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Non-perturbative predictions for cold atom gases with tunable interactions

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[Cooper, Chien, Mihaila, Dawson and Timmermans, Phys. Rev. Lett. **105**, 240402 (2010)]

Auxiliary Field (AF) formalism for Bose gases

Consider the classical action:

$$S[\,\phi,\phi^*\,] = \int [\mathrm{d}x]\,\mathcal{L}[\,\phi,\phi^*\,]$$

with the Lagrangian

$$\mathcal{L}[\,\phi,\phi^*\,] = \frac{i\hbar}{2}\Big[\,\phi^*(x)\,\big(\partial_t\,\phi(x)\big) - \big(\partial_t\,\phi^*(x)\big)\,\phi(x)\,\Big] - \phi^*(x)\Big\{ -\frac{\hbar^2\nabla^2}{2m} - \mu_0 \Big\}\,\phi(x) - \frac{\lambda_0}{2}\,|\,\phi(x)|^4$$

We introduce the auxiliary Lagrangian

$$\mathcal{L}_{\mathrm{aux}}[\phi,\phi^*,\chi,A,A^*] = \frac{1}{2\lambda_0}\,\big[\,\chi(x) - \lambda_0\,\cosh\theta\,|\phi(x)|^2\,\big]^2 - \frac{1}{2\lambda_0}\,\big|\,A(x) - \lambda_0\,\sinh\theta\,\phi^2(x)\,\big|^2$$

Leading Order Auxiliary Field (LOAF) equations

"Gap" equations

$$\frac{\chi' + \mu}{\lambda \cosh^2 \theta} = |\phi|^2 + \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\epsilon_k + \chi'}{2\omega_k} [2n(\beta\omega_k) + 1] - \frac{1}{2} \right\}$$

and

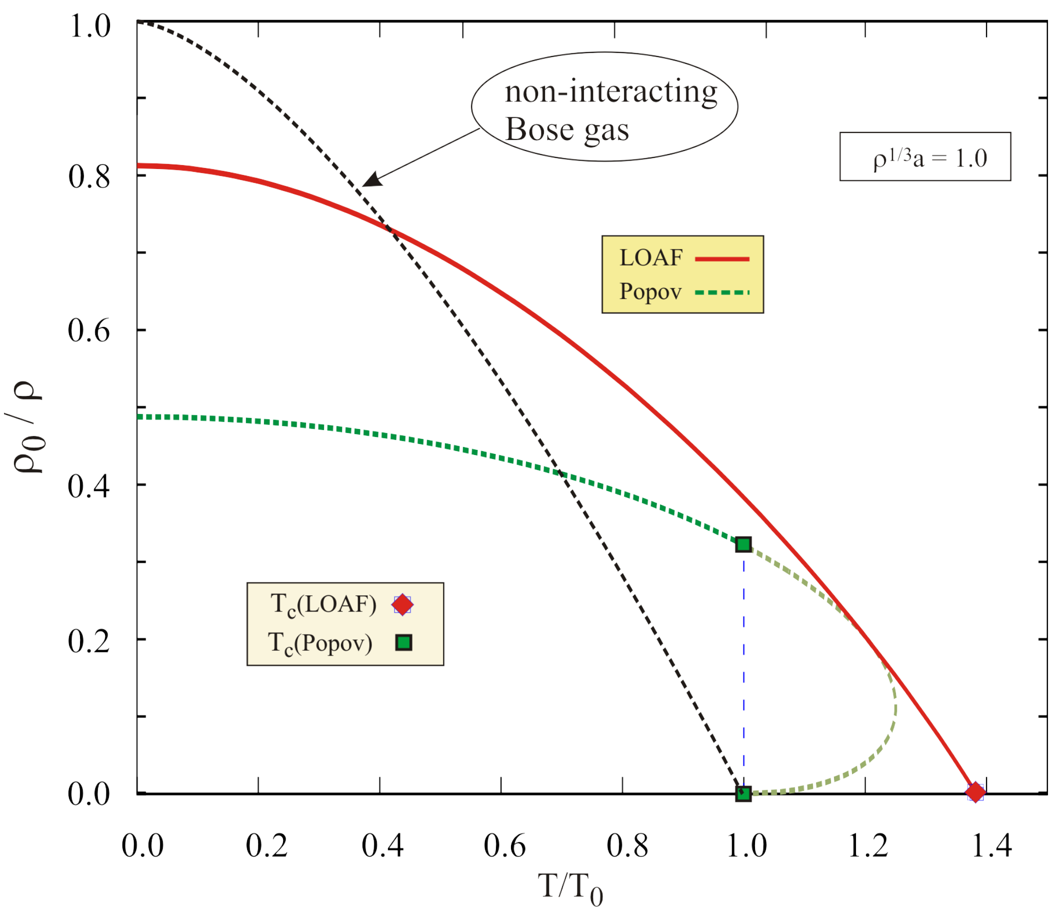
$$\frac{A'}{\lambda \sinh^2 \theta} = \phi^2 + A' \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{2n(\beta\omega_k) + 1}{2\omega_k} - \frac{1}{2\epsilon_k} \right\}$$

with the dispersion relation $\omega_k^2 = (\epsilon_k + \chi')^2 - |A'|^2$

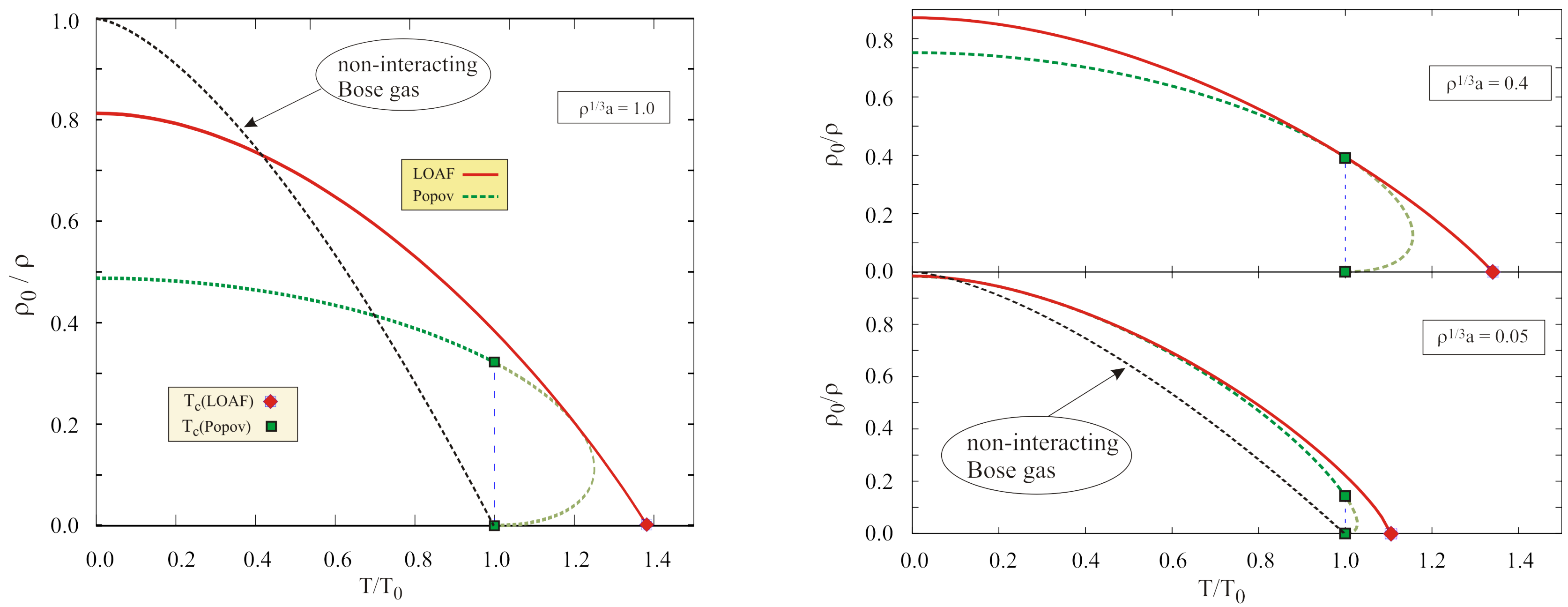
and $\chi' = \chi \cosh\theta - \mu$

$A' = A \sinh\theta$

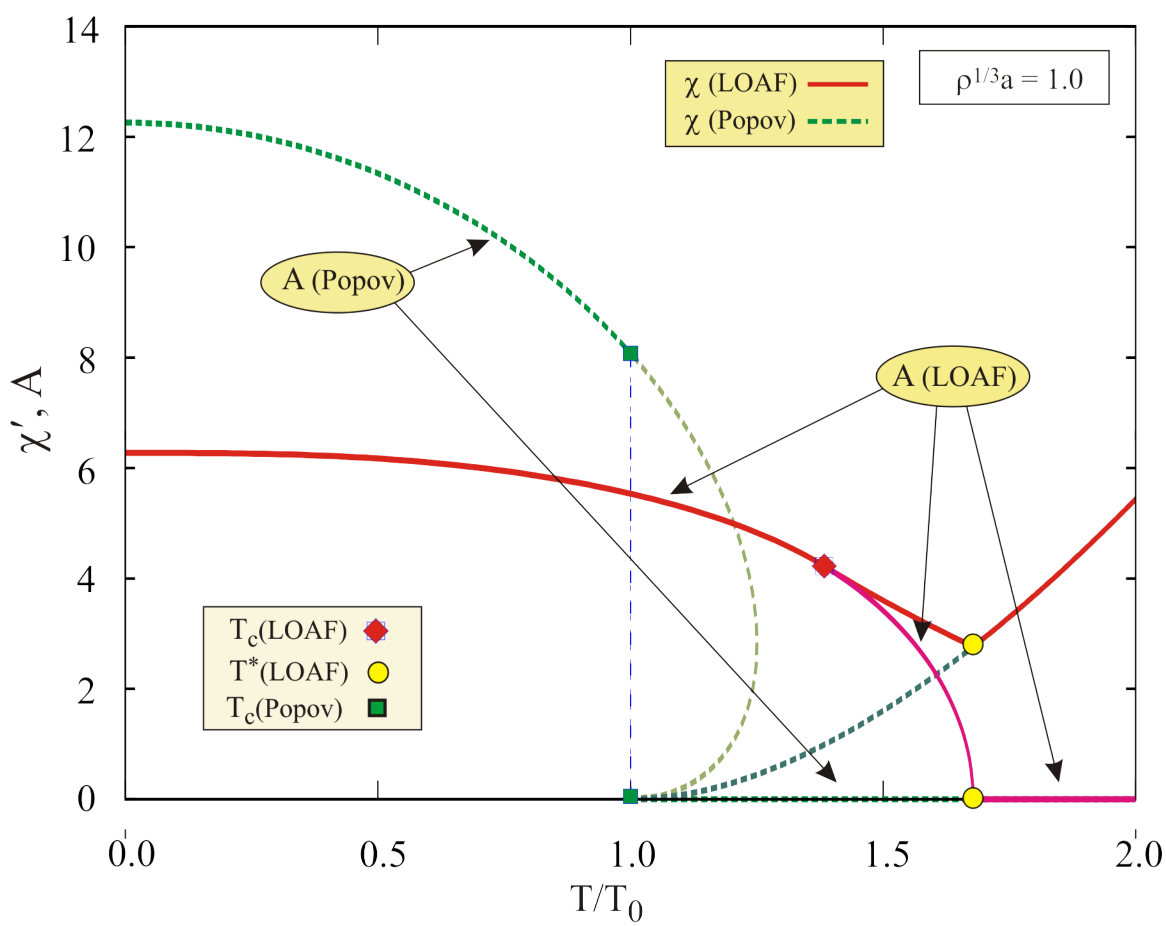
Condensate density



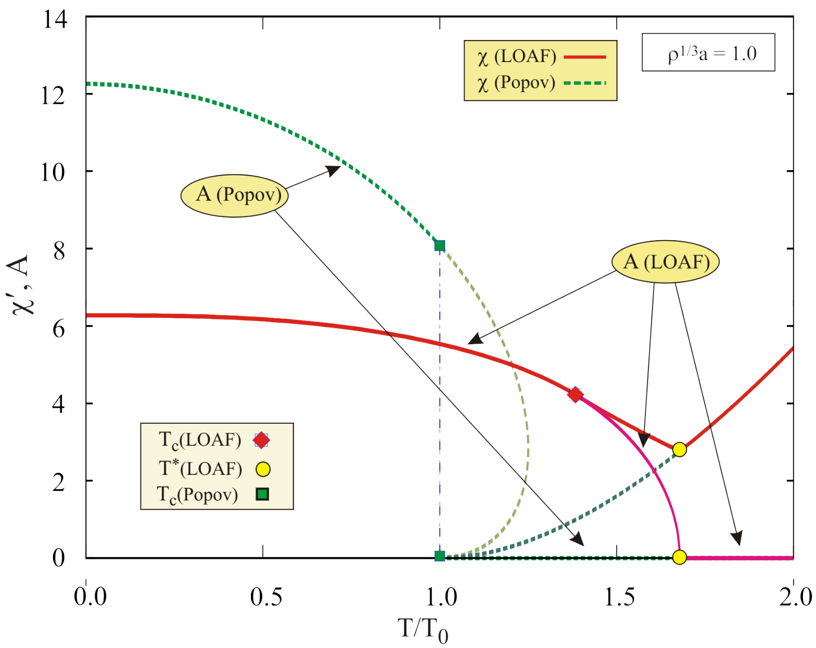
Condensate density

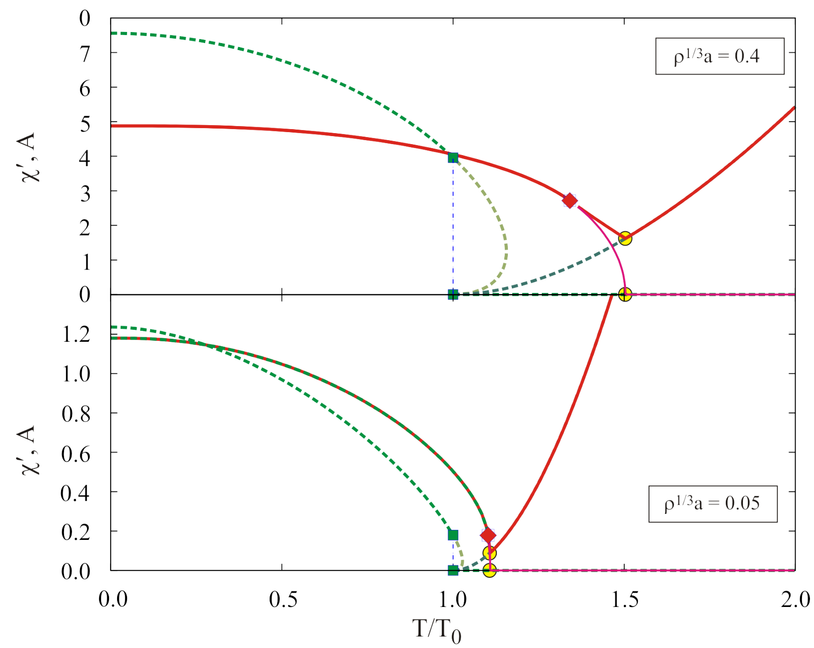


Normal and anomalous *auxiliary*-field densities

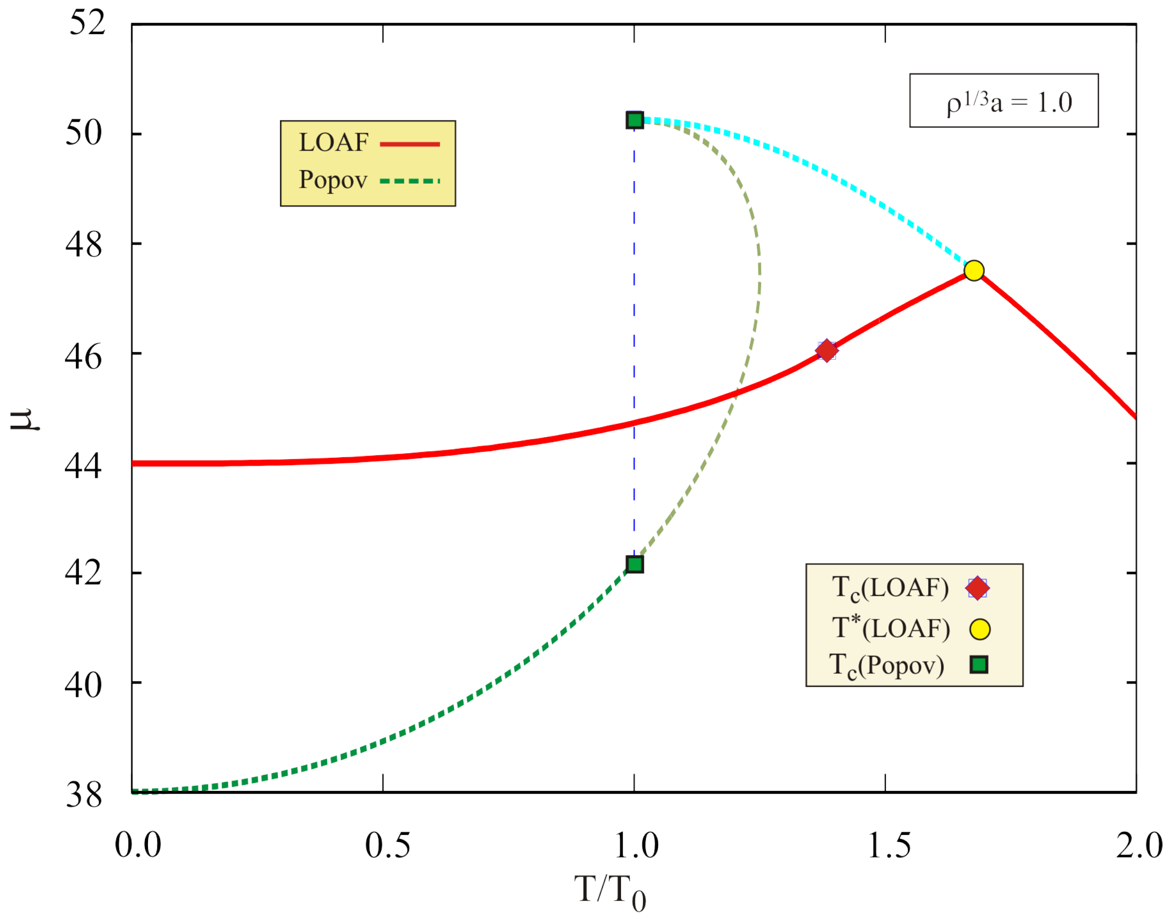


Normal and anomalous *auxiliary*-field densities

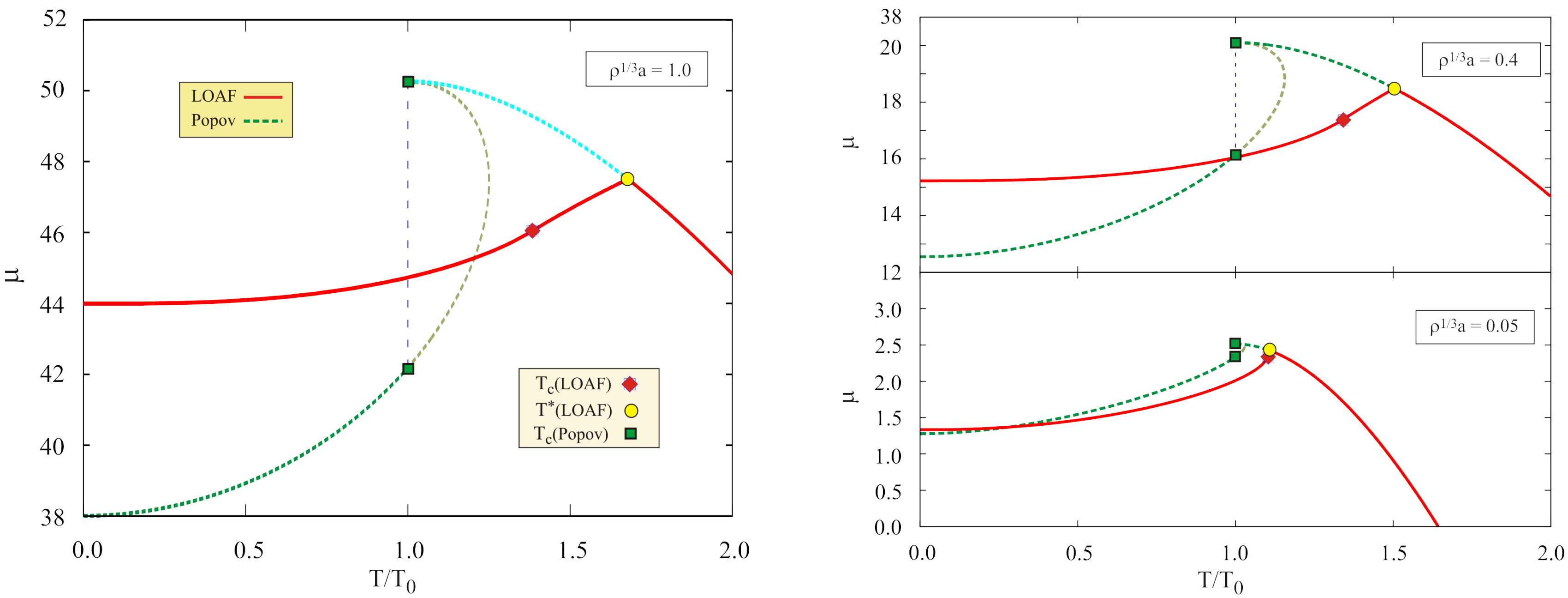




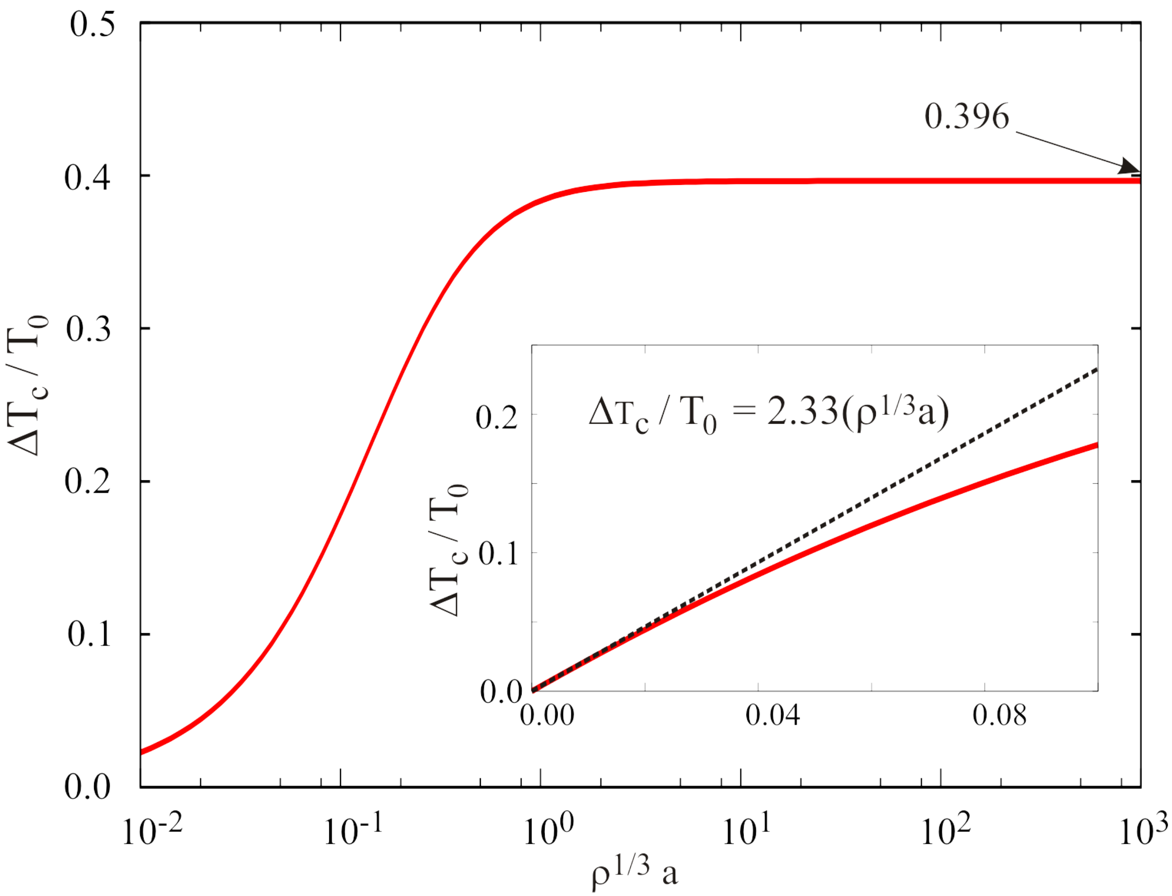
Chemical potential



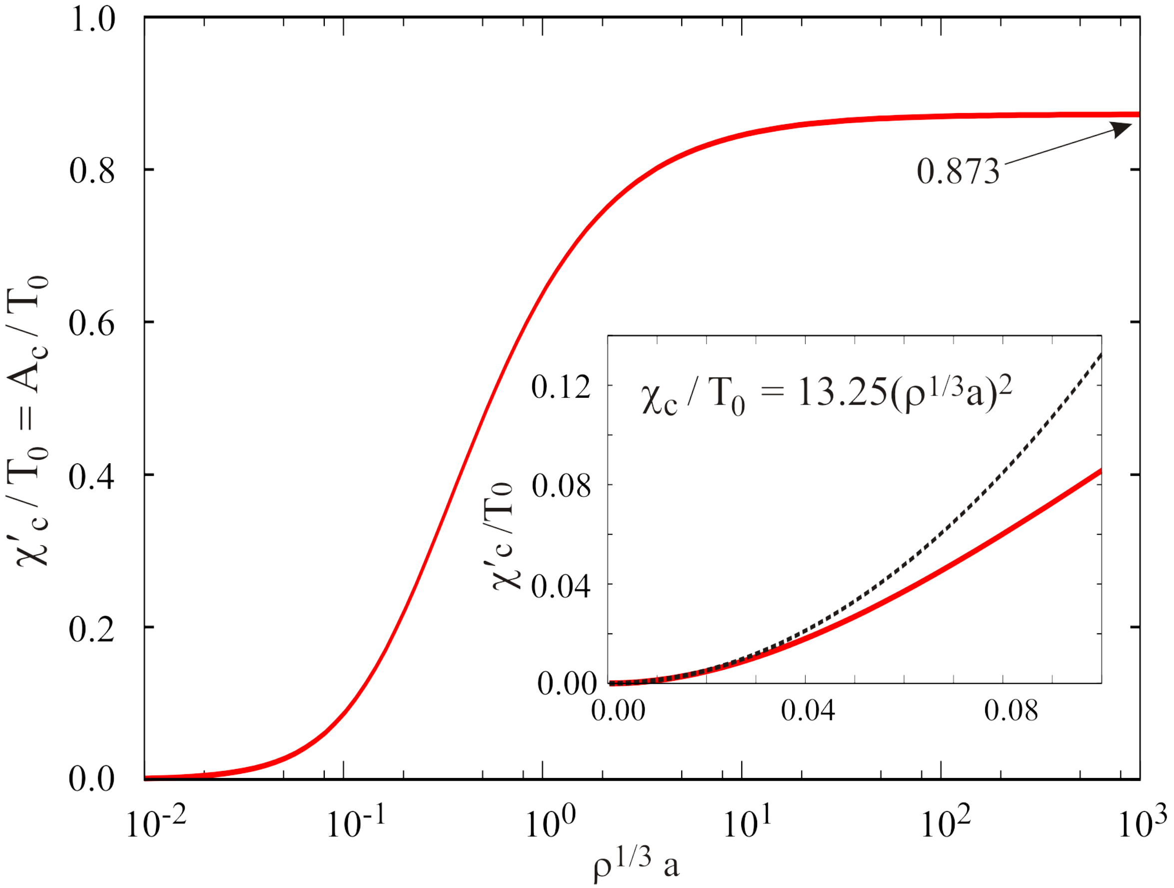
Chemical potential



$\Delta T_c/T_0$ in the unitarity limit



Critical densities in the unitarity limit



Conclusions (Bose gas)

Auxiliary Field Formalism

- ✓ Second-order phase transition
- ✓ LOAF recovers Large N results for the critical theory
- ✓ Non-perturbative formalism
- ✓ Rigorous machinery for NLO

Auxiliary Field (AF) formalism for Fermi gases

Consider the classical action: $$S[\,\phi,\phi^*\,] = \int [\mathrm{d}x]\,\mathcal{L}[\,\phi,\phi^*\,]$$

with the Lagrangian

$$\mathcal{L}[\,\psi,\psi^*\,] = \sum_\sigma \Big\{ \frac{1}{2}\Big[\,\psi^*_\sigma(x)\,\frac{\partial\psi_\sigma(x)}{\partial\tau} + \psi_\sigma(x)\,\frac{\partial\psi^*_\sigma(x)}{\partial\tau}\,\Big] + \psi^*_\sigma(x)\,\Big[-\gamma\nabla^2 - \mu_\sigma\Big]\,\psi_\sigma(x) + \frac{\lambda_0}{2}\,\psi^*_\sigma(x)\,\psi^*_{-\sigma}(x)\,\psi_{-\sigma}(x)\,\psi_\sigma(x)\Big\}\,.$$

We introduce the auxiliary Lagrangian $\mathcal{L}_{\mathrm{aux}}[\psi,\chi,\Delta] = -\mathcal{L}_\chi[\psi,\chi] + \mathcal{L}_\Delta[\psi,\Delta]\,,$

$$\mathcal{L}_\chi[\psi,\chi] = \frac{1}{2\lambda_0}\sum_\sigma \big[\,\chi_\sigma(x) - \lambda_0\,\rho_\sigma(x)\,\sin\theta\,\big]\big[\,\chi_{-\sigma}(x) - \lambda_0\,\rho_{-\sigma}(x)\,\sin\theta\,\big]$$

$$\mathcal{L}_\Delta[\psi,\Delta] = \frac{1}{2\lambda_0}\sum_\sigma \big[\,\Delta_\sigma(x) - \lambda_0\,\kappa_\sigma(x)\,\cos\theta\,\big]\big[\,\Delta^*_{-\sigma}(x) - \lambda_0\,\kappa^*_{-\sigma}(x)\,\cos\theta\,\big]$$

Leading Order Auxiliary Field (LOAF) equations

"Gap" equations

$$\frac{1}{\xi \cos^2\theta} = \frac{2}{\pi}\int_0^\infty k^2\,\mathrm{d}k\left\{\frac{1}{k^2} - \frac{1-2\,n_{\omega_k,T}}{\omega_k}\right\}$$

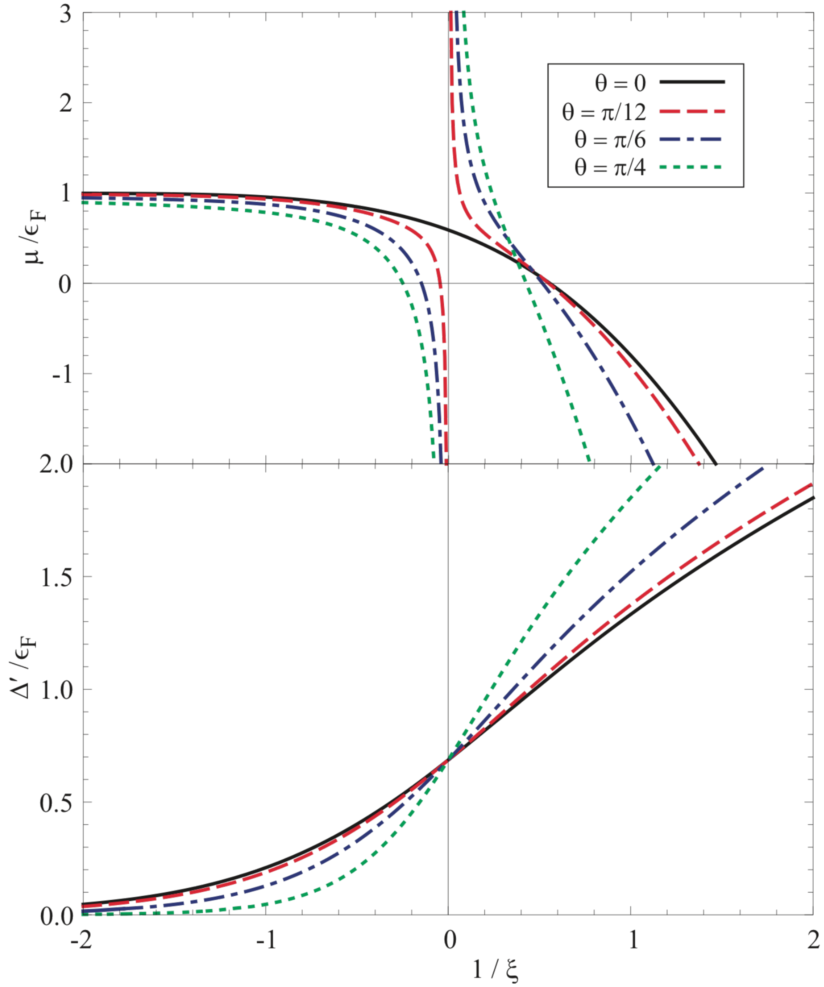
and

$$1 = \frac{3}{2}\int_0^\infty k^2\,\mathrm{d}k\left\{1 - \frac{k^2+\chi'}{\omega_k}\left[1-2\,n_{\omega_k,T}\right]\right\}$$

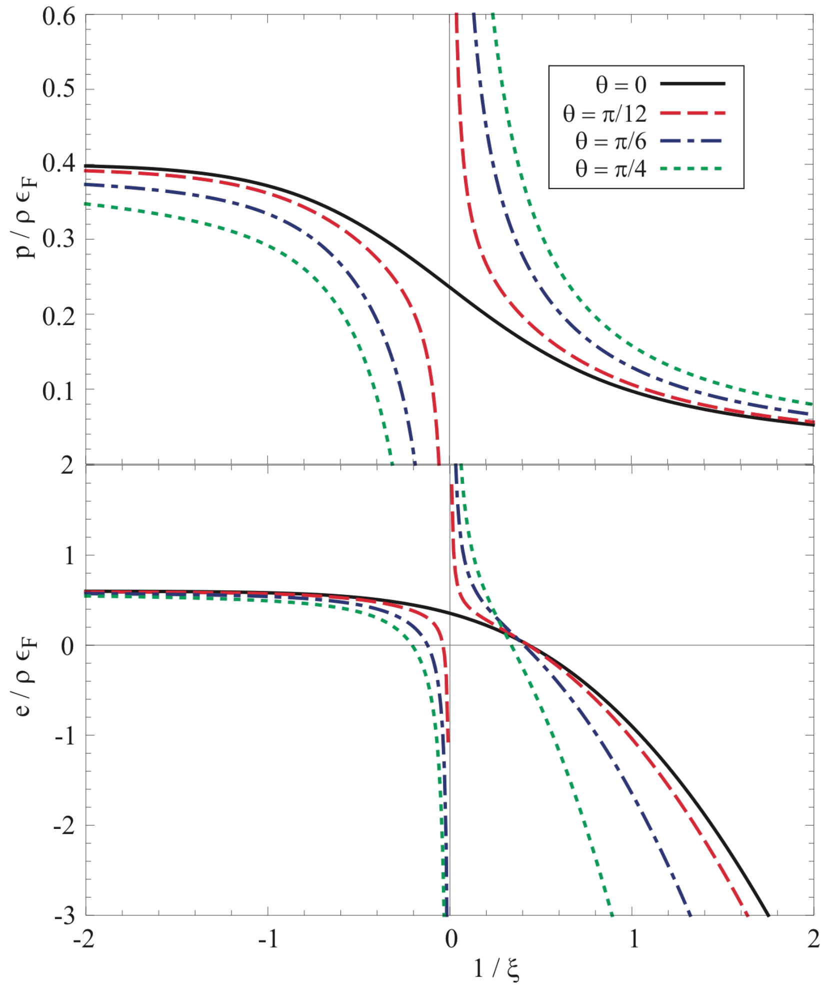
with the dispersion relation $\omega_k^2 = (k^2+\chi')^2 + |\Delta|^2$

and $\chi' = \frac{4}{3\pi}\,\xi\,\sin^2\theta - \mu$

Fermi gas: zero temperature



Fermi gas: zero temperature



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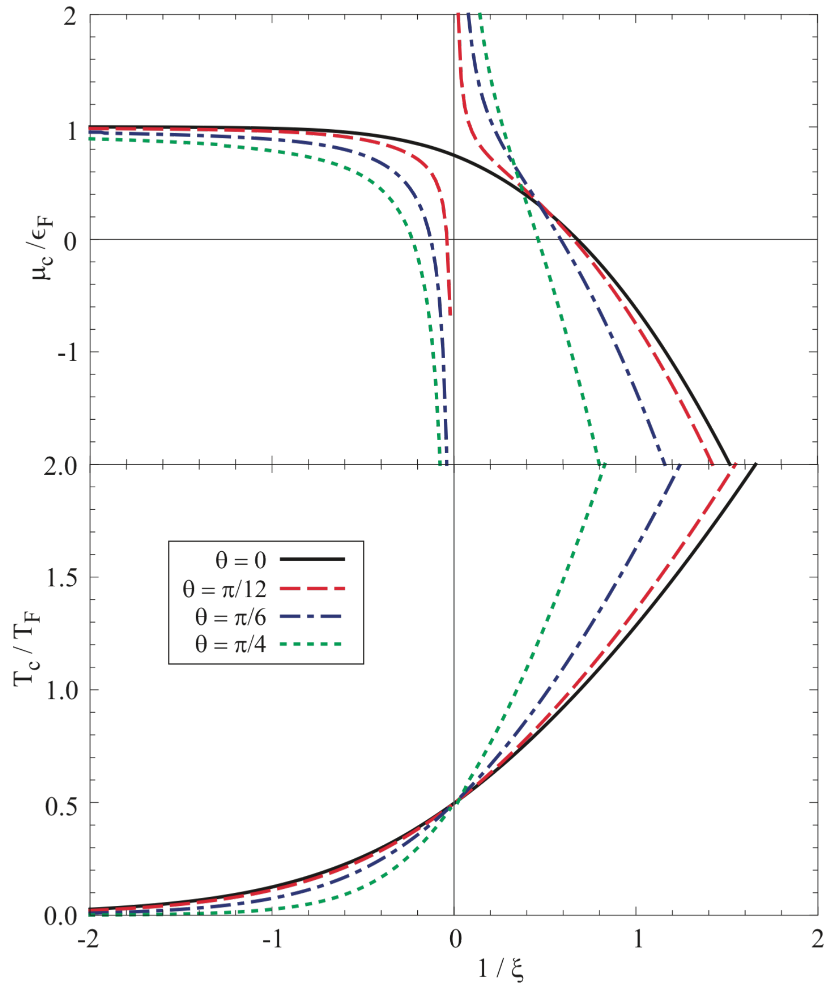
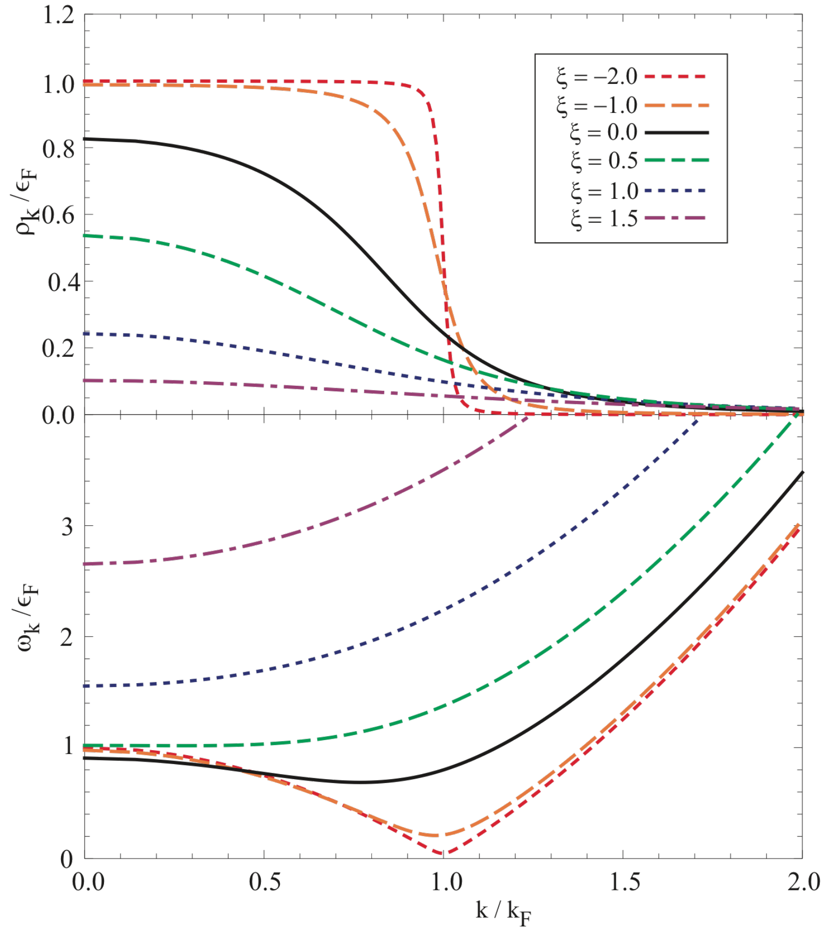
Fermi gas: critical temperature



Fermi gas: LOAF gives the BCS ansatz



Fermi gas: Tan's relations

Fermion momentum distribution

$$\rho(k) \to \frac{C}{k^4}$$

Energy density variation with the inverse scattering length

$$\frac{de}{da^{-1}} = -\frac{\gamma}{2\pi} C$$

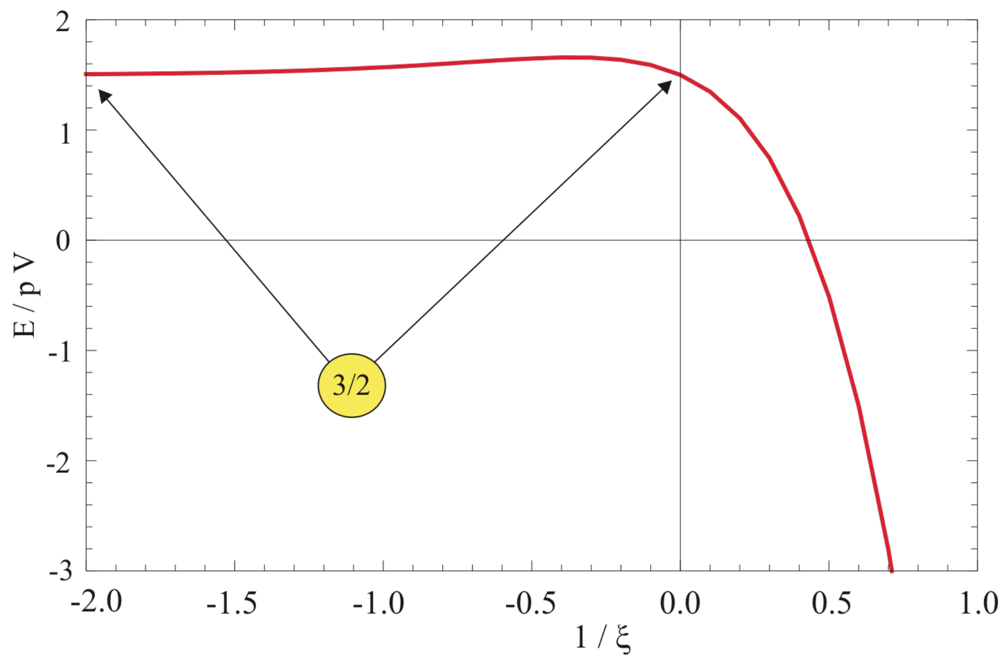
LOAF predictions

$$\rho(k) = \frac{C_{\mathrm{LOAF}}}{k^4} + \mathcal{O}\left(\frac{1}{k^6}\right)$$

$$C_{\mathrm{LOAF}} = \frac{\Delta'^2}{4\gamma^2}$$

$$\frac{de}{da^{-1}} = -\frac{\partial p}{\partial a^{-1}} = -\frac{\Delta'^2}{8\pi\,\gamma} = -\frac{\gamma}{2\pi} C_{\mathrm{LOAF}}$$

Fermi gas: equation of state at T=0



Conclusions (Fermi gas)

Auxiliary Field Formalism

- ✓ BCS ansatz is the only physical auxiliary-field theory
- ✓ LOAF gives Leggett's equations at zero temperature
- ✓ At finite-temperature: theory of Randeria *et al.*
- ✓ Non-perturbative formalism
- ✓ Rigorous machinery for NLO

Conclusions

Auxiliary Field Formalism

- ✓ Integrated framework for Bose and Fermi gases
- ✓ Non-perturbative formalism
- ✓ Rigorous machinery for NLO
- ✓ Applicable to non-equilibrium studies