Nearly Perfect Fluidity in Cold Atomic Gases

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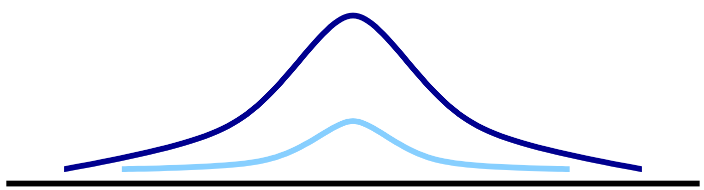
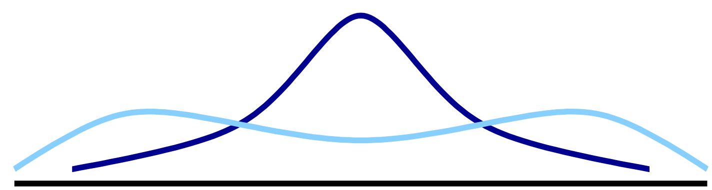
Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



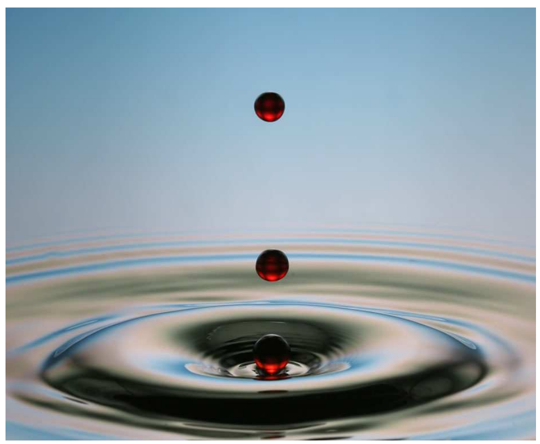
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



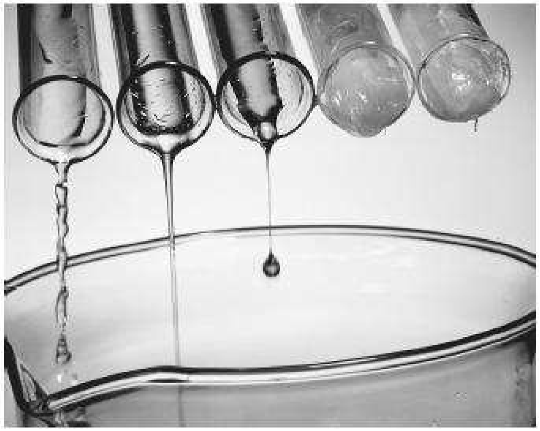
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\,\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive   dissipative   2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

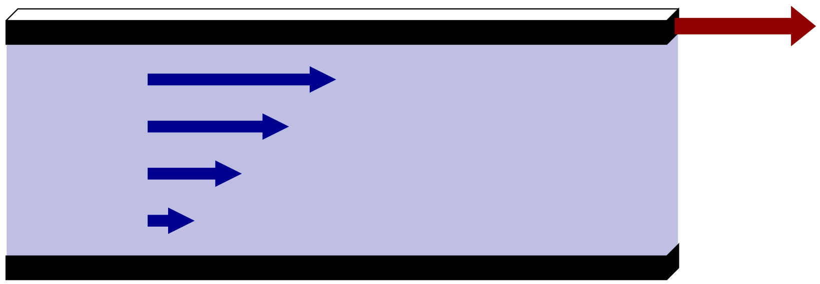
Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$Re^{-1} = \underbrace{\frac{\eta}{\hbar n}}_{\text{fluid property}} \times \underbrace{\frac{\hbar}{m v L}}_{\text{flow property}}$$

Consider $mvL \sim \hbar$: Hydrodynamics requires $\eta/(\hbar n) < 1$

Shear viscosity

Viscosity determines shear stress ("friction") in fluid flow

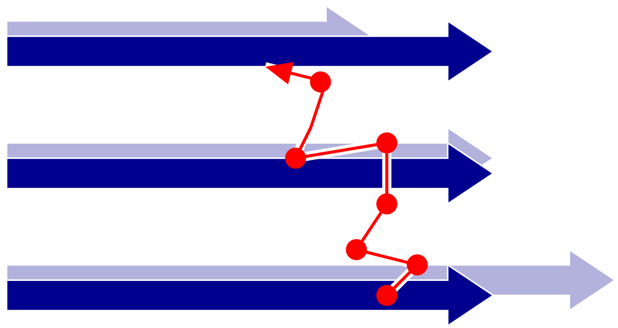


$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$



Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

independent of density!

Shear viscosity

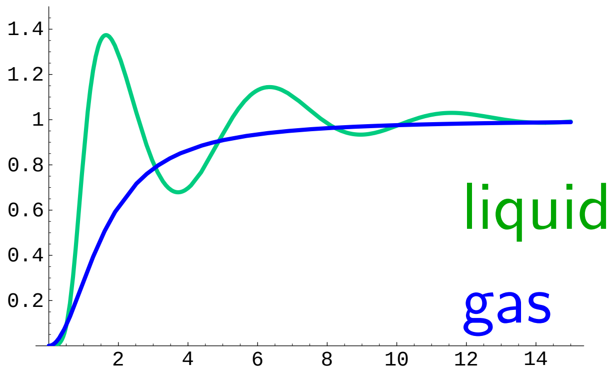
non-interacting gas ($\sigma \to 0$): $\boxed{\eta \to \infty}$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas: $\boxed{\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar}$

but: kinetic theory not reliable!

what happens if the gas condenses into a liquid?



Eyring, Frenkel:

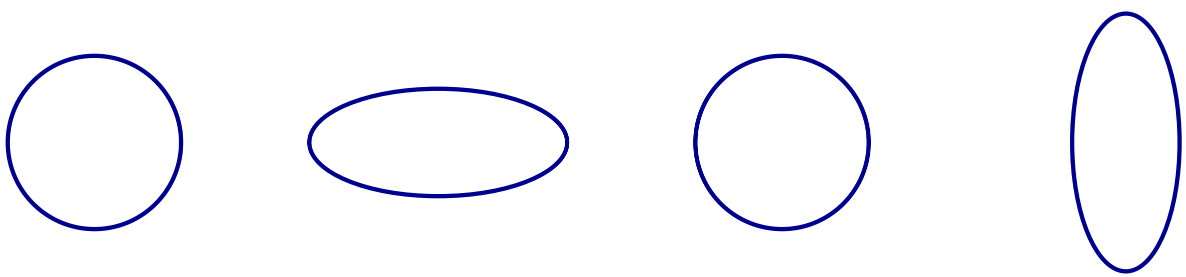
$$\boxed{\eta \simeq hn \exp(E/T) \geq hn}$$

Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



Holographic duals: Transport properties

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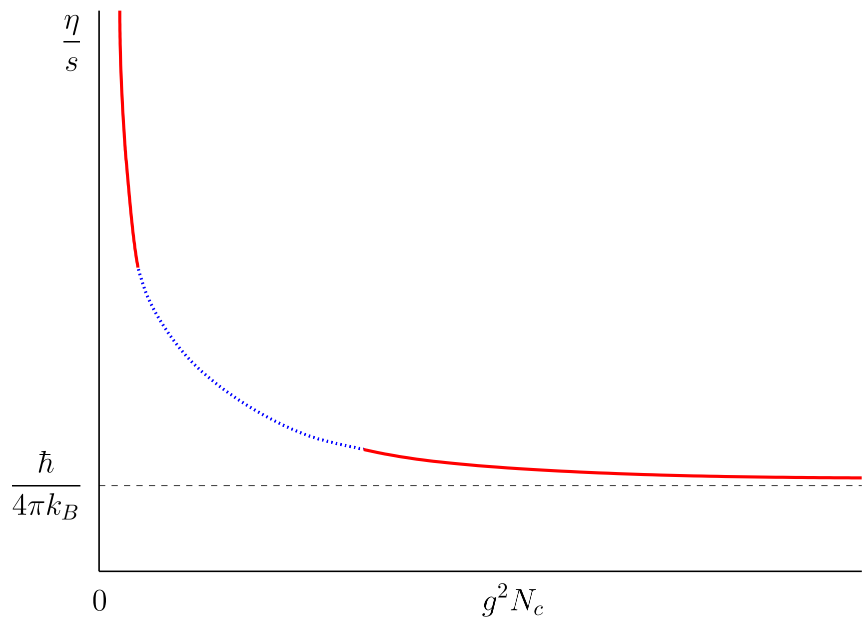
Strong coupling limit

$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$$

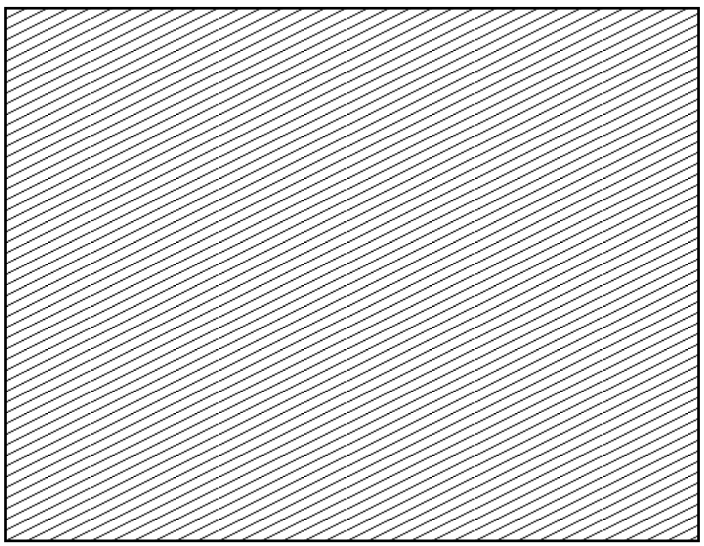
Son and Starinets (2001)



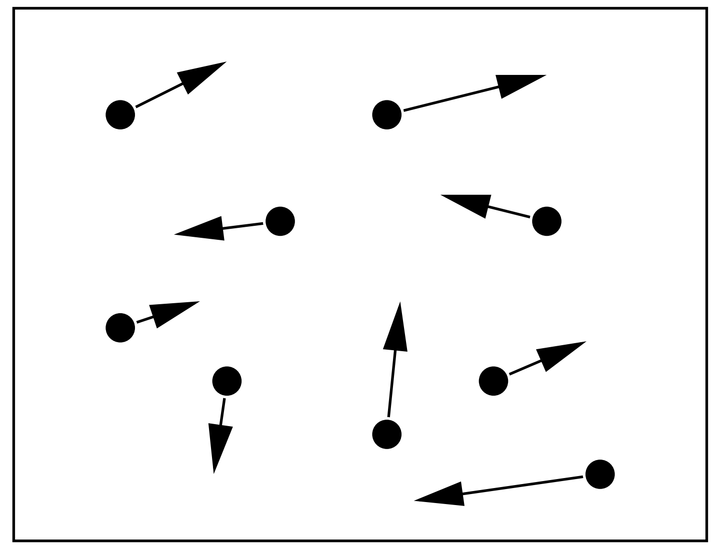
Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

Kinetics vs No-Kinetics



AdS/CFT low viscosity goo



pQCD kinetic plasma

Effective theories for fluids (Here: Weak coupling QCD)

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad \omega < T$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \qquad \omega < g^4 T$$

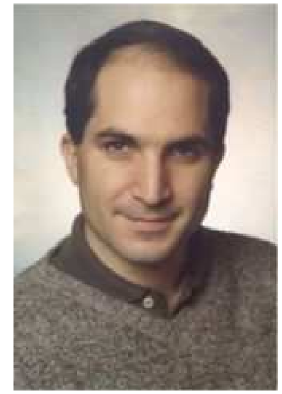
Effective theories for fluids (Unitary Fermi Gas, $T > T_F$)

$$\mathcal{L} = \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2M}\right)\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2$$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad \omega < T$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad \omega < T\left(\frac{T_F}{T}\right)^{3/2}$$

Effective theories (Strong coupling)



$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

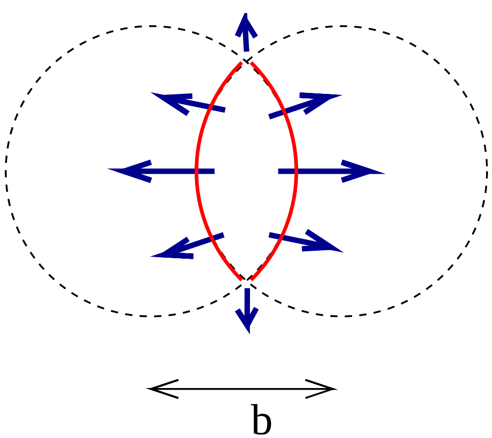
$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \;\; (\omega < T)$$

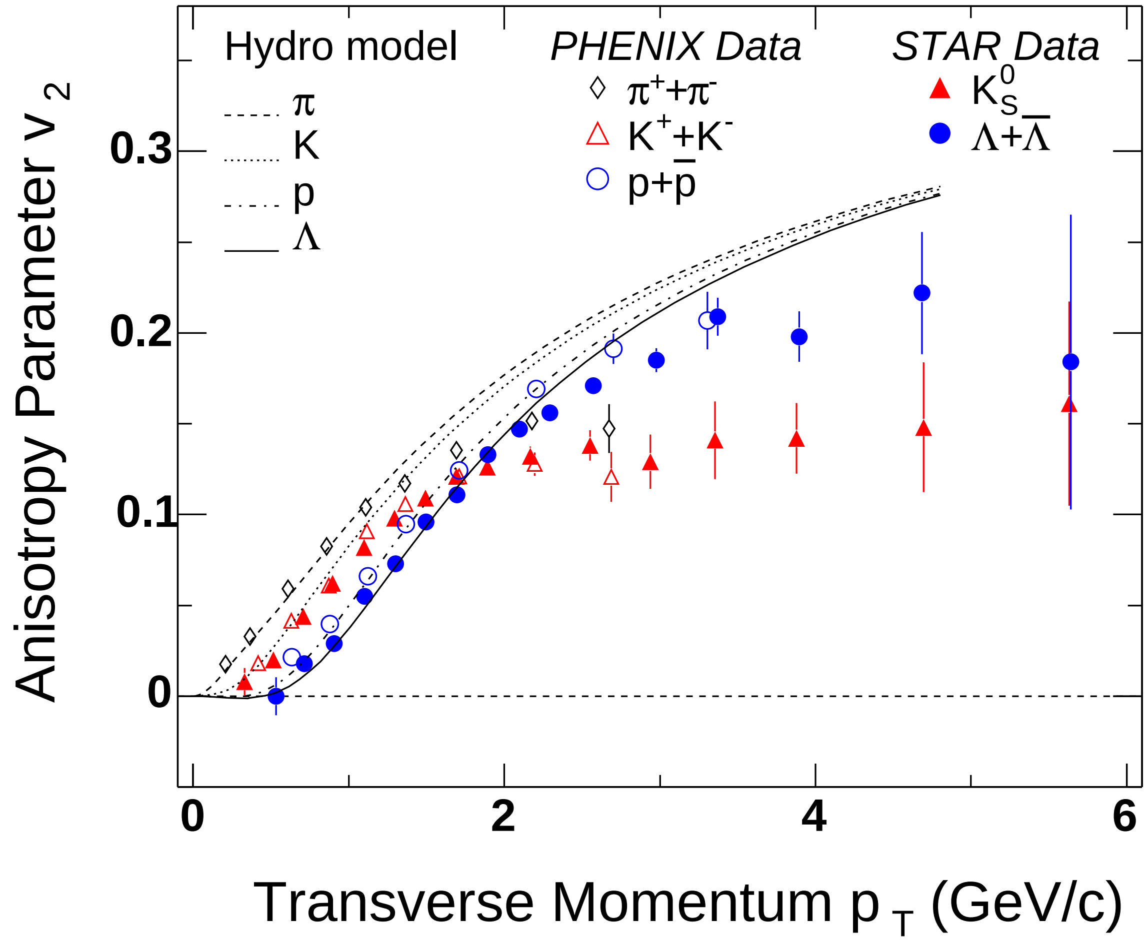
Nearly perfect fluidity in QCD: Elliptic Flow of the QGP

Hydrodynamic expansion converts

coordinate space anisotropy

to momentum space anisotropy

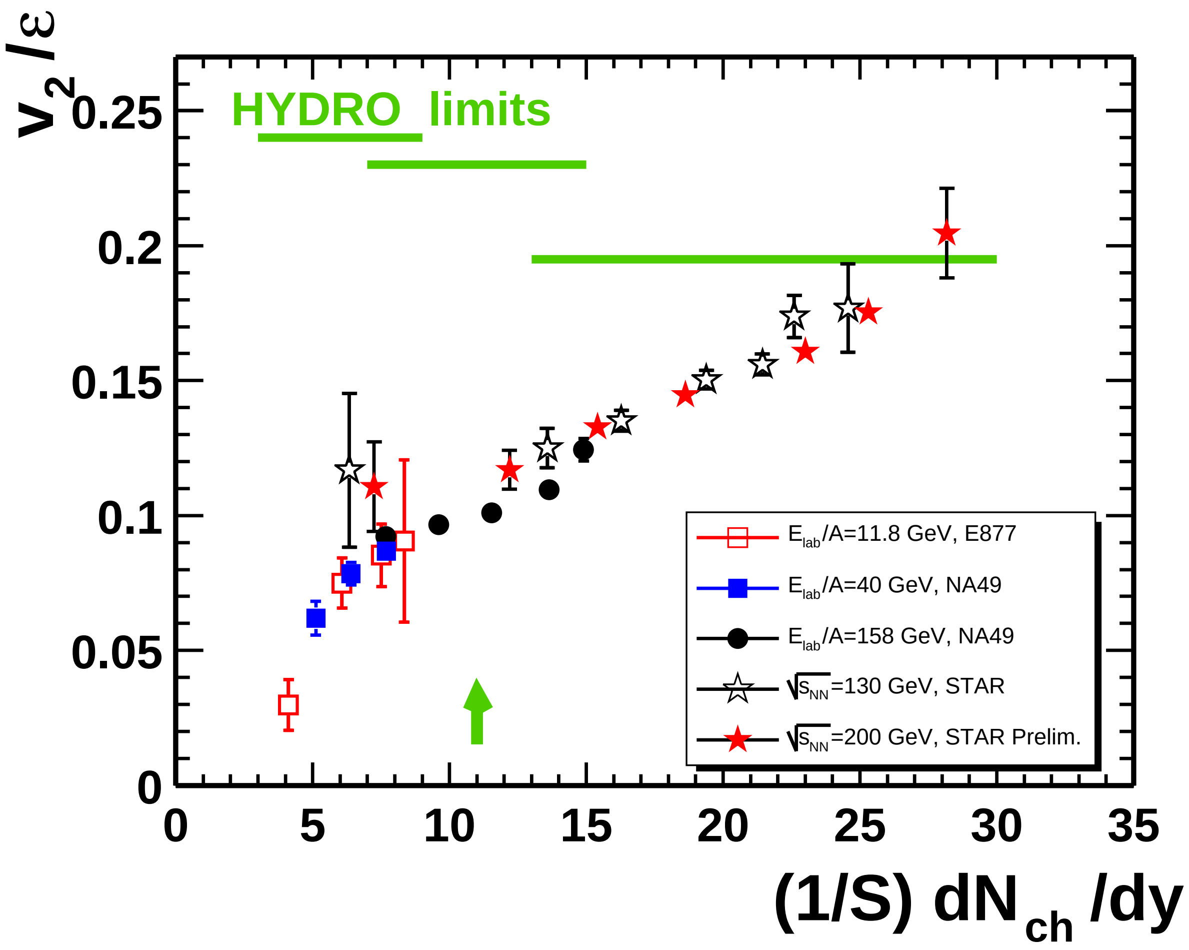




source: U. Heinz (2005)

$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \dots)$$

Elliptic flow: initial entropy scaling



source: U. Heinz (2005)

Viscosity and Elliptic Flow

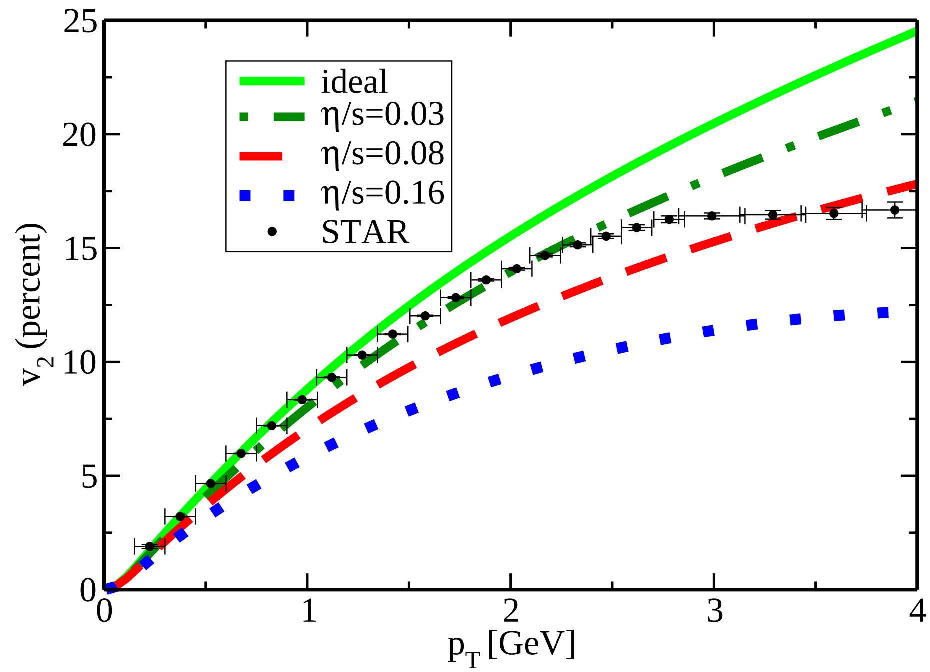
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$

(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

conservative bound $\boxed{\frac{\eta}{s} < 0.4}$

Dilute Fermi gas: field theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth: HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \Big(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \Big) - \frac{1}{C_0} \phi^* \phi \ ,$$

Low T ($T < T_c \sim \mu$): Pairing and superfluidity, $\langle \phi \rangle \neq 0$

Linear response and kinetic theory

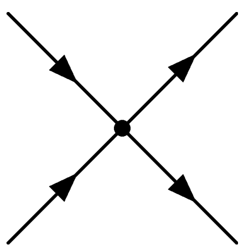
Consider background metric $g_{ij}(t,\mathbf{x}) = \delta_{ij} + h_{ij}(t,\mathbf{x})$. Linear response

$$\delta\Pi^{ij} = \frac{\delta\Pi^{eq}_{ij}}{\delta h_{ij}} h^{ij} - \frac{1}{2} G_R^{ijkl} h_{kl}$$

Kubo relation: $\eta(\omega) = \frac{1}{\omega}\mathrm{Im}G_R^{xyxy}(\omega,0)$

Kinetic theory: Boltzmann equation $(T > T_F)$

$$\left(\frac{\partial}{\partial t} + \frac{p^i}{m}\frac{\partial}{\partial x^i} - \left(g^{il}\dot{g}_{lj}p^j + \Gamma^i_{jk}\frac{p^j p^k}{m}\right)\frac{\partial}{\partial p^i}\right) f(t,\mathbf{x},\mathbf{p}) = C[f]$$

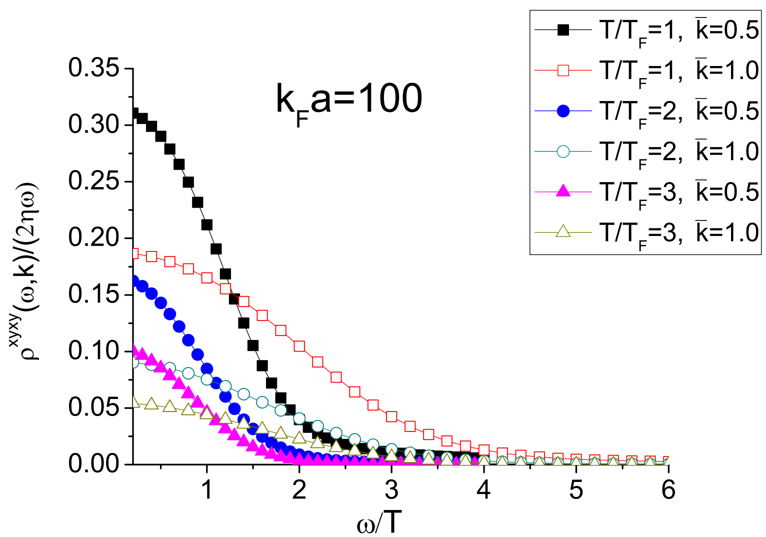
$C[f] =$ 

Kinetic theory

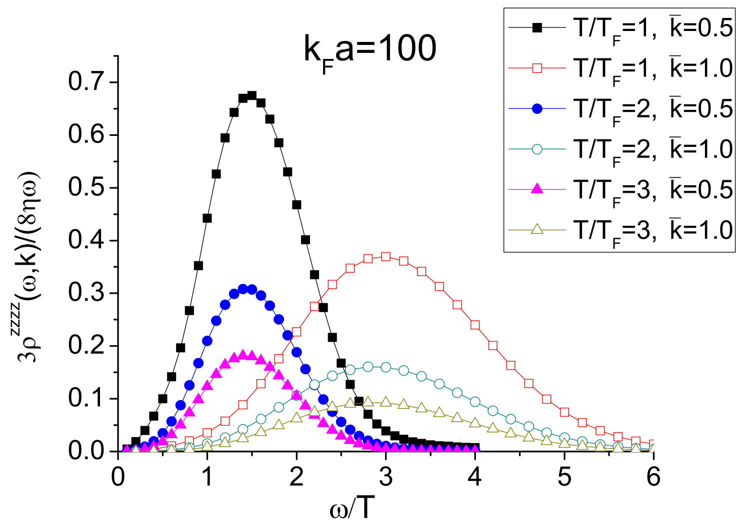
linearize $f = f_0 + \delta f$, solve for δf, $\hookrightarrow \delta\Pi_{ij}$, $\hookrightarrow G_R$, $\hookrightarrow \eta(\omega)$

$$\eta(\omega) = \frac{\eta}{1+\omega^2\tau_R^2} \qquad \eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2} \qquad \tau_R = \frac{\eta}{nT}$$

shear channel



sound channel



Braby, Chao, T.S. (2010)

Shear viscosity: Sum rules

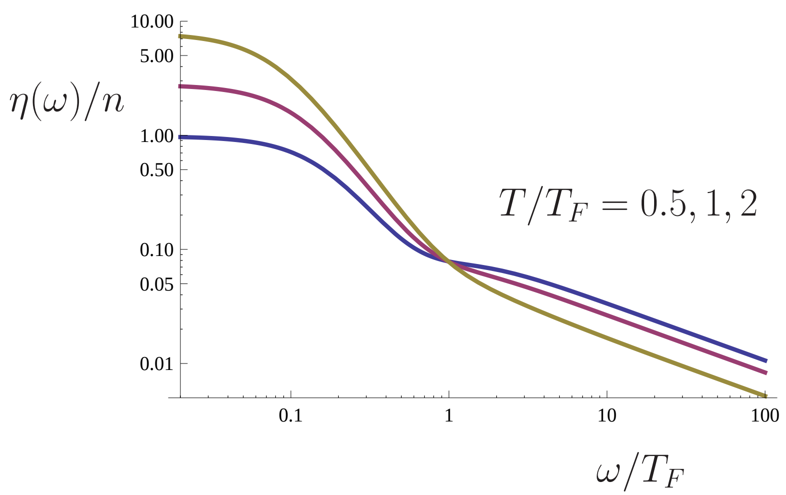
Randeira & Taylor proved the sum rules (corrected by Enss & Zwerger)

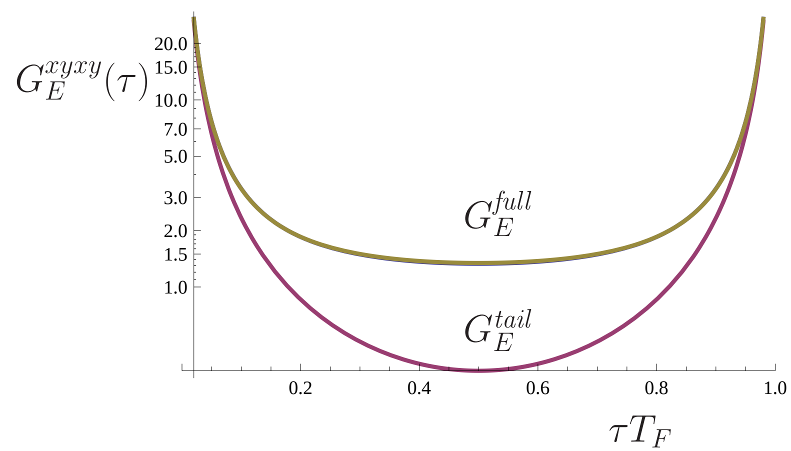
$$\frac{1}{\pi}\int dw\left[\eta(\omega)-\frac{C}{15\pi\sqrt{m\omega}}\right]=\frac{\mathcal{E}}{3}-\frac{C}{10\pi m a}$$

$$\frac{1}{\pi}\int dw\,\zeta(\omega)=\frac{1}{72\pi m a^2}\left(\frac{\partial C}{\partial a^{-1}}\right)$$

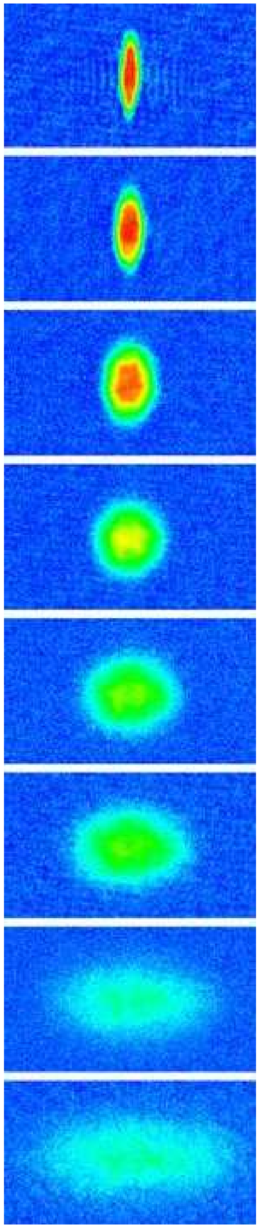
where C is Tan's contact, $\rho(k)\sim C/k^4$.

Sum rules constrain spectral function and euclidean correlator

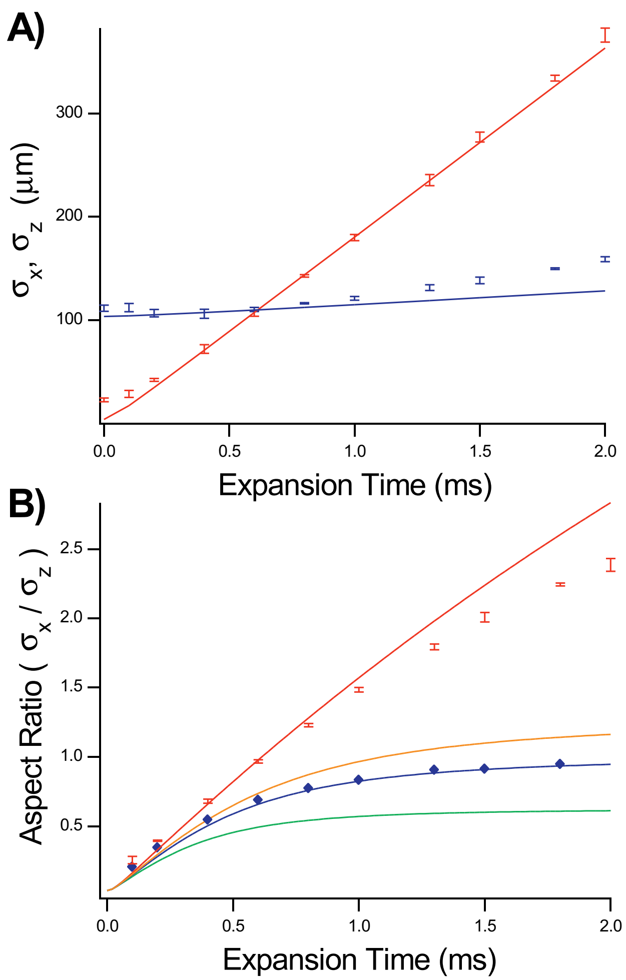




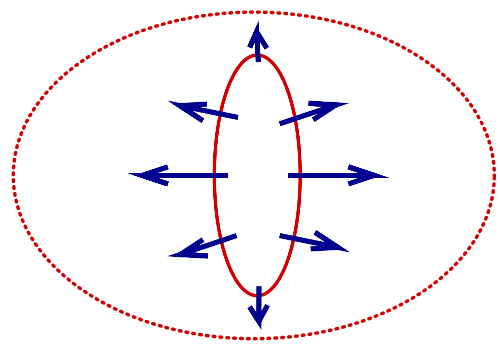
Almost ideal fluid dynamics



O'Hara et al. (2002)

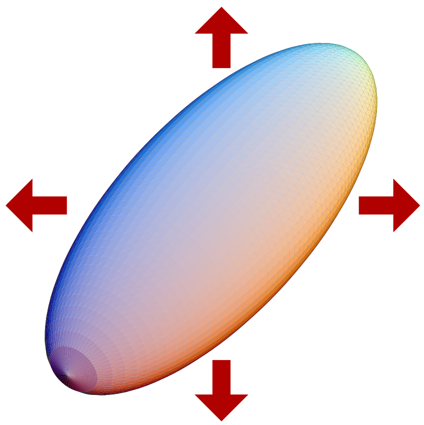


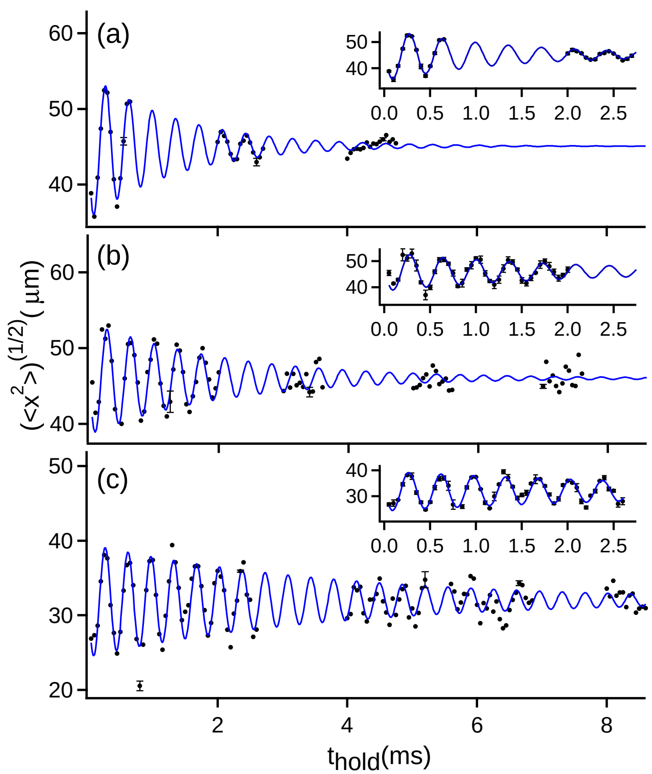
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Hydrodynamics: Collective modes

Radial breathing mode





Ideal fluid hydrodynamics ($P = \frac{2}{3}\mathcal{E}$)

$$
\begin{aligned}
\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) &= 0 \\
\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} &= -\frac{\vec{\nabla}P}{mn} - \frac{\vec{\nabla}V}{m}
\end{aligned}
$$

Hydro frequency at unitarity

$$
\omega = \sqrt{\frac{10}{3}}\,\omega_\perp
$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

Damping of collective mode

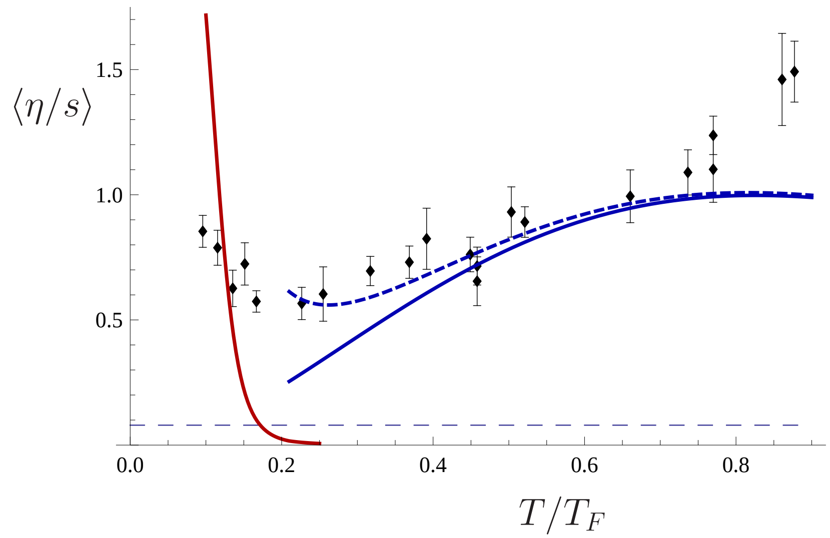
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

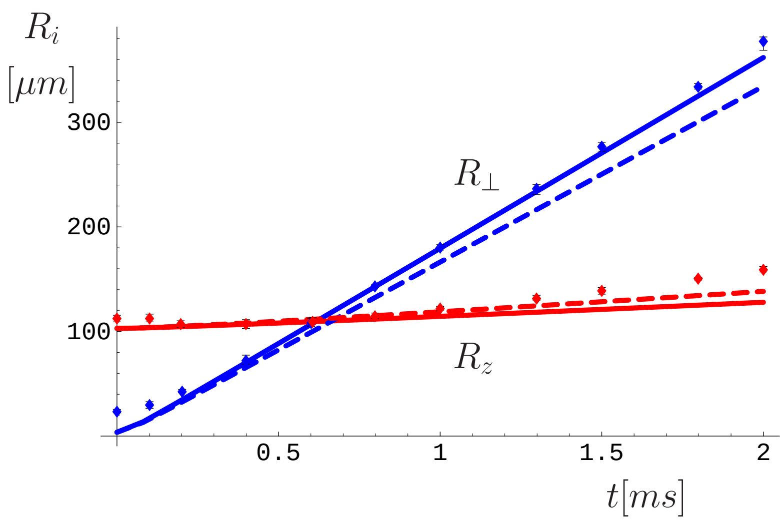
$$\boxed{\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{E_0}{E_F}\frac{N}{S}}$$

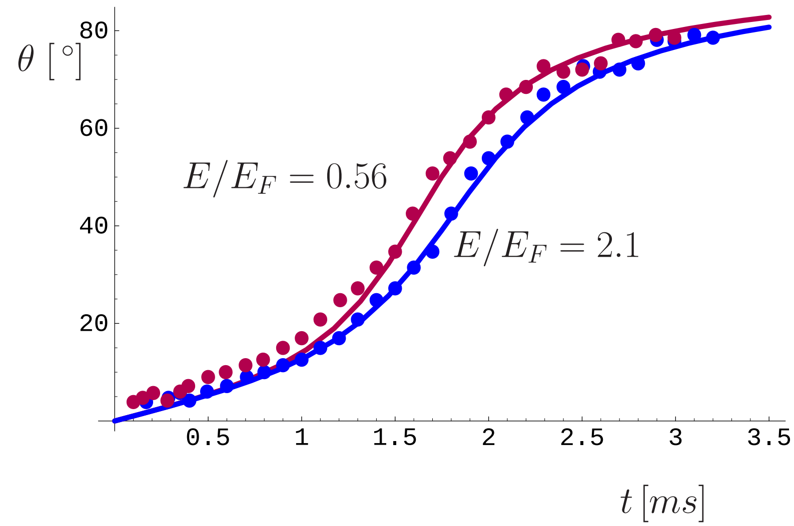


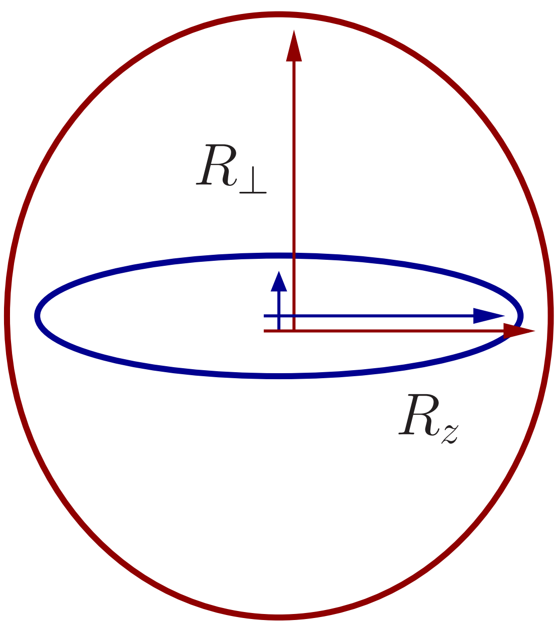
Schaefer (2007), see also Bruun, Smith

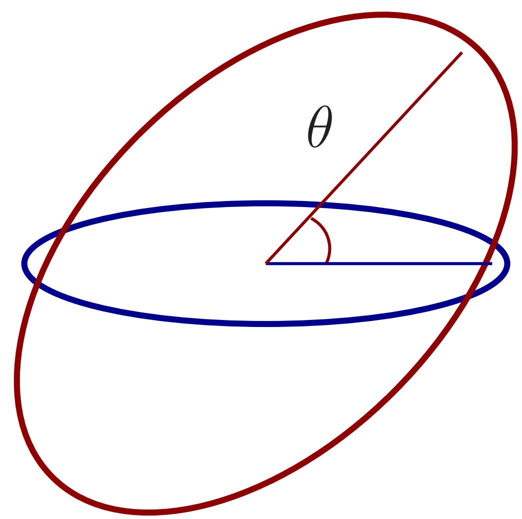
$T \ll T_F$ $\qquad$ $T \gg T_F$, $\tau_R \simeq \eta/P$

Hydrodynamics: Free expansion and rotation









O'Hara et al (2002), Clancy et al (2007)

Scaling Flows

Universal equation of state $$P = \frac{2}{3}\mathcal{E}$$

Equilibrium density profile

$$n_0(x) = n(\mu(x), T) \qquad \mu(x) = \mu_0 \left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

Scaling Flow: Stretch and rotate profile

$$\mu_0 \to \mu_0(t), \qquad T \to T_0(\mu_0(t)/\mu_0), \qquad R_x \to R_x(t), \ \ldots$$

Linear velocity profile

$$\boxed{\vec{v}(x,t) = (\alpha_x x, \alpha_y y, \alpha_z z) + \alpha \vec{\nabla}(xy) + \vec{\omega} \times \vec{x}}$$

"Hubble flow"

Scaling hydrodynamics

Write $R_i(t) = b_i(t) R_i(0)$. Euler equation

$$\frac{\ddot{b}_\perp}{b_\perp} = \frac{\omega_\perp^2}{(b_\perp^2 b_{||})^{2/3}} \frac{1}{b_\perp^2} \qquad b_\perp(\omega_\perp t \gg 1) \sim \sqrt{\frac{3}{2}}\, \omega_\perp t$$

Dissipation breaks scaling behavior $(\nabla_i P/n = a_i x_i)$

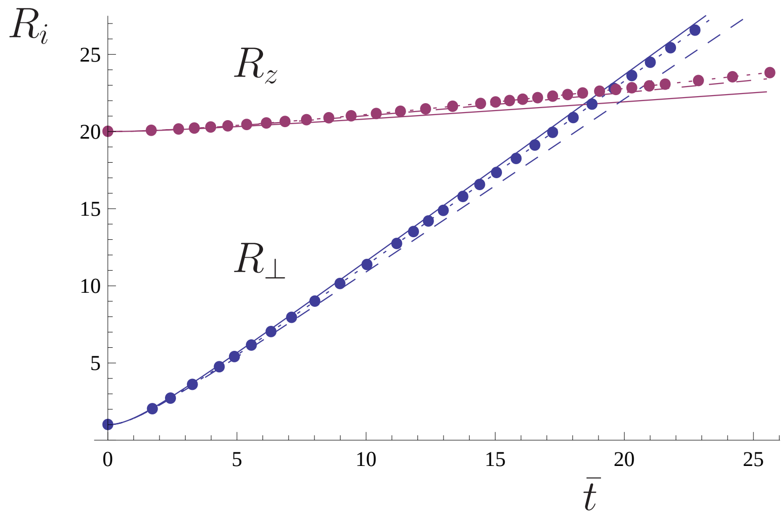
$$\frac{\ddot{b}_\perp}{b_\perp} = a_\perp - \frac{2\beta\omega_\perp}{b_\perp^2} \left(\frac{\dot{b}_\perp}{b_\perp} - \frac{\dot{b}_x}{b_x} \right) \qquad \textit{friction}$$

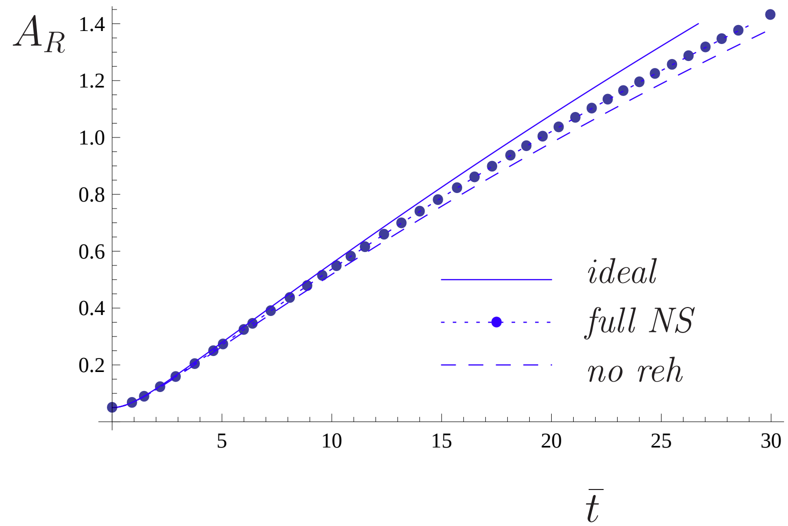
$$\dot{a}_\perp = ideal + \frac{8\beta\omega_\perp^2}{3b_\perp} \left(\frac{\dot{b}_\perp}{b_\perp} - \frac{\dot{b}_z}{b_z} \right)^2 \qquad \textit{heating}$$

$$\beta = \frac{\langle \eta \rangle}{N} \frac{E_F}{E_0} \frac{1}{(3N\lambda)^{1/3}}$$

Navier-Stokes: Numerical results

Full 3-D hydro with $\eta = \alpha_n n$ and $\alpha_n = const$.





Reheating leads to reacceleration. Dissipation causes characteristic curvature of $A_R(t) \equiv R_\perp / R_z$.

Issues: i) Dilute corona $\eta \sim T^{3/2} \to \nabla_i \delta\Pi_{ij} = 0$. No force (?)
ii) $Kn \sim (b_{||}/b_\perp)^{1/3}$ drops $\to$ No freezeout (?)

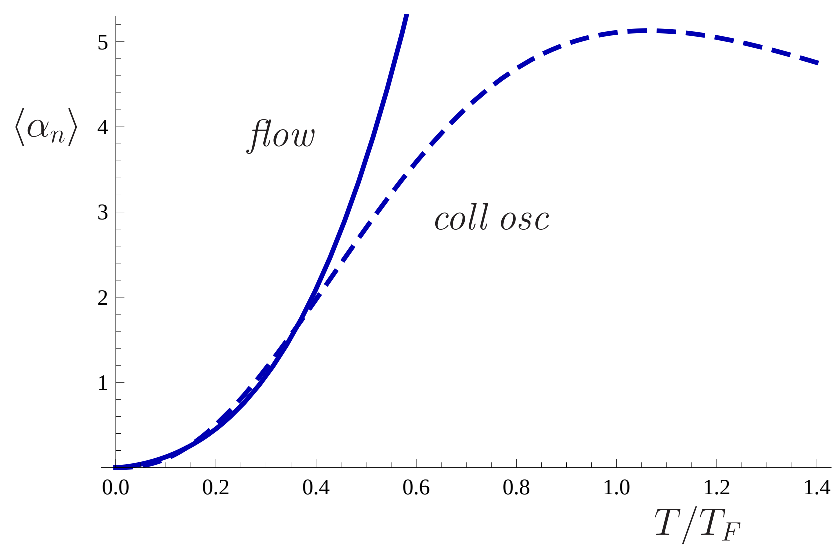
Relaxation time model

In real systems stress tensor does not relax to Navier-Stokes form instantaneously. Consider

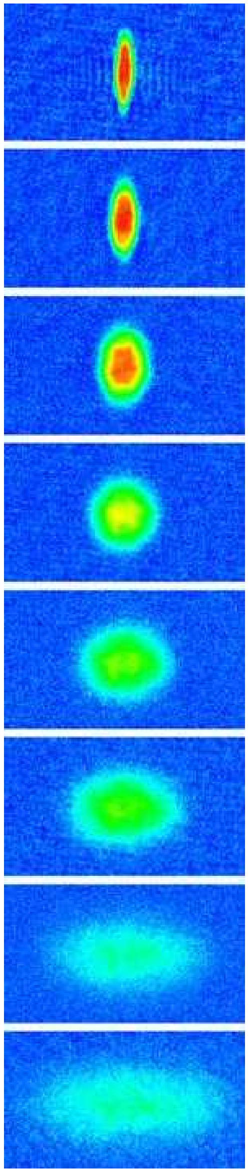
$$\tau_R \frac{\partial}{\partial t}\delta\Pi_{ij} = \delta\Pi_{ij} - \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial\cdot v\right)$$

In kinetic theory $\tau_R \simeq (\eta/n)\, T^{-1}$

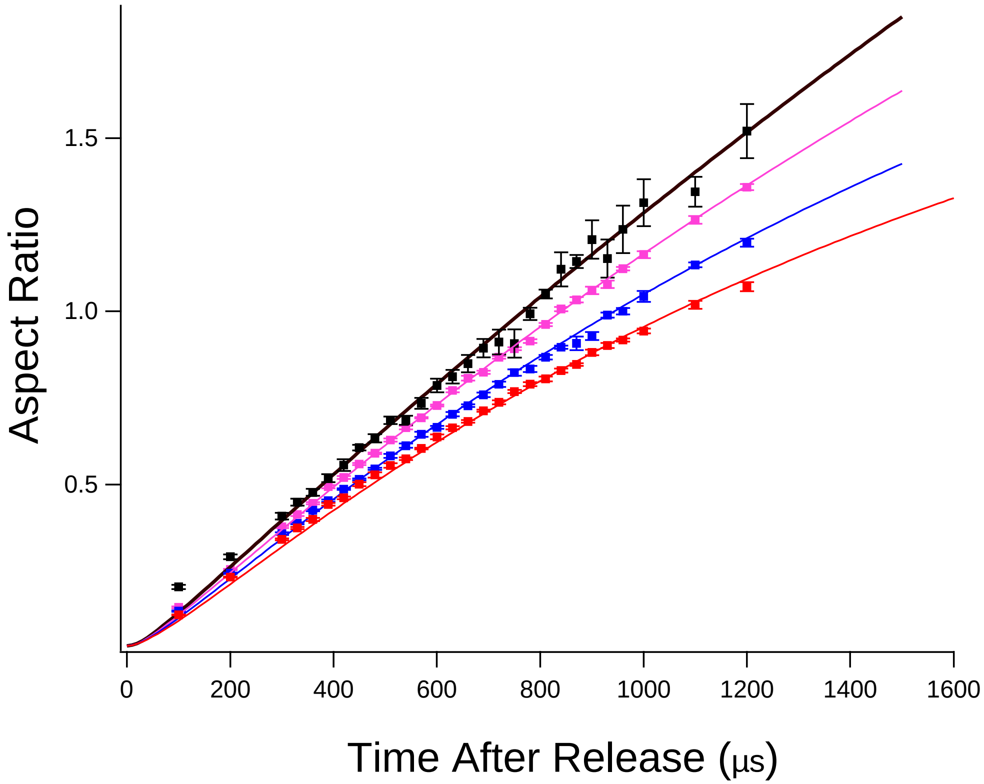
- disspiation from $\eta \sim (mT)^{3/2}$: corona excerts drag force.
- find $\langle\alpha_n\rangle \sim T^3$
- system dependence



Elliptic flow: High T limit



Quantum viscosity $\eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$



$$\eta = \eta_0 (mT)^{3/2}$$

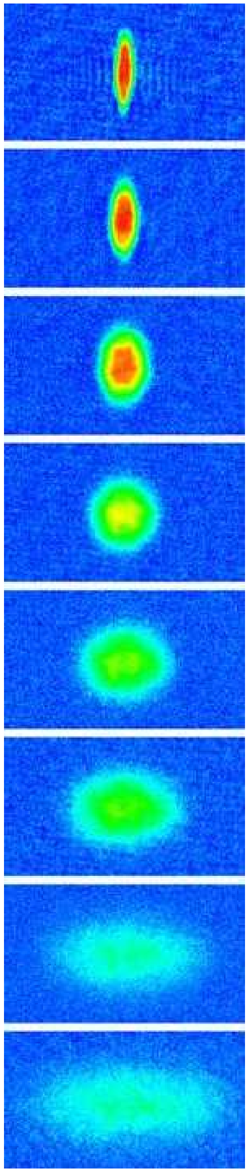
$$\tau_R = \eta / P$$

Cao et al., Science (2010)

fit: $\eta_0 = 0.33 \pm 0.04$

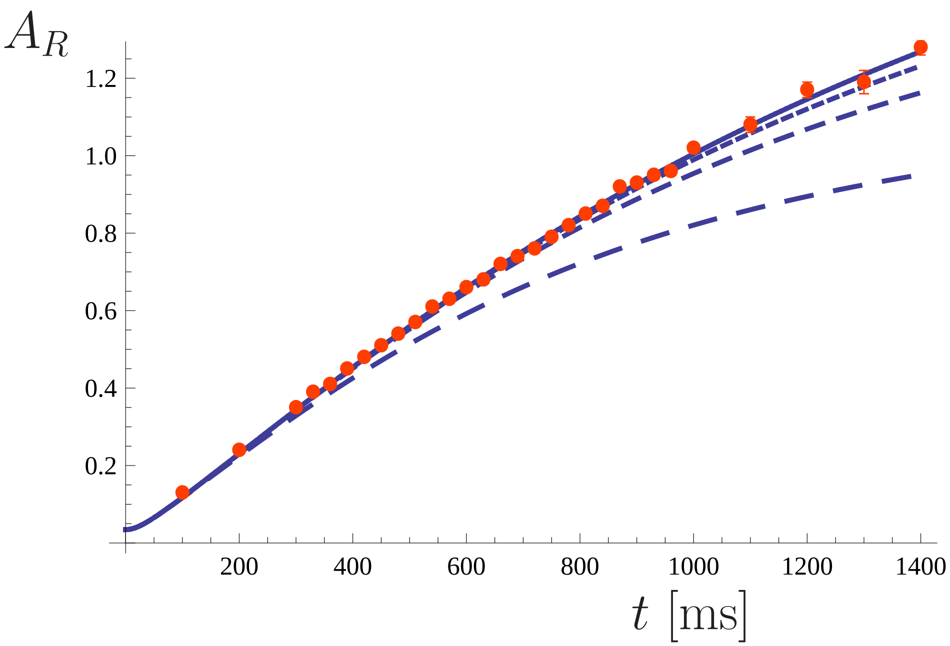
theory: $\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$

Elliptic flow: Freezeout?



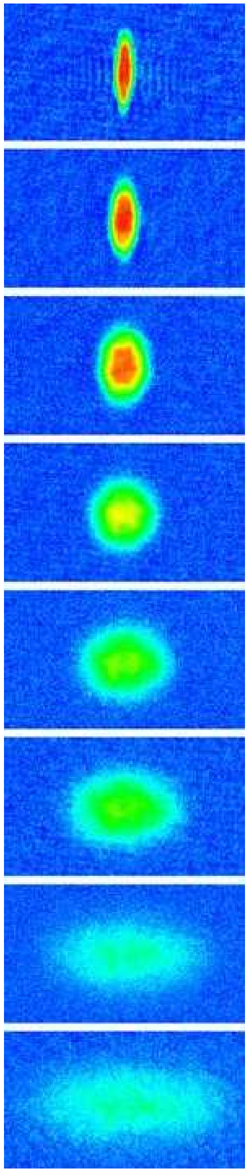
switch from hydro to (weakly collisional) kinetics

at scale factor $b_\perp^{fr} = 1, 5, 10, 20$

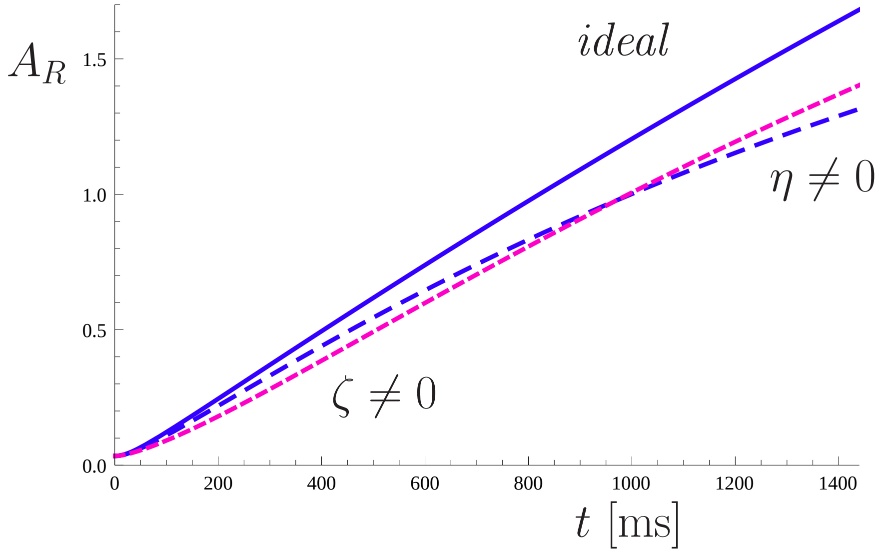


no freezeout seen in the data

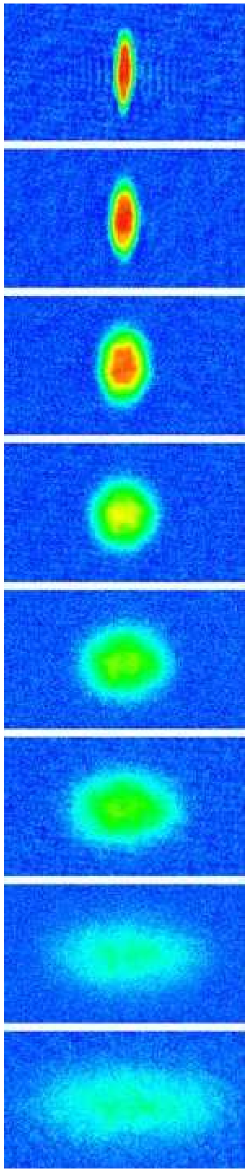
Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

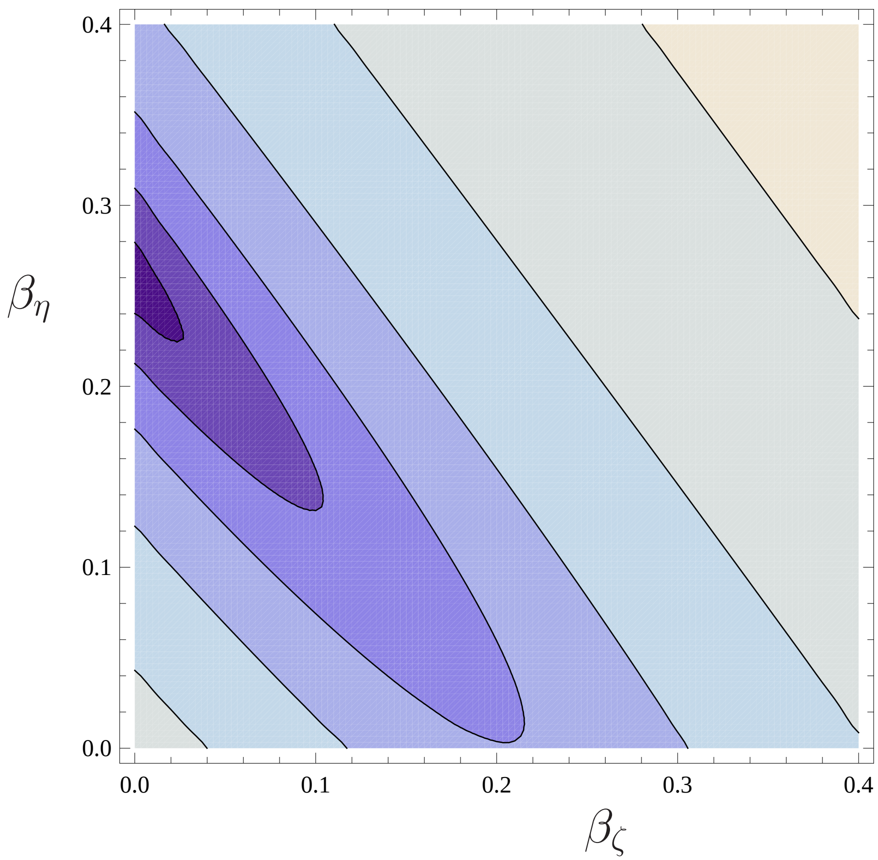


Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

$$\beta_{\eta,\zeta} = (\eta, \zeta) \frac{E_F}{E} \frac{1}{(3\lambda N)^{1/3}}$$

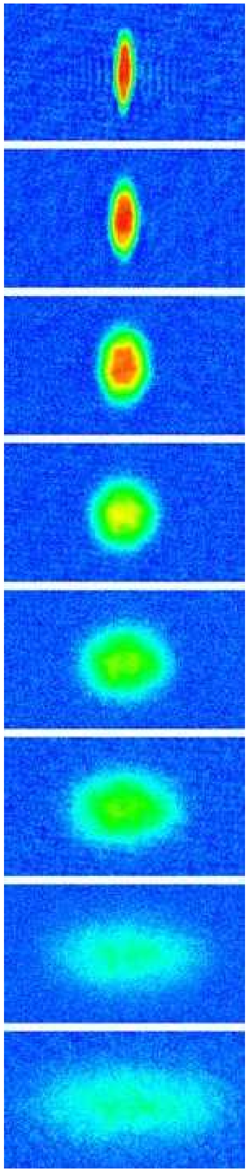


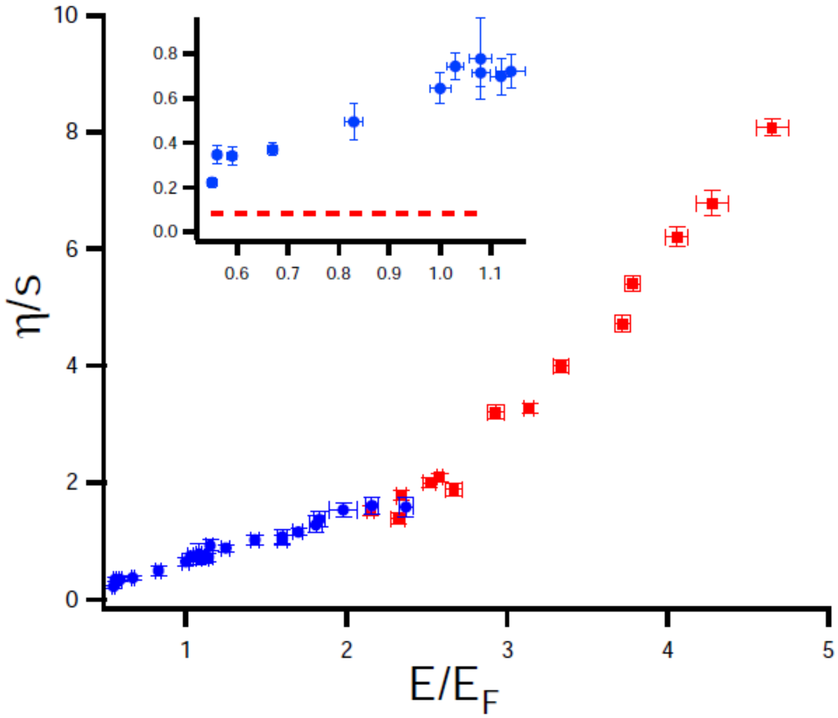
$\eta \gg \zeta$

Viscosity to entropy density ratio

consider both collective modes (low T)

and elliptic flow (high T)





Cao et al., Science (2010)

$$\eta/s \le 0.4$$

Outlook

The unitary Fermi gas is an important model system for other strongly correlated quantum fluids in nature.

The equation of state has been determined to a few percent.

Transport properties are more difficult: Kinetic theory at $T \gg T_F$ and $T \ll T_F$. Sum rules constrain spectral fct at all T.

Experimental determination of transport properties: Collective modes give $\langle \eta/s \rangle < 0.4$. Local analysis requires second order hydro or hydro+kinetic.