Vortex lattices throughout the BCS-BEC crossover

G.C. Strinati
Dipartimento di Fisica, Università di Camerino

Program INT-11-1 on:
"Fermions from Cold Atoms to Neutron Stars: Benchmarking the Many-Body Problem"
Institute for Nuclear Theory, Seattle (U.S.A.),
March 14-May 20, 2011.

Summary of the talk:

Summary of the talk:

[1] Experimental evidence of superfluid behavior in trapped Fermi gases across BCS-BEC crossover $\Longleftarrow$ vortices (MIT, Nature, June 2005)

Summary of the talk:

[1] Experimental evidence of superfluid behavior in trapped Fermi gases across BCS-BEC crossover $\Longleftarrow$ vortices (MIT, Nature, June 2005)

[2] No theoretical explanation of this experiment has been attempted thus far !

Summary of the talk:

[1] Experimental evidence of superfluid behavior in trapped Fermi gases across BCS-BEC crossover $\Longleftarrow$ vortices (MIT, Nature, June 2005)

[2] No theoretical explanation of this experiment has been attempted thus far !

[3] Perform ab-initio calculation of vortices ($T = 0$) throughout BCS-BEC crossover $\Longleftarrow$ need to simplify the Bogoliubov-de Gennes equations

Summary of the talk:

[1] Experimental evidence of superfluid behavior in trapped Fermi gases across BCS-BEC crossover $\Longleftarrow$ vortices (MIT, Nature, June 2005)

[2] No theoretical explanation of this experiment has been attempted thus far !

[3] Perform ab-initio calculation of vortices ($T = 0$) throughout BCS-BEC crossover $\Longleftarrow$ need to simplify the Bogoliubov-de Gennes equations

[4] Develop a LAPA version, more accurate for phase (P) than for amplitude (A) of local gap

Summary of the talk:

[1] Experimental evidence of superfluid behavior in trapped Fermi gases across BCS-BEC crossover $\Longleftarrow$ vortices (MIT, Nature, June 2005)

[2] No theoretical explanation of this experiment has been attempted thus far !

[3] Perform ab-initio calculation of vortices ($T = 0$) throughout BCS-BEC crossover $\Longleftarrow$ need to simplify the Bogoliubov-de Gennes equations

[4] Develop a LAPA version, more accurate for phase (P) than for amplitude (A) of local gap

[5] Rotation activates an "Orbital Breached-Pair Phase" on the BCS side of unitarity.

This work is:

This work is:

- ♣ Done in collaboration with S. Simonucci and P. Pieri (Camerino)

This work is:

- ♣ Done in collaboration with S. Simonucci and P. Pieri (Camerino)
- ♣ Still in progress (unpublished)

This work is:

- Done in collaboration with S. Simonucci and P. Pieri (Camerino)
- Still in progress (unpublished)
- Possibly amenable to extensions and useful for diverse physical problems.

The MIT experimental paper (2005):

Vol 435|23 June 2005|doi:10.1038/nature03858 nature

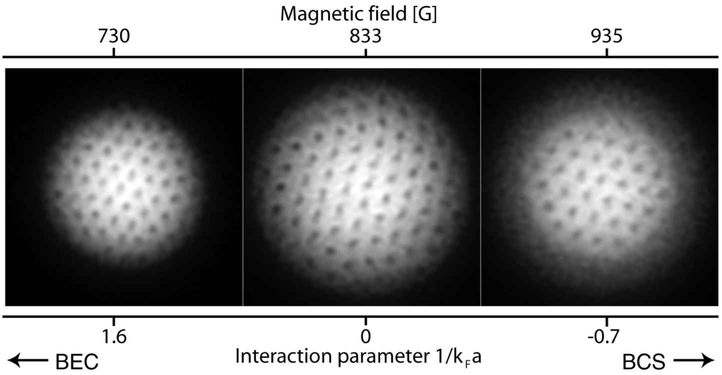
ARTICLES

Vortices and superfluidity in a strongly interacting Fermi gas

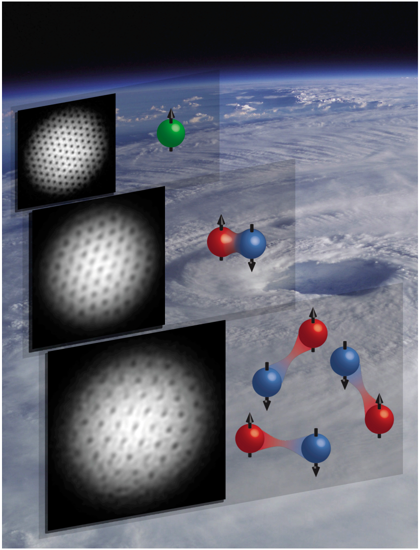
M. W. Zwierlein[1], J. R. Abo-Shaeer[1]†, A. Schirotzek[1], C. H. Schunck[1] & W. Ketterle[1]

Quantum degenerate Fermi gases provide a remarkable opportunity to study strongly interacting fermions. In contrast to other Fermi systems, such as superconductors, neutron stars or the quark-gluon plasma of the early Universe, these gases have low densities and their interactions can be precisely controlled over an enormous range. Previous experiments with Fermi gases have revealed condensation of fermion pairs. Although these and other studies were consistent with predictions assuming superfluidity, proof of superfluid behaviour has been elusive. Here we report observations of vortex lattices in a strongly interacting, rotating Fermi gas that provide definitive evidence for superfluidity. The interaction and therefore the pairing strength between two ^{6}Li fermions near a Feshbach resonance can be controlled by an external magnetic field. This allows us to explore the crossover from a Bose–Einstein condensate of molecules to a Bardeen–Cooper–Schrieffer superfluid of loosely bound pairs. The crossover is associated with a new form of superfluidity that may provide insights into high-transition-temperature superconductors.

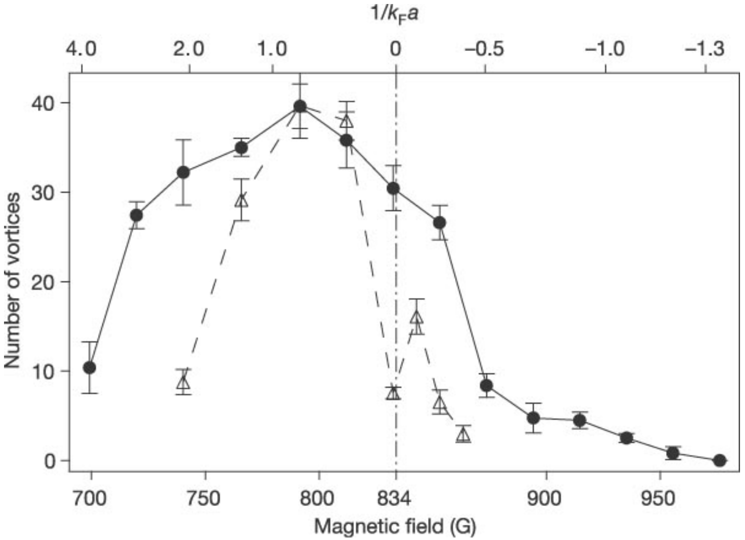
Selection of experimental vortices:



… and its fancy version:

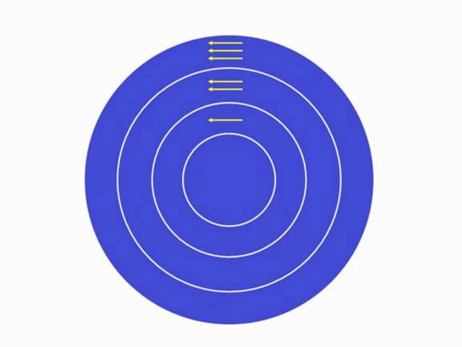


Main experimental results:



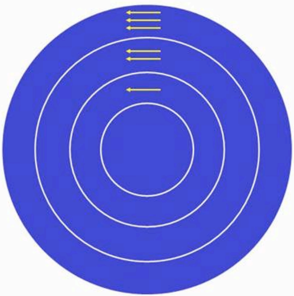
Normal vs superfluid vortices:

Normal vs superfluid vortices:

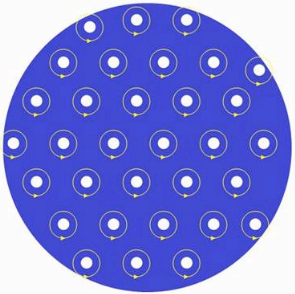


normal vortex

Normal vs superfluid vortices:



normal vortex



superfluid vortices

Theoretical approach: The BdG equations

$$\begin{pmatrix} \mathcal{H}(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -\mathcal{H}(\mathbf{r})^* \end{pmatrix} \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix} = \epsilon_\nu \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix}$$

Theoretical approach: The BdG equations

$$\begin{pmatrix} \mathcal{H}(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -\mathcal{H}(\mathbf{r})^* \end{pmatrix} \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix} = \epsilon_\nu \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix}$$

$\mathcal{H}(\mathbf{r}) = -\nabla^2/(2m) + i\mathrm{A}(\mathbf{r}) \cdot \nabla/m + V(\mathbf{r}) - \mu$

$V(\mathbf{r})$ = trapping potential

$\mathrm{A}(\mathbf{r}) = m\,\mathbf{\Omega} \times \mathbf{r}$ = "vector potential" (rotat. frame)

μ = chemical potential

Theoretical approach: The BdG equations

$$\begin{pmatrix} \mathcal{H}(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -\mathcal{H}(\mathbf{r})^* \end{pmatrix} \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix} = \epsilon_\nu \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix}$$

$\mathcal{H}(\mathbf{r}) = -\nabla^2/(2m) + i\mathrm{A}(\mathbf{r}) \cdot \nabla/m + V(\mathbf{r}) - \mu$

$V(\mathbf{r})$ = trapping potential

$\mathrm{A}(\mathbf{r}) = m\,\mathbf{\Omega} \times \mathbf{r}$ = "vector potential" (rotat. frame)

μ = chemical potential

$\Delta(\mathbf{r})$ = local gap function, determined by

$$\Delta(\mathbf{r}) = g \sum_\nu u_\nu(\mathbf{r}) v_\nu(\mathbf{r})^* \left[1 - 2f_F(\epsilon_\nu)\right]$$

$f_F(\epsilon) = (e^{\epsilon/(k_B T)} + 1)^{-1}$ = Fermi function ($T \to 0^+$)

Their implementation for vortex patterns:

Their implementation for vortex patterns:

- Implementation of BdG equations for a single vortex is OK [Sensarma, Randeria, and Ho, PRL 96, 090403 (2006)] ...

Their implementation for vortex patterns:

- Implementation of BdG equations for a single vortex is OK [Sensarma, Randeria, and Ho, PRL 96, 090403 (2006)] ...
- But for many (≈ 50) vortices it is challenging ! [see Castin *et al.* (2006), but only in BCS limit]

Their implementation for vortex patterns:

- Implementation of BdG equations for a single vortex is OK [Sensarma, Randeria, and Ho, PRL 96, 090403 (2006)] ...
- But for many (≈ 50) vortices it is challenging ! [see Castin *et al.* (2006), but only in BCS limit]
- Typically, with $\approx 10^6$ particles $\Longrightarrow$ diagonalize $\approx 10^7 \times 10^7$ matrices to obtain $\{u_\nu(\mathbf{r}), v_\nu(\mathbf{r})\}$

Their implementation for vortex patterns:

- Implementation of BdG equations for a single vortex is OK [Sensarma, Randeria, and Ho, PRL 96, 090403 (2006)] ...
- But for many (≈ 50) vortices it is challenging ! [see Castin *et al.* (2006), but only in BCS limit]
- Typically, with $\approx 10^6$ particles $\implies$ diagonalize $\approx 10^7 \times 10^7$ matrices to obtain $\{u_\nu(\mathbf{r}), v_\nu(\mathbf{r})\}$
- Computer time $\approx 10^4$ hours / convergence cycle

Their implementation for vortex patterns:

- Implementation of BdG equations for a single vortex is OK [Sensarma, Randeria, and Ho, PRL 96, 090403 (2006)] ...
- But for many (≈ 50) vortices it is challenging ! [see Castin *et al.* (2006), but only in BCS limit]
- Typically, with $\approx 10^6$ particles $\Longrightarrow$ diagonalize $\approx 10^7 \times 10^7$ matrices to obtain $\{u_\nu(\mathbf{r}), v_\nu(\mathbf{r})\}$
- Computer time $\approx 10^4$ hours / convergence cycle

$\Longrightarrow \approx 10^6$ hours are needed for 100 iterations !

Their implementation for vortex patterns:

- Implementation of BdG equations for a single vortex is OK [Sensarma, Randeria, and Ho, PRL 96, 090403 (2006)] ...
- But for many (≈ 50) vortices it is challenging ! [see Castin *et al.* (2006), but only in BCS limit]
- Typically, with $\approx 10^6$ particles $\Longrightarrow$ diagonalize $\approx 10^7 \times 10^7$ matrices to obtain $\{u_\nu(\mathbf{r}), v_\nu(\mathbf{r})\}$
- Computer time $\approx 10^4$ hours / convergence cycle

  $\Longrightarrow \approx 10^6$ hours are needed for 100 iterations !
- Thus for many vortices an alternative procedure is strongly needed !

Generalized Gross-Pitaevskii equation:

Generalized Gross-Pitaevskii equation:

- Express BdG eqs. in terms of Green's functions :

Generalized Gross-Pitaevskii equation:

- Express BdG eqs. in terms of Green's functions :

$$-\mathcal{G}_{21}(\mathbf{r},\mathbf{r}';\omega_n) = \int d\mathbf{r}''\tilde{\mathcal{G}}_0(\mathbf{r}'',\mathbf{r};-\omega_n)\Delta(\mathbf{r}'')^*\mathcal{G}_{11}(\mathbf{r}'',\mathbf{r}';\omega_n)$$

Generalized Gross-Pitaevskii equation:

- Express BdG eqs. in terms of Green's functions :

$$-\mathcal{G}_{21}(\mathbf{r},\mathbf{r}';\omega_n) = \int d\mathbf{r}''\tilde{\mathcal{G}}_0(\mathbf{r}'',\mathbf{r};-\omega_n)\Delta(\mathbf{r}'')^*\mathcal{G}_{11}(\mathbf{r}'',\mathbf{r}';\omega_n)$$

where $\omega_n = (2n+1)\pi/\beta$ (n integer) and

$$[i\omega_n - \mathcal{H}(\mathbf{r})]\,\tilde{\mathcal{G}}_0(\mathbf{r},\mathbf{r}';\omega_n) = \delta(\mathbf{r}-\mathbf{r}')$$

Generalized Gross-Pitaevskii equation:

- Express BdG eqs. in terms of Green's functions :

$$-\mathcal{G}_{21}(\mathbf{r},\mathbf{r}';\omega_n) = \int d\mathbf{r}''\tilde{\mathcal{G}}_0(\mathbf{r}'',\mathbf{r};-\omega_n)\Delta(\mathbf{r}'')^*\mathcal{G}_{11}(\mathbf{r}'',\mathbf{r}';\omega_n)$$

where $\omega_n = (2n+1)\pi/\beta$ (n integer) and

$$[i\omega_n - \mathcal{H}(\mathbf{r})]\,\tilde{\mathcal{G}}_0(\mathbf{r},\mathbf{r}';\omega_n) = \delta(\mathbf{r}-\mathbf{r}')$$

with

$$\frac{\Delta(\mathbf{r})^*}{g} = -\frac{1}{\beta}\sum_n e^{i\omega_n\eta}\,\mathcal{G}_{21}(\mathbf{r},\mathbf{r};\omega_n)$$

Two entangled coarse-graining procedures:

Two entangled coarse-graining procedures:

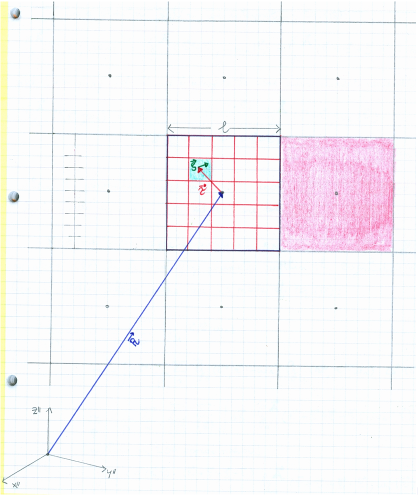
- Write $\Delta(\mathbf{r}'') = \tilde{\Delta}(\mathbf{R})\, e^{2i\,\mathbf{q}(\mathbf{R},\tau)\cdot(\mathbf{R}+\tau+\rho)}$

  with $\mathbf{r}'' = \mathbf{R} + \tau + \rho$

Two entangled coarse-graining procedures:

- Write $\Delta(\mathbf{r}'') = \tilde{\Delta}(\mathbf{R})\, e^{2i\,\mathbf{q}(\mathbf{R},\tau)\cdot(\mathbf{R}+\tau+\rho)}$

  with $\mathbf{r}'' = \mathbf{R} + \tau + \rho$



A "local" Fulde - Ferrel phase:

A "local" Fulde - Ferrel phase:

- Treat the small (green) volume as it were a homogeneous system with a Fulde-Ferrel phase of wave vector $\mathbf{q} = \mathbf{q}(\mathbf{R}, \tau)$:

A "local" Fulde - Ferrel phase:

- Treat the small (green) volume as it were a homogeneous system with a Fulde-Ferrel phase of wave vector $\mathbf{q} = \mathbf{q}(\mathbf{R}, \tau)$:

$$\mathcal{G}_{11}(\mathbf{k}, \omega_n|\mathbf{q}) = \frac{u(\mathbf{k}|\mathbf{q})^2}{i\omega_n - E_+(\mathbf{k}|\mathbf{q})} + \frac{v(\mathbf{k}|\mathbf{q})^2}{i\omega_n + E_-(\mathbf{k}|\mathbf{q})}$$

A "local" Fulde - Ferrel phase:

- Treat the small (green) volume as it were a homogeneous system with a Fulde-Ferrel phase of wave vector $\mathbf{q} = \mathbf{q}(\mathbf{R}, \tau)$:

$$\mathcal{G}_{11}(\mathbf{k}, \omega_n|\mathbf{q}) = \frac{u(\mathbf{k}|\mathbf{q})^2}{i\omega_n - E_+(\mathbf{k}|\mathbf{q})} + \frac{v(\mathbf{k}|\mathbf{q})^2}{i\omega_n + E_-(\mathbf{k}|\mathbf{q})}$$

$$\left.\begin{array}{l} u(\mathbf{k}|\mathbf{q})^2 \\ v(\mathbf{k}|\mathbf{q})^2 \end{array}\right\} = \frac{1}{2}\left(1 \pm \frac{\xi(\mathbf{k}|\mathbf{q})}{E(\mathbf{k}|\mathbf{q})}\right)$$

A "local" Fulde - Ferrel phase:

- Treat the small (green) volume as it were a homogeneous system with a Fulde-Ferrel phase of wave vector $\mathbf{q} = \mathbf{q}(\mathbf{R}, \tau)$:

$$\mathcal{G}_{11}(\mathbf{k}, \omega_n|\mathbf{q}) = \frac{u(\mathbf{k}|\mathbf{q})^2}{i\omega_n - E_+(\mathbf{k}|\mathbf{q})} + \frac{v(\mathbf{k}|\mathbf{q})^2}{i\omega_n + E_-(\mathbf{k}|\mathbf{q})}$$

$$\left.\begin{array}{l} u(\mathbf{k}|\mathbf{q})^2 \\ v(\mathbf{k}|\mathbf{q})^2 \end{array}\right\} = \frac{1}{2}\left(1 \pm \frac{\xi(\mathbf{k}|\mathbf{q})}{E(\mathbf{k}|\mathbf{q})}\right)$$

$$\xi(\mathbf{k}|\mathbf{q}) = \frac{\mathbf{k}^2}{2m} - \mu + \frac{\mathbf{q}^2}{2m} - \frac{\mathbf{A}\cdot\mathbf{q}}{m} \quad , \quad E(\mathbf{k}|\mathbf{q}) = \sqrt{\xi(\mathbf{k}|\mathbf{q})^2 + |\Delta(\mathbf{q})|^2}$$

$$E_\pm(\mathbf{k}|\mathbf{q}) = E(\mathbf{k}|\mathbf{q}) \pm \frac{\mathbf{k}}{m}\cdot(\mathbf{q} - \mathbf{A}) \quad , \quad E_-(\mathbf{k}|\mathbf{q}) = E_+(-\mathbf{k}|\mathbf{q})$$

An Orbital Breached - Pair Phase:

An Orbital Breached - Pair Phase:

- Note that $E_+(\mathbf{k}|\mathbf{q}) < 0$ for $k_1 \leq |\mathbf{k}| \leq k_1$ with

$$\frac{k_{1,2}(\mathbf{q})^2}{2m} = \frac{(\mathbf{q}-\mathbf{A})^2}{2m} + \left(\mu + \frac{\mathbf{A}^2}{2m}\right) \pm \sqrt{4\,\frac{(\mathbf{q}-\mathbf{A})^2}{2m}\left(\mu + \frac{\mathbf{A}^2}{2m}\right) - |\Delta|^2}$$

An Orbital Breached - Pair Phase:

- Note that $E_+(\mathbf{k}|\mathbf{q}) < 0$ for $k_1 \leq |\mathbf{k}| \leq k_1$ with

$$\frac{k_{1,2}(\mathbf{q})^2}{2m} = \frac{(\mathbf{q}-\mathbf{A})^2}{2m} + \left(\mu + \frac{\mathbf{A}^2}{2m}\right) \pm \sqrt{4\,\frac{(\mathbf{q}-\mathbf{A})^2}{2m}\left(\mu + \frac{\mathbf{A}^2}{2m}\right) - |\Delta|^2}$$

- Occupation number:

$$n(\mathbf{k}|\mathbf{q}) = u(\mathbf{k}|\mathbf{q})^2\; f_F(E_+(\mathbf{k}|\mathbf{q})) + v(\mathbf{k}|\mathbf{q})^2\; [1 - f_F(E_-(\mathbf{k}|\mathbf{q}))]$$

$\Longrightarrow \quad n(\mathbf{k}|\mathbf{q}) = 1 \quad \text{if} \quad E_+(\mathbf{k}|\mathbf{q}) < 0 \quad !$

The Urban-Schuck paper (2008):

The Urban-Schuck paper (2008):

A related problem was considered by Urban and Schuck, but with $\mathbf{q} = 0 \iff$ NO vortices :

The Urban-Schuck paper (2008):

A related problem was considered by Urban and Schuck, but with $\mathbf{q} = 0 \iff$ NO vortices :

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **78**, 011601(R) (2008)

Pair breaking in rotating Fermi gases

Michael Urban[1] and Peter Schuck[1,2]

[1]*Institut de Physique Nucléaire, CNRS-IN2P3 and Université Paris-Sud, 91406 Orsay Cedex, France*

[2]*Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS and Université Joseph Fourier, Maison des Magistères, Boîte Postale 166, 38042 Grenoble Cedex, France*

(Received 17 April 2008; published 9 July 2008)

We study the pair-breaking effect of rotation on a cold Fermi gas in the BCS-BEC crossover region. In the framework of BCS theory, which is supposed to be qualitatively correct at zero temperature, we find that in a trap rotating around a symmetry axis, three regions have to be distinguished: (A) a region near the rotational axis where the superfluid stays at rest and where no pairs are broken, (B) a region where the pairs are progressively broken with increasing distance from the rotational axis, resulting in an increasing rotational current, and (C) a normal-fluid region where all pairs are broken and which rotates like a rigid body. Due to region B, density and current do not exhibit any discontinuities.

DOI: 10.1103/PhysRevA.78.011601 PACS number(s): 03.75.Kk, 03.75.Ss, 67.85.De, 67.85.Lm

The Werthamer approach - PR (1963) :

The Werthamer approach - PR (1963) :

- Aimed at extending the Gorkov's derivation of the Ginzburg-Landau (GL) equations far from T_c, but still in the BCS (weak-coupling) limit

The Werthamer approach - PR (1963) :

- Aimed at extending the Gorkov's derivation of the Ginzburg-Landau (GL) equations far from T_c, but still in the BCS (weak-coupling) limit
- Similar in spirit to ours: The phase of $\Delta(\mathbf{r})$ varies more rapidly than its amplitude

The Werthamer approach - PR (1963) :

- Aimed at extending the Gorkov's derivation of the Ginzburg-Landau (GL) equations far from T_c, but still in the BCS (weak-coupling) limit
- Similar in spirit to ours: The phase of $\Delta(\mathbf{r})$ varies more rapidly than its amplitude
- But, does not extend to the BEC side and misses the breached-pair phase on the BCS side (it treats the vector potential as a perturbation)

The Werthamer approach - PR (1963) :

- Aimed at extending the Gorkov's derivation of the Ginzburg-Landau (GL) equations far from T_c, but still in the BCS (weak-coupling) limit
- Similar in spirit to ours: The phase of $\Delta(\mathbf{r})$ varies more rapidly than its amplitude
- But, does not extend to the BEC side and misses the breached-pair phase on the BCS side (it treats the vector potential as a perturbation)
- Yet, we can compare with and recover his (formal) results for the extended GL equations.

· · · after some manipulations · · ·

$\cdots$ after some manipulations $\cdots$

$\cdots$ we end up with a generalized Gross-Pitaevskii equation for the local gap function $\Delta(\mathbf{r})$ throughout the BCS-BEC crossover, under some (moderate) local "phase & amplitude" assumptions:

$\cdots$ after some manipulations $\cdots$

$\cdots$ we end up with a generalized Gross-Pitaevskii equation for the local gap function $\Delta(\mathbf{r})$ throughout the BCS-BEC crossover, under some (moderate) local "phase & amplitude" assumptions:

$$-\frac{m}{4\pi a_F}\Delta(\mathbf{r}) = \mathcal{I}_0[\Delta(\mathbf{r})|\mathbf{r}]\,\Delta(\mathbf{r}) + \mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}]\,\frac{\nabla^2}{2m}\Delta(\mathbf{r}) - \mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}]\,\frac{\mathbf{A}(\mathbf{r})}{m}\cdot\nabla\Delta(\mathbf{r})$$

$\cdots$ after some manipulations $\cdots$

$\cdots$ we end up with a generalized Gross-Pitaevskii equation for the local gap function $\Delta(\mathbf{r})$ throughout the BCS-BEC crossover, under some (moderate) local "phase & amplitude" assumptions:

$$-\frac{m}{4\pi a_F}\Delta(\mathbf{r}) = \mathcal{I}_0[\Delta(\mathbf{r})|\mathbf{r}]\,\Delta(\mathbf{r}) + \mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}]\,\frac{\nabla^2}{2m}\Delta(\mathbf{r}) - \mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}]\,\frac{\mathbf{A}(\mathbf{r})}{m}\cdot\nabla\Delta(\mathbf{r})$$

where $\mathcal{I}_0[\Delta(\mathbf{r})|\mathbf{r}]$ and $\mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}]$ are local functionals of $\Delta(\mathbf{r})$:

$$\mathcal{I}_0[\Delta(\mathbf{r})|\mathbf{r}] = \int_0^\infty \frac{dk}{4\pi^2} \left[\frac{k^2}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}} - 2m \right] - \int_{k_1(\mathbf{q}=0)}^{k_2(\mathbf{q}=0)} \frac{dk}{4\pi^2} \left[\frac{k^2}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}} - \frac{m\,k}{|\mathbf{A}(\mathbf{r})|} \right]$$

$$\mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}] = \int_0^\infty \frac{dk}{8\pi^2} \frac{k^2 \left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)}{\left[\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2\right]^{3/2}} - \int_{k_1(\mathbf{q}=0)}^{k_2(\mathbf{q}=0)} \frac{dk}{8\pi^2} \left[\frac{k^2 \left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)}{\left[\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2\right]^{3/2}} - \frac{m^2\,k}{|\mathbf{A}(\mathbf{r})|^3} \right]$$

In the BEC limit ($|\Delta| \ll |\mu|$):

In the BEC limit ($|\Delta| \ll |\mu|$):

$$\mathcal{I}_0[\Delta(\mathbf{r})|\mathbf{r}] \longrightarrow -\frac{m}{4\pi a_F} + \frac{m^2 a_F}{8\pi}\left(\mu_B - 2V(\mathbf{r})\right) - \frac{m^3 a_F^3}{16\pi}|\Delta(\mathbf{r})|^2$$

$$\mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}] \longrightarrow \frac{m^2 a_F}{4\pi}$$

In the BEC limit ($|\Delta| \ll |\mu|$):

$$\mathcal{I}_0[\Delta(\mathbf{r})|\mathbf{r}] \longrightarrow -\frac{m}{4\pi a_F} + \frac{m^2 a_F}{8\pi}\left(\mu_B - 2V(\mathbf{r})\right) - \frac{m^3 a_F^3}{16\pi}|\Delta(\mathbf{r})|^2$$

$$\mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}] \longrightarrow \frac{m^2 a_F}{4\pi}$$

$\Longrightarrow$ one recovers the Gross-Pitaevskii equation for composite bosons of mass m_B and scattering length $a_B = 2a_F$ in the rotating frame :

In the BEC limit ($|\Delta| \ll |\mu|$):

$$\mathcal{I}_0[\Delta(\mathbf{r})|\mathbf{r}] \longrightarrow -\frac{m}{4\pi a_F} + \frac{m^2 a_F}{8\pi}\left(\mu_B - 2V(\mathbf{r})\right) - \frac{m^3 a_F^3}{16\pi}|\Delta(\mathbf{r})|^2$$

$$\mathcal{I}_1[\Delta(\mathbf{r})|\mathbf{r}] \longrightarrow \frac{m^2 a_F}{4\pi}$$

$\Longrightarrow$ one recovers the Gross-Pitaevskii equation for composite bosons of mass m_B and scattering length $a_B = 2a_F$ in the rotating frame :

$$-\frac{\nabla^2}{2m_B}\Phi(\mathbf{r}) + i\,\frac{2\mathbf{A}(\mathbf{r})}{m_B}\cdot\nabla\,\Phi(\mathbf{r}) + 2V(\mathbf{r})\,\Phi(\mathbf{r})$$
$$+\frac{4\pi a_B}{m_B}|\Phi(\mathbf{r})|^2\,\Phi(\mathbf{r}) = \mu_B\,\Phi(\mathbf{r})$$

Additional local quantities :

Additional local quantities :

- The density:

$$
\begin{aligned}
n(\mathbf{r}) &= \int_0^\infty \frac{dk}{4\pi^2}\, k^2 \left[1 - \frac{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}}\right] \\
&+ \int_{k_1(\mathbf{q}=0)}^{k_2(\mathbf{q}=0)} \frac{dk}{2\pi^2} k^2 \left(\frac{k^2}{2m} - \mu(\mathbf{r})\right) \left[\frac{1}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}} - \frac{m}{k|\mathbf{A}(\mathbf{r})|}\right]
\end{aligned}
$$

Additional local quantities :

- The density:

$$n(\mathbf{r}) = \int_0^\infty \frac{dk}{4\pi^2}\, k^2 \left[1 - \frac{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}}\right]$$

$$+ \int_{k_1(\mathbf{q}=0)}^{k_2(\mathbf{q}=0)} \frac{dk}{2\pi^2} k^2 \left(\frac{k^2}{2m} - \mu(\mathbf{r})\right) \left[\frac{1}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}} - \frac{m}{k|\mathbf{A}(\mathbf{r})|}\right]$$

- The current (here $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})| e^{i\varphi(\mathbf{r})}$):

$$\mathbf{J}(\mathbf{r}) = \frac{n(\mathbf{r})}{m} \left(\frac{\nabla\varphi(\mathbf{r})}{2} - \mathbf{A}(\mathbf{r})\right) + \frac{2}{m} \int \frac{d\mathbf{k}}{(2\pi)^3}\, \mathbf{k}\; f_F[E_+(\mathbf{k}; \nabla\varphi(\mathbf{r}), \mathbf{A}(\mathbf{r}))]$$

Additional local quantities :

- The density:

$$n(\mathbf{r}) = \int_0^\infty \frac{dk}{4\pi^2} k^2 \left[1 - \frac{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}} \right]$$

$$+ \int_{k_1(\mathbf{q}=0)}^{k_2(\mathbf{q}=0)} \frac{dk}{2\pi^2} k^2 \left(\frac{k^2}{2m} - \mu(\mathbf{r})\right) \left[\frac{1}{\sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2}} - \frac{m}{k|\mathbf{A}(\mathbf{r})|} \right]$$

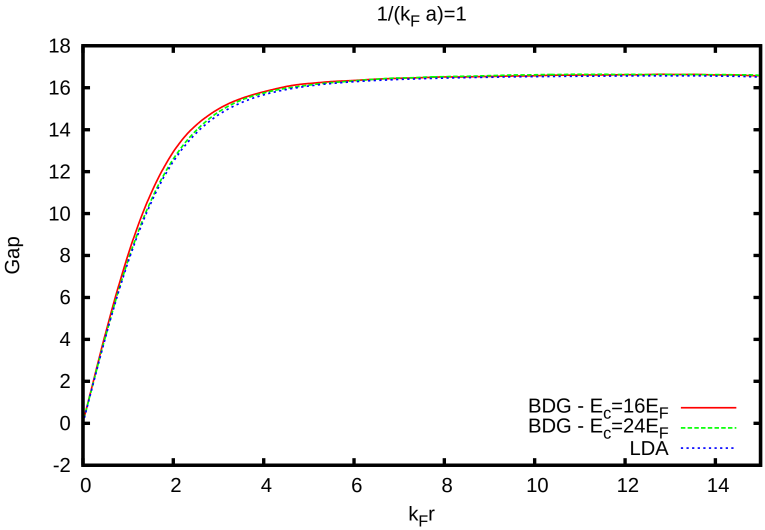
- The current (here $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})| e^{i\varphi(\mathbf{r})}$):

$$\mathbf{J}(\mathbf{r}) = \frac{n(\mathbf{r})}{m} \left(\frac{\nabla\varphi(\mathbf{r})}{2} - \mathbf{A}(\mathbf{r}) \right) + \frac{2}{m} \int \frac{d\mathbf{k}}{(2\pi)^3} \mathbf{k} \, f_F[E_+(\mathbf{k}; \nabla\varphi(\mathbf{r}), \mathbf{A}(\mathbf{r}))]$$

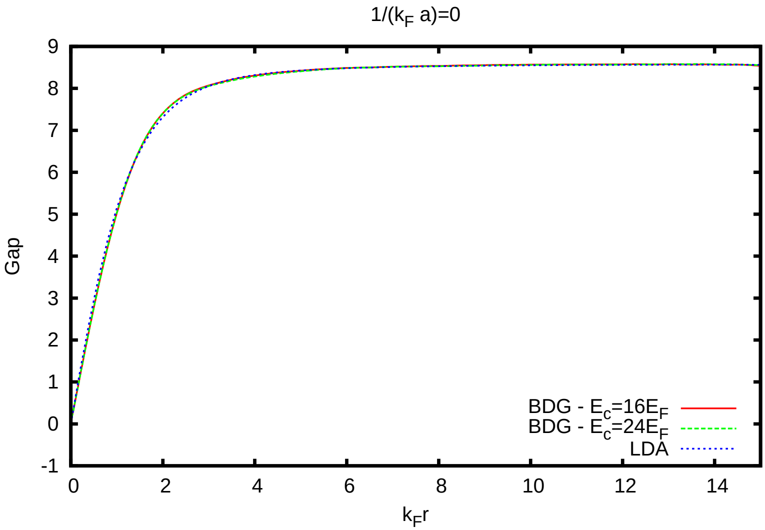
$$E_+(\mathbf{k}; \nabla\varphi(\mathbf{r}), \mathbf{A}(\mathbf{r})) = \sqrt{\left(\frac{k^2}{2m} - \mu(\mathbf{r})\right)^2 + |\Delta(\mathbf{r})|^2} + \frac{\mathbf{k}}{m} \cdot \left(\frac{\nabla\varphi(\mathbf{r})}{2} - \mathbf{A}(\mathbf{r}) \right)$$

Test G-GPE with 1 vortex BEC $\rightarrow$ BCS:

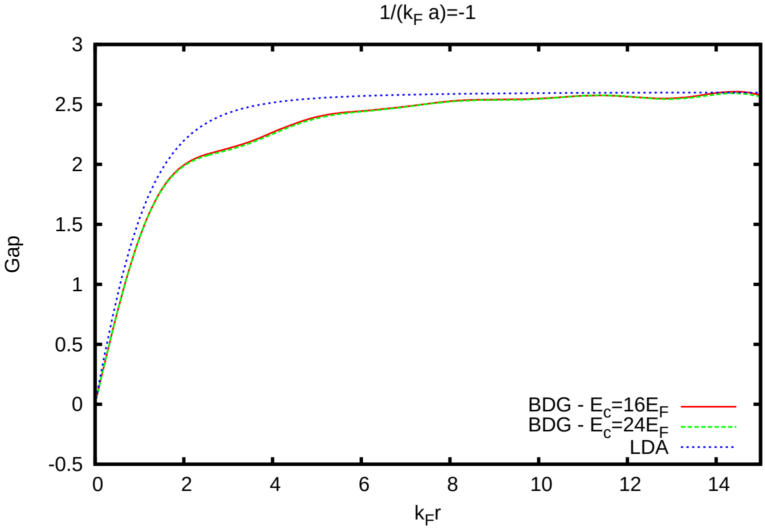
Test G-GPE with 1 vortex BEC → BCS:



good results in the BEC limit !

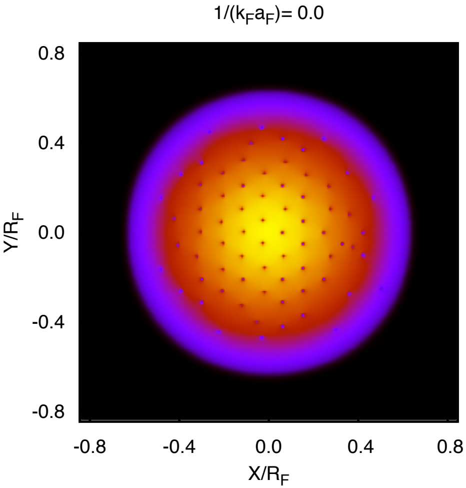


good results also at unitarity !

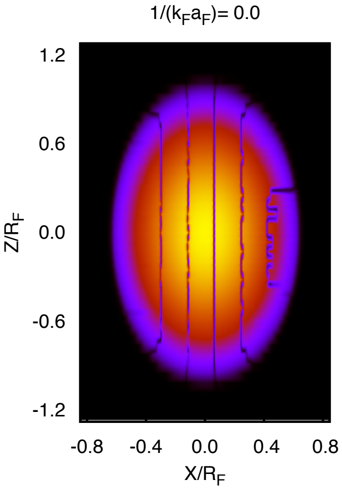


miss the Friedel oscillations on the BCS side !

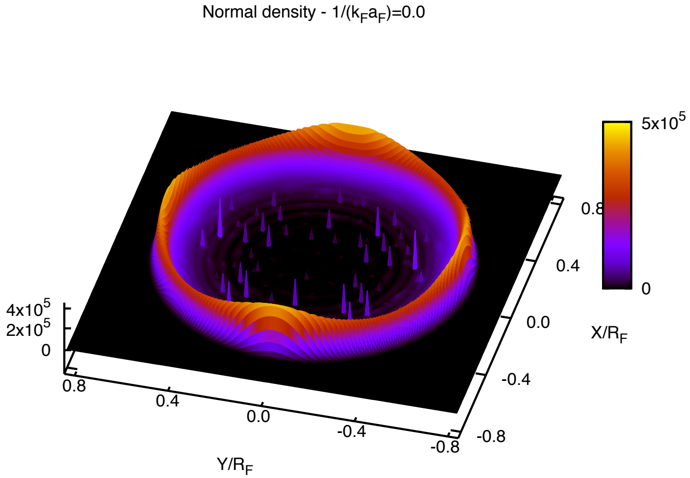
G-GPE: Results for vortex patterns - 1



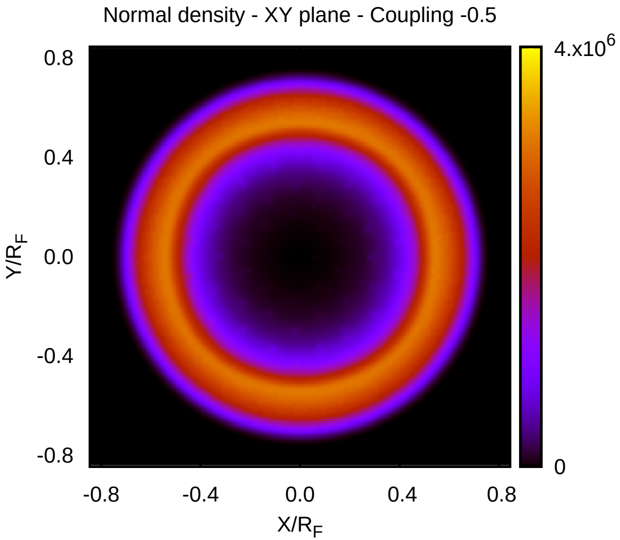
G-GPE: Results for vortex patterns - 2



G-GPE: Results for vortex patterns - 3



G-GPE: Results for vortex patterns - 4



Implementing G-GPE for vortex patterns:

Implementing G-GPE for vortex patterns:

- Discretize 3D space (typically, $201 \times 201 \times 41$ points for a cigar-shape trap)

Implementing G-GPE for vortex patterns:

- Discretize 3D space (typically, $201 \times 201 \times 41$ points for a cigar-shape trap)
- Discretize the operators ∇ and ∇^2 accordingly

Implementing G-GPE for vortex patterns:

- Discretize 3D space (typically, $201 \times 201 \times 41$ points for a cigar-shape trap)
- Discretize the operators ∇ and ∇^2 accordingly
- Reduce the nonlinear G-GPE to a system of the type:

$$\begin{cases} \mathcal{F}_1(\Delta_1, \cdots, \Delta_{\mathcal{N}}) = 0 \\ \cdots \\ \mathcal{F}_{\mathcal{N}}(\Delta_1, \cdots, \Delta_{\mathcal{N}}) = 0 \end{cases}$$

where $\Delta_j = \Delta(x_j, y_j, z_j)$ with $j = (1, \cdots, \mathcal{N})$.

Implementing G-GPE for vortex patterns:

- Discretize 3D space (typically, $201 \times 201 \times 41$ points for a cigar-shape trap)
- Discretize the operators ∇ and ∇^2 accordingly
- Reduce the nonlinear G-GPE to a system of the type:

$$\begin{cases} \mathcal{F}_1(\Delta_1, \cdots, \Delta_{\mathcal{N}}) = 0 \\ \cdots \\ \mathcal{F}_{\mathcal{N}}(\Delta_1, \cdots, \Delta_{\mathcal{N}}) = 0 \end{cases}$$

where $\Delta_j = \Delta(x_j, y_j, z_j)$ with $j = (1, \cdots, \mathcal{N})$.

- Solve this system with Newton's method, using as "initial condition" the *ansatz*:

$$\Delta_j^{(0)} = \Delta_{TF}(\mu - V(x_j, y_j, z_j); a_F) \times \prod_{v=1}^{N_v} \frac{[(x_j - X_v) + i(y_j - Y_v)]}{\sqrt{r_v^2 + (x_j - X_v)^2 + (y_j - Y_v)^2}}$$

where

$\Delta_{TF}(\mu - V(x_j, y_j, z_j); a_F)$ is the Thomas-Fermi result;

$\{\mathbf{X_v}, \mathbf{Y_v}\}$ are the initial positions of the vortices in the $x - y$ plane (use a triangular mesh with size given by the Feynman's theorem);

$\mathbf{r_v}$ is the vortex size ($\cong k_F^{-1}$ when $-1 \lesssim (k_F a_F)^{-1} \lesssim +1$)

where

$\Delta_{TF}(\mu - V(x_j, y_j, z_j); a_F)$ is the Thomas-Fermi result;

$\{\mathbf{X_v}, \mathbf{Y_v}\}$ are the initial positions of the vortices in the $x - y$ plane (use a triangular mesh with size given by the Feynman's theorem);

$\mathbf{r_v}$ is the vortex size ($\cong k_F^{-1}$ when $-1 \lesssim (k_F a_F)^{-1} \lesssim +1$)

- Typically, each iteration (Newton method) takes $\approx 15sec$

where

$\Delta_{TF}(\mu - V(x_j, y_j, z_j); a_F)$ is the Thomas-Fermi result;

$\{\mathbf{X_v}, \mathbf{Y_v}\}$ are the initial positions of the vortices in the $x - y$ plane (use a triangular mesh with size given by the Feynman's theorem);

$\mathbf{r_v}$ is the vortex size ($\cong k_F^{-1}$ when $-1 \lesssim (k_F a_F)^{-1} \lesssim +1$)

- Typically, each iteration (Newton method) takes $\approx 15 sec$
- Less than 10^3 iterations are needed for given coupling

where

$\Delta_{TF}(\mu - V(x_j, y_j, z_j); a_F)$ is the Thomas-Fermi result;

$\{\mathbf{X_v}, \mathbf{Y_v}\}$ are the initial positions of the vortices in the $x - y$ plane (use a triangular mesh with size given by the Feynman's theorem);

$\mathbf{r_v}$ is the vortex size ($\cong k_F^{-1}$ when $-1 \lesssim (k_F a_F)^{-1} \lesssim +1$)

- Typically, each iteration (Newton method) takes $\approx 15 sec$
- Less than 10^3 iterations are needed for given coupling
- About one hour is enough to obtain $\Delta(\mathbf{r})$ and related quantities for given coupling (recall that the corresponding estimated time with the full BdG equations was $\approx 10^6$ hours !!!).

Three types of calculations:

Three types of calculations:

- Exclude the Orbital Breached-Pair Phase

Three types of calculations:

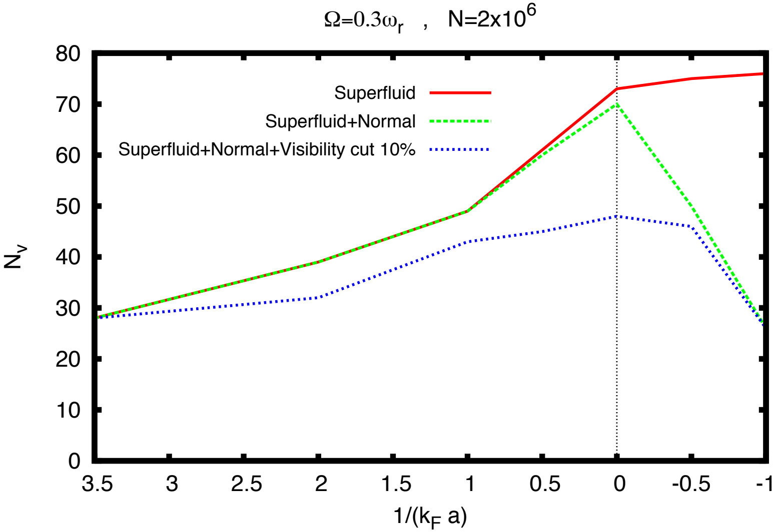
- Exclude the Orbital Breached-Pair Phase
- Include the Orbital Breached-Pair Phase

Three types of calculations:

- Exclude the Orbital Breached-Pair Phase
- Include the Orbital Breached-Pair Phase
- To compare with experiment, introduce in addition a "visibility filter" in the density profile

  (typically, a sharp cut is applied at the edge of the cloud where the density is reduced, say, to 10% of the trap center).

Three types of calculations:



Comparison with MIT experiment:

Comparison with MIT experiment:

To compare with MIT experiment, play a little bit with the:

Comparison with MIT experiment:

To compare with MIT experiment, play a little bit with the:

(i) Effective rotational frequency of the trap $(\Omega_{\text{eff}} \lesssim \Omega_{\text{nominal}})$

Comparison with MIT experiment:

To compare with MIT experiment, play a little bit with the:

(i) Effective rotational frequency of the trap $(\Omega_{\mathrm{eff}} \lesssim \Omega_{\mathrm{nominal}})$

(ii) Total particle number N

Comparison with MIT experiment:

To compare with MIT experiment, play a little bit with the:

(i) Effective rotational frequency of the trap $(\Omega_{\rm eff} \lesssim \Omega_{\rm nominal})$

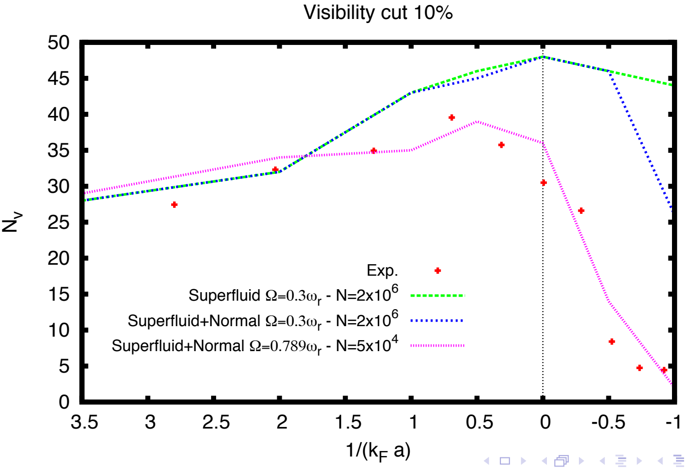
(ii) Total particle number N $\Longrightarrow$

Comparison with MIT experiment:

To compare with MIT experiment, play a little bit with the:

(i) Effective rotational frequency of the trap $(\Omega_{\text{eff}} \lesssim \Omega_{\text{nominal}})$

(ii) Total particle number N $\Longrightarrow$



Possible extensions of the LAPA method:

Possible extensions of the LAPA method:

- ♣ Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c

Possible extensions of the LAPA method:

- ♣ Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]
- Try to introduce correlations beyond mean field at least in non-kinetic terms

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]
- Try to introduce correlations beyond mean field at least in non-kinetic terms [reasonable]

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]
- Try to introduce correlations beyond mean field at least in non-kinetic terms [reasonable]
- Set up a time-dependent version

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]
- Try to introduce correlations beyond mean field at least in non-kinetic terms [reasonable]
- Set up a time-dependent version [$\approx$ dream]

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]
- Try to introduce correlations beyond mean field at least in non-kinetic terms [reasonable]
- Set up a time-dependent version [$\approx$ dream]
- Compare with poly-tropic version(s) of GP eqn. across the BCS-BEC crossover ($\leftrightarrow$ power-law dependence of chemical potential on density)

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]
- Try to introduce correlations beyond mean field at least in non-kinetic terms [reasonable]
- Set up a time-dependent version [$\approx$ dream]
- Compare with poly-tropic version(s) of GP eqn. across the BCS-BEC crossover ($\leftrightarrow$ power-law dependence of chemical potential on density) [?]

Possible extensions of the LAPA method:

- Recover the Ginzburg-Landau equations in the weak-coupling BCS limit close to T_c [check]
- Introduce imbalanced spin populations and look for FFLO phases in definite geometries [feasible]
- Try to introduce correlations beyond mean field at least in non-kinetic terms [reasonable]
- Set up a time-dependent version [$\approx$ dream]
- Compare with poly-tropic version(s) of GP eqn. across the BCS-BEC crossover ($\leftrightarrow$ power-law dependence of chemical potential on density) [?]
- Suggestions would be welcome !

Conclusions:

Conclusions:

- ♣ We have developed a LAPA approximation to the BdG equations, which is much less dramatic than the Thomas-Fermi approximation.

Conclusions:

- ♣ We have developed a LAPA approximation to the BdG equations, which is much less dramatic than the Thomas-Fermi approximation.
- ♣ This approximation has been tested to work well for a single vortex.

Conclusions:

- We have developed a LAPA approximation to the BdG equations, which is much less dramatic than the Thomas-Fermi approximation.
- This approximation has been tested to work well for a single vortex.
- It has made possible also solutions with large vortex patterns.

Conclusions:

- ♣ We have developed a LAPA approximation to the BdG equations, which is much less dramatic than the Thomas-Fermi approximation.
- ♣ This approximation has been tested to work well for a single vortex.
- ♣ It has made possible also solutions with large vortex patterns.
- ♣ The presence of “Orbital Breached-Pair Phase” has been evidenced in the MIT experiment.

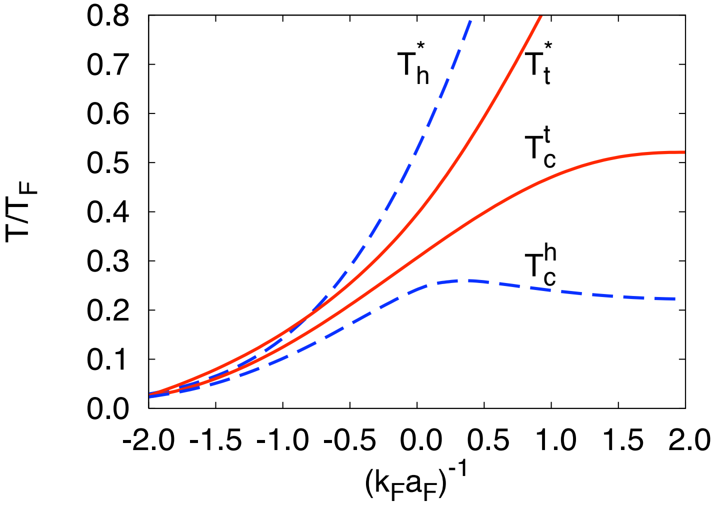
Conclusions:

- ♣ We have developed a LAPA approximation to the BdG equations, which is much less dramatic than the Thomas-Fermi approximation.
- ♣ This approximation has been tested to work well for a single vortex.
- ♣ It has made possible also solutions with large vortex patterns.
- ♣ The presence of "Orbital Breached-Pair Phase" has been evidenced in the MIT experiment.
- ♣ Future applications: moment of inertia , $\cdots$

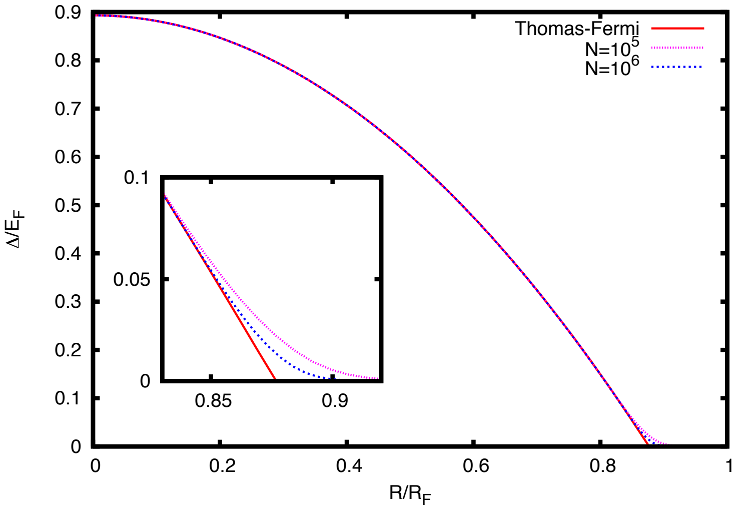
Conclusions:

- We have developed a LAPA approximation to the BdG equations, which is much less dramatic than the Thomas-Fermi approximation.
- This approximation has been tested to work well for a single vortex.
- It has made possible also solutions with large vortex patterns.
- The presence of "Orbital Breached-Pair Phase" has been evidenced in the MIT experiment.
- Future applications: moment of inertia , $\cdots$
- Thank you for your attention !

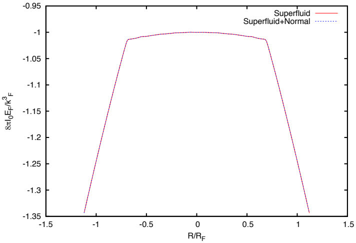
Where G-GPE works in the phase diagram:

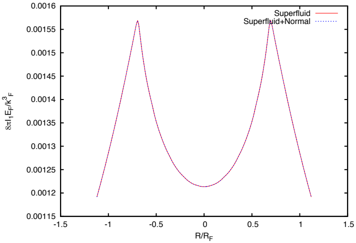


Thomas-Fermi vs Generalized GPE ($\Omega = 0$, unitarity limit):



“Coefficients” $\mathcal{I}_0$ and $\mathcal{I}_1$ of the G-GPE:

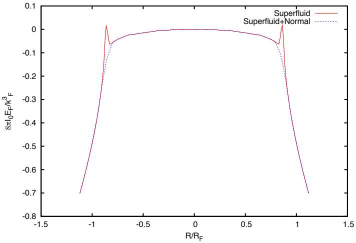


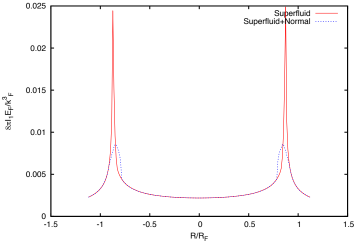


$\mathcal{I}_0$ when $(k_F a_F)^{-1} = +1.0$

$\mathcal{I}_1$ when $(k_F a_F)^{-1} = +1.0$

“Coefficients” $\mathcal{I}_0$ and $\mathcal{I}_1$ of the G-GPE:

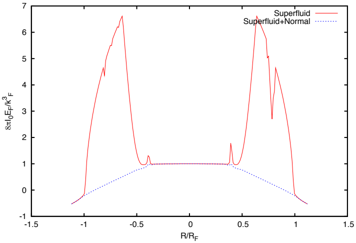


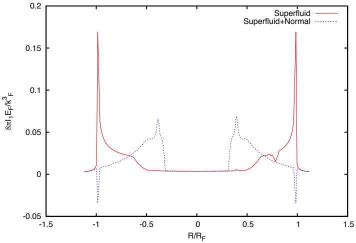


$\mathcal{I}_0$ when $(k_F a_F)^{-1} = 0.0$

$\mathcal{I}_1$ when $(k_F a_F)^{-1} = 0.0$

“Coefficients” $\mathcal{I}_0$ and $\mathcal{I}_1$ of the G-GPE:





$\mathcal{I}_0$ when $(k_F a_F)^{-1} = -1.0$

$\mathcal{I}_1$ when $(k_F a_F)^{-1} = -1.0$