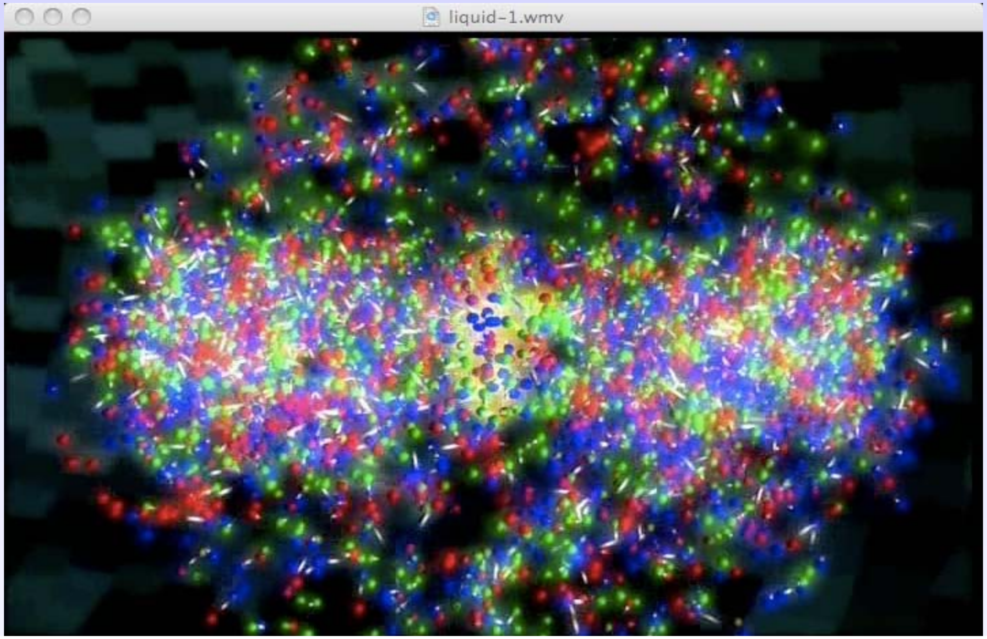
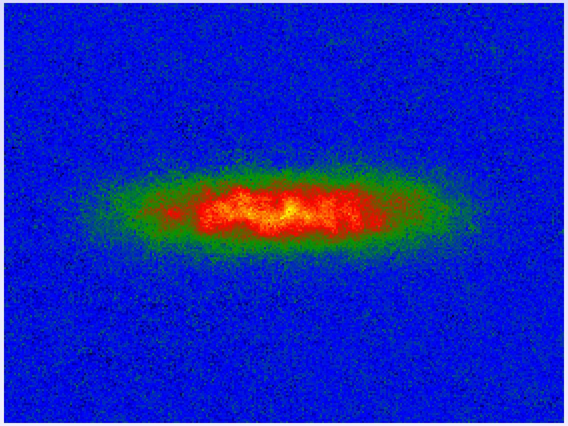
Quantum hydrodynamics in a strongly interacting Fermi gas

Duke Physics
Atom Cooling and Trapping

John E. Thomas



Quark-gluon plasma T = 10^{12} K BIG BANG
Computer simulation of RHIC collision



Ultracold atomic gas
T = 10^{-7} K

JETLab Group

Duke Physics
Atom Cooling and Trapping

Students:

Yingyi Zhang
Chenglin Cao
Ethan Elliot
Willie Ong
Chinyun Cheng
Arun Jaganathan

Post Docs:

Haibin Wu
Ilya Arakelian
James Joseph

J. E. Thomas

Support:

ARO
NSF
DOE
AFOSR

Ken O'Hara*
Mike Gehm*
Stephen Granade*
Staci Hemmer*
Joe Kinast*
Bason Clancy*
Le Luo*
Andrey Turlapov*
Xu Du*
Jessie Petricka*

Outline

- *Introduction: Optically trapped Fermi gases:*

– Universal behavior

- *Thermodynamics of strongly-interacting Fermi gases:*

– Global entropy and energy

– Temperature calibration

- *Quantum viscosity in strongly-interacting Fermi gases:*

– Shear forces and heating in collective modes and expanding gases

– Comparison to the minimum viscosity conjecture

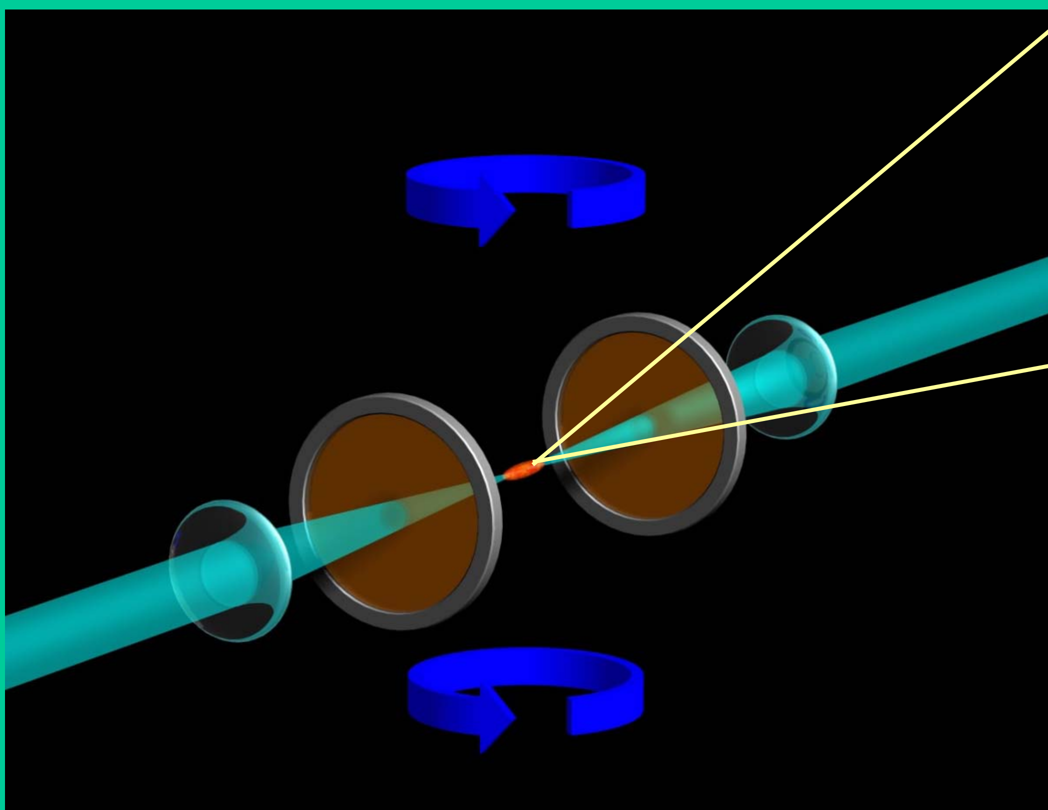
– Vanishing *Bulk* viscosity

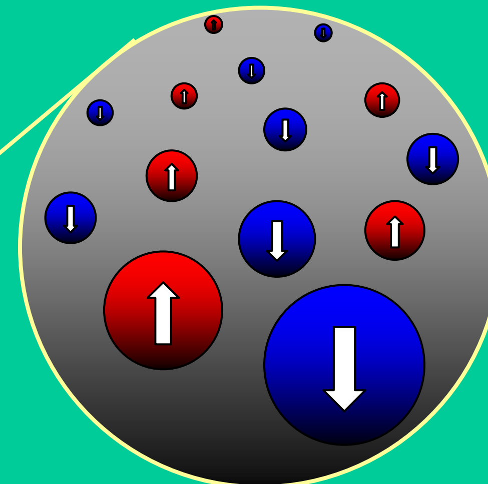
- *Shock Waves in strongly-interacting Fermi gases*

– Nonlinear hydrodynamics in quantum matter

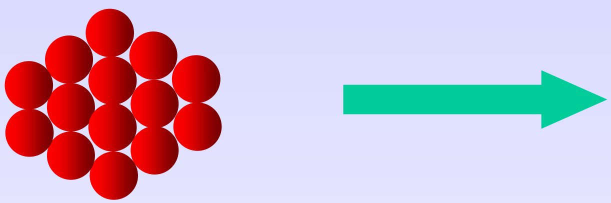
Optically Trapped Fermi Gas

Fermionic $^{6}\mathrm{Li}$

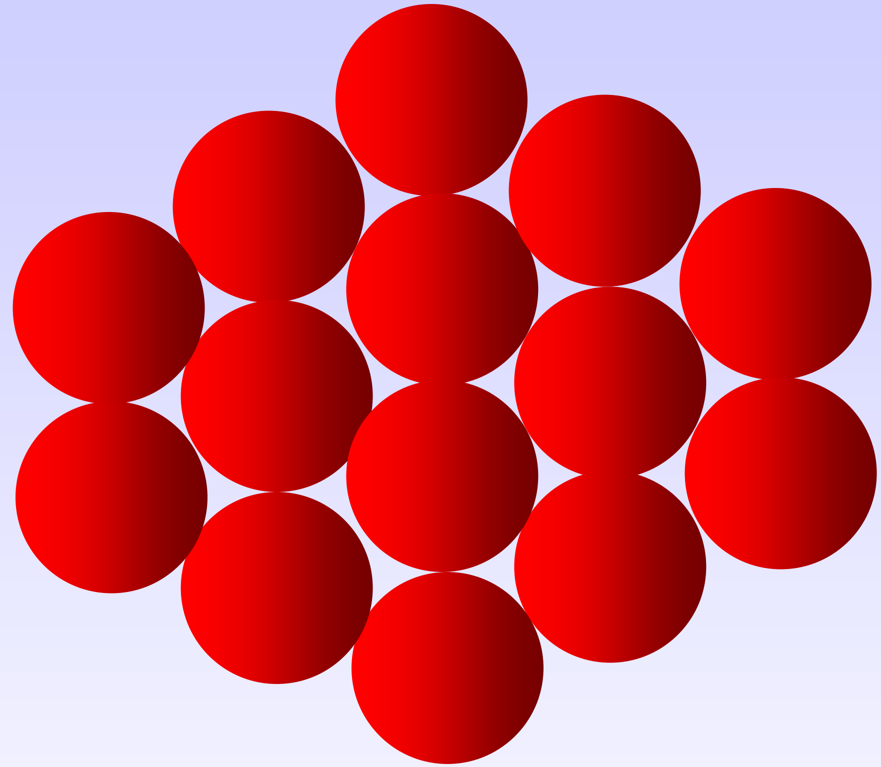




Magic of a *Universal* Strongly Interacting Fermi Gas



Compressed
“Balloons”



Expanded “Balloons”

Density and temperature of the system *set* the length scale of the interactions

The Minimum Viscosity Conjecture—String Theory

Viscosity—Hydrodynamics

$$\boxed{\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}}$$

Kovtun et al., PRL 2005

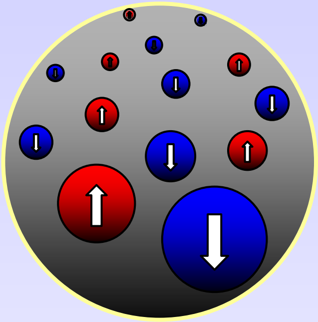
Entropy density—Thermodynamics

Minimum defines a *Perfect* normal fluid

In a ^{6}Li gas we can *measure* η and s.

Thermodynamics of Strongly Interacting Fermi gases

Duke Physics
Atom Cooling and Trapping



- Ground State Energy
- Finite temperature: Energy and Entropy
- *Temperature calibration*

"Universal" – independent of the microscopic interactions

Energy E measurement

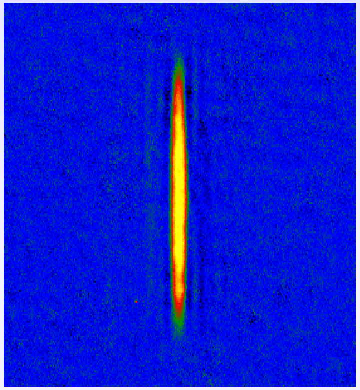
Duke Physics
Atom Cooling and Trapping

Universal **Gas obeys the Virial Theorem**

Thomas (2005)
Castin (2004)
Werner and Castin (2006)
Son (2007)

In a HO potential: $E = 2\langle U \rangle$

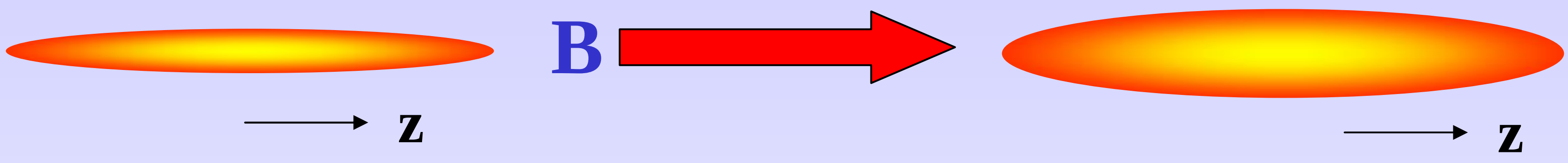
Energy per particle $\boxed{\mathrm{E} = 3m\omega_z^2 \langle z^2 \rangle}$



For a *universal* quantum gas,
the energy E is determined
by the *cloud size*

Measuring the Energy E versus Entropy S by Adiabatic Sweep of Magnetic Field B

Duke Physics
Atom Cooling and Trapping



Start 834 G

End 1200 G

<u>Strongly interacting at 834 G:</u>
Energy **E_S known from cloud size**
— Universal Fermi gas

<u>Weakly interacting at 1200 G:</u>
Entropy **S_W known from cloud size**
— Weakly Interacting Fermi gas

<u>Energy Measurement:</u>

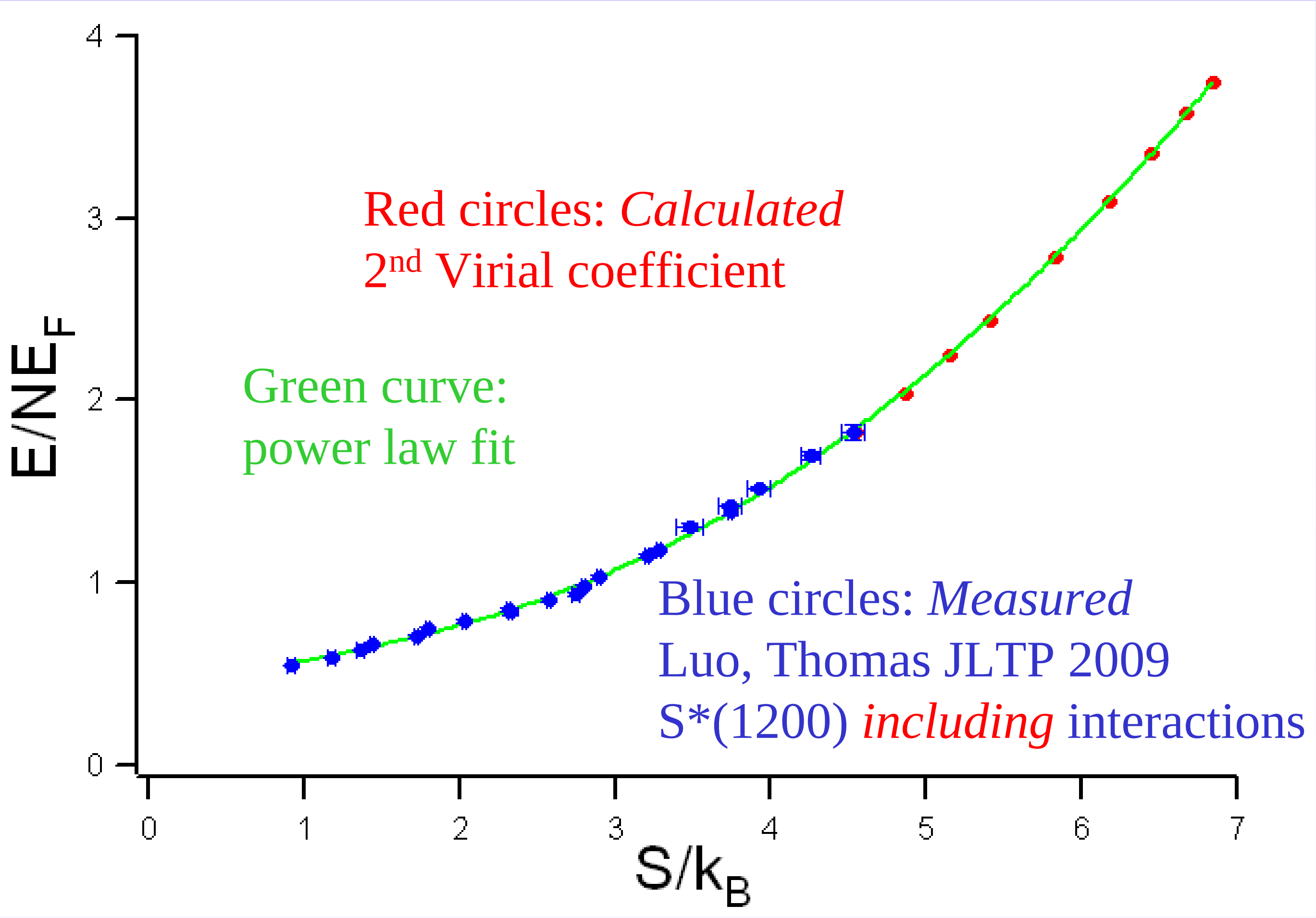
$$E_S = 3m\omega_z^2 \langle z^2 \rangle_{834G}$$

<u>Adiabatic:</u>

$$S_S = S_W$$

Energy per particle versus Entropy per Particle

Duke Physics
Atom Cooling and Trapping



Temperature Calibration

Power law fit to global E versus S data:

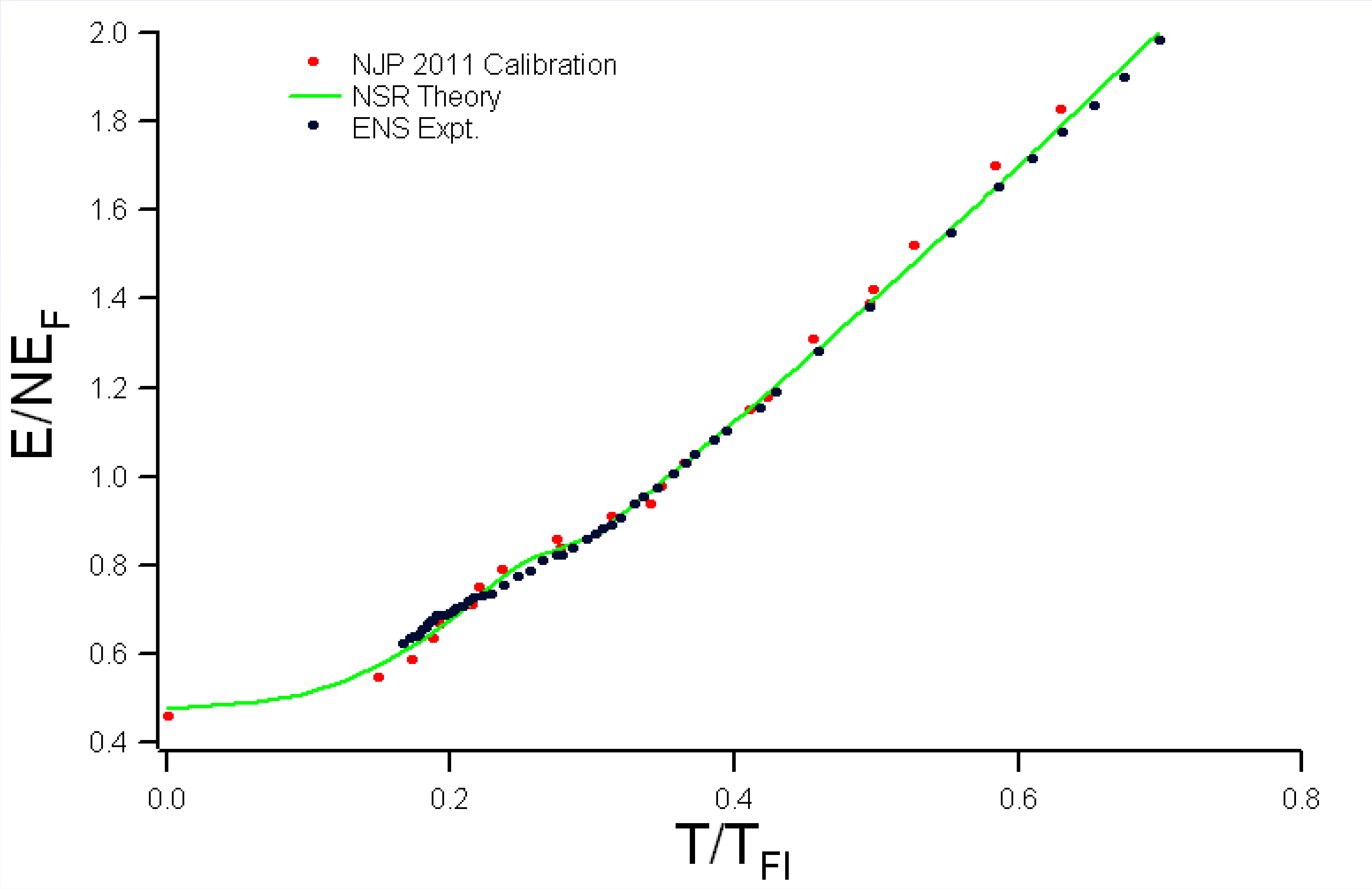
$$E_<(S) = E_0 + aS^b; \quad 0 \leq S \leq S_c$$

$$E_>(S) = E_1 + cS^d; \quad S \geq S_c$$

Temperature from: $T = \dfrac{\partial E}{\partial S}$

Energy versus Temperature

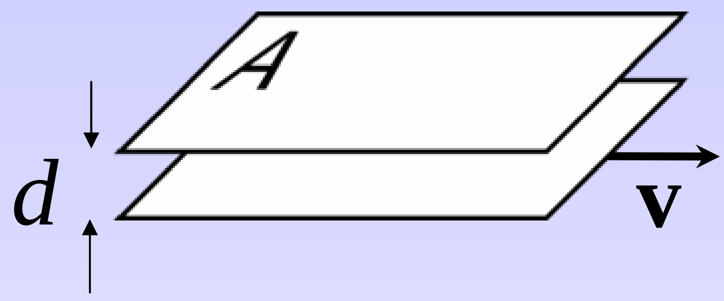
Duke Physics
Atom Cooling and Trapping



"Quantum" Viscosity Hydrodynamics

Quantum Viscosity

Shear forces



$$\mathbf{F} = \eta \frac{A}{d} \mathbf{v}$$

Viscosity scale:

$$\eta = \frac{p}{\sigma}$$

$$p = \hbar k$$

$$\sigma = \frac{4\pi}{k^2}$$

$$\boxed{\eta \propto \hbar k^3}$$

Quantum scale—requires Planck's constant!

Quantum Viscosity at Low and High Temperature

Duke Physics
Atom Cooling and Trapping

$$\eta \propto \hbar k^3$$

<u>Low Temperature</u>

$$T \le T_\mathrm{F}$$

$$k \approx k_\mathrm{F} \approx 1/L$$

$$\boxed{\eta \approx \hbar n}$$

<u>High Temperature</u>

$$T \ge T_\mathrm{F}$$

$$k \approx k_\mathrm{Thermal} \approx \sqrt{2mk_\mathrm{B}T}/\hbar$$

$$\boxed{\eta \propto T^{3/2}/\hbar^2}$$

Entropy density scale: $s \approx n k_\mathrm{B}$

Low temperature: $\boxed{\eta / s \approx \hbar / k_\mathrm{B}}$ ➡ String theory limit

Universal Shear Viscosity

Duke Physics
Atom Cooling and Trapping

$$\boxed{\eta(\mathbf{x},t)=\alpha(\theta)\hbar n(\mathbf{x},t)}$$

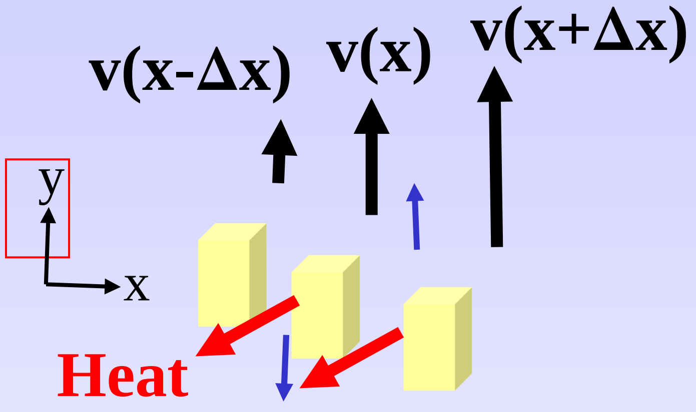
Measuring Universal Shear Viscosity
at Low and at High Temperature:

Breathing Mode and Elliptic Flow

Viscous Hydrodynamics

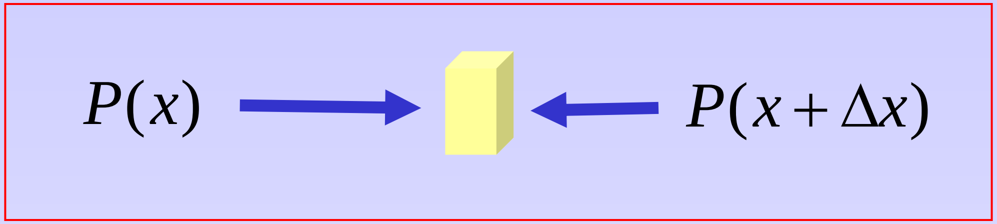
Duke Physics

Atom Cooling and Trapping



- **Shear force at *each* surface** $\eta \dfrac{\partial \mathrm{v}_{\mathrm{y}}}{\partial x}$
- ***Net* shear force on *volume element*** $\dfrac{\partial}{\partial x}\left(\eta \dfrac{\partial \mathrm{v}_{\mathrm{y}}}{\partial x}\right)$
- **Friction *heating* at each surface** $\dot{q} = \dfrac{\eta}{2}\left(\dfrac{\partial \mathrm{v}_{\mathrm{y}}}{\partial x}\right)^2$

Pressure Forces with Heating



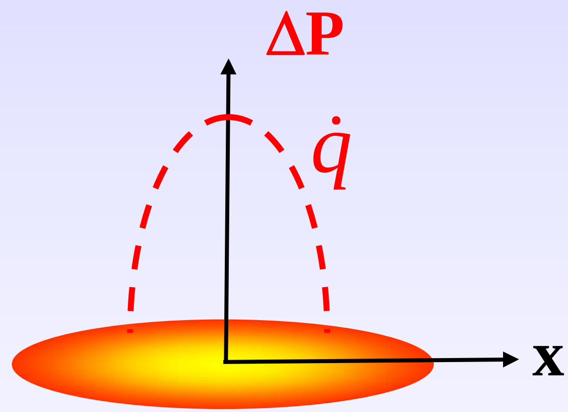
Scalar pressure gradient: *Outward* force—expands after release.

Friction force: Inward—*slows* the flow

Friction *Heating:*

$$\dot{q} = \frac{\eta}{2}\left(\frac{\partial \mathrm{v}_{\mathrm{y}}}{\partial x}\right)^2$$

*The viscosity must **vanish** at the cloud edges*



Heating *gradient*: *Outward* pressure force that *speeds* the flow!

Hydrodynamic *Forces*

Duke Physics
Atom Cooling and Trapping

- **Net Force with Friction:**

$$\boxed{m\left(\partial_t + \mathbf{v}\cdot\nabla\right)\mathrm{v}_\mathrm{i} = f_i + \sum_j \frac{\partial_j\left(\eta\sigma_{ij} + \varsigma\sigma'_{ij}\right)}{n} - \partial_i U_{trap}}$$

Force arising from scalar pressure: $f_i = -\dfrac{\partial_i P}{n}$

Shear viscosity: η $\quad \sigma_{ij} = \partial_j \mathrm{v}_i + \partial_i \mathrm{v}_j - \tfrac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}$

Bulk viscosity: ς $\quad \sigma'_{ij} = \delta_{ij}\nabla\cdot\mathbf{v}$

- **Initial Condition:** $f_i(t=0) = \partial_i U_{Trap}\left(\mathbf{x}\right) = m\omega_i^2 x_i$

Universal Pressure with Heating

Duke Physics
Atom Cooling and Trapping

- **Friction Heating rate per unit volume**

$$\dot{q} = \tfrac{1}{2}\eta \sum_{ij} \sigma_{ij}^2 + \varsigma (\nabla \cdot \mathbf{v})^2$$

- **Energy conservation:** $\left(\partial_t + \mathbf{v}\cdot\nabla + \tfrac{5}{3}\nabla\cdot\mathbf{v}\right)\mathcal{E} = \dot{q}$

- **Universal Pressure:** $P = \tfrac{2}{3}\mathcal{E}$ Ho, PRL 2004

$$\boxed{\left(\partial_t + \mathbf{v}\cdot\nabla + \tfrac{5}{3}\nabla\cdot\mathbf{v}\right)P = \tfrac{2}{3}\dot{q}}$$

Cao, Elliot,Wu, Joseph, Petricka, Schaefer, and Thomas
Science **331**, 58 (2011)

Universal Viscous Hydrodynamics

Equation for $f_i = -\dfrac{\partial_i P}{n}$

0

$$\left(\partial_t + \mathbf{v}\cdot\nabla + \tfrac{2}{3}\nabla\cdot\mathbf{v}\right)f_i + \sum_j (\partial_i \mathrm{v}_j) f_j - \tfrac{5}{3}(\partial_i \nabla\cdot\mathbf{v})\frac{P}{n} = -\tfrac{2}{3}\frac{\partial_i \dot{q}}{n}$$

Scale transformation: $n(x,y,z,t) = \dfrac{n\left(\frac{x}{b_x},\frac{y}{b_y},\frac{z}{b_z}\right)}{b_x(t)b_y(t)b_z(t)}$

$$\mathrm{v}_i = x_i \frac{\dot{b}_i}{b_i} \qquad f_i = a_i(t) m \omega_i^2 x_i$$

$$b_i(0) = 1; \quad \dot{b}_i(0) = 0; \quad a_i(0) = 1$$

Extracting the *Shear* Viscosity

Duke Physics
Atom Cooling and Trapping

$$\boxed{\eta(\mathbf{x},t) = \alpha(\theta)\hbar n(\mathbf{x},t)} \qquad \theta = \frac{T}{T_F(n)}$$

$$\dot{a}_i + 2\frac{\dot{b}_i}{b_i}a_i + \frac{2}{3}\sum_j \frac{\dot{b}_j}{b_j}a_i = \frac{\hbar\overline{\alpha}}{3m\omega_i^2\langle x_i^2\rangle_0 b_i^2(t)}\sum_{ij}\sigma_{ij}^2$$

$$\frac{\ddot{b}_i}{b_i} = (a_i - 1_{trap})\omega_i^2 - \frac{\hbar\overline{\alpha}}{m\langle x_i^2\rangle_0 b_i^2(t)}\sigma_{ii} \qquad \partial_j \mathrm{v}_i = \delta_{ij}\frac{\dot{b}_i}{b_i}$$

$$b_i(0) = 1; \quad \dot{b}_i(0) = 0; \quad a_i(0) = 1$$

Trap-averaged Viscosity coefficient

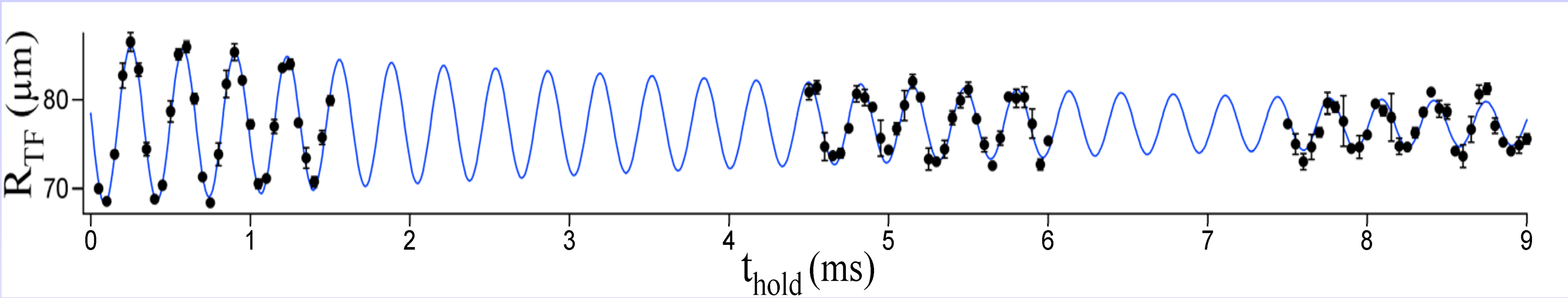
$$\boxed{\overline{\alpha} \equiv \frac{1}{N\hbar}\int d^3\mathbf{x}\,\eta(\mathbf{x},t) = \frac{1}{N}\int d^3\mathbf{x}\,n(\mathbf{x},t)\alpha(\theta)}$$

Duke Physics
Atom Cooling and Trapping

Precision Measurement of Viscosity at *Low* Temperature: Breathing Mode

Damping of the Breathing Mode

Duke Physics
Atom Cooling and Trapping



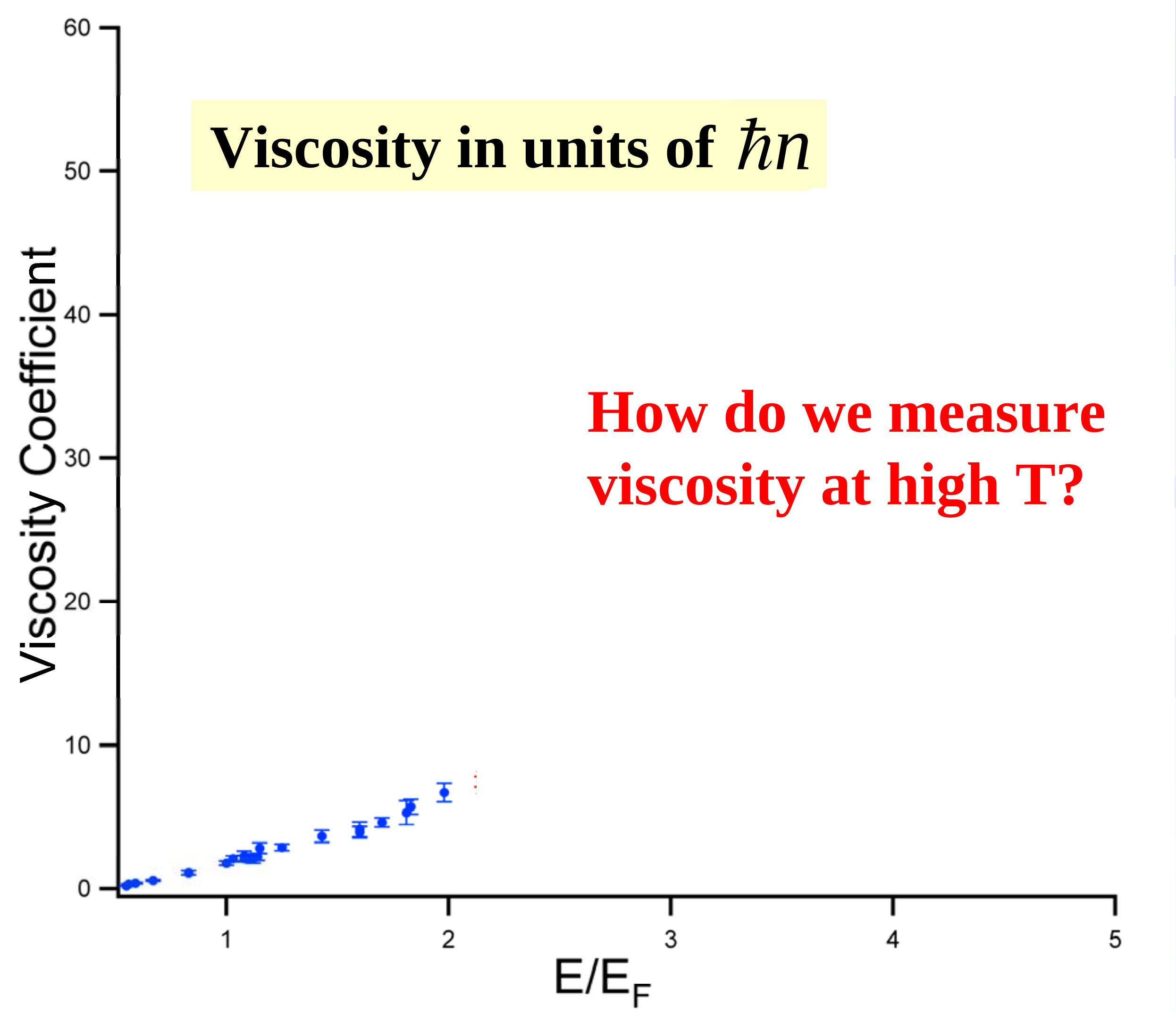
For viscous damping: $\eta = \alpha \hbar n$

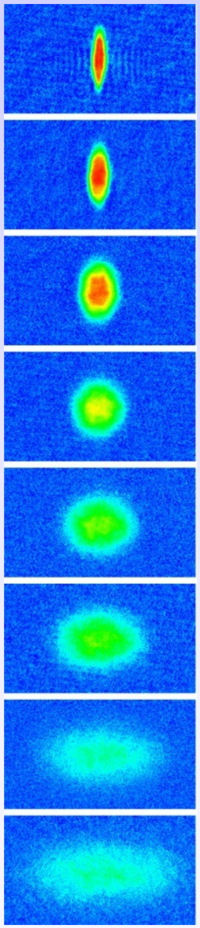
Damping rate: $\dfrac{1}{\tau} = \dfrac{\hbar \overline{\alpha}}{3m\left\langle x^2 \right\rangle_0}$

- Measure trap-averaged *viscosity coefficient* $\overline{\alpha}$

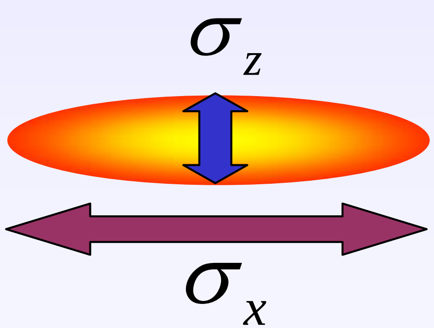
Viscosity Coefficient: Low Temperature

Duke Physics
Atom Cooling and Trapping





High Temperature Quantum Viscosity in Elliptic Flow

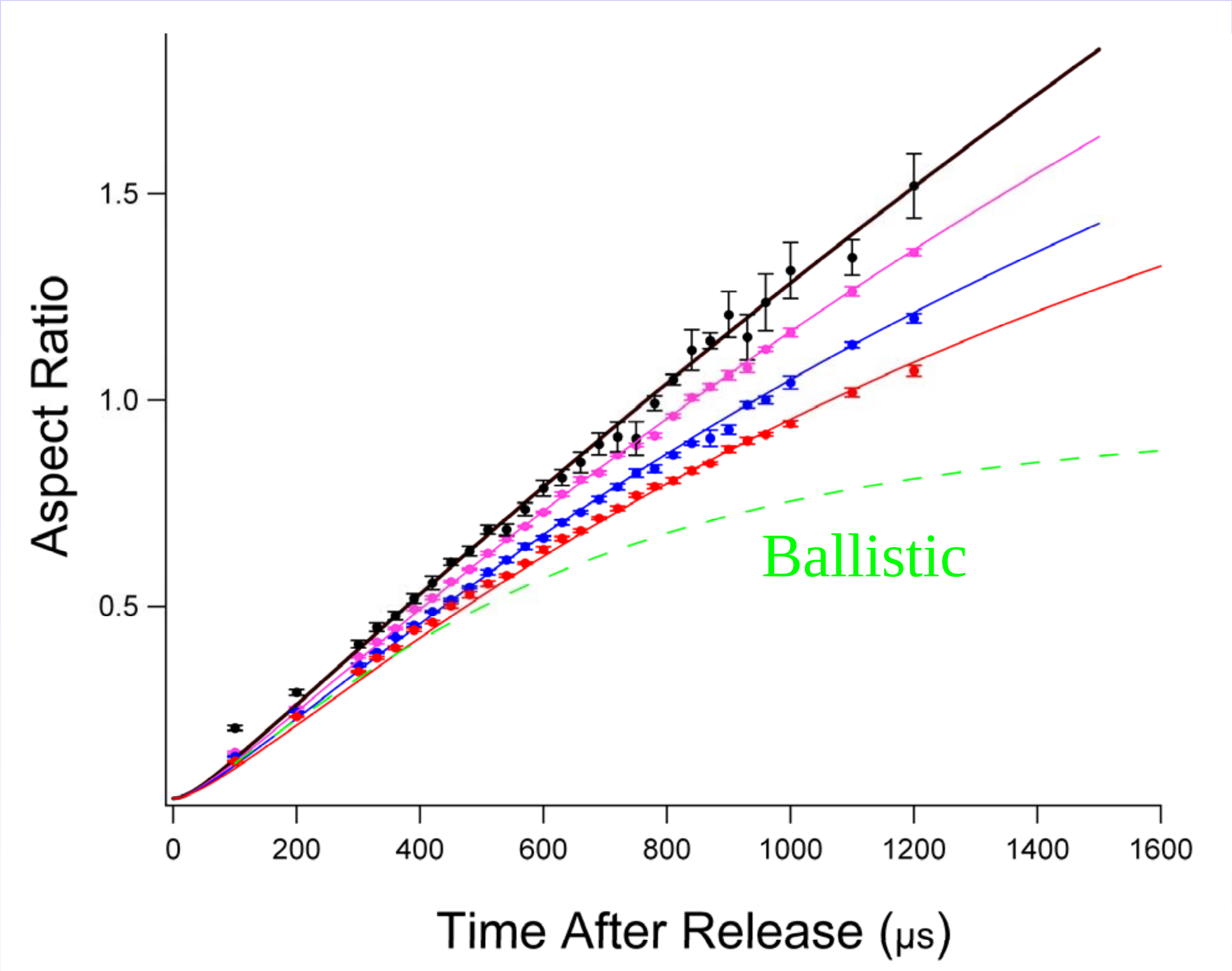


- **Measure *Aspect Ratio*:**

$$\boxed{\frac{\sigma_x}{\sigma_z}}$$

Expansion Dynamics: Elliptic Flow

Duke Physics
Atom Cooling and Trapping



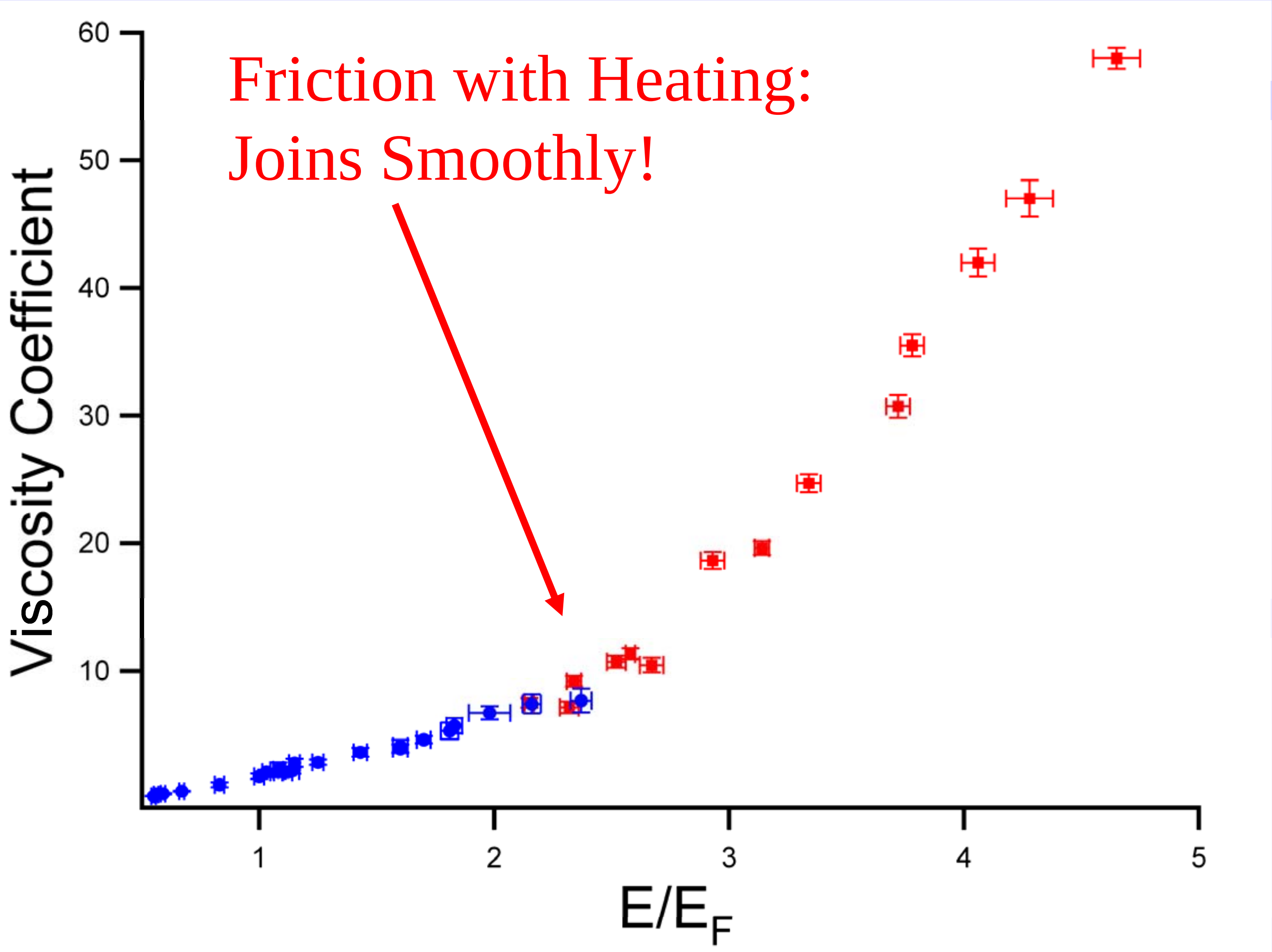
$$\frac{E}{E_F}$$

0.6
2.3
3.3
4.6

High and Low Temperature Data

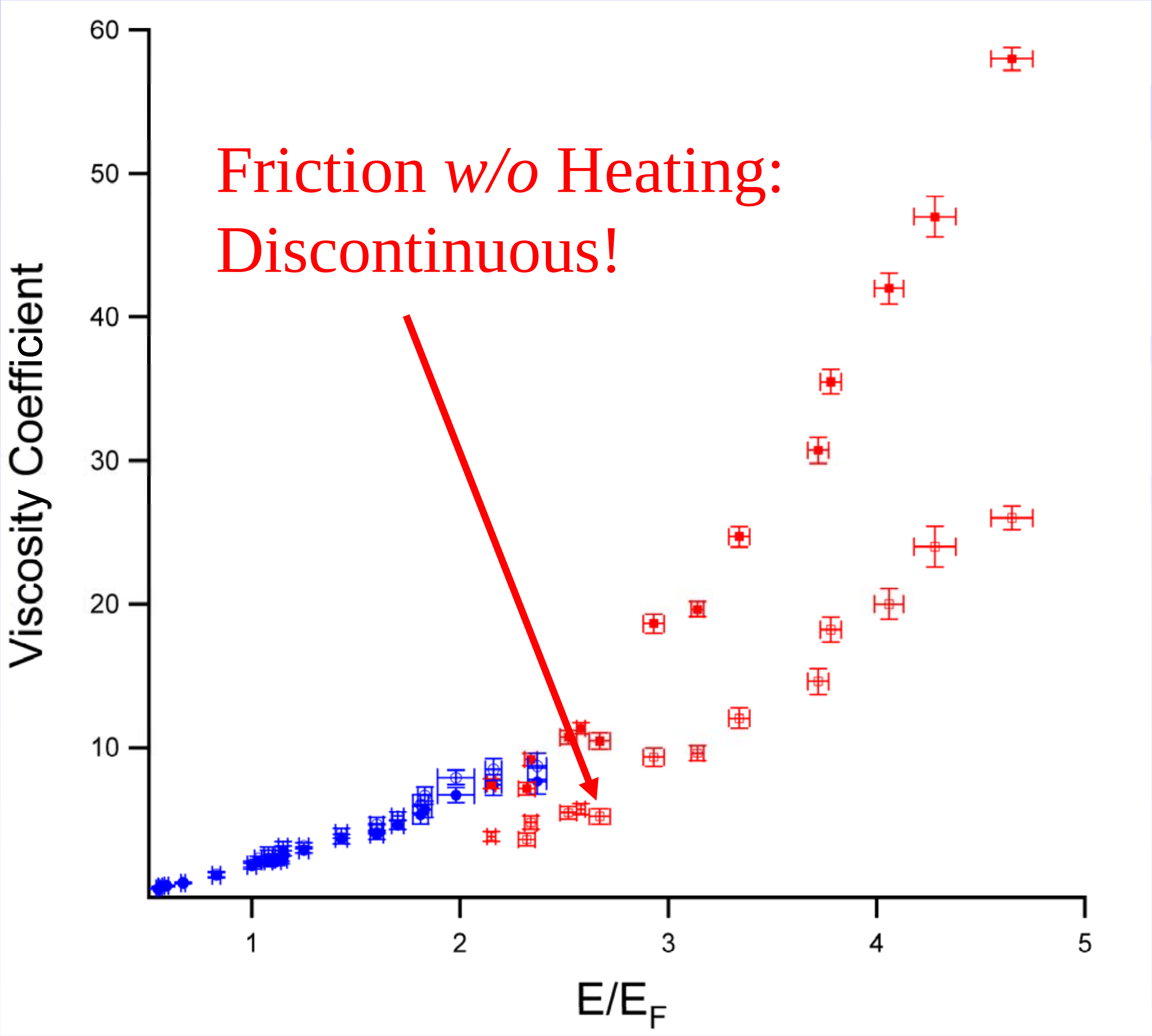
Duke Physics
Atom Cooling and Trapping

Viscosity in units of $\hbar n$



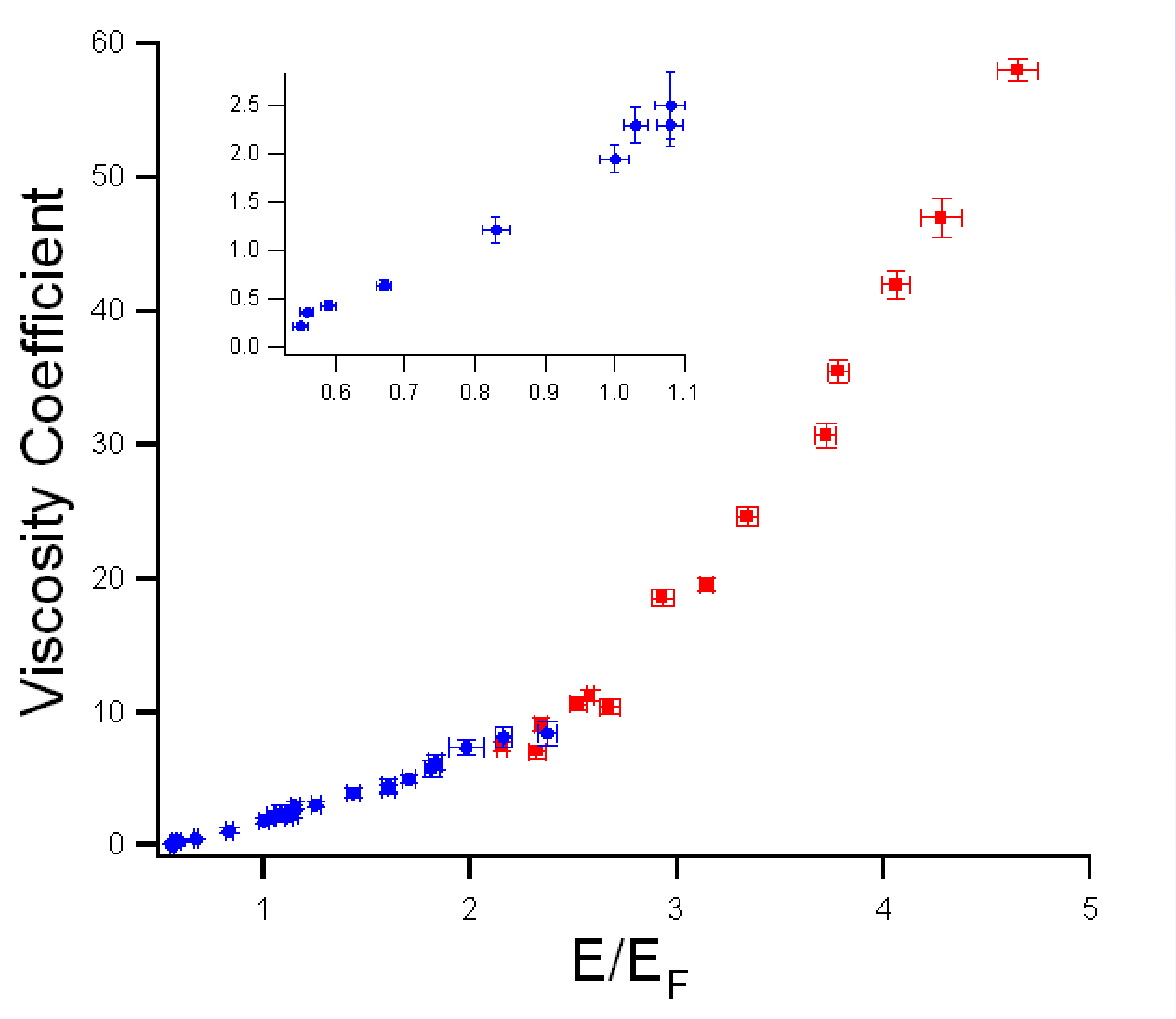
Effect of the *Heating* Rate

Duke Physics
Atom Cooling and Trapping

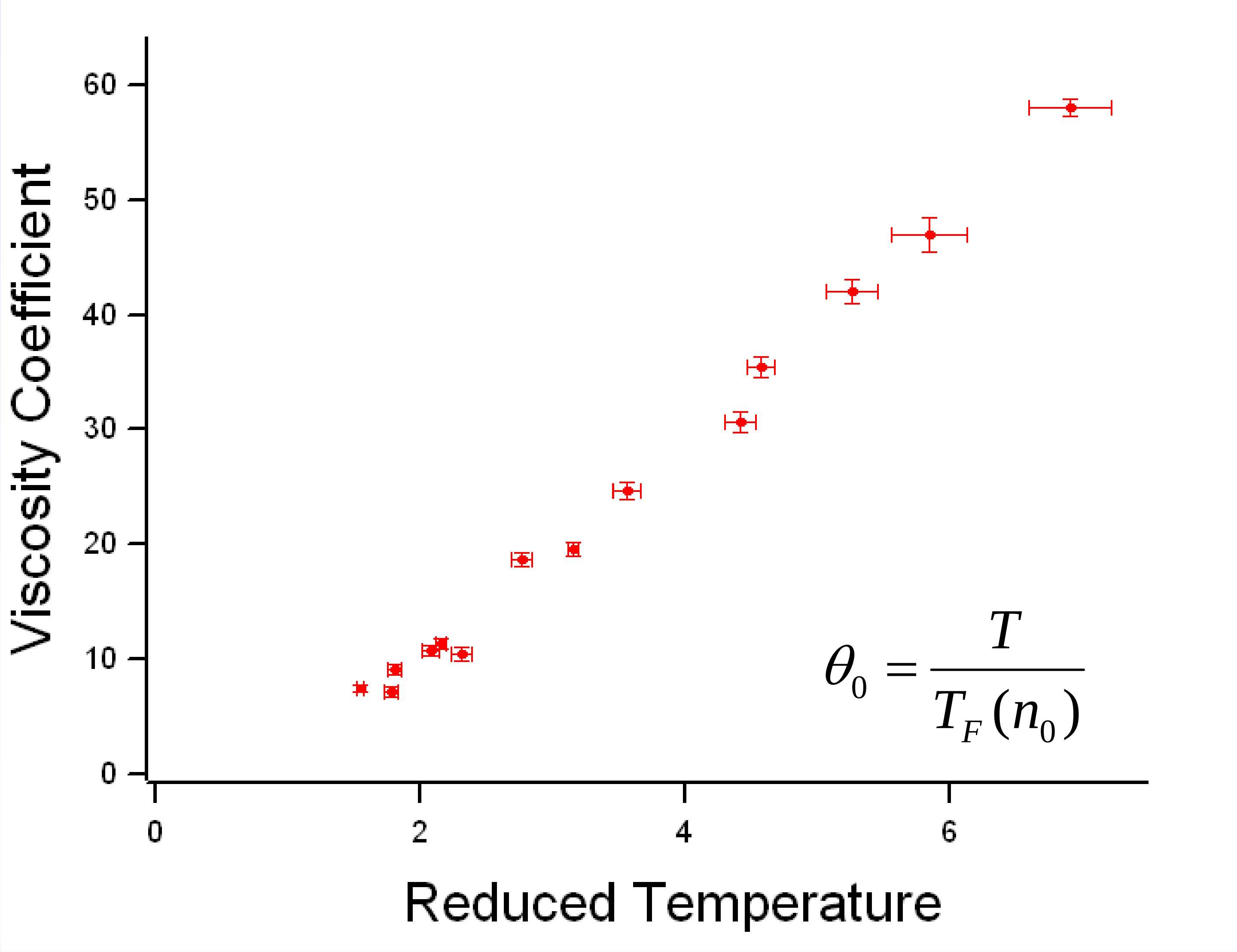


High and Low Temperature Data

Duke Physics
Atom Cooling and Trapping



Universal High Temperature Scaling



Ratio of the Shear Viscosity to the Entropy Density

Duke Physics
Atom Cooling and Trapping

$$\frac{\eta}{s} = \frac{\alpha \hbar n}{s} = \frac{\alpha \hbar}{\frac{\mathrm{s}}{\mathrm{n}}}$$

$$\boxed{\frac{\eta}{s} = \frac{\hbar}{k_B} \frac{\langle \alpha \rangle}{\mathrm{S/k_B}}}$$

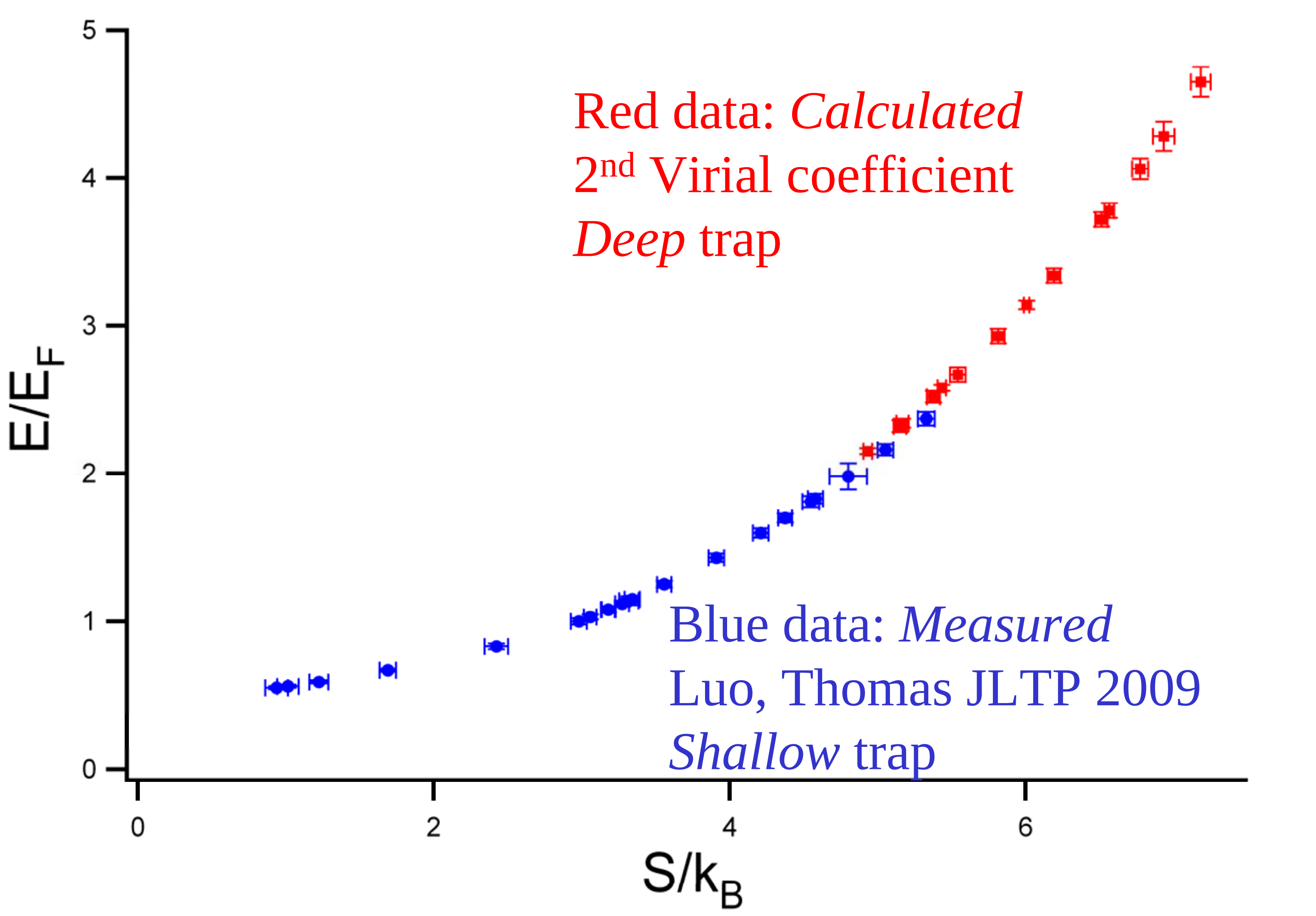
Trap averaged viscosity coefficient

Average entropy per particle

JLTP **150**, 567 (2008)

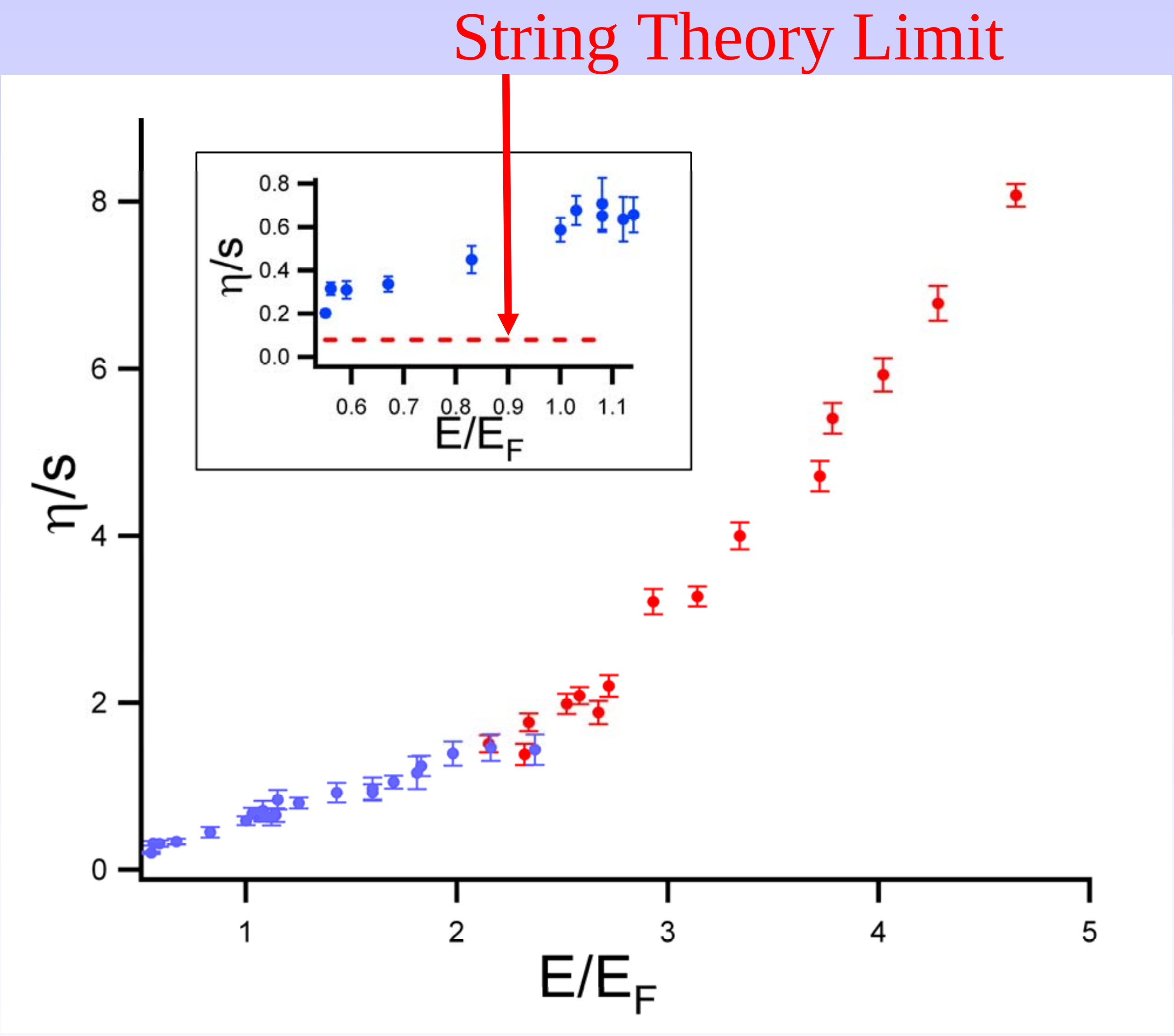
Energy per particle versus Entropy per Particle

Duke Physics
Atom Cooling and Trapping



Ratio of the Viscosity to the Entropy

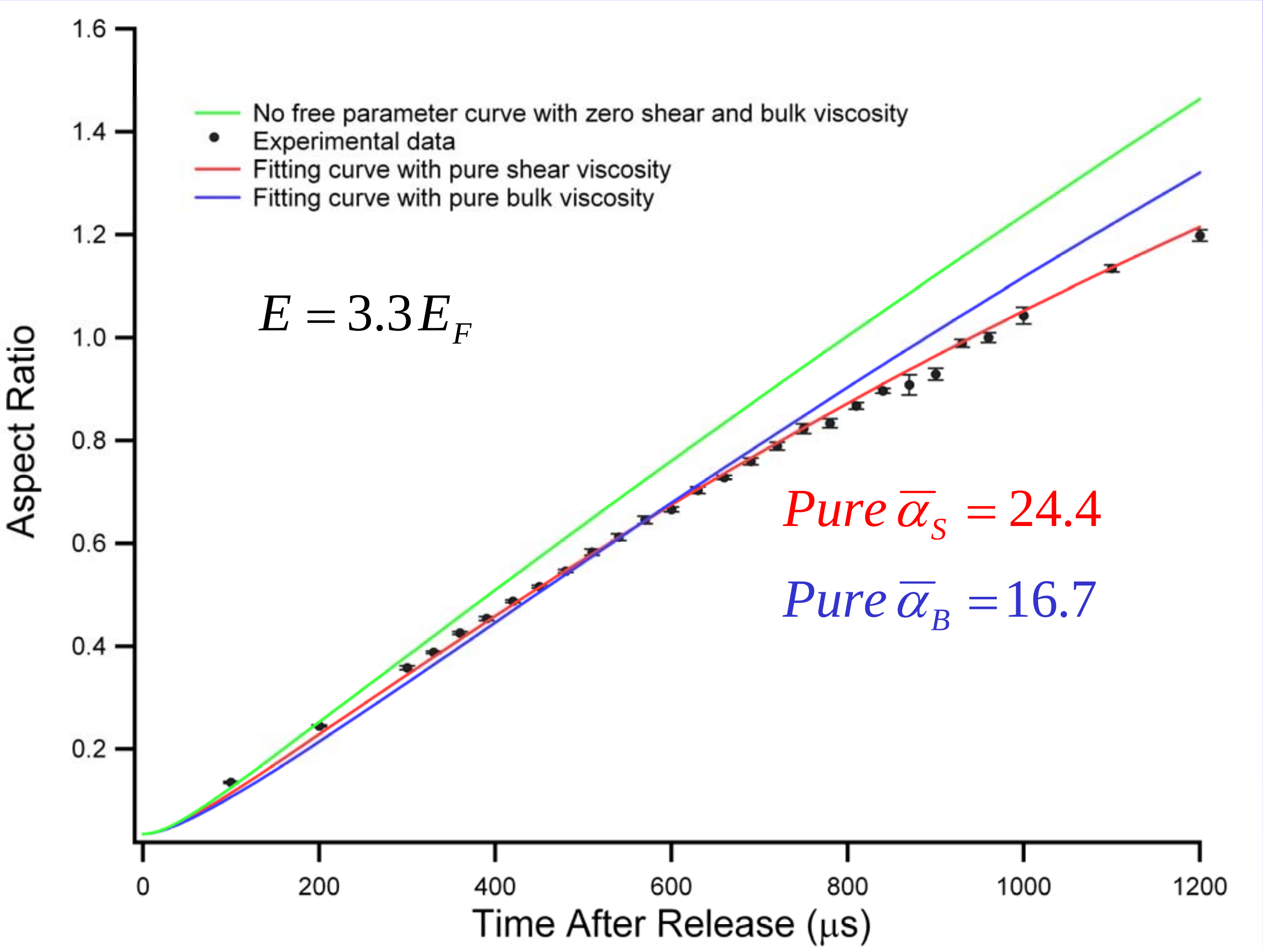
Duke Physics
Atom Cooling and Trapping



What about *Bulk* Viscosity?

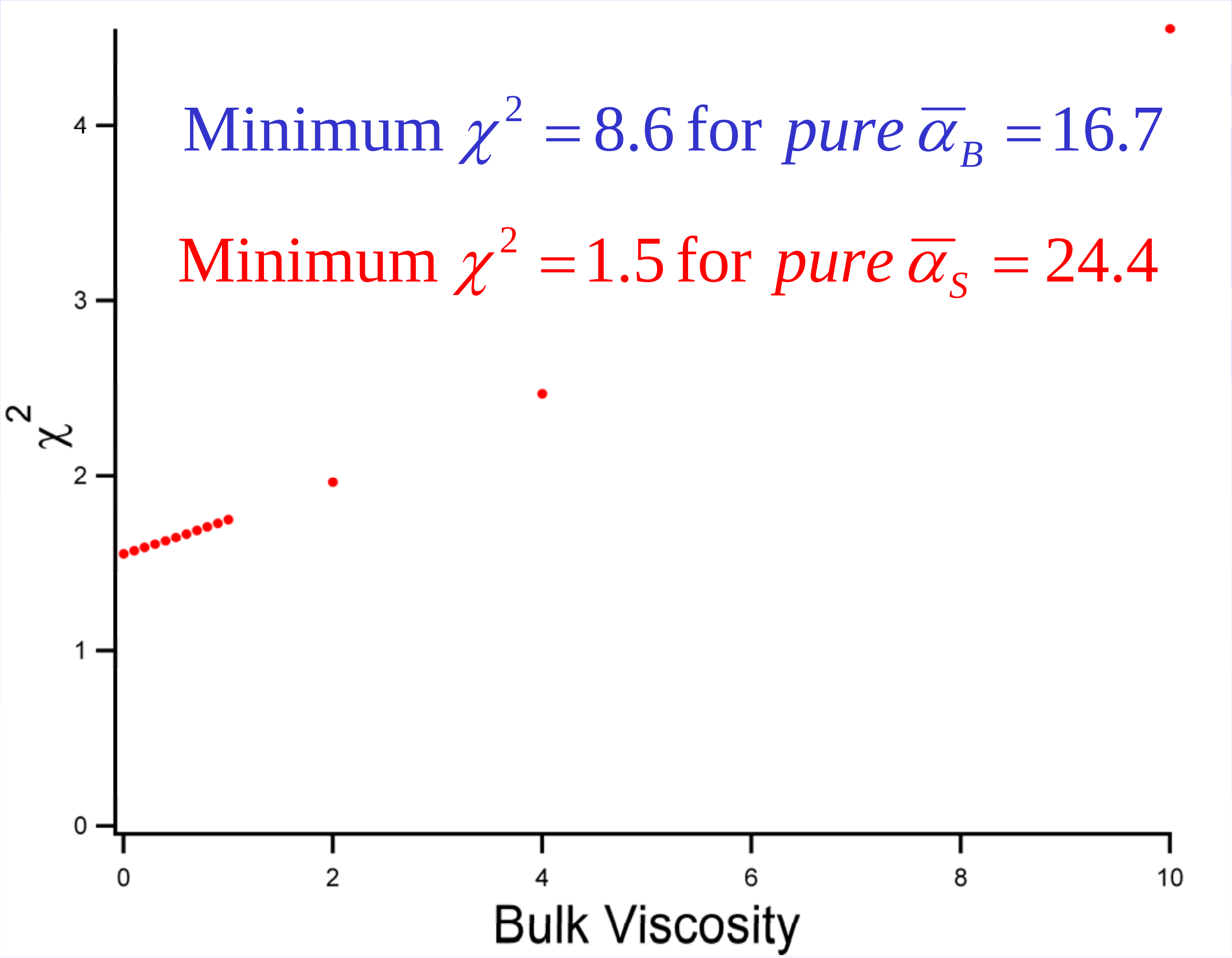
Duke Physics

Atom Cooling and Trapping



Vanishing Bulk Viscosity

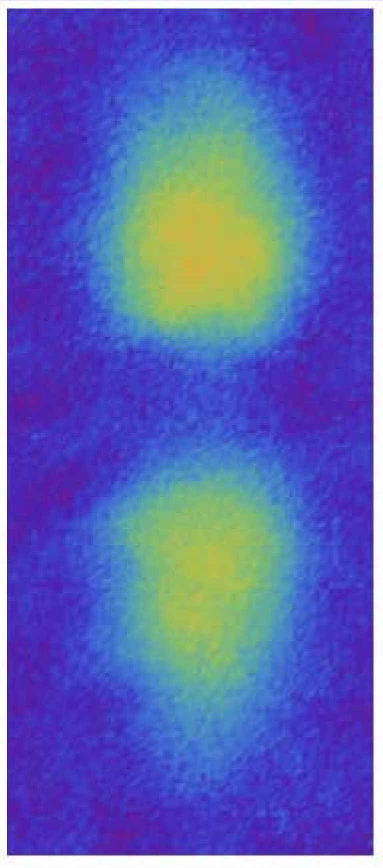
- *Two* parameter fit, *optimum* shear viscosity for each bulk viscosity



Shock waves in Fermi gases

Duke Physics
Atom Cooling and Trapping

Colliding Fermi gas clouds—LHC!

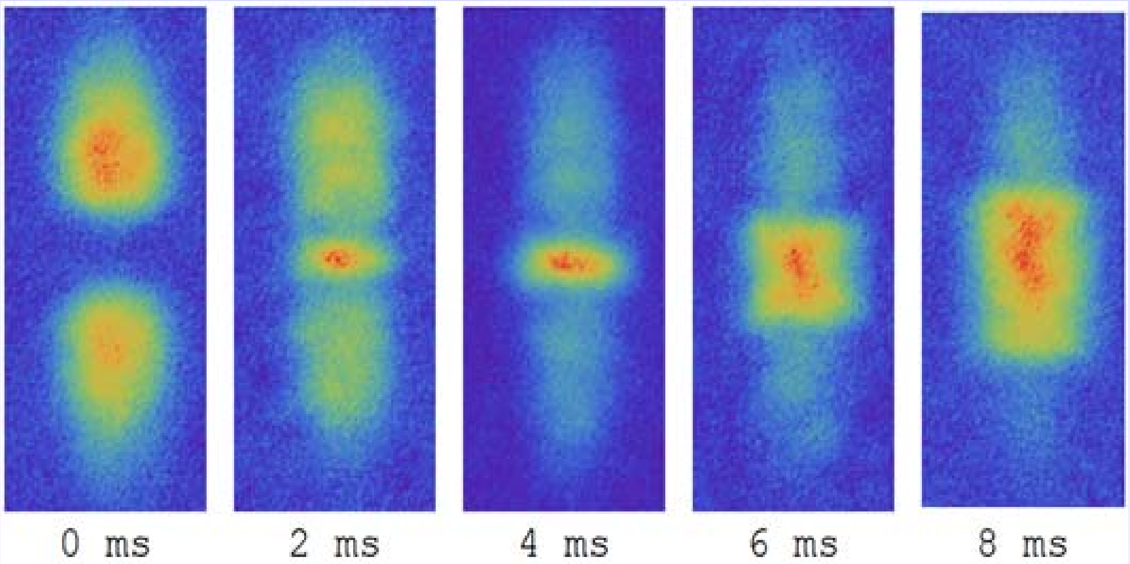


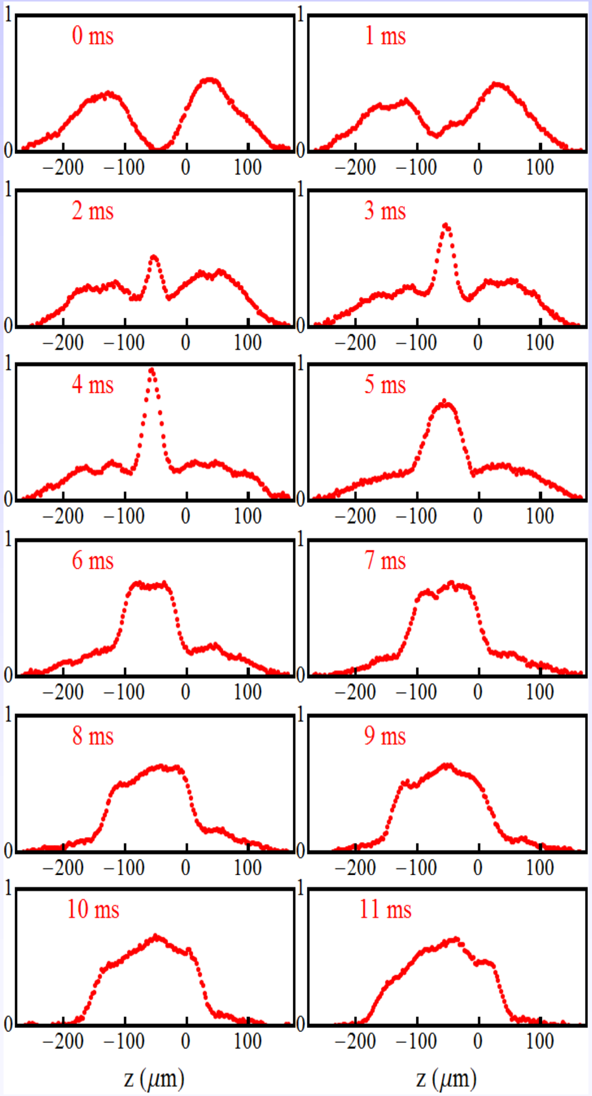
Nonlinear hydrodynamics of strongly interacting *quantum* matter.

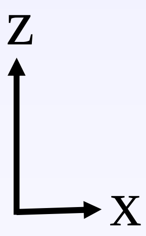
Shock waves in Fermi gases

Duke Physics
Atom Cooling and Trapping

Colliding Fermi gas clouds







Integrate along x

One Dimensional Model

Duke Physics
Atom Cooling and Trapping

Force per atom: $m(\partial_t \mathrm{v}_z + \mathrm{v}_z \partial_z \mathrm{v}_z) = -\partial_z\left(\mu_{1D}(z) + \frac{1}{2} m\omega_z^2 z^2\right)$

$$\mu_{3D} = \mu_G - U_{trap}(\mathbf{x}) \propto n_{3D}^{2/3} \qquad n_{3D} \propto [\mu_G - \tfrac{1}{2} m\overline{\omega}^2 r^2]^{3/2}$$

$$n_{1D} = \iint dx\,dy\, n_{3D}(x,y,z) \propto [\mu_G - \tfrac{1}{2} m\omega_z^2 z^2]^{5/2} \propto \mu_{1D}^{5/2}$$

$$\mu_{1D} = C_1 n_{1D}^{2/5} \qquad C_1 \propto \hbar\omega_\perp l_\perp^{2/5} \qquad l_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$$

$$\boxed{\partial_t \mathrm{v}_z = -\partial_z\left(\tfrac{1}{2}\mathrm{v}_z^2 + C n_{1D}^{2/5} + \tfrac{1}{2}\omega_z^2 z^2\right)}$$

Nonlinear hydrodynamics

Duke Physics
Atom Cooling and Trapping

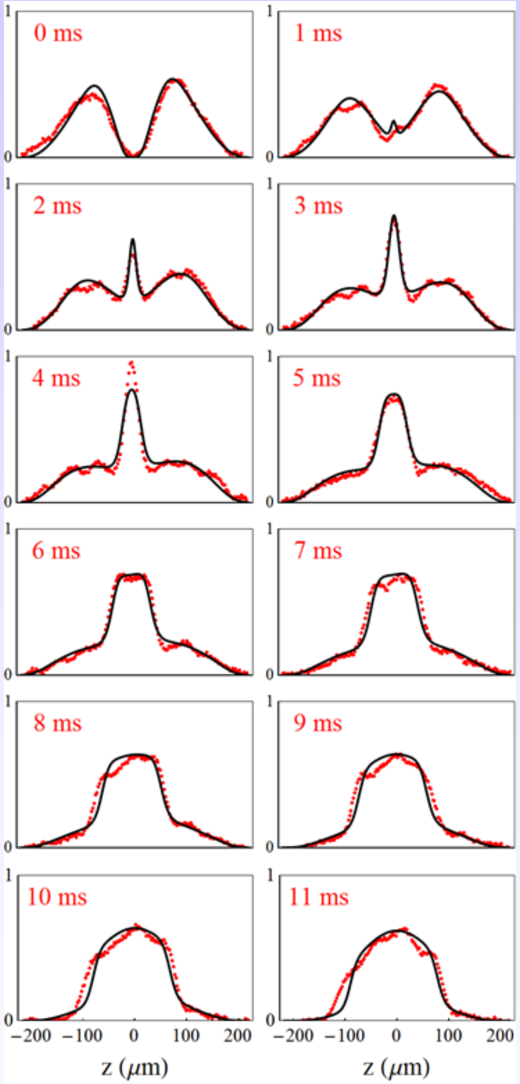
$$\partial_t \mathrm{v} = -\partial_z\left(\tfrac{1}{2}\mathrm{v}^2 + Cn^{2/5} + \tfrac{1}{2}\omega_z^2 z^2\right) + \nu\,\frac{\partial_z(n\partial_z \mathrm{v})}{\mathrm{n}}$$

Kinetic viscosity: $\nu = \dfrac{\overline{\alpha}\hbar n}{nm} = \overline{\alpha}\,\dfrac{\hbar}{m}$ $\qquad \nu = 10\,\dfrac{\hbar}{m}$

Strongly interacting quantum matter:

- Nonlinear dynamics
- Dissipation arising from viscosity
- Dispersion arising from quantum pressure

$$-\frac{1}{\sqrt{n}}\frac{\hbar^2}{2m}\nabla^2\sqrt{n}$$



Summary

Duke Physics
Atom Cooling and Trapping

- Thermodynamics of strongly-interacting Fermi gases:
  - Tests of non-perturbative many-body theory
  - Temperature calibration from E(S)
- Transport: Minimum viscosity hydrodynamics:
  - Shear viscosity versus reduced temperature
  - Minimum η/s 5 times the minimum viscosity conjecture
  - Bulk viscosity vanishes for high temperature expansion
- Future
  - Dependence of shear viscosity on interaction strength
  - Precision measurement of the *bulk* viscosity
  - Nonlinear hydrodynamics and shock waves

MOVING TO NC STATE UNIVERSITY!
SUMMER, 2011

Universal Behavior at T = 0

Duke Physics
Atom Cooling and Trapping

Interparticle spacing L is the *only* length scale: Set by the density n.

Ideal Fermi Gas

$a = 0$

$$E_{\text{ideal}} = \frac{3}{5} E_F(n)$$

Universal Fermi Gas

$a >> L >> R$

$$\boxed{\mathrm{E}_{\text{gnd}} = (1+\beta)\, E_{\text{ideal}}}$$

Bertsch 1998, Baker 1999, Heiselberg 2001

Theory: Carlson (2008) $\beta = -0.60(1)$

Experiment: JLTP (2009) $\beta = -0.62(2)$