Efimov effect, Unitary Fermi gas, and "beyond"

Yusuke Nishida (MIT)

INT program on
"Fermions from Cold Atoms to Neutron Stars: Benchmarking the Many-Body Problem"

April 15, 2011

I am (we are) interested in physics in the unitarity limit ($a \to \infty$)

Why interesting ?

- Strongest interaction

⇒ nontrivial phenomena

⇒ challenging problems (unitary Fermi gas)

- Scale invariance

⇒ constrain properties of unitary Fermi gas

⇒ Efimov effect for bosons

- Universality

⇒ atomic, condensed matter, nuclear phsics

Physics at unitarity

- Strong interaction
- Scale invariance
- Universality

Are there other such interesting systems ?

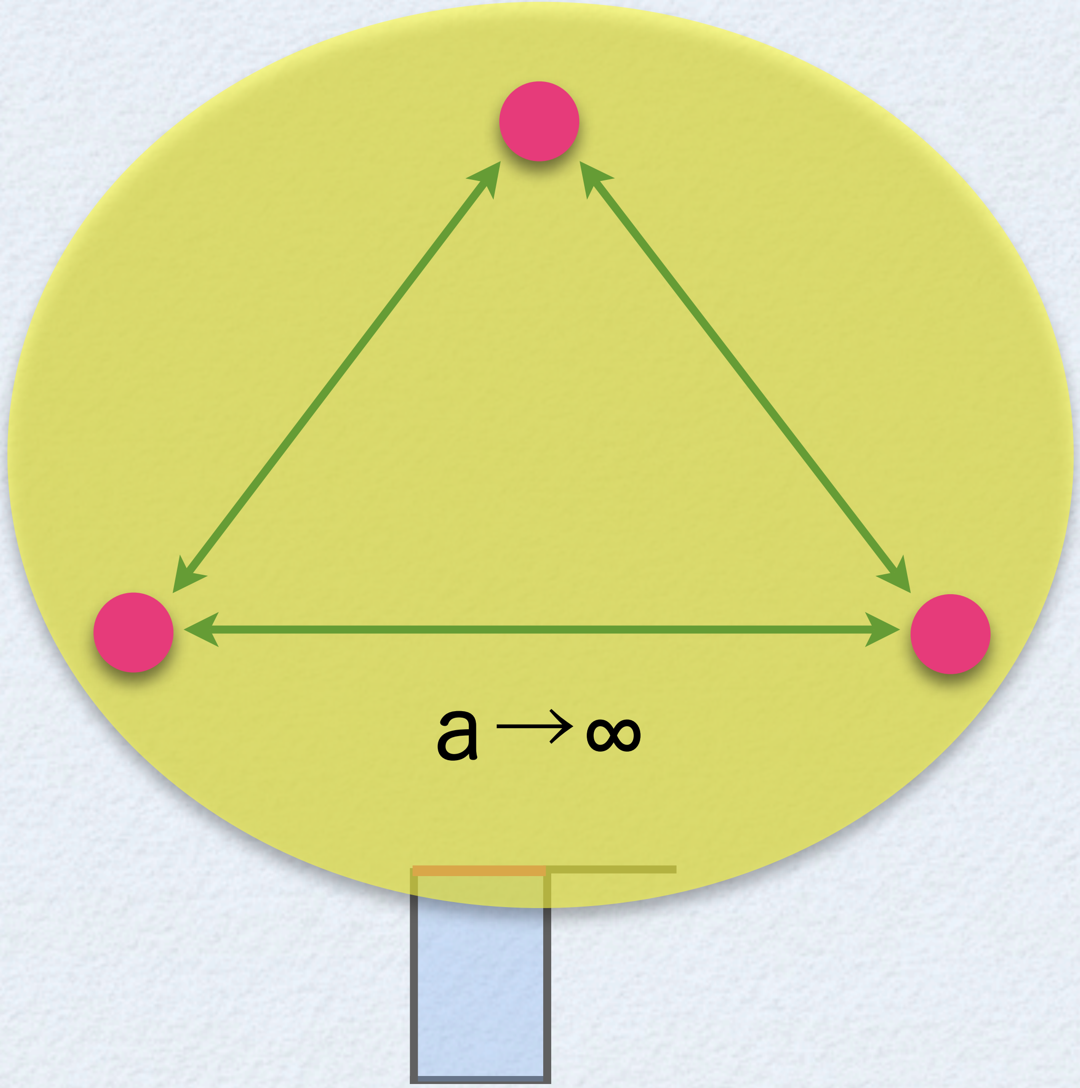
Does the unitarity physics (unitary Fermi gas & Efimov effect) exist in other than d=3 ?

Efimov effect beyond 3D

Efimov effect

When 2 bosons interact with infinite "a",
3 bosons always form a series of bound states

Efimov (1970)



Efimov effect

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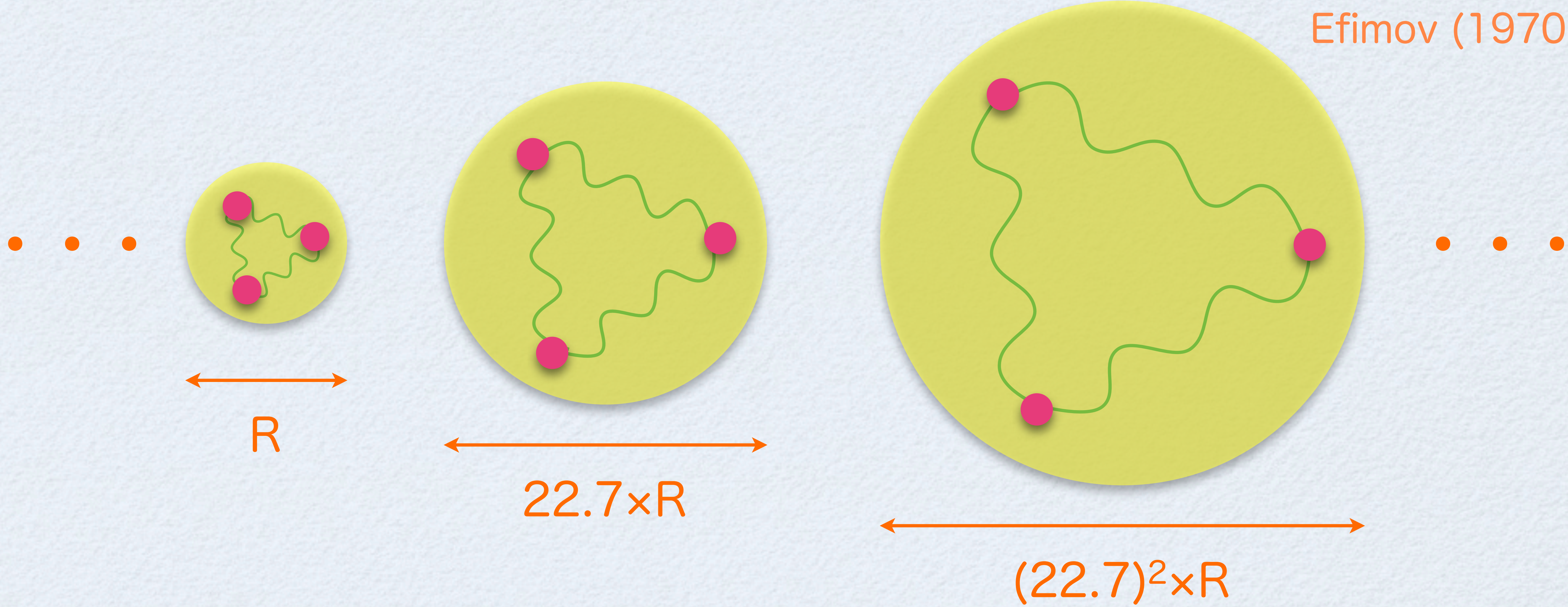
Efimov (1970)



Discrete scaling symmetry

RG limit cycle (K. Wilson 1971, P. Bedaque et al. 1999)

Efimov effect

When 2 bosons interact with infinite "a",
3 bosons always form a series of bound states

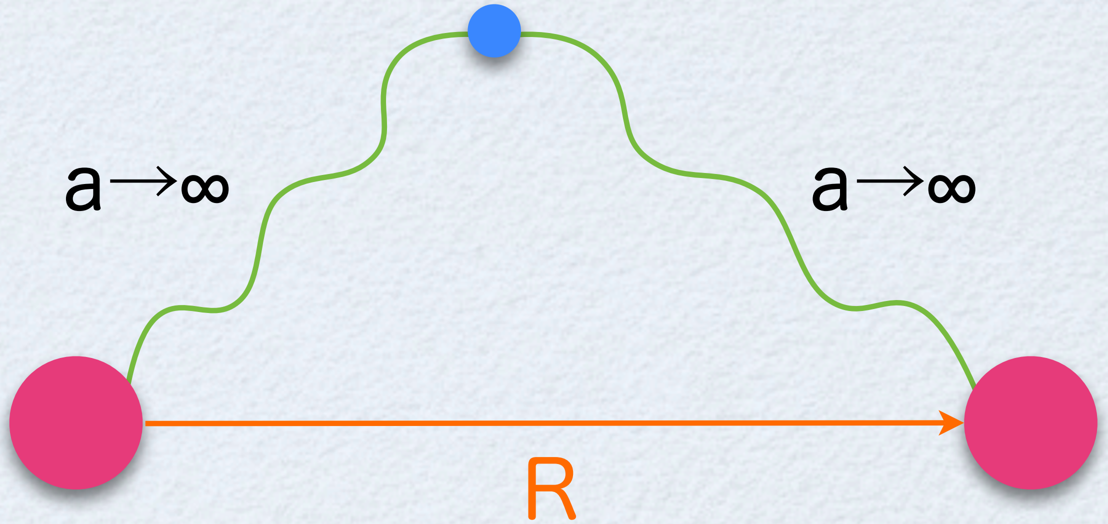
Efimov (1970)

Exchange of a light particle

Scale invariance

Effective potential
between heavy particles

$$V(R) \propto -\frac{\hbar^2}{2mR^2}$$



Quantum anomaly

Efimov effect (discrete scale invariance)

$$E_{n+1}/E_n = \text{const}$$

Efimov effect in D≠3 ?

PHYSICAL REVIEW A VOLUME 19, NUMBER 2 FEBRUARY 1979

Binding of three identical bosons in two dimensions

L. W. Bruch* and J. A. Tjon

Instituut voor Theoretische Fysica, Rijksuniversiteit Utrecht, Utrecht, The Netherlands

(Received 11 August 1978)

Qualitative features are discussed for the binding of three identical bosons interacting through pair potentials in two dimensions. Two special cases, known to yield pathologies in three dimensions, are examined using the Faddeev equation for the bound states. The Thomas effect does not occur: in the model which is treated, the trimer binding energy is finite for a zero-range force with a finite dimer energy. The

PHYSICAL REVIEW B VOLUME 22, NUMBER 3 1 AUGUST 1980

Nonexistence of the Efimov effect in two dimensions

T. K. Lim

Department of Physics and Atmospheric Science, Drexel Univeristy, Philadelphia, Pennsylvania 19104

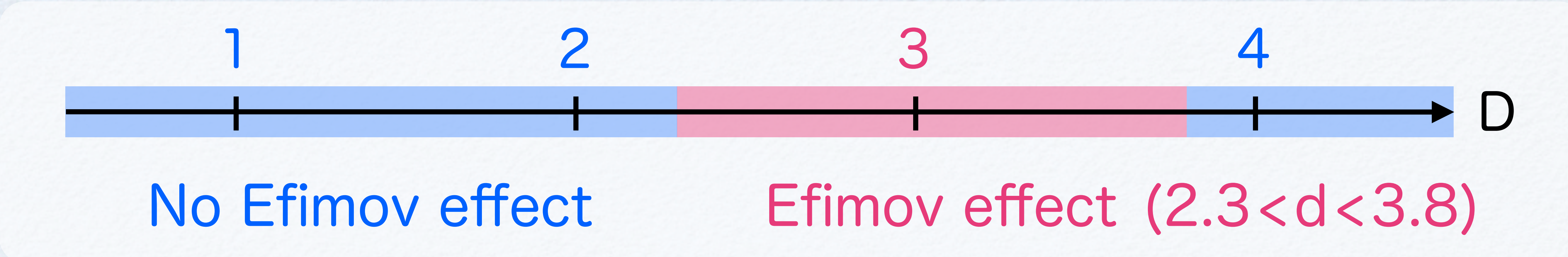
P. A. Maurone

Department of Physics, Villanova University, Villanova, Pennsylvania

(Received 17 March 1980)

Using the method first employed by Efimov for three bosons in three dimensions, we have found that the pathological condition discovered by him does not occur in two dimensions.

Efimov effect in D≠3 ?



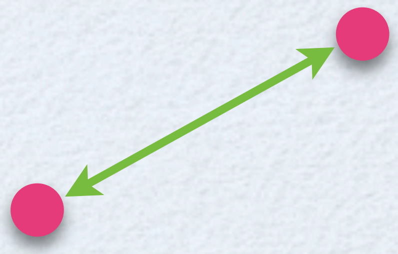
Nielsen et al., Phys. Rept. (2001)

"3 spatial dimensions" are not essential

but "3 relative coordinates" are essential !

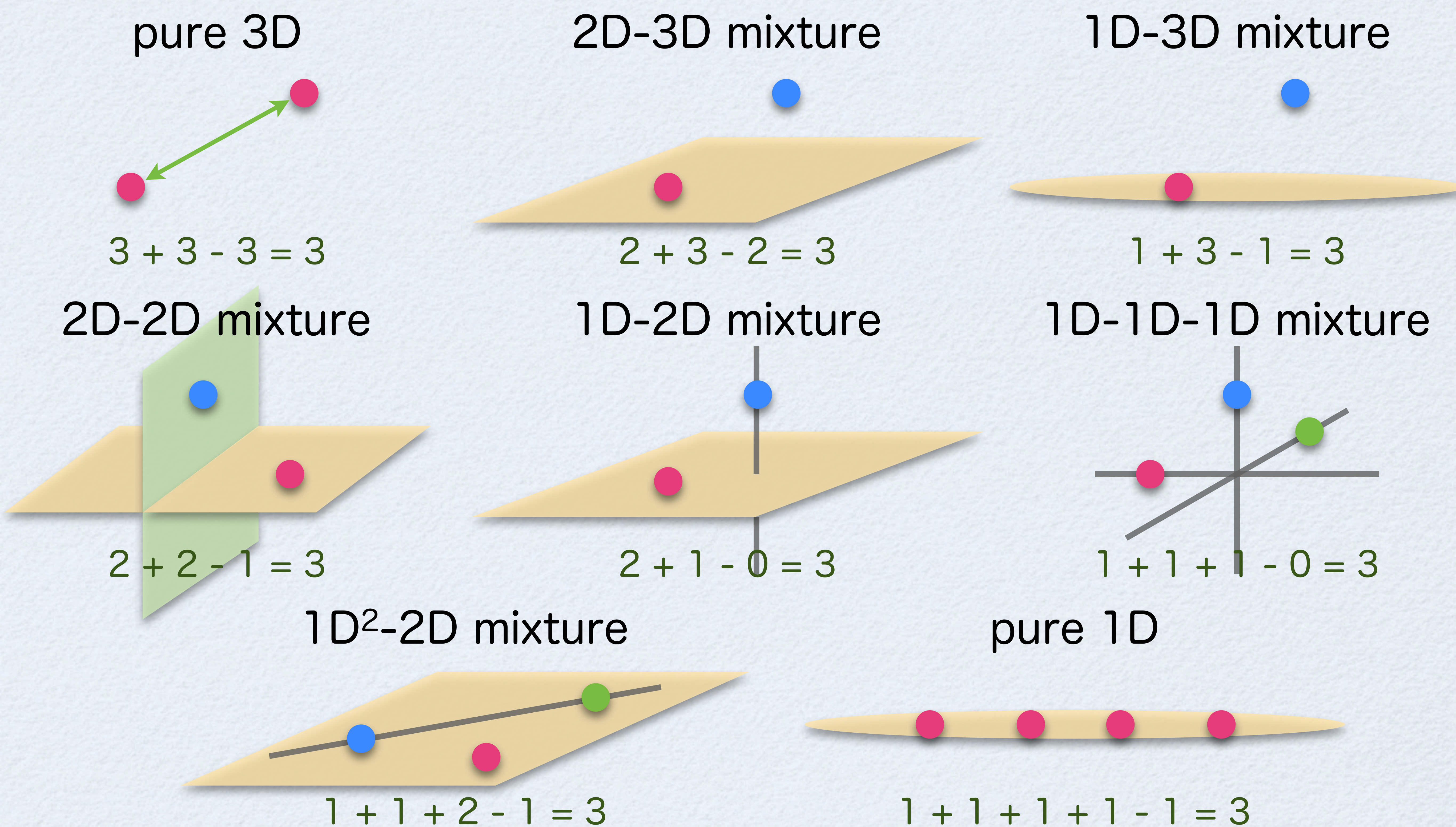
E.g. 3 + 3 - 3 = 3

$$\sum_{i=1}^{N} d_i - d_{\mathrm{com}} = d_{\mathrm{rel}}$$



Family of universal systems

Y. N. & S. Tan
PRL 101 (2008)

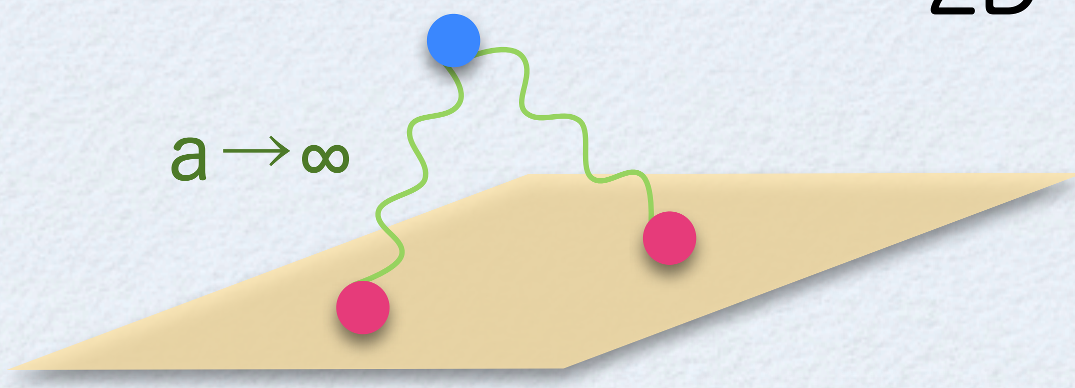


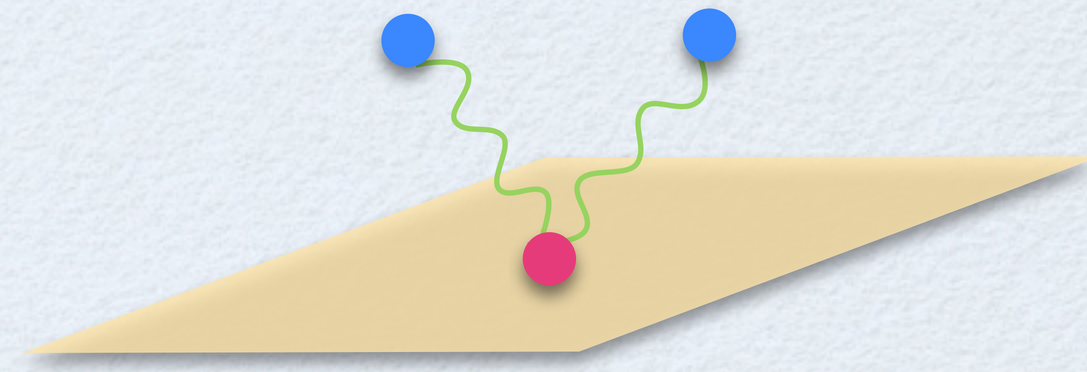
New fields of universal few-body & many-body physics

Efimov effect “beyond” 3D

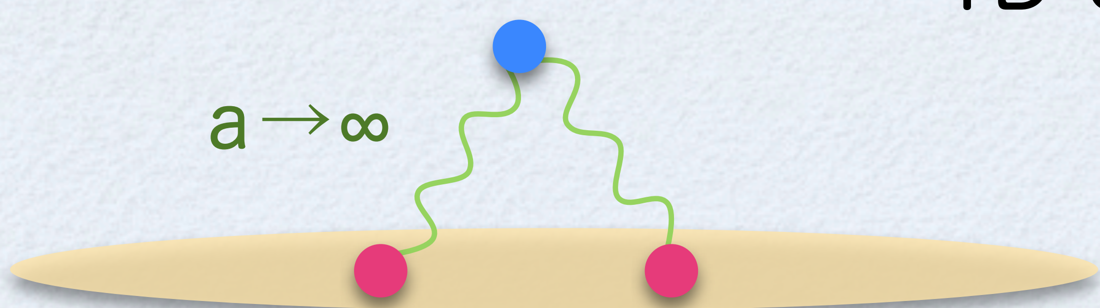
Y. N. & D.T. Son (2010)
Y. N. & S. Tan (2011)

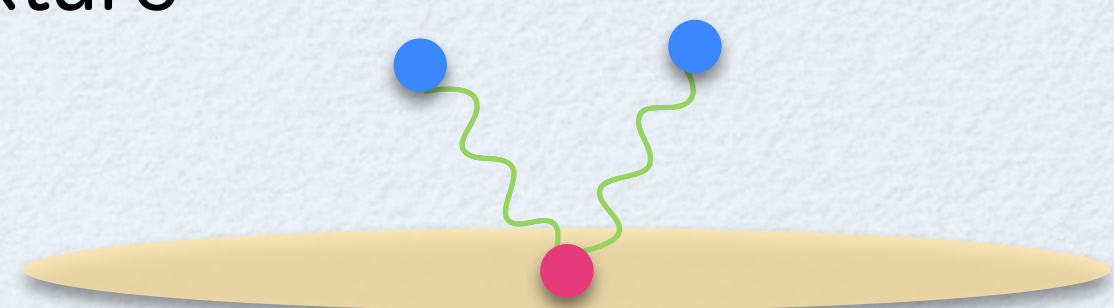
2D-3D mixture



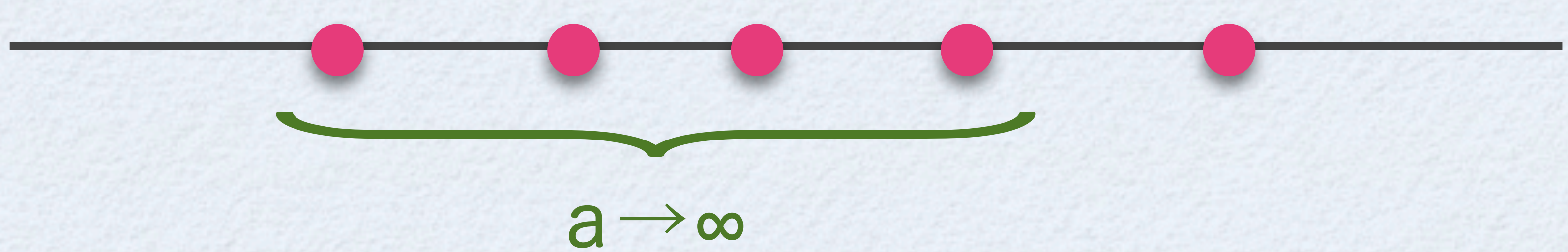


1D-3D mixture





1D with 4-body interaction

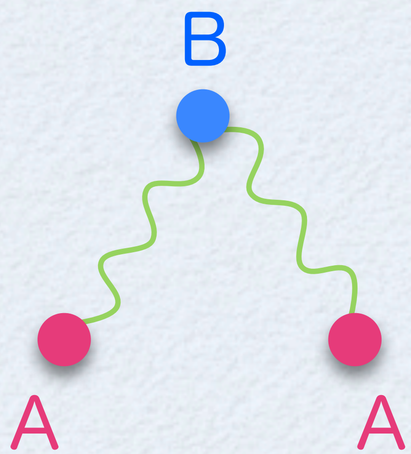


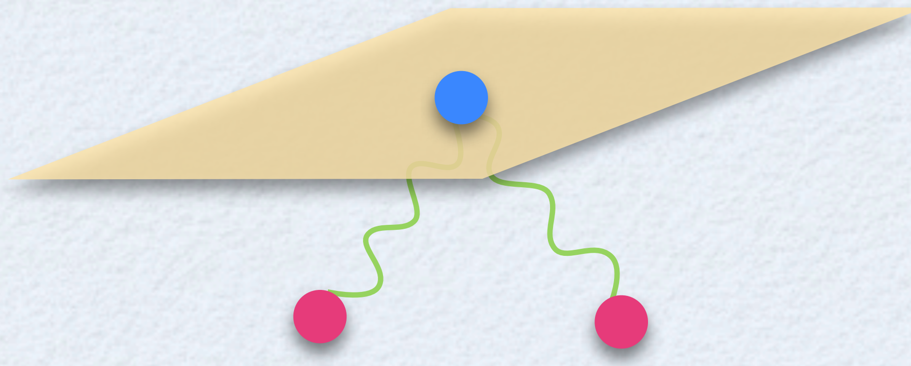
$$\frac{R_{n+1}}{R_n} = 12.4$$

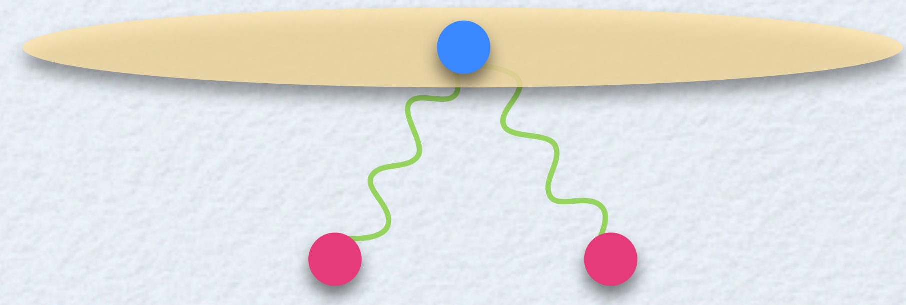
Liberation of Efimov physics (1970~) from 3D !!!

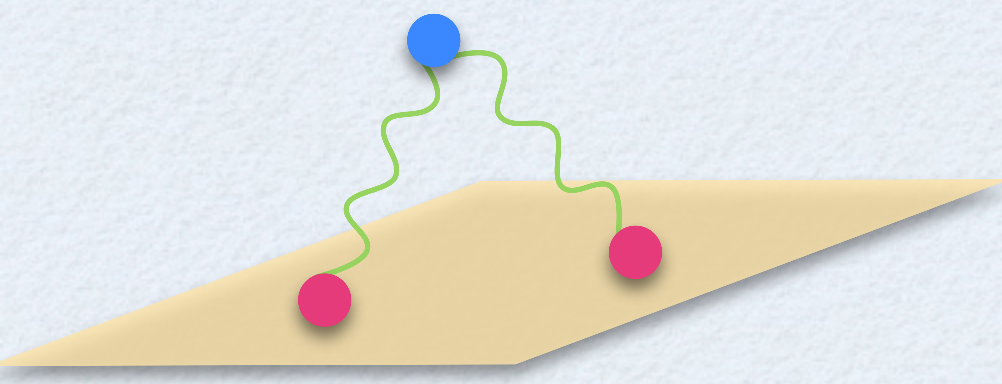
Efimov effect "beyond" 3D

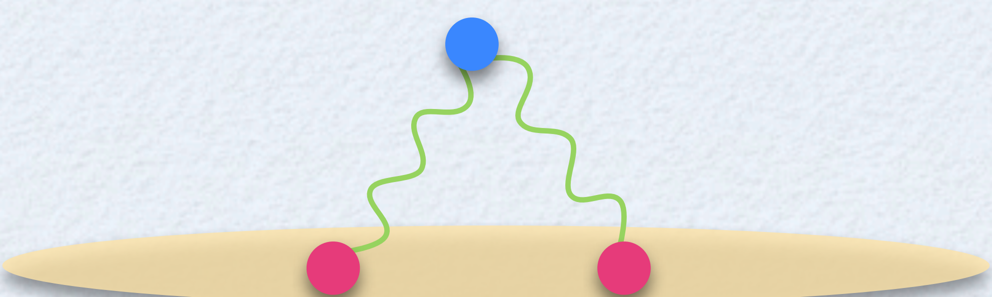
Y. N. & S. Tan (2011)











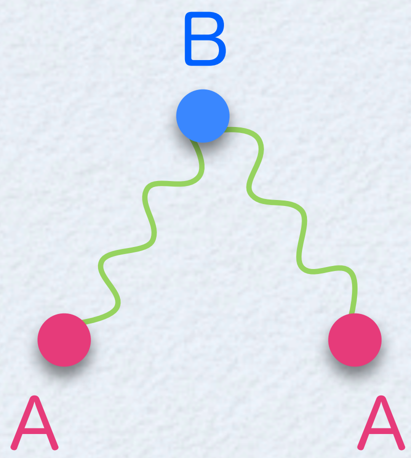
A \ B		3D	2D	1D
3D	boson	○	○	○
	fermion	13.6	28.5	155
2D	boson	○	○	○
	fermion	6.35	11.0	×
1D	boson	○	○	—
	fermion	2.06	×	

A ⇒ bosons : Efimov effect occurs for any mass ratio m_A/m_B

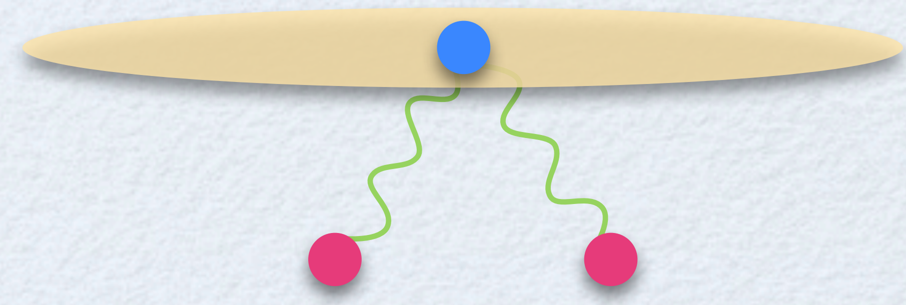
A ⇒ fermions : Efimov effect occurs above a critical m_A/m_B

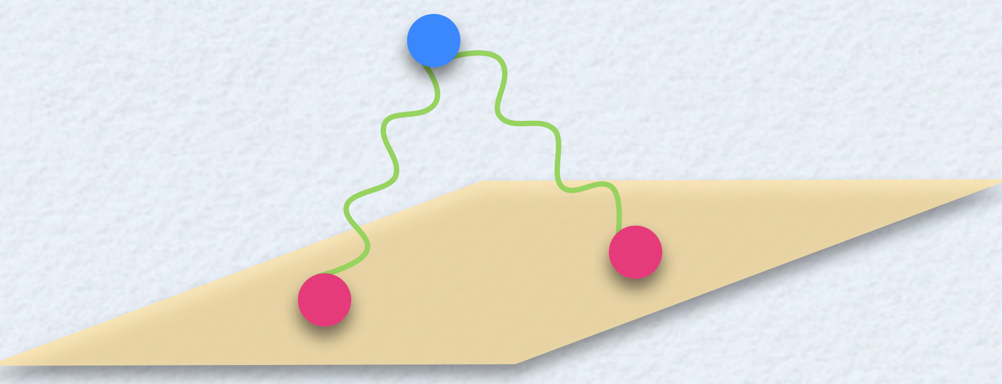
Efimov effect "beyond" 3D

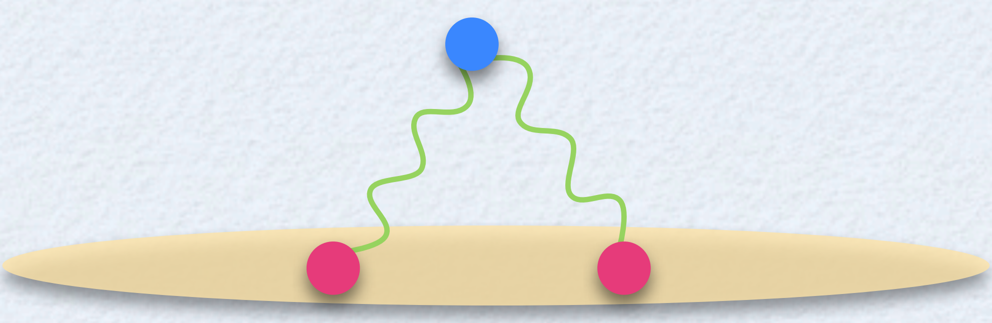
Y. N. & S. Tan (2011)



$$V_{\text{eff}} \sim -\frac{\hbar^2}{m_B R^2} + \frac{\hbar^2}{m_A R^2}$$







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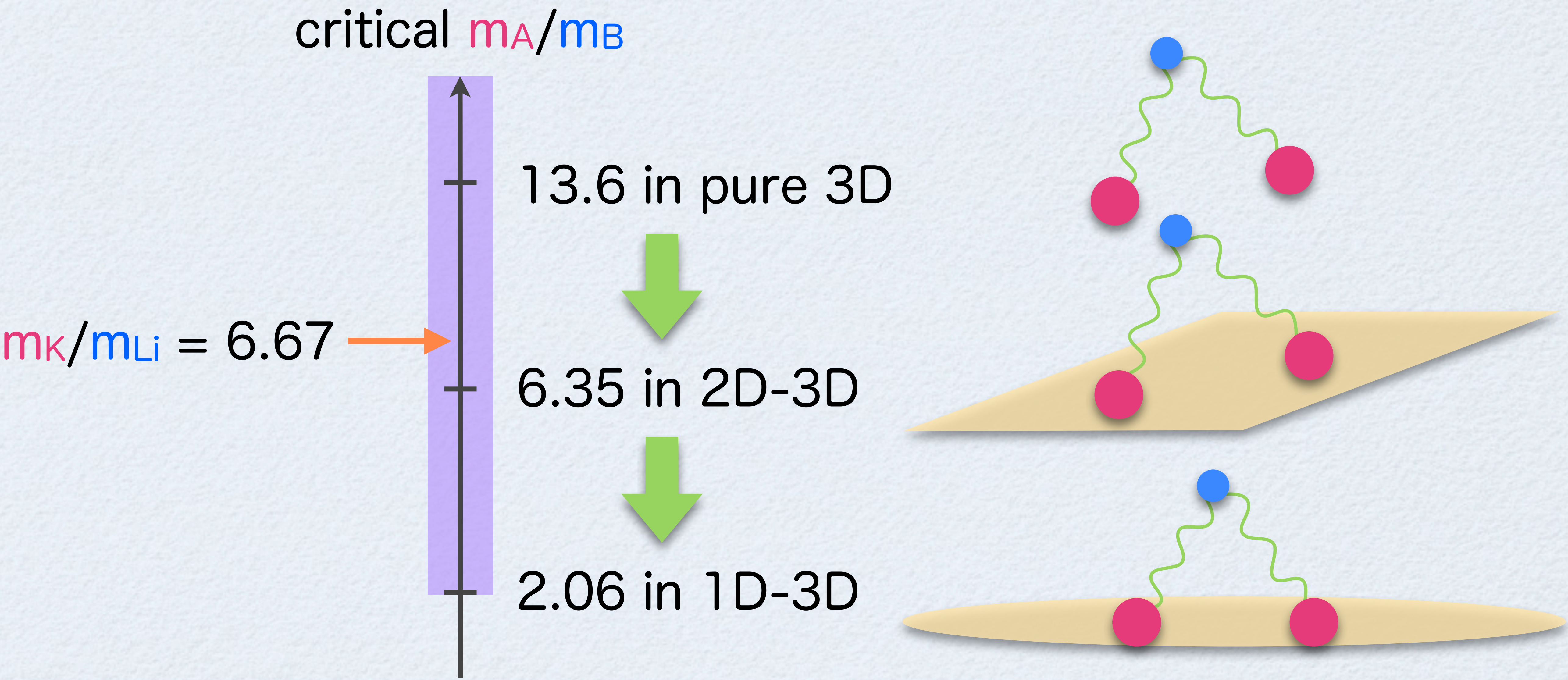
Implication for ^{40}K-^{6}Li mixture

Y. N. & S. Tan
PRA 79 (2009)

5 experimental groups (MIT, Amsterdam, Paris, Munich, Innsbruck)

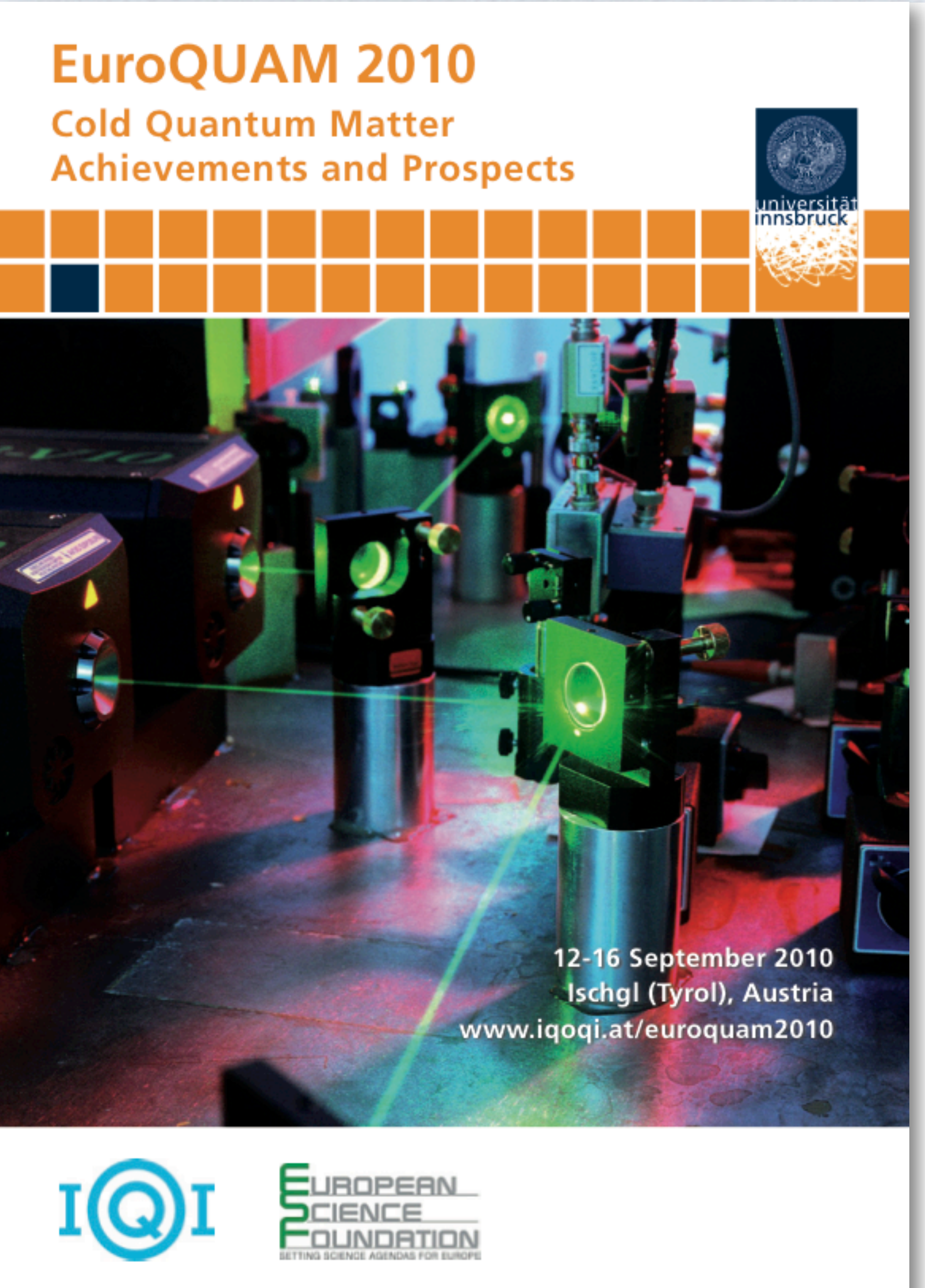
study Fermi-Fermi mixture of $A={}^{40}K$ & $B={}^{6}Li$



Confinement induces the Efimov effect !!!

Implication for ^{40}K-^{6}Li mixture

Ongoing experiment by the Amsterdam group



Towards Fermi-Fermi Systems in Mixed Dimensions

Frederik M. Spiegelhalder

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Antje Ludewig, Tobias Tiecke
and Jook Walraven

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Valckenierstraat 65, NL-1018 XE Amsterdam

Fermi-Fermi systems, in particular the ^{6}Li-^{40}K system, are attracting more and more attention with several experiments running and more being currently set up worldwide. We are now beginning to understand well the basic scattering properties of the Li-K system. Several interspecies Feshbach resonances have been found and characterized [1, 2]. Heteronuclear molecules have been created at several interspecies resonances [3, 4]. But still many effects that are only observable in a mixture of different species have not been observed yet. One significant difference to the single-species experiments is the possibility to apply species-specific optical trapping potentials [5]. Several theoretical proposals suggest that by confining one species using species-selective optical lattices, interesting new phases could be realized [7]. For example one could confine K in a one-dimensional optical lattice and thus create a mixed-dimensional system, where K is confined to two dimensions with Li being confined in 3D. A long-lived universal trimer state is expected to be observed in such a system [6]. Here we present our recent experimental efforts towards realization of a quasi two-dimensional trap for K immersed in a three-dimensional Li gas.

[1] Wille et al. Phys. Rev. Lett. **100** (2008) 053201
[2] Tiecke et al. Phys. Rev. Lett. **104** (2010) 053202

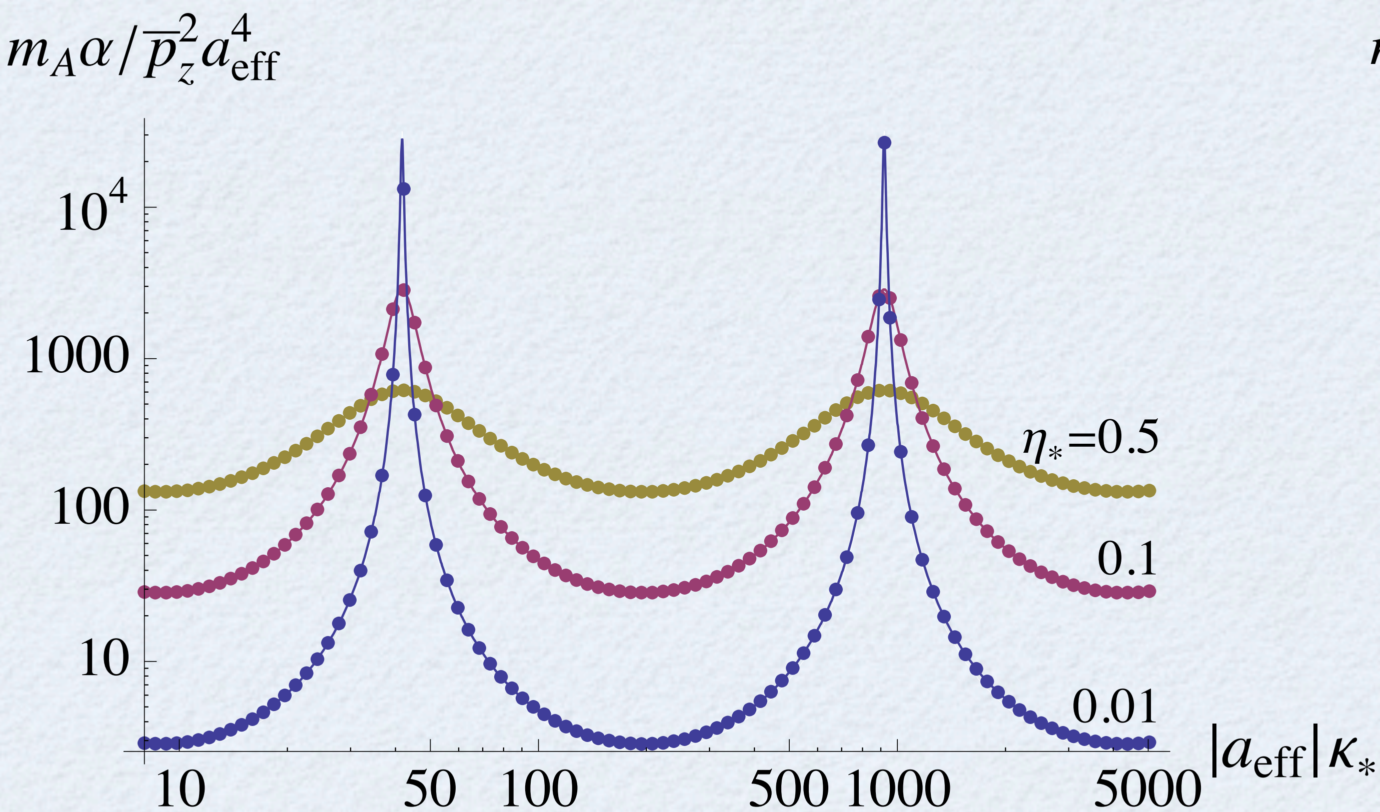
3-body recombination rate

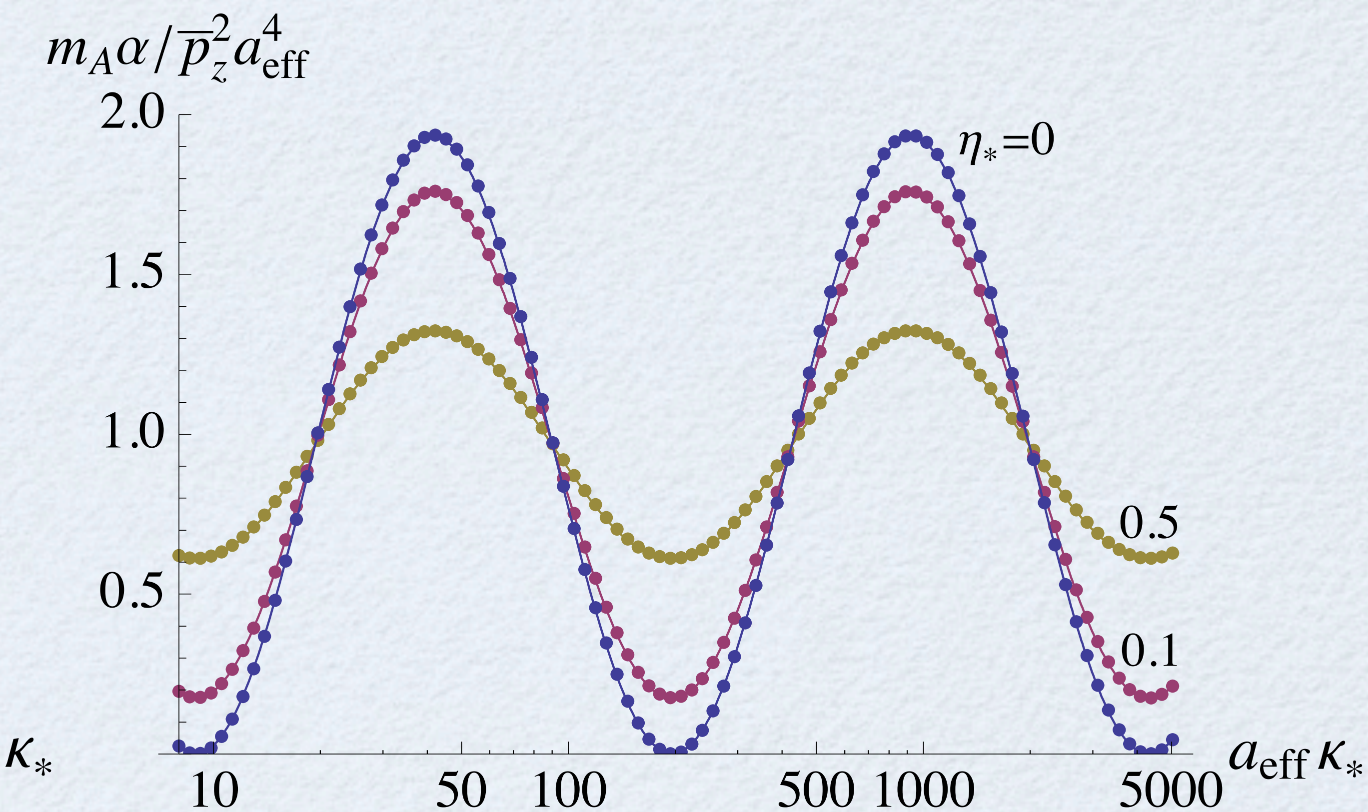
Y. N. & S. Tan
PRA 79 (2009)

3-body recombination (A+A+B->A+AB) results in atom losses

$$\dot{n}_A \approx -2\,\alpha\, n_A^2 n_B$$

Its rate constant α has the characteristic log-periodic behaviors with the scaling factor $\lambda = 22.0$ for A=^{40}K in 1D & B=^{6}Li in 3D





induced Efimov resonances for $a_{\mathrm{eff}}<0$ deconstructive interferences for $a_{\mathrm{eff}}>0$

Efimov parameter $\kappa_* \approx 1.91/l$ & width parameter $\eta_* \sim (r_0/l)^{2.39} \ll 1$

If observed, the first evidence of the Efimov effect of fermions !!!

Experimental realization of mixed dimensions

First experiment @ Florence

Bose-Bose mixture of A=^{41}K in 2D & B=^{87}Rb in 3D

PRL **104**, 153202 (2010) PHYSICAL REVIEW LETTERS week ending 16 APRIL 2010

Scattering in Mixed Dimensions with Ultracold Gases

G. Lamporesi,[1] J. Catani,[1,2] G. Barontini,[1] Y. Nishida,[3] M. Inguscio,[1,2] and F. Minardi[1,2]
[1]*LENS—European Laboratory for Nonlinear Spectroscopy and Dipartimento di Fisica, Università di Firenze, via Nello Carrara 1, I-50019 Sesto Fiorentino, Italy*
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[3]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
(Received 26 January 2010; published 14 April 2010)

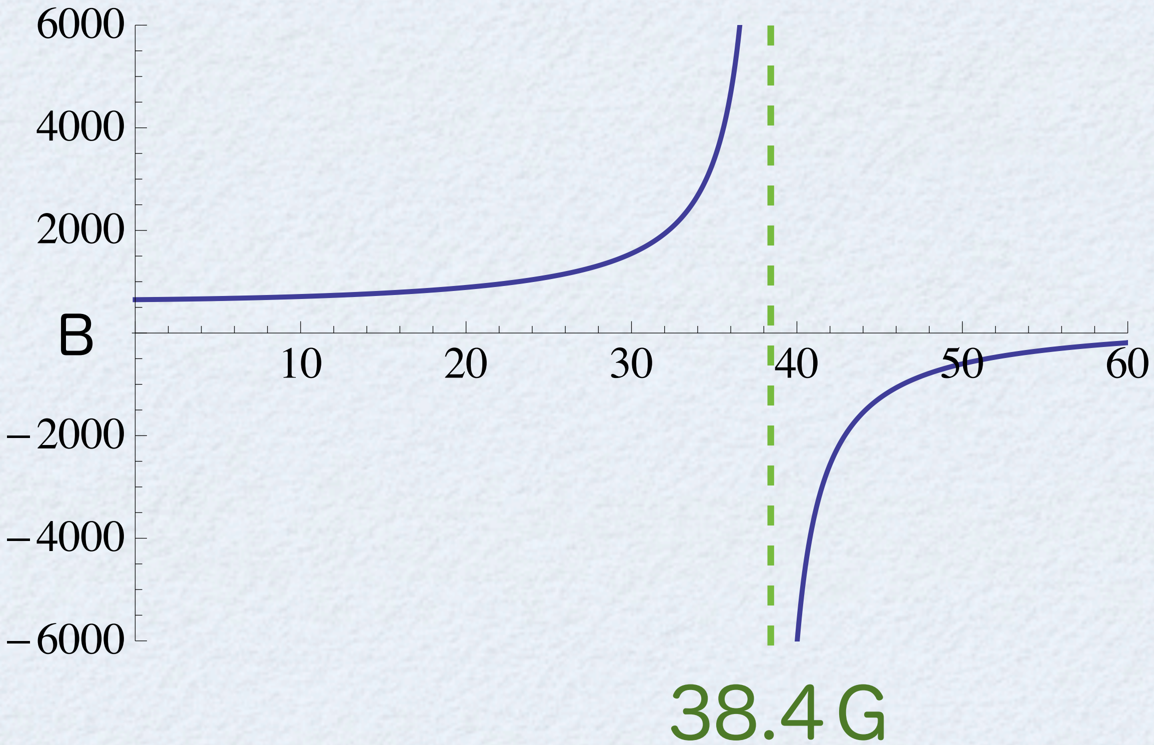
We ex… partners
live in d… species-
selective … magnetic
field in pr… a series of
resonance… etic field
values co… with the
theoretic…

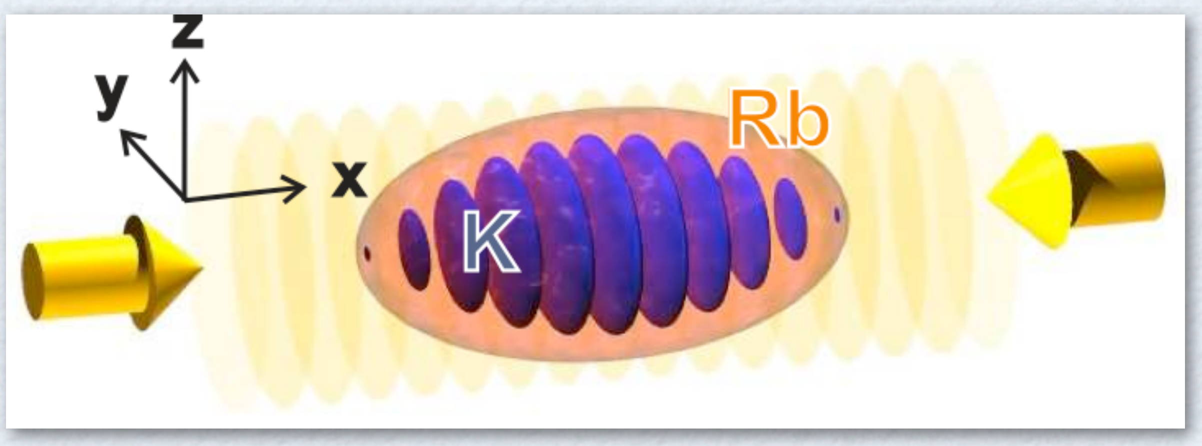
DOI: 10.1… 67.85.−d

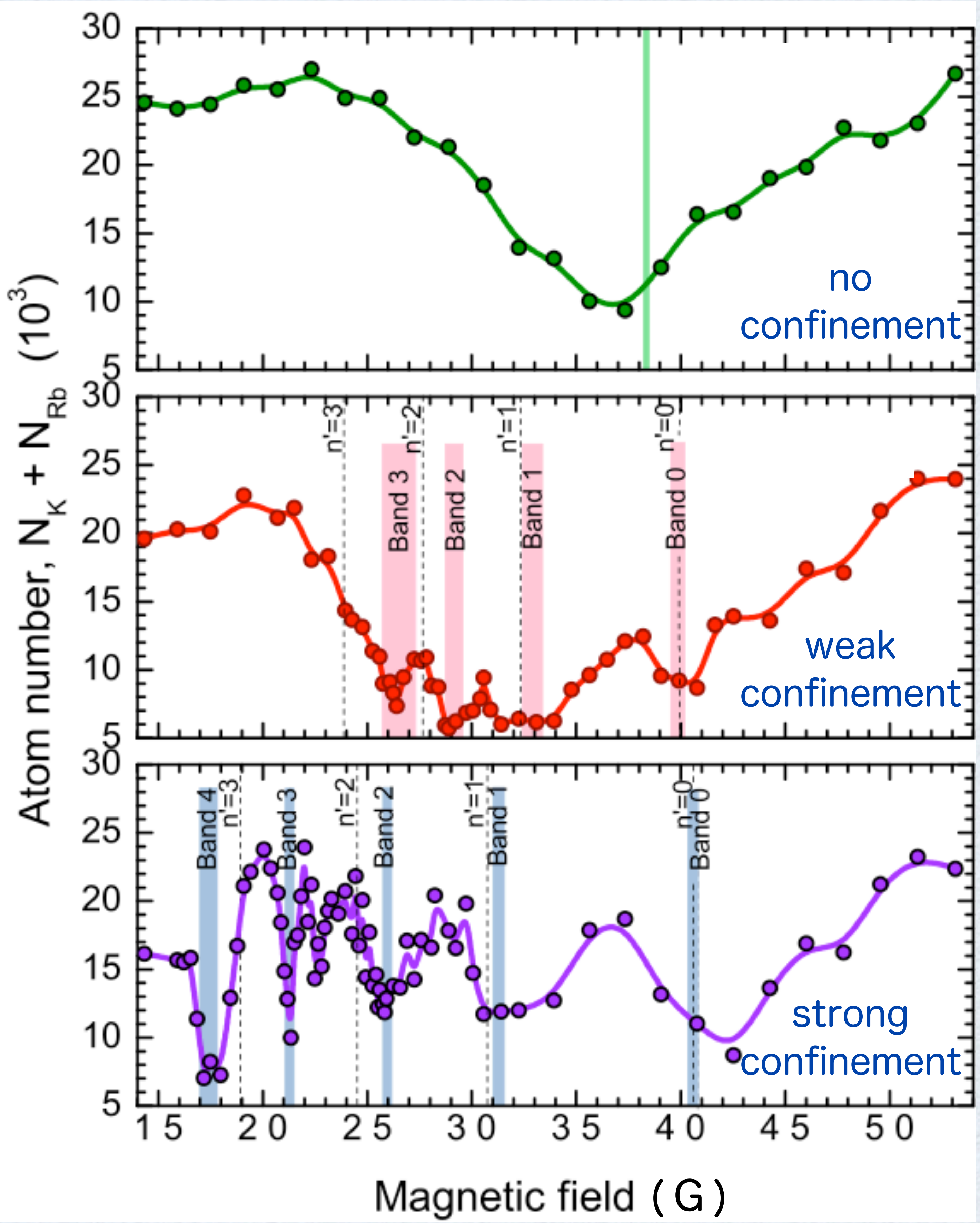
Degenerate atomi… nces, induced by the
tems with unpreceden… resonances are pecu-
control, achieved by … ne collisional partner
tentials as well as sca… e in 3D homogeneous
model and control tig… absent for confined
experimental investig… equencies. Important
dimensionality, since … ensional resonances

First experiment @ Florence

3D scattering length a (a_0)





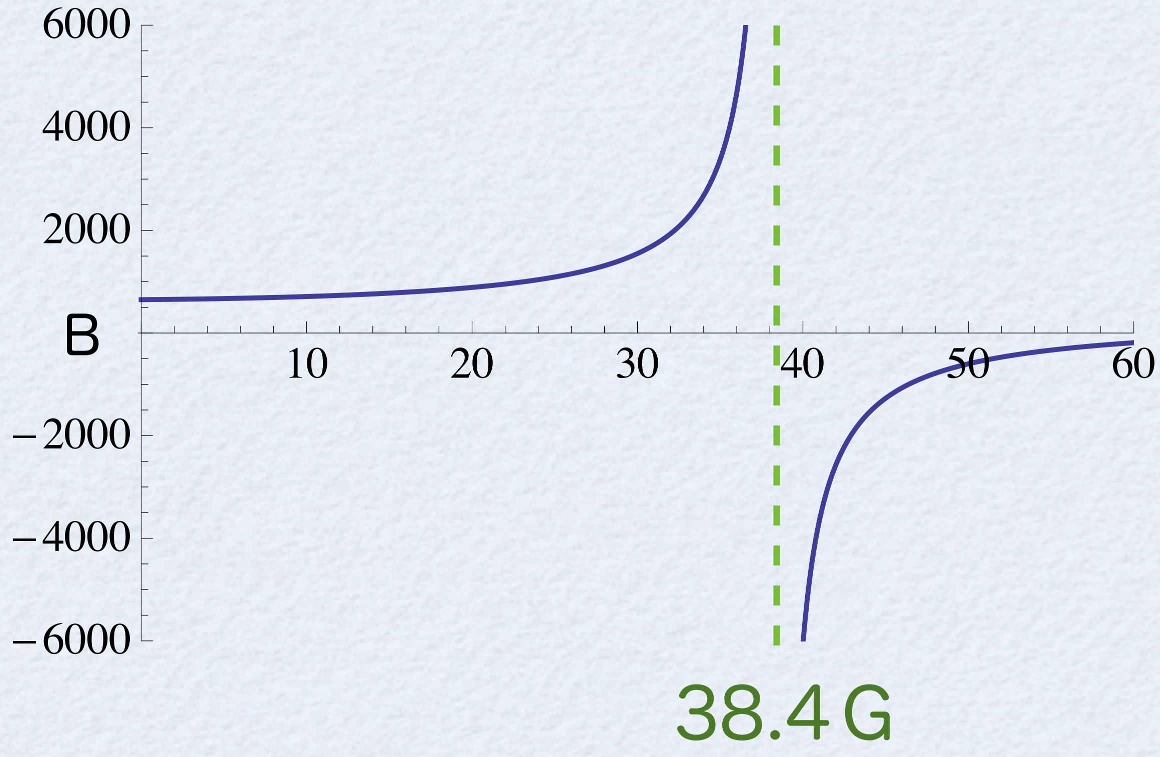


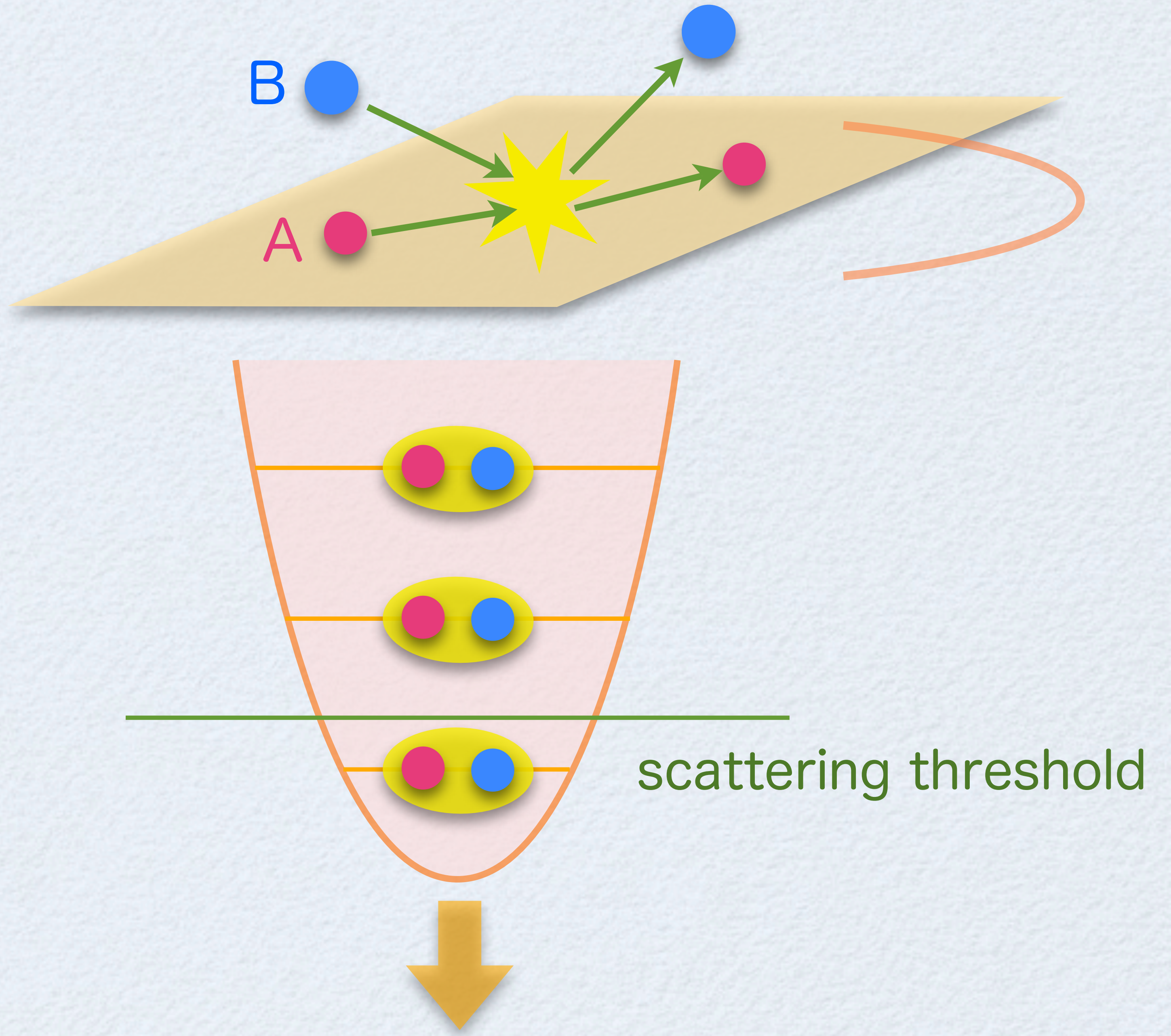
A=^{41}K in 2D & B=^{87}Rb in 3D

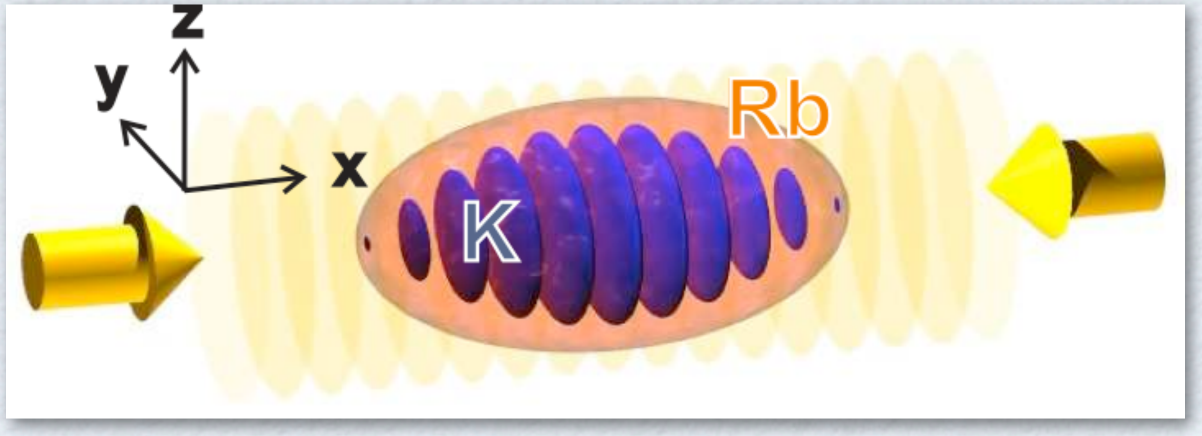
Multiple resonances in mixed D

Y. N. & S. Tan
PRA 79 (2010)

3D scattering length a (a_0)







$$\left(\frac{1}{2}+n\right)\hbar\omega_M - \frac{\hbar^2}{ma^2} = \frac{1}{2}\hbar\omega_A$$

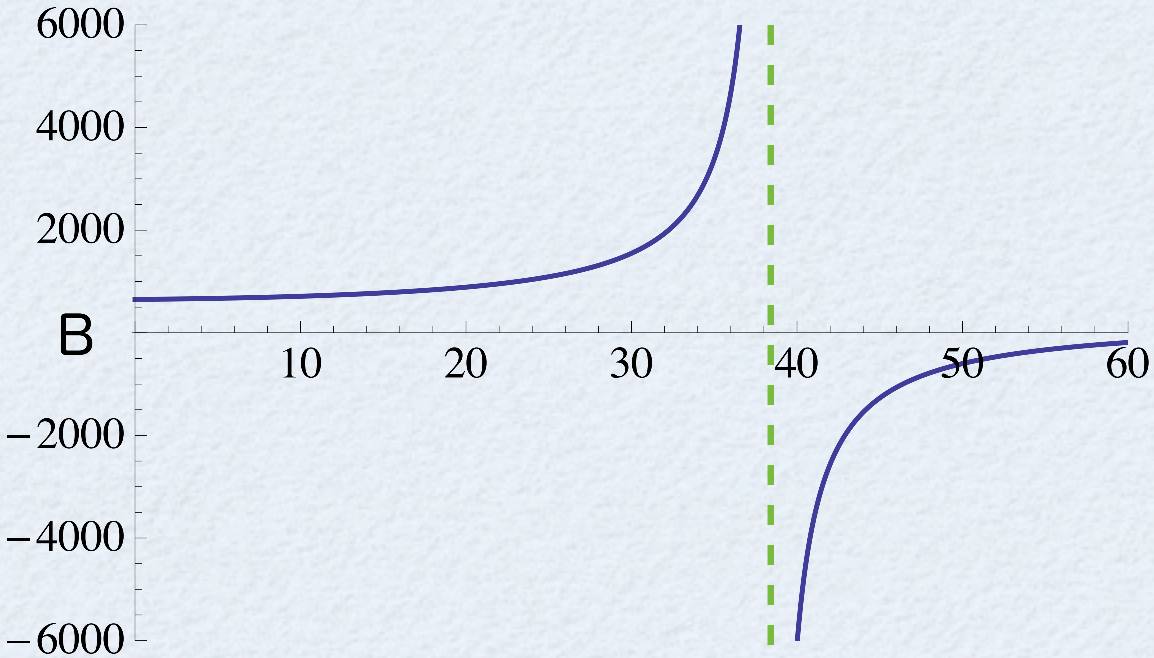
A=^{41}K in 2D & B=^{87}Rb in 3D

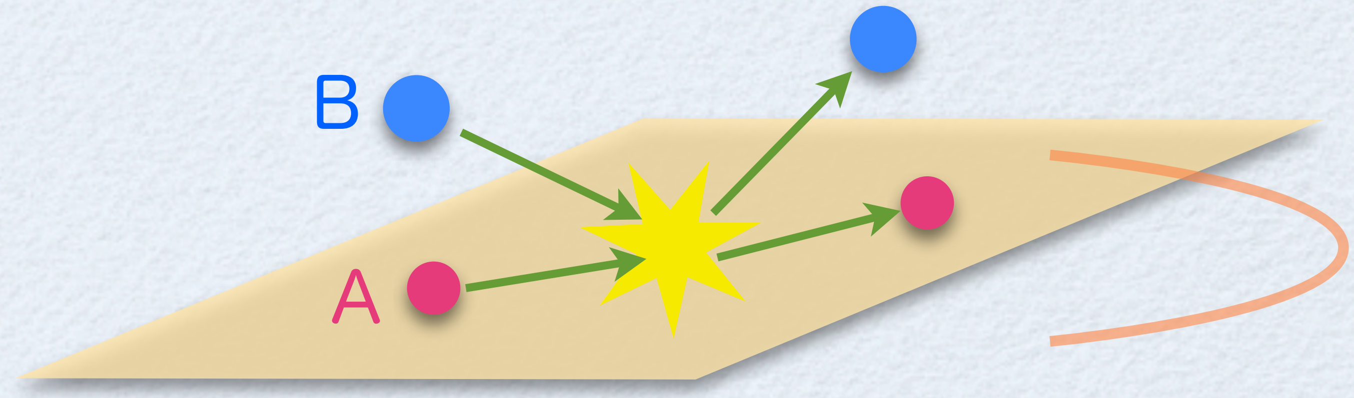
See also 0D-3D case;
P. Massignan & Y. Castin, PRA 74 (2006)

Multiple resonances in mixed D

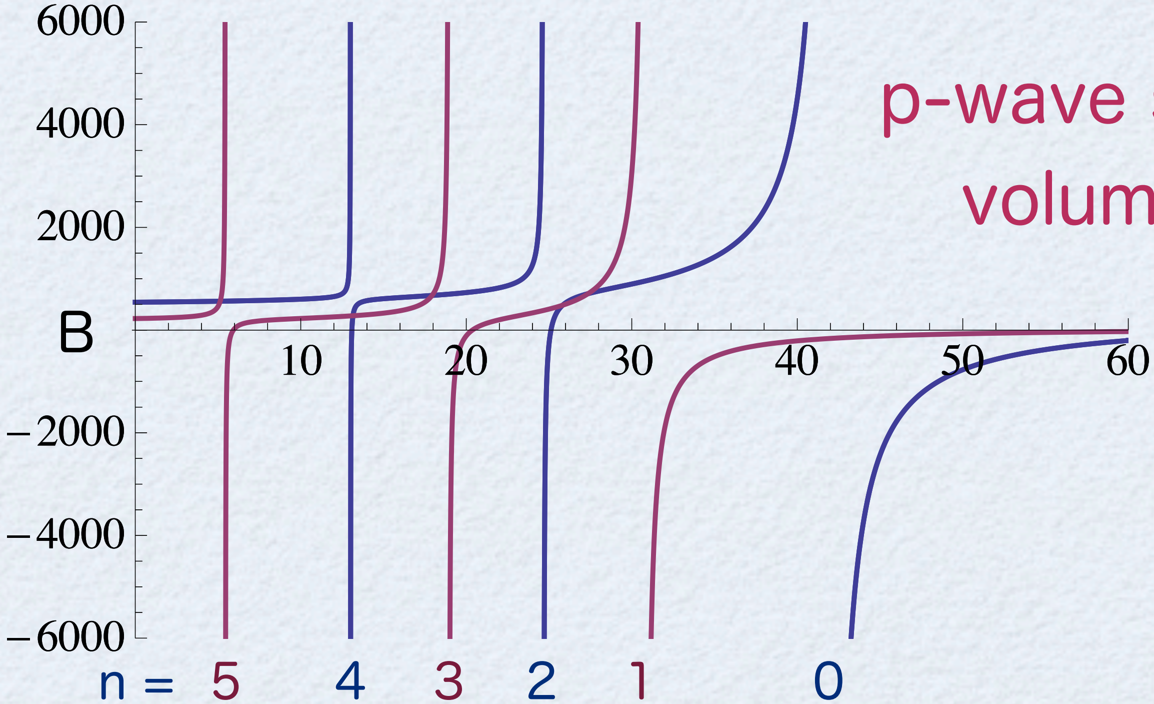
Y. N. & S. Tan
PRA 79 (2010)

3D scattering length a (a_0)

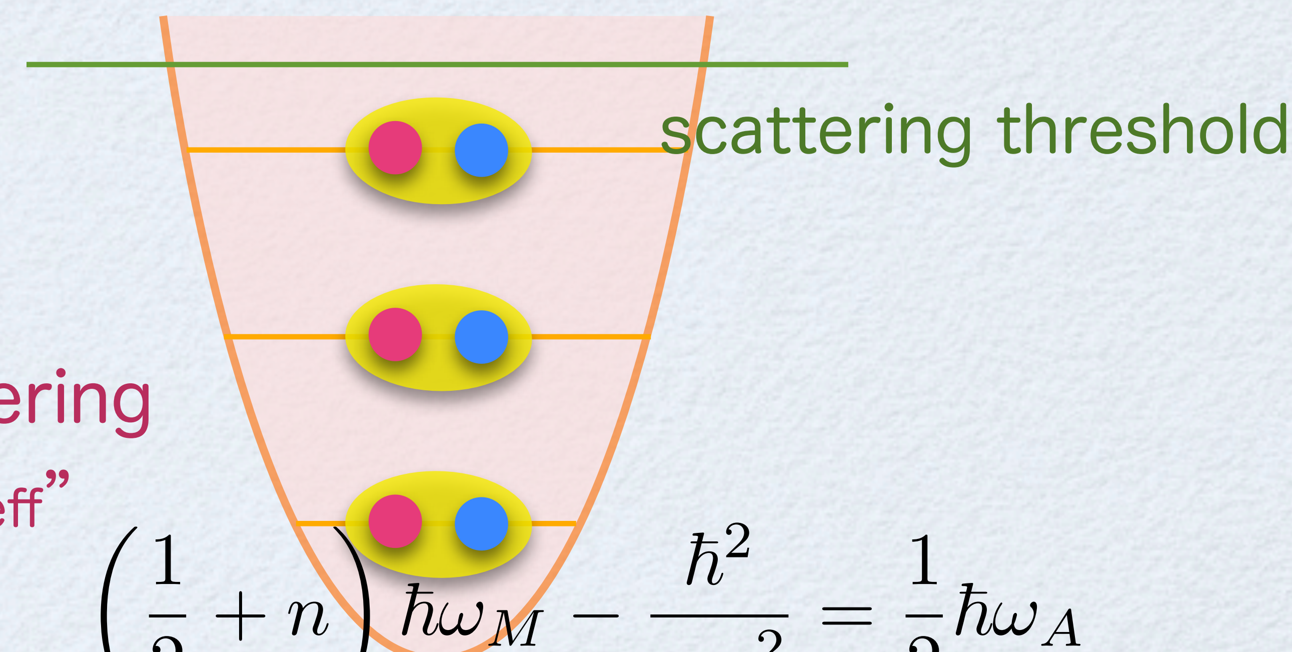




effective scattering length "a_{eff}" in 2D-3D mixture



p-wave scattering volume "v_{eff}"

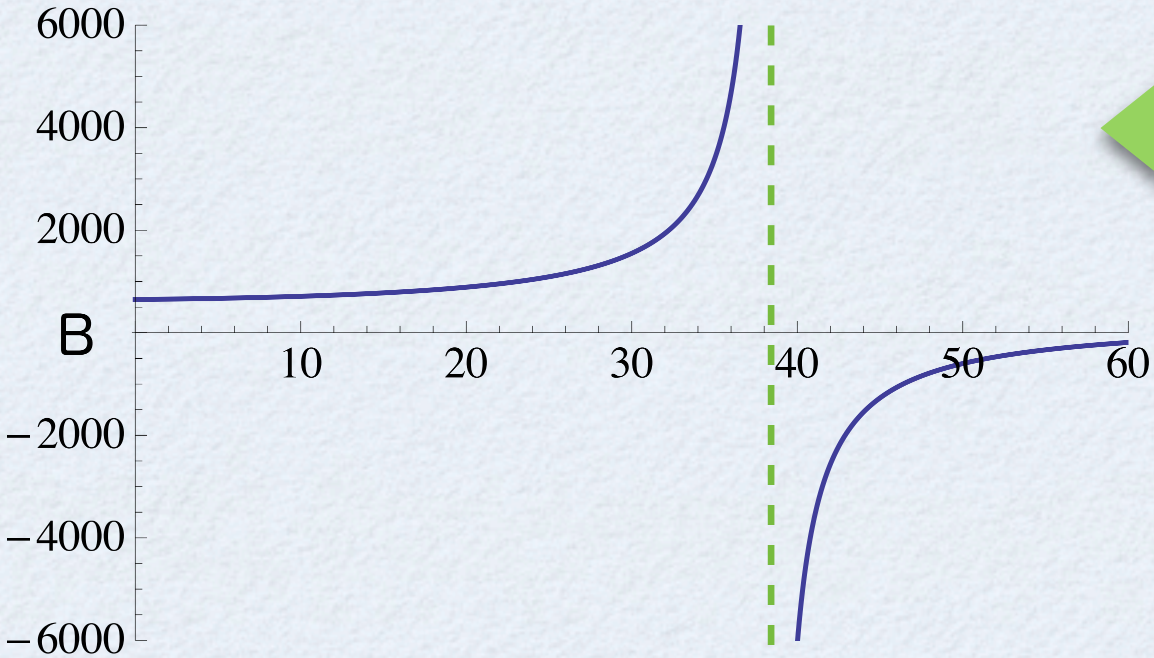


$$\left(\frac{1}{2}+n\right)\hbar\omega_M - \frac{\hbar^2}{ma^2} = \frac{1}{2}\hbar\omega_A$$

See also 0D-3D case;
P. Massignan & Y. Castin, PRA 74 (2006)

Multiple resonances in mixed D

3D scattering length a (a_0)



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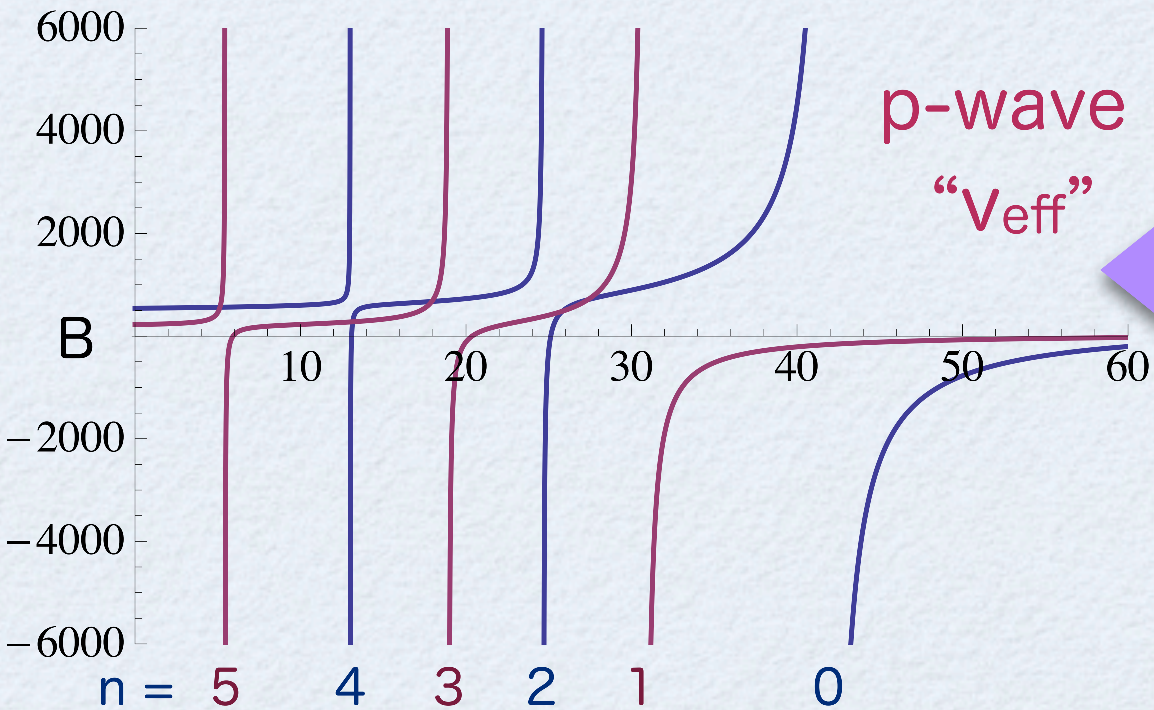
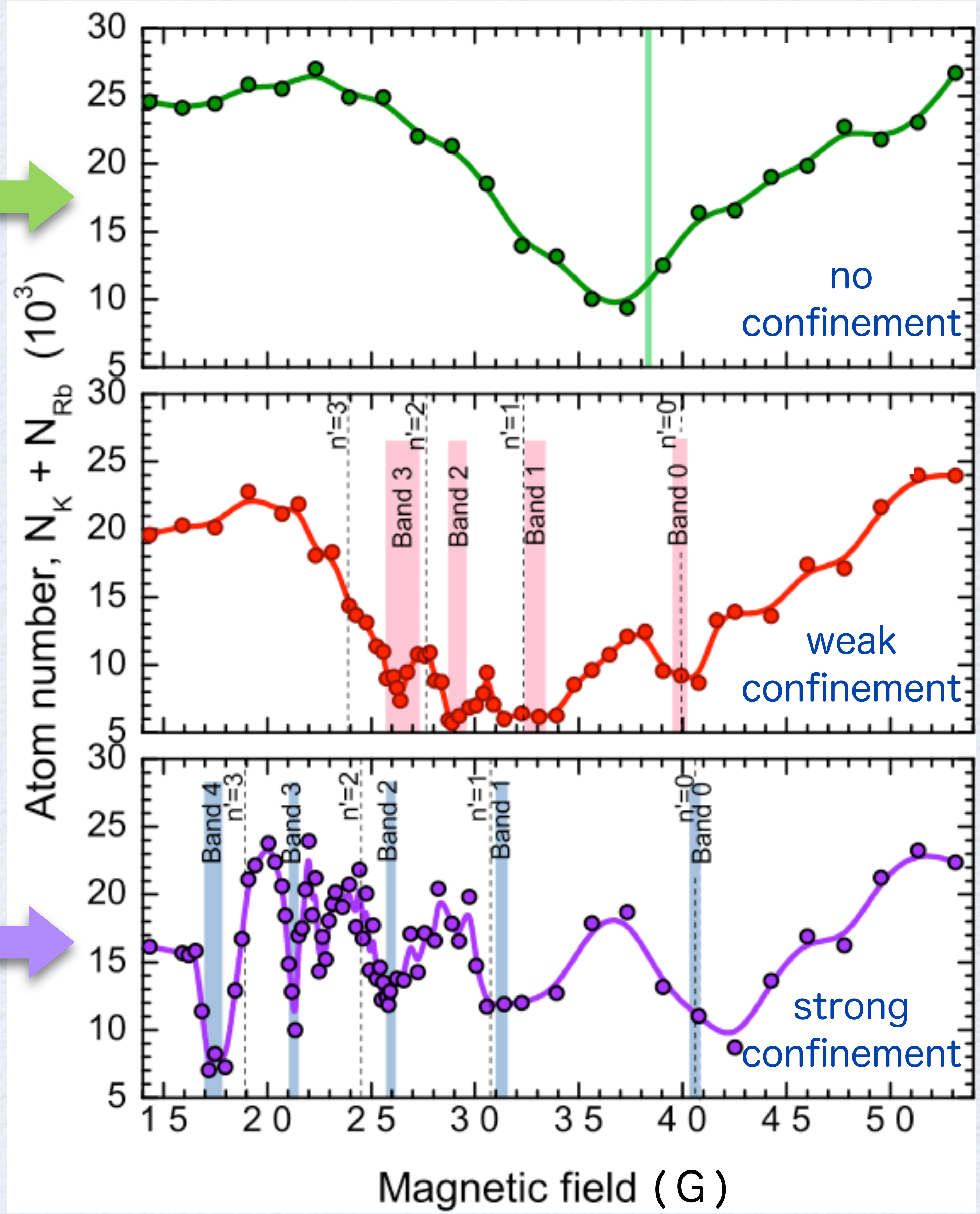
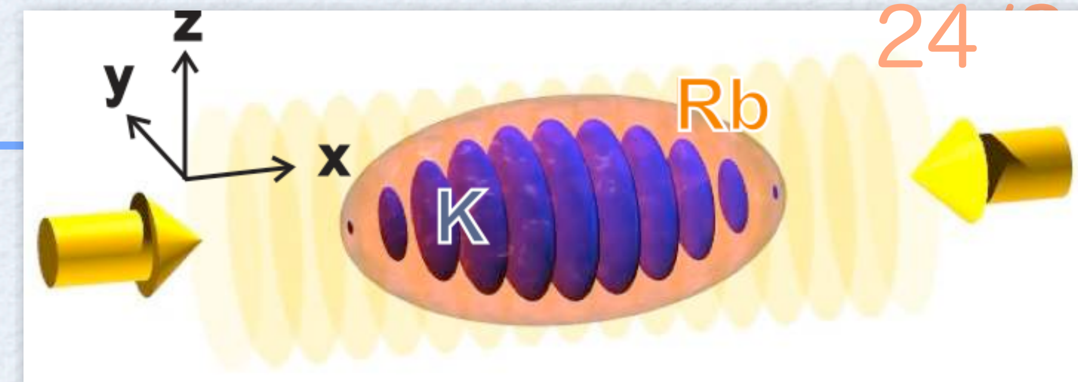




Applications of mixed D



Florence group experiment

- observed 2-body scattering resonances in mixed D
- demonstrated the tunability of interaction strength

Important first step toward a rich variety of physics

✓ Efimov physics in mixed dimensions

Y. N. & S. Tan, PRL 101 (2008), PRA 79 (2009)

✓ If A atoms are confined in 0D

➡ Impurities resonant in s-wave, p-wave, ... channels

- Anderson localization [P. Massignan & Y. Castin, PRA 74 (2006)]
- Infrared catastrophe / Kondo physics [work in progress with E. Demler]

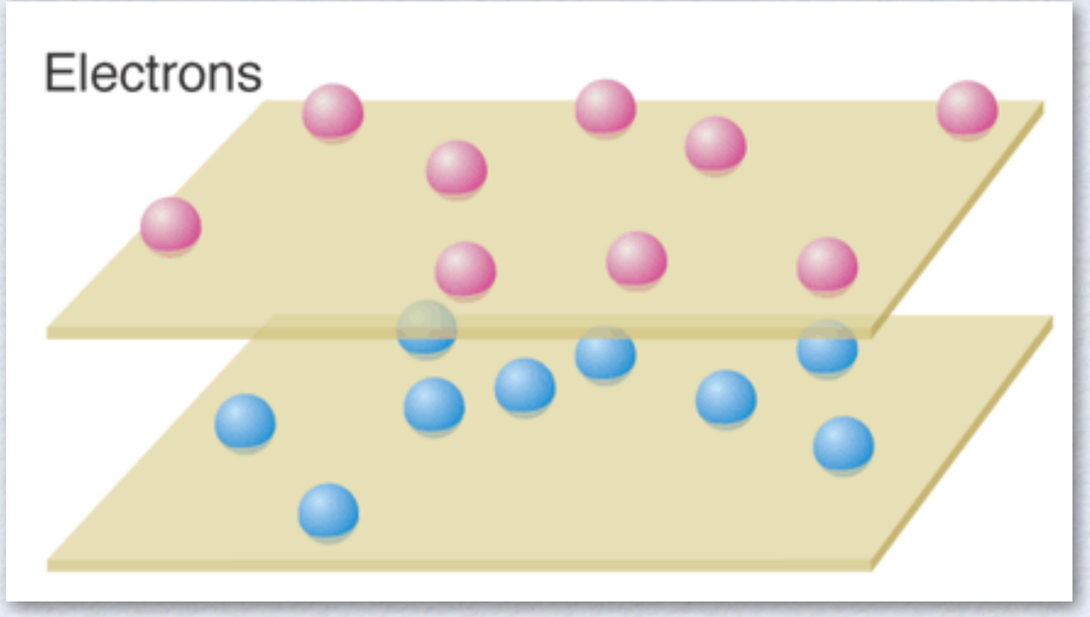
✓ Rich many-body physics ➡ next part of this talk

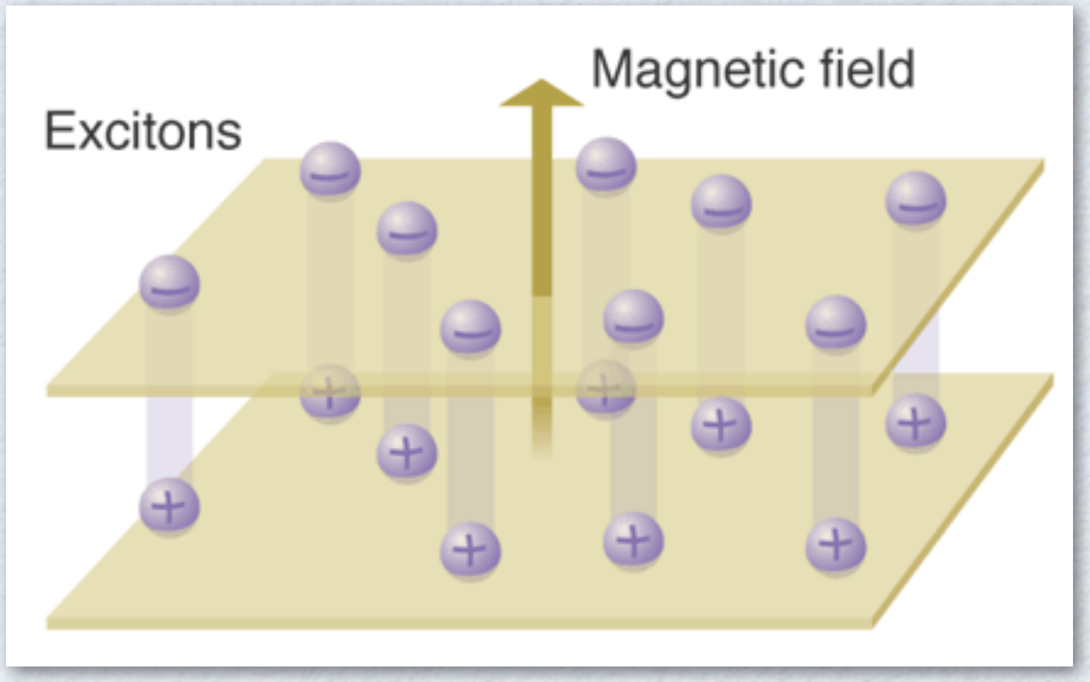
Many-body physics in mixed dimensions

What’s unique in mixed D ?

- In condensed matter, layered systems have rich physics because of long-range Coulomb interaction

  E.g. Interlayer exciton superfluidity in bilayer systems

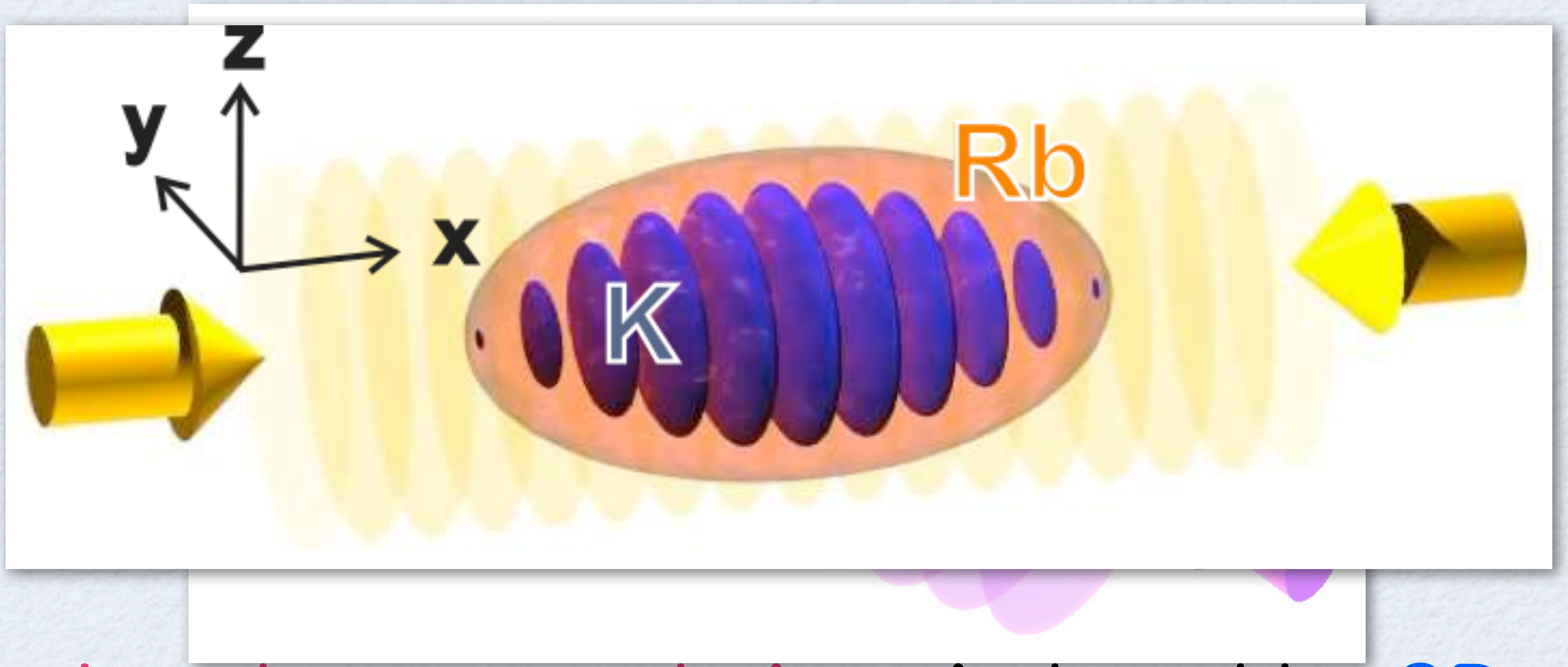




J. P. Eisenstein, Science (2004)

What's unique in mixed D ?

- In condensed matter, layered systems have rich physics because of long-range Coulomb interaction
- Multi-layers can be easily created by a 1D optical lattice
- However, separated layers decouple in neutral atoms



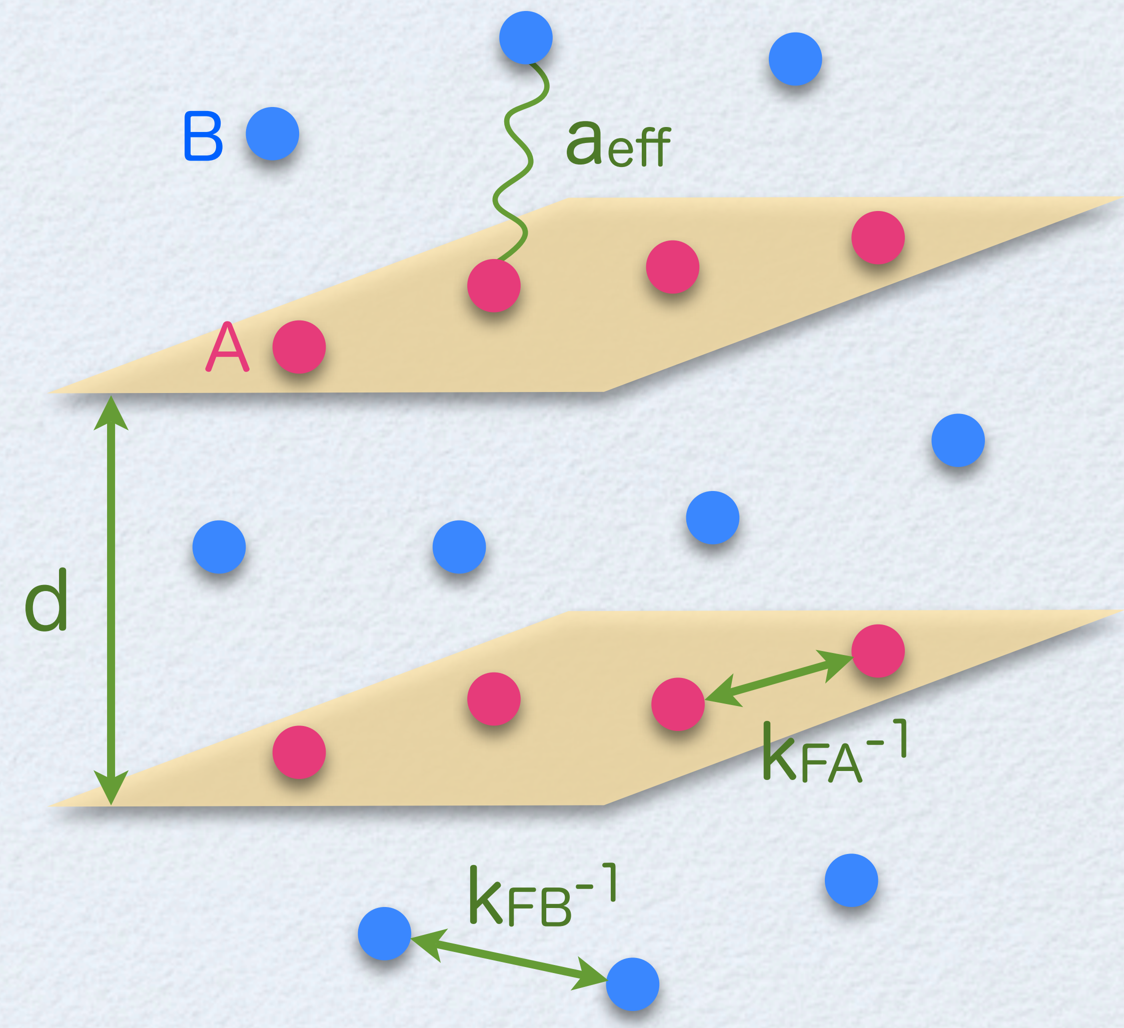
→ Interlayer correlations induced by 3D atoms can lead to rich physics !!!

Bilayer Fermi-Fermi mixture

Y. N., PRA 82 (2010)

parameters of the system

- a_{eff}
- k_{FA}
- k_{FB}
- m_A/m_B
- T
- d

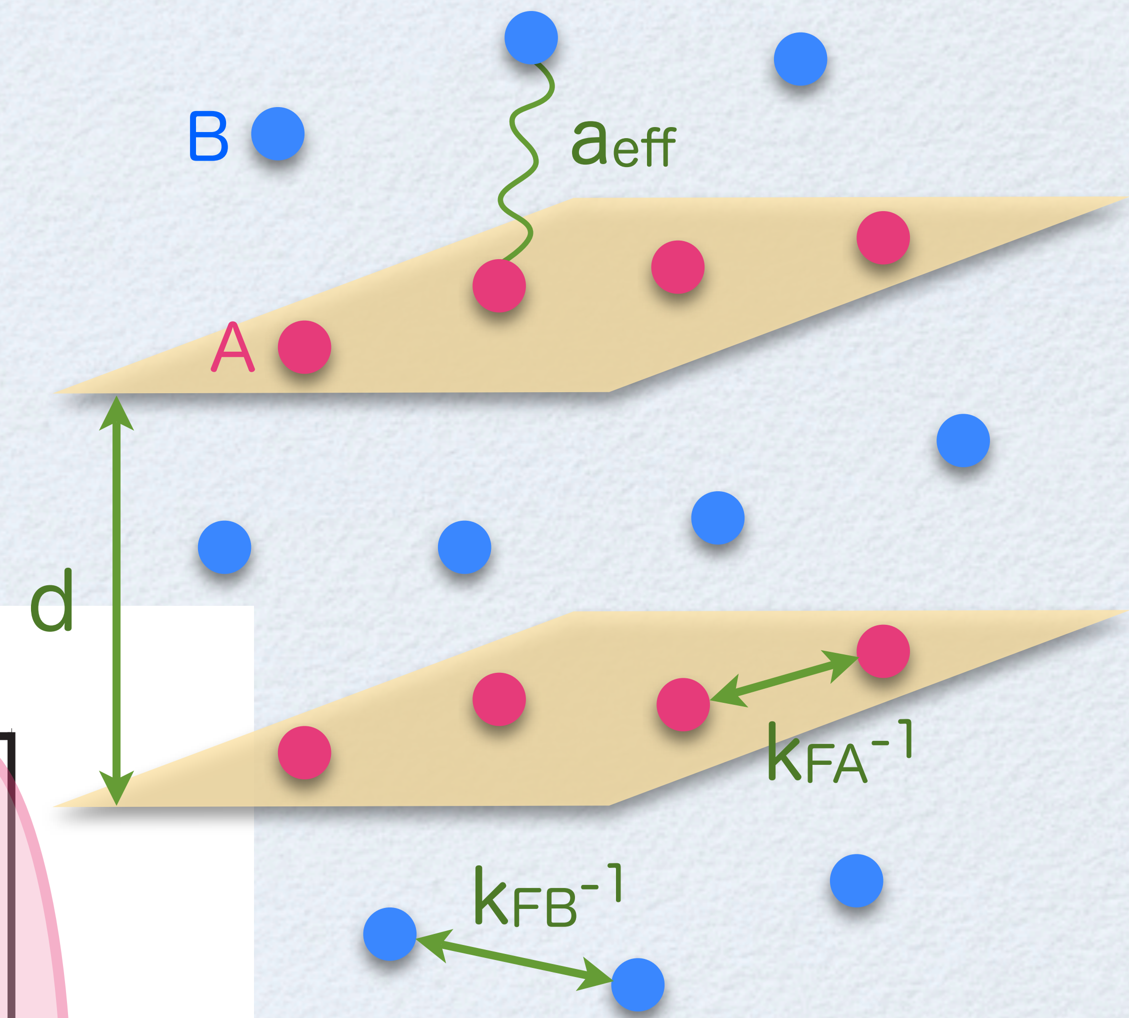


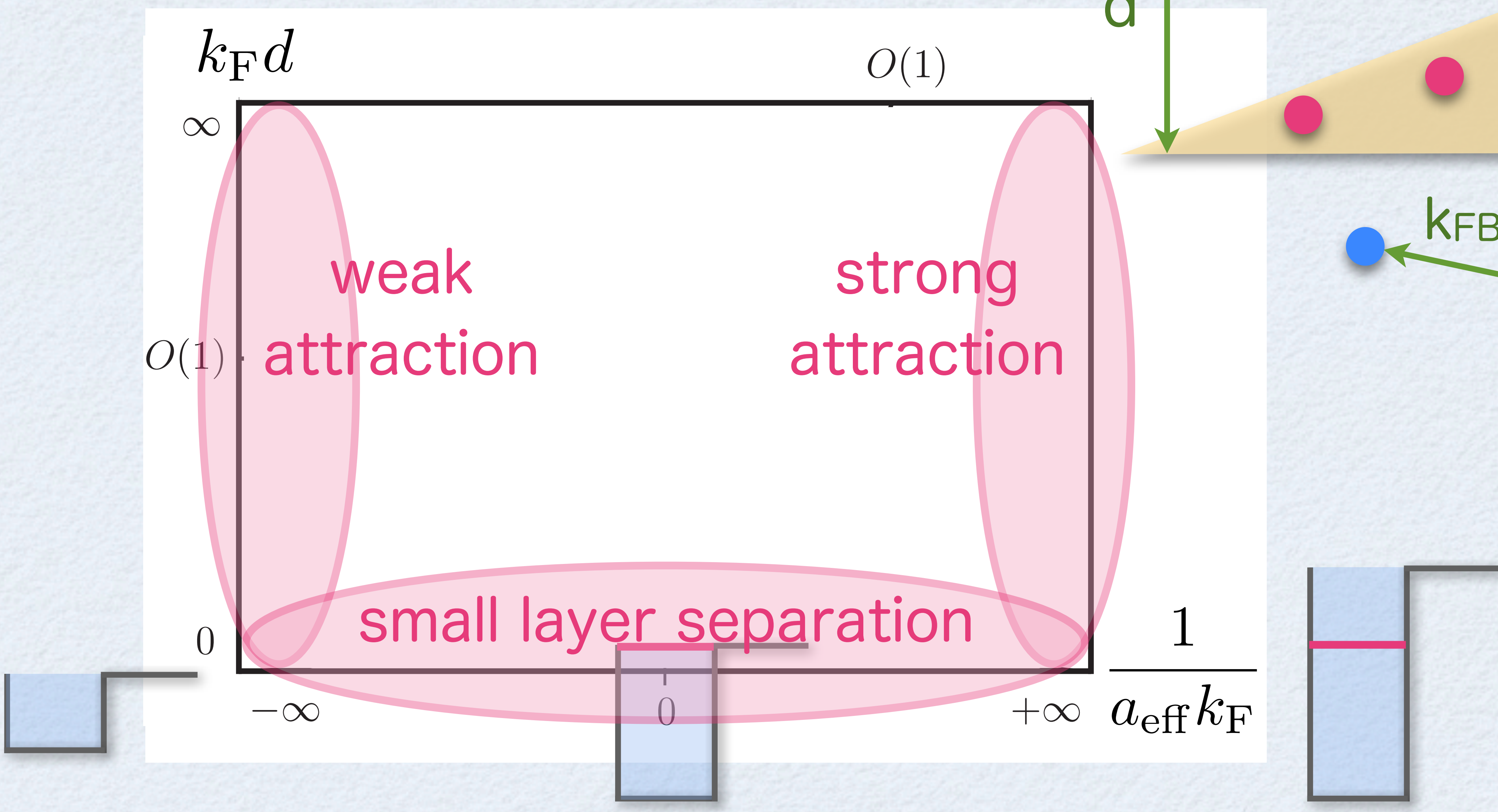
Bilayer Fermi-Fermi mixture

Y. N., PRA 82 (2010)

parameters of the system

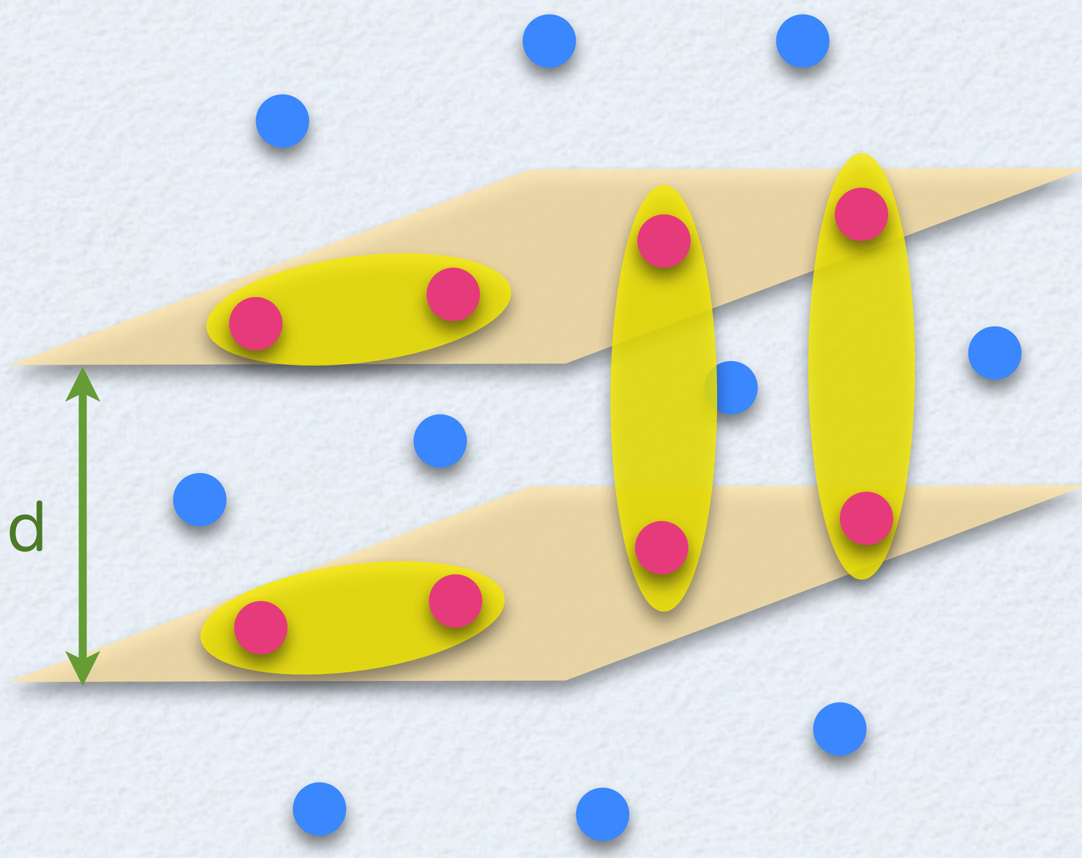
- a_{eff}
- k_{FA} } k_F
- k_{FB}
- $m_A/m_B < 6.35$
- $T=0$
- d

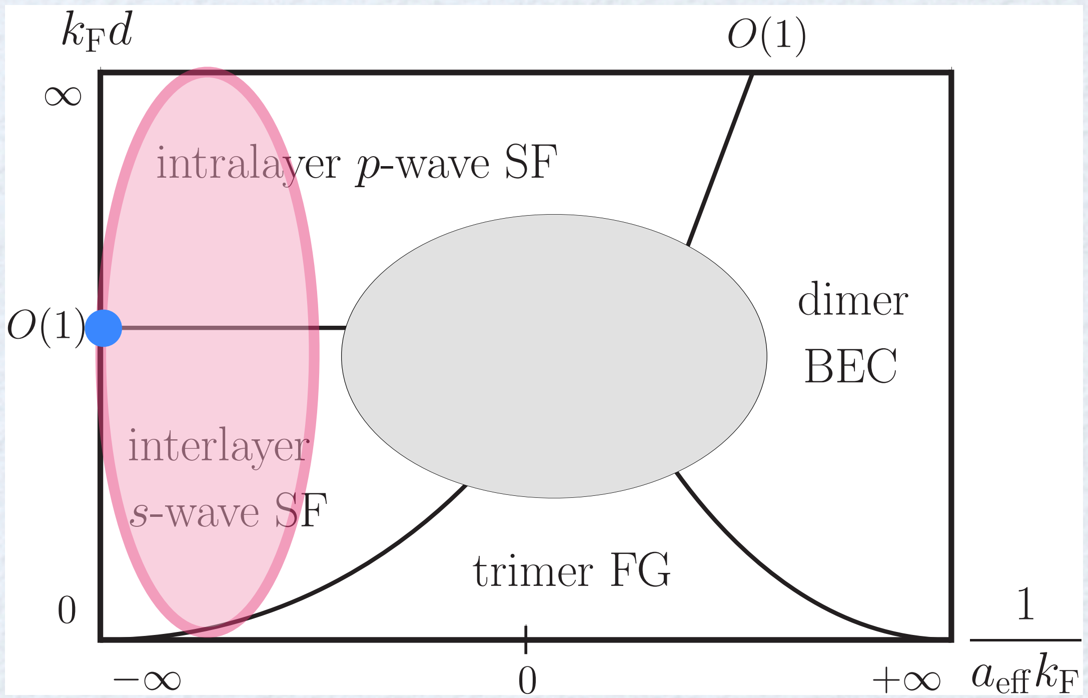


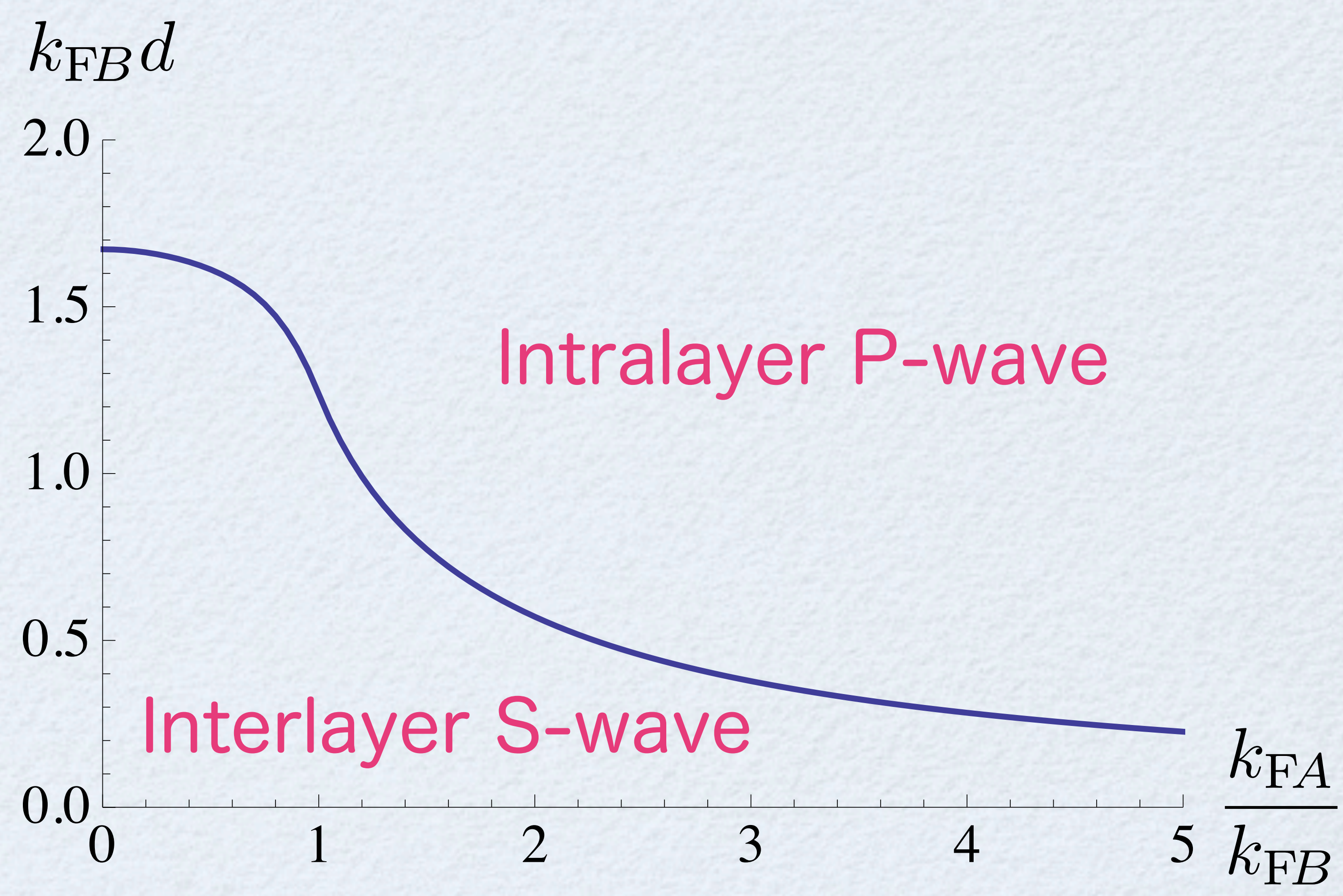


Weak attraction (BCS) limit

B atoms in 3D induce an effective attraction between A atoms in 2D





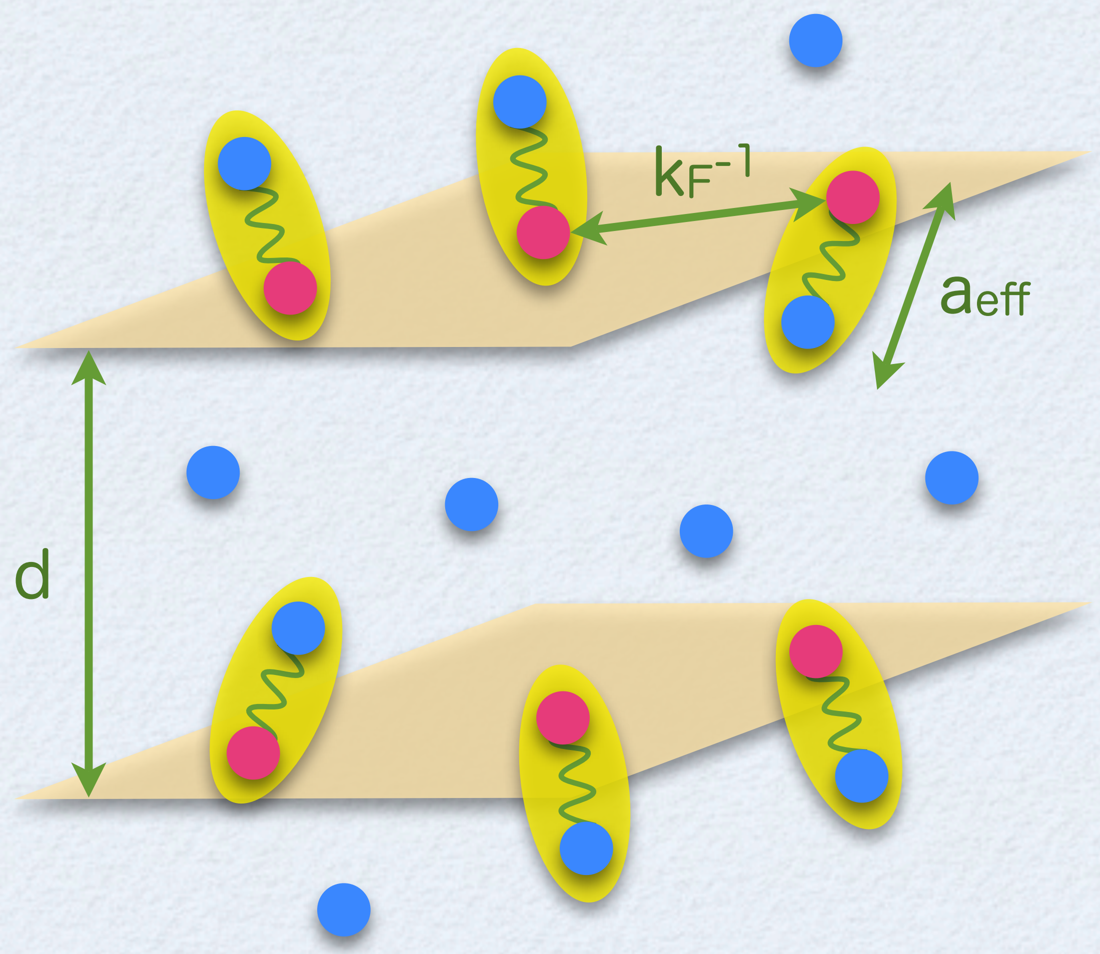


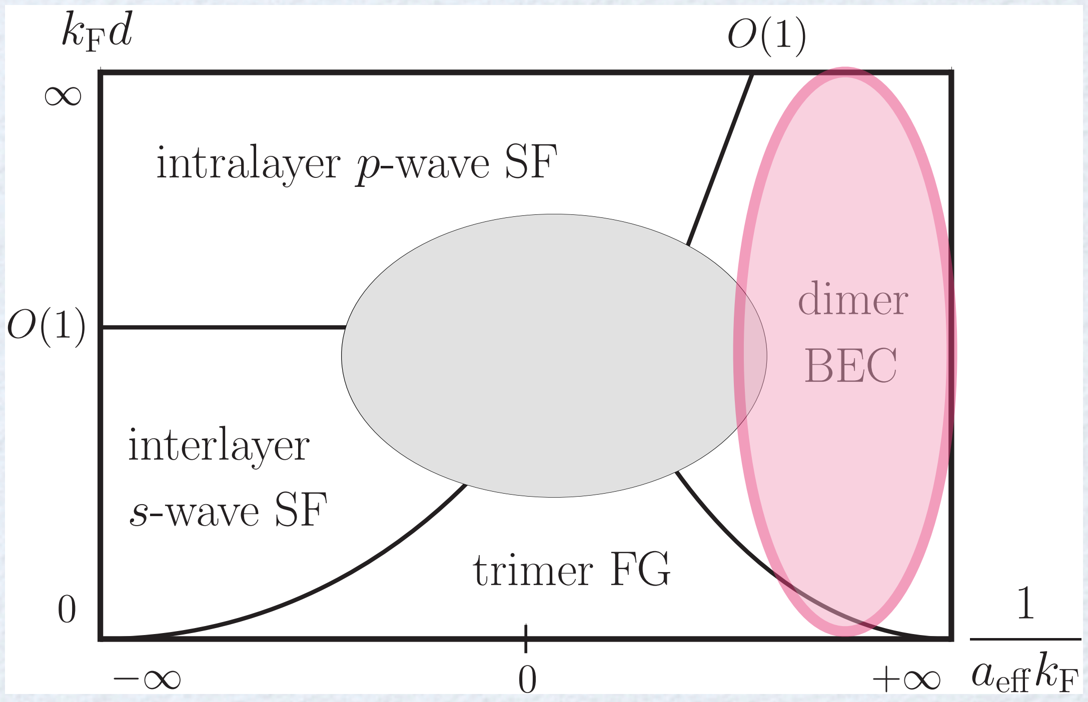
large d	small d
Intralayer P-wave	Interlayer S-wave

Strong attraction (BEC) limit

A atoms in 2D capture

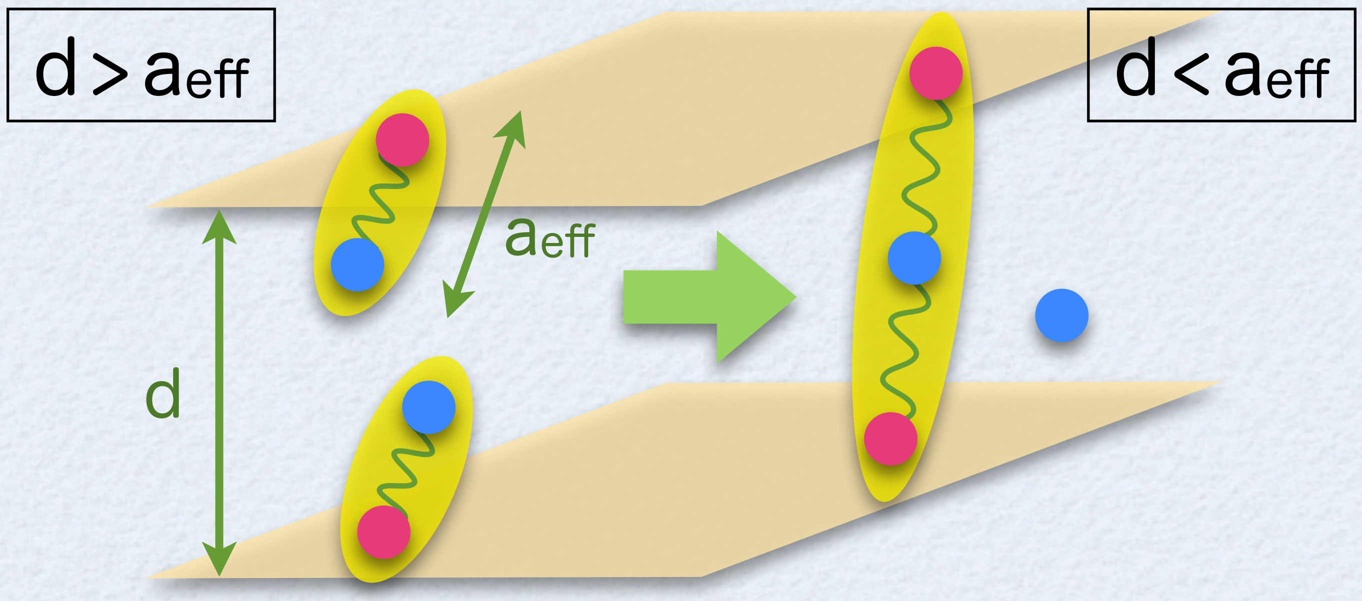
B atoms from 3D to form dimers

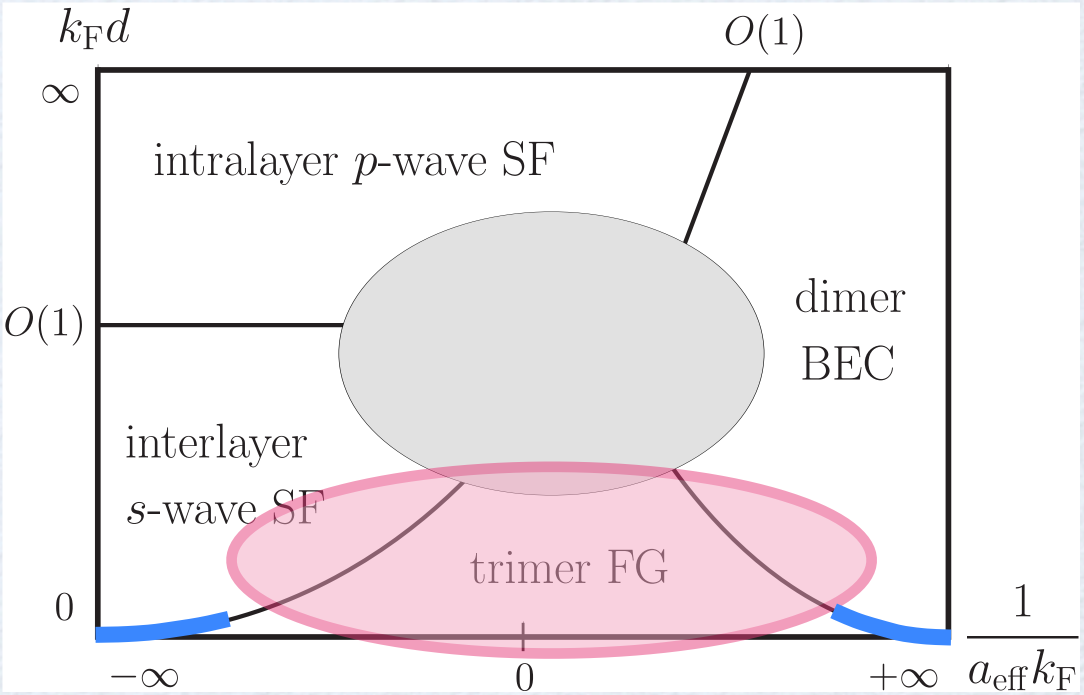


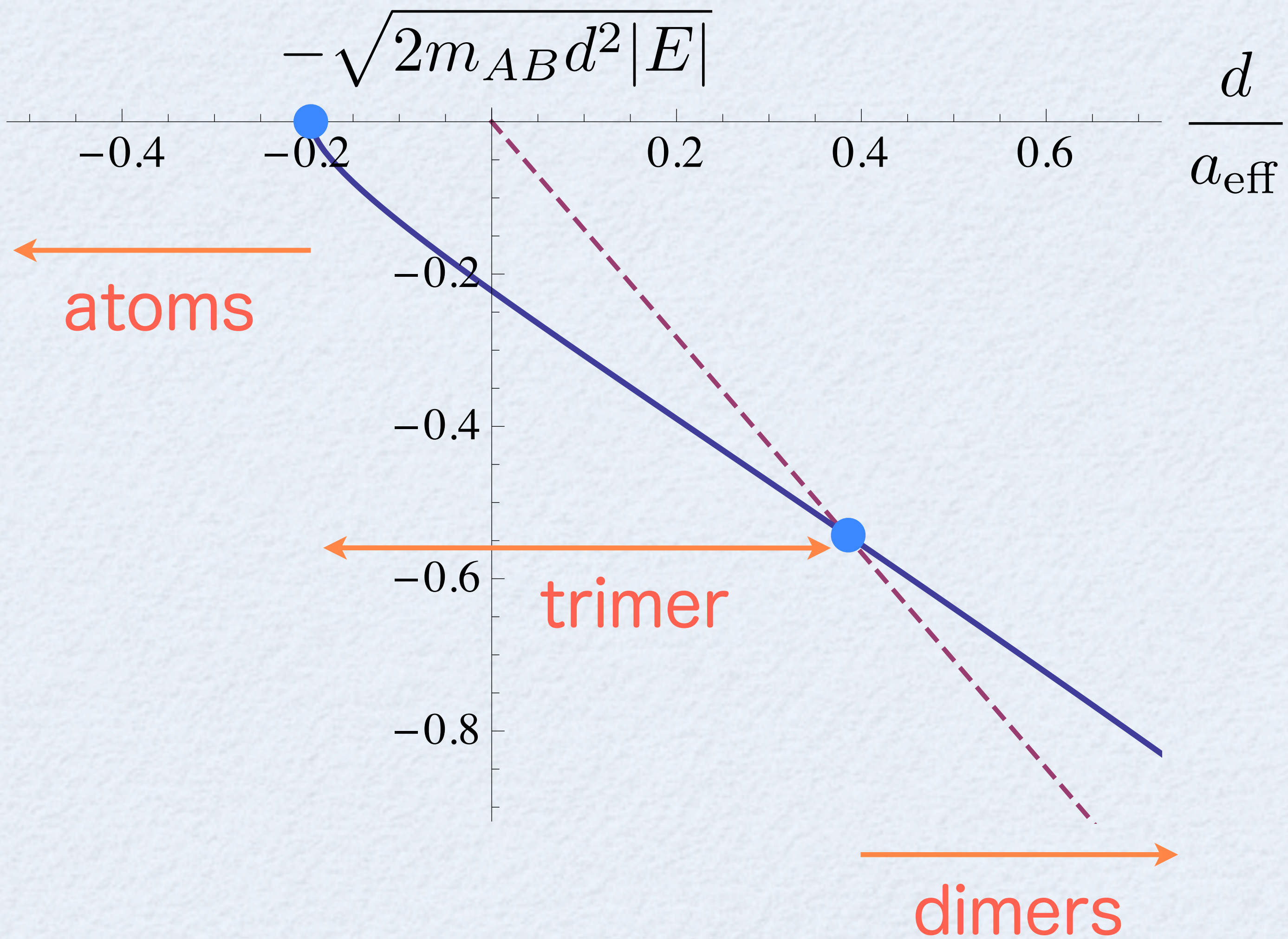


Dimer BEC in each 2D layer

Interlayer trimer formation





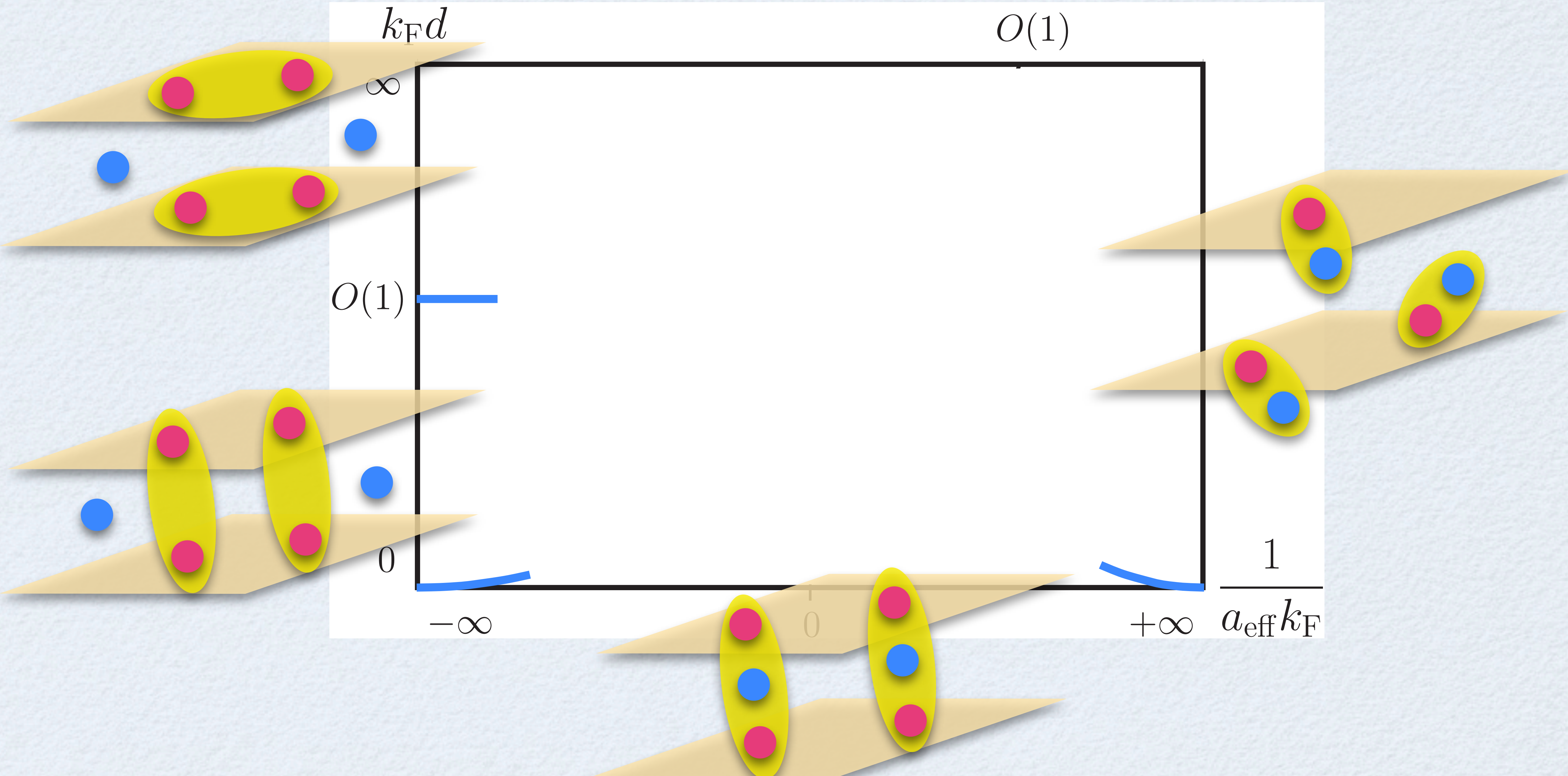


Trimer Fermi gas
bridging 2 layers

Phases of bilayer Fermi gas

Y. N., PRA 82 (2010)

Very rich but “minimal” phase diagram !!!



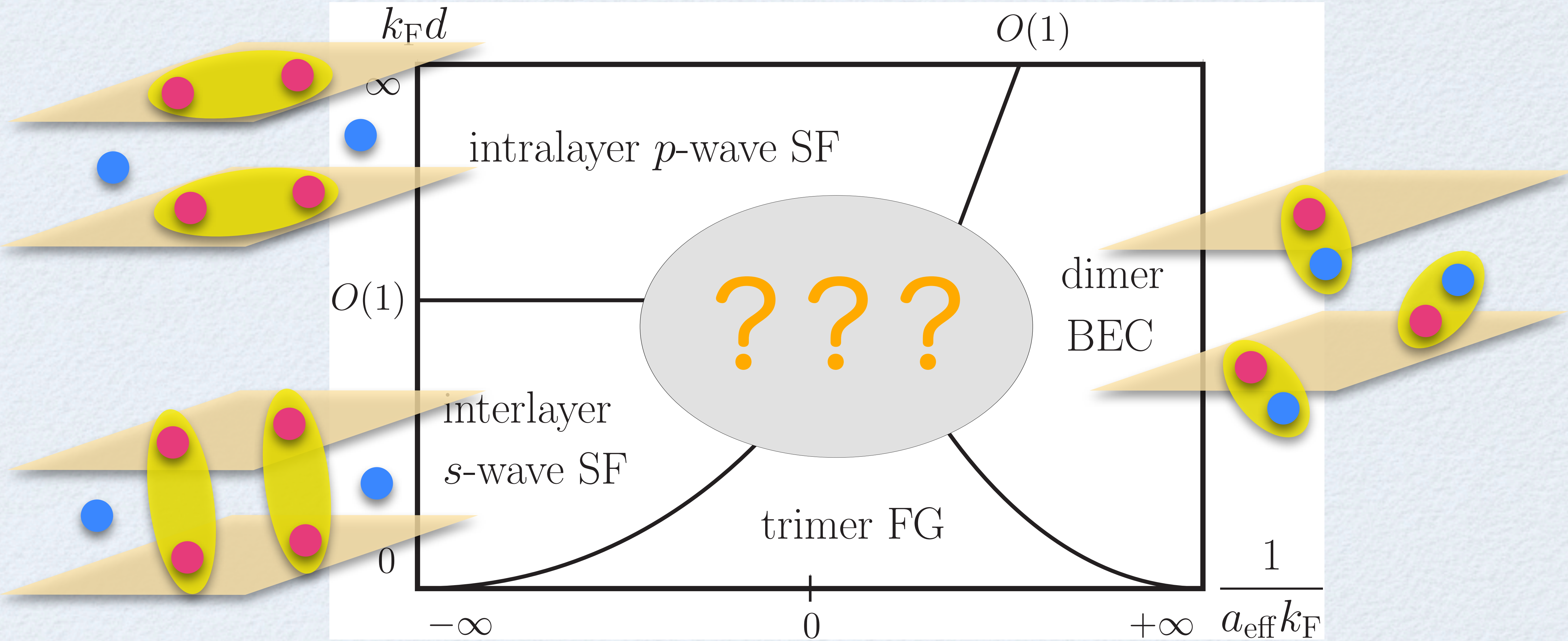
Highlighted in Viewpoint of Physics 3, 58 (2010)

“The complexities of simplicity” (P. Bedaque)

Phases of bilayer Fermi gas

Y. N., PRA 82 (2010)

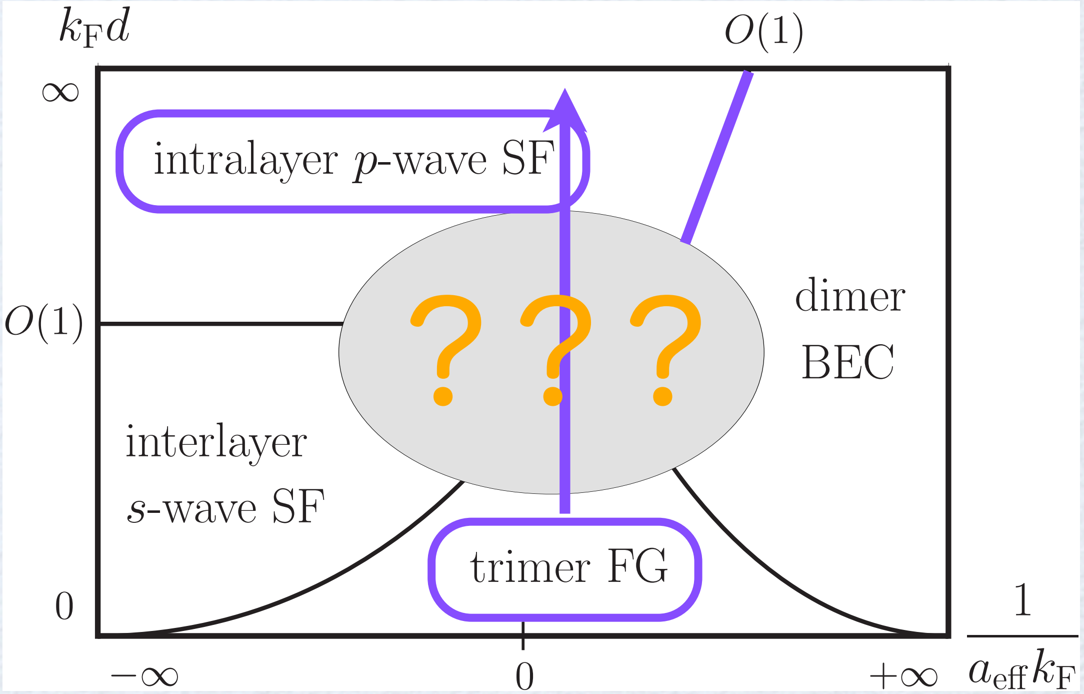
Very rich but "minimal" phase diagram !!!



Previously, s-wave pairings and dimers have been realized but p-wave pairings and trimers are difficult due to instability

→ Enlarge the scope of ultracold atoms

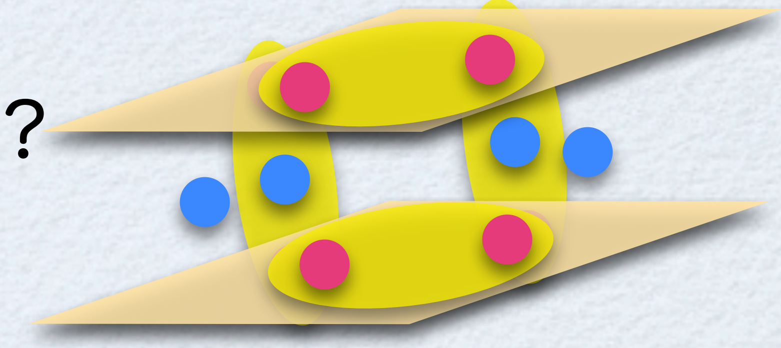
Future prospects



Q1. How does the trimer gas evolve into a gas of their fermionic constituents?

(Cf. BEC→BCS crossover for dimers)

- How many phases appear?
- What is the nature of quantum phase transition(s)?
- Implications for nuclear-quark matter transition?



Q2. What are the critical temperature and experimental signatures of superfluid phases?

- At weak coupling $|a_{eff} k_F| \ll 1$, $\Delta \sim \varepsilon_F \, e^{-\#/(a_{eff} k_F)^2}$
  How large it can be at strong coupling before the dimerization?
- Chiral p-wave superfluid in 2D is topological
  (Majorana fermions, non-abelian statistics, ...)

Summary

Efimov effect and Unitary Fermi gas exist in 8 (7 new) classes of systems :

- ✓ 2-body in pure 3D
- ✓ 2-body in 2D-3D mixture
- • 2-body in 1D-3D mixture
- • 2-body in 2D-2D mixture
- • 2-body in 1D-3D mixture
- • 3-body in 1D-1D-1D mixture
- • 3-body in $1D^2$-2D mixture
- ✓ 4-body in pure 1D

  Y. N. & D.T. Son, PRA (2010)

- Confinement-induced Efimov effect
- Experimental realization (Florence, Amsterdam?, ...)
- Many-body phase diagram of multi-layered Fermi gas

Very rich new fields of universal few & many-body physics

Backup slides

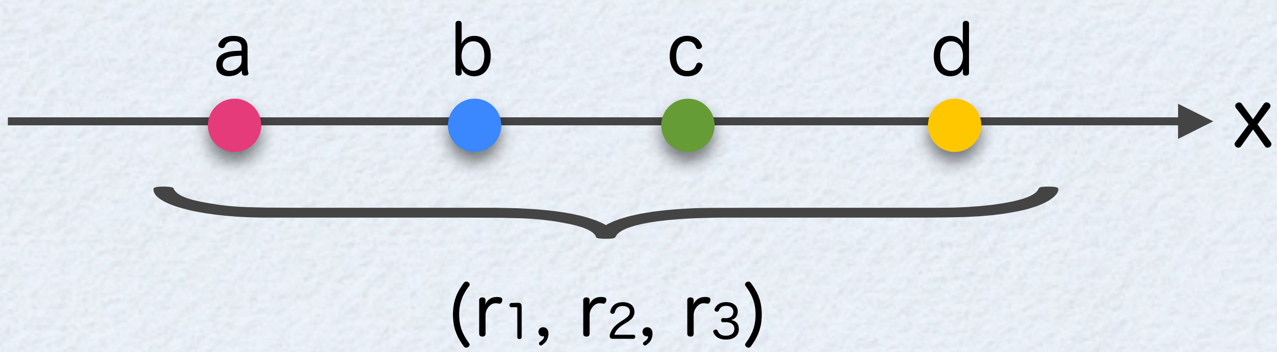
"BCS-BEC" crossover in one dimension

Lattice model

4-component (σ=a,b,c,d) fermions on a 1D lattice

$$H = -t \sum_{\langle xy \rangle, \sigma} c^\dagger_{x\sigma} c_{y\sigma} - U \sum_x c^\dagger_{xa} c^\dagger_{xb} c^\dagger_{xc} c^\dagger_{xd} c_{xd} c_{xc} c_{xb} c_{xa}$$

Scattering of 4 particles with all different components



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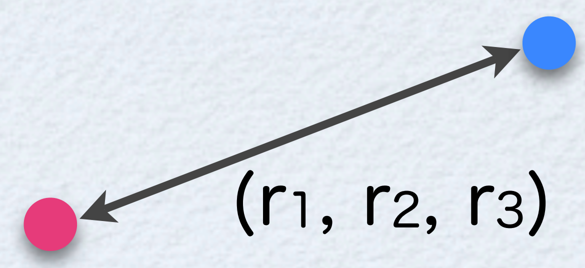
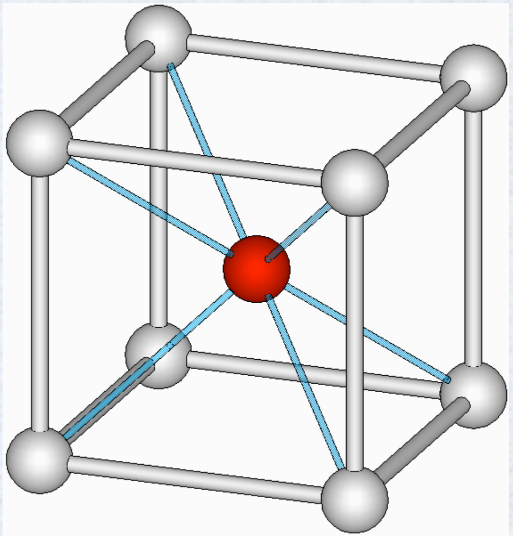


Scattering of 2 particles in 3D

$$\left[-t \sum_{i=1}^{4} \Delta_i - \delta_{\boldsymbol{r},\boldsymbol{0}} U \right] \Psi(\boldsymbol{r}) = E\,\Psi(\boldsymbol{r}) \quad \text{with} \quad \boldsymbol{r} = (r_1, r_2, r_3)$$

1 particle moving in a body-centered cubic lattice



Mapping to 2-body scattering in 3D

4-body scattering in 1D

$$\left[-t\sum_{i=1}^{4}\Delta_i - \delta_{\boldsymbol{r},\boldsymbol{0}}\,U\right]\Psi(\boldsymbol{r}) = E\,\Psi(\boldsymbol{r})$$

Schrödinger equation in 3D !

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_{\boldsymbol{r}}^2\,\Psi(\boldsymbol{r}) = E\,\Psi(\boldsymbol{r})$$

Correspondence between 4-body interaction in 1D
and 2-body interaction in 3D

Physical meaning of |r| is different :

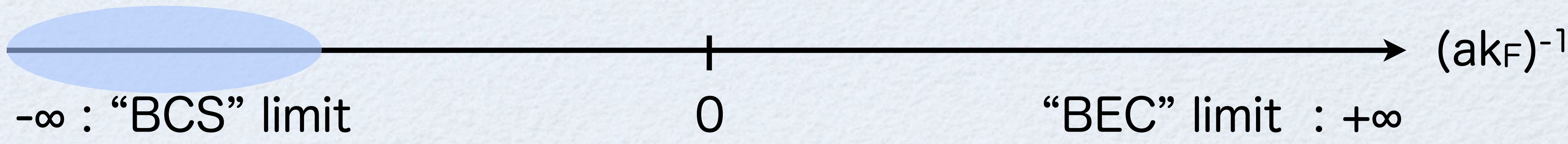
hyperradius of 4 particles in 1D vs. separation of 2 particles in 3D

Both are characterized by scattering length “a” in the same way

$$\Psi(|\boldsymbol{r}|\to\infty)\big|_{E=0} \to \frac{1}{|\boldsymbol{r}|} - \frac{1}{a} \qquad \text{with} \qquad \frac{l}{a} = \frac{\Gamma\left(\frac{1}{4}\right)^4}{4\pi^2} - \frac{8\pi t}{U}$$

Resonance (a=∞) is achieved at $\frac{U}{t} = 32\pi^3/\Gamma\left(\frac{1}{4}\right)^4 \approx 5.742$

Weak coupling ("BCS") limit



$$\mathcal{H} = \sum_\sigma \psi_\sigma^\dagger \left(-\frac{\nabla^2}{2m} - \mu \right) \psi_\sigma - c_0\, \psi_a^\dagger \psi_b^\dagger \psi_c^\dagger \psi_d^\dagger \psi_d \psi_c \psi_b \psi_a \quad \text{with} \quad c_r = -\frac{4\pi a}{m}$$

linearization around Fermi points : $\psi_\sigma(x) \simeq e^{ik_{\mathrm{F}}x}\psi_\sigma^{\mathrm{R}}(x) + e^{-ik_{\mathrm{F}}x}\psi_\sigma^{\mathrm{L}}(x)$

⇒ $\mathcal{H}_{\mathrm{U(1)\,charge}} + \mathcal{H}_{\mathrm{SU(4)\,spin}}$

- Tomonaga-Luttinger liquid

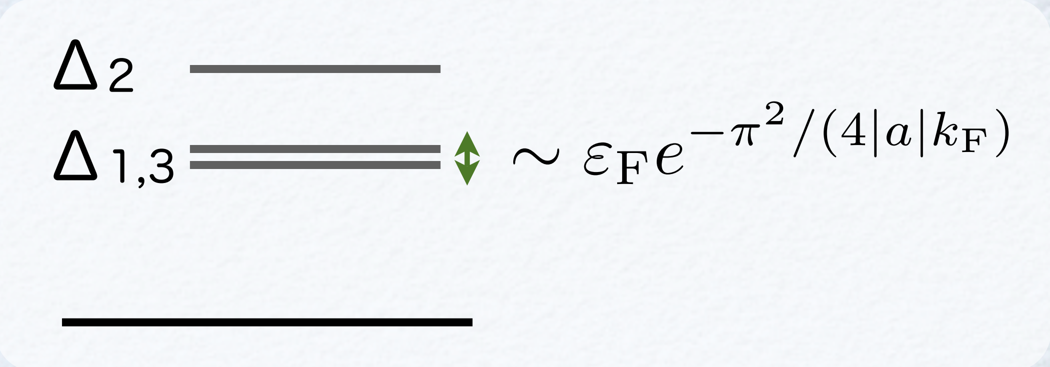
  with gapless dispersion ($E = v_s |k|$) :

$$v_{\mathrm{s}} = \left(1 - \frac{6|a|k_{\mathrm{F}}}{\pi^2} + \cdots \right) v_{\mathrm{F}}$$

- Marginally relevant coupling

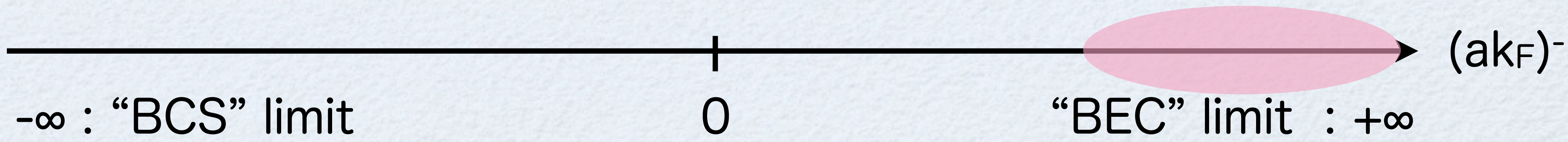
  develops 3 spin gaps (f=1,2,3) :

$$\Delta_f \propto \varepsilon_{\mathrm{F}} e^{-\pi^2/(8|a|k_{\mathrm{F}})} \sin\left(\frac{f\pi}{4}\right)$$



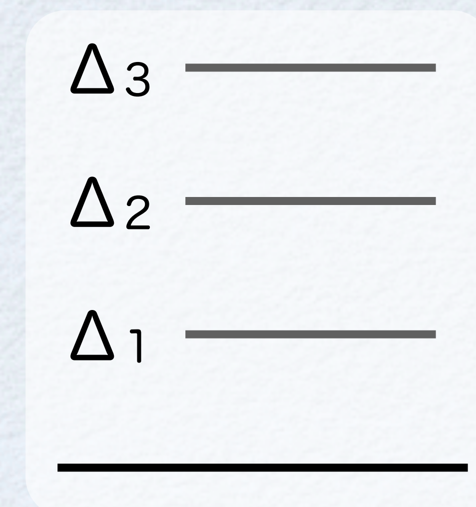
Andrei & Lowenstein, PLB 90 (1980)

Strong coupling ("BEC") limit

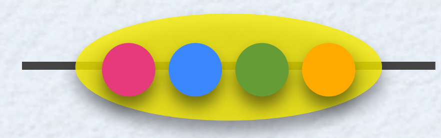


4 fermions form a tightly-bound tetramer : $E_0 = -1/(2ma^2)$

→ Fermions are largely gapped : $\Delta_f \to \dfrac{f}{8ma^2}$



- Dilute Bose gas of tetramers



$$\mathcal{H}_{\mathrm{tetramer}} = \phi^\dagger \left(-\frac{\nabla^2}{2M} - 4\mu \right) \phi - \frac{1}{M a_{tt}} \phi^\dagger \phi^\dagger \phi \phi$$

with tetramer-tetramer scattering length : $a_{tt} = -\eta a \quad (\eta > 0)$

→ Its low-energy physics is governed by Tomonaga-Luttinger liquid with sound velocity :

$$v_{\mathrm{s}} = \left(1 - \eta \frac{2ak_{\mathrm{F}}}{\pi} + \cdots \right) \frac{v_{\mathrm{F}}}{4}$$

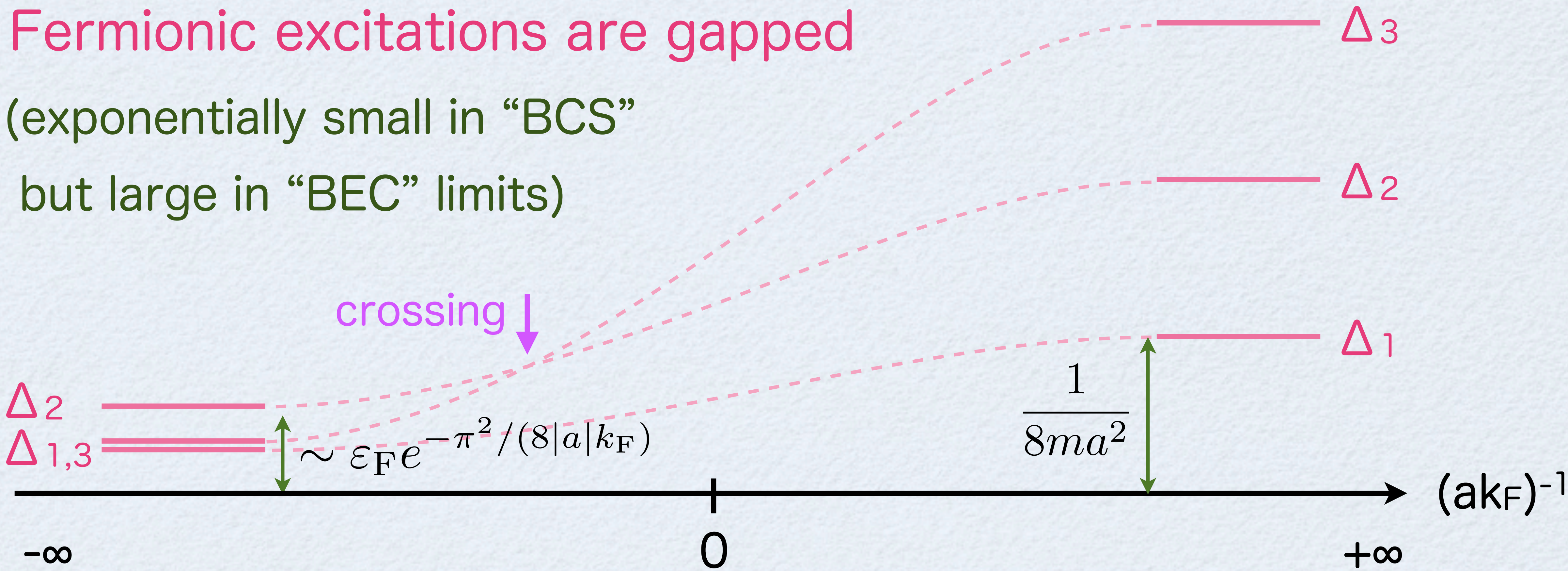
Lieb & Liniger, Phys.Rev. 130 (1963)

"BCS-BEC" crossover ?

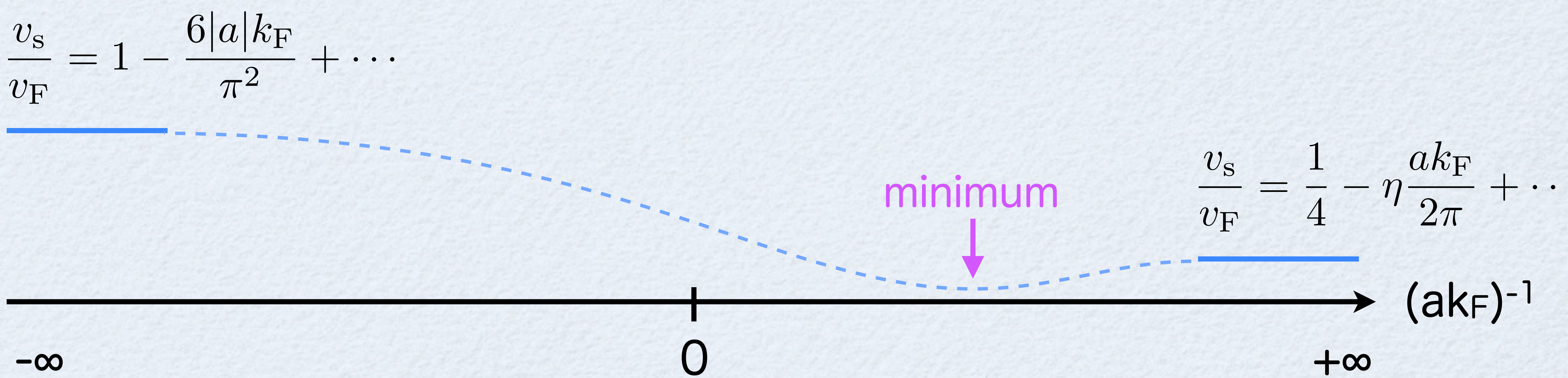
- Fermionic excitations are gapped

  (exponentially small in "BCS"

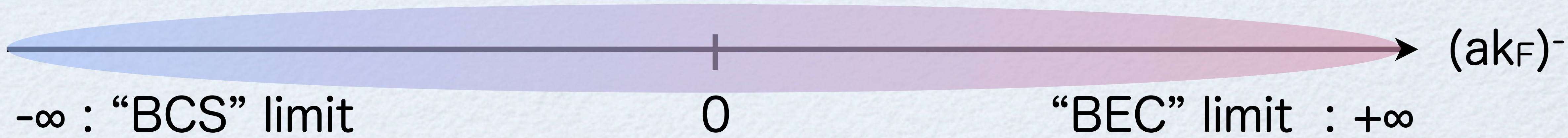
  but large in "BEC" limits)



- Gapless sound mode with linear dispersion $E = v_s|k|$



Unitarity limit



Universal parameter ξ for 1D unitary Fermi gas :

$$\mathcal{E}_{\text{unitary}} = \xi \times \mathcal{E}_{\text{free}} \quad \left(= \xi \, \frac{\pi^2}{96m} n^3\right)$$

$$\Rightarrow \quad \frac{\mathcal{E}}{\varepsilon_{\mathrm{F}} n} = \frac{1}{3}\xi \qquad \frac{P}{\varepsilon_{\mathrm{F}} n} = \frac{2}{3}\xi \qquad \frac{\mu}{\varepsilon_{\mathrm{F}}} = \xi \qquad \frac{v_{\mathrm{s}}}{v_{\mathrm{F}}} = \sqrt{\xi}$$

Currently no reasonable estimate of ξ in 1D,

but numerical simulations or the ε expansion would help ...

$$\xi|_{d\to\frac{2}{3}} \to 1 \quad \text{and} \quad \xi|_{d\to\frac{4}{3}} \to 0 \quad \Rightarrow \quad \xi \approx 0.5 \text{ in 1D ???}$$

Exact relationships in 1D

0. contact density

$$\mathcal{C} \equiv \langle (mc_0)^2 \psi_a^\dagger \psi_b^\dagger \psi_c^\dagger \psi_d^\dagger \psi_d \psi_c \psi_b \psi_a \rangle$$

1. momentum distribution tail

$$\rho_\sigma(k \to \infty) \to \frac{\sqrt{3}}{4\pi} \frac{\mathcal{C}}{k^2}$$

2. energy relation

$$\mathcal{E} = \sum_\sigma \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{k^2}{2m} \left(\rho_\sigma(k) - \frac{\sqrt{3}}{4\pi} \frac{\mathcal{C}}{k^2} \right) + \frac{\mathcal{C}}{4\pi m a}$$

3. adiabatic relation

$$\frac{d\mathcal{E}}{da} = \frac{\mathcal{C}}{4\pi m a^2}$$

4. pressure relation

$$P = 2\mathcal{E} + \frac{\mathcal{C}}{4\pi m a}$$

5. virial theorem

$$E = 2\langle V_\omega \rangle - \int dx \, \frac{\mathcal{C}(x)}{8\pi m a}$$

6. “quadruplet” density within hyperradius < R→0

$$\mathcal{N}_4(R) \equiv \int_{|\boldsymbol{r}|<R} d\boldsymbol{r} \, \langle \psi_a^\dagger \psi_a(x_a) \psi_b^\dagger \psi_b(x_b) \psi_c^\dagger \psi_c(x_c) \psi_d^\dagger \psi_d(x_d) \rangle \to \frac{\mathcal{C}}{4\pi} R$$