Pairing Pseudogap in Strongly Interacting Fermi Gases

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Pseudogap in Strongly Interacting Fermi Gases

- definition
- significance
- signatures
- status: theory & experiment
- implications for other fields

Recent review:
M. Randeria, W. Zwerger & M. Zwierlein,
"BCS-BEC Crossover and Unitary Fermi Gas"
Chapter in book edited by W. Zwerger (Springer, 2011)

Pseudogap:

Suppression of low-energy spectral weight in the normal state (T > Tc)

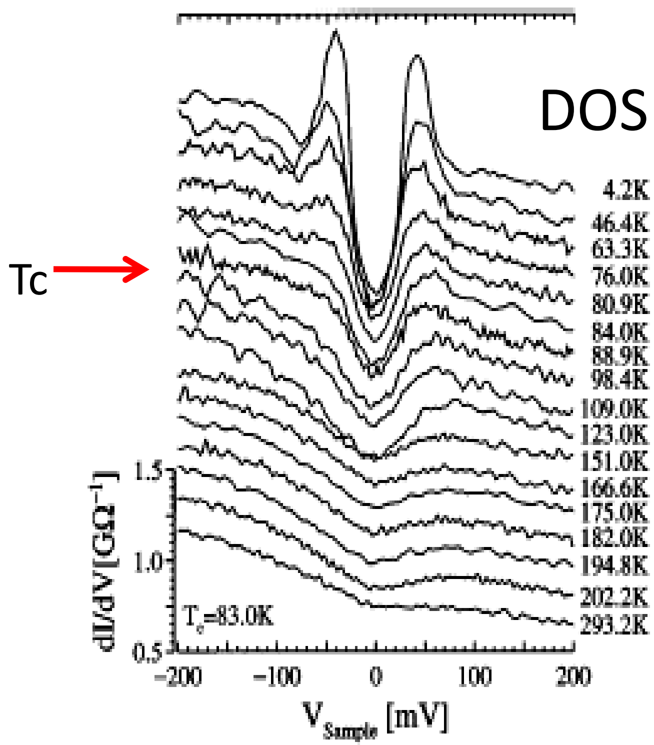
This terminology originates from papers like:

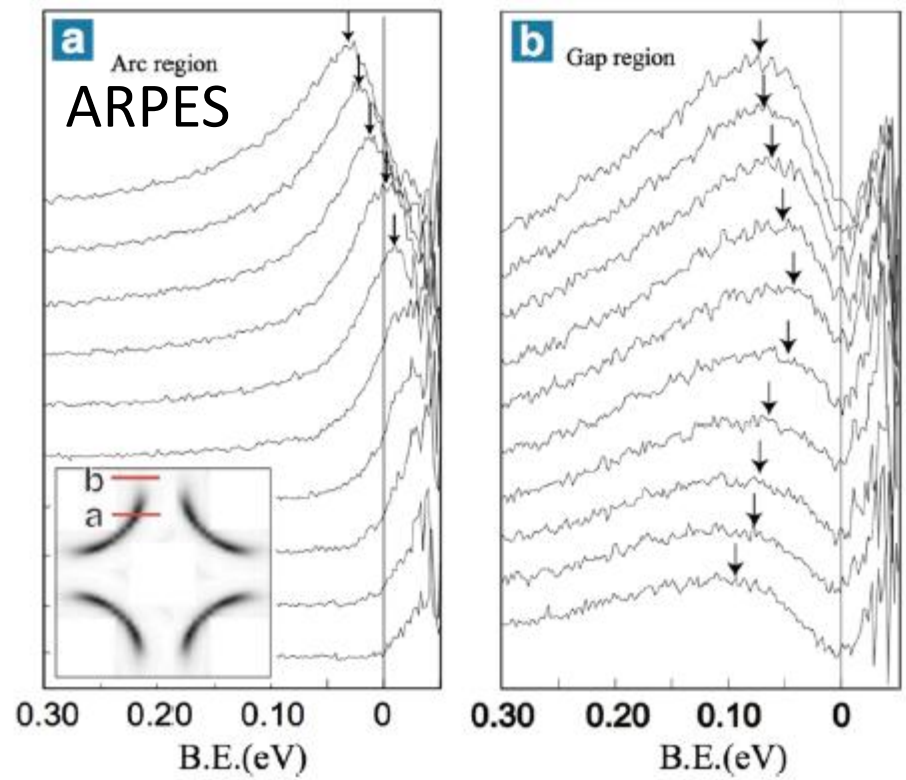
Ding *et al*, "Spectroscopic evidence for a pseudogap in the normal state of underdoped high-Tc superconductors" Nature 382, 51 (1996)

Loeser *et al.* "Excitation gap in the normal state of underdoped Bi2212" Science 273, 325 (1996)

High Tc Pseudogap: A first look

T = 140K > Tc = 90K





Renner et al, PRL (1998)

Kanigel, Chatterjee, MR et al, PRL (2008)

Physics of HighTc pseudogap is much more complicated than the problem of our interest here (see later)

Pairing Pseudogap:

Suppression of low-energy spectral weight
due to incoherent pairing
in the normal state (T > Tc)

- single-particle spectral function
- single-particle density of states
- spectral weight for singlet to triplet transitions

- "hard" gap = zero spectral weight
- pseudogap = suppression, not necessarily zero

The "Pairing pseudogap" question related to two important themes in condensed matter physics:

- What drives the SC transition: amplitude or phase?
  - * amplitude → BCS mean field theory
  - * phase fluctuations → breakdown of MF description

- Are there sharp, gapless quasiparticles in a normal Fermi system?
  - * yes → Landau's Fermi liquid theory
  - * no → breakdown of Fermi liquid paradigm

Amplitude v/s Phase

- Conventional BCS SCs:
  phase transition by gap (amplitude) collapse

$$2\Delta = 3.5 k_B T_c$$

$$\Delta \sim T_c \ll \rho_s \sim \epsilon_f$$

ρ_s = superfluid stiffness

- Many interesting SC systems exhibit

$$T_c \sim \rho_s \ll \Delta$$

Transition driven by fluctuations of the phase
-- underdoped High Tc cuprates
Uemura (1989), Emery, Kivelson (1995),
Broun et al, PRL (2007)
Hetel, Lemberger & MR, Nature Phys. (2007)
-- SC-Insulator transitions
Bouadim, Loh, MR, & Trivedi, Nature Phys. (2011)
Sacape et al, Nature Phys & Nature Comm (2011)

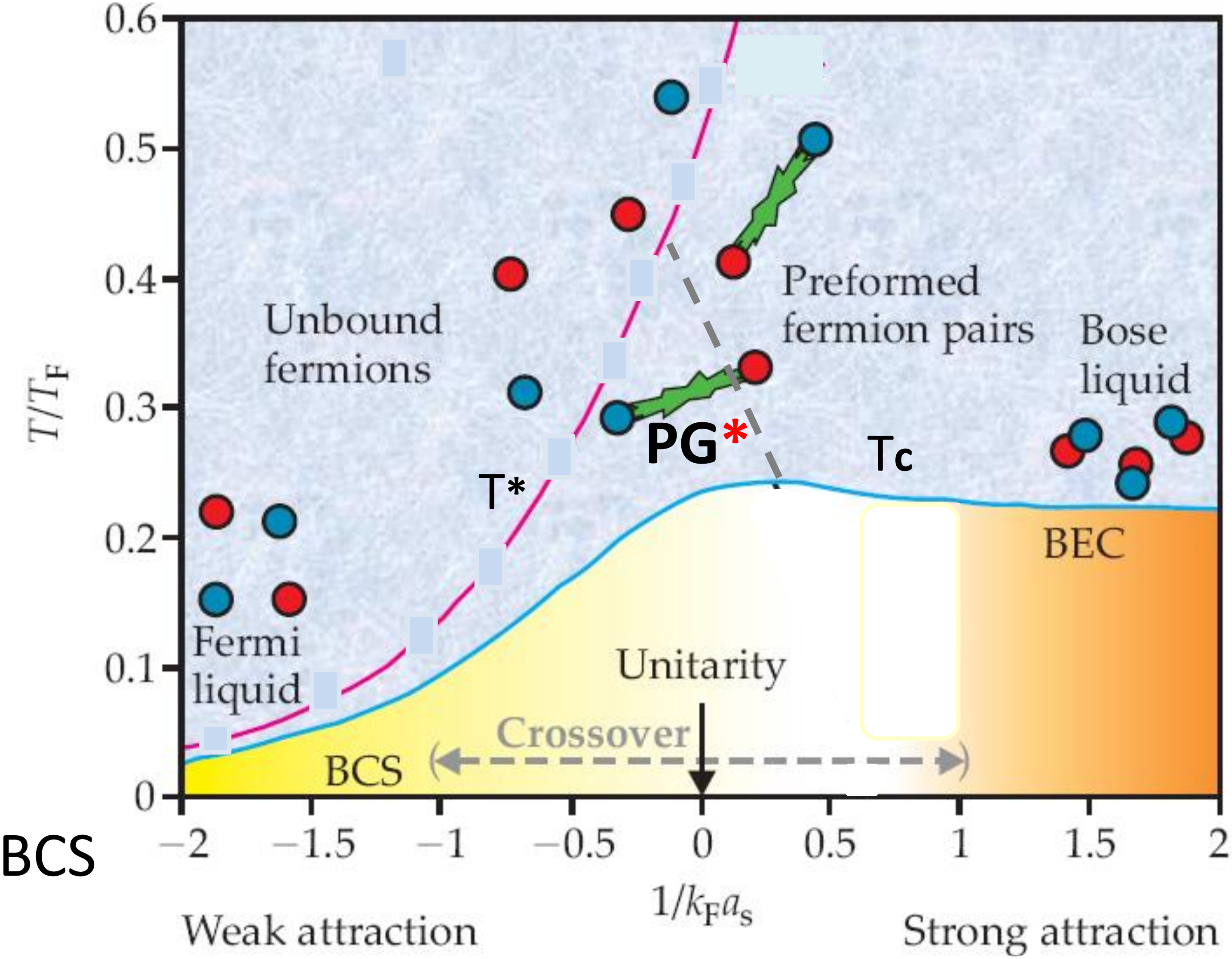
Gap will Survive Above Tc

Prehistory

Does the normal state of a short coherence length superconductor show deviations from Landau Fermi liquid description?

* MR, Trivedi, Moreo & Scalettar, PRL 69, 2001 (1992)
* Trivedi & MR, PRL 75, 381 (1995)
* MR, "Precursor Pairing Correlations and Pseudogaps" Varenna Lectures (1997) cond-mat/9710223

BCS-BEC crossover



BCS

BEC

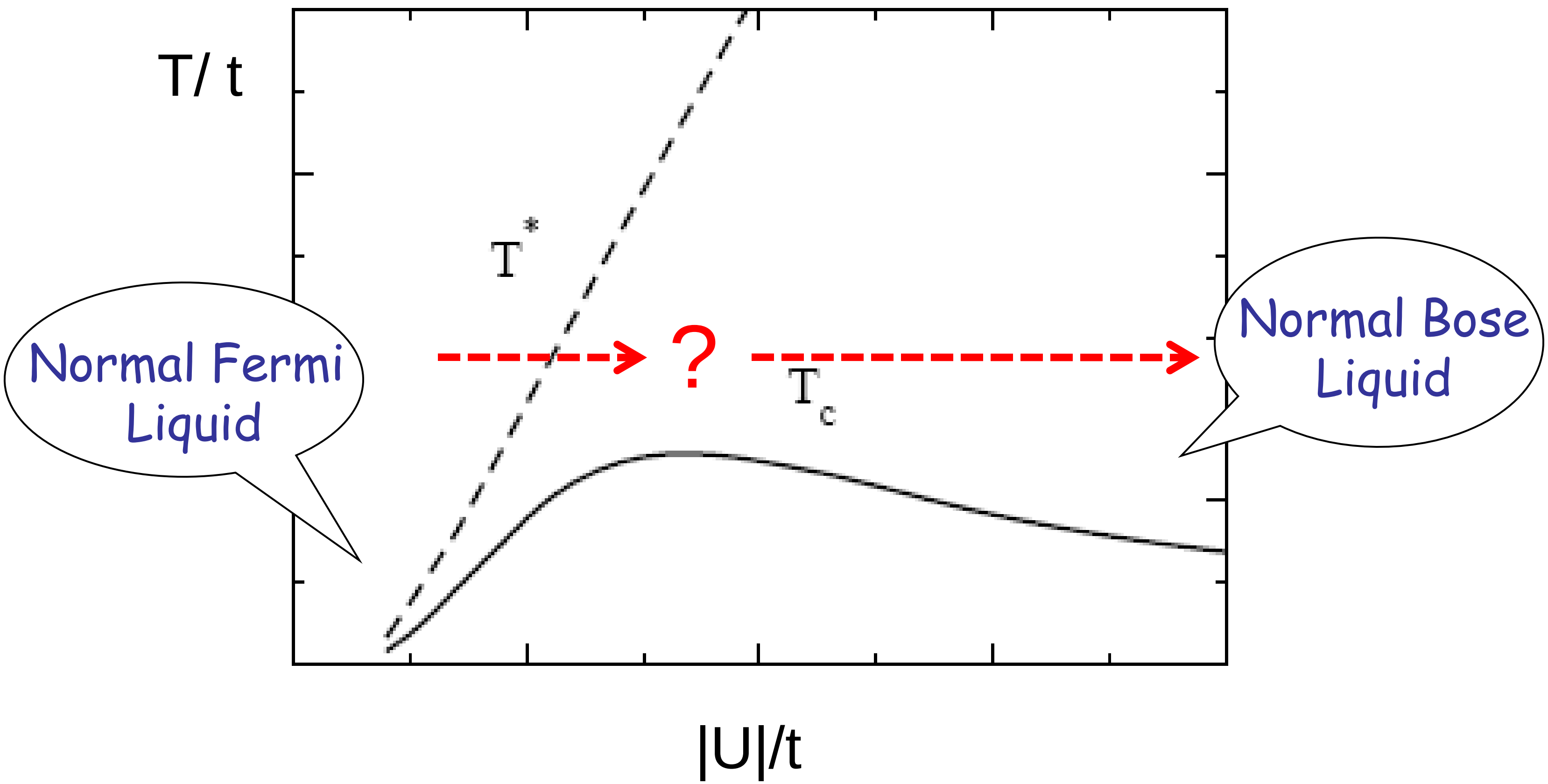
* Pairing pseudogap Breakdown of Fermi Liquid Description

Note:

$T \not\ll T_F$

Based on Sa de Melo, MR & Engelbrecht, PRL (1993)

from: Sa de Melo, Phys.Today (Oct. 2008)



Pairing Pseudogap in
2D Lattice model of BCS-BEC crossover

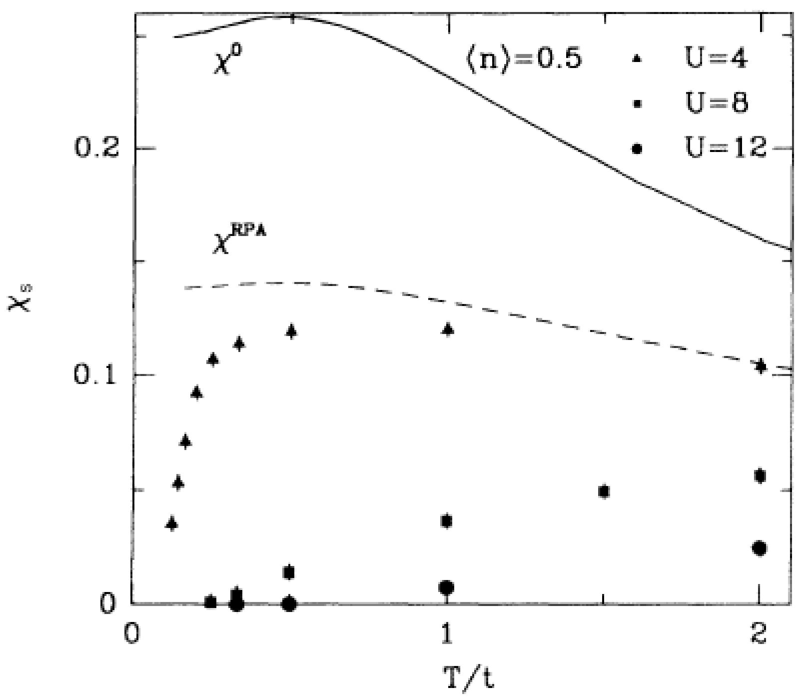
- 2D simpler for QMC
- 2D enhances pseudogap effect
  -- fluctuations lower Tc
- no fermion sign problem (Hirsch)

PRL (1992)
PRL (1995)

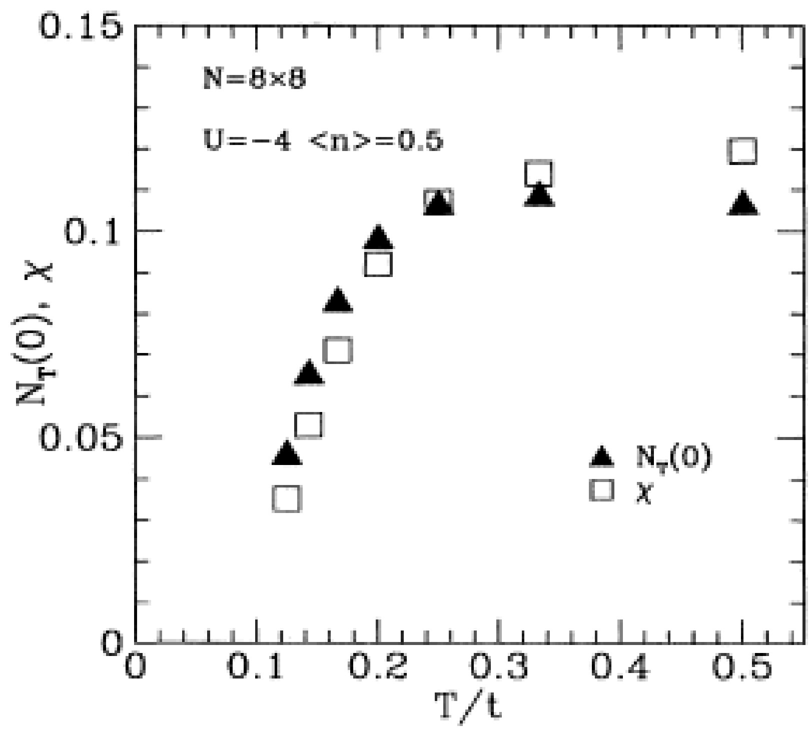
Pairing Pseudogap in 2D Attractive Hubbard model

Spin susceptibility & single-particle density of states

$\chi_s(T)$



$\chi_s(T) \sim N(0;T)$



PRL 69, 2001 (1992)
PRL 75, 381 (1995)

Pairing Pseudogap

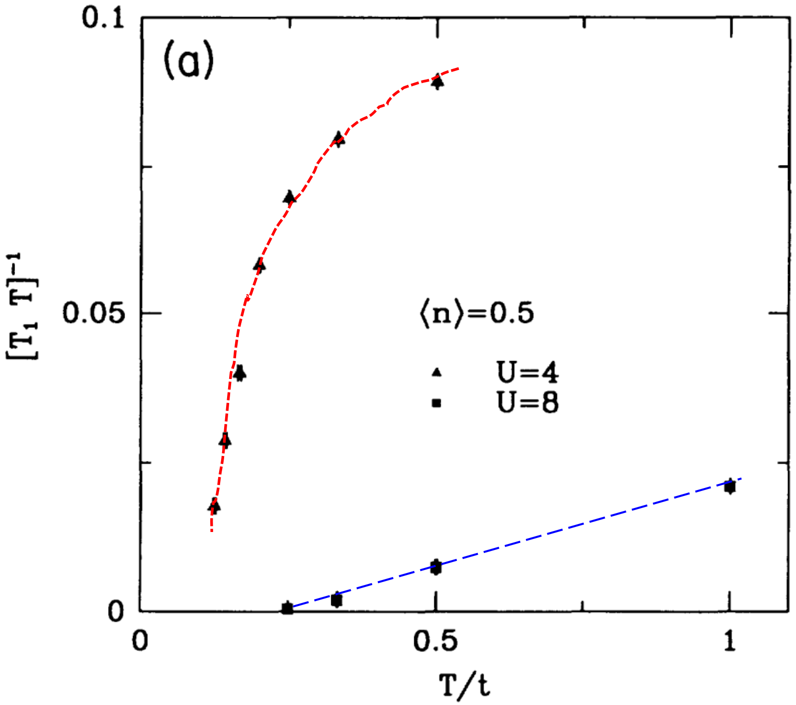
Dynamic Spin Susceptibility

$$\frac{1}{T_1 T} = \lim_{\omega \to 0} \sum_{\mathbf{q}} \frac{\chi''(\mathbf{q},\omega)}{\omega}$$

$$\chi'' = \mathrm{Im}\,\chi_{\uparrow,\downarrow}(q,\omega)$$

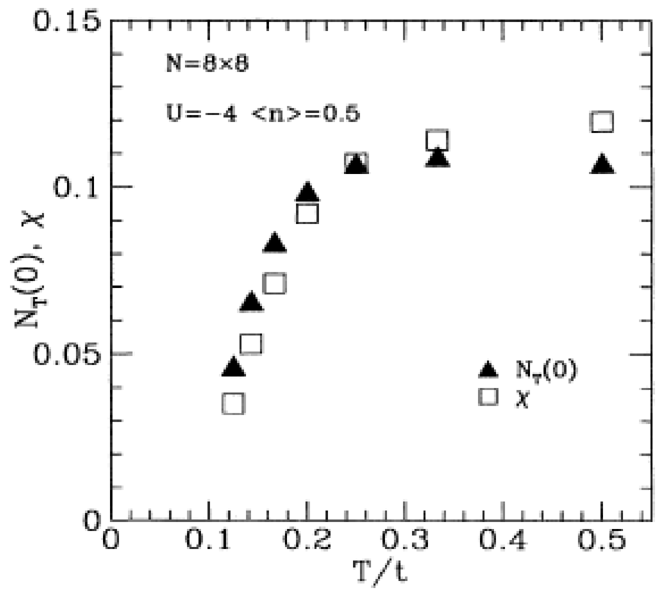
↓

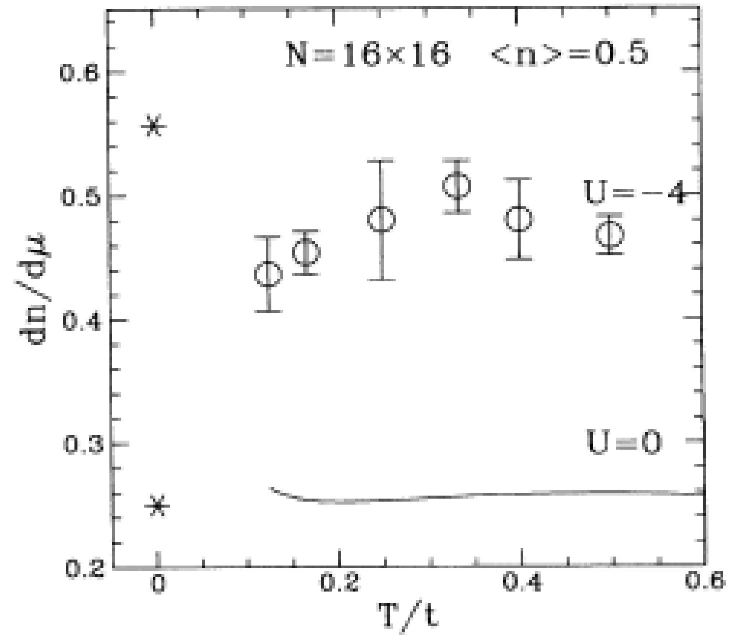
"spin" Bragg in cold atoms



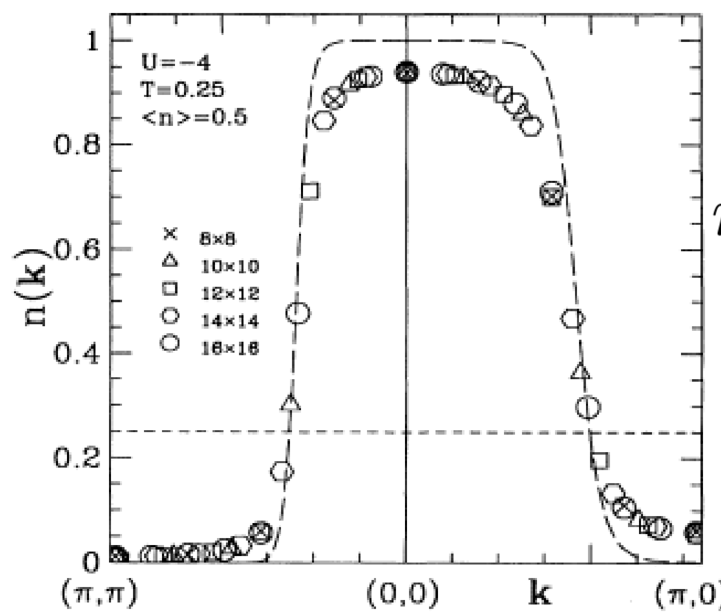
PRL 69, 2001 (1992)
PRL 75, 381 (1995)

- Pseudogap in $\chi_s(T)$ & $N(0)$
- No signature in $dn/d\mu$
- Fermi degeneracy: $n(\mathbf{k})$





$$\frac{dn}{d\mu}$$



$$n(\mathbf{k})$$

PRL 69, 2001 (1992)
PRL 75, 381 (1995)

Fast-forward to the present...

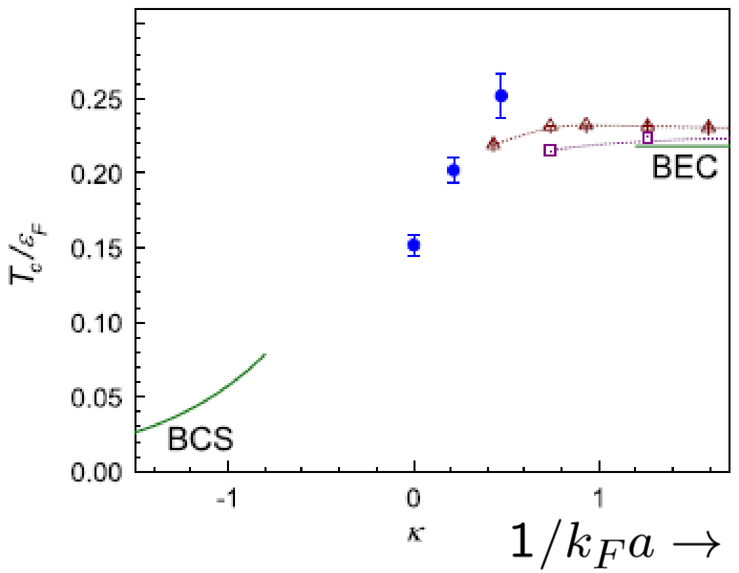
Pairing pseudogap in the normal state of the strongly interacting Fermi gas?

- How large is the effect in various observables?
- How large is the temperature regime? $T_c < T < T^*$
- How large is the Interaction regime? $1/k_F a = 0$

→ Small positive values?

Quantum Monte Carlo Estimate of Tc

$$T_c \simeq 0.15 - 0.20 T_F$$



Burovski et al, PRL (2008)

→Cannot discuss the question of "Fermi liquid or not" in asymptotic low T regime $T \ll T_F$

Fermion Spectral function:

$$A(\mathbf{k},\omega) = \frac{1}{\pi}\left|\mathrm{Im}G(\mathbf{k},\omega + i0^{+})\right|$$

= Probability density of making a single-fermion excitation with momentum **k** at an energy ω

Green's function:

$$G(\mathbf{k},\omega) = \frac{1}{\omega - \frac{k^2}{2m} + \mu - \Sigma(\mathbf{k},\omega)}$$

Self Energy:

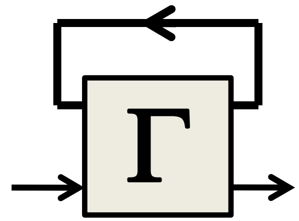
$$\Sigma(\mathbf{k},\omega) = \Sigma'(\mathbf{k},\omega) + i\Sigma''(\mathbf{k},\omega)$$

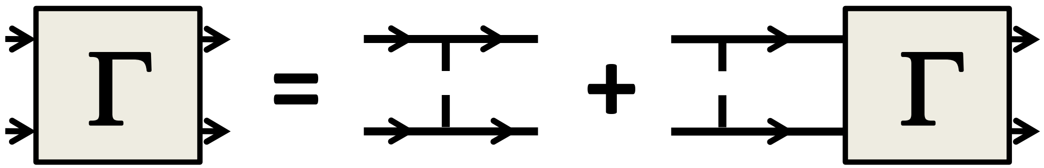
Σ': renormalized dispersion

Σ'': scattering rate ~ 1/(lifetime)

Diagrammatic approximations

Pair Fluctuations





- No small parameter
- No organizing principle;
  other than conservation laws,
  sum rules, limiting cases,
  and comparison with expt & QMC
- e.g., ignores Gorkov-MB diagrams

pseudogap in A(**k**,ω)

- Strinati: G_0G_0
  PRB (2002), PRL (2011)
- Ohashi: G_0G_0
  PRA (2010)
- Levin: GG_0
  PRL (2009)
  Rep. Prog. Phys. (2009)
- Zwerger & Haussmann: GG
  PRA (2010)

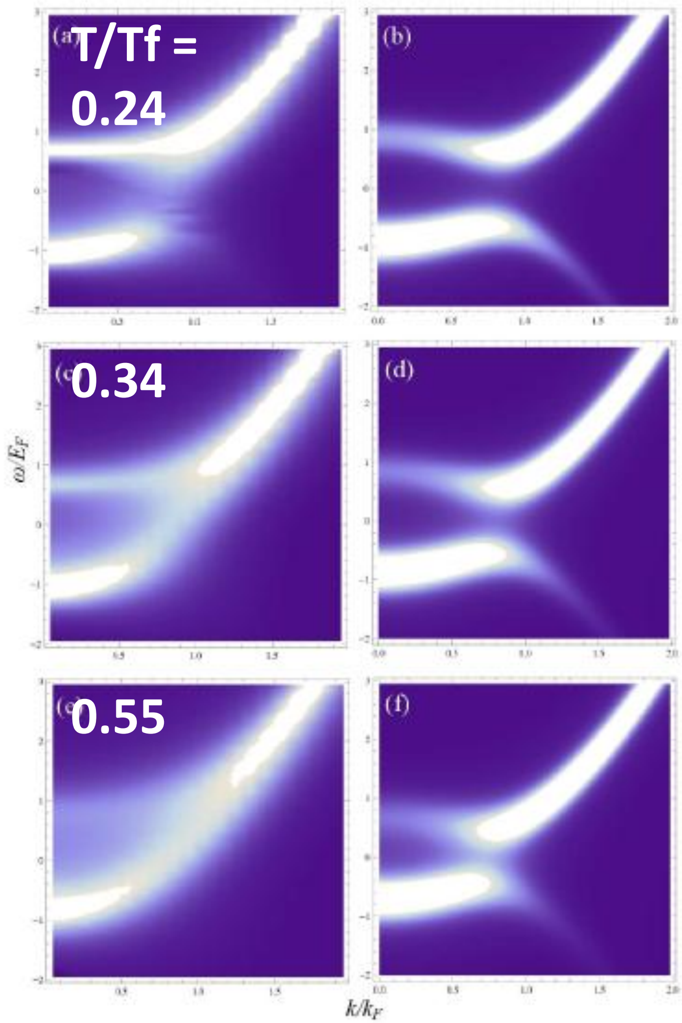
Virial expansion

Hu et al PRL, PRA (2010)

only valid for fugacity

z < 1 $(\mu < 0)$

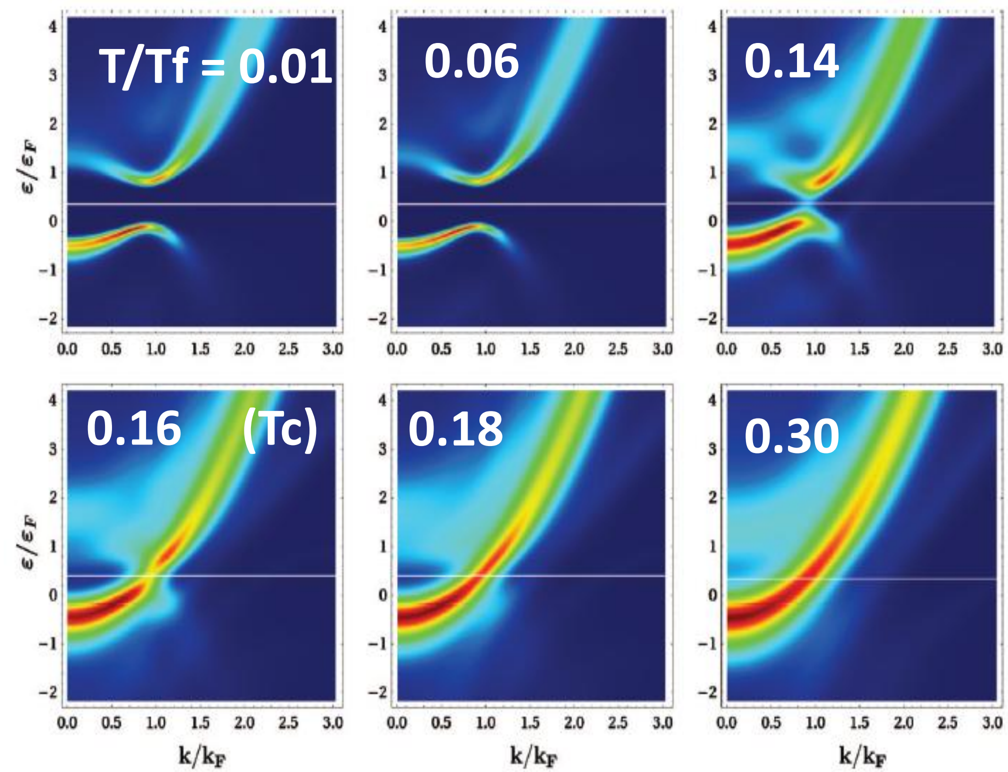
Pair Fluctuation Approximations $A(\mathbf{k},\omega)$ $1/k_F a = 0$



G0G0
→ pseudogap
But quantitative differences

GG0

Chien et al, PRA 81, 023622 (2010)



GG → No pseudogap

Haussmann et al, PRA 80, 063612 (2009)

○ Analytic continuation: $\left.\begin{array}{c}\text{imaginary } \tau \\ \text{or } i(2n+1)\pi/\beta\end{array}\right\} \to \text{real } \omega$

Technical challenge for <u>all</u> calculations:
diagrammatic approximations and Quantum Monte Carlo

$$G(\mathbf{k},\tau) = -\int_{-\infty}^{\infty} d\omega \, \frac{\exp(-\omega\tau)}{1+\exp(-\beta\omega)} \, \mathcal{A}(\mathbf{k},\omega)$$

Inverting the Laplace transform is an ill-posed problem

* Maximum entropy
* Pade approximants

…

$A(\mathbf{k},\omega)$ $1/k_F a = 0$

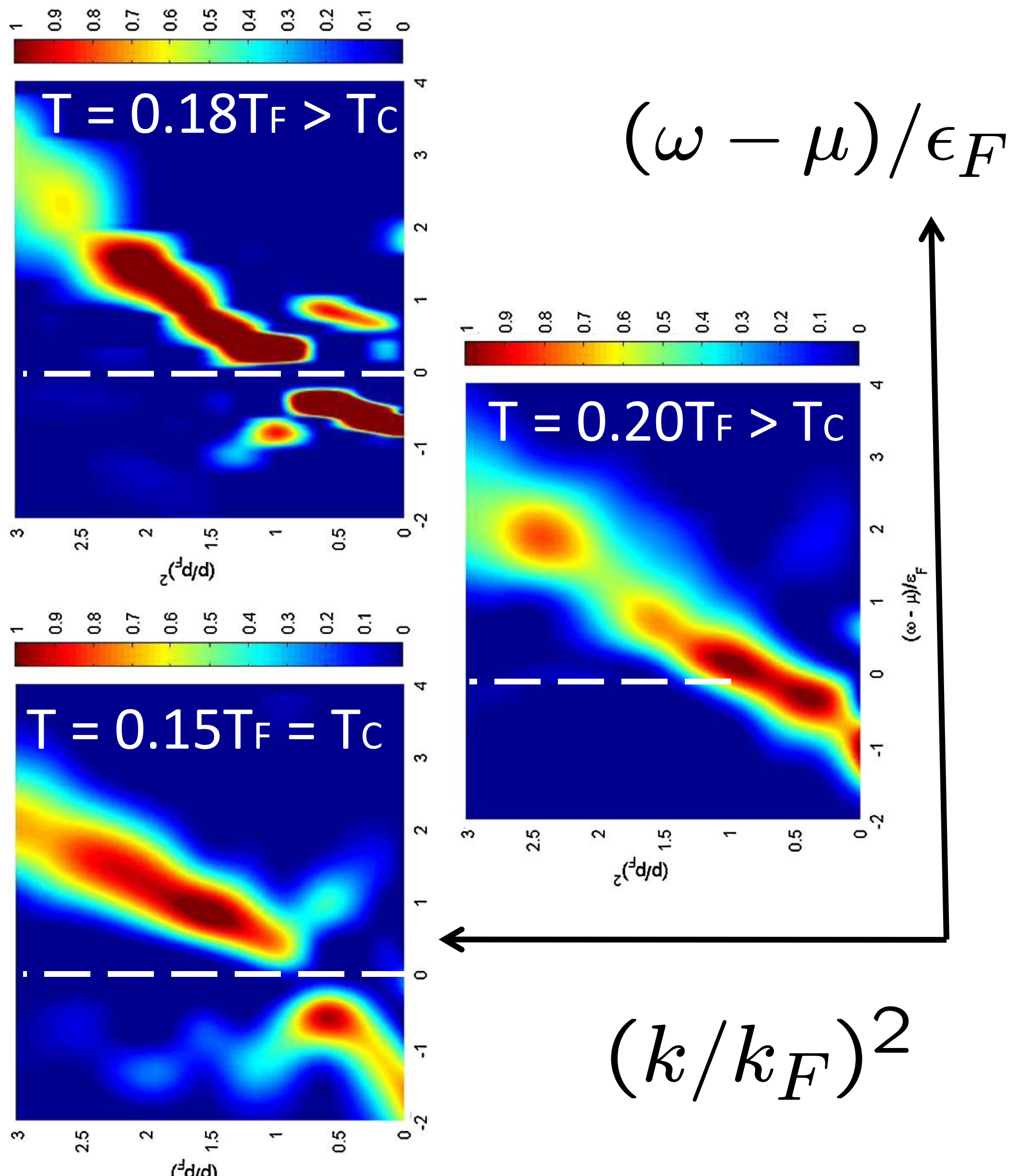
QMC
+ Analytic
Continuation

→ pseudogap

$T_c < T < T^*$

$T_c = 0.15 T_F$

$T^* \simeq 0.20 T_F$



Magierski et al,
PRL 103, 210403 (2009)

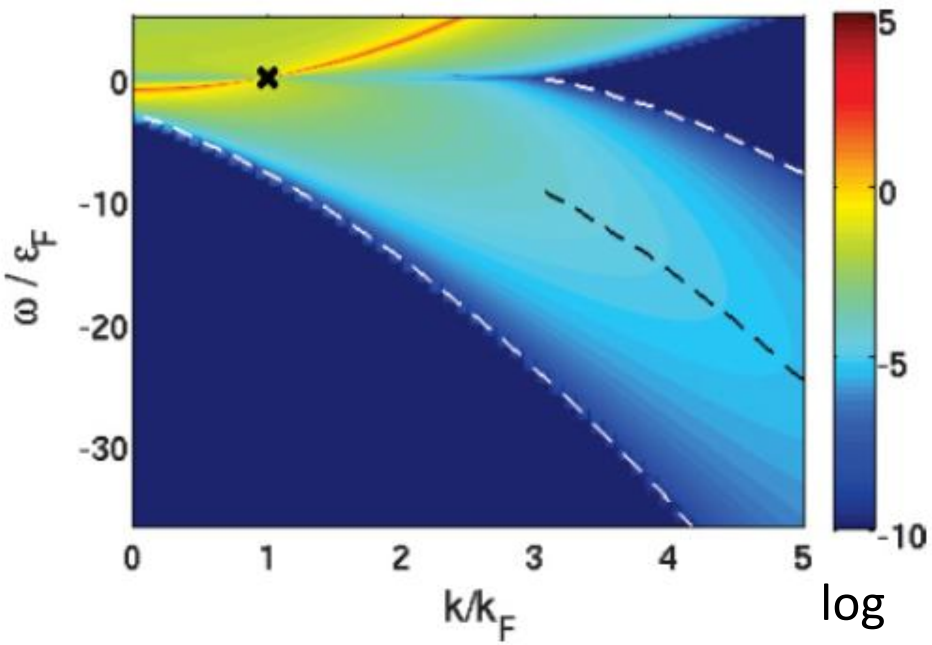
Universal back-bending of large-k dispersion in all phases

$$n(\mathbf{k}) = \int_{-\infty}^{+\infty} f(\omega) A(\mathbf{k},\omega)$$

$$n(\mathbf{k}) \approx C/k^4 \ \ (k \gg k_F)$$

S. Tan (2006)

$$\Rightarrow A(k \gg k_F, \omega < 0) \neq 0$$



Incoherent spectral weight

-- center $\omega \approx -k^2/2m$

-- width $\sim v_F k$

-- area $= C/k^4$

→ Only near-k_F back-bending is signature of Pairing pseudogap

Schneider & MR
PRA 81, 021601(R) (2010)

Brief Review of experiments*

- Spectroscopy → Pseudogap: "yes"
- Thermodynamics → Pseudogap: "no"?
- Transport → no sharp quasiparticles

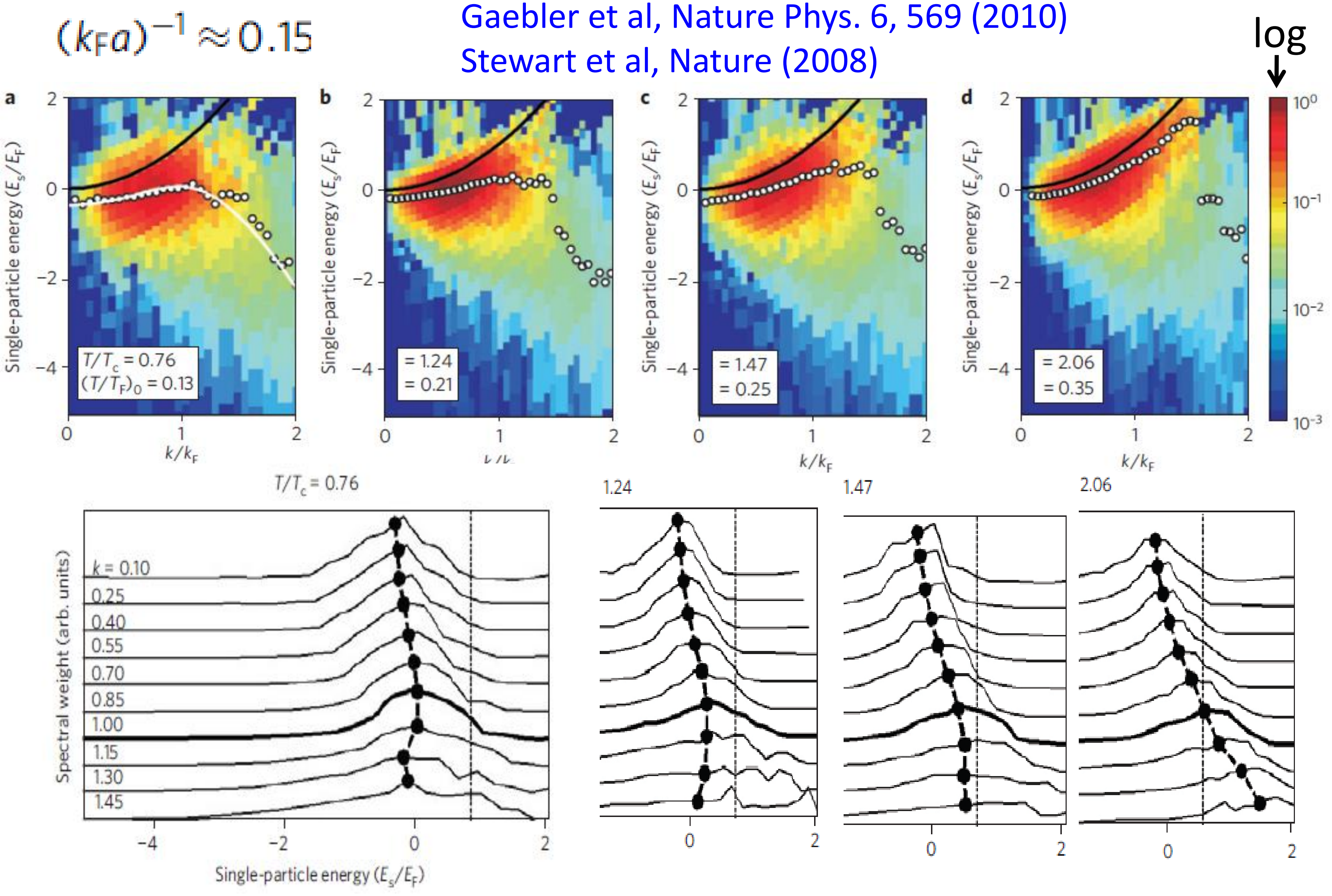
*All the experimentalists are/will be speaking at the conference!

k-Resolved RF Spectroscopy (“ARPES”) experiments

$(k_F a)^{-1} \approx 0.15$

Gaebler et al, Nature Phys. 6, 569 (2010)

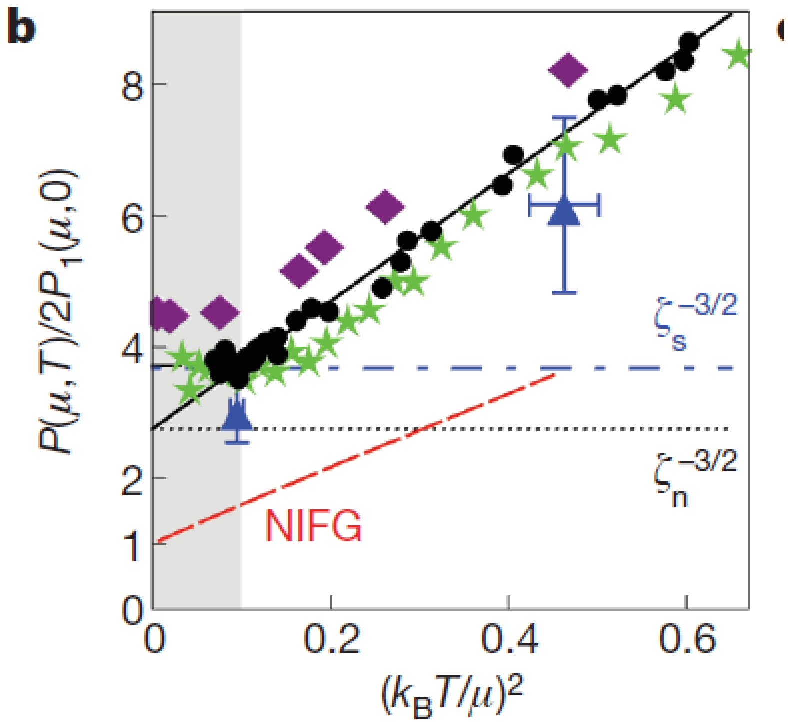
Stewart et al, Nature (2008)



Thermodynamics:

Equation of state:

Not very sensitive to Pseudogap physics



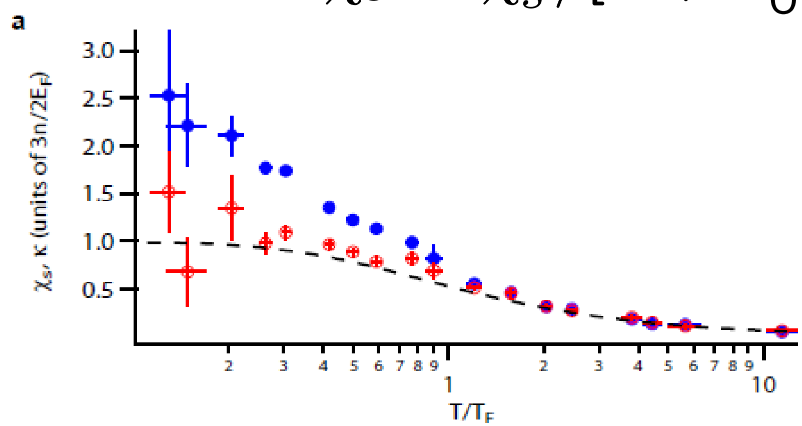
Nascimbene et al, Nature 463, 1057 (2010)

Compressibility
& Spin susceptibility

$$\chi_s = \sigma_s / D_s$$

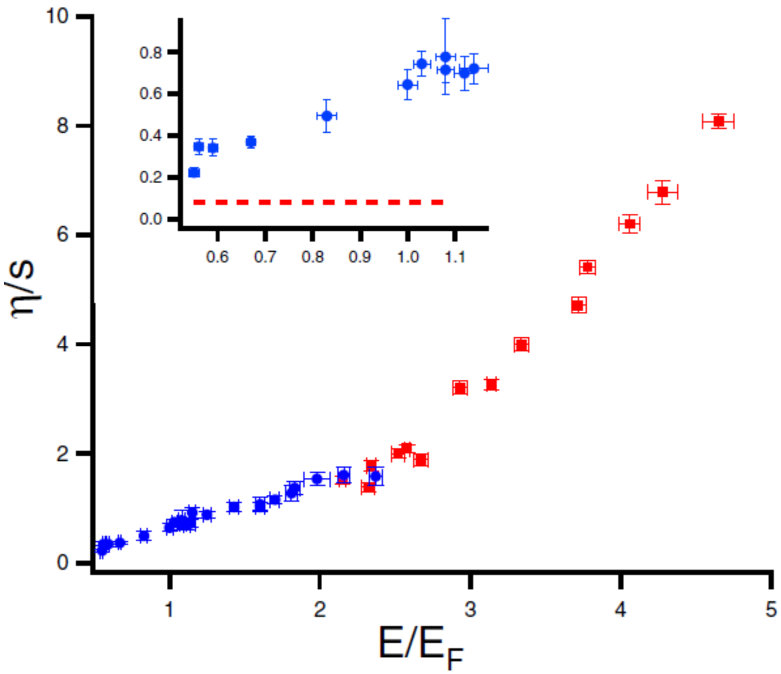
Quite apart from lack of T-dep, Expect suppression in a Fermi Gas with attractive interactions

$$\chi_s = \chi_s^0/[1 + F_0^a]$$

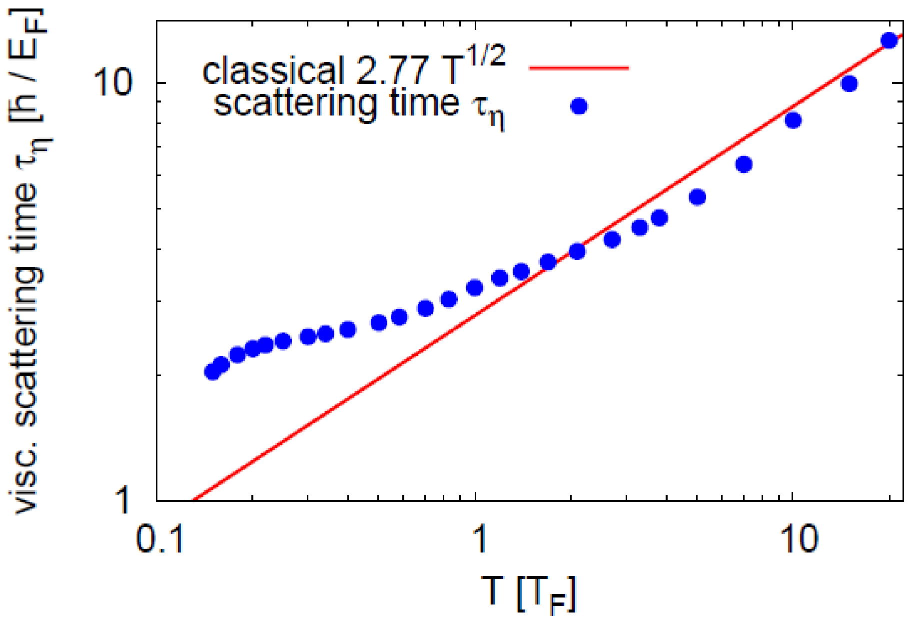


Sommer et al, Nature 472, 201 (2011)

Transport:
Low Viscosity → Ill-defined quasiparticles



Cao et al, Science (2011)



Enss et al, Ann. Phys. (2011)

Expt. →

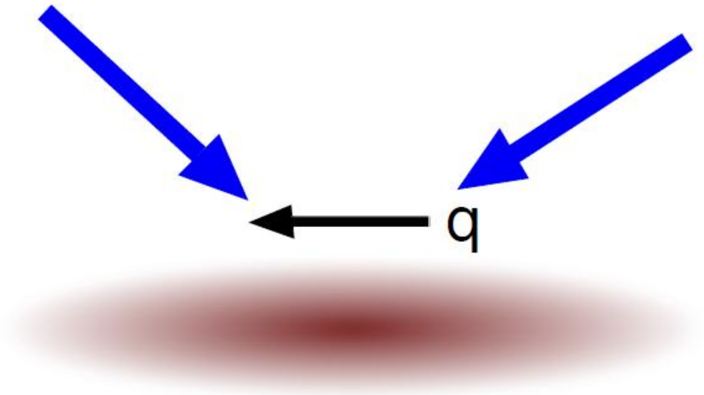
$\eta/s \approx$ 5 X KSS bound

Theory → $1/\tau_\eta \sim \epsilon_F$

Proposal to measure $\eta(\omega;T)$ at unitarity

via two-photon Bragg spectroscopy

Sum-Rule $\Rightarrow \zeta(\omega,T)=0$



$$\eta(\omega)=\lim_{q\to 0}\frac{3\omega^3}{4q^4}\,\mathrm{Im}\chi_{\rho\rho}(\mathbf{q},\omega)$$

Analog of "Optical Conductivity" for Cold Atoms

Taylor & MR, PRA 81, 053610 (2010)

A few remarks on
High Tc superconductors ...

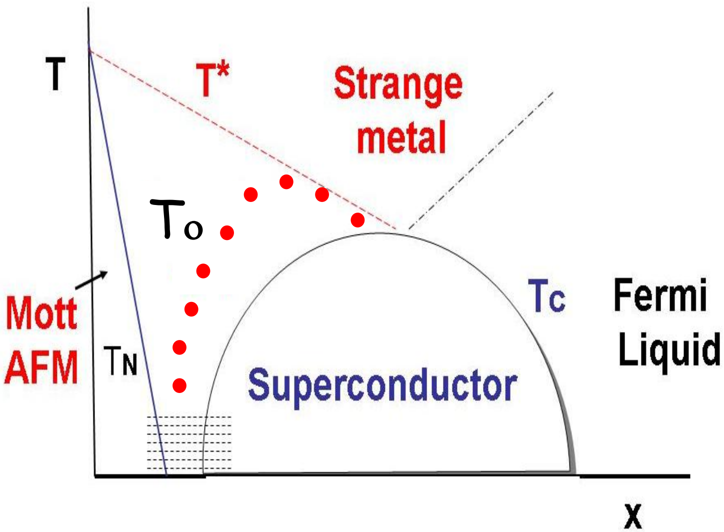
High Tc Superconductors

- Highest known Tc (in K)
  - * cuprates
- Charged electrons
- Repulsive interactions
- d-wave SC
- doped Mott insulator
- competing orders: AFM, stripes, …
- single band
- repulsion U >> bandwidth
- $\xi \sim 10$ A
- $T_c \sim \rho_s \ll \Delta$ (underdoped)
- anomalous normal states
  - strange metal
  - pseudogap

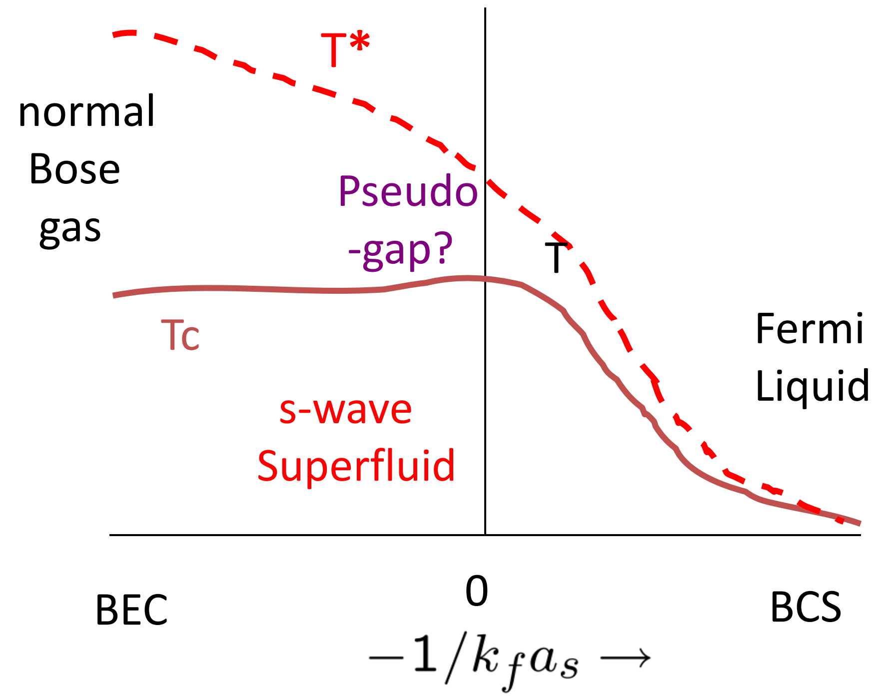
BCS-BEC crossover

- Highest known $T_c/E_f \sim 0.2$
  - * ultracold atomic gases
- Neutral Fermi atoms
- Attractive interactions
- s-wave SF
- only pairing instability
- single band
- attraction >> E_f
- $\xi \sim 1/k_f$
- $T_c \sim \rho_s \ll \Delta$ (for $a_s > 0$)
- pairing pseudogap
- Mean-field theory fails for Tc

High Tc Cuprates



BCS-BEC crossover



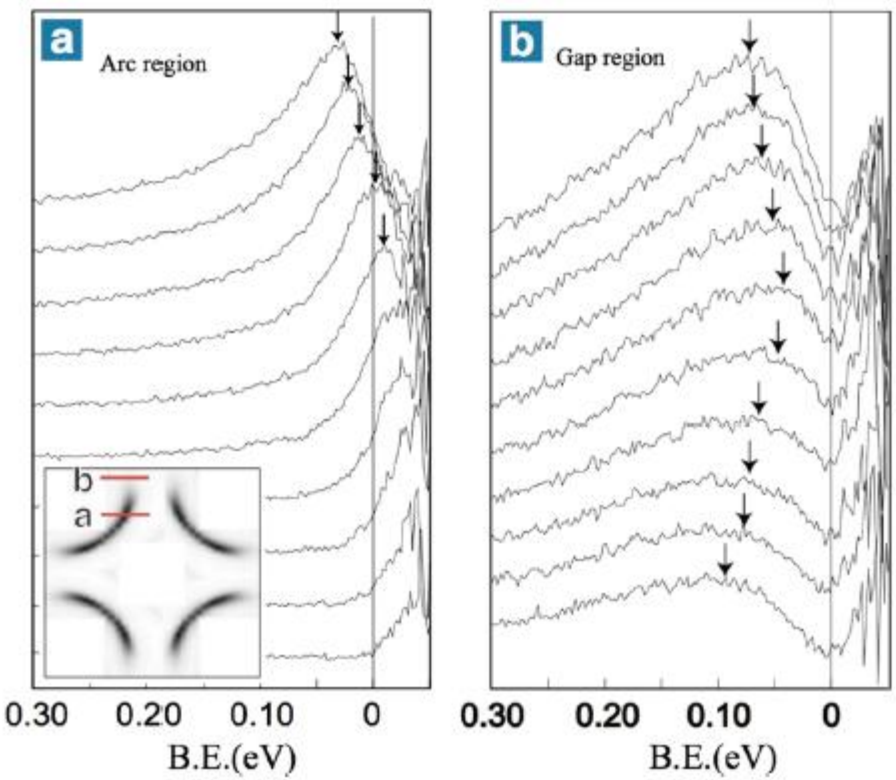
Cuprate pseudogap is more complex:
Proximity to Mott insulator
& competing order parameters

Pairing signals strong only upto To
Is T* a crossover or phase transition?

BCS-BEC
Pseudogap can only be related to Precursor pairing

ARPES in High Tc SC

T = 140K > Tc = 90K



Kanigel, Chatterjee, MR et al, PRL (2008)

Quantitative Analysis requires:

- knowing μ

$$f(\omega)A(\mathbf{k},\omega) \rightarrow A(\mathbf{k},\omega)$$

Line-shape analysis
→extract Self-energy

-- scattering rates
-- mass renormalization
-- gaps & pseudogaps

Review: Campuzano, Norman, MR in *Superconductivity* (eds Bennemann & Ketterson) Vol. 2, 923(Springer, 2004).

Thermodynamic Signatures of the Pseudogap in High Tc Cuprates

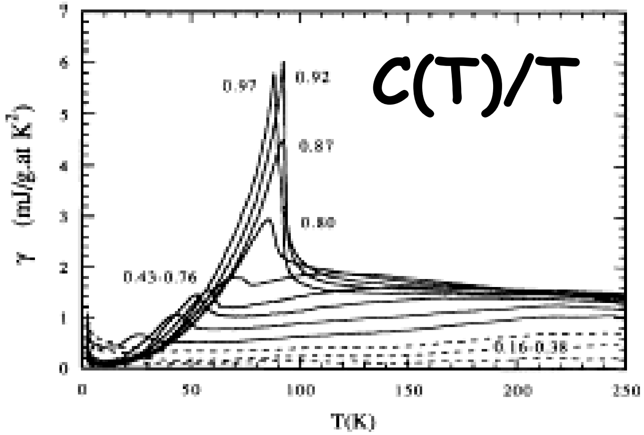


FIG. 4. Electronic specific heat coefficient $\gamma(x,T)$ vs T for $YBa_2Cu_3O_{6+x}$ relative to $YBa_2Cu_3O_6$. Values of x are 0.16, 0.29, 0.38, 0.43, 0.48, 0.57, 0.67, 0.76, 0.80, 0.87, 0.92, and 0.97.

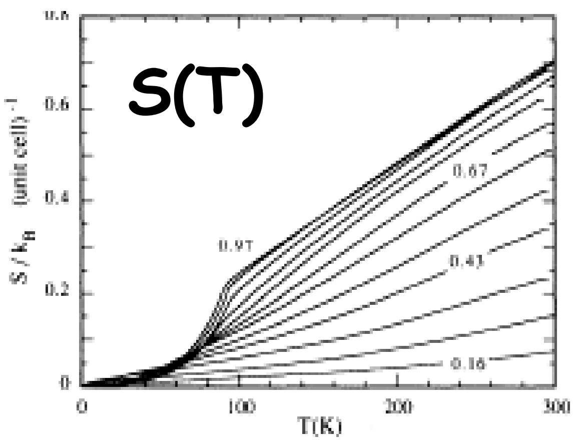
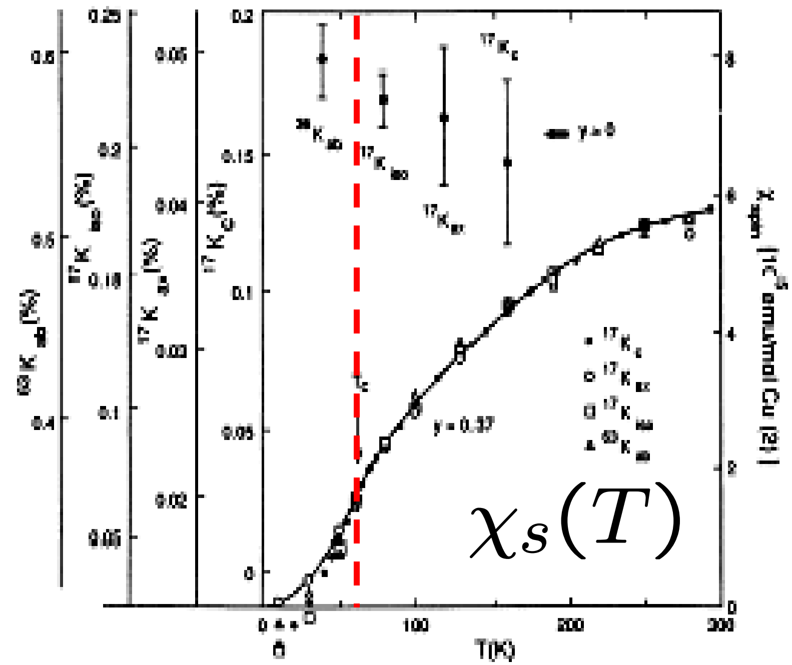


FIG. 5. Electronic entropy $S(x,T)$ for $YBa_2Cu_3O_{6+x}$. Values of x as in Fig. 4.

Loram et al, PRL (1993)



NMR Knight shift Takigawa et al, PRB (1991)

Very useful to see T-evolution
Through Tc and into SC state

conclusion

Pairing Pseudogap:

Suppression of low-energy spectral weight due to incoherent pairing in the normal state ($T > T_c$)

The end