Relaxation of a high-energy quasiparticle in a 1D Bose liquid

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in collaboration with

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Fermions from cold atoms to neutron stars:
benchmarking the many-body problem

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Outline

- motivation
- introduction: 3D vs 1D
- momentum distribution in 1D
- relaxation rate in 1D
- crunching numbers
- summary

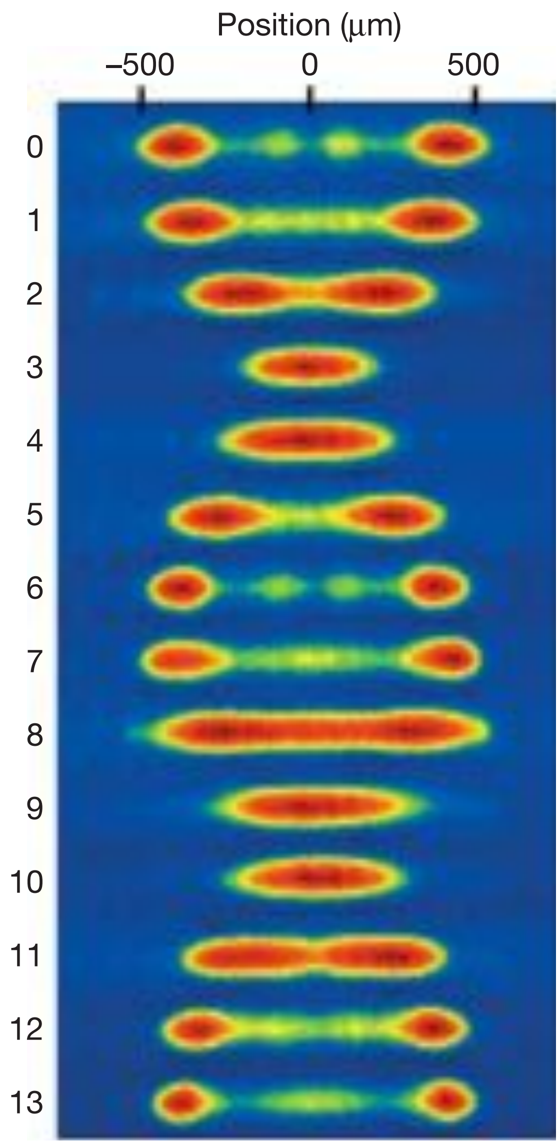
this work: S. Tan, M.P., and L. Glazman, PRL 105, 090404 (2010)

related: I. Mazets, T. Schumm, and J. Schmiedmayer, PRL 100, 210403 (2008)
I. Mazets and J. Schmiedmayer, PRA 79, 061603 (2009)
I. Mazets, arXiv: 1102.3934

nature Vol 440|13 April 2006|doi:10.1038/nature04693

A quantum Newton's cradle

Toshiya Kinoshita[1], Trevor Wenger[1] & David S. Weiss[1]



Here we report the preparation of out-of-equilibrium arrays of trapped one-dimensional (1D) Bose gases, each containing from 40 to 250 ^{87}Rb atoms, which do not noticeably equilibrate even after thousands of collisions. Our results are probably explainable by the well-known fact that a homogeneous 1D Bose gas with point-like collisional interactions is integrable.

In summary, we have watched the time evolution of non-equilibrium trapped 1D Bose gases, which are almost integrable systems. We find no evidence of redistribution of momentum, from the Tonks–Girardeau gas limit to the intermediate coupling regime. That is, we observe thousands of parallel 1D Bose gases, each with hundreds of atoms colliding thousands of times without approaching equilibrium.

"...changes in the distribution with time are attributable to known loss and heating"

nature Vol 440|13 April 2006|**doi:10.1038/nature04693**

A quantum Newton's cradle

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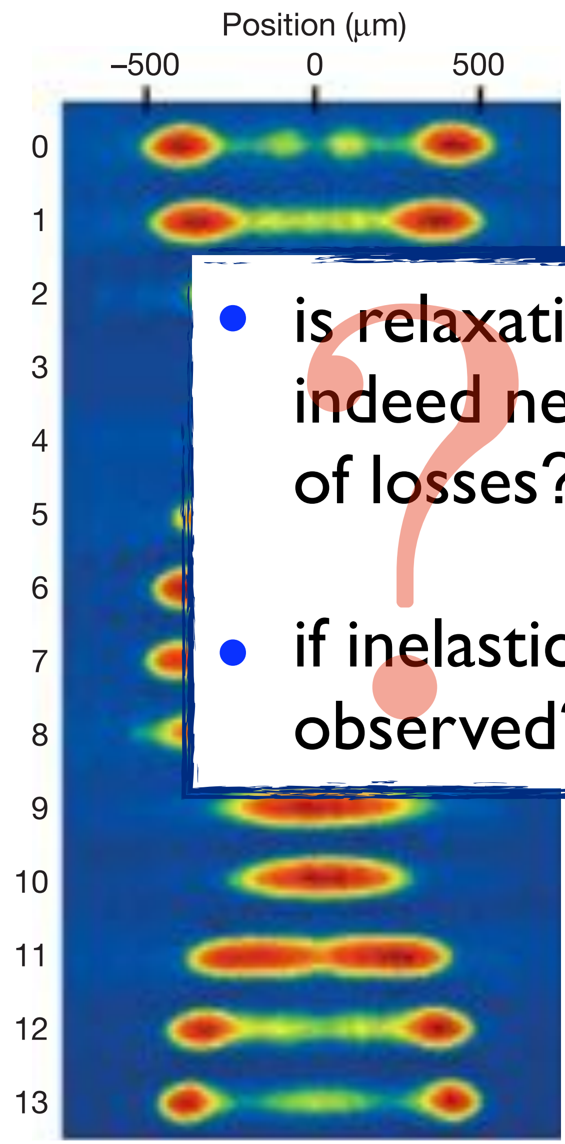

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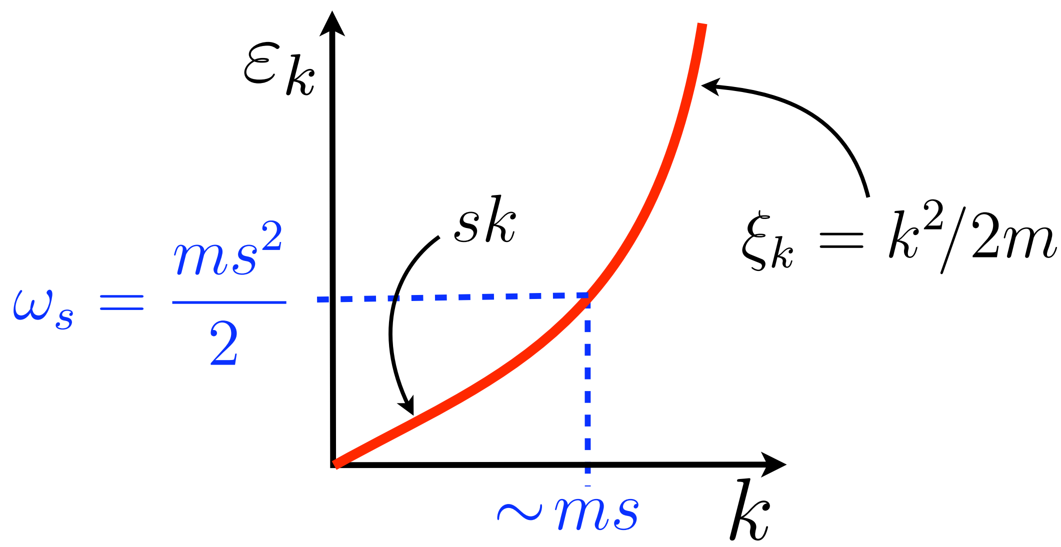
- is relaxation due to deviations from the integrability indeed negligible compared with other mechanisms of losses?
- if inelastic relaxation is strong, why it was not observed?

"...changes in the distribution with time are attributable to known loss and heating"

3D

BEC transition at a finite temperature

Elementary excitations: Bogolubov's quasiparticles



Rate of relaxation by 2-body collisions:

$$\Gamma_k \propto \max\{\varepsilon_k^5, \varepsilon_k^4 T\} \text{ for } k \ll ms$$

independent of either k or T for $k \gg ms$
(thus insensitive to BEC)

Beliaev (1958)
Andreev and Khalatnikov (1963)
Hohenberg and Martin (1965)

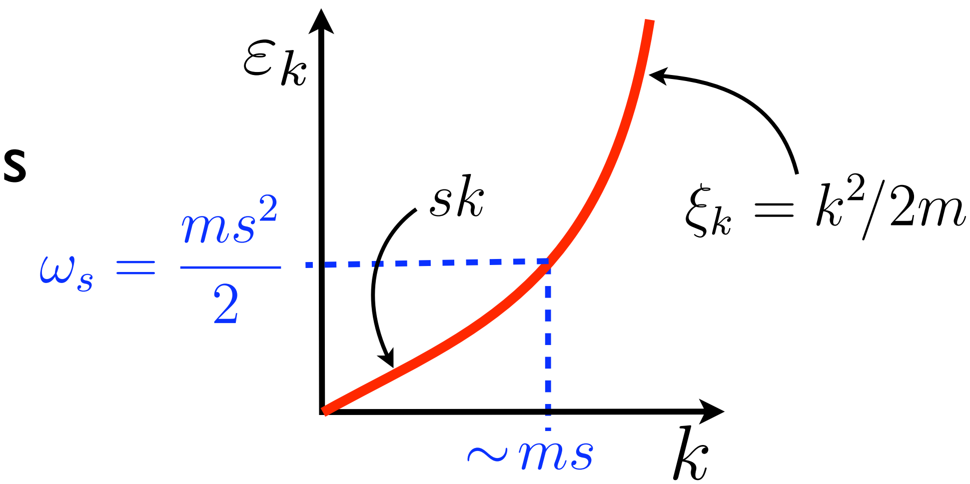
1D

no BEC, except for free bosons at zero temperature (for repulsive interaction)

quasicondensate instead of a condensate, with a finite spread in a momentum space

V. Popov (1972)

Elementary excitations:
Bogolubov's quasiparticles



exact eigenstates: Lieb and Liniger (1963)
quasiparticles: see, e.g., C. Mora and Y. Castin, PRA 67, 053615 (2003)

Relaxation is due to 3-body collisions

$$\Gamma_k \text{ at } k \gg ms \text{ depends strongly on } T \text{ and } k$$

S. Tan, M.P., and L. Glazman, PRL 105, 090404 (2010)

Energy scales

n (concentration)

s (sound velocity)

m (mass)

$\longrightarrow$

$$\omega_s = ms^2/2$$

$$T_0 = 2n^2/m$$

$$T_s = \sqrt{\omega_s T_0} = ns$$

$(k_B = \hbar = 1)$

For a weak repulsive interaction

$$\omega_s \ll T_s \ll T_0$$

ω_s: interaction energy per particle

T_s: interaction temperature: at $T \lesssim T_s$ interaction is important

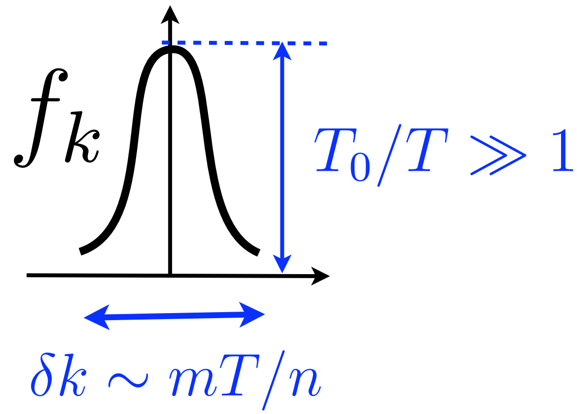
T_0: quantum degeneracy temperature

Momentum distribution in 1D

noninteracting bosons, $\xi_k \ll T \ll T_0 = 2n^2/m$

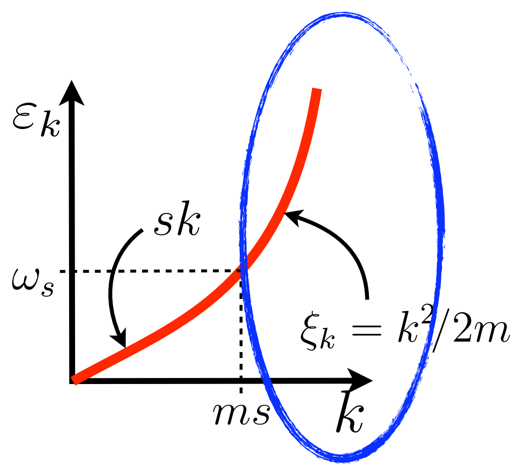
$$f_k = \langle \psi_k^\dagger \psi_k \rangle = \frac{1}{e^{(\xi_k - \mu)/T} - 1} \approx \frac{T}{\xi_k + \mu_0}$$

$$\mu_0(T) = -\mu = T^2/T_0$$



Applicable to interacting system as long as

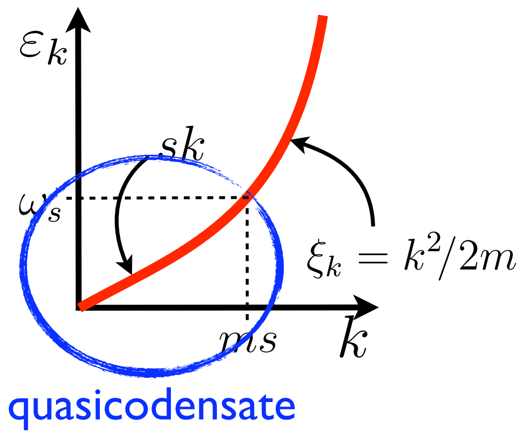
$$\mu_0 \gg \omega_s = ms^2/2 \;\Leftrightarrow\; T \gg T_s$$



At $T \gg T_s$,
the majority of the occupied states have $k \gg ms$

Momentum distribution in 1D

interacting bosons, $T \ll T_s,\ k \lesssim ms$



density fluctuations are
suppressed by the interactions

Tool: hydrodynamic description of long-wavelength excitations

Popov (1972)
Haldane (1981)

$$\psi(x) \approx \sqrt{n}\, e^{i\vartheta(x)}\,,\ \ \rho(x) = n + \pi^{-1}\partial_x\varphi$$

$$[\varphi(x), \vartheta(y)] = i(\pi/2)\,\mathrm{sgn}(x-y)$$

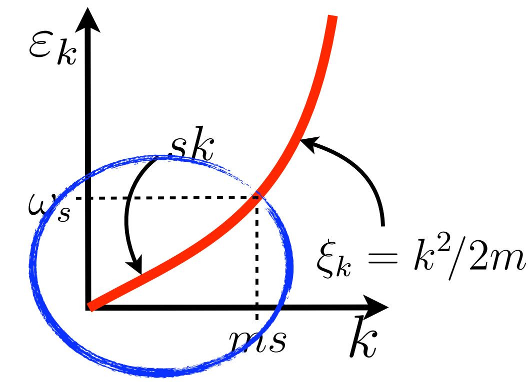
Effective Hamiltonian

$$H = \frac{n}{2m}\int dx\left[\kappa^{-2}(\partial_x\varphi)^2 + (\partial_x\vartheta)^2\right]$$

$$\kappa = \frac{\pi T_0}{2T_s} = \frac{\pi n}{ms} \gg 1$$

Momentum distribution in 1D

interacting bosons, $T \ll T_s,\ k \lesssim ms$



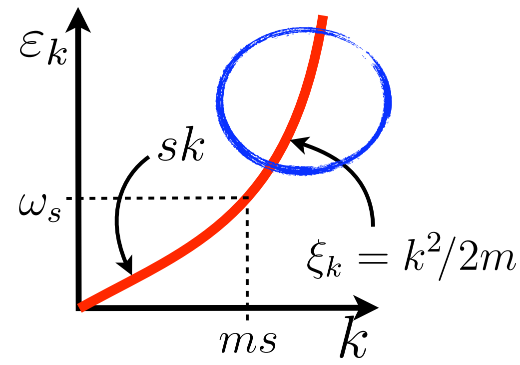
$$f_k \sim |ms/k|^{1-1/2\kappa}, \quad T \ll sk \ll \omega_s$$

$$f_k = \frac{2T}{4\xi_k + \mu_0} (T_0/T)^{-1/2\kappa}, \quad sk \ll \min\{T, \omega_s\}$$

- $\omega_s \lesssim T \ll T_S$: f_k is a Lorentzian, cf. free bosons
- At $k \sim ms$ and $T \sim T_s$ we have $f \sim T_s/\omega_s = 2\kappa/\pi$ (both for hydrodynamics and free bosons)
- There is no room for power-low dependence at $T \gg \omega_s$

Momentum distribution in 1D

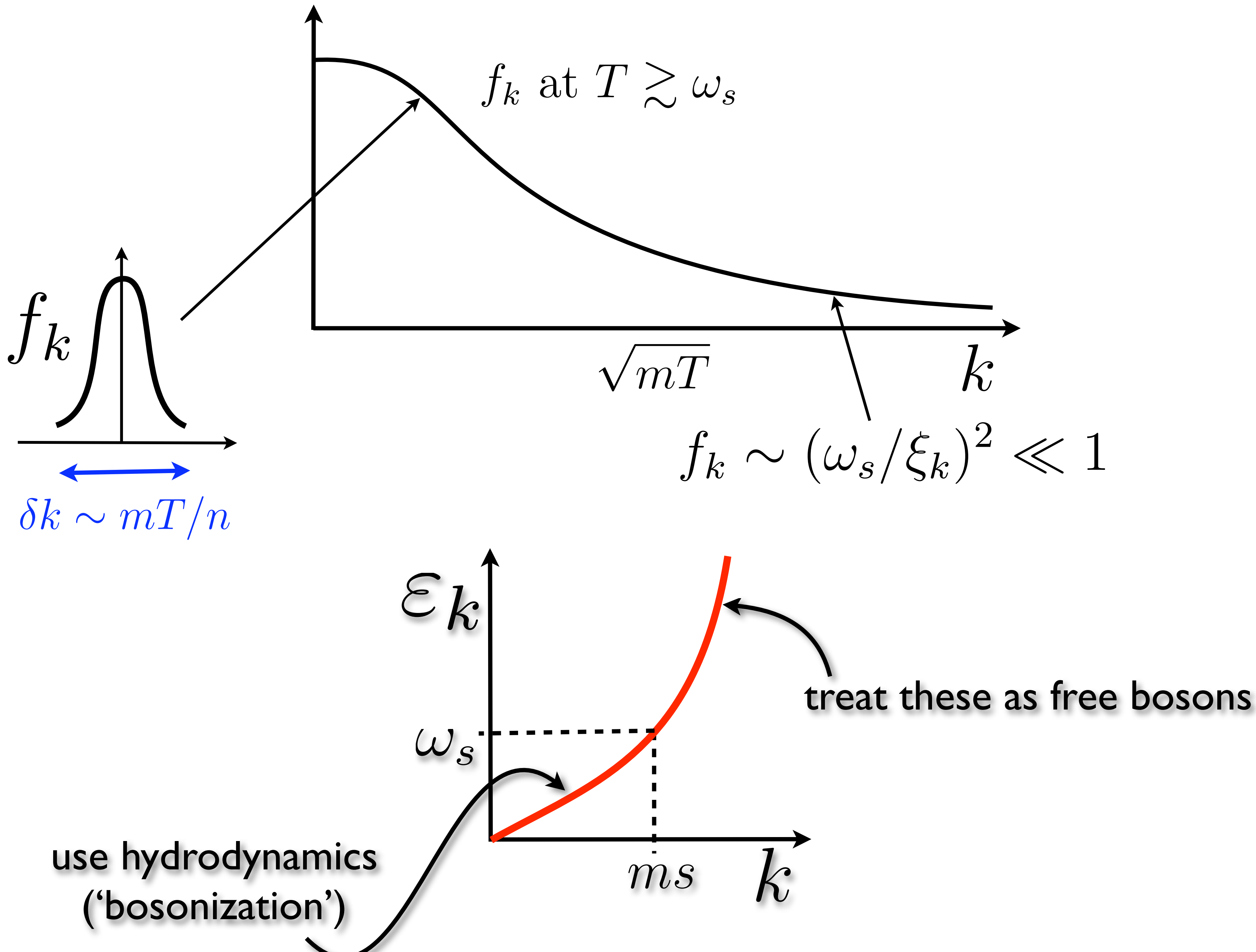
interacting bosons, $\xi_k \gg \max\{\omega_s, T\}$



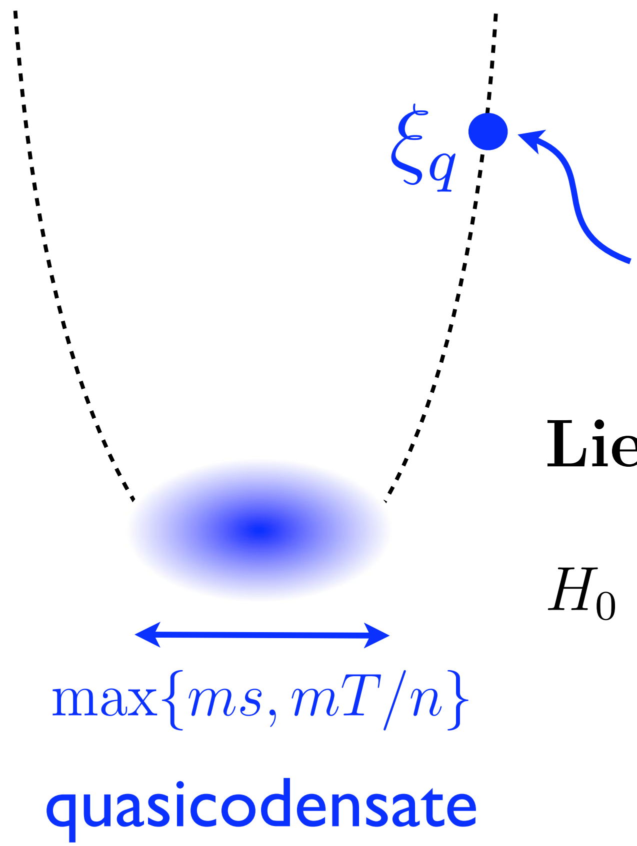
$$f_k = dE_2/d\xi_k \sim (nc/\xi_k)^2 \sim (\omega_s/\xi_k)^2 \ll 1$$

second-order correction to the ground state energy

Momentum distribution in 1D



The model



Lieb-Liniger model:

$$H_0 = \int dx\, \psi^\dagger(x) \left(-\frac{1}{2m} \frac{d^2}{dx^2} \right) \psi(x) + \frac{c}{2} \int dx : \rho^2(x) :$$

$$\gamma = mc/n \ll 1 \text{ (weak interaction)}$$

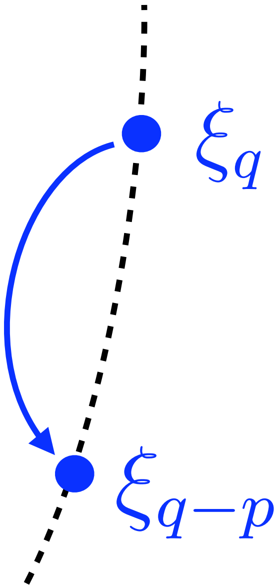
sound velocity: $s = (n/m)\sqrt{\gamma}$

Integrability-breaking perturbation:

$$V = \frac{\alpha}{m} \int dx : \rho^3(x) : , \quad \alpha \ll 1$$

(the model can be justified microscopically)

Relaxation rate



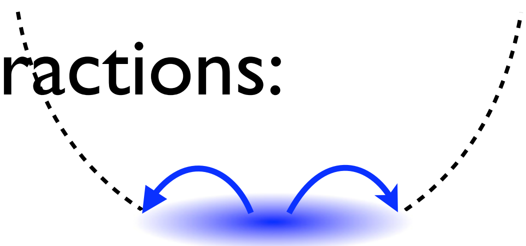
energy transfer:

$$\omega = \xi_q - \xi_{q-p} \approx qp/m$$

momentum transfer:

$$p \approx m\omega/q \to 0 \text{ for } q \to \infty$$

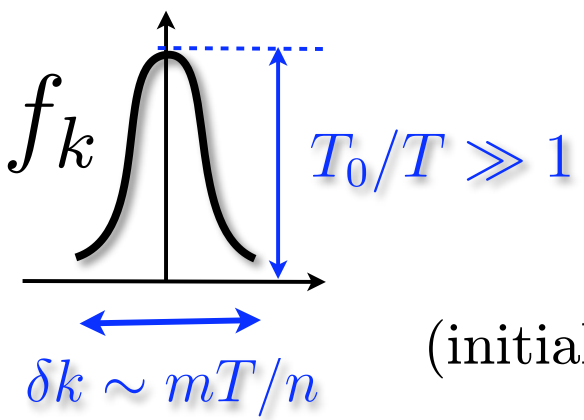
w/out interactions:



$$\Gamma_q \propto \frac{\alpha^2}{mq} \int d\omega \int \prod_{i=1}^{4} dk_i \, f_{k_1} f_{k_2} (f_{k_3} + 1)(f_{k_4} + 1)$$
$$\times \, \delta(k_1 + k_2 - k_3 - k_4) \, \delta(\xi_{k_1} + \xi_{k_2} - \xi_{k_3} - \xi_{k_4} + \omega)$$

Relaxation by small energy transfer

$$\Gamma_q \propto \frac{\alpha^2}{mq}\int d\omega \int \prod_{i=1}^{4} dk_i\, f_{k_1} f_{k_2}(f_{k_3}+1)(f_{k_4}+1)$$
$$\times\, \delta(k_1+k_2-k_3-k_4)\,\delta(\xi_{k_1}+\xi_{k_2}-\xi_{k_3}-\xi_{k_4}+\omega)$$



Main contribution comes from

$$|k_i| \lesssim mT/n$$

(initial and final states are within the quasicondensate)

$$\Gamma_q \sim \frac{\alpha^2}{mq}(T_0/T)^4\,(\delta k)^3 \sim \alpha^2 T_0 \left(\frac{T_0}{\xi_q}\right)^{1/2}\frac{T_0}{T}$$

diverges at $T \to 0$

corresponds to a **small** energy transfer $\omega \lesssim (\delta k)^2/m \sim \mu_0$

Relaxation by small energy transfer

$$\Gamma_q \sim \alpha^2 T_0 \left(\frac{T_0}{\xi_q}\right)^{1/2} \frac{T_0}{T} \xrightarrow[T \to 0]{} 0$$

unphysical order of limits: $\gamma \to 0$ first, $T \to 0$ after

interacting bosons:

$\Gamma_q \propto 1/T$ is applicable as long as $T \gg T_s$

At $\omega_s \ll T \ll T_s$ bosonization yields

$$\Gamma_q \sim \alpha^2 T_0 \left(\frac{T_0}{\xi_q}\right)^{1/2} \frac{T_0}{T_s} \left(\frac{T}{T_s}\right)^2$$

$\max\{\Gamma_q\}$ is reached at $T \sim T_s$

$$\Gamma_{\max} \sim \alpha^2 T_0 \left(\frac{T_0}{\xi_q}\right)^{1/2} \frac{T_0}{T_s}$$

Relaxation by large energy transfer

$$\Gamma_q \propto \frac{\alpha^2}{mq}\int d\omega \int \prod_{i=1}^{4} dk_i\, f_{k_1} f_{k_2} (f_{k_3}+1)(f_{k_4}+1)$$
$$\times\, \delta(k_1+k_2-k_3-k_4)\,\delta(\xi_{k_1}+\xi_{k_2}-\xi_{k_3}-\xi_{k_4}+\omega)$$

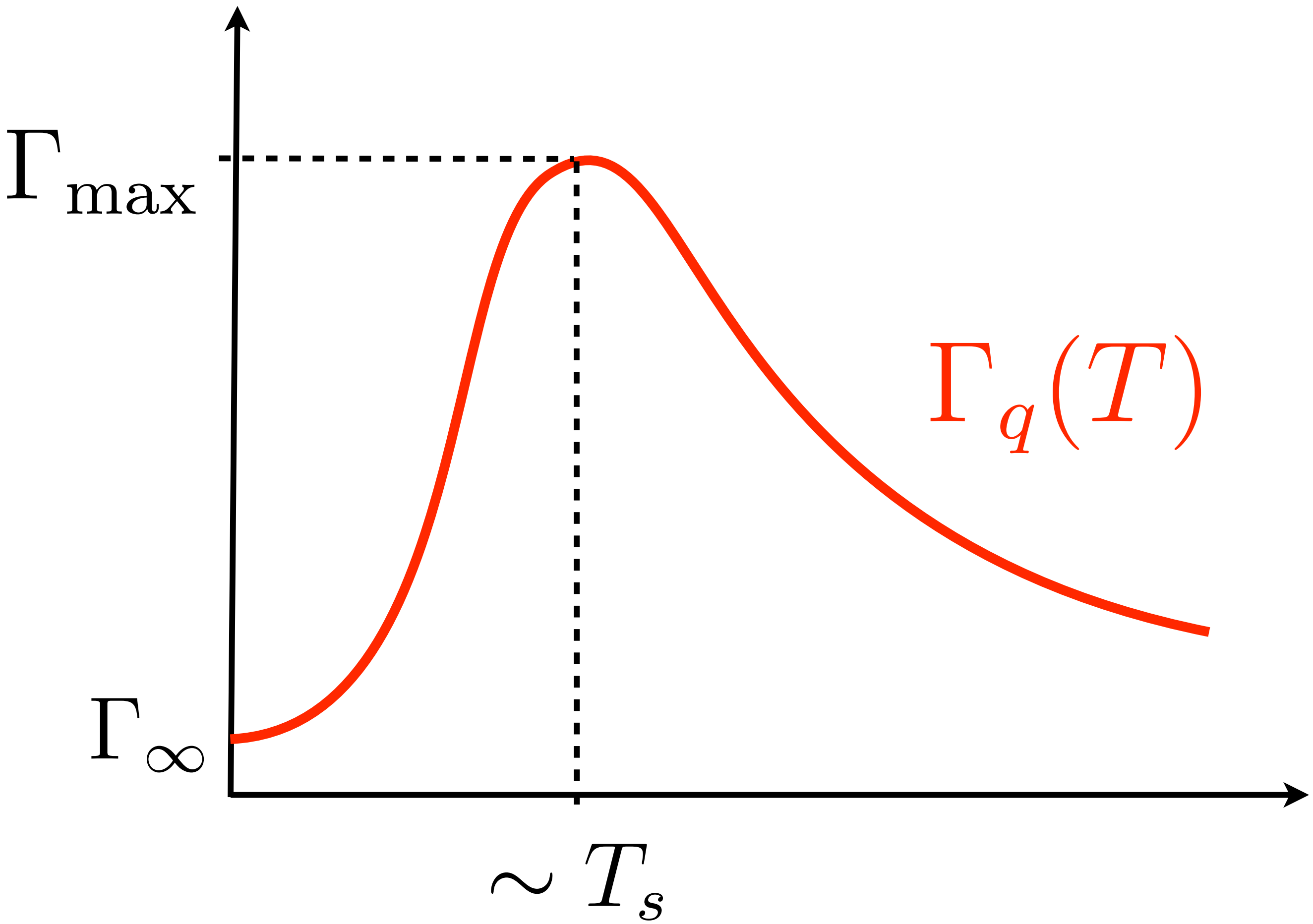
Subleading contribution: final states well outside the quasicondensate

$$\Gamma_\infty \sim \frac{\alpha^2 n^2}{mq}\int d\omega \int dk_3 dk_4\, \delta(k_3+k_4)\,\delta(\xi_{k_3}+\xi_{k_4}-\omega) \;\sim \alpha^2 T_0$$

- independent of q (hence the notation Γ_∞)
- corresponds to a **large** energy transfer $\mu_0 \ll \omega \lesssim \xi_q$
- (almost) independent of the interaction strength

I. Mazets, T. Schumm, and J. Schmiedmayer, PRL 100, 210403 (2008)

Relaxation rate



Cold atoms in a cylindrical trap

Interaction in 3D: $V_{3D}(\boldsymbol{r}) = 4\pi(a/m)\delta(\boldsymbol{r})$

Projection onto the lowest subband of transverse quantization yields

$$\gamma = 2a/na_r^2, \quad \alpha = 18\ln(4/3)(a/a_r)^2$$

$$a_r = (m\omega_r)^{-1/2} \gg a$$

Main mechanism of **losses**:
3-body recombination processes

$$\Gamma_R = \beta n^2/a_r^4$$

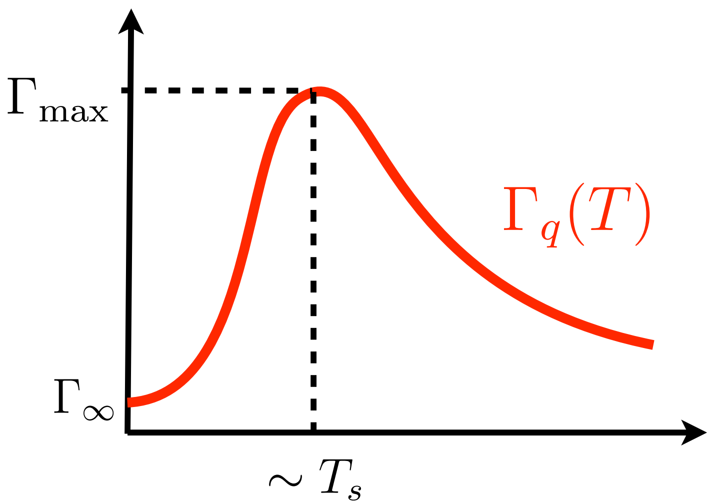
$$\Gamma_\infty/\Gamma_R = 10.3 \underbrace{a^4/(m\beta)}_{=2.1 \text{ for } ^{87}\text{Rb}} \approx 20$$

For $\omega_r/2\pi = 15\,\text{kHz}$ and $n = 7\,\mu\text{m}^{-1}$ we have $\gamma = 0.2,\ T_s = 120\,\text{nK}$.

With $\xi_q/T_s = \omega_r/\xi_q = 2.4,$ this gives $\Gamma_{\max}/\Gamma_R \sim 100$

(these are realistic numbers)

Summary



Strong nonmonotonic $T-$ dependence (unlike in 3D), with a max at $T \sim T_s = ns$

$\Gamma_q \gg \Gamma_R \;\; \Rightarrow$

inelastic collisions due to deviations from the integrability should be observable in ultracold atomic gases

S. Tan, M.P., and L. Glazman, PRL 105, 090404 (2010)