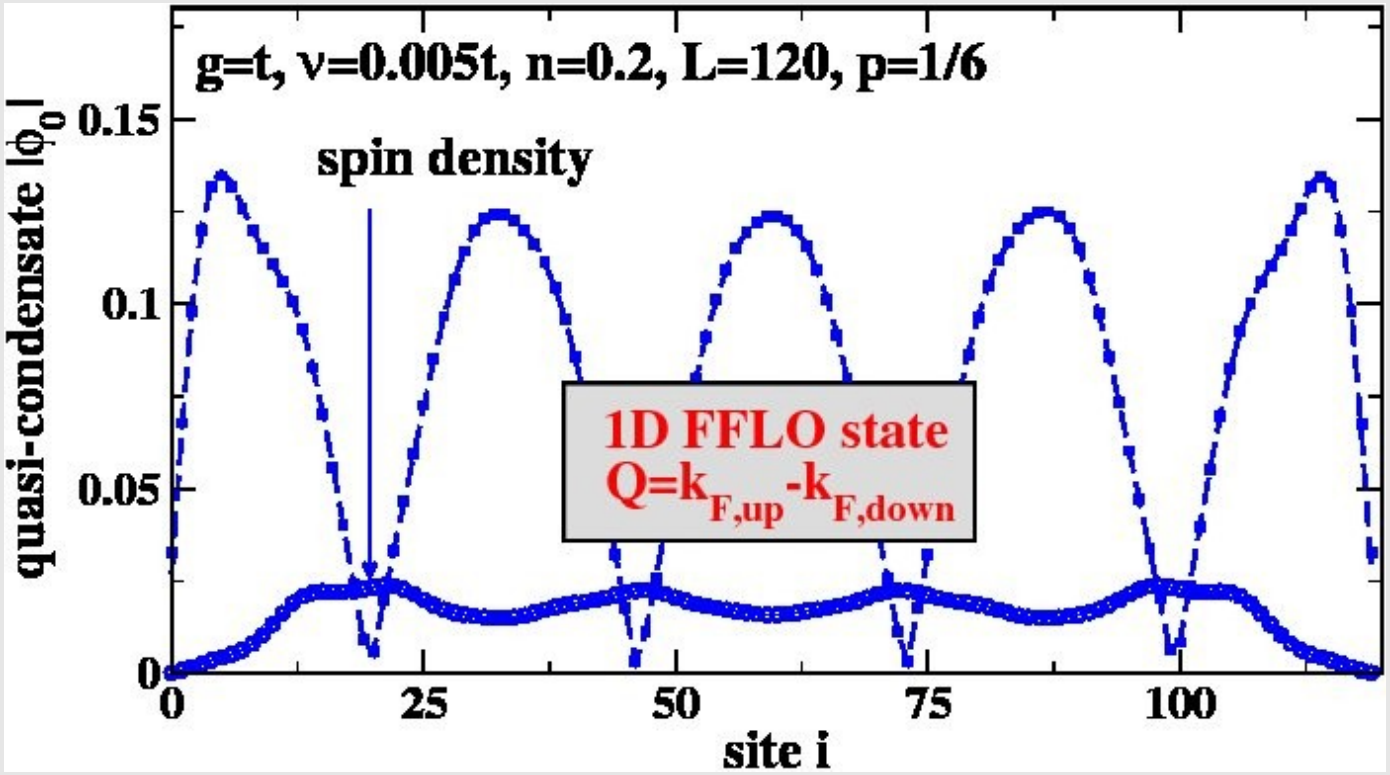
FFLO physics in spin-polarized Fermi gases in 1D



Fabian Heidrich-Meisner (LMU Munich)

Adrian Feiguin (U Wyoming), Giuliano Orso (Paris)

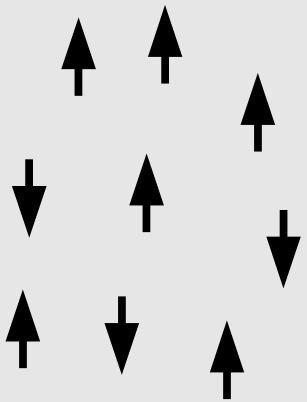
Uli Schollwöck (LMU), Wilhelm Zwerger (TU Munich)

Ground-state of spin imbalanced Fermi gases?

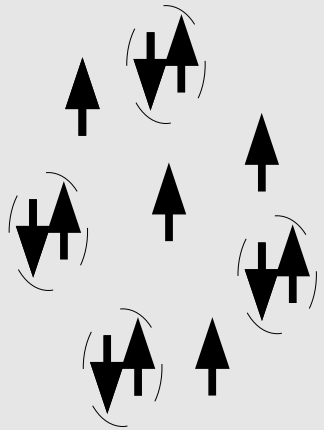
Son, Stephanov PRA 2006,
Moon, Nikolic, Sachdev PRL 2007
Sheehy, Radzihovsky PRL 2006,
Bukgac, Forbes PRL 2008, …
many others!

$$p = \frac{(N_\uparrow - N_\downarrow)}{(N_\uparrow + N_\downarrow)}$$

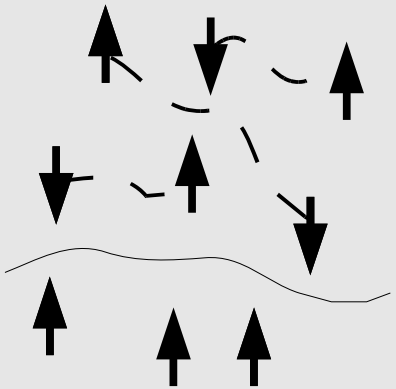
Normal gas
(preformed pairs?)

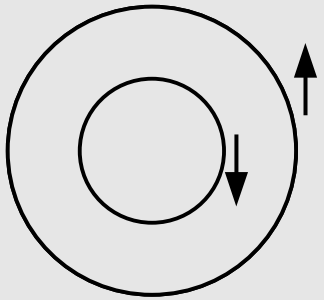


Bose-Fermi mixtures



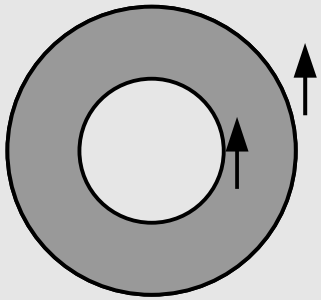
Phase separation:
Fully polarized Fermi gas & Balanced SF





Sarma phase

Predicted for T>0
Gubbels, Romans, Stoof PRL 2006



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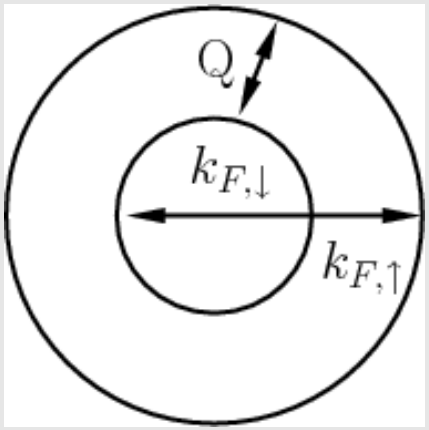
Ground-state of spin imbalanced Fermi gases?

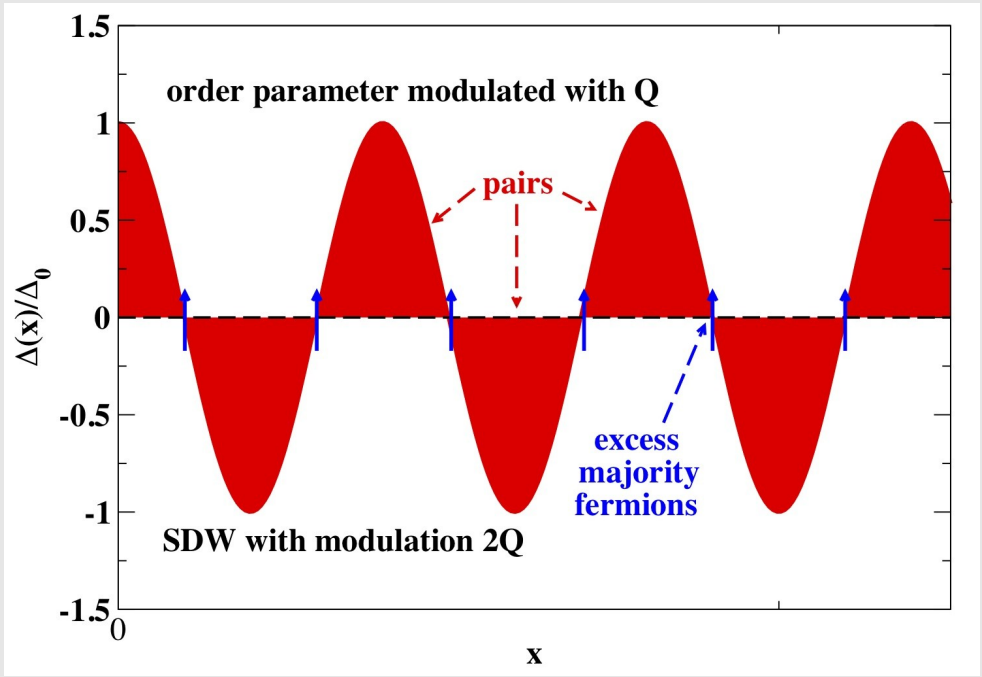


Fulde-Ferrell-Larkin-Ovchinnikov superfluid

Fulde & Ferrell PR 1964; Larkin & Ovchinnikov Zh. Eksp. Teor. Fiz. 1964

Cooper pairs w/ finite center-of-mass momentum

$$\Delta(\mathbf{r}) \sim \Delta_0 \cos(\mathbf{Q}\mathbf{r})$$



- **1D: perfect nesting**

$$Q = k_{F,\uparrow} - k_{F,\downarrow}$$

- **Exact methods available:**
  Bethe ansatz, DMRG, QMC

$$\left|\langle c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} c_{j\downarrow} c_{j\uparrow} \rangle\right| \sim \left|\cos(Q(x))\right| x^{-\Delta}$$

Yang PRB 2001

At weak coupling/ small polarization: Domain-wall state

Machida, Nakanishi PRB 1984, Buzdin, Polonski Sov. Phys. Lett. 1983 ...

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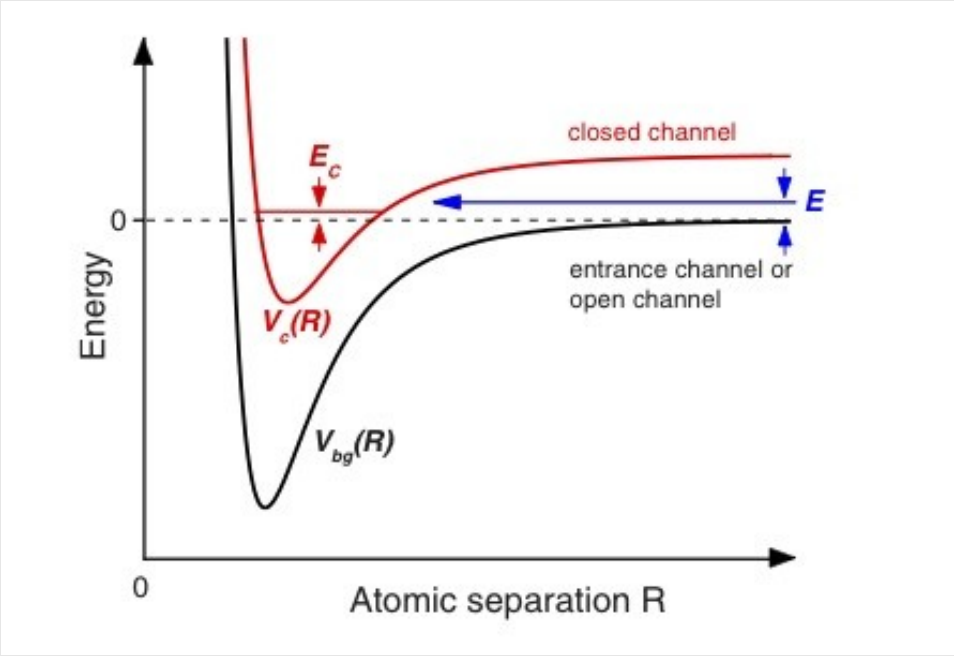
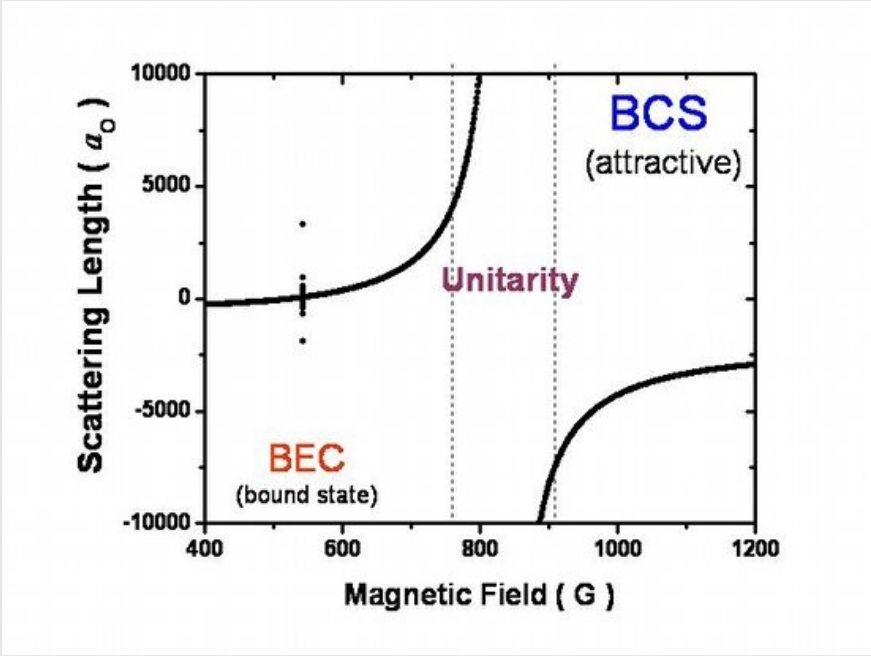
Taking a polarized Fermi gas across a resonance: Competing phases?



Chin, Grimm, Julienne, Tiesinga Rev. Mod. Phys. 2010



From http://atomcool.rice.edu/
Partridge et al Phys. Rev. Lett. 2005

We will show that in 1D
"partially polarized = FFLO"
is not generally correct

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UW@Seattle – April 4, 2011

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Outline

1) Density matrix renormalization group method – DMRG

2) FFLO state in the 1D Hubbard model with U<0

3) Trapped gas: Density profiles, the 2010 Rice experiment
   1D vs. coupled chains and vs. 3D

4) BCS-BEC crossover of a spin imbalanced gas in 1D

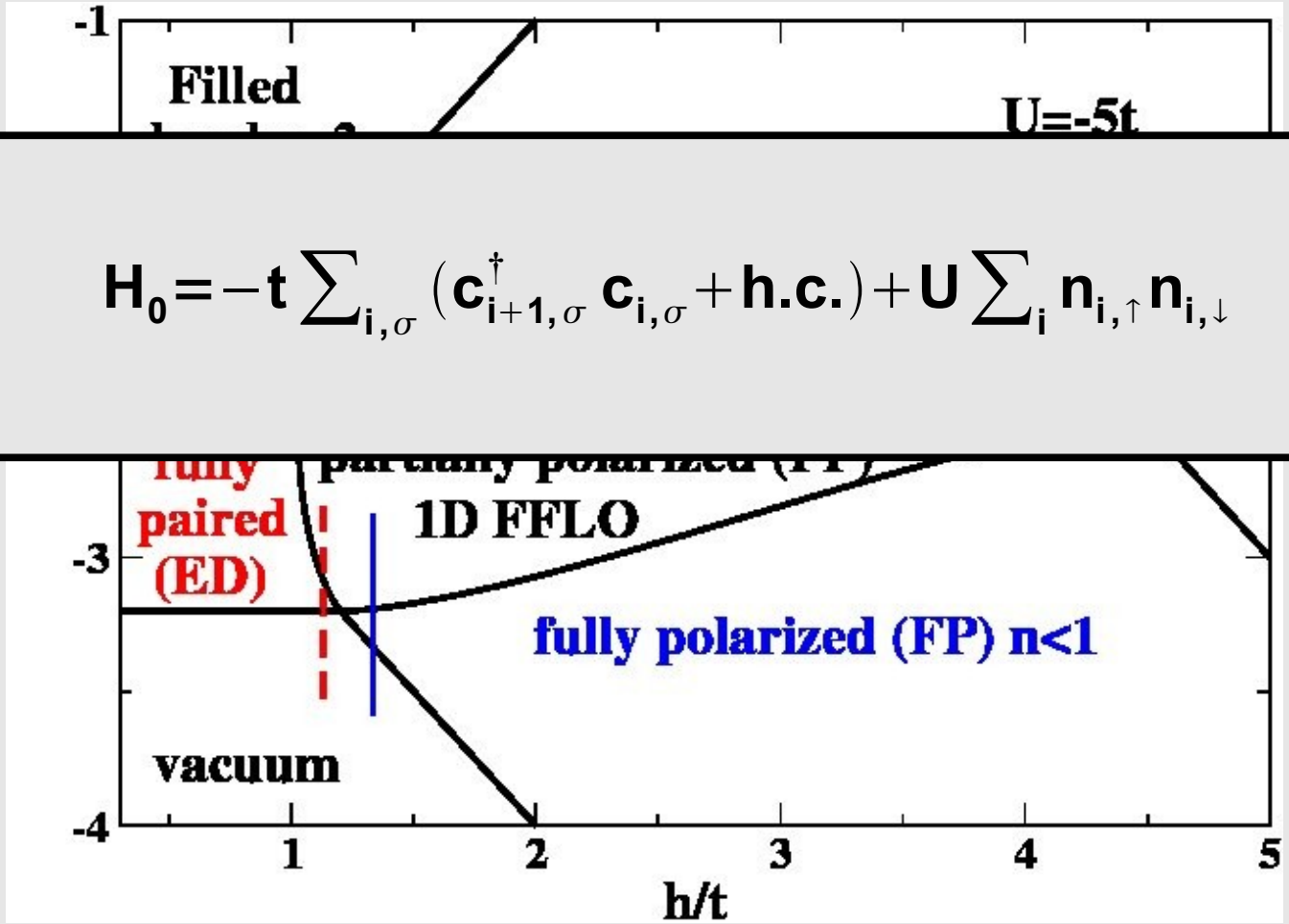
5) Open questions: Experimental observation of
   FFLO correlations, dimensional crossover 1D → 2D/3D

One-channel: 1D U<0 Hubbard model

$$H_0 = -t \sum_{i,\sigma} \left(c^{\dagger}_{i+1,\sigma} c_{i,\sigma} + \text{h.c.} \right) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

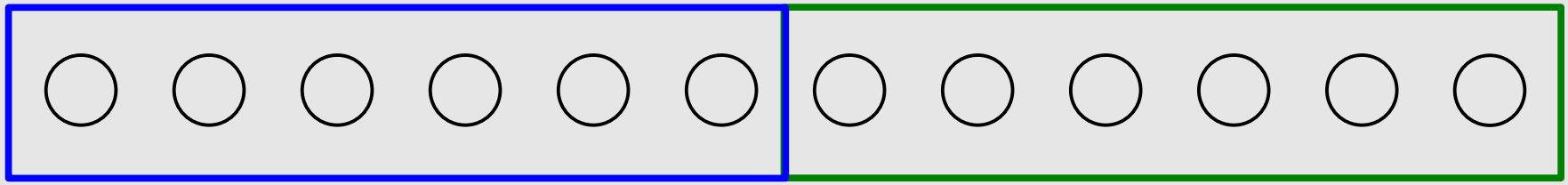
$N_\uparrow$=const

$N_\downarrow$=const



Essler et al. CUP 2005; Continuum: Orso PRL 2007, Hu et al. PRL 2007

What does DMRG do?



System A, L_A:
basis states $|i\rangle_A$

Environment B, L_B:
basis states $|r\rangle_B$

$$|\psi\rangle_0 = \sum_{i,r}^{\dim} \psi_{ir} |i\rangle_A |r\rangle_B \Rightarrow |\psi\rangle_{0,m} = \sum_{\alpha}^{m} \lambda_\alpha |\alpha\rangle_A |\alpha\rangle_B$$

Schmidt decomposition (SVD of ψ_{ir})

Reduced density matrix:

$$\rho_A = \mathrm{tr}_B |\psi_0\rangle\langle\psi_0| = \sum_{\alpha}^{m} \lambda_\alpha^2 |\alpha\rangle_{AA}\langle\alpha|$$

Discarded weight:

$$\delta\rho = \sum_{m+1}^{\dim} \lambda_\alpha^2 \ll 1$$

White Phys. Rev. Lett. 1992;
Schollwöck Rev. Mod. Phys. 2005, Ann. Phys. 2011

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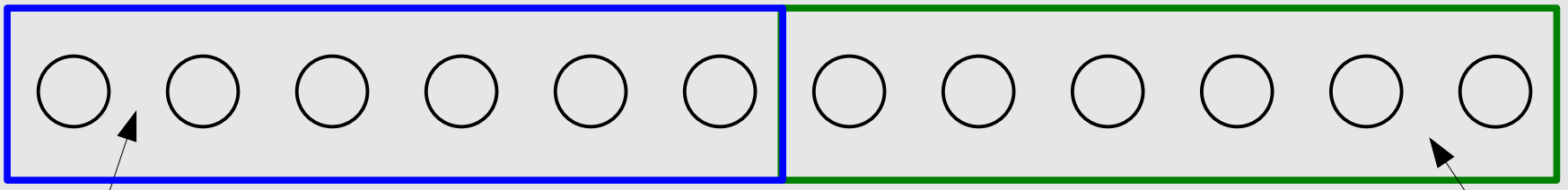
Density matrix renormalization group



System A
dim(basis)=m

Environment B

$$H_0|\psi\rangle_0 = E_0|\psi\rangle_0$$

White Phys. Rev. Lett. 1992
Schollwöck Rev. Mod. Phys. 2005

Matrix-Product-State dimension m
Rommer, Östlund PRL 1995, PRB 1997

$$\rho_A = \mathrm{tr}_B |\psi_0\rangle\langle\psi_0|$$

$$S_{vN} = -\mathrm{tr}[\rho_A \ln(\rho_A)]$$

$$m \propto e^{S_{vN}(L)}$$

Works for ground states in 1D - mildly entangled wave functions:

S_{vN} ~ const. ($L_A > \xi$)

S_{vN} ~ log L_A (critical systems)

Area law:

$$S_{vN} \sim \partial V \sim L^{d-1}$$

Review: Eisert, Cramer, Plenio RMP 2010

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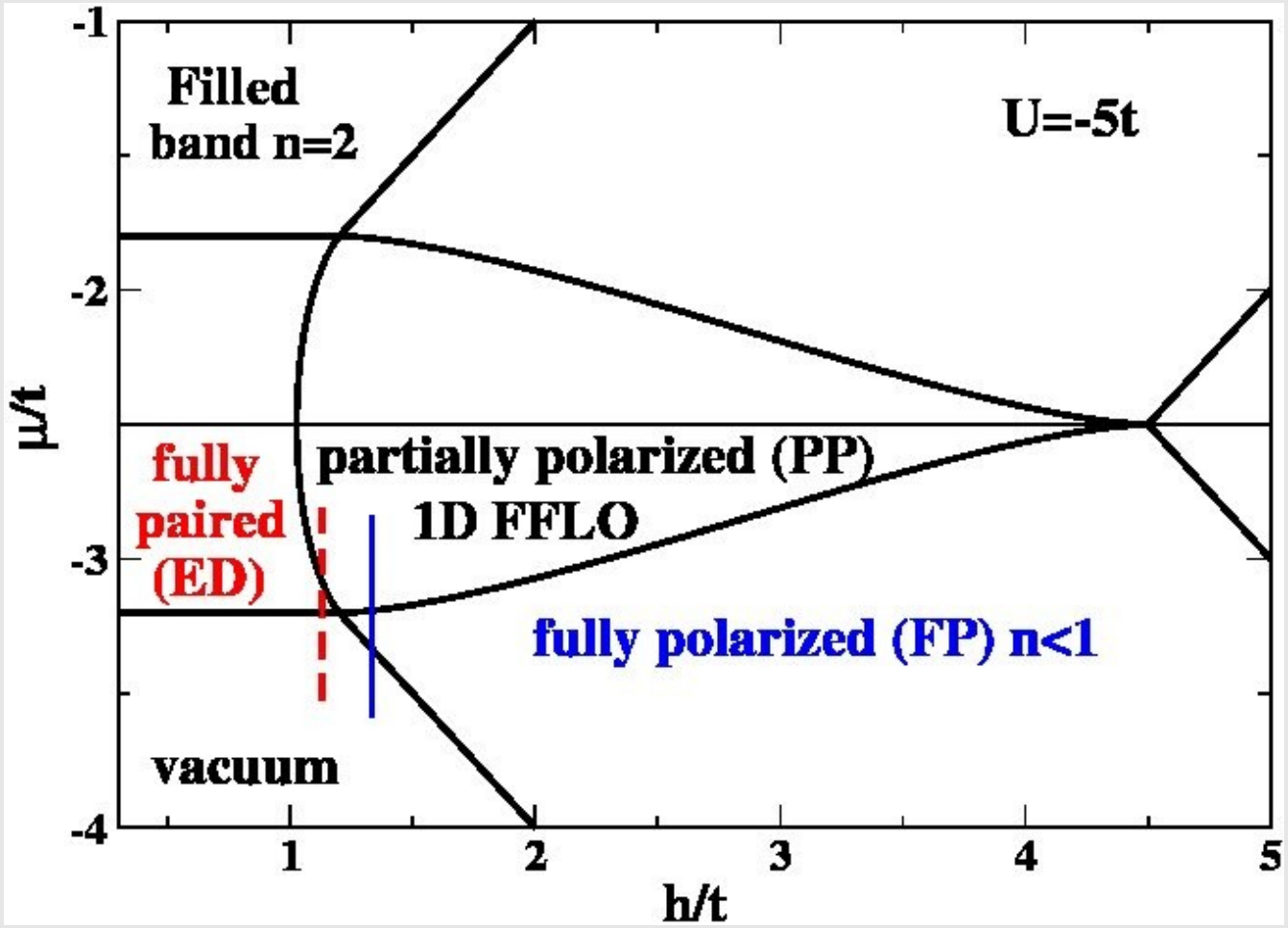
2) One-channel: 1D U<0 Hubbard model



Density
n=N/L

Polarization
$p=(N_\uparrow - N_\downarrow)/N$

$h = 2(\mu_\uparrow - \mu_\downarrow)$

Goal: Show that partially polarized phase is FFLO

HM, Orso, Feiguin PRA 2010, Essler et al. CUP 2005; **Continuum: Orso PRL 2007, Hu et al. PRL 2007**

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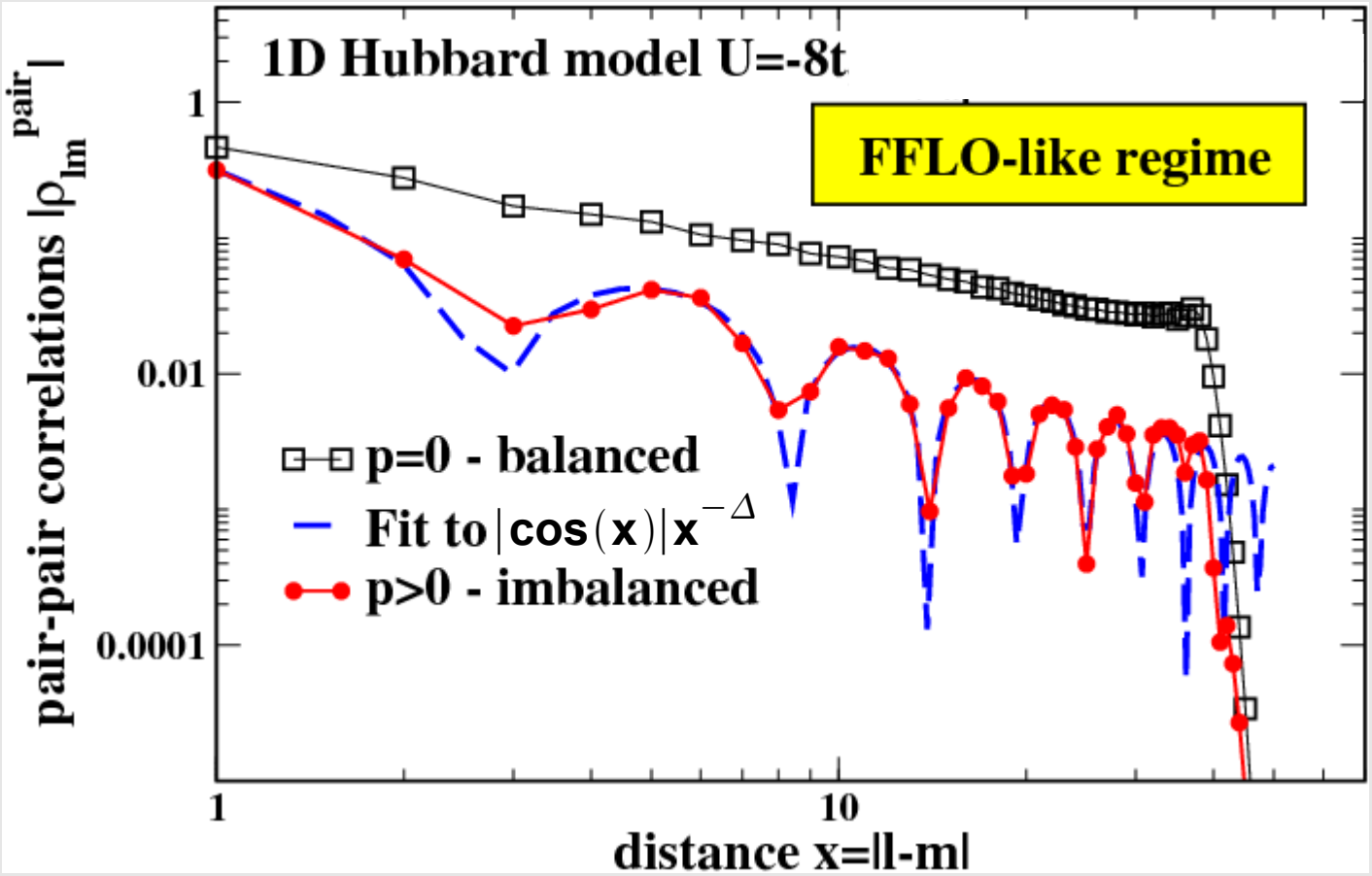
One-channel: FFLO correlations



FFLO correlations as predicted from bosonization!

$$|\rho_{ij}| = |\langle c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} c_{j\downarrow} c_{j\uparrow} \rangle| \sim |\cos(Q(i-j))| x^{-\Delta}$$

Yang PRB 2001; Feiguin & HM PRB 2007

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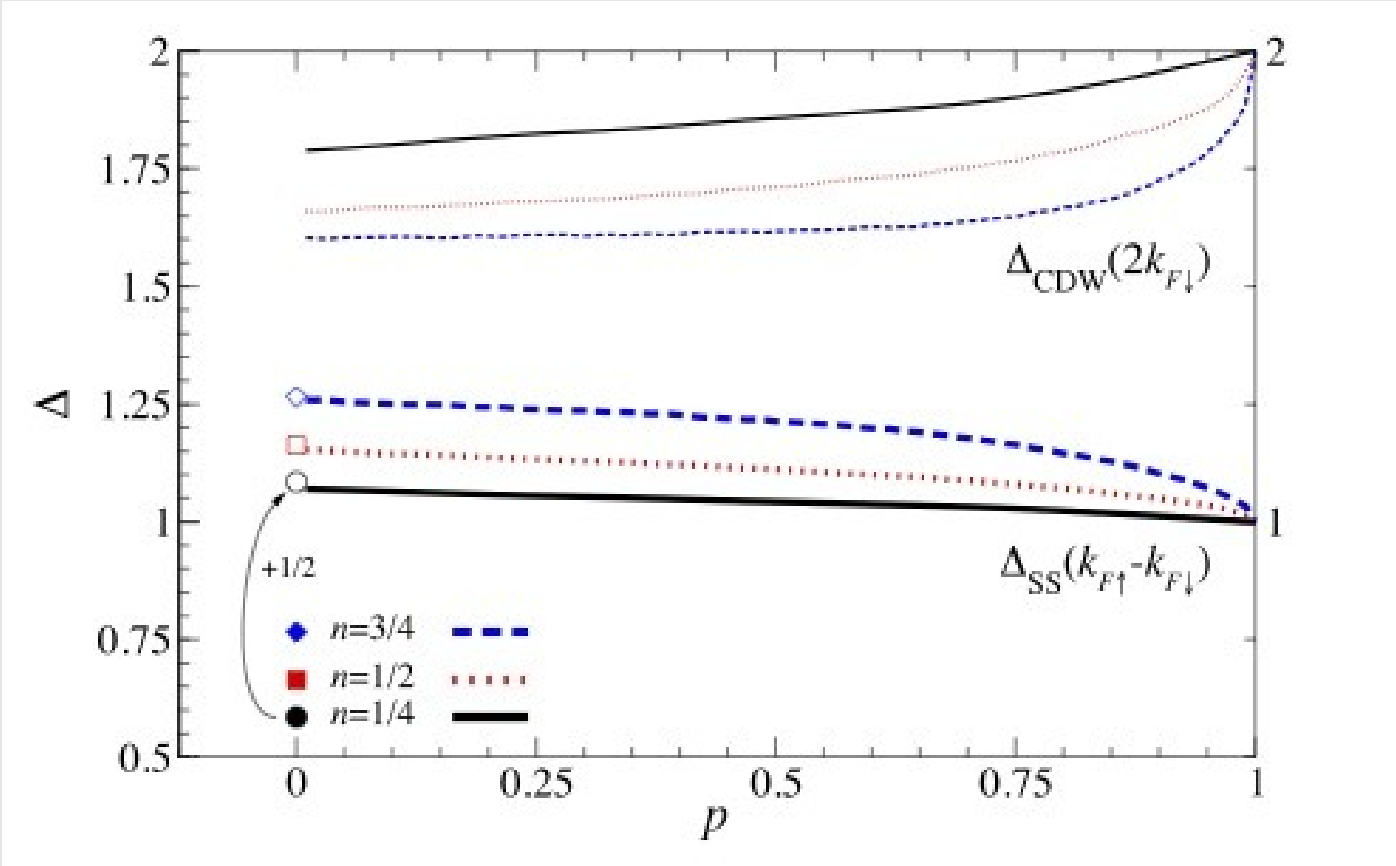
One-channel: FFLO correlations



FFLO at any finite imbalance!

$$|\rho_{ij}| = |\langle c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} c_{j\downarrow} c_{j\uparrow} \rangle| \sim |\cos(Q(i-j))| x^{-\Delta}$$

Yang PRB 2001; Feiguin & HM PRB 2007

Lüscher, Noack, Läuchli PRA 2008:
FFLO slowest decaying correlation!

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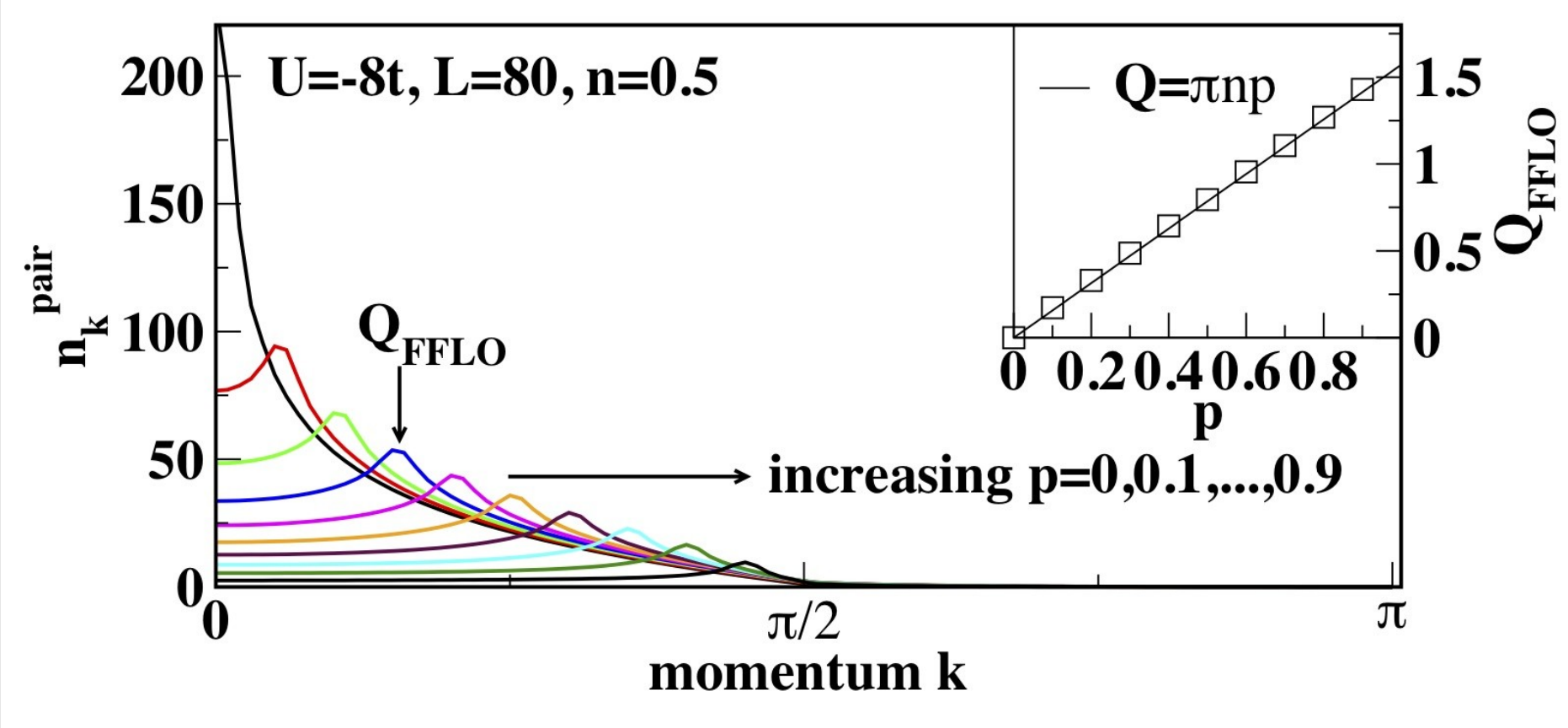
Momentum distribution of pairs

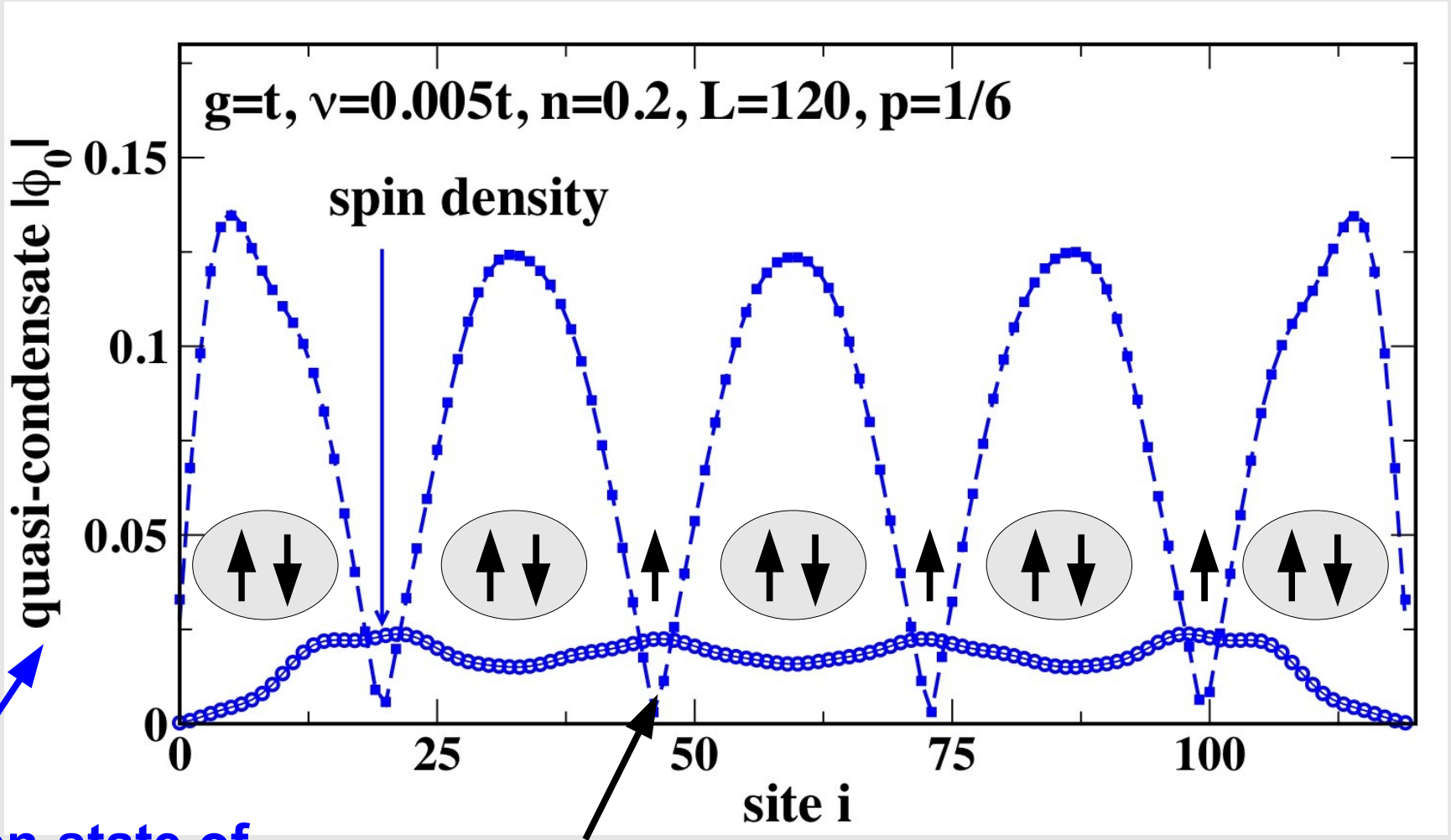


Finite momentum peaks scale with p as expected

$$\rho_{ij} = \langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{j\downarrow} c_{j\uparrow} \rangle \qquad Q = k_{F,\uparrow} - k_{F,\downarrow} = \pi n p$$

Feiguin & HM PRB 2007, Feiguin, HM, Orso, Zwerger Lect. Not. Phys. in press
Rizzi et al. PRA 2008, Batrouni et al PRL 2008, Casula et al. PRA 2008

The FFLO state: Real-space structure



eigen-state of

$$\rho_{ij} = \langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{j\downarrow} c_{j\uparrow} \rangle$$

"order parameter" changes its sign

… oops … this is actually for the two-channel case, but looks nice :-)

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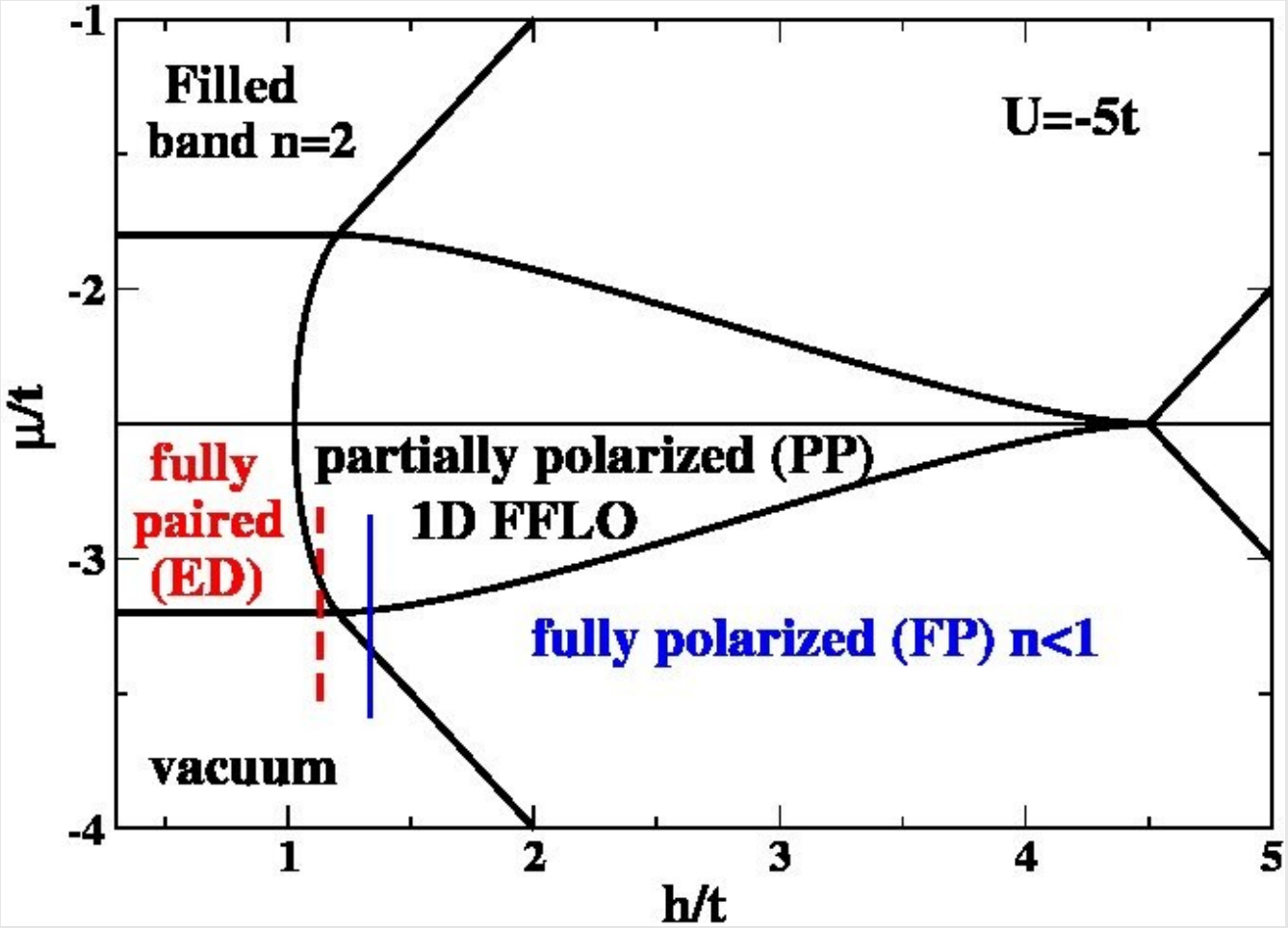
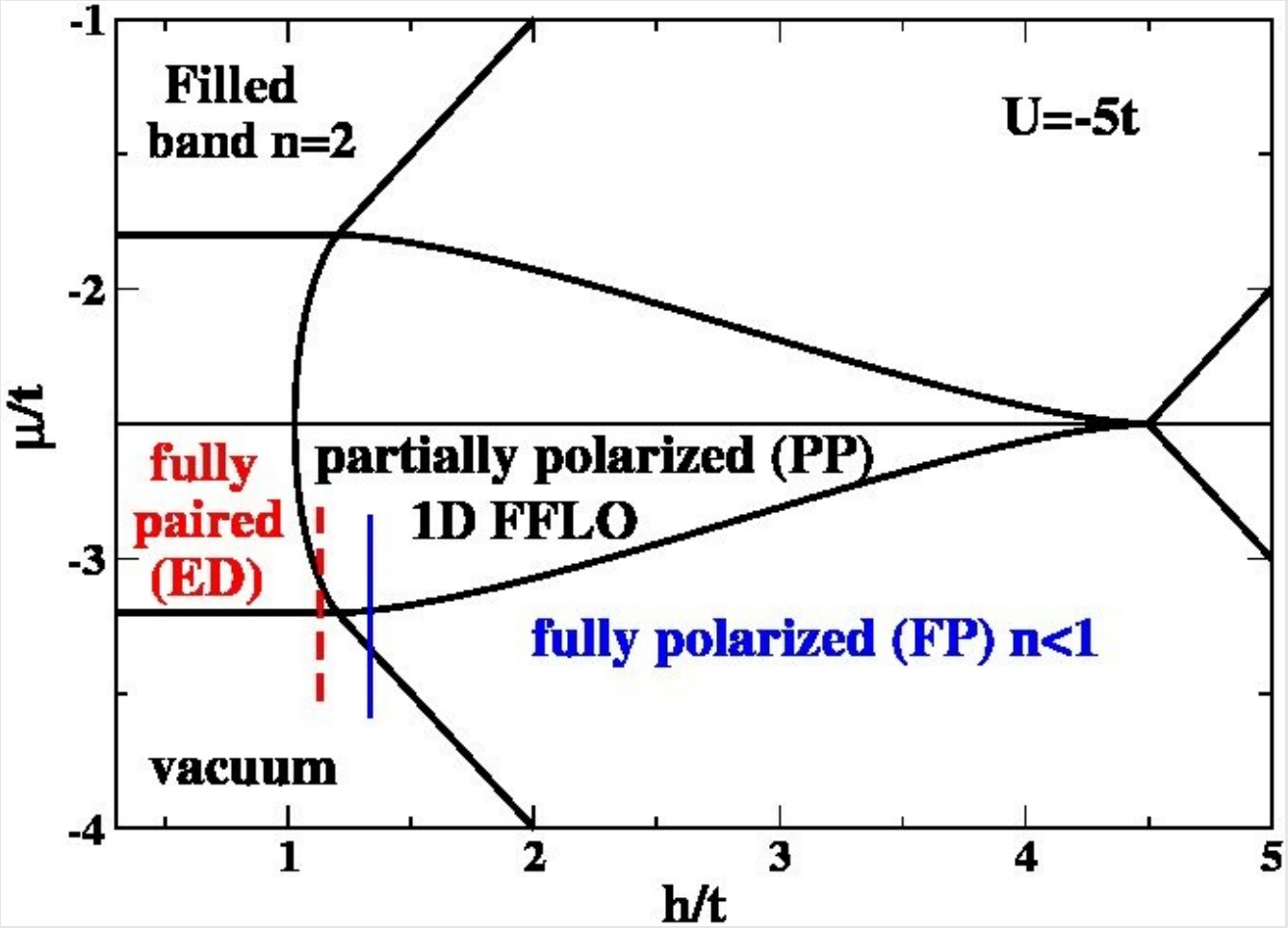
Adding the trap: Local density approximation



$$H = H_0 + V\sum_i (i-i_0)^2 n_i \rightarrow \begin{cases} \mu_i = \mu_c - V(i-i_0)^2 \\ h_i = h_c = \text{const} \end{cases}$$

Orso PRL 2007, Hu et al. PRL 2007, HM, Orso, Feiguin, PRA 2010

Adding the trap: Local density approximation



Balanced superfluid at *low n* in trap!

Spin gap:

$$\Delta \sim \exp(-\pi k_F a_1/2)$$

Pairing enhanced by high DOS at ε_F

(small n, $k_F \to 0$)

$$DOS_{1D} \sim 1/k_F$$

$$DOS_{3D} \sim k_F$$

$$H = H_0 + V\sum_i (i-i_0)^2 n_i \longrightarrow \mu_i = \mu_c - V(i-i_0)^2$$

$$h_i = h_c = \text{const}$$

Orso PRL 2007, Hu et al. PRL 2007, HM, Orso, Feiguin, PRA 2010

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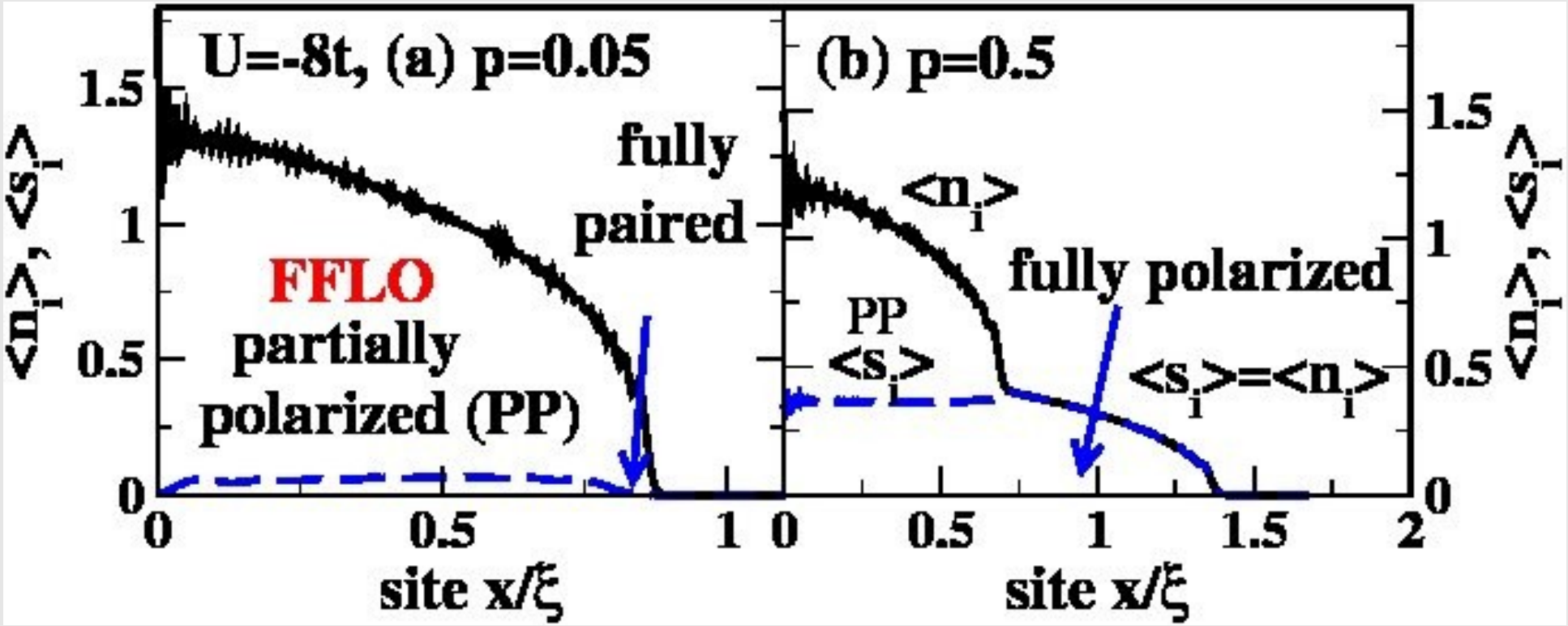
Harmonic trap: Shell structure

DMRG $H_{trap} = V \sum_i n_i (i - i_0)^2 ; s_i = n_{i\uparrow} - n_{i\downarrow} ; \xi = 1/\sqrt{V}$



PP phase always in core!

$p<p_c$: Fully paired wings, $p>p_c$: Fully polarized wings

HM & Feiguin PRB 2007; HM, Orso, Feiguin, PRA 2010

Continuum: Orso PRL 2007, Hu et al. PRL 2007

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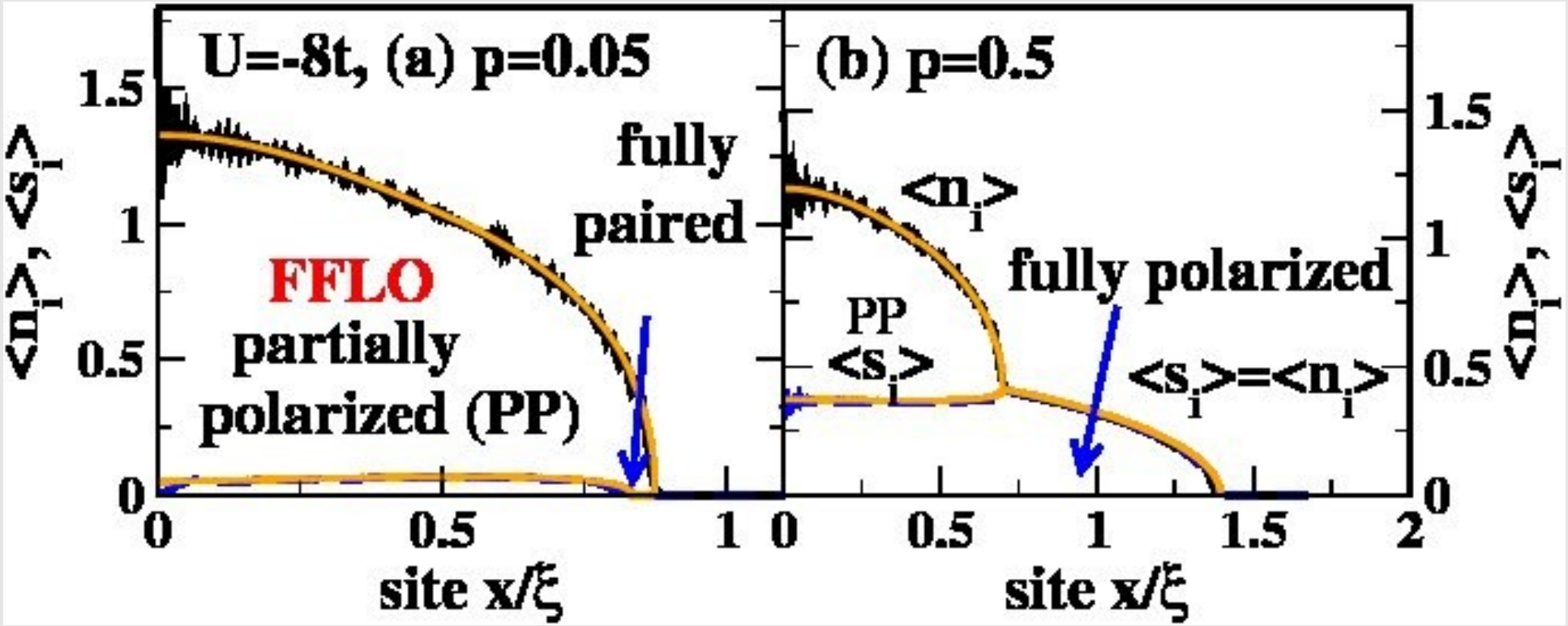
Harmonic trap: Shell structure

DMRG $H_{trap} = V \sum_i n_i (i - i_0)^2 ; s_i = n_{i\uparrow} - n_{i\downarrow} ; \xi = 1/\sqrt{V}$ LDA



DMRG & (Bethe-ansatz)+LDA agree
Requires large N~160

HM & Feiguin PRB 2007; HM, Orso, Feiguin, PRA 2010
Continuum: Orso PRL 2007, Hu et al. PRL 2007; Casula et al. PRA 2008

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Fermionic gases from ^{6}Li, the 1D Rice experiment

"Spin" states: hyperfine states F=1/2

$$m_F = -1/2 : |2\rangle = |\downarrow\rangle$$

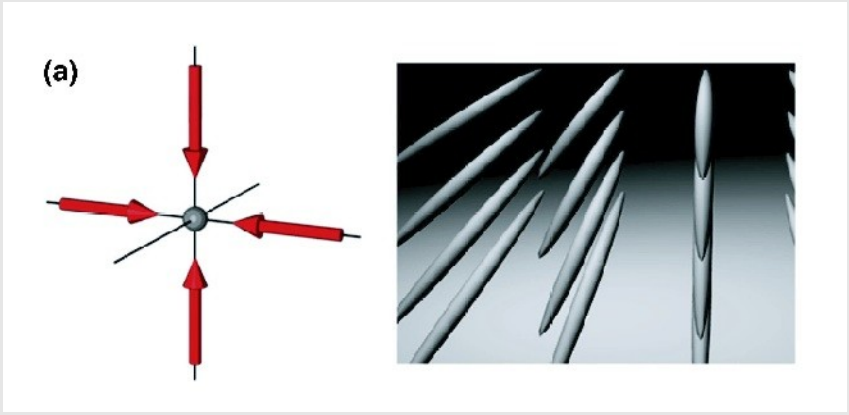
$$m_F = +1/2 : |1\rangle = |\uparrow\rangle$$

$$p = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$$

Conditions for 1D

$$t < \epsilon_F, T$$

$$\epsilon_F = N_\uparrow \hbar\omega_z < \hbar\omega_\perp$$



$$\omega_\perp/\omega_z \sim 10^3; N_\uparrow \sim 120$$

$$t/k_B = 17\,\mathrm{nK}; \epsilon_F/k_B \approx 1.2\,\mu\mathrm{K}$$

$$T \approx 175\,\mathrm{nK}$$

Bloch, Dalibard, Zwerger Rev. Mod. Phys. 2008

From Liao et al. Nature (2010) – Rice

→ **Incorporate trapping potential**

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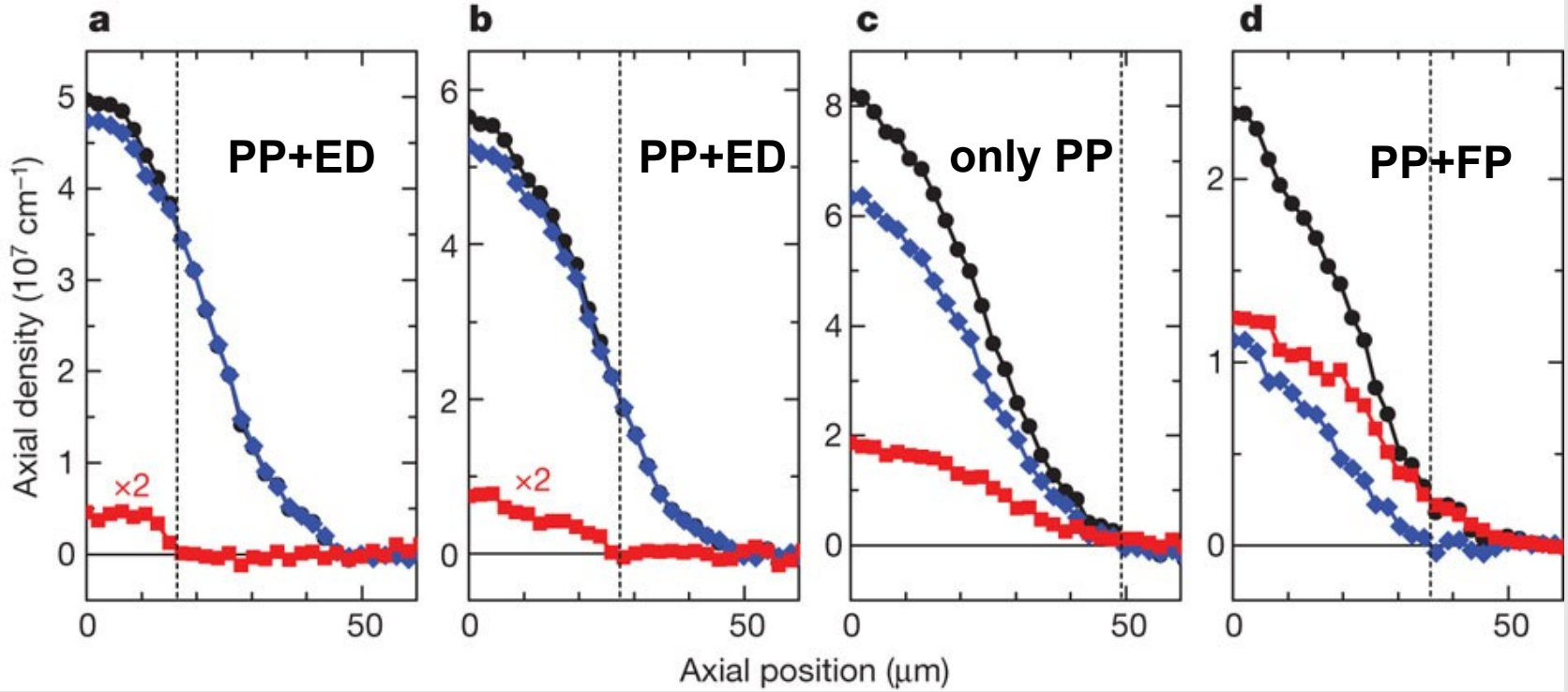
Experimental status: The 1D Rice experiment



Black: majority fermions (↑)
Blue: minority fermions (↓)
Red: density difference

1D tubes, continuum Liao et al. Nature 2010

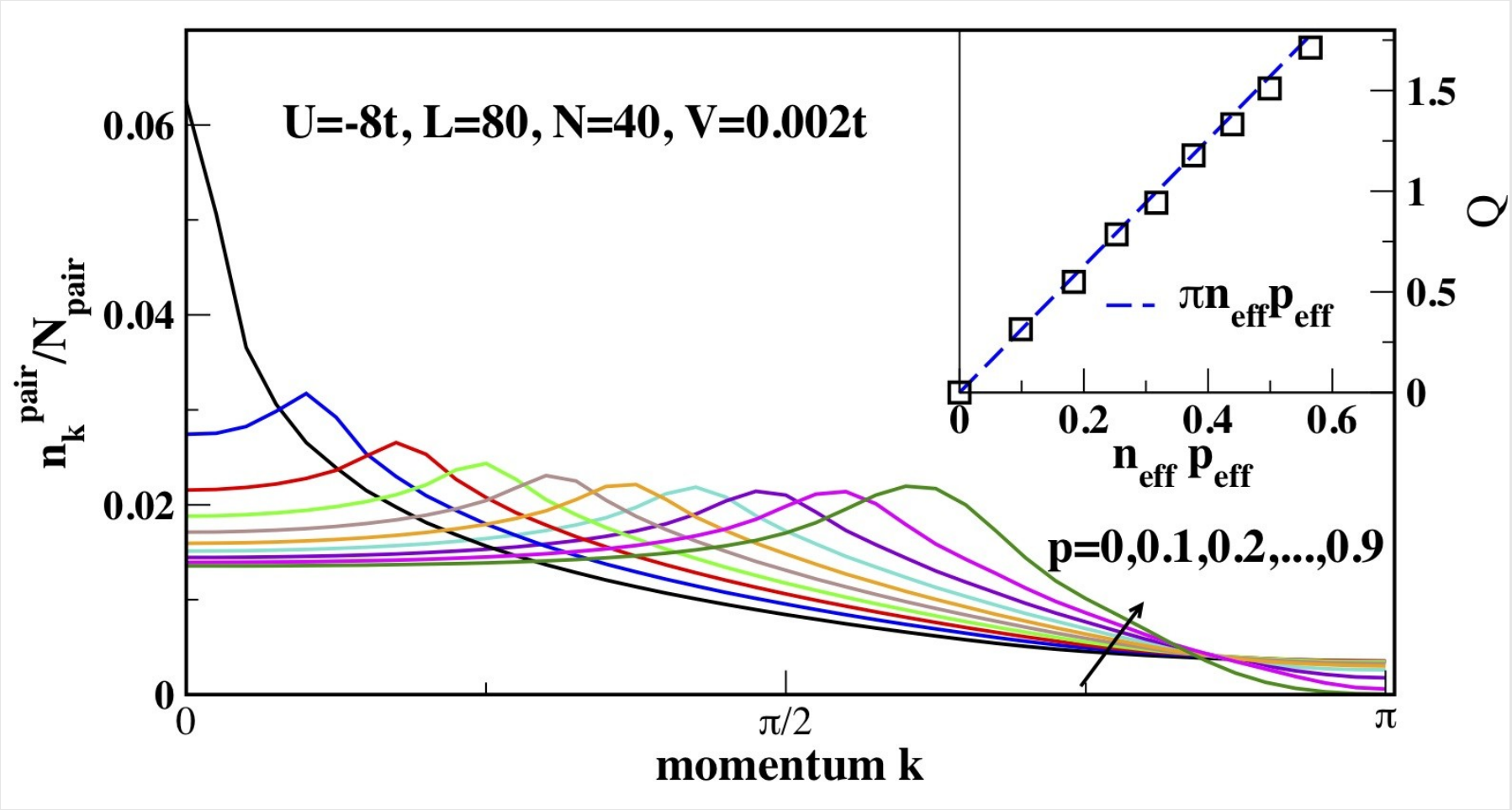
Realizes the Gaudin-Yang model!

$$P_c = 0.13 \pm 0.03$$

Reasonable agreement with Bethe-ansatz+LDA

Orso PRL 2007; Hu et al PRL 2007 ; Kakashvilii, Bolech PRA 2009 ← Finite T

Pair correlations “stable” in the presence of trap



p_{eff}=p – p(fully polarized wings)

Feiguin & HM PRB 2007

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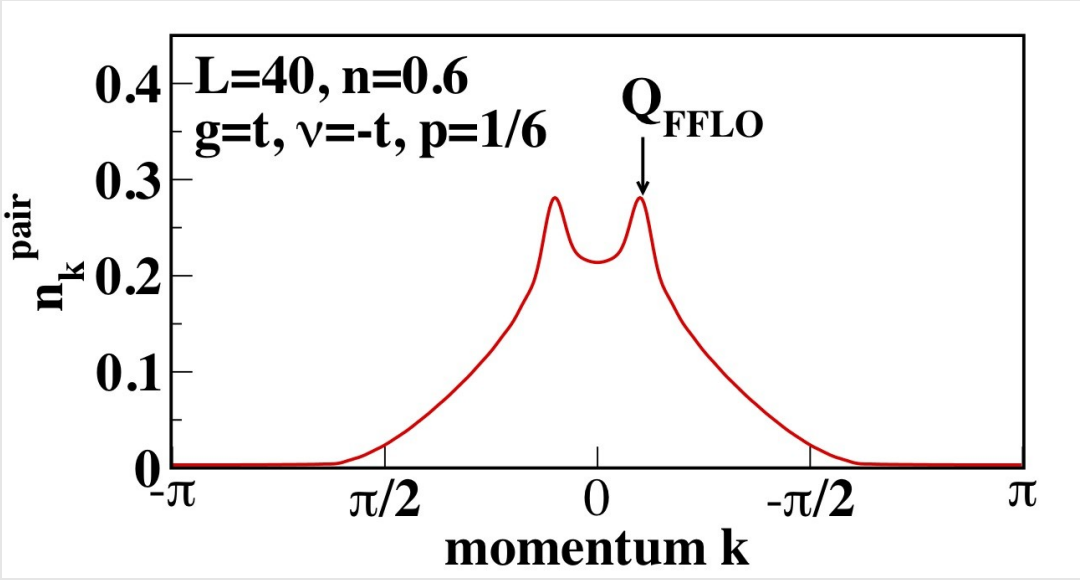
Detecting the FFLO state

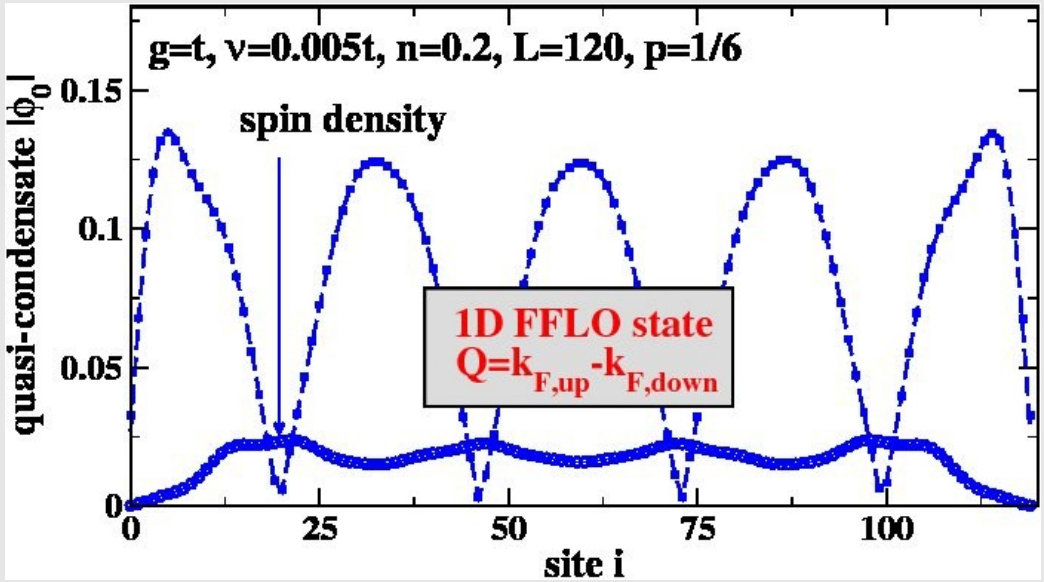


Direct probes

- **Time-of-flight**
  Yang PRL 2005
- **Noise correlations**
  Lüscher, Noack, Läuchli PRA 2008
- **rf-spectroscopy**
  Bakhtiari et al PRL 2008

Indirect probes

- **2Q - spin density wave**
  Nakanishi, Machida PRB 1984
  Feiguin & HM PRB 2007
  Roscilde et al. NJP 2009
- **Modulation spectroscopy**
  Korolyuk, Massel, Törmä PRL 2010
- **Collective modes**
  Edge & Cooper PRL 2009



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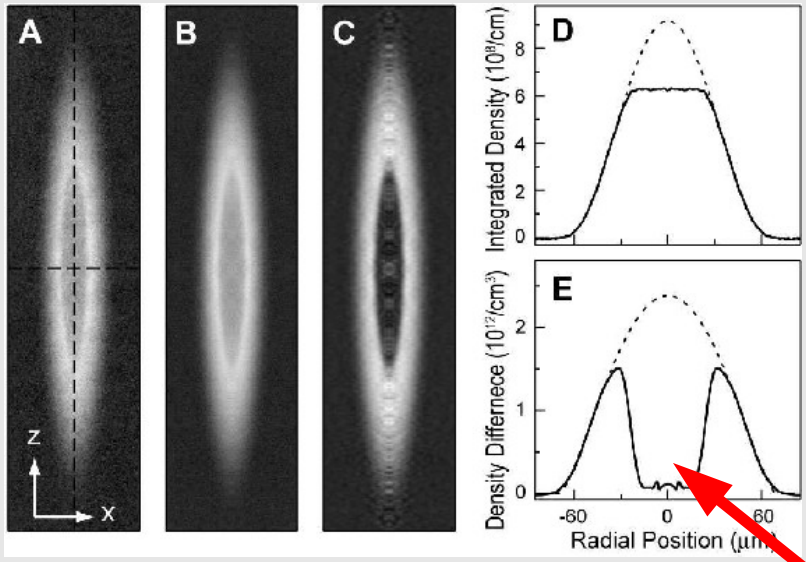
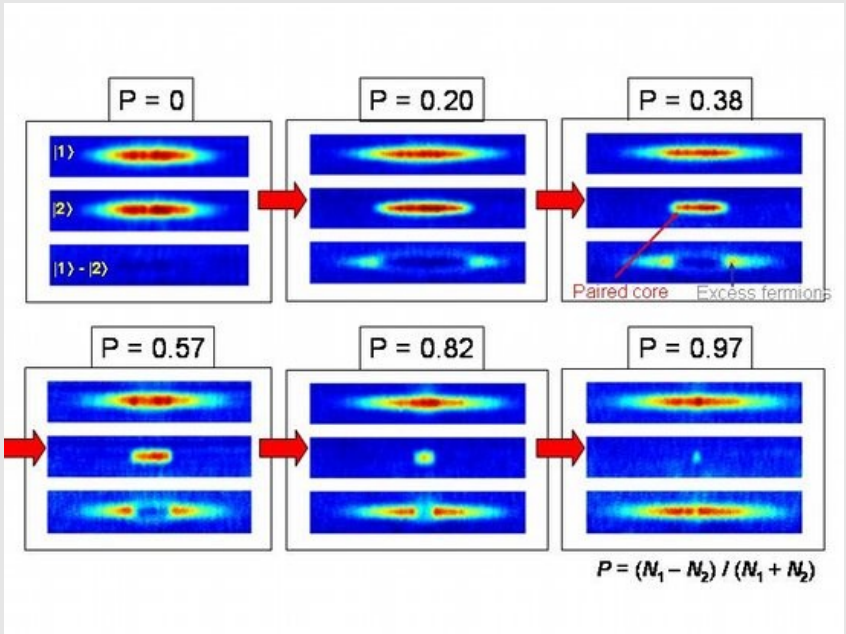
Experiments: 3D case



Ketterle group @ MIT, Hulet group @ Rice 2006



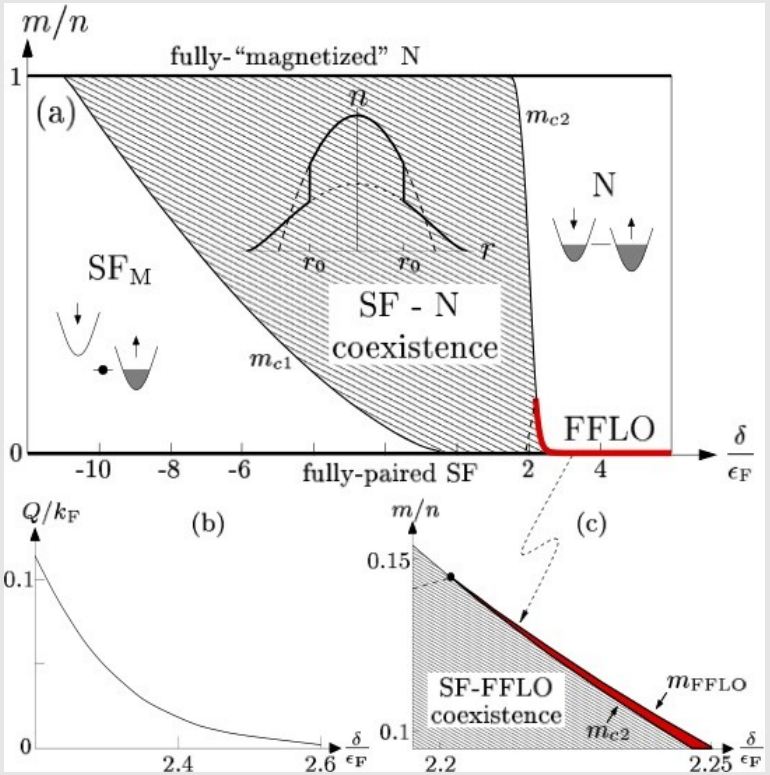
$p^{core} = 0$

3D: unpolarized core, fully polarized wing

Theory, e.g. de Silva, Mueller PRA 2006

Theory: FFLO phase "small" (?)

see, e.g. Sheehy & Radzihovsky 2006; Bulgac & Forbes PRL 2007; Yoshida & Yip PRA 2007; ...



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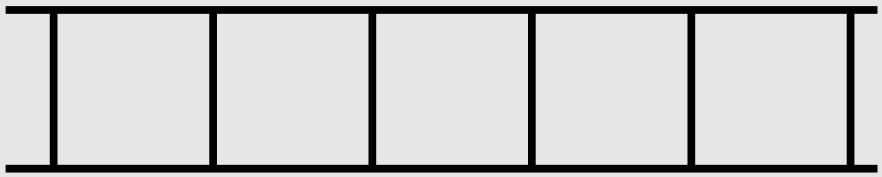
Hubbard model on two-leg ladder geometry

$$H = -t_{\perp} \sum_i \left(c^{\dagger}_{1,i,\sigma} c_{2,i+1,\sigma} + \text{h.c.} \right) - t_{\parallel} \sum_{l=1,2;i} \left(c^{\dagger}_{l,i,\sigma} c_{l,i+1,\sigma} + \text{h.c.} \right)$$

$$+ U \sum_{l=1,2;i} n_{l,i,\uparrow} n_{l,i,\downarrow} + V \sum_{l,i} n_{l,i} (i - i_0)^2 - \mu N - hnp/2$$

Two-leg ladder
Exact solution with DMRG



Feiguin & HM PRL 2010

Reflection symmetry: four "bands" (2x spin up, 2x spin down) labeled by transverse momentum & spin (k_y=0,π, σ=↑,↓)

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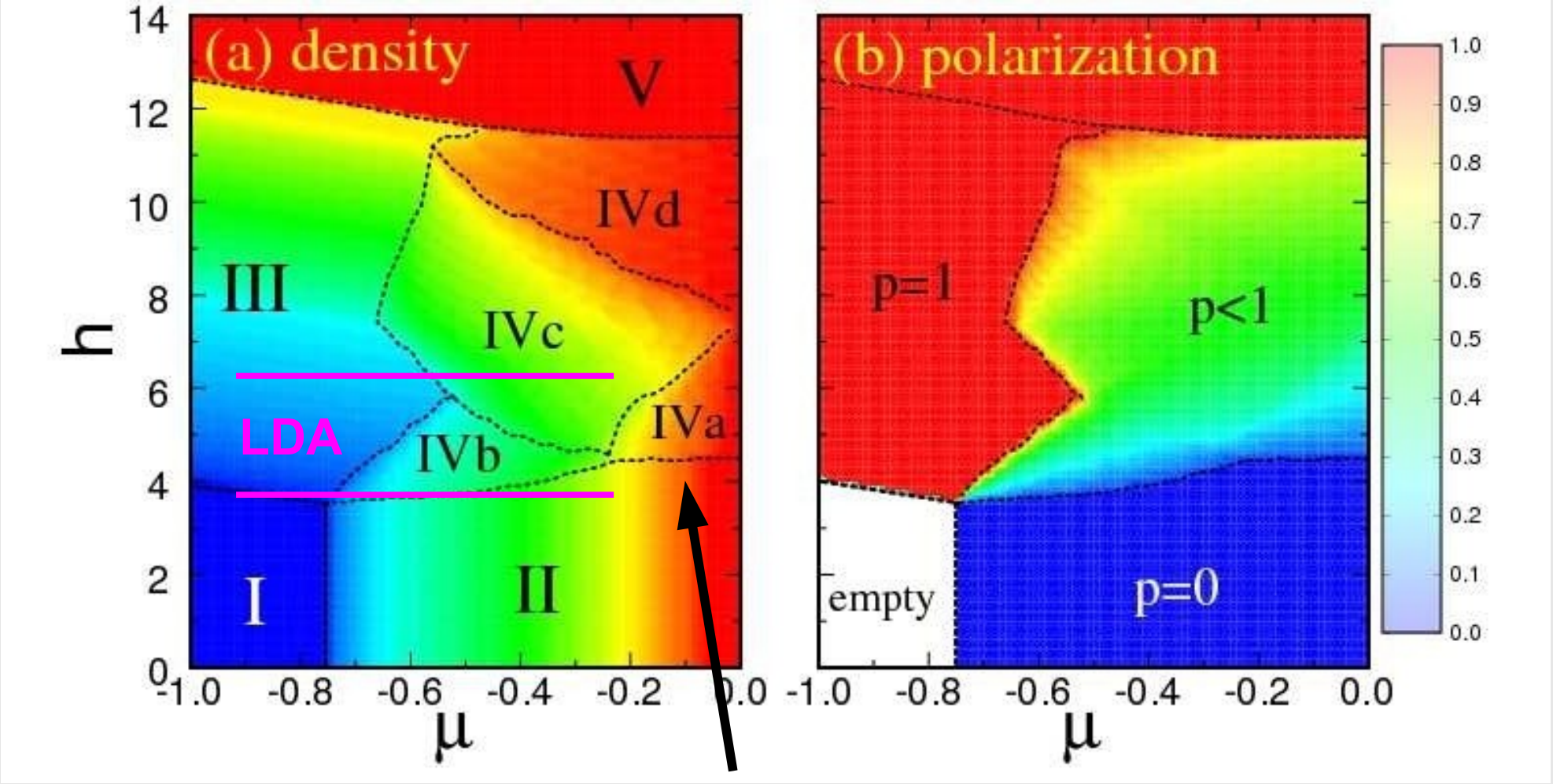
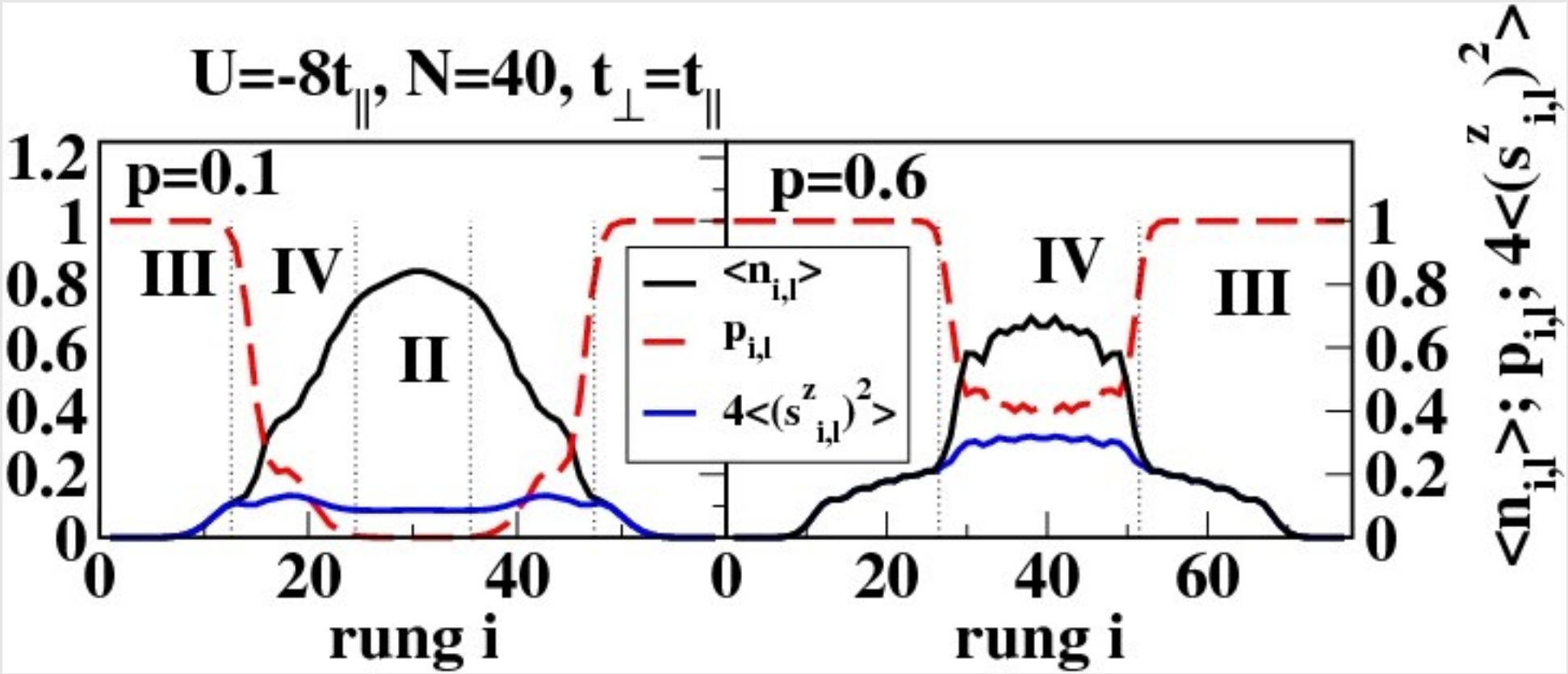
Ladder geometry: Phase diagram



Phase IV: partially polarized, FFLO

→ h increases with μ, as in 2D and 3D

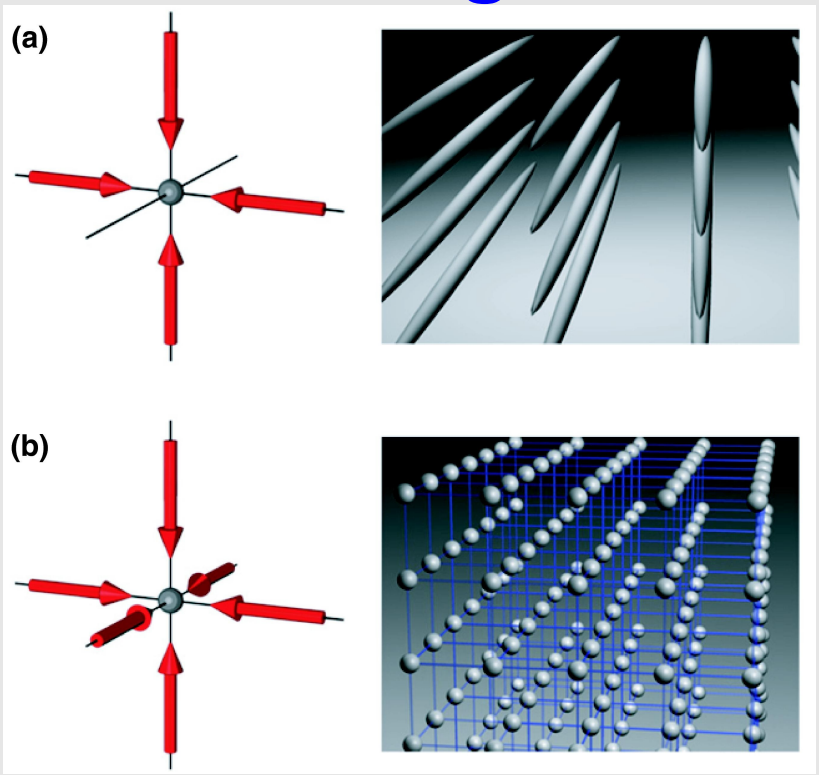
Ladder geometry: Trap & phase separation



Small p: core unpolarized, similar to 2D/3D!
→ Very weak t_{perp} is sufficient!

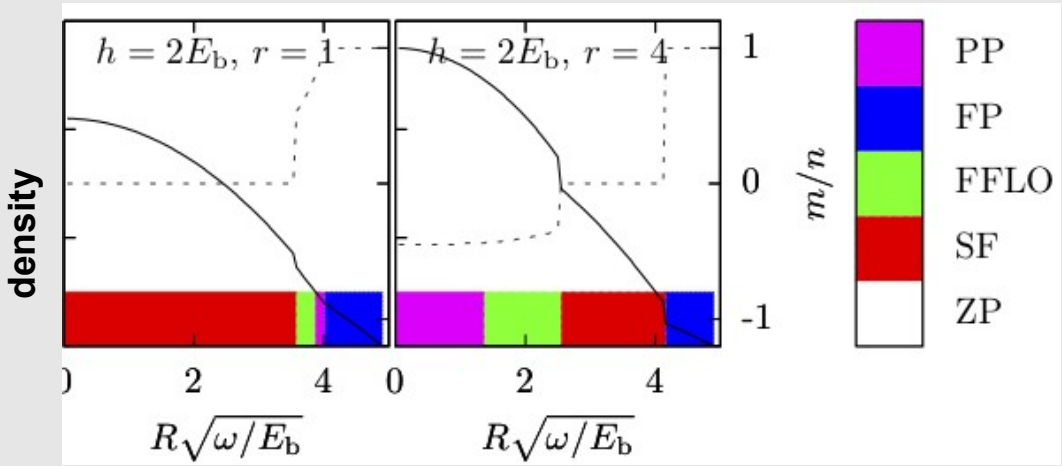
Feiguin & HM Phys. Rev. Lett. 2009

Going from 1D to 2D (… and 3D)



Bloch, Dalibard, Zwerger RMP 2008

Phase separation in 2D from mean-field theory:



Conduit, Conlon, Simons PRA 2008
Moreo & Scalapino PRL 2007
Yanase PRA 2009
Koga & Werner arXiv:1002.2958;
Pei, Dukelsky, Nazarewicz PRA 2010; ...

Weakly coupled chains:

→Incommensurate FFLO (?)

→Competition between FFLO and normal Fermi gas

Parish et al. PRL 2007; Zhao & Liu PRA 2008
Lüscher, Noack, Läuchli PRA 2008

→ FFLO in outer shell
Also true in 3D

de Silva, Mueller PRA 2006

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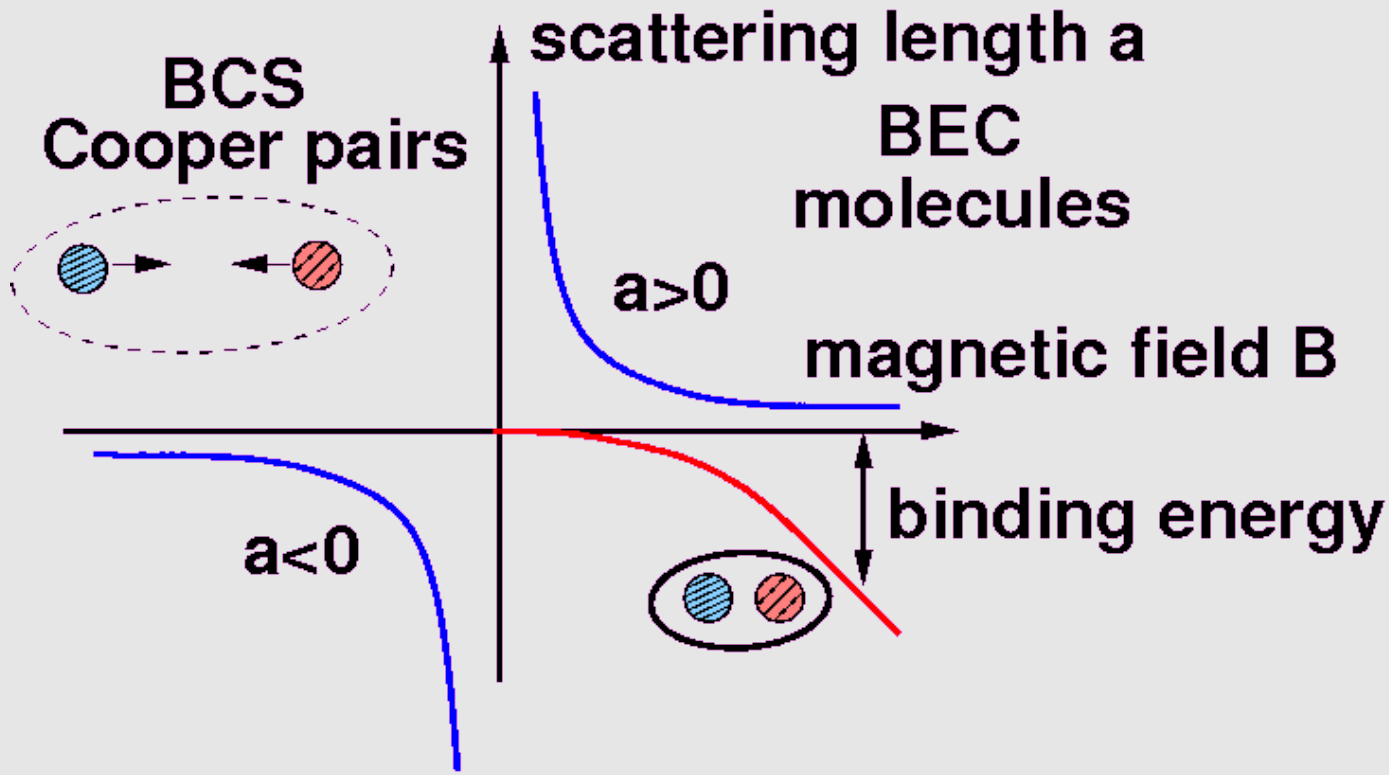
Stability of 1D FFLO against presence of molecules

Bloch, Dalibard, Zwerger RMP 2008 Giorgini, Pitaevski, Stringari RMP 2009 Ketterle, Zwierlein 2008



Competition between FFLO (BCS side) & (Composite) Bose-Fermi mixture (BEC side)

Experiments@ETH Moritz et al. PRL 2005 1D: confinement induced resonance Olshanii PRL 1998

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BCS-BEC crossover of a spin-imbalanced Fermi gas in 1D: Bose-Fermi resonance model

$$H = -t\sum_i (c^\dagger_{i,\sigma} c_{i+1,\sigma} + \text{h.c.}) - \frac{t}{2}\sum_i (m^\dagger_i m_{i+1} + \text{h.c.})$$

$$-(\nu + 3t)\sum_i n^{mol}_i + g\sum_i (m^\dagger_i c_{i,\uparrow} c_{i,\downarrow} + \text{h.c.})$$

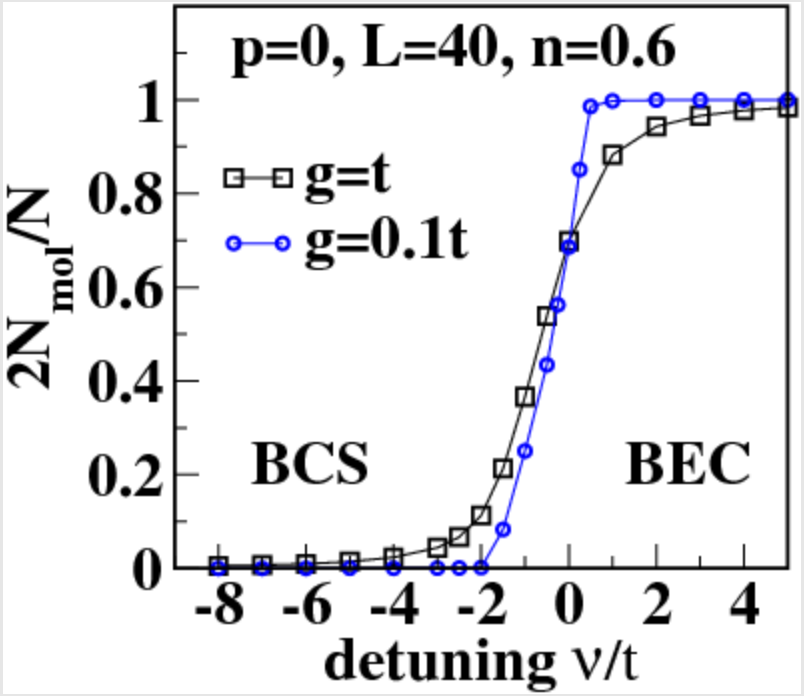
Detuning molecular level

"Feshbach" coupling

Holland et al. PRL 2001;
Timmermans et al. Phys. Lett. A 2001
Recati, Fuchs, Zwerger PRA 2005
Fuchs, Recati, Zwerger PRL 2004, Tokatly PRL 2004
Sachdev & Yang PRB 2006; Citro & Orignac PRL 2005

BCS limit $\nu \ll -1$

BEC limit $\nu \gg 1$



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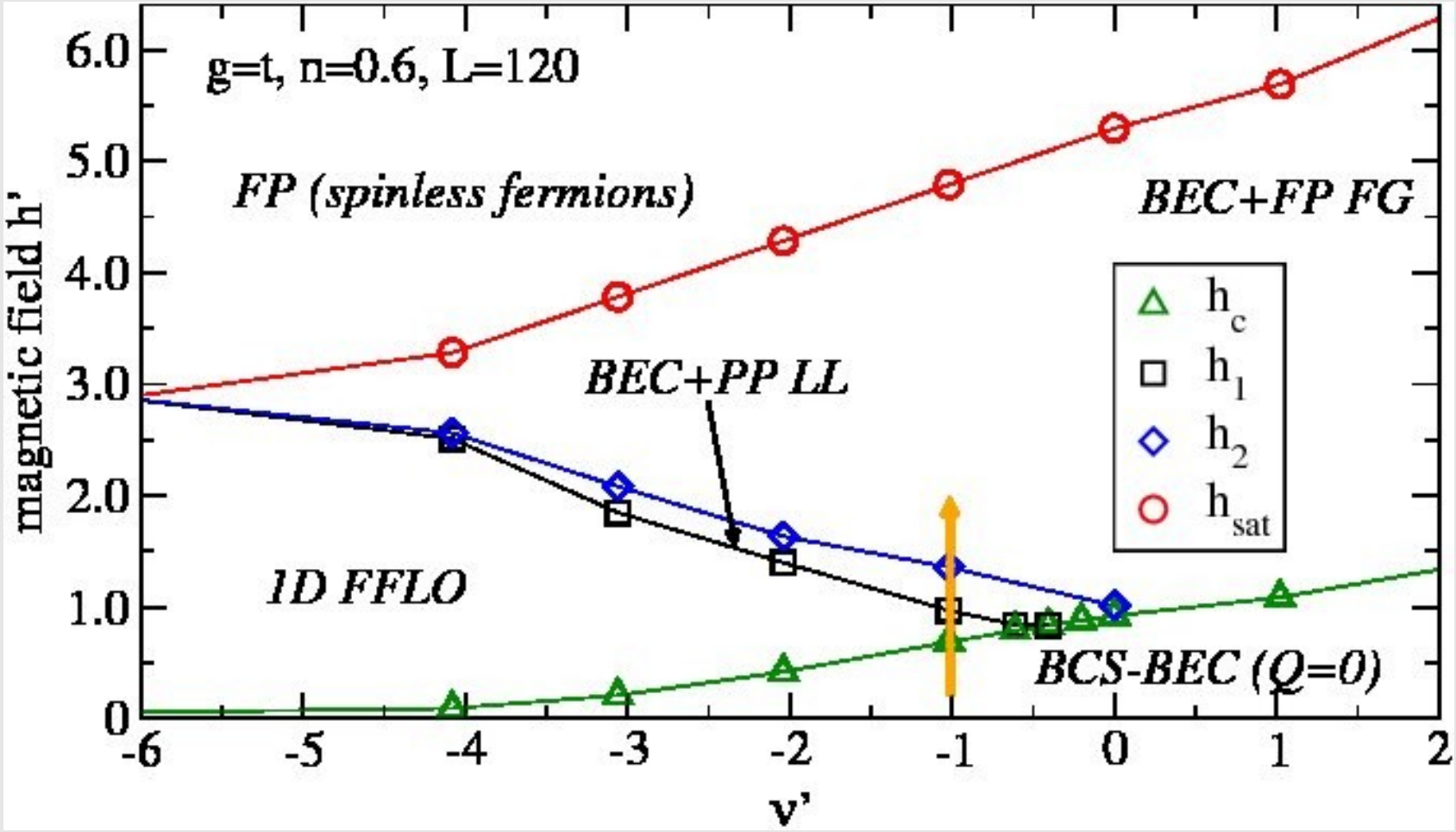
BFRM: Phase diagram from DMRG



FFLO breaks down: below a critical polarization & close to resonance

HM, Feiguin, Schollwöck, Zwerger PRA 2010

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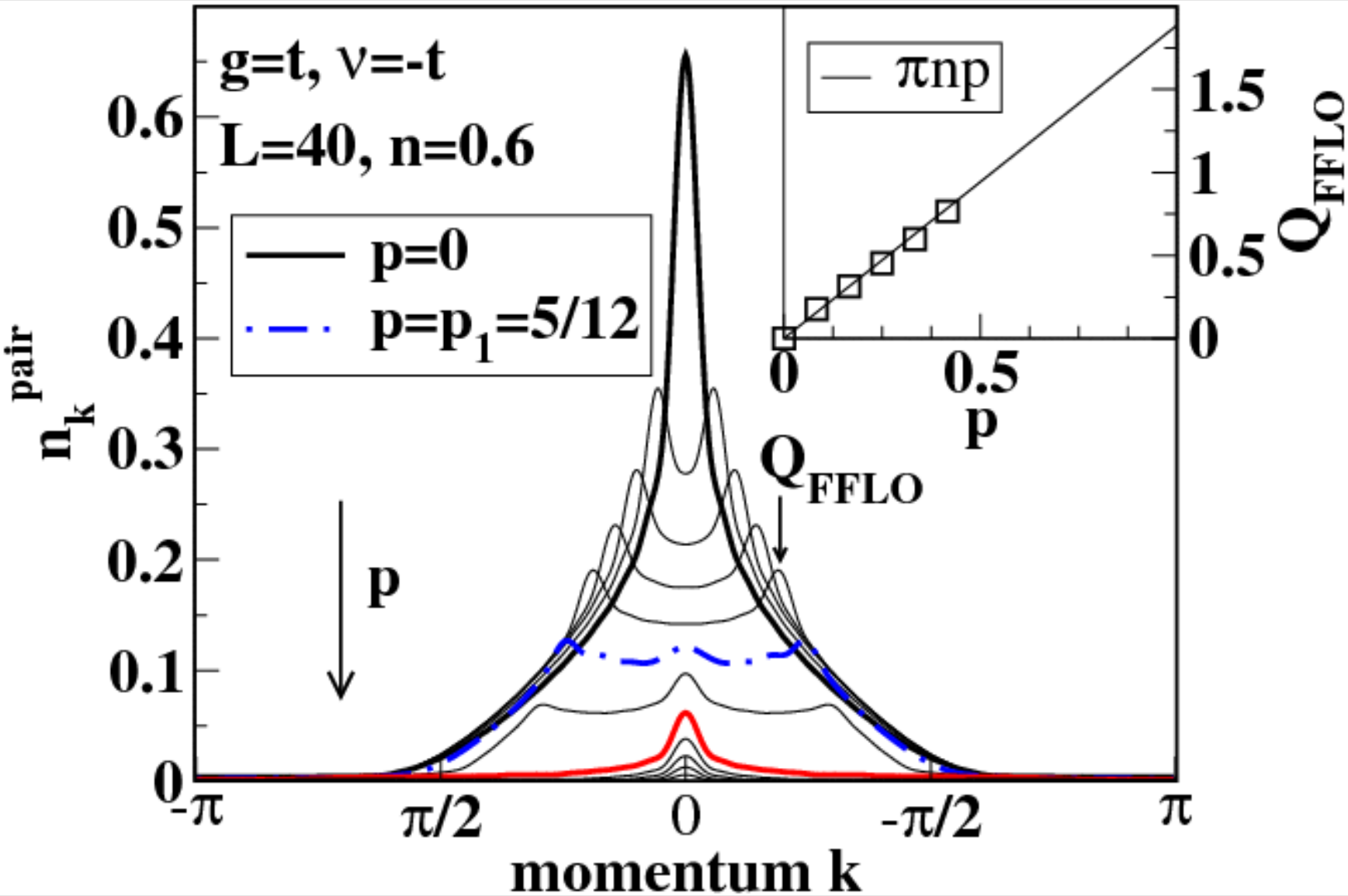
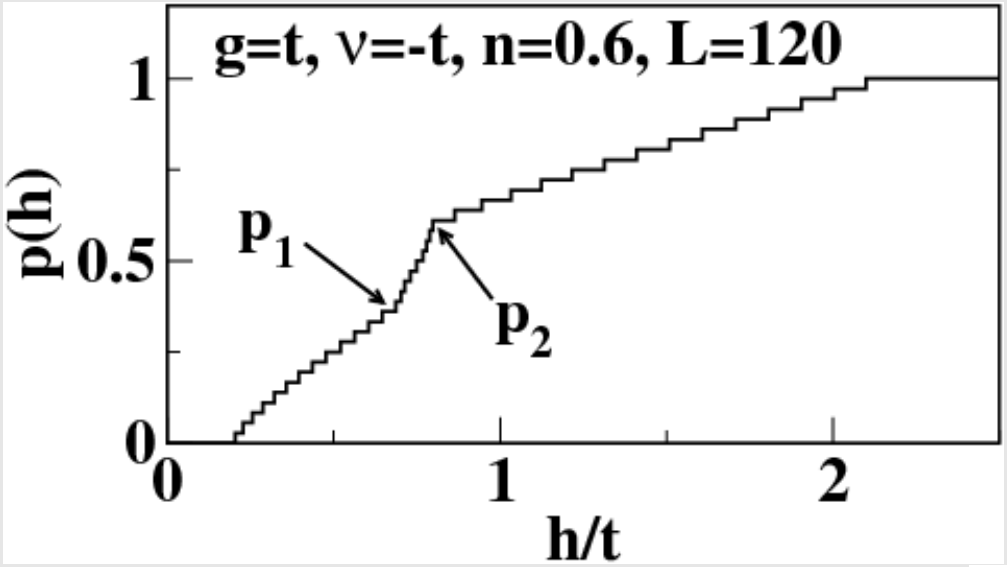
Pair correlations in the crossover region



Polarization and number of molecules



Three phases at $0 < p < 1$

"magnetic" field:

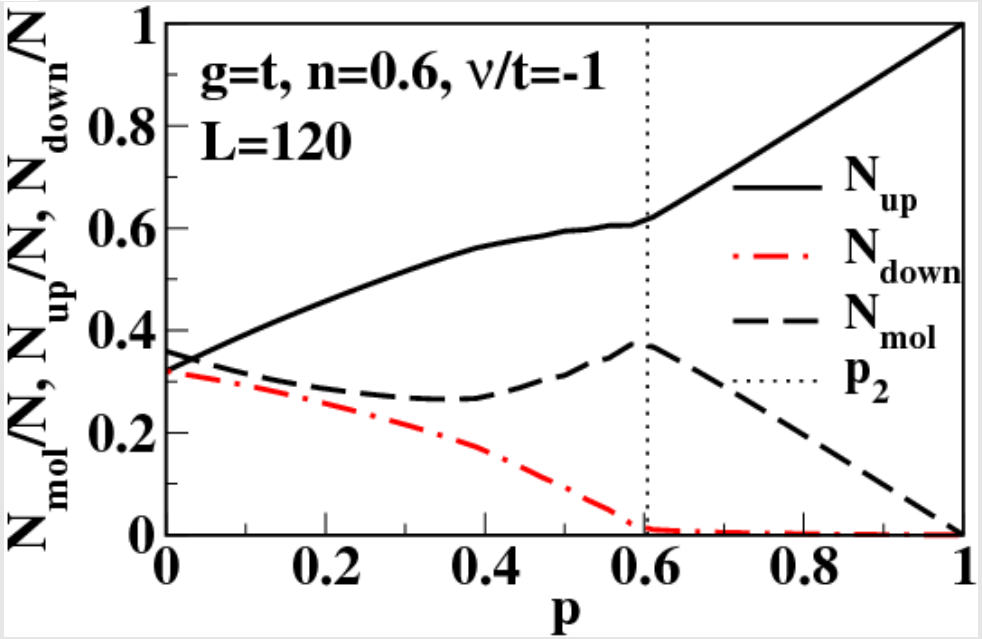
$$H_{field} = -h\left(N_{\uparrow} - N_{\downarrow}\right)$$

At $p \geq p_2$: $N_{\downarrow} = 0$

$p > p_2$: composite bosons

& spin-less fermions

BEC+FP FG



HM, Feiguin, Schollwöck, Zwerger PRA 2010

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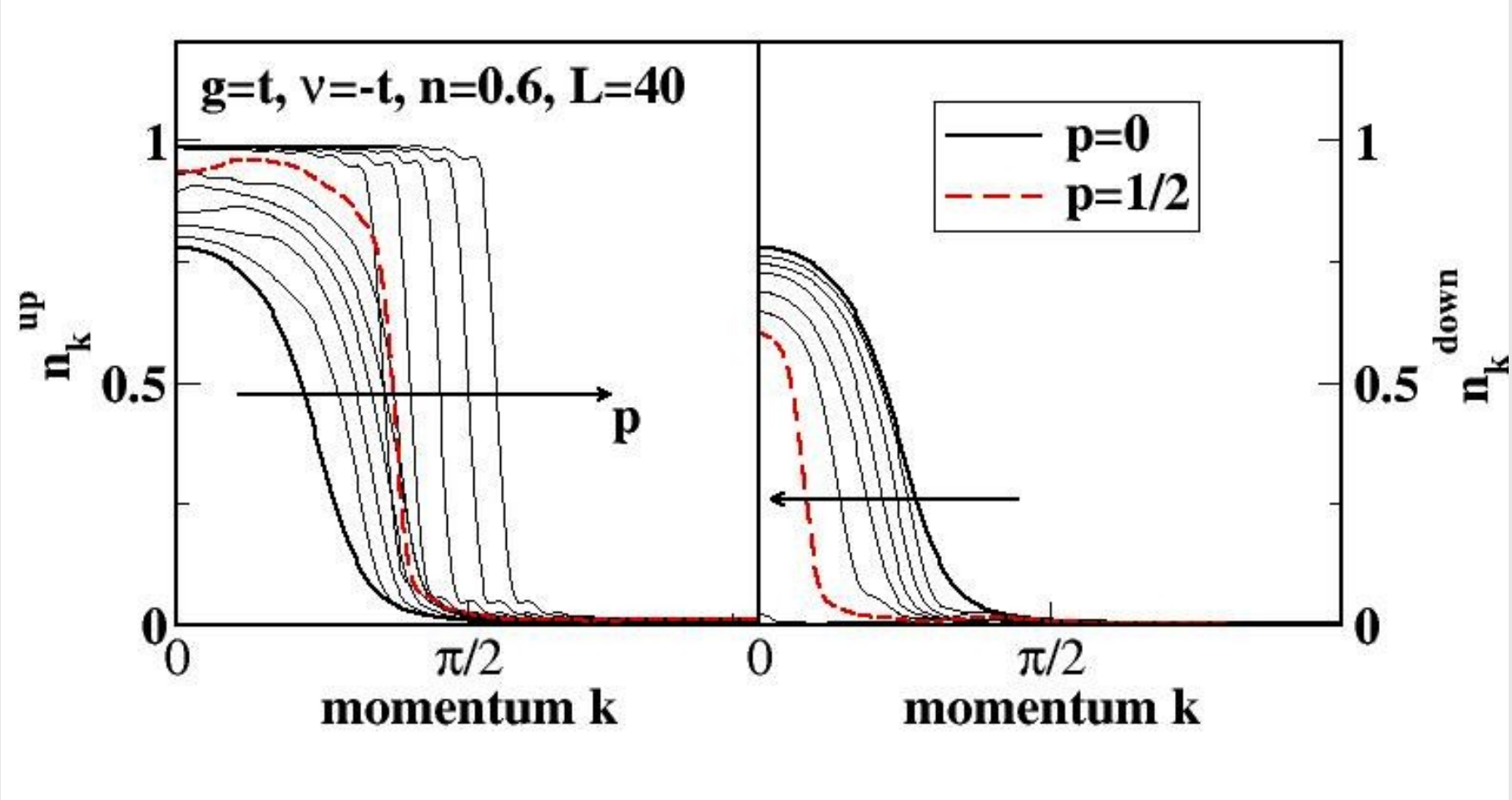
MDF of fermionic components



For $p>p_2$: all minority fermions bound in molecules:

FFLO disappears

HM, Feiguin, Schollwöck, Zwerger PRA 2010

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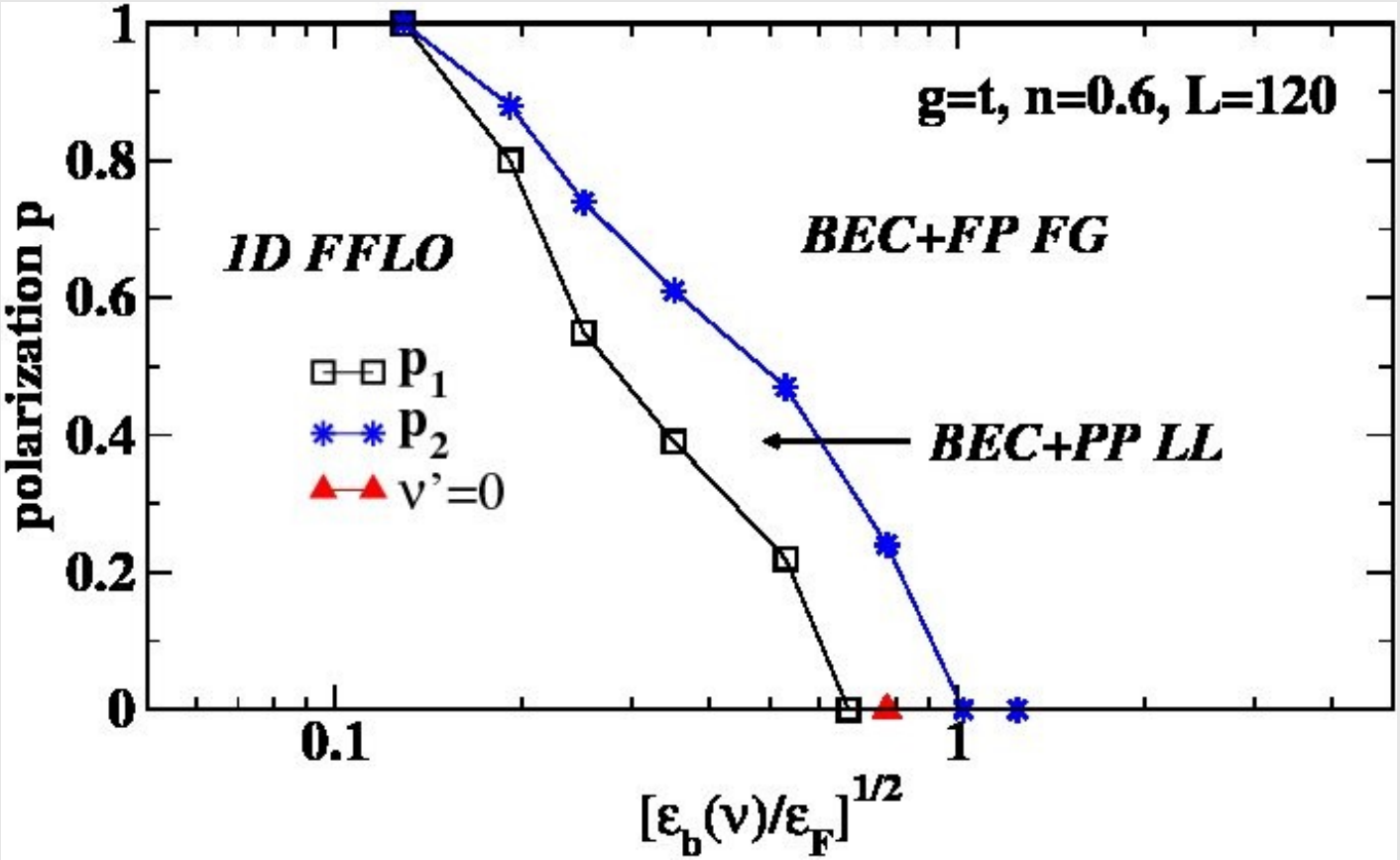
BFRM: Phase diagram at finite densities



For FFLO : ratio binding energy / Fermi energy : $\epsilon_b/\epsilon_F < c(n,g)$

Quantitative value of constant matters!

HM, Feiguin, Schollwöck, Zwerger PRA 2010

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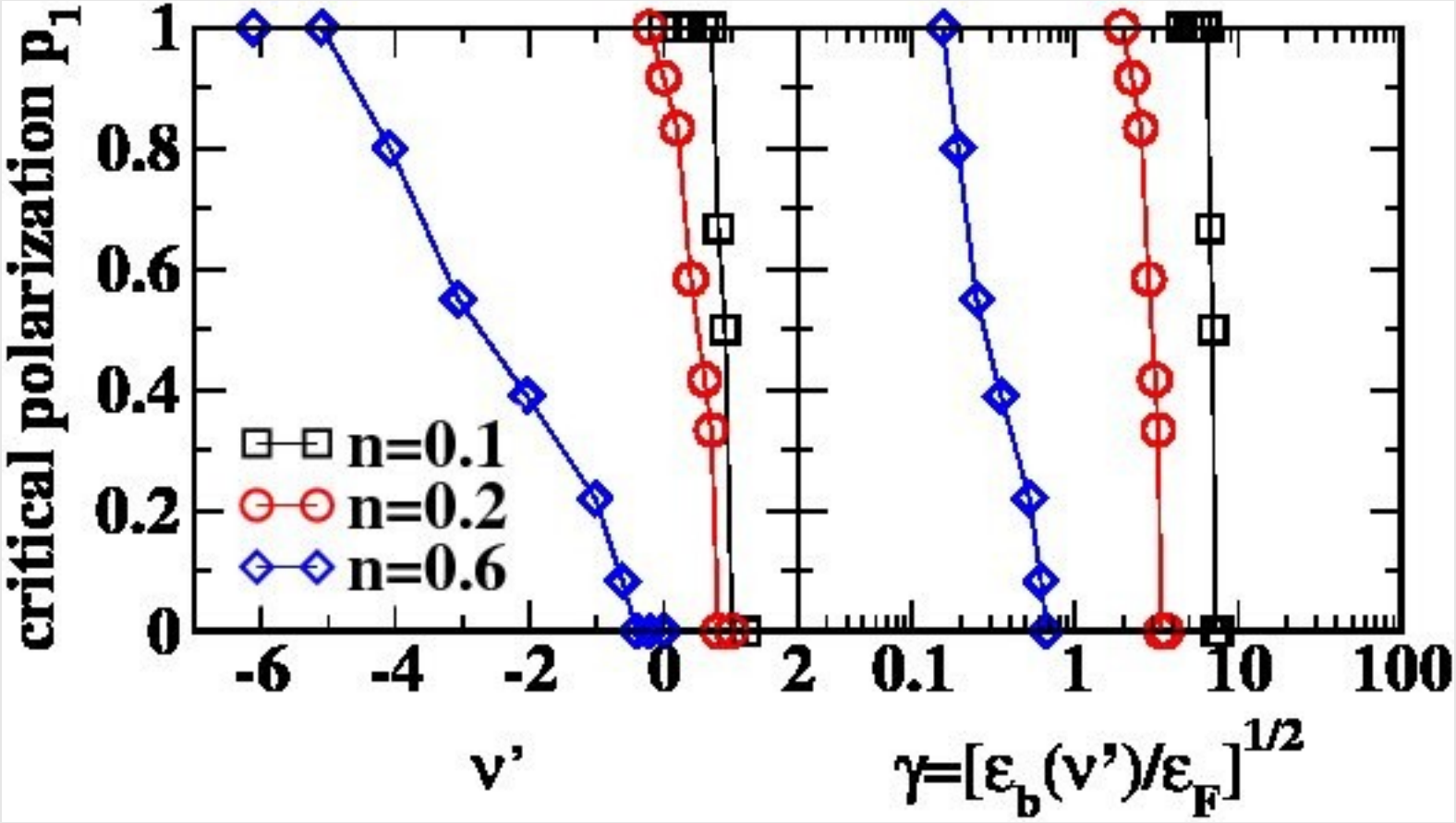
FFLO phase boundary at low densities



Low density stabilizes FFLO: Survives beyond resonance

HM, Feiguin, Schollwöck, Zwerger PRA 2010
Consistent with 3-body study Baur, Shumway, Mueller PRA 2010

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Summary

- **One-channel: Partially polarized phase in 1D is FFLO-like**
- **Rice experiment: Realizes Gaudin-Yang**

  Liao et al Nature 2010; Orso PRL 2007; Hu, Liu, Drummond PRL 2007
- **Bose-Fermi resonance model/ two-channel: Competition between FFLO & Bose-Fermi mixture**
- **Detecting the FFLO correlations: The big goal**

  … work in progress ...

Thanks for funding:

Review 1D FFLO:
Feiguin, HM, Orso, Zwerger
Lect. Not. Physics, in press

Thank you for your attention!

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