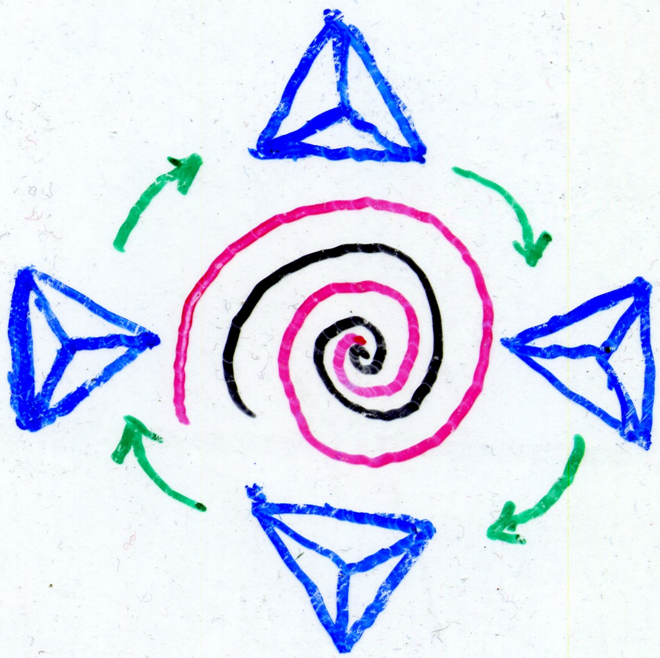
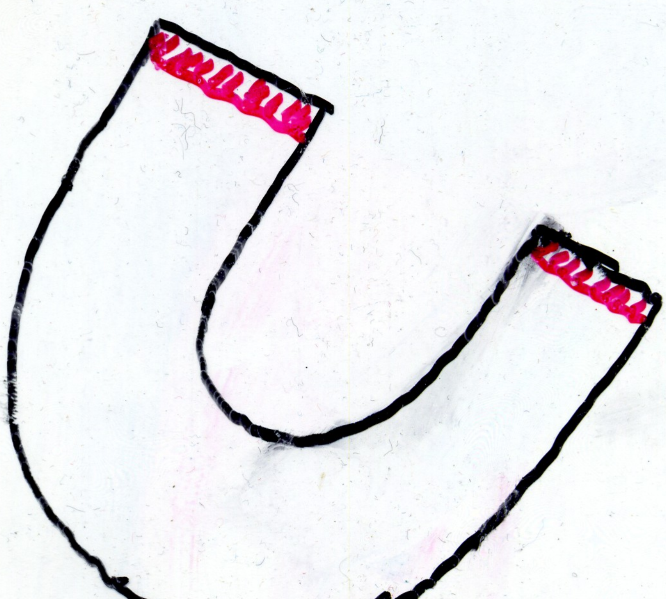
Vortex Molecules





Ryan Barnett
Eugene Demler
Ashvin Vishwanath

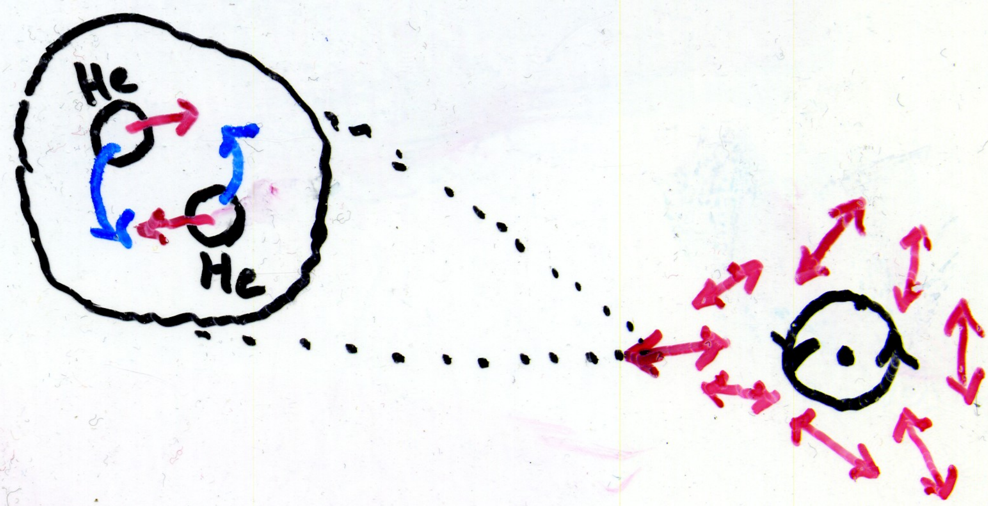
Vortex Chemistry

Varieties of Vortices

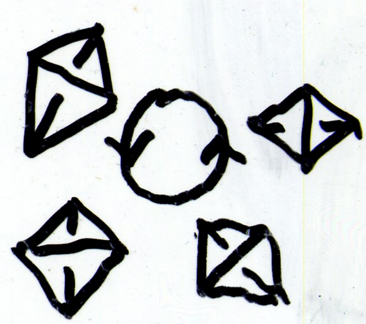
He^4

Circulation $= \dfrac{nh}{m_{He}}$

He^3

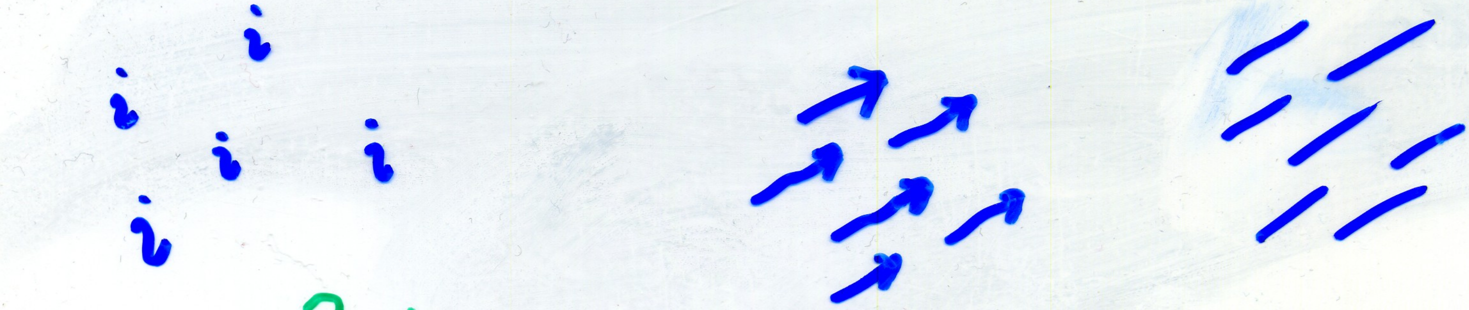


Condensates of Atoms with Spins [Laser-Trapped]

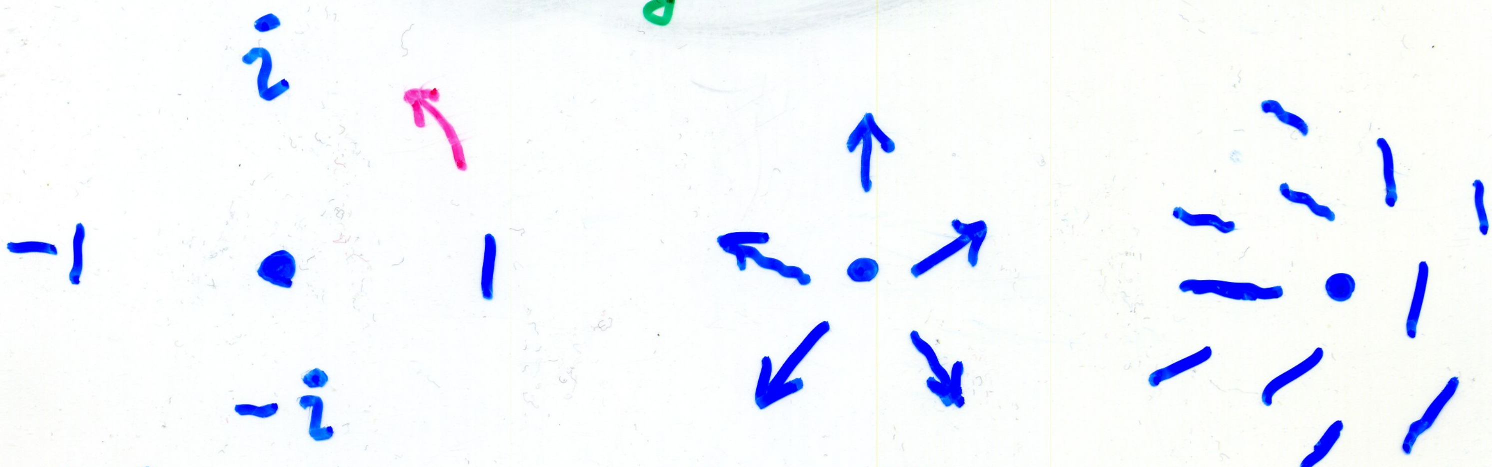


Quantum Hall States — fancy vortices & magnetization textures with fractional charges

Defects in Different Fields



Homogeneous Fields



Vortex | Spin Texture | Disclinatio

Phase Field | Planar Vector Field | Line Field

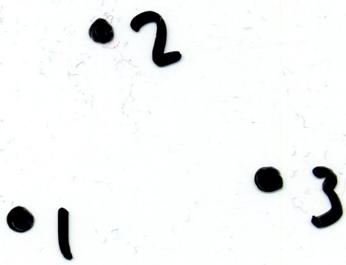
"Noncommutative" Defects

"Tetrahedron Field"

Neat Properties

Finite or Noncommutative Sets of Charges

Non-additive Interactions



$\left.\begin{matrix}2,3\\1,2\\1,3\end{matrix}\right\}$ repel

$1,2,3\}$ attract

"Telecatalysis"

Before After

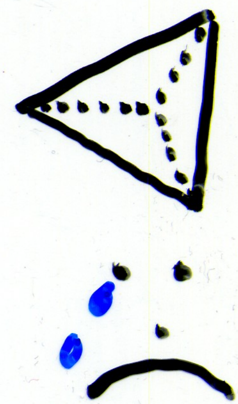
Vortices

&

Symmetry
(Perfect or Not)

Imperfect Symmetry

$\uparrow \vec{B}$ (Polarizing Field)



Total energy:

$$\iiint K(\nabla X)^2 + V(X) + \epsilon V_{A\text{-}S}(X)\, d^3$$

Elastic Energy

Asymmetric Energy

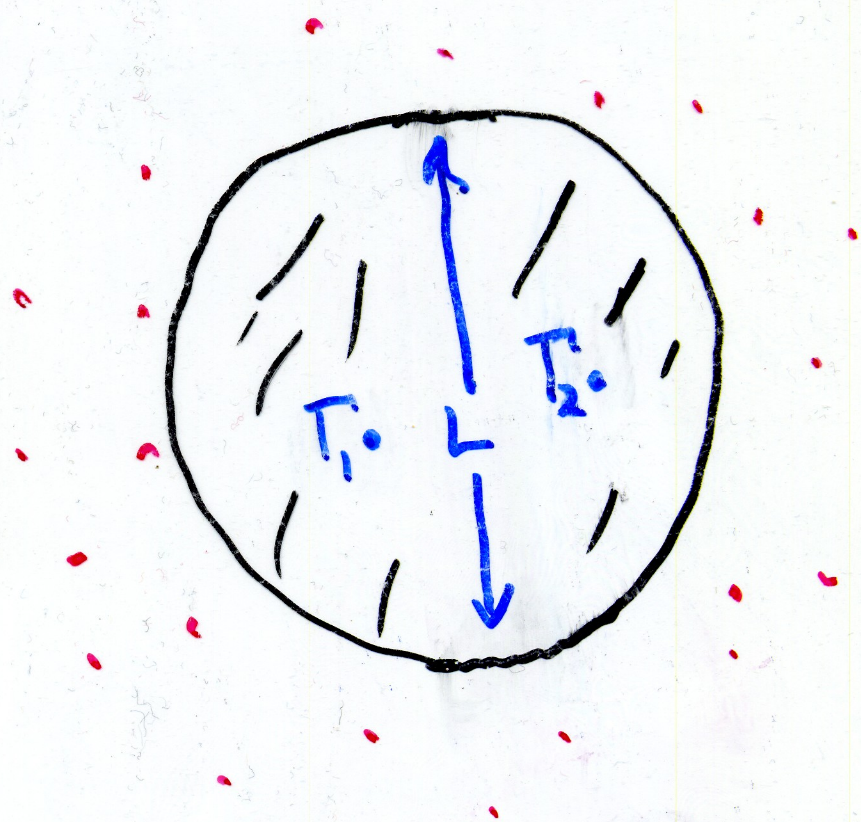
Does $V_{A\text{-}S}$ ruin everything?

Confinement Force

(see Volovik: Universe in a Helium Droplet)

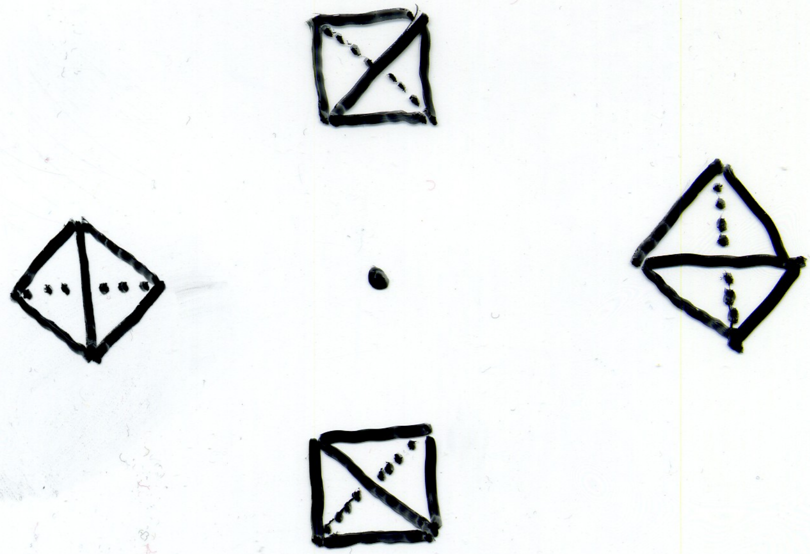
Γ_1, Γ_2 not $\vec{B}$ Compatible

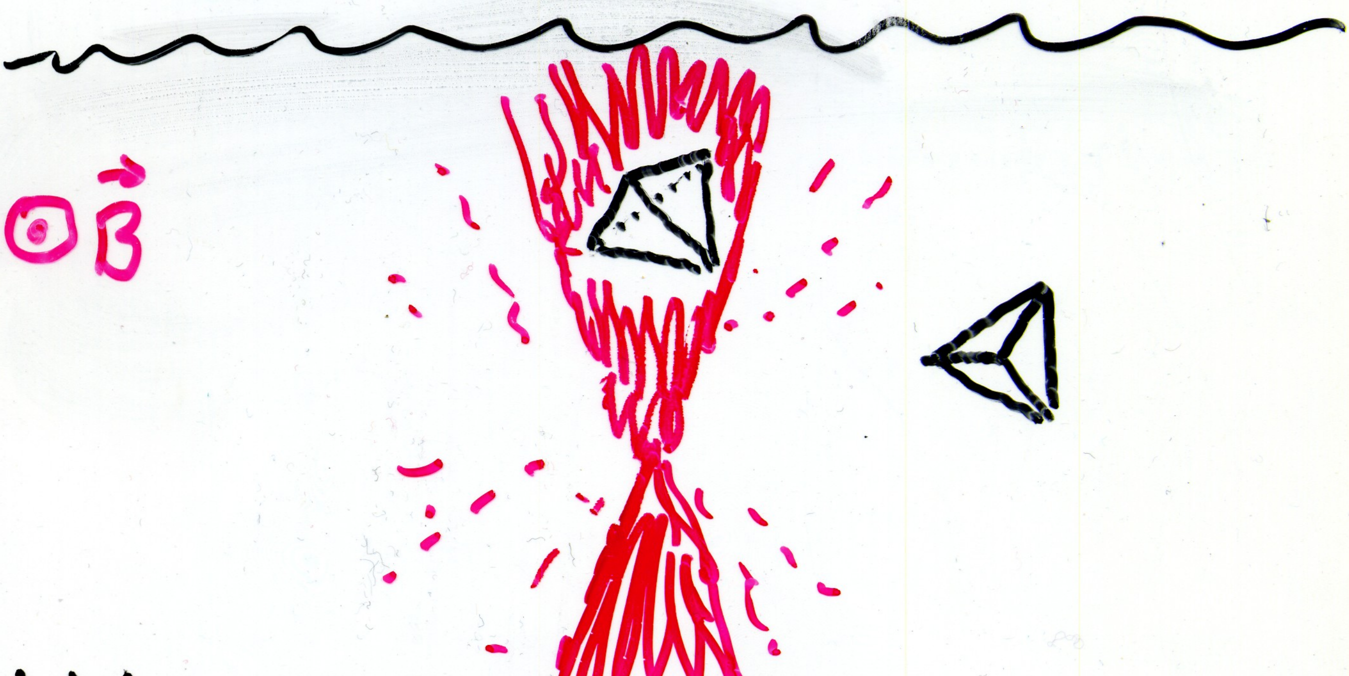
$\Gamma = \Gamma_1 \Gamma_2 \in G_B$ "Magnetic Charge"



$$E = \epsilon n L^2$$

A Vortex w/o B





Prohibitive with Magnetic Field

Two Vortices can be surrounded by good tetrahedra

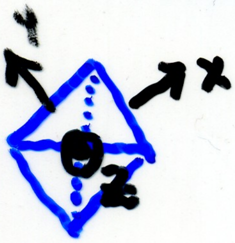
Defects in Symmetric Phase

Defects in Asymmetric Phase

$\mathcal{S}_{\text{tetrahedron}} \subseteq SU_2$

$\{2\pi n/3\}$

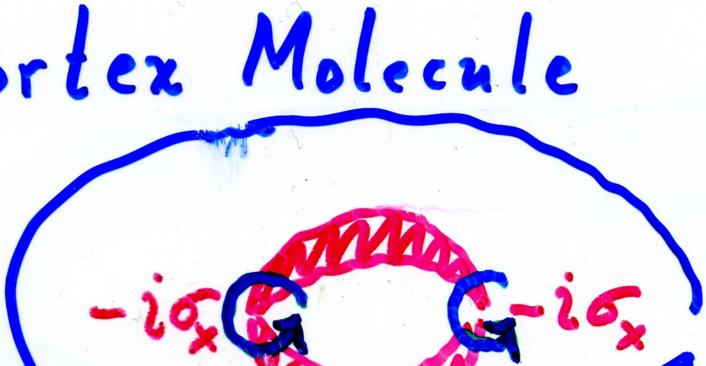
Examples:



120° rotation $\longrightarrow$ $2\pi\left(\frac{1}{3}+2k\right)$

180° rotation $\not\longrightarrow$

Vortex Molecule



$(-i\sigma_x)^2 = -\mathbb{1} = e^{2\pi i \frac{(\hat{x}+\hat{y}+2\hat{z})}{2\sqrt{3}}}$

Structure of Symmetry Group Leads to interesting effects even when symmetry is imperfect:

Vortex Molecules

Require

a) Long-range confining forc

Produced by

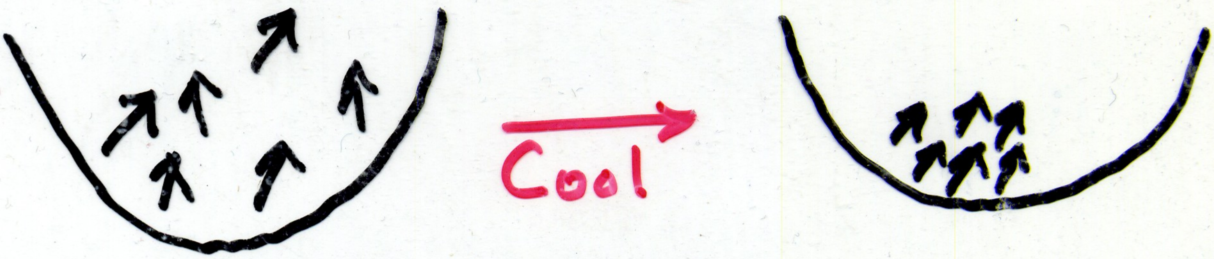
$V_{A\text{-}S}$, asymmetry

b) Separating force

2 Mechanisms:

Short-range "Coulomb" Repulsion / Dynamics

Spinor Condensates



Collective Spinor $\Psi = \begin{pmatrix} \psi_f \\ \vdots \\ \psi_{-f} \end{pmatrix}$

Minimizes spin-dependent interaction energy

$$F = \iint d^2r \, \frac{4\pi\hbar^2}{m} \sum_{\substack{S_{tot}=0 \\ \text{even}}}^{2f} a_{S_{tot}} \, \psi^\dagger_{(r)} \psi^\dagger_{(r)} P_{S_{tot}} \psi_{(r)} \cdots$$

$$- \mu \, \psi^\dagger \psi$$

Spin 1 Atoms in Condensates

$\beta \propto a_2 - a_0$

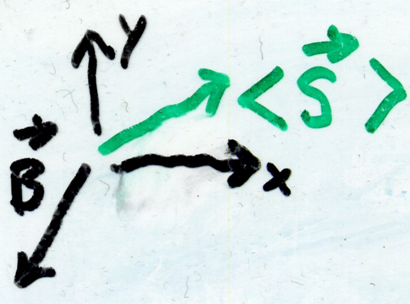
$|\langle \vec{S} \rangle| = 1$ e.g. $|S_z = 1\rangle$ — ^{87}Rb

$\langle \vec{S} \rangle = 0$ e.g. $|S_z = 0\rangle$ — ^{23}Na

Ferromagnetic ⟵ $\beta = 0$ ⟶ Polar

(Stamper-Kurn, Chapman, Sengstock) (Ketterle)

Ferromagnetic Phase



Spin 1:

Dynamics & Mass of Spin Vortices

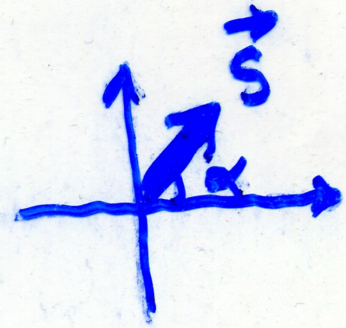
Vortex Types in Spin 1 Condensates

Three-Species Mixture

Spin Picture

$$e^{i\theta}\begin{pmatrix}\sqrt{n_1}\,e^{-i\alpha}\\ \sqrt{n_0}\\ \sqrt{n_{-1}}\,e^{i\alpha}\end{pmatrix}$$

Ground States

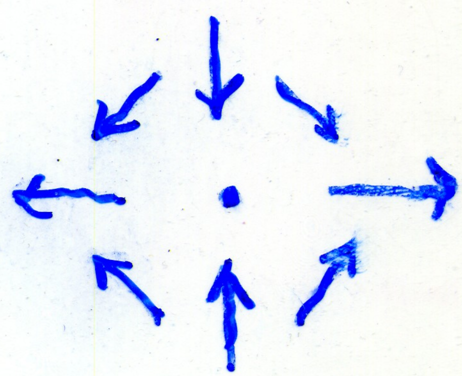


Charge Vortex

$Q_C = 1$

Spin Vortex

$Q_S = +1$



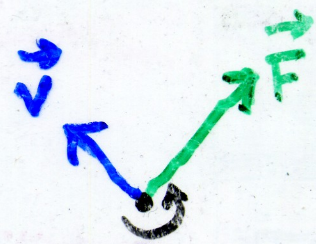
Sadler et al. Nature (2006)

Ohmi & Machida J. Phys Soc. Jpn (1999)

Ho Phys. Rev. L. (1998)

Newtonian Vortex Orbits?

The velocity of an ordinary vortex is determined:



$$\vec{v} \propto \hat{z} \times \vec{F}$$

A vortex cannot move if there is no force.

How do spin vortices move?

— Markus Greiner

"Newton's first law"

Inertia

$$F_M \propto n_1 - n_{-1} = 0$$

"Newton's second law"

Forces

$$\vec{F}_{12} \propto \frac{Q_{s1} Q_{s2} \hat{r}}{r}$$

$$\vec{F}_{12} = m_v \vec{a}_1$$

arxiv 0902.3685

Energy Function

$$E = \int \frac{\hbar^2}{2m} |\nabla \Psi|^2$$

energy of currents

$$+ \frac{1}{2} \alpha |\Psi|^4 - \mu |\Psi|^2$$

repulsive energy and chemical potential

$$+ \frac{1}{2} \beta |\langle \vec{S} \rangle|^2$$

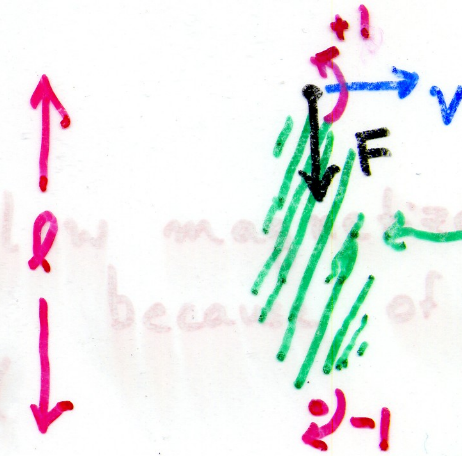
spin-dependent energy $\beta < 0$

$$+ q \langle S_z^2 \rangle$$

quadratic Zeeman

Kinetic Energy and Mass

Energy Density
(Numerics)



low magnetization because of phase mismatch

[Chang et al. Nat. Phys. 1 (2005)]

$$E_{spin} \propto |\beta|\, \ell^2$$

$$F \propto |\beta|\, \ell$$

$$v \propto \hat{z} \times \vec{F} \propto \beta \ell$$

$$E_{spin} \propto \frac{v^2}{|\beta|} = E_{kinetic}$$

$$m_v \propto \frac{1}{|\beta|}$$

Mass Estimate

$$m_v \sim \frac{w}{\Delta a}\, m_{atom}$$

$$\sim 10^{-21}\ \mathrm{kg}$$

for Rb^{87}

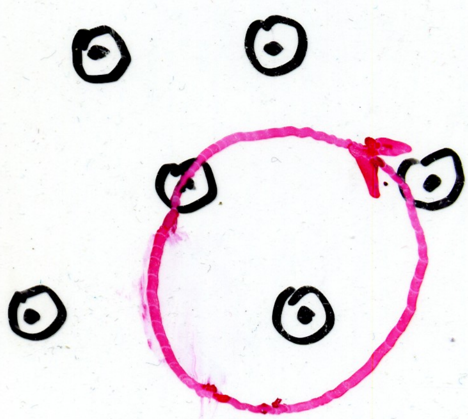
$w = 1\ \mu m$

Measuring the Mass

$$v = \text{const.} \sim \sqrt{\frac{n_1 \hbar^2}{2\pi m_v m_a}}$$

Kepler Orbits

$$\vec{B}_{eff} \propto \text{Magnetization}$$



$$\tau = \frac{m_{vortex}}{\hbar(n_1 - n_{-1})}$$

$\sim .3$ sec for

$n_1 - n_{-1} = 5\% \, n$

Cyclotron Orbits

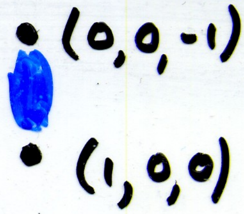
Phase Space

	Order of Eq. of Motion	Degrees of Freedom
Charge Vortex	1st order	$\vec{r}$
Spin Vortex	2nd order	$\vec{r} = \frac{1}{2}(\vec{r}_+ + \vec{r}_-)$ $\vec{p} \propto \hat{z} \times (\vec{r}_+ - \vec{r}_-)$

Massive Vortices - Summary

1. Spin vortices are made of two <u>confined</u> subvortices.

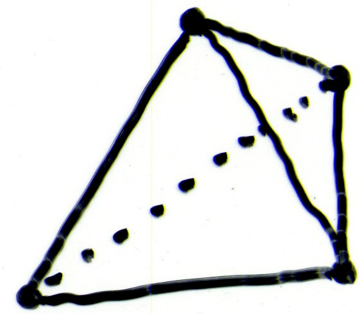
2. Spin vortices obey Newton's laws; their Kinetic Energy is stored in real space, in the confining band



3. Their mass can be measured by dynamical experiments.

Higher Spin

$$\begin{pmatrix} \psi_f \\ \vdots \\ \psi_{-f} \end{pmatrix} =$$



?

Geometry of Spinor Condensates

For a spin f state χ_0

Find all $2f$ coherent states which are orthogonal.

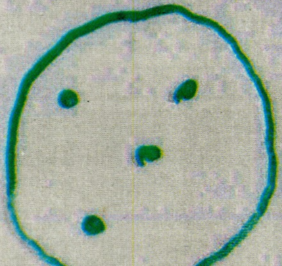
Def of coherent, depends on $\hat{n}$:

$$\hat{n}\cdot\vec{S}|\hat{n}\rangle = f|\hat{n}\rangle$$

The $2f$ solutions to

$$\langle\hat{n}|\chi_0\rangle = 0$$

make a geometric shape:



$f=2$

F. Klein etc.

Barnett, Turner, Demler Phys. rev. lett. Vol. 97

Spin roots distinguish between spinors which can't be rotated into each other:

$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} .66+.28i \\ .64+.05i \\ -.28+.07i \end{pmatrix}$
$\begin{pmatrix} .5 \\ .71 \\ .5 \end{pmatrix}$	$\begin{pmatrix} .66+.28i \\ -.05+.64i \\ -.28+.07i \end{pmatrix}$

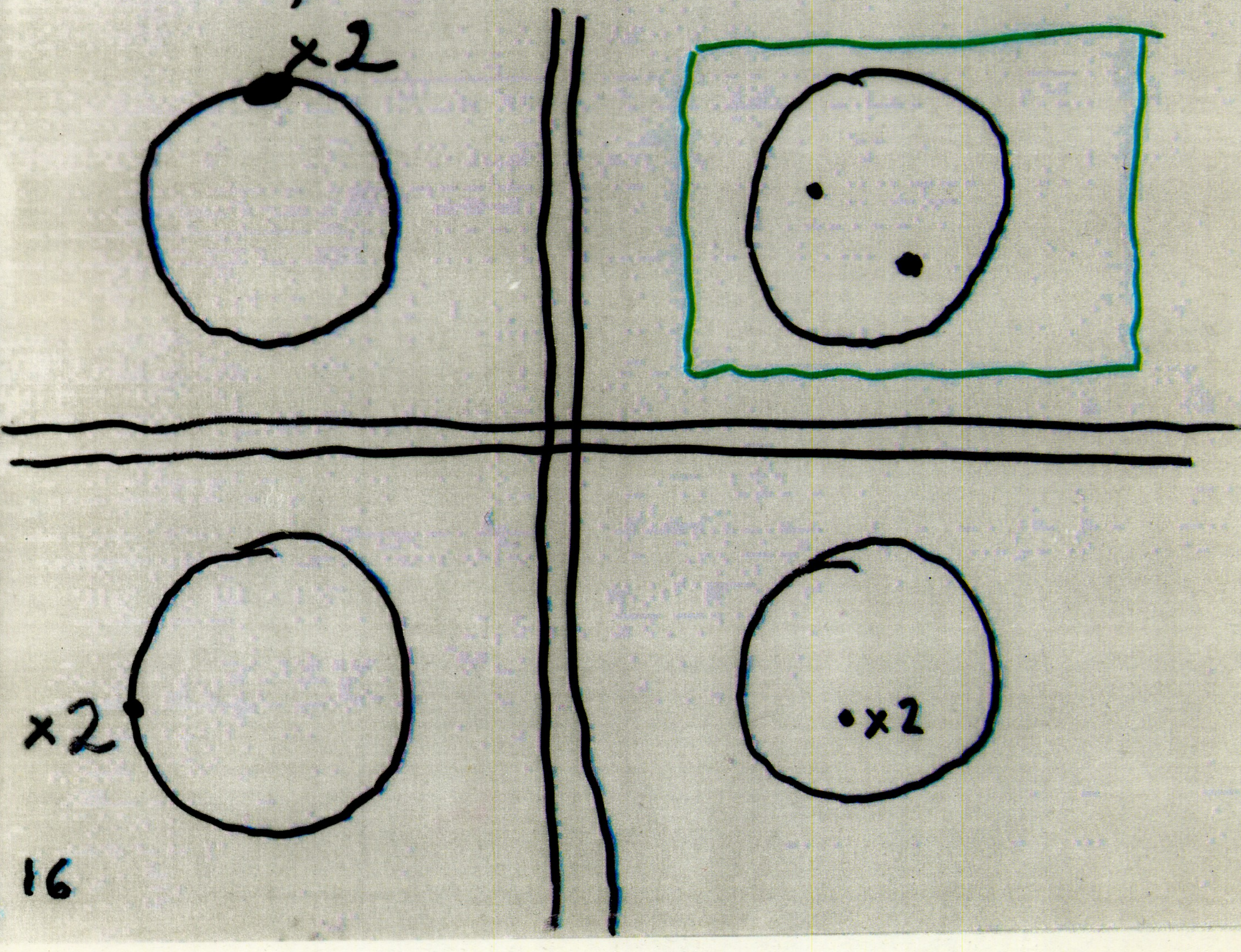
Orthogonal coherent states:

$$|\hat{n}\rangle \propto \begin{pmatrix} 1 \\ \sqrt{2}\,\zeta \\ \zeta^2 \end{pmatrix} \qquad \zeta = e^{i\phi}\tan\frac{\theta}{2}$$

$$\langle \hat{n} | \begin{smallmatrix} 0 \\ 0 \\ 1 \end{smallmatrix} \rangle = 0 \quad \leftrightarrow \quad \zeta^2 = 0$$

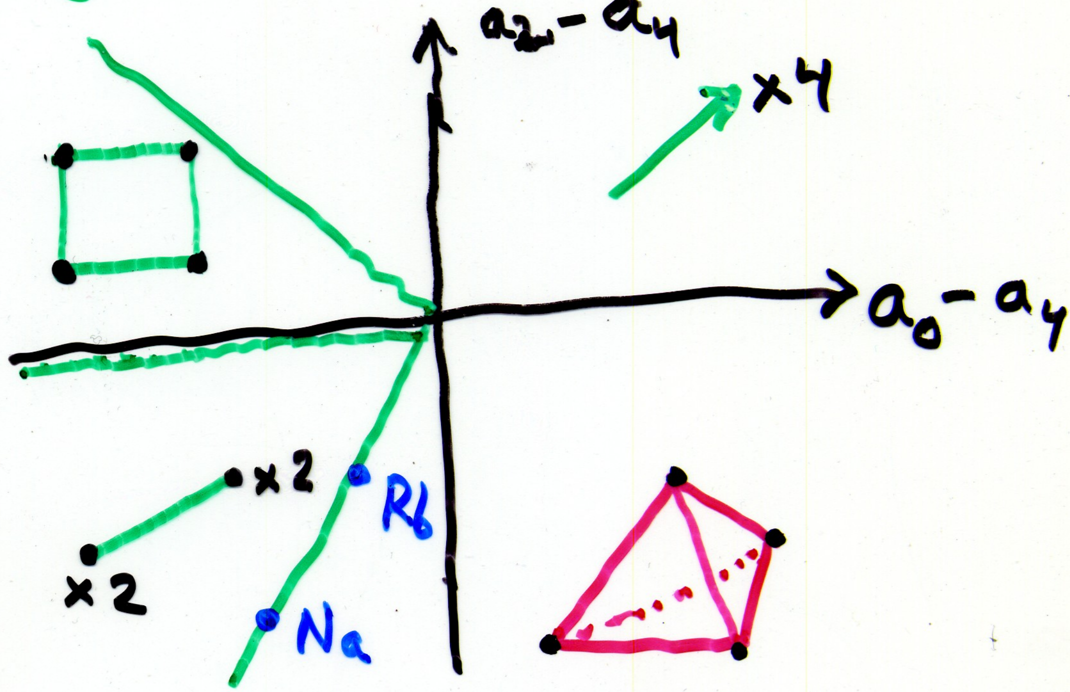
Double root at $\zeta = 0$

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is the only orthogonal coherent state, but counts w/ multiplicity 2.



Spin 2 Phase Diagram

E.g. ^{87}Rb $f=2$



a's : Spin-Dependent Interactions

Refs: Theory Ciobanu, Yip, Ho: PRA 61 033607
Song Semenoff, Zhou / Turner Barnett Demler Vishwanath
PRL, Apr. 2007

Expts. Schmaljohann et al PRL Jan. 2004
Widera et al. New J. Phys. 8

Spinor Vortices

$$\Psi(\phi) = e^{-i\alpha \hat{n}\cdot\vec{F}\frac{\varphi}{2\pi}} e^{i\theta\frac{\varphi}{2\pi}} \chi_0$$

Spin Current   Particle Current

Visualize the
Spin current as
a field of tetrahedra.

Don't forget the
phases!

Classification

$$\Gamma = \left(e^{-i\alpha\frac{\hat{n}\cdot\vec{\sigma}}{2}}, \theta \right)$$

$$E = \frac{\hbar^2 n}{4\pi m}\left(2\alpha^2 + \theta^2\right)\ln\frac{R}{a_c}$$

1/3 Vortex: Zhang, Mäkelä, Suominen,
Chin. Phys. Lett. Vol. 22 (cond-mat/0305489)
Semenoff, Zhou Phys. Rev. Lett. Vol. 98

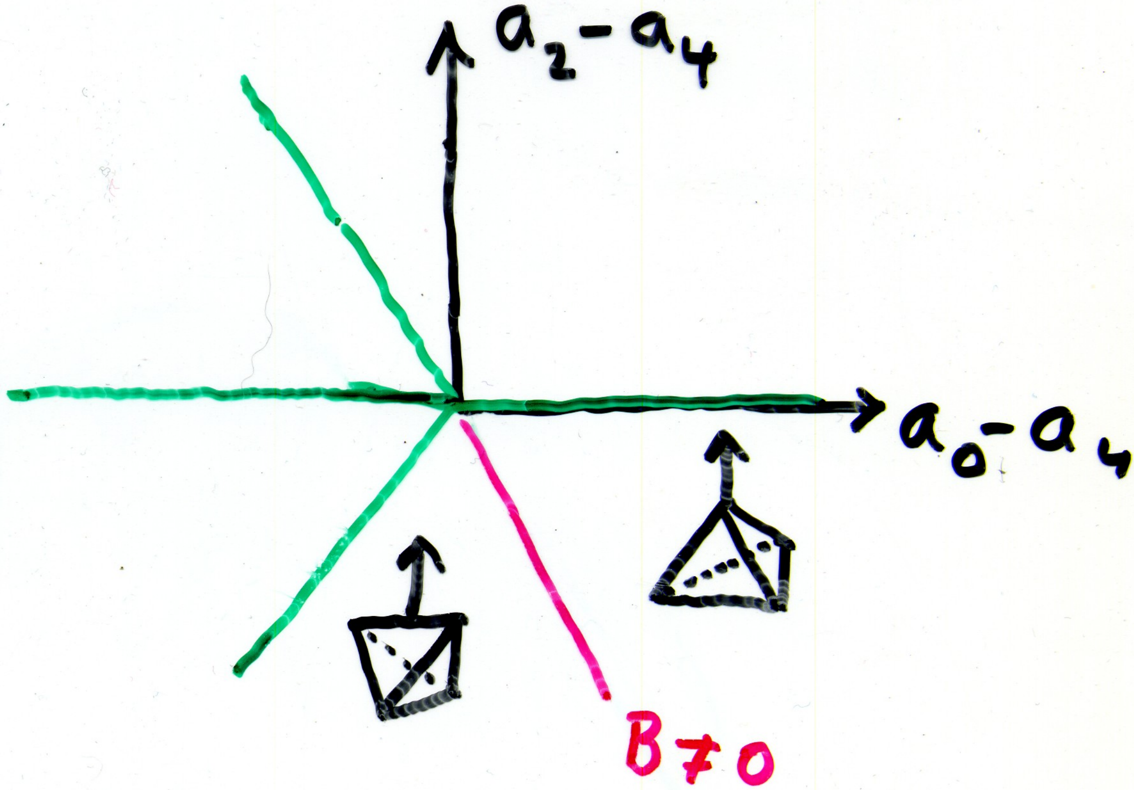
Vortex Molecules

Turner & Demler

Phys. Rev. B Vol. 79

Response to Magnetic Fields

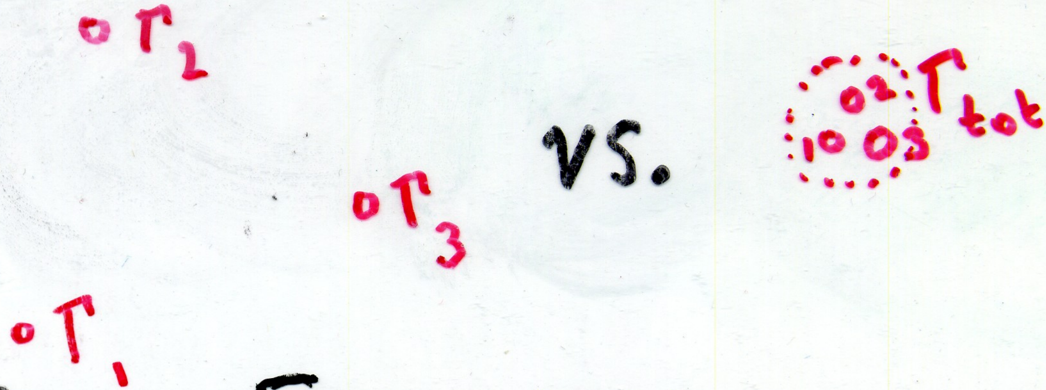
$$\Delta H \propto B^2 \, \psi^+ F_z^2 \, \psi$$



Different Symmetries Preferred

Short-range force:

Repulsive or Attractive?

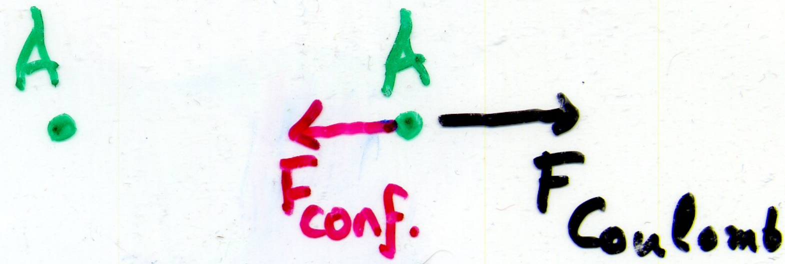


Compare Energies:

$$\sum \alpha_{\Gamma_i}^2 \lessgtr \alpha_{\Gamma_{tot}}^2$$

$$E \approx \text{const.} + \frac{K}{2\pi}\left(\sum \alpha_{\Gamma_i}^2 - \alpha_{\Gamma_{tot}}^2\right) \ln \frac{L}{a}$$

Analyzing AA molecule



Confinement Force

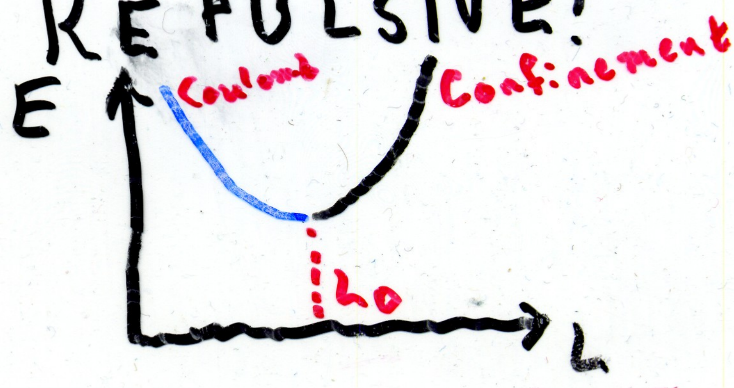
$$E_{conf} \sim \epsilon(B)\, n L^2$$

Coulomb Force

$\alpha_{T_1} = \alpha_{T_2} = \pi$ $\qquad$ $\alpha_{T_{tot}} = 2\pi$

$$E_{Cou} = \frac{K}{2\pi}\left[\pi^2 + \pi^2 - (2\pi)^2\right] \ln\frac{L}{a}$$

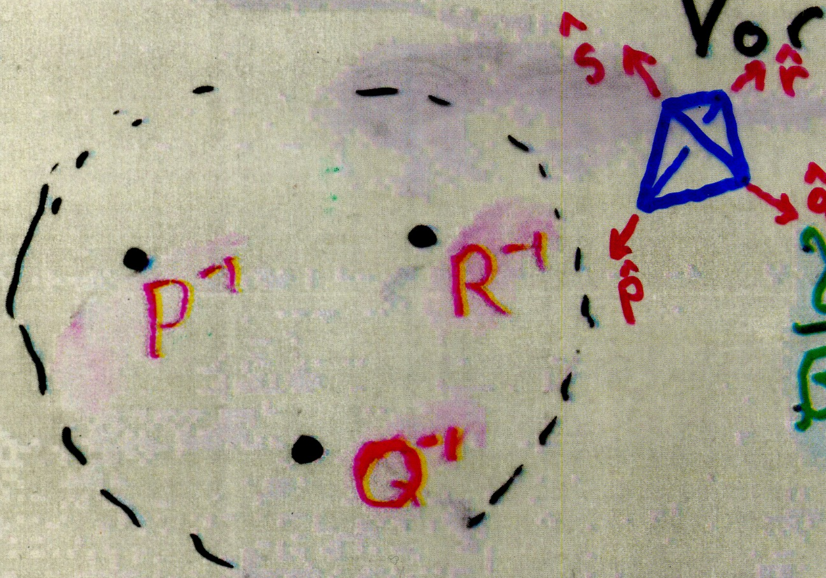
REPULSIVE!



$$L_0 \propto \frac{1}{\sqrt{\epsilon(B)}} \propto \frac{1}{B^2}$$

$\sim 1\,\mu m$ when $B = 1G$

A Doubly-Quantized Stable Vortex ?!?!

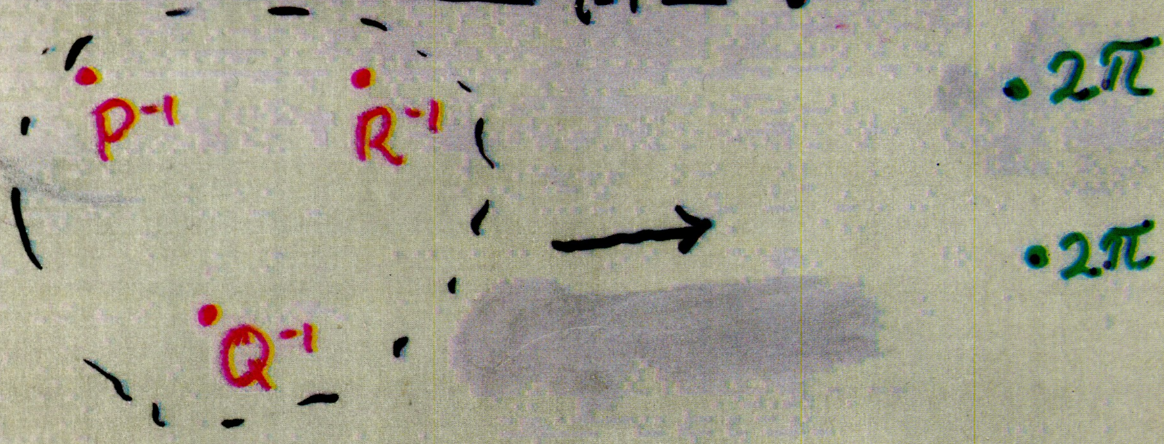


$\frac{\gamma}{\beta} < 4$ [Order 2 Symmetric Ground State]

Algebra

$$\left(e^{\frac{\pi i}{3}\hat{P}\cdot\vec{\sigma}}, \frac{4\pi}{3}\right)\left(e^{\frac{\pi i}{3}\hat{Q}\cdot\vec{\sigma}}, \frac{4\pi}{3}\right)\left(e^{\frac{\pi i}{3}\hat{R}\cdot\vec{\sigma}}, \frac{4\pi}{3}\right) = \left(\begin{pmatrix}1&0\\0&1\end{pmatrix}, 4\pi\right)$$

Can this happen?



Beyond Rough Estimates:

Need Detailed Calculation to determine whether

$(P^{-1}, \frac{4\pi}{3})$ is stable

[$(P^{-1}, \frac{4\pi}{3}) = (R^{-1}, \frac{2\pi}{3})(Q^{-1}, \frac{2\pi}{3})$

Energy/$\pi K \ln L$

$2\left(\frac{\alpha}{2\pi}\right)^2 + \left(\frac{\theta}{2\pi}\right)^2$

$= \frac{2}{9} + \frac{4}{9}$ $\qquad \frac{2}{9} + \frac{1}{9}$ $\qquad \frac{2}{9} + \frac{1}{9}$

Both Sides the Same at this Level]

Calculation would determine either

$(P^{-1}, \frac{4\pi}{3})$ is stable $\Rightarrow$ Metastable 4π Vortex

$(P^{-1}, \frac{4\pi}{3})$ is unstable $\Rightarrow$ Abrikosov lattice has a 3D texture ("Nonabelian")

Vortex Molecules:

Breaking the Symmetry of the order Parameter

$\Rightarrow$ Confined Vortices

Many examples can be understood

Graphically.

(but for some questions, more detail is needed)