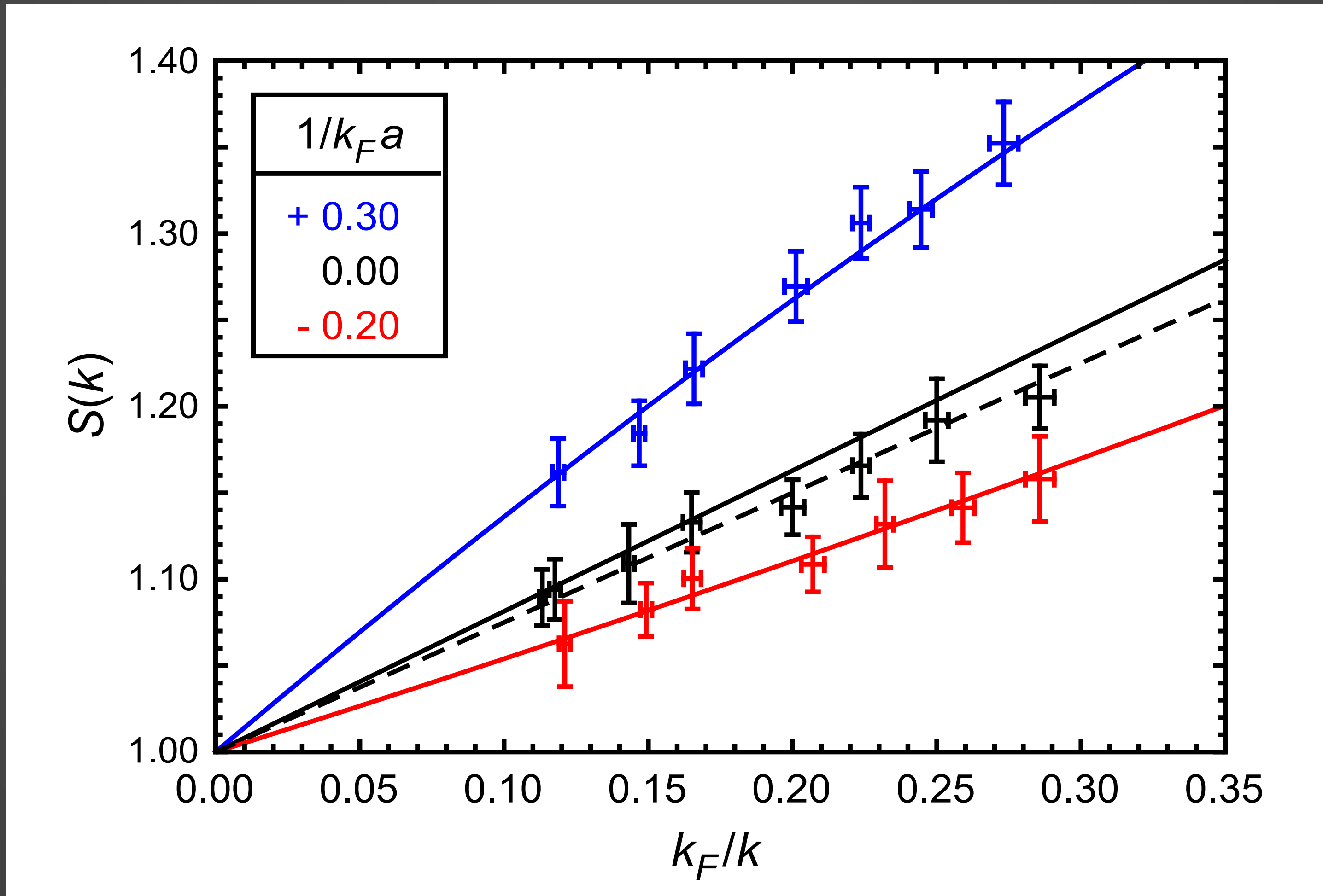
$$k_F = (48N)^{1/6}\sqrt{\frac{m\bar{\omega}}{\hbar}} \qquad \bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$$

- With our crossed trap we can tune over the range

$$\bar{\omega} = 2\pi \times (38 \rightarrow 252)\,\mathrm{s}^{-1}$$

Universal $S(k)$

- We vary k/k_F over the range 3.5 – 9.1 and also vary B to achieve the desired value of $1/(k_F a)$ for each point

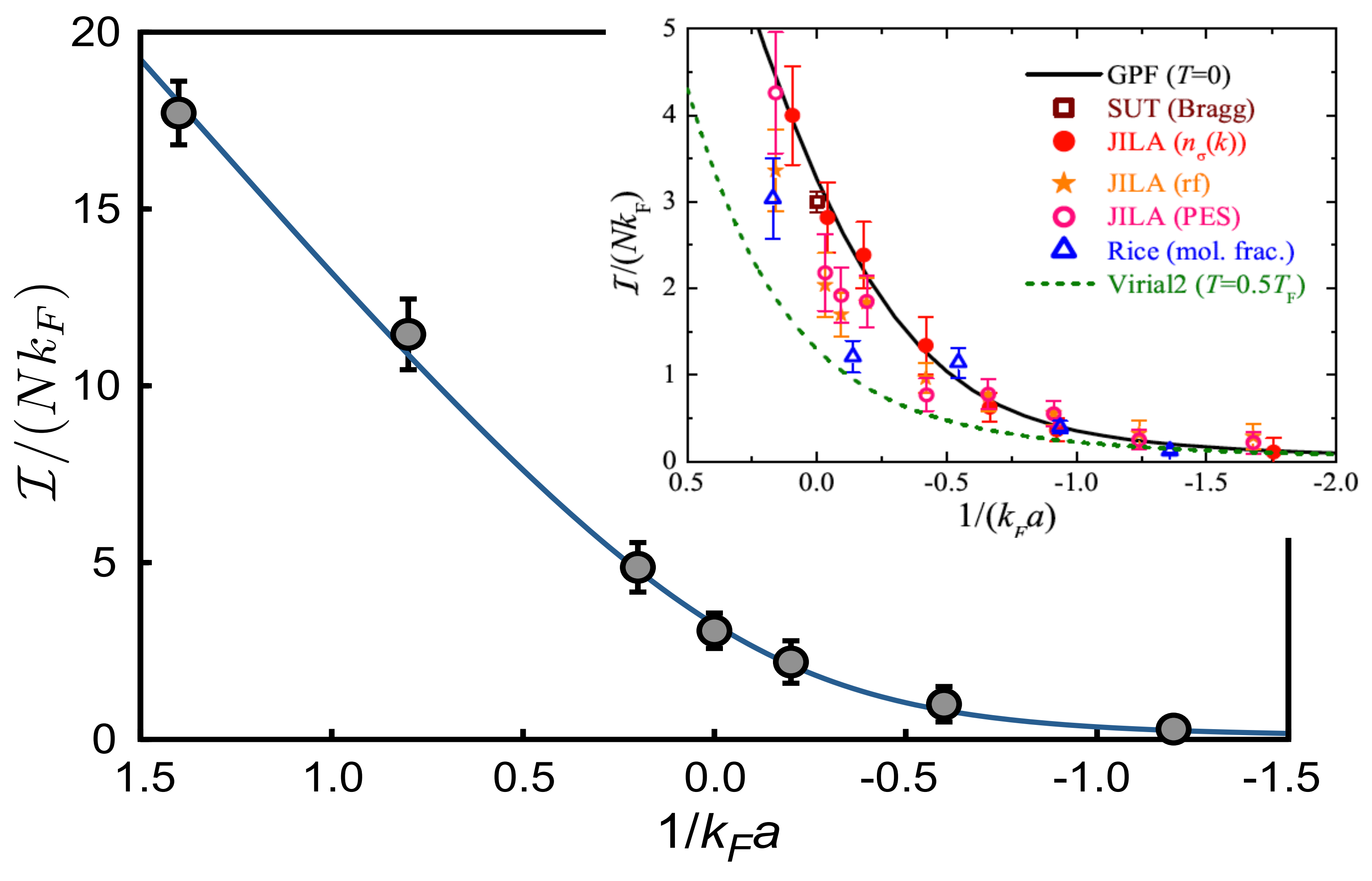


Kuhnle *et al.*, PRL **105**, 070402 (2010)

Contact – Interaction Dependence

Interaction dependence

- Our measurements of $S(k)$ with the universal relation allow us to extract the C as a function of $1/k_Fa$ …



Contact – Temperature Dependence

Contact vs. T/T_F

- Contact and $S(k)$ are linearly related, at unitarity:

$$S_{\uparrow\downarrow}(k) = \frac{\mathcal{I}}{4Nk_F}\left(\frac{k_F}{k}\right)$$

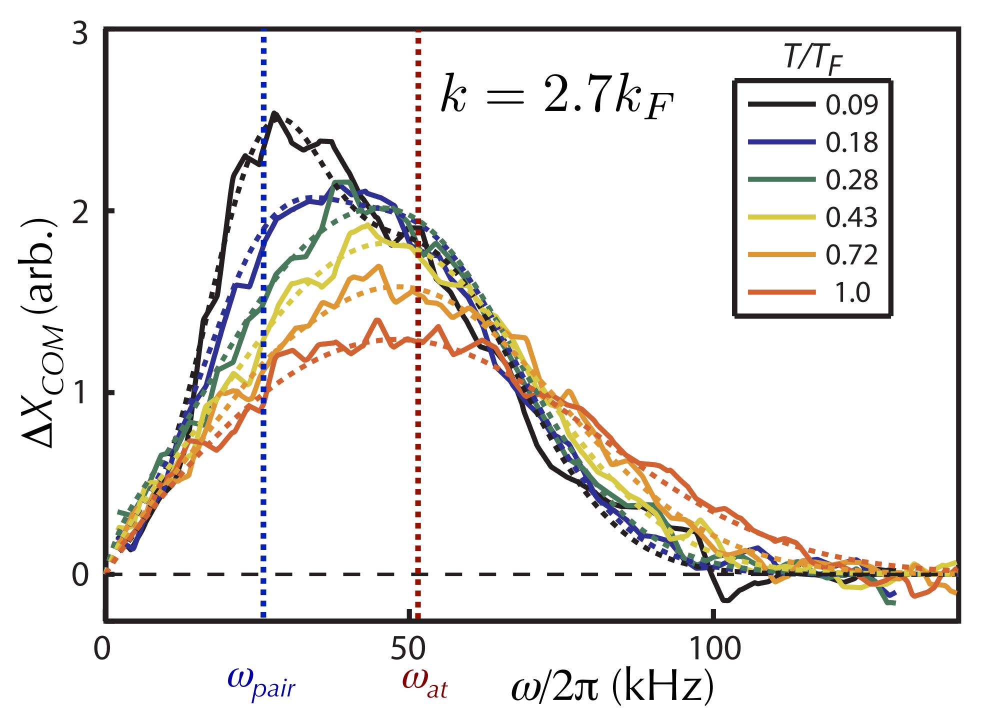
➔ $S(k)$ will show how $\mathcal{C}$ depends on temperature

- We use a modified setup for this - Bragg beams intersect at an angle of 50° to reduce k - (ω_{at} = 51kHz)

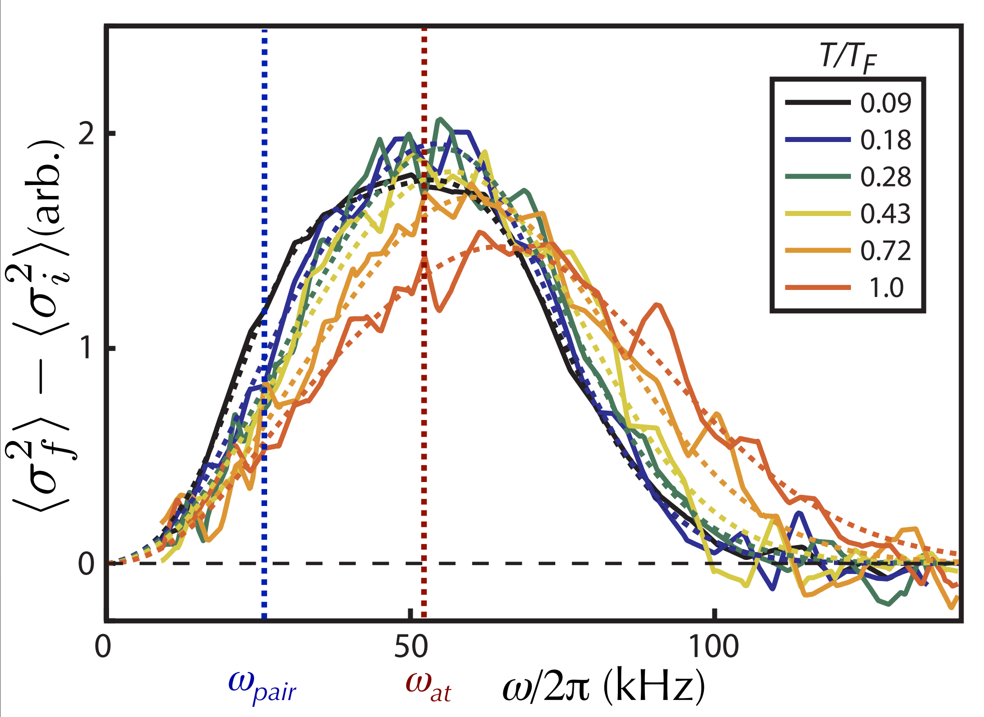
T dependence of *S*(*k*,*ω*)

- Pairing is prominent at low temperatures: $\frac{1}{k_F a} = 0$

Momentum transferred



Energy transferred

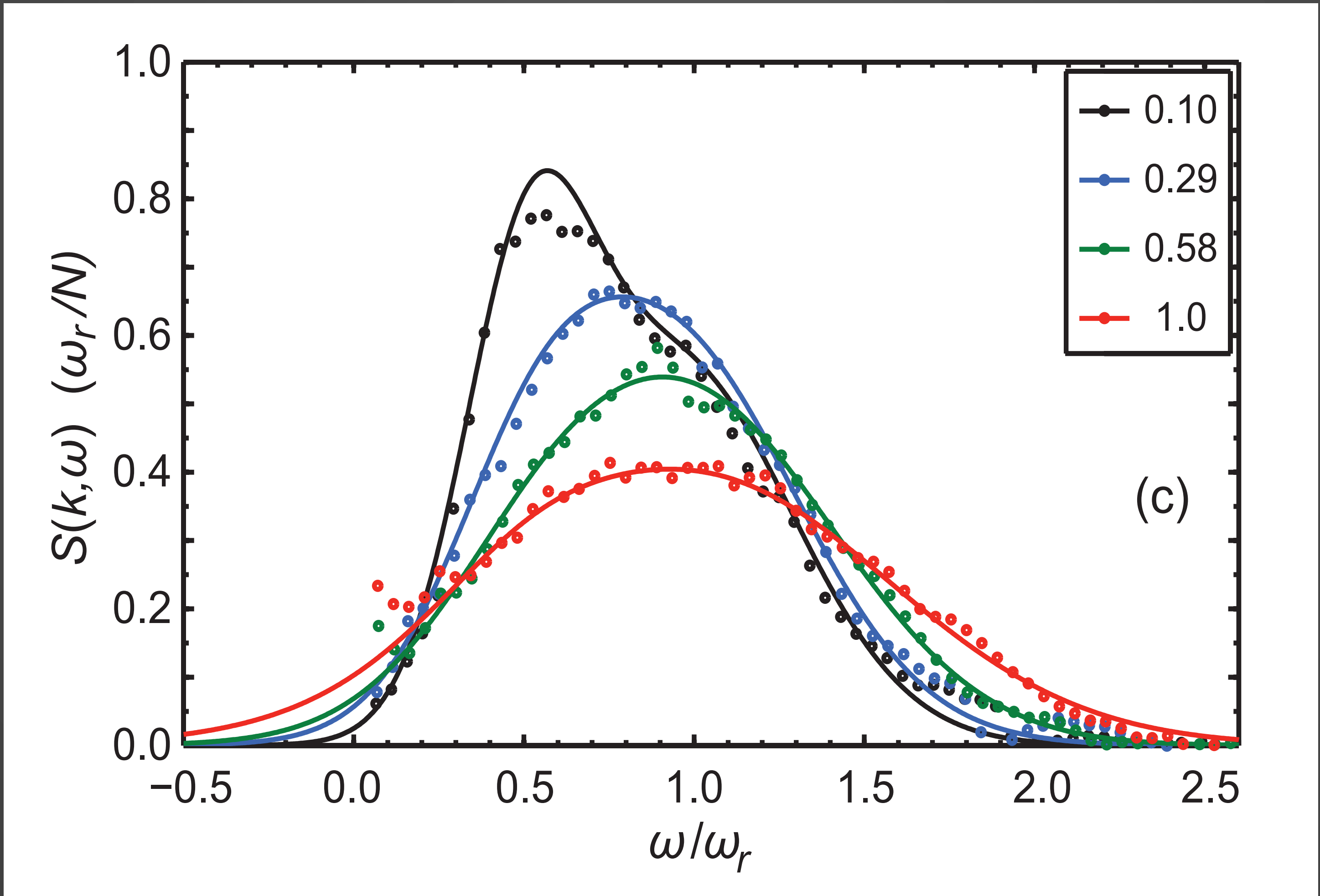


$$\Delta P \propto \Delta X_{COM} \propto k \left[1 - \exp\left(\frac{\hbar\omega}{k_B T}\right)\right] S(k,\omega)$$

$$\Delta E \propto \langle \sigma_f^2 \rangle - \langle \sigma_i^2 \rangle \propto \omega \left[1 - \exp\left(\frac{\hbar\omega}{k_B T}\right)\right] S(k,\omega)$$

T dependence of $S(k,\omega)$

- Using both the first and second moments we get two measures of $S(k,\omega)$ which average to give…



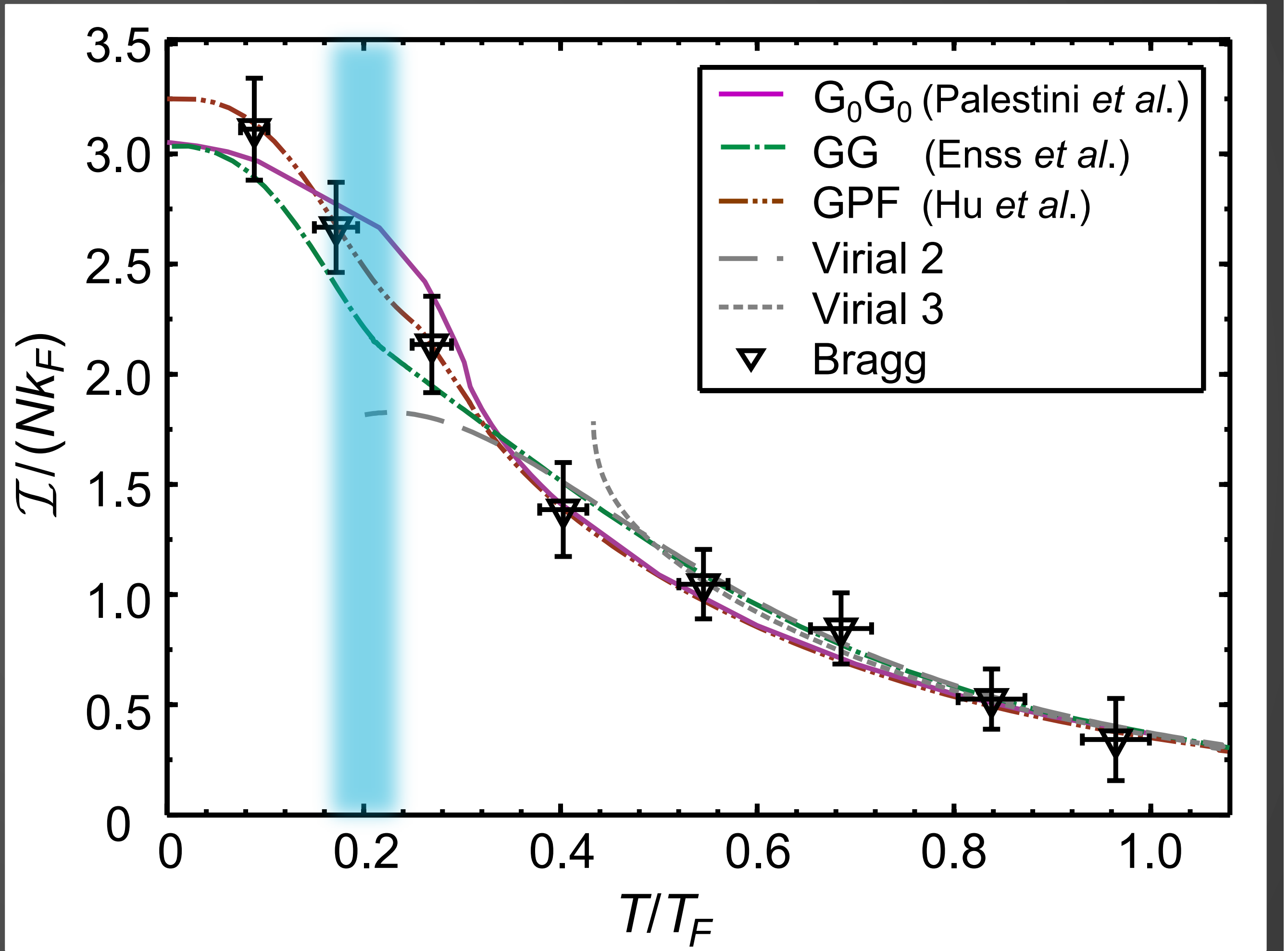
$$\frac{1}{k_F a} = 0$$

Contact at Unitarity

- Integrating $S(k,\omega)$ gives $S(k)$ and hence C
- Theory based on t-matrix and virial expansion

Palestini, PRA **82**, 021605(R) (2010)
Hu, NJP **13**, 035007 (2010)
Enss, Ann Phys. **326**, 770 (2011)

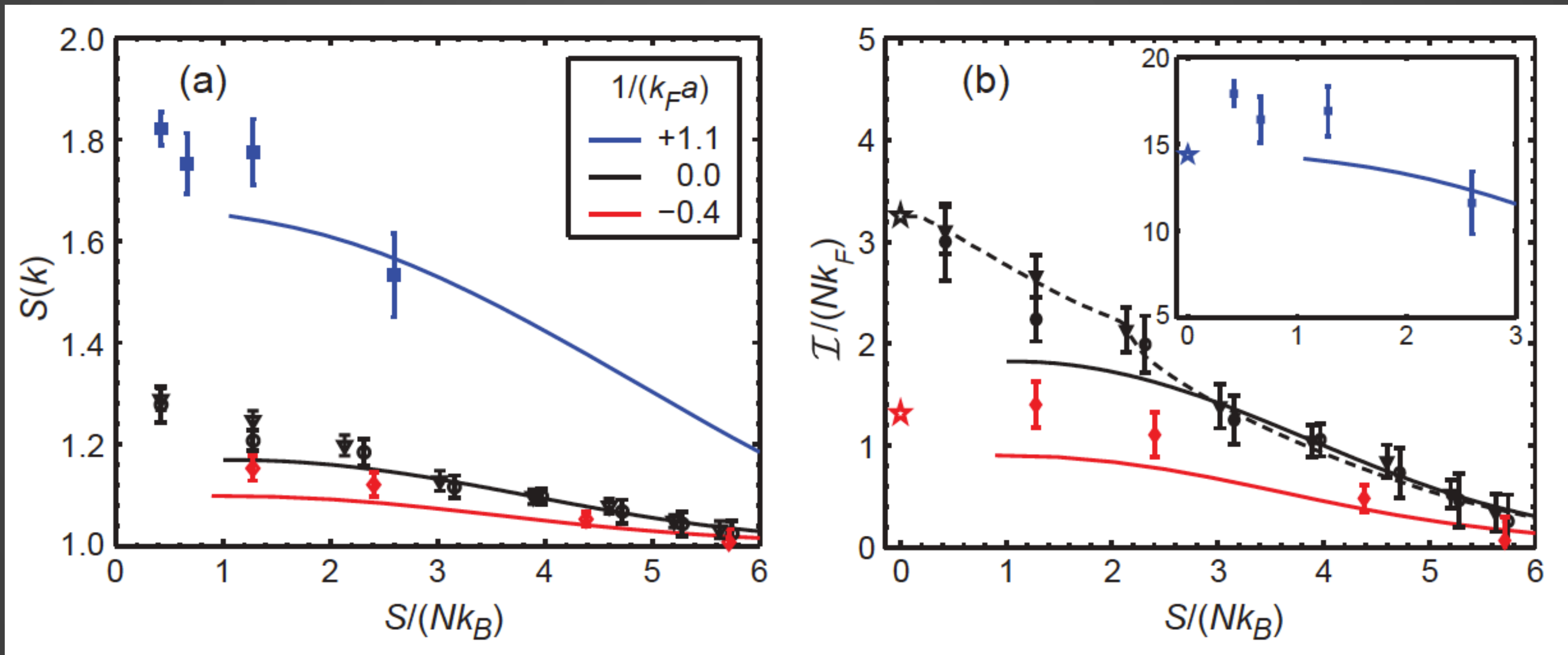
- Short-range pair correlations exist well above T_c



Kuhnle *et al.*, PRL **106**, 170402 (2011)

BEC and BCS

- Away from unitarity T is difficult to define
- Instead we measure entropy after adiabatic sweep to unitarity or far BCS limits



- ΔE measured after long hold time

Towards Spatially Resolved Bragg Spectroscopy

Bragg Spectroscopy

- Fluctuations/drift in the cloud/trap position prior to the Bragg pulse are a dominant noise source

- However, there is a way to reduce their impact…

- We can effectively make two simultaneous measurements of $S(k,\omega)$ from a single image…

(i) $\Delta P \propto k\, S(k,\omega)$ (ii) $\Delta E \propto \omega\, S(k,\omega)$

Bragg Spectroscopy

- Quantitatively for a short Bragg pulse we have

$$\Delta P = m\frac{\Delta X_{COM}}{\tau} \qquad \Delta E = \frac{m}{2}\frac{\Delta\sigma_x^2}{\tau^2}$$

where τ is the time of flight after the Bragg pulse and

$$\Delta X_{COM} = \langle x_f \rangle - \langle x_i \rangle$$

$$\begin{aligned}\Delta\sigma_x^2 &= \langle \sigma_{x_f}^2 \rangle - \langle \sigma_{x_i}^2 \rangle = \langle (x_f - \langle x_i \rangle)^2 \rangle - \langle \sigma_{x_i}^2 \rangle \\ &= \langle x_f^2 \rangle - 2\langle x_f \rangle \langle x_i \rangle + \langle x_i \rangle^2 - \langle \sigma_{x_i}^2 \rangle\end{aligned}$$

- We can therefore measure both the momentum and energy transferred in a single shot

Bragg Spectroscopy

- With these we can take the ratio $\Delta P/\Delta E$ to eliminate $S(k,\omega)$ and find a quadratic equation for the cloud centre of mass before the Bragg pulse

$$\frac{\Delta P}{\Delta E} = \frac{k}{\omega}\frac{S(k,\omega)}{S(k,\omega)} = \frac{2\tau(\langle x_f\rangle - \langle x_i\rangle)}{\langle x_f^2\rangle - 2\langle x_f\rangle\langle x_i\rangle + \langle x_i\rangle^2 - \langle\sigma_{x_i}^2\rangle}$$

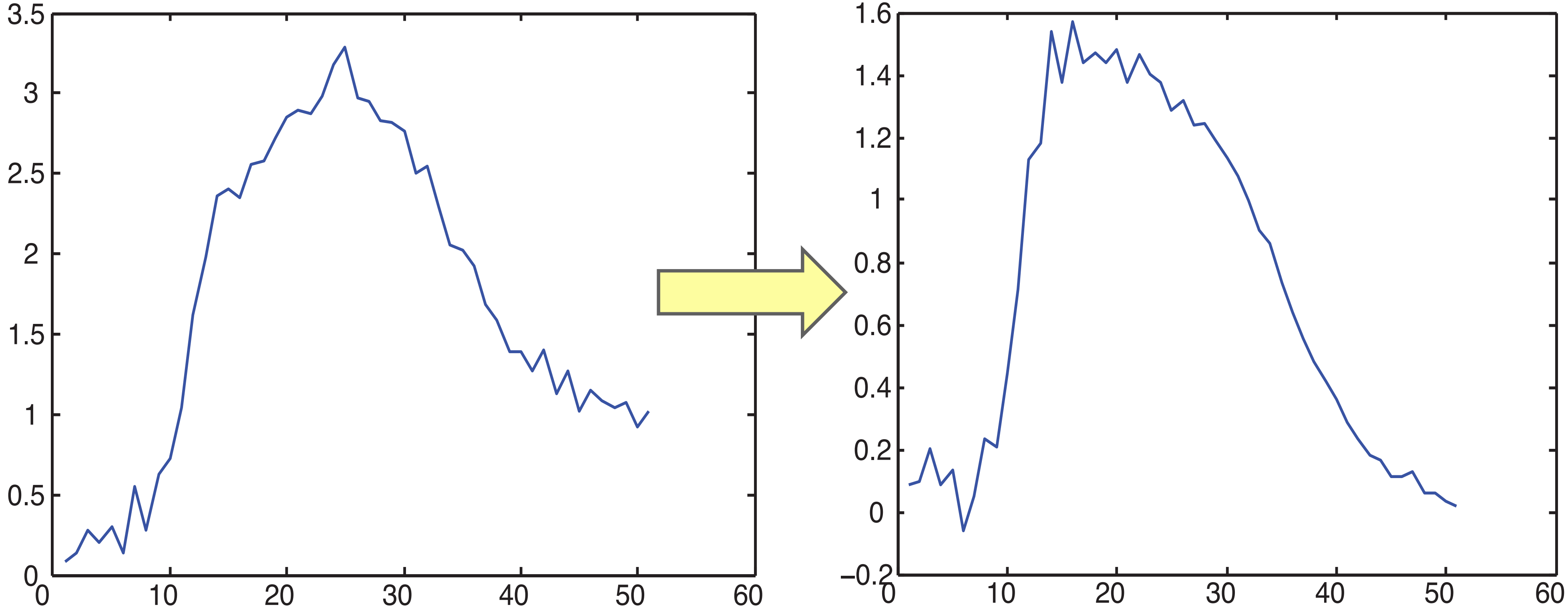
- For this to work we need:

$$k, \omega, \tau \text{ and } \langle\sigma_{x_i}^2\rangle$$

- We can obtain the initial mean square width from a reference cloud, taken with an $\omega = 0$ Bragg pulse (as the centre of mass of the reference image is known)

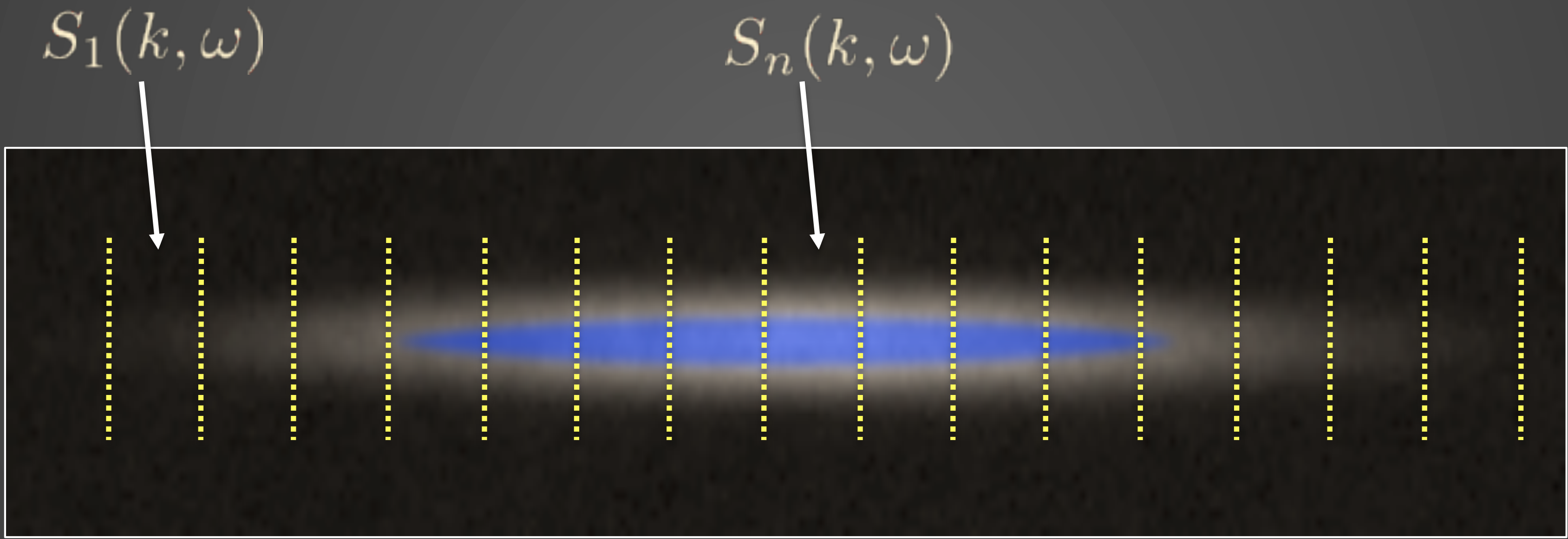
Bragg Spectroscopy

- Solving $\langle x_i \rangle$ for each image allows us to plot the corrected centre of mass displacement: $\langle x_f \rangle - \langle x_i \rangle$



Bragg Spectroscopy

- Improved precision lets us measure the response through different longitudinal slices of the cloud

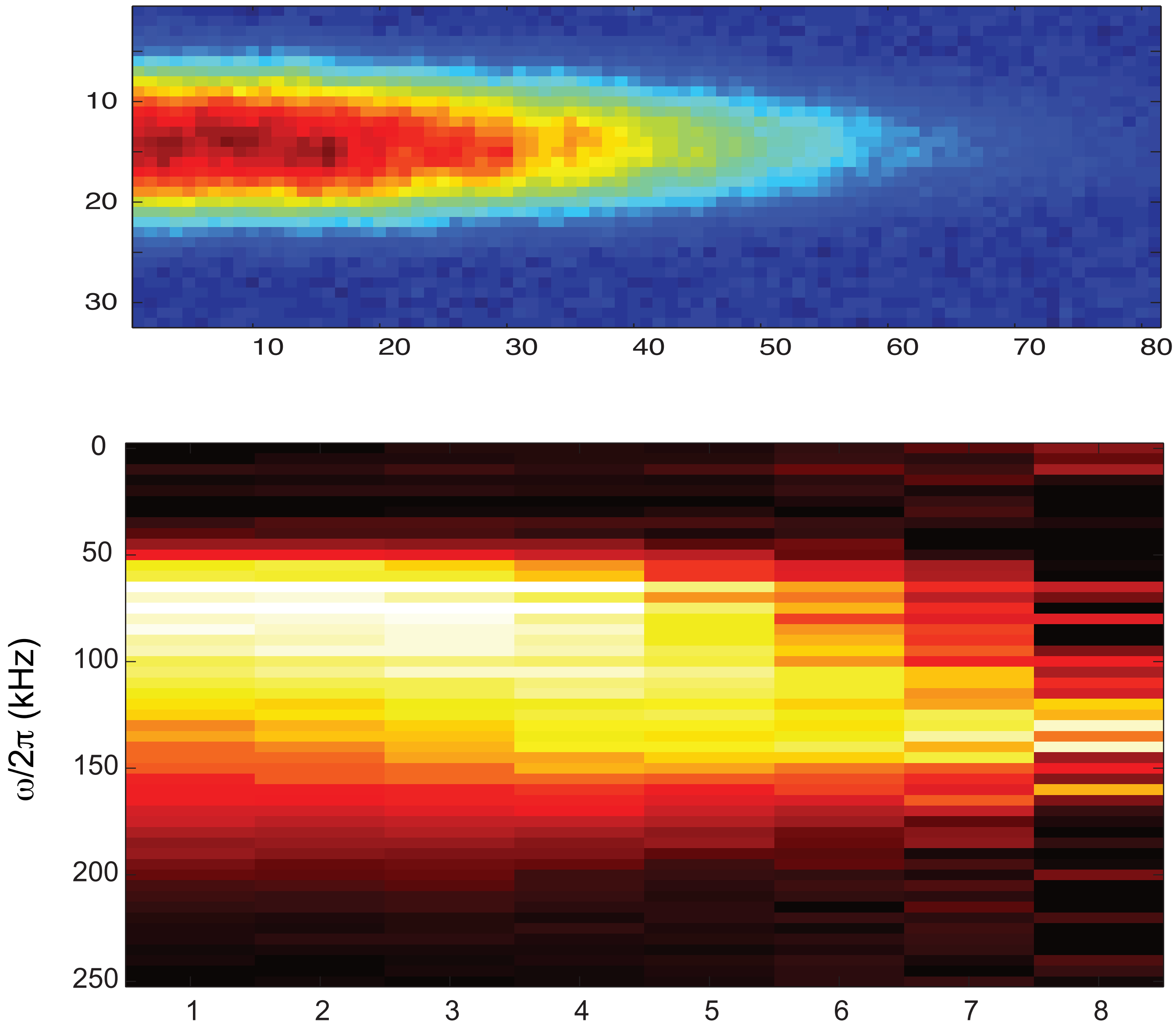


- We also use a short time of flight so the different slices do not mix too much

Bragg Spectroscopy

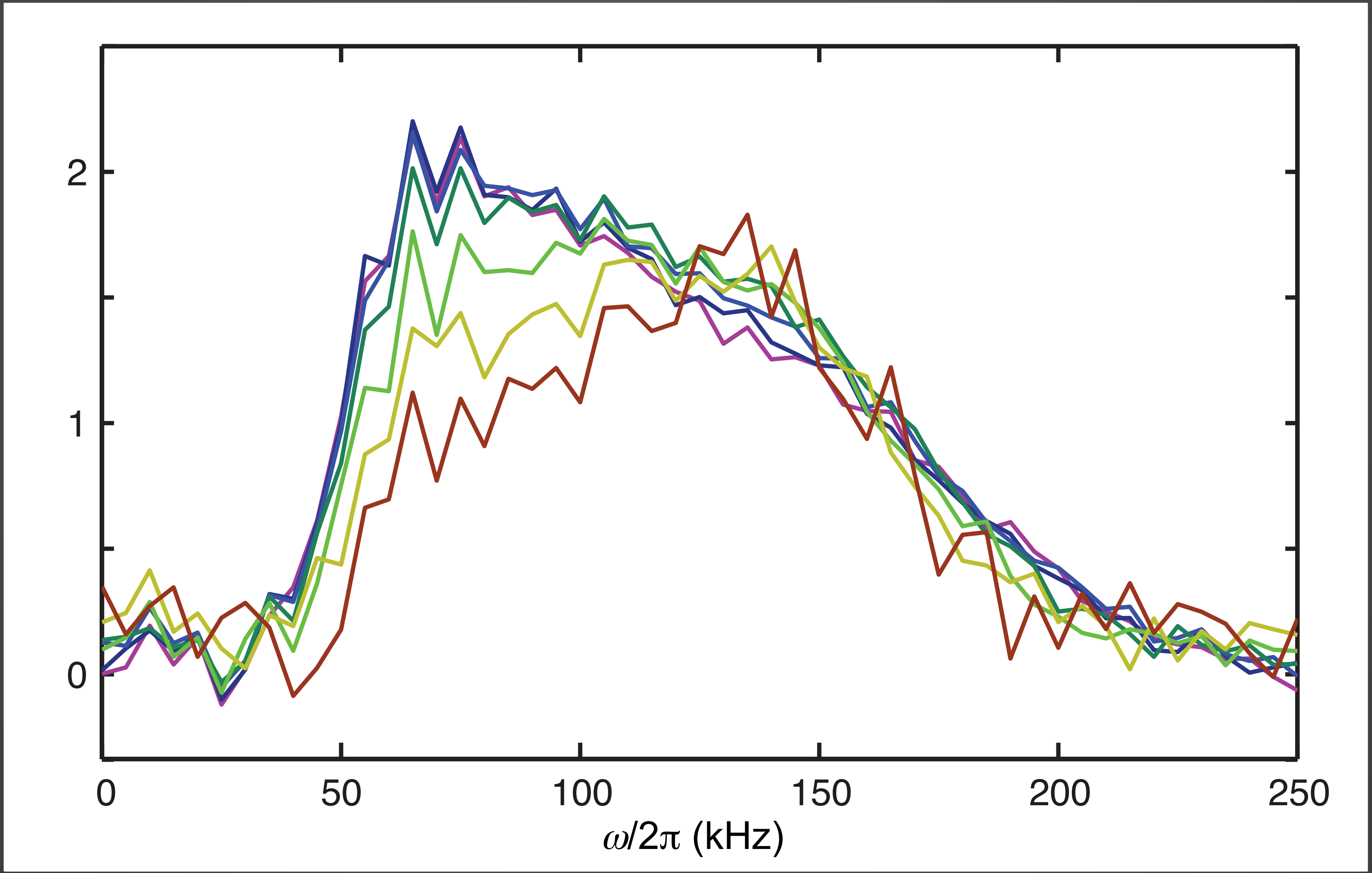
- Choosing symmetric slices around the cloud centre we average over two sides to obtain longitudinal spectra

$\omega_r/2\pi = 130\,\mathrm{kHz}$



Bragg Spectroscopy

- "Qualitative" preliminary spectra for slices with increasing distance from the trap centre



Conclusions and Outlook

- Bragg spectroscopy provides as useful tool to study the contact in a strongly interacting Fermi gas
- We are working towards resolving the Bragg response from different spatial regions in the cloud
- A number of issues still remain before the corrected method is quantitatively accurate
- Extracting homogeneous contact…

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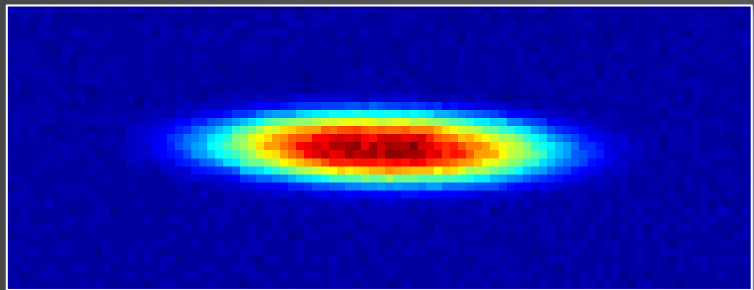
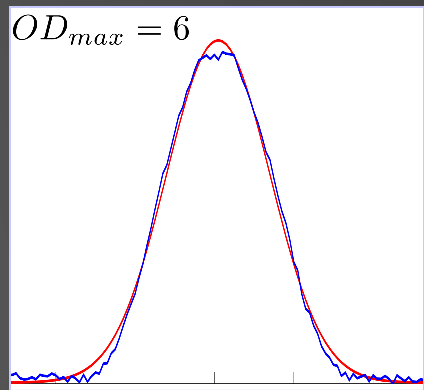
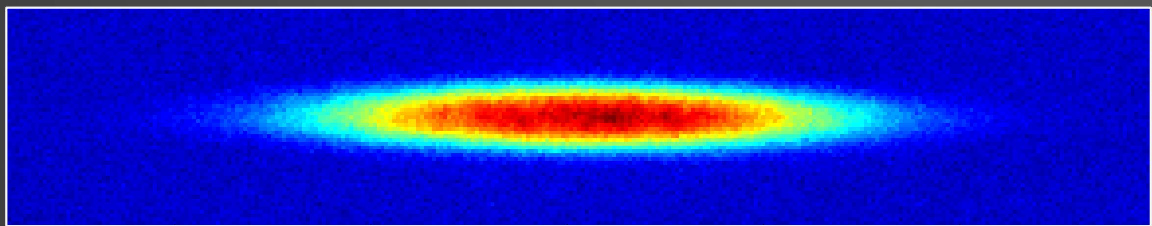
Imaging

- Improved resolution (~2.8 μm) and sensitivity

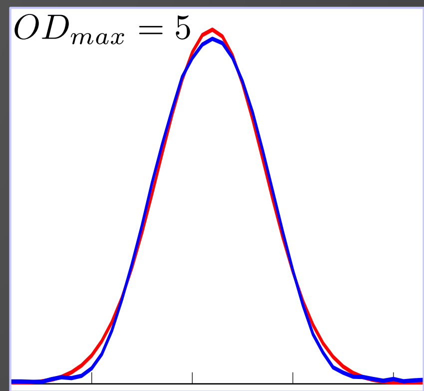


Old - side imaging



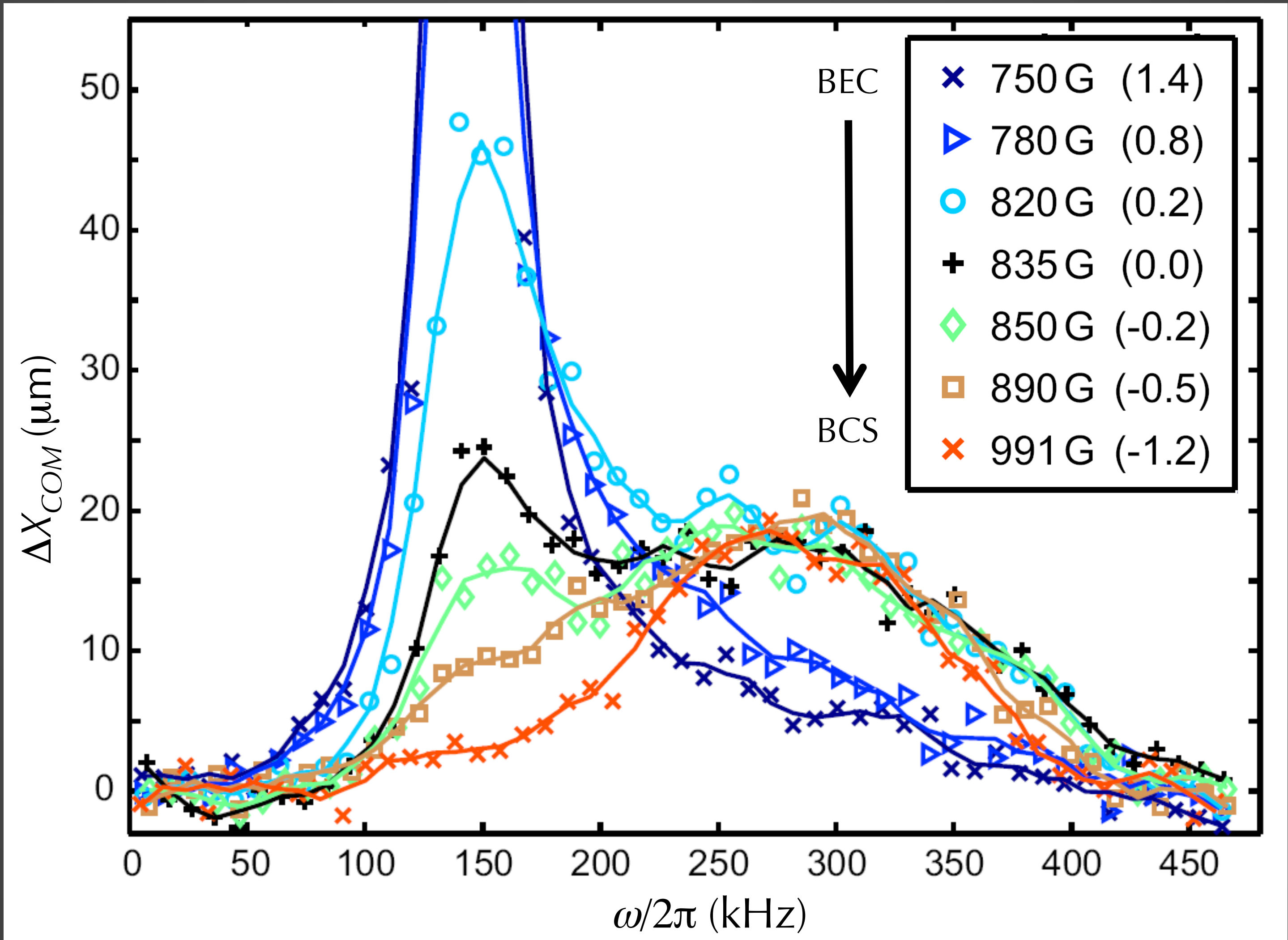


New - top imaging



Bragg Spectroscopy

- Previously measured spectra in the BEC-BCS crossover



Veeravalli *et al.*, PRL **101**, 250403 (2008)