Neutrino emissivity in $^3P_2$ superfluid neutron matter

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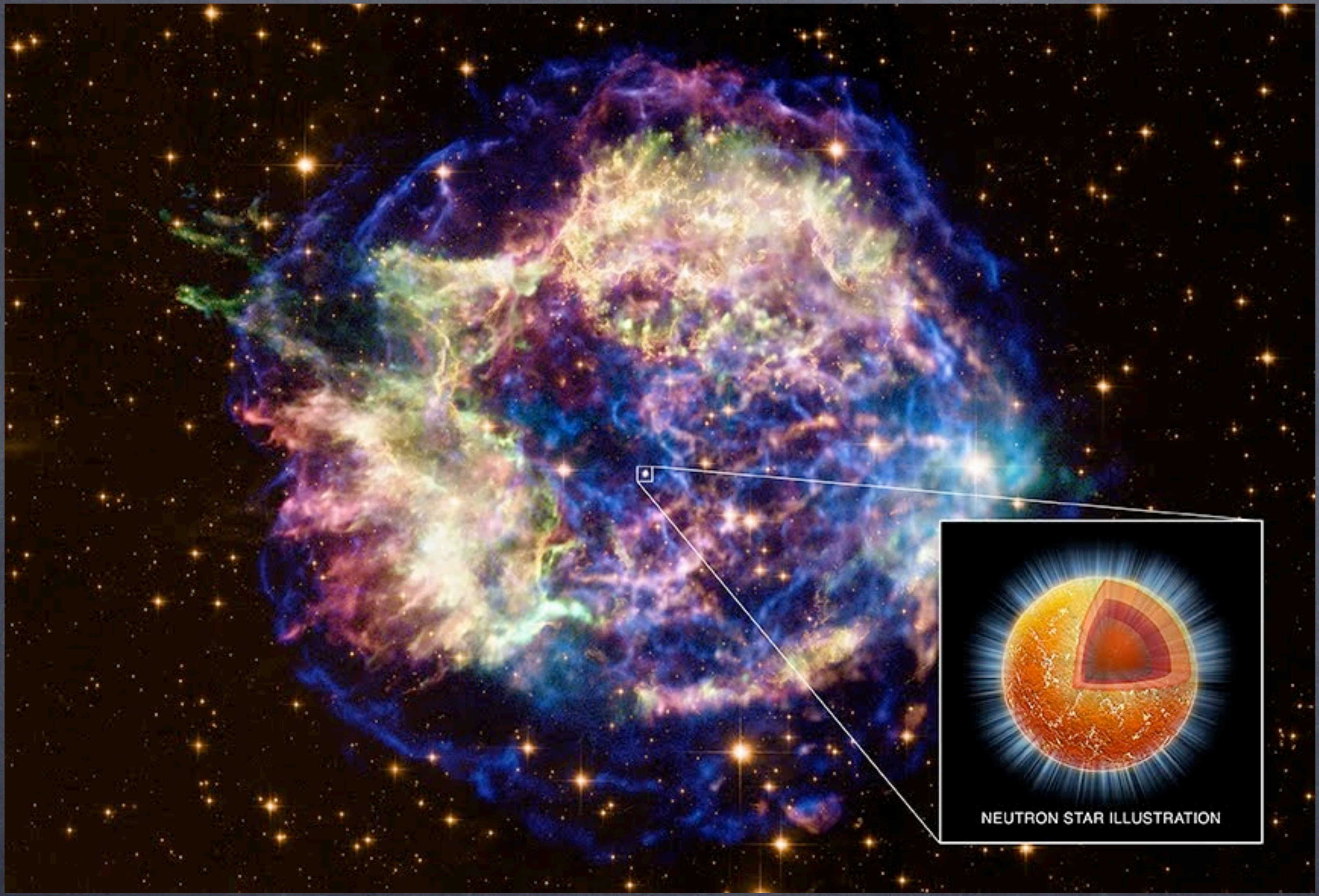
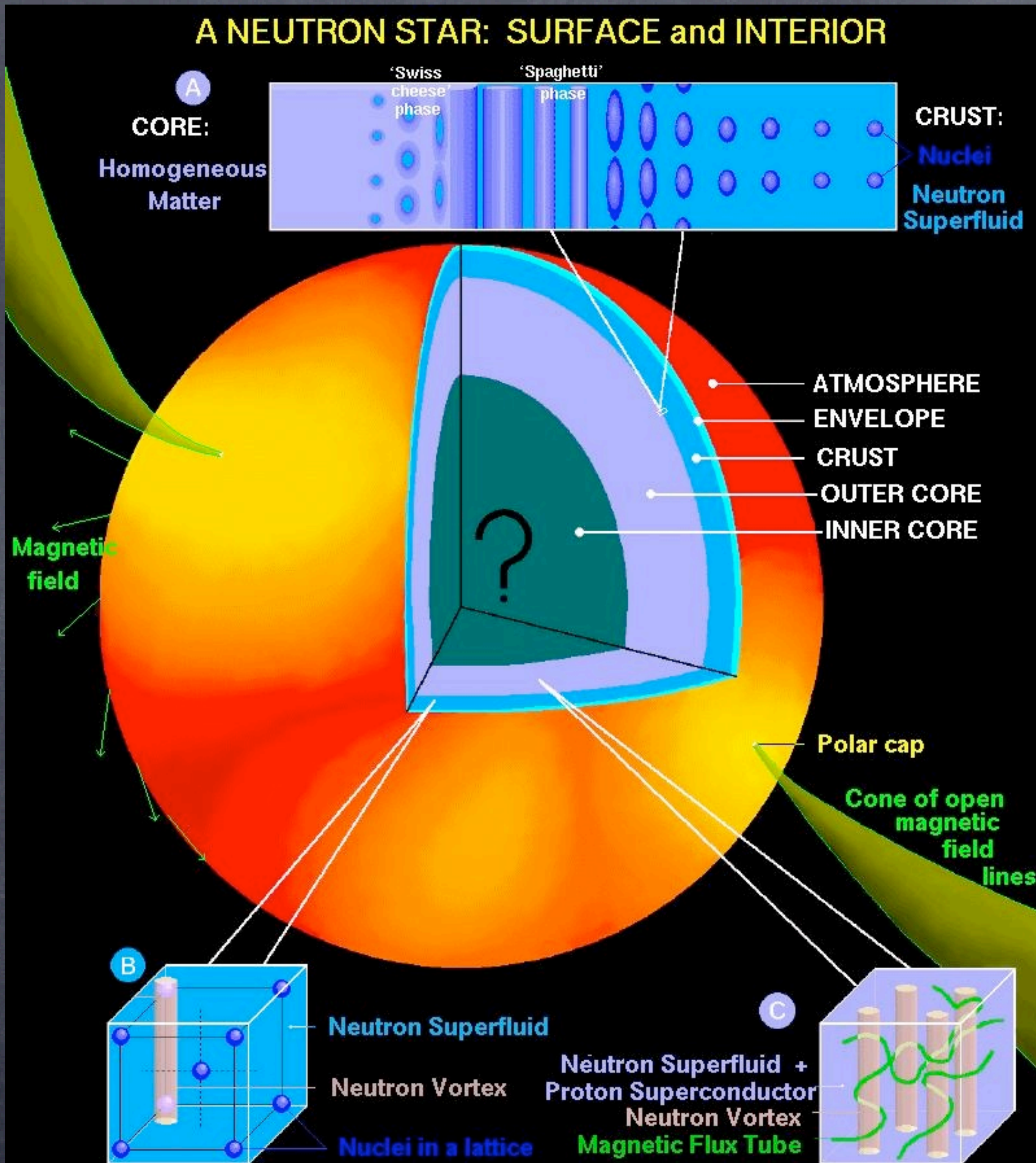
Cas A, brightest radio source



associated with SN 1680



consider simplified structure

Background

-- Neutron star cools by neutrino emissions

$$n \to p + e + \bar{\nu}, \quad p + e \to n + \nu,$$

$$n + n \to n + p + e + \bar{\nu}, \ldots$$

-- Recent (10 yrs) rapid (4%) cooling

Heinke and Ho, 2010

-- superfluidity ? Page et al., 2010

Cooper pairing $n + n \to \phi + \bar{\nu} + \nu$

fast cooling

Minimal cooling paradigm

- Cooling of hot neutron star by modified Urca
- $^3P_2$ Cooper pair breaking dominates near T_c
- At $T \ll T_c$, neutrons are gapped
- Goldstone modes become important

$^3P_2$ gap $\quad i\langle n^T \sigma_2 \sigma_l \overleftrightarrow{\nabla}_m n \rangle \neq 0$

breaks rotational symmetry

$$SU_S(2) \otimes SO_L(3) \times U(1) \xrightarrow{\text{spin-orbit}} SO_J(3) \times U(1)$$

$$\Big\downarrow \langle n^T \cdots n \rangle$$

discrete

3 rotational, 1 usual Nambu-Goldstone modes

Order parameter

$$\Sigma = U\Delta$$

$$U = e^{i2\phi/f_0}$$

$$\Delta = \xi\Delta_0\xi, \quad \xi = e^{i\boldsymbol{J}\cdot\boldsymbol{\pi}/f}$$

rescaled constants

Exact form of Δ_0 not known.

We choose a simple form

$$\Delta_0 = \overline{\Delta}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \end{pmatrix}$$

$$
\begin{aligned}
\mathcal{L} =& \frac{f_0^2}{8}\left[\partial_0 U \partial_0 U^\dagger - v_0^2 \partial_i U \partial_i U^\dagger\right] \\
&+ \frac{f^2}{8\overline{\Delta}^2}\left[\mathrm{Tr}[\partial_0 \Delta \partial_0 \Delta^\dagger] - v^2 \mathrm{Tr}[\partial_i \Delta \partial_i \Delta^\dagger] - w^2 \partial_i \Delta^\dagger_{ik} \partial_j \Delta_{kj}\right] \\
&+ i H_V Z_0^0 (U \partial_0 U^\dagger - \partial_0 U U^\dagger) \\
&+ i H_A Z_i^0 \mathrm{Tr}[J^i (\Delta \partial_0 \Delta^\dagger - \partial_0 \Delta \Delta^\dagger)] \; + \; \cdots ,
\end{aligned}
$$

Bedaque, Rupak, Savage 2003

Need to fix: $f_0, v_0, f, v, w, H_V, H_A$

Non-diagonal due to the "w" term.

Diagonalize $\pi_i(p) = K_{ia}(p)\alpha_a$

angulons

We match EFT to microscopic theory

$$\mathcal{L}_S^N = n^\dagger [i\partial_0 + \frac{\nabla^2}{2M^*} + \mu] n - g_{nn} R_{ijkl} (n^T \sigma_2 \sigma_k \overleftrightarrow{\nabla}_l\, n)^\dagger (n^T \sigma_2 \sigma_i \overleftrightarrow{\nabla}_j\, n) \ ,$$

and

$$\mathcal{L}_W^N = C_V Z_0^0\ n^\dagger n + C_A Z_i^0\ n^\dagger \sigma^i n - g_{Z\overline{\nu}\nu}\ Z_\mu^0\ \overline{\nu} \gamma^\mu (1-\gamma_5) \nu,$$

with

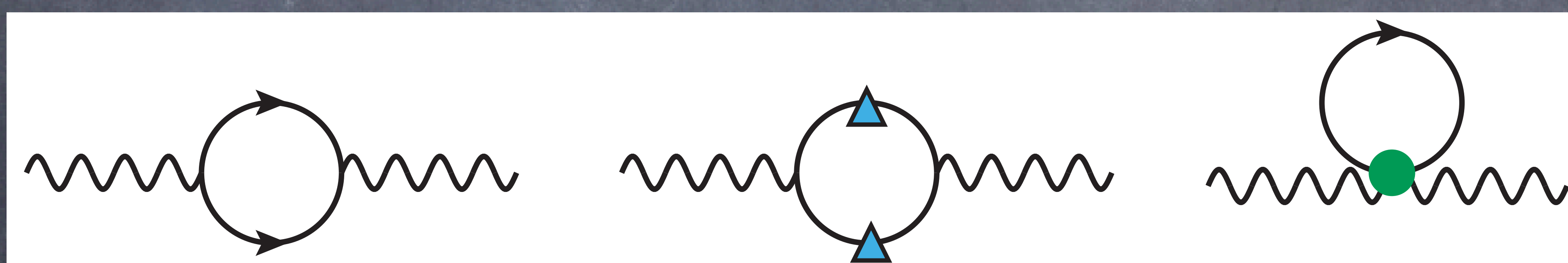
$$C_V^2 = \frac{G_F M_Z^2}{2\sqrt{2}} = g_{Z\overline{\nu}\nu}^2,$$

$$C_A^2 = (g_A + \Delta_s)^2 C_V^2$$

Match fictitious currents

$$\partial_\mu n \to D_\mu n = \partial_\mu n + iA_\mu n, \quad \partial_\mu U \to D_\mu U = \partial_\mu U + 2iA_\mu U$$

density of states



$$f_0^2 = \frac{Mk_F}{\pi^2}\,, \quad v_0^2 \;=\; \frac{v_F^2}{3}$$

well known, Son 2002

Fictitious gauge spin rotation $\quad n \to e^{i\frac{\boldsymbol{\sigma}}{2}.\boldsymbol{\theta}} n$

$$D_\mu n = \partial_\mu n + i B^i_\mu \frac{\sigma^i}{2} n \;\; , \;\; D_\mu \Delta = \partial_\mu \Delta + i B^i_\mu J^i \Delta$$

$$f^2 \;=\; \frac{M k_F}{3\pi^2} \;\; , \;\; v^2 \;=\frac{1}{5} v_F^2 \;\; , \;\; w^2 \;=\frac{1}{5} v_F^2$$

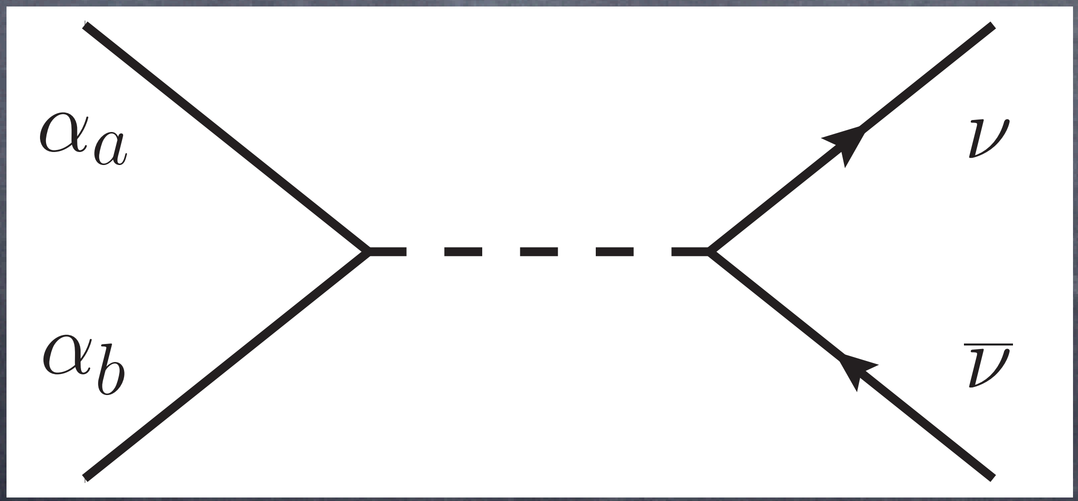
From fictitious field couplings

$$H_V \;=\; -\frac{f_0^2}{4}\, C_V \;\; , \;\; H_A \;=-\frac{f^2}{4\bar{\Delta}^2}\, C_A$$

Neutrino processes

$$\mathcal{L}_{Z^0} = -f_0 C_V Z_0^0 \partial_0 \phi - f C_A \sqrt{\frac{3}{2}} Z_i^0 K_{ia} \partial_0 \alpha_a$$
$$+ \frac{C_A}{2} Z_k^0 \epsilon^{ijk} K_{ia}(p) K_{jb}(k) \alpha_a(p) \partial_0 \alpha_b(k) + \cdots$$

Angulon annihilation dominates $\alpha_a \alpha_b \rightarrow \overline{\nu}\nu$



Emissivity

$$\mathcal{E}_{ab} = \int d\Gamma_{p,k;p_\nu,p_{\overline{\nu}}} \; n[E_a(p)] \; n[E_b(k)] \; (E_\nu + E_{\overline{\nu}}) \sum_{s,s'} |\mathcal{M}_{ab}|^2$$

$$(2\pi)^4 \; \delta^{(4)}(p + k - p_\nu - p_{\overline{\nu}})$$

$$\sim 10^{17} \; T_9^9 \left(\frac{0.15}{v}\right)^3 \; \mathrm{erg\ cm}^{-3} \; \mathrm{s}^{-\mathrm{s}}$$

Whats realistic for v, T? At $T_9 \sim 0.3$

Electron bremsstrahlung $\mathcal{E}_e \sim 10^{10} \; \mathrm{erg\ cm}^{-3} \; \mathrm{s}^{-1}$

Angulons annihilation $\mathcal{E}_{\alpha\alpha} \sim 10^{12} \; \mathrm{erg\ cm}^{-3} \; \mathrm{s}^{-1}$

Luminosity

-- for k_F = 300 MeV, M_n = 940 MeV

$$v = \frac{1}{\sqrt{5}} \frac{k_F}{M_n} \sim 0.15$$

$$L_\nu^{(\alpha)} \sim \frac{4\pi}{3} R^3 \mathcal{E} \sim 4 \times 10^{35}\ T_9^9 \left(\frac{0.15}{v}\right)^3 \ \mathrm{erg} \cdot \mathrm{s}^{-1}$$

Heat capacity

$$C_V^{(\alpha)} = \frac{4\pi^2 k_B^4 V}{15(\hbar c)^3} T^3 \sim 1.3 \times 10^{32}\ R_{10}^3 T_9^3\ \mathrm{erg} \cdot \mathrm{K}^{-1}$$

Electrons $L_\nu^{(e)} \sim 10^{40}\ T_9^8\ \frac{\mathrm{erg}}{\mathrm{s}}, \quad C_V^{(e)} \sim 10^{40}\ T_9\ \mathrm{erg} \cdot \mathrm{K}^{-1}$

dominate

Page, Geppert, Weber 2006

Luminosity using polytrope

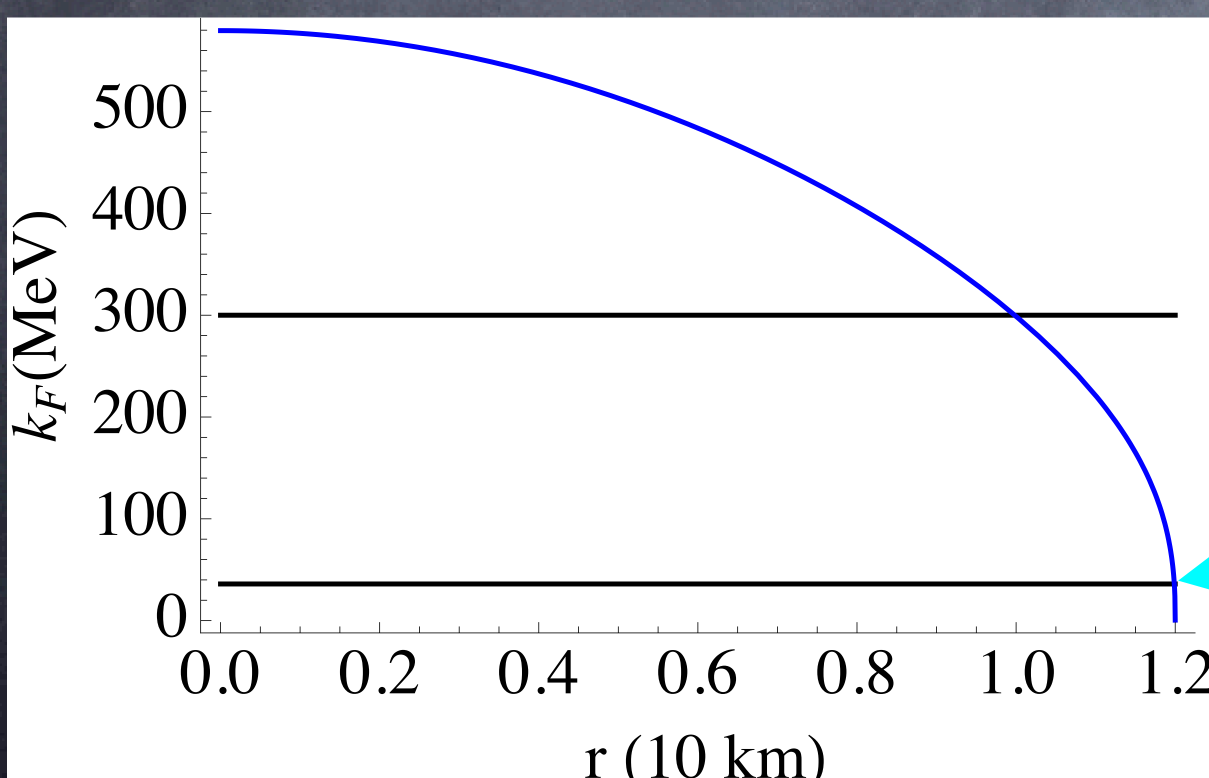
-- $P(r) = K\ [\rho(r)]^{1+1/n}$

-- $\Delta \sim \frac{3.52}{2}\ T_c \sim 0.08\ \text{MeV}$

-- $k_F \gg \sqrt{2M_n\Delta} \sim 12\ \text{MeV}$

Page et al., 2010

-- polytrope n=1

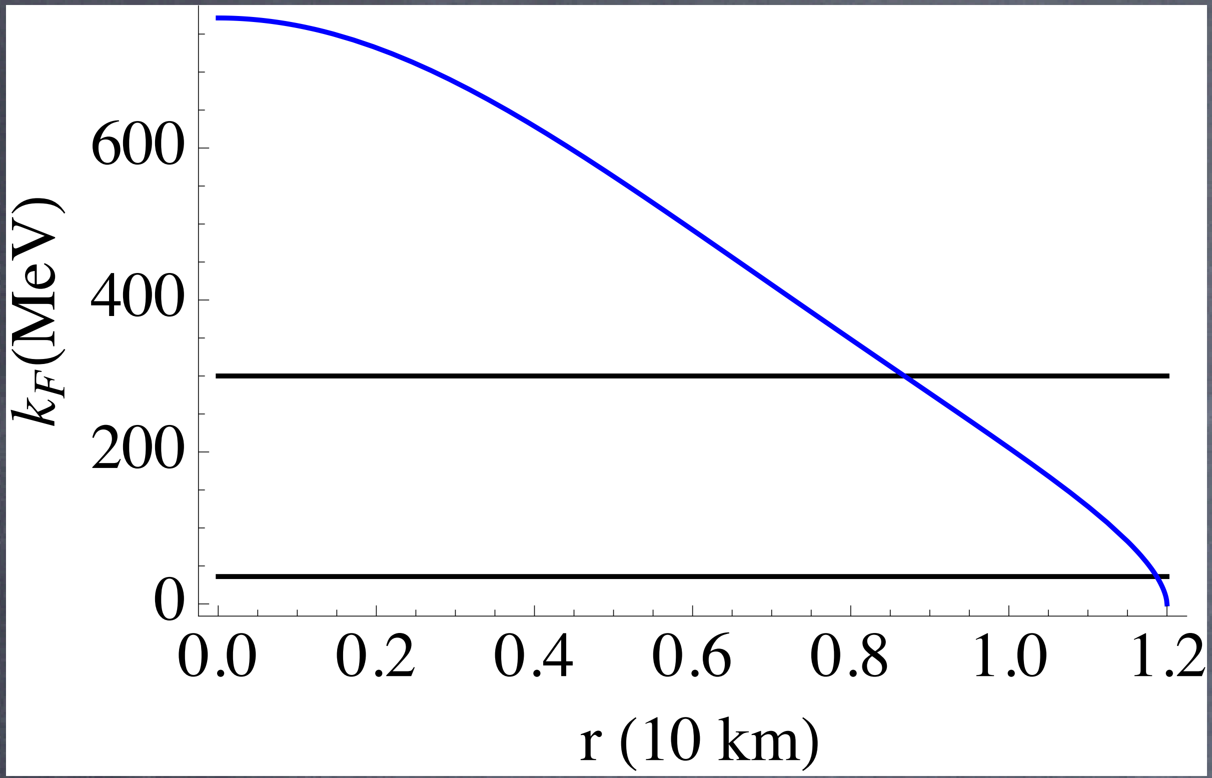


$$L_\nu = 4\pi \int_0^{r_0} dr r^2 \mathcal{E}_{\alpha\alpha}$$

$$\sim 6 \times 4 \times 10^{35}\ T_9^9\ \frac{\text{erg}}{\text{s}}$$

$$r_0 = r(k_F = 36\ \text{MeV})$$

-- polytrope n=2



$$L_\nu = 4\pi \int_0^{r_0} dr r^2 \mathcal{E}_{\alpha\alpha}$$

$$\sim 22 \times 4 \times 10^{35}\ T_9^9\ \frac{\text{erg}}{\text{s}}$$

Reasonable $\quad L_\nu \sim (0.1 - 10) \times 4 \times 10^{35}\ T_9^9\ \frac{\text{erg}}{\text{s}}$

Cooling

$$C\frac{dT}{dt} = -L_\nu$$

-- electrons dominated

$$\frac{dT_9}{dt} = -10^{-9}\frac{L_9^{(e)}}{C_9^{(e)}}T_9^7 = -10^{-9}\ T_9^7 \sim 10^{-14}\ \mathrm{s}^{-1}$$

-- angulons

$$\frac{dT_9}{dt} = -10^{-9}\frac{L_9^{(\alpha)}}{C_9^{(\alpha)}}T_9^6 \approx -10^{-9}(0.1-10)\frac{3\times10^3}{R_{10}^3}T_9^6$$

$$\sim (10^{-11} - 10^{-9})\ \mathrm{s}^{-1}$$

comparison only

Used T_9 = 0.2, R_{10} = 1.2

Shut off neutrons when T = 0.5 T_c

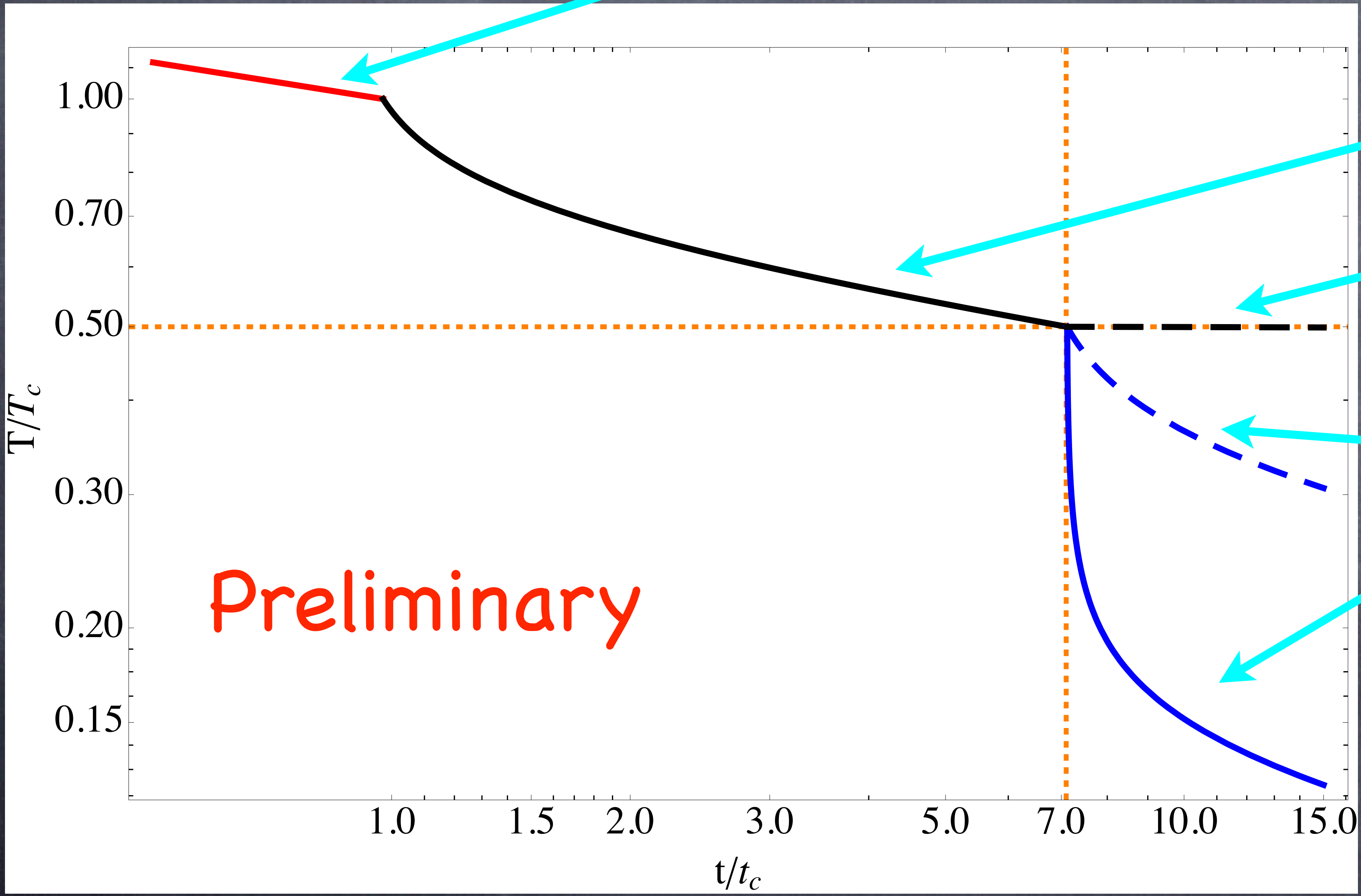
$T_c \sim 0.5\ T_9,\ \ t_c \sim 330$ yrs

modified Urca

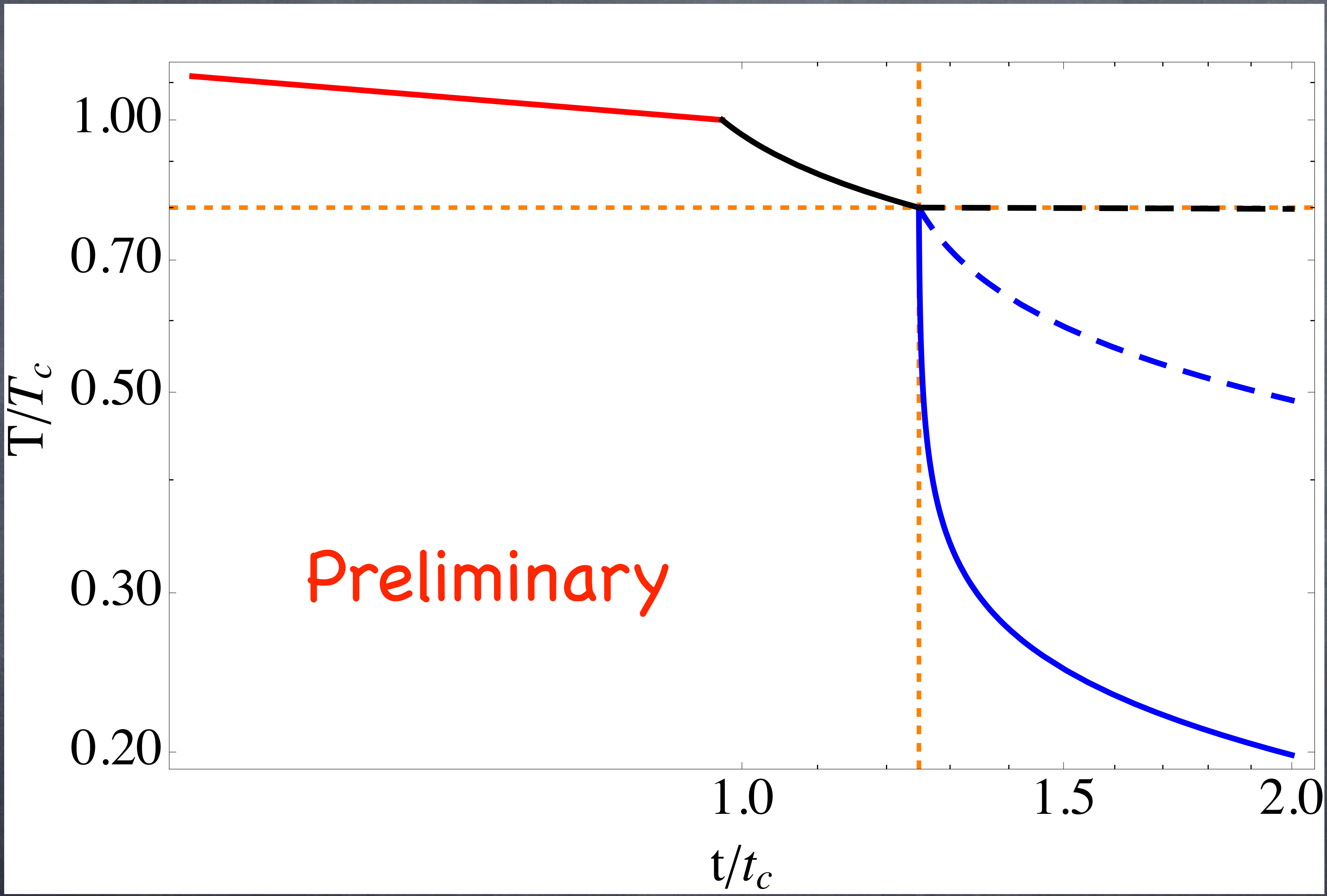
pair breaking

electrons

angulons



Shut off neutrons when T = 0.8 T_c



Summary

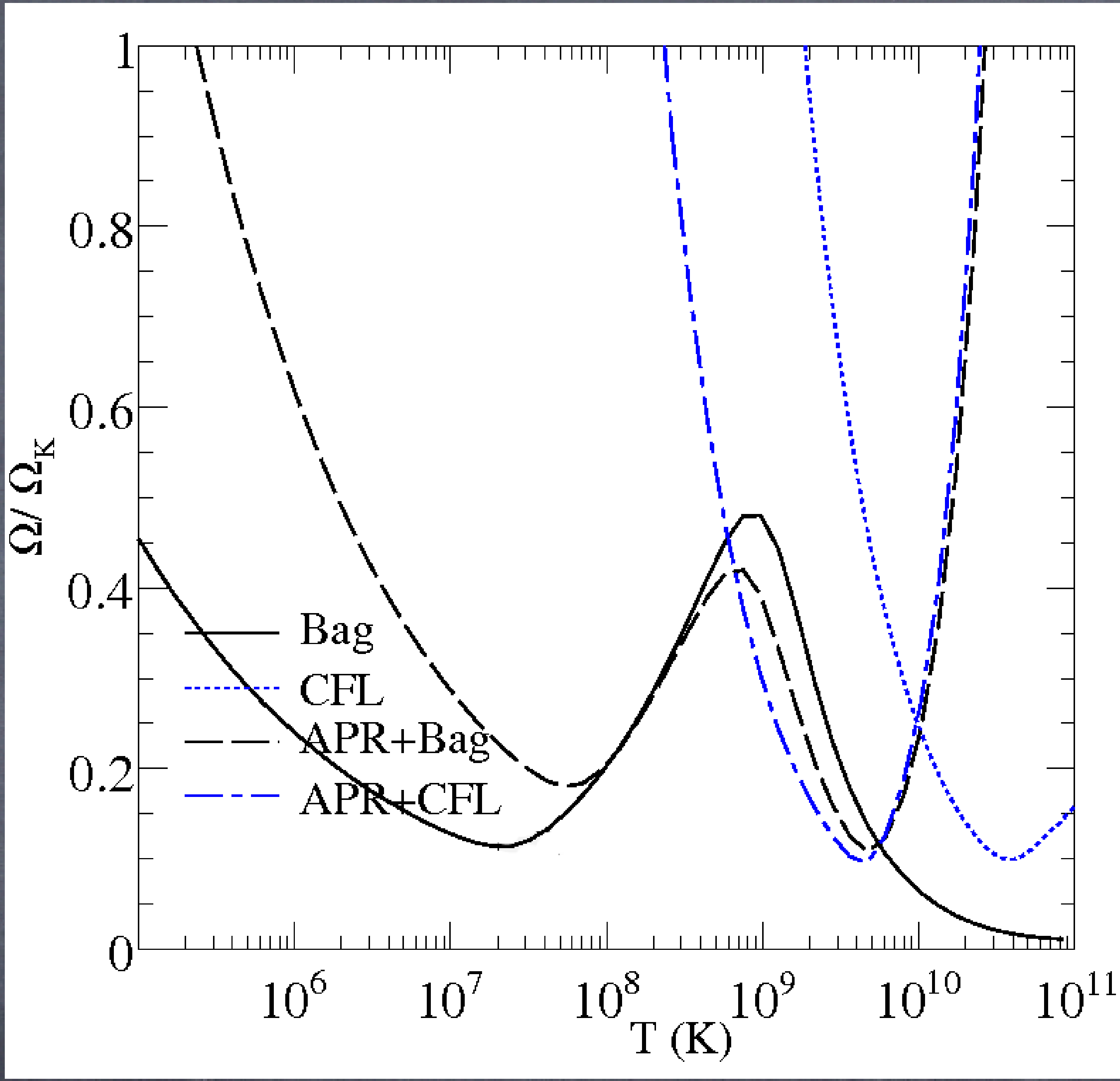
- Theory for neutrino emissivity in $^3P_2$ superfluid
- Truly model-independent calculation lacking
- No apparent angulon signature in cooling
- r-mode damping Yang et al., 2011

r-mode heat source

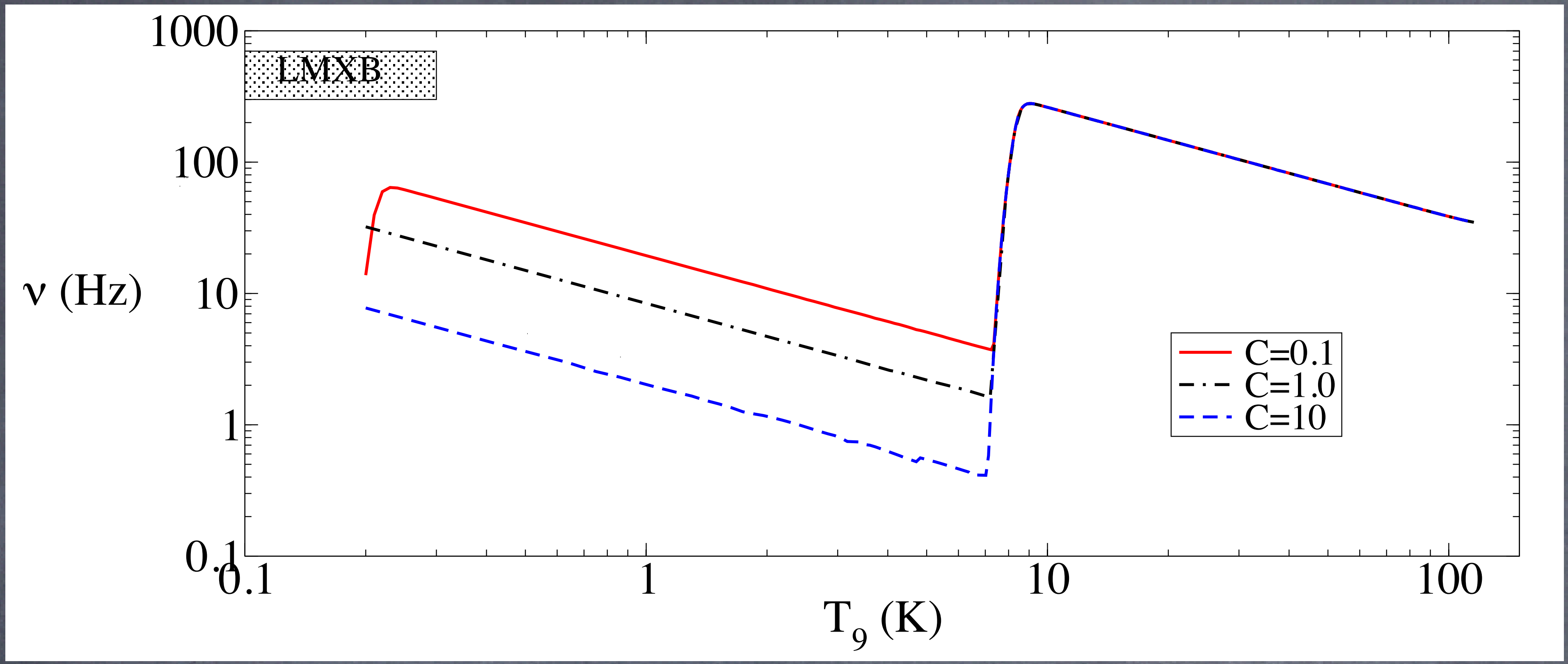
Rossby waves

$$C\frac{dT}{dt} = -L + H,$$

$$H \sim E_r/\tau_{\mathrm{damping}}$$



Jaikumar, Rupak, Steiner, 2008



Rupak, Jaikumar, 2010

Backup

$$\mathcal{M}_{ab} = \frac{G_F \tilde{C}_A}{4\sqrt{2}} \; [H_{abk} E_b(k) + H_{bak} E_a(p)] \; \overline{\nu} \gamma^k (1 - \gamma_5) \nu$$

$$H_{abk} = \sum_{i,j} \varepsilon_{ijk} K_{ia}(p) K_{jb}(p)$$

(no sum implied),