

# Effects of interactions on Bose-Einstein condensation

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Experiment

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Theory w/

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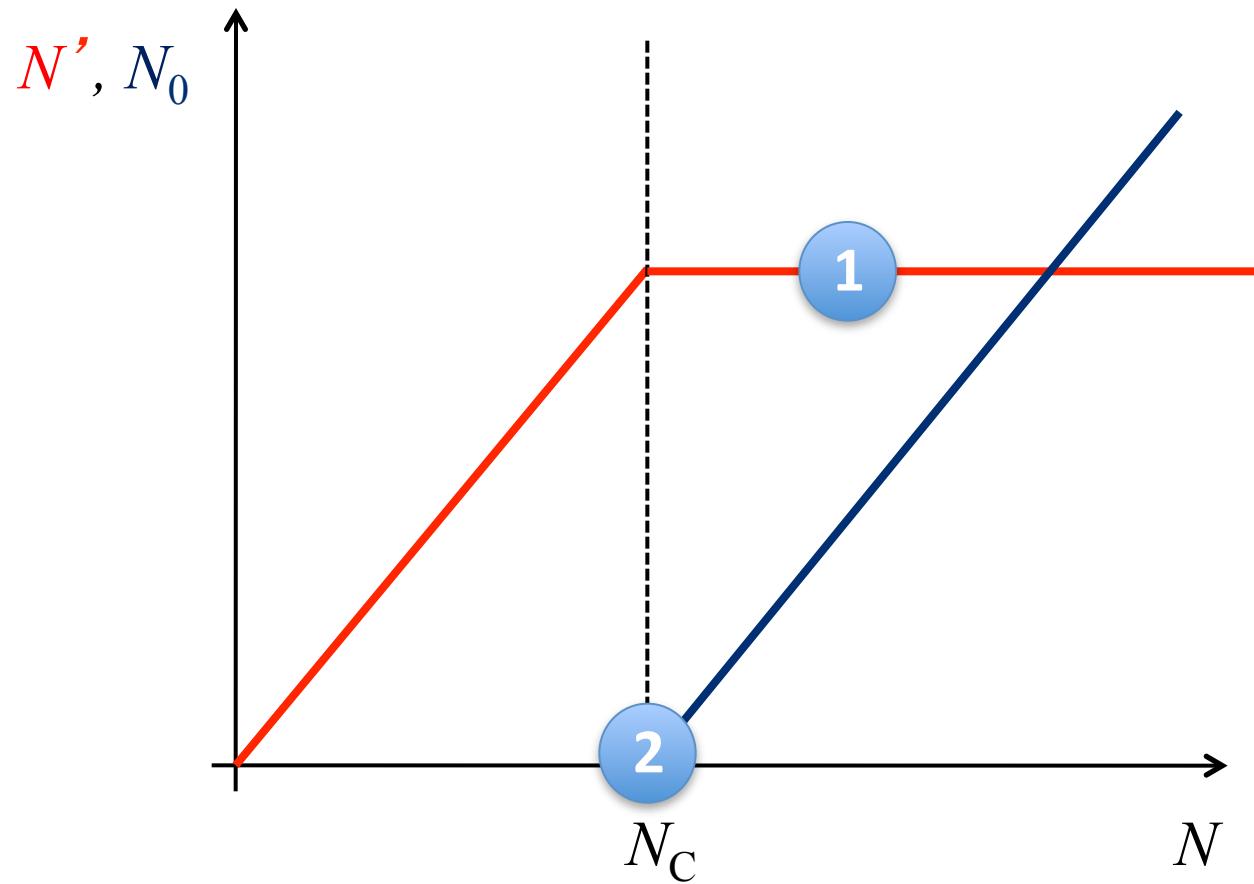
INT Seattle, May 19 2011



Engineering and Physical Sciences  
Research Council

# Basic (Einstein's) picture of BEC

3D, ideal gas, constant  $T$ , vary  $N$ :

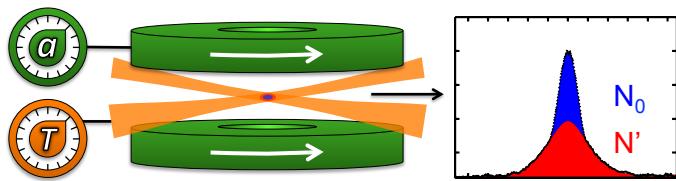


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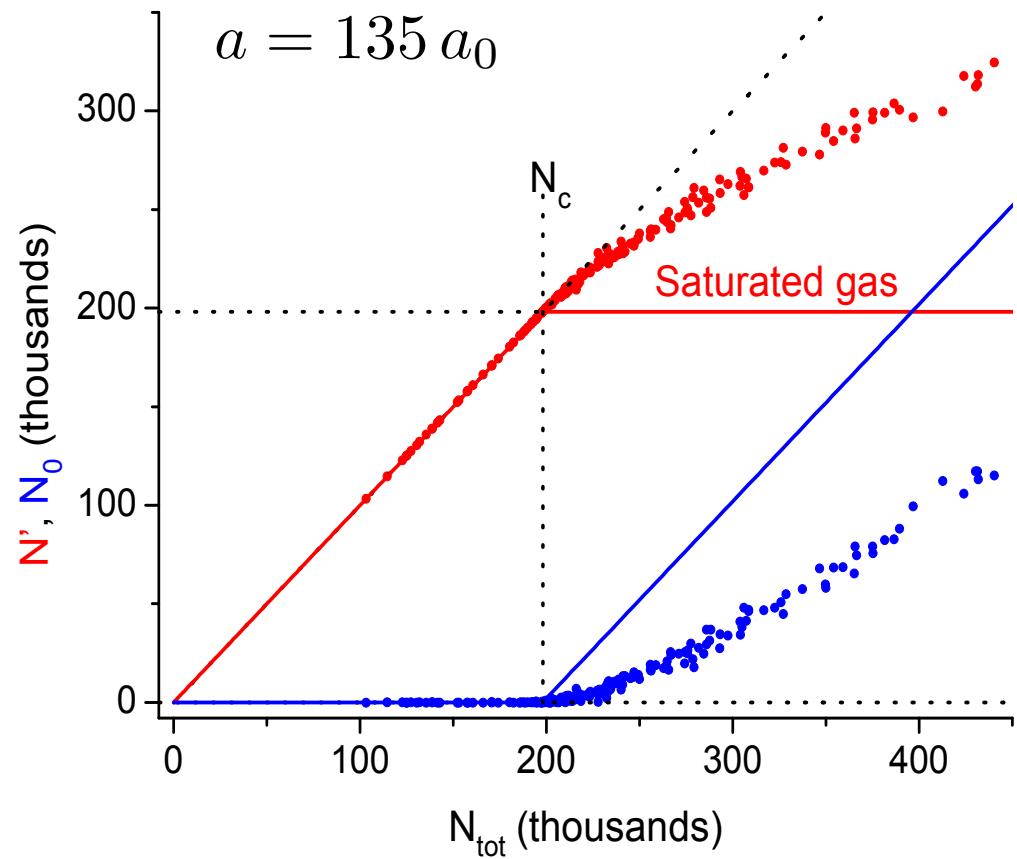
Non-equilibrium effects

# Saturation?

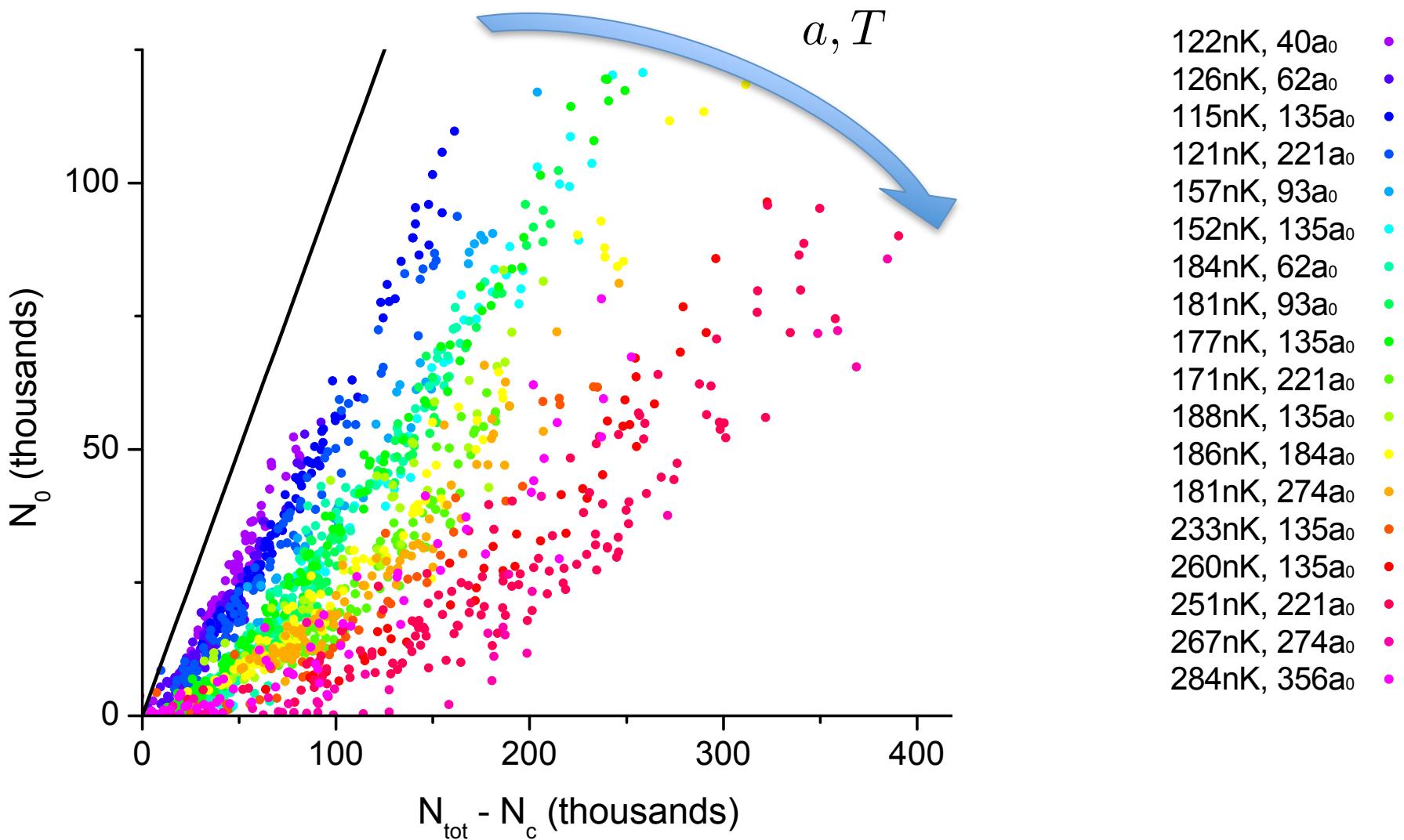
Not even close...



$^{39}\text{K}$ , broad Feshbach at 402 G

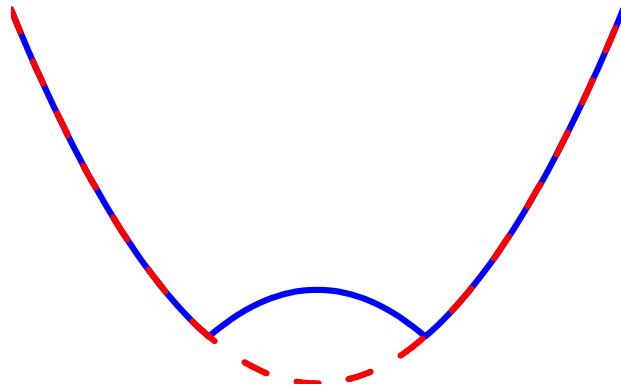


# Dependence on interactions and temperature



# Simplest theory

Hartree-Fock, thermal atoms interact only w/ the denser BEC,  
feel a Mexican hat potential



$$\frac{N'}{N_c} \approx 1 + 1.37 \frac{\mu_0}{k_B T}$$

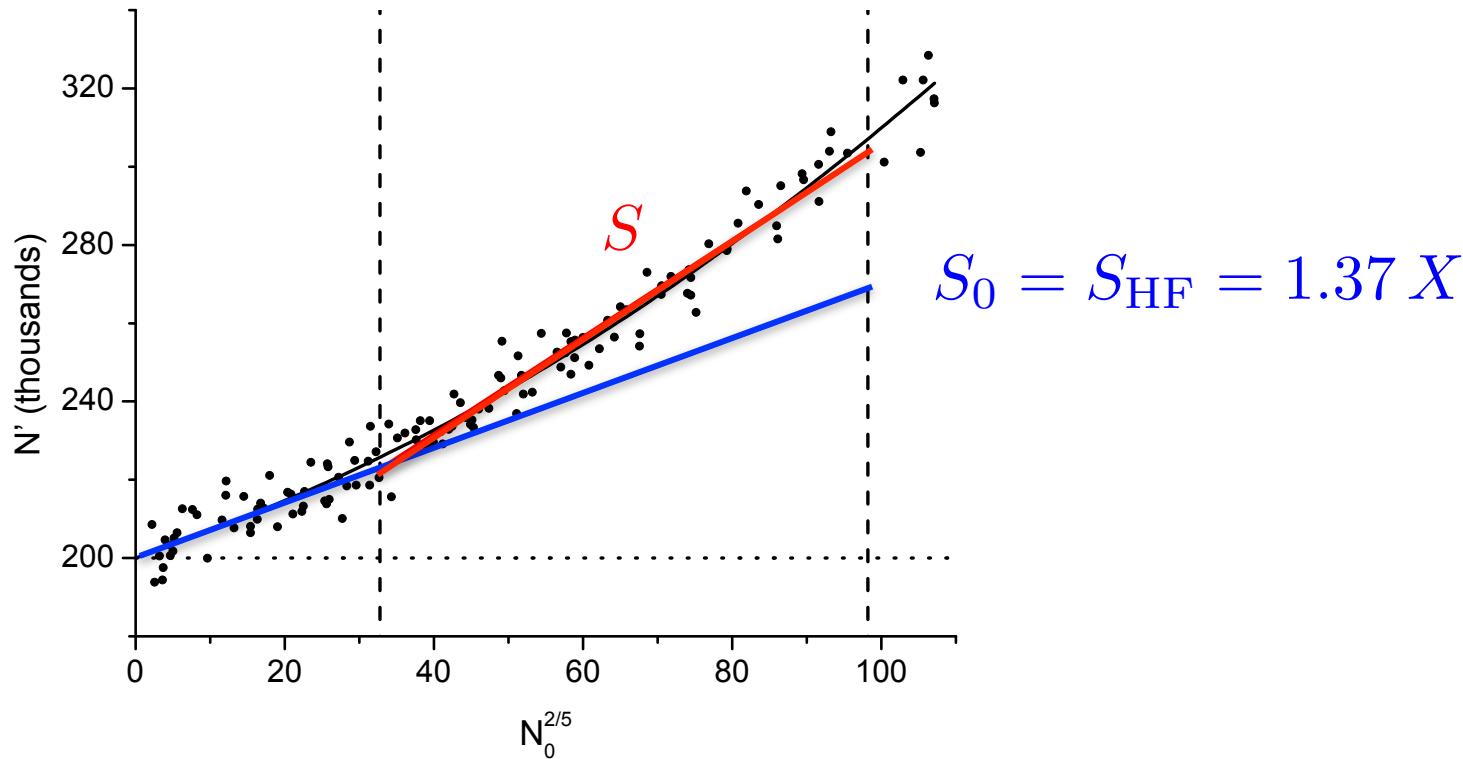
Following Stringari et al. cca 1996

$$\mu_0 \propto (N_0 a)^{2/5}$$

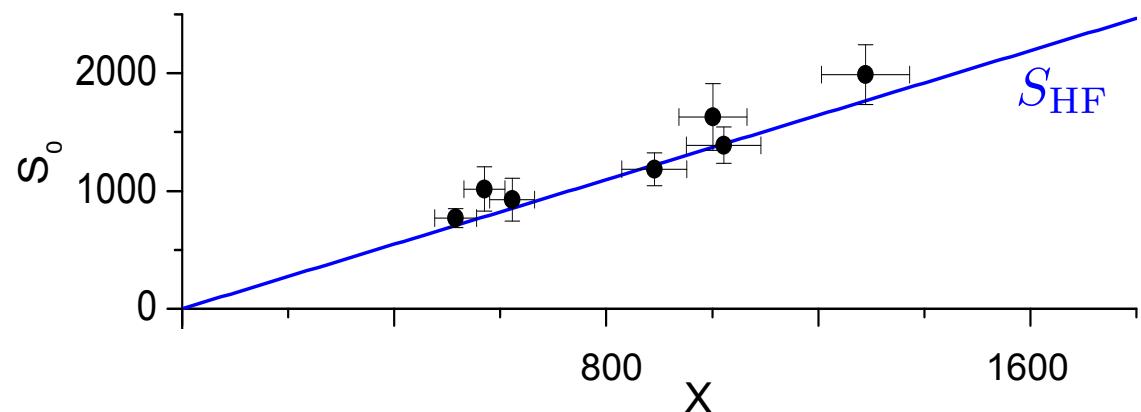
Non-saturation  
slope:

$$S_{\text{HF}} = \frac{dN'}{dN_0^{2/5}} = 1.37 X \quad X = \xi a^{2/5} T^2$$

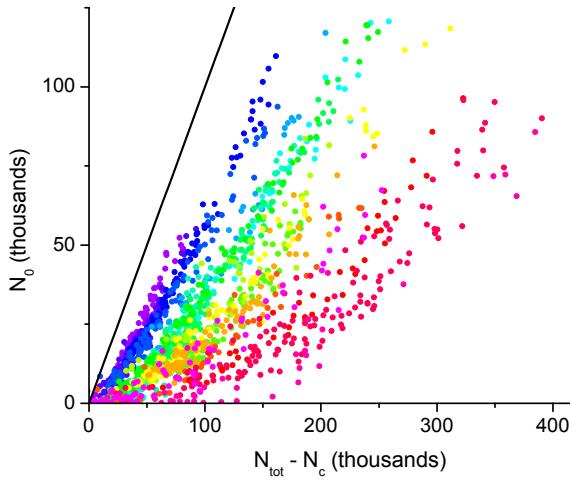
# Comparison with experiments



vary  $a, T$   
 $X \propto a^{2/5} T^2$

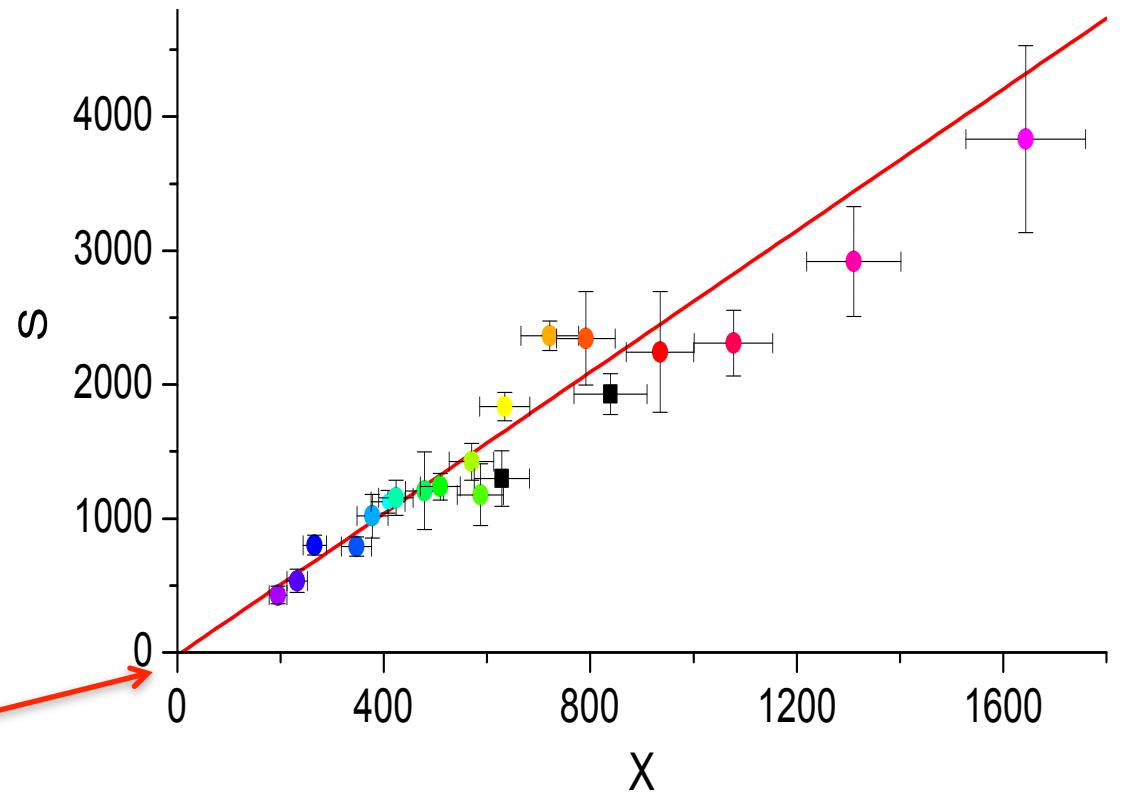
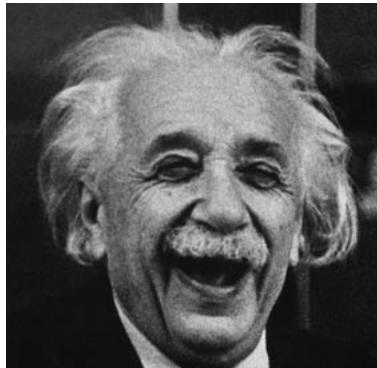


# Saturation in the non-interacting limit



all  $^{39}\text{K}$  data  
+ some  $^{87}\text{Rb}$

$$X \propto a^{2/5} T^2$$



$$S(0) = -20 \pm 100 \quad dS/dX = 2.6(3)$$

# Critical point of a dilute Bose gas

Discussed since Lee & Yang 1957...

- Ideal gas:  $n\lambda_0^3 = \zeta(3/2) \approx 2.612$

- Uniform system:

$$\frac{\Delta T_c}{T_c^0} \approx +1.8 \frac{a}{\lambda_0} + \dots$$

Stoof et al., Baym et al.,  
Prokof'ev & Svistunov,  
Arnold & Moore...

beyond-MF effect!

- Harmonic trap:

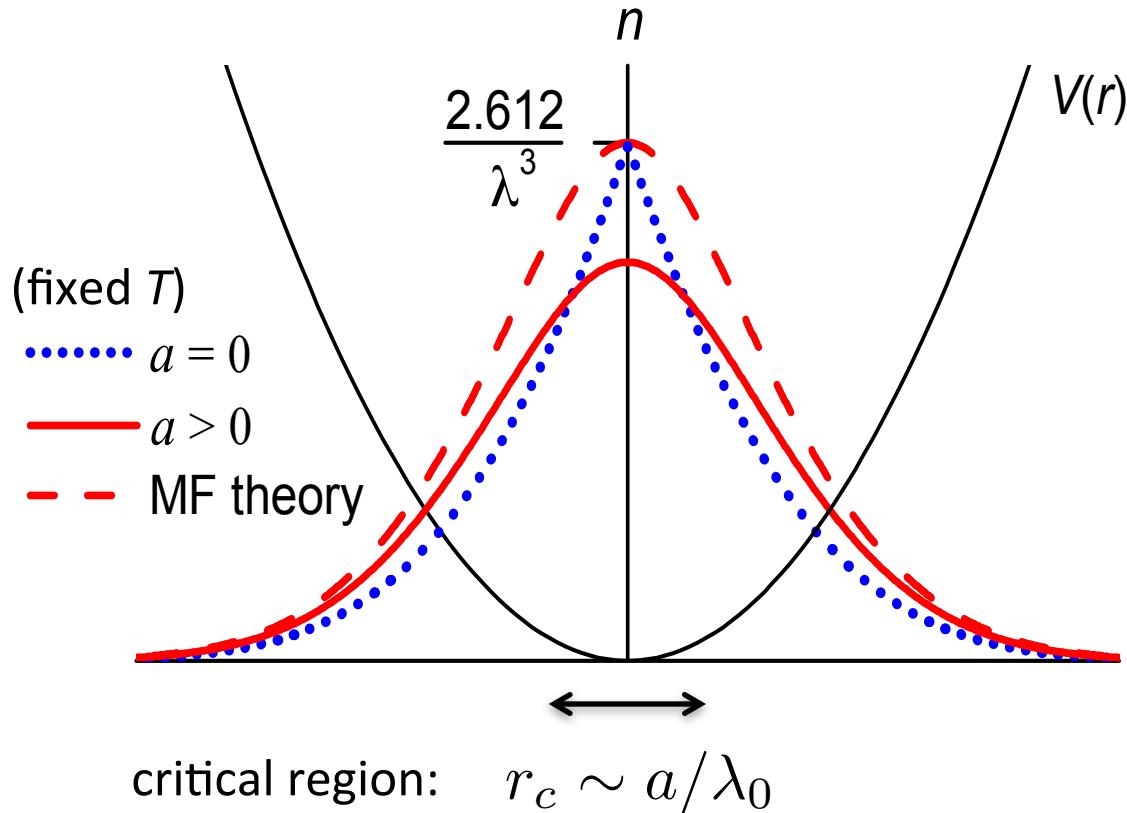
$$\frac{\Delta T_c}{T_c^0} \approx -3.426 \frac{a}{\lambda_0} + \dots$$

Stringari et al.

MF!      beyond-MF

# Two competing effects in a trapped gas

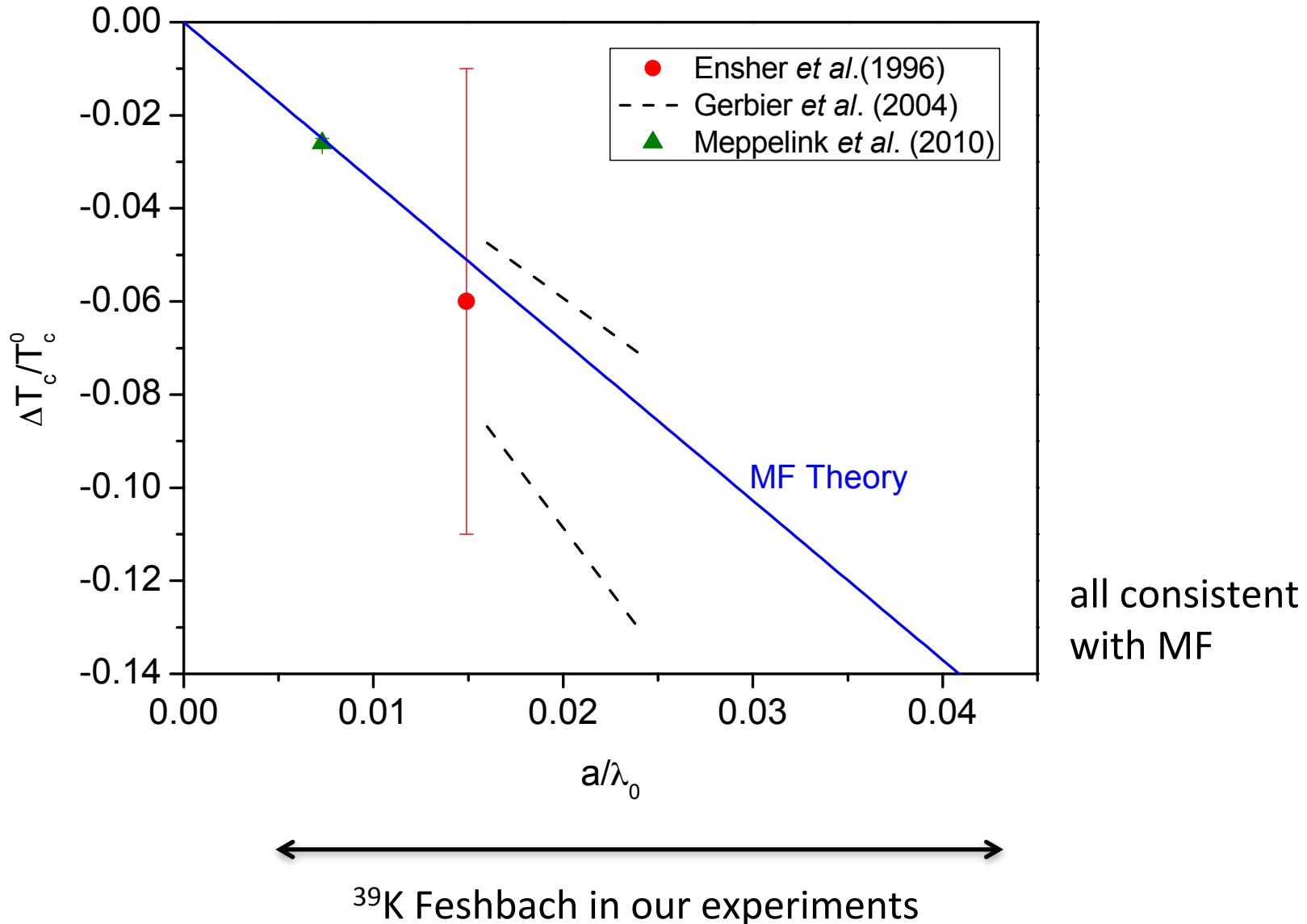
at the critical point...



MF exact  
to 1<sup>st</sup> order

$$\frac{\Delta T_c}{T_c^0} = -3.426 \frac{a}{\lambda_0} + \dots$$

# Previous $T_c$ measurements

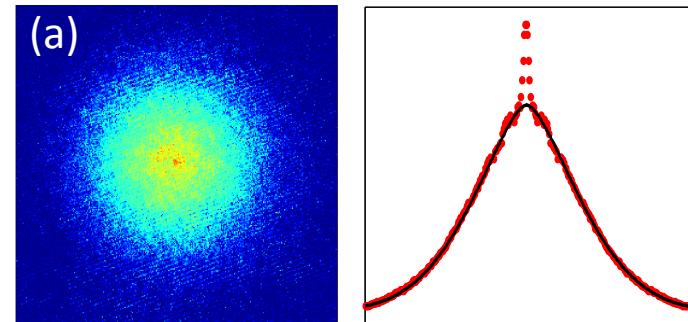


# What we do differently...

## 1. Zoom-in on the critical point (precision)

interactions off during TOF,  
detect very small condensates

$$N_0/N \approx 0.14\%$$



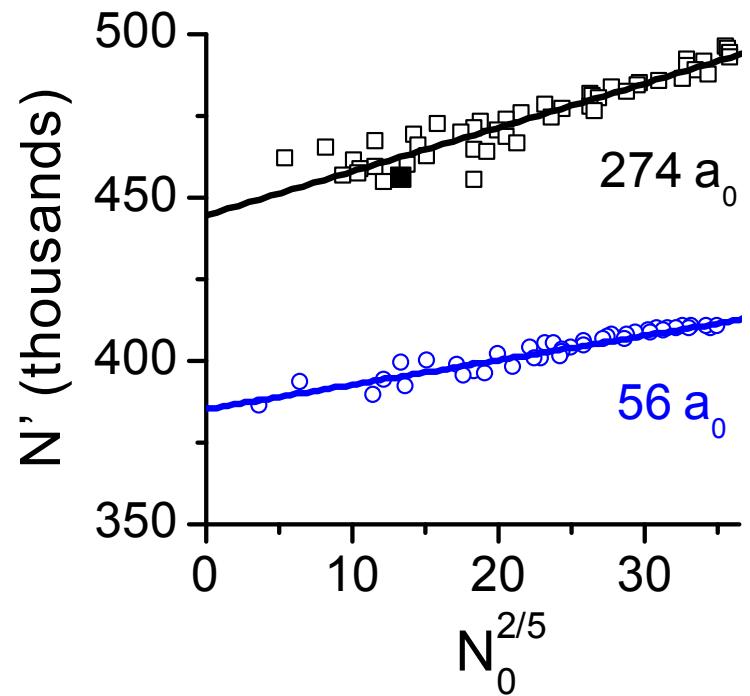
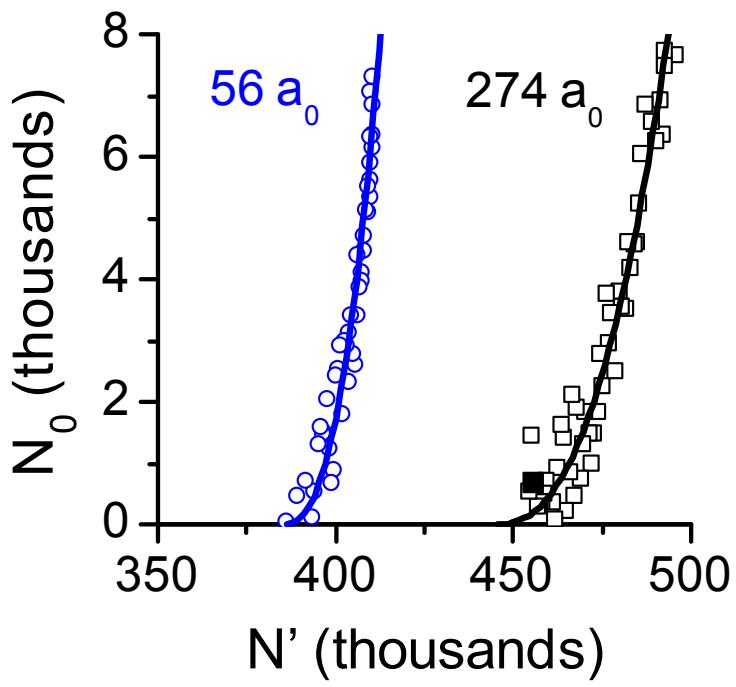
## 2. Differential measurements (accuracy)

Usually:      measure       $T_c(N, \omega, a)$       [or  $N_c(T, \omega, a)$  ]  
calculate       $T_c(N, \omega, a = 0)$

Our work: measure both  $T_c(N, \omega, a)$  and  $T_c(N, \omega, \text{small } a)$   
(need thermal equilibrium)

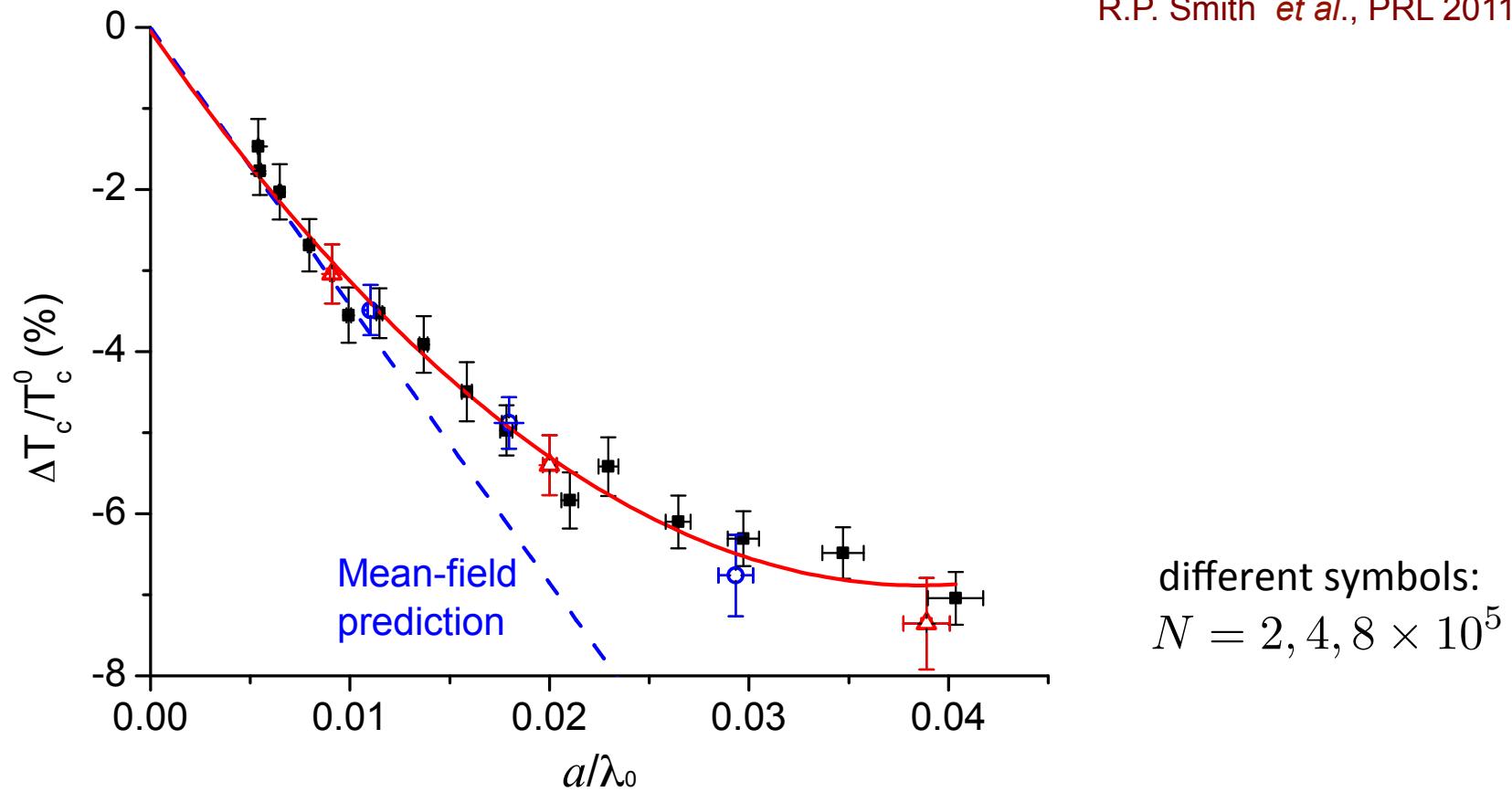
# (Differential) critical point

Same  $T, \omega$ , different  $a$ :



# Beyond-MF $T_c$ shift

R.P. Smith *et al.*, PRL 2011



$$\frac{\Delta T_c}{T_c^0} = -3.426 \frac{a}{\lambda_0}$$

$$\frac{\Delta T_c}{T_c^0} = -3.5(3) \frac{a}{\lambda_0} + 46(5) \left( \frac{a}{\lambda_0} \right)^2$$

small print: neglect logarithmic corrections

# (Non-)Equilibrium?

What does it take?

$$\gamma_{\text{el}} \tau > 3 \quad 5 \quad t_{\text{hold}} > \tau > 1/\omega$$

Not the whole story if we have **constant dissipation**...

**relevant  $\tau$  depends on the measurement precision**

$$0.01 N_c \rightarrow \tau = 0.01 / \gamma_{\text{loss}}$$

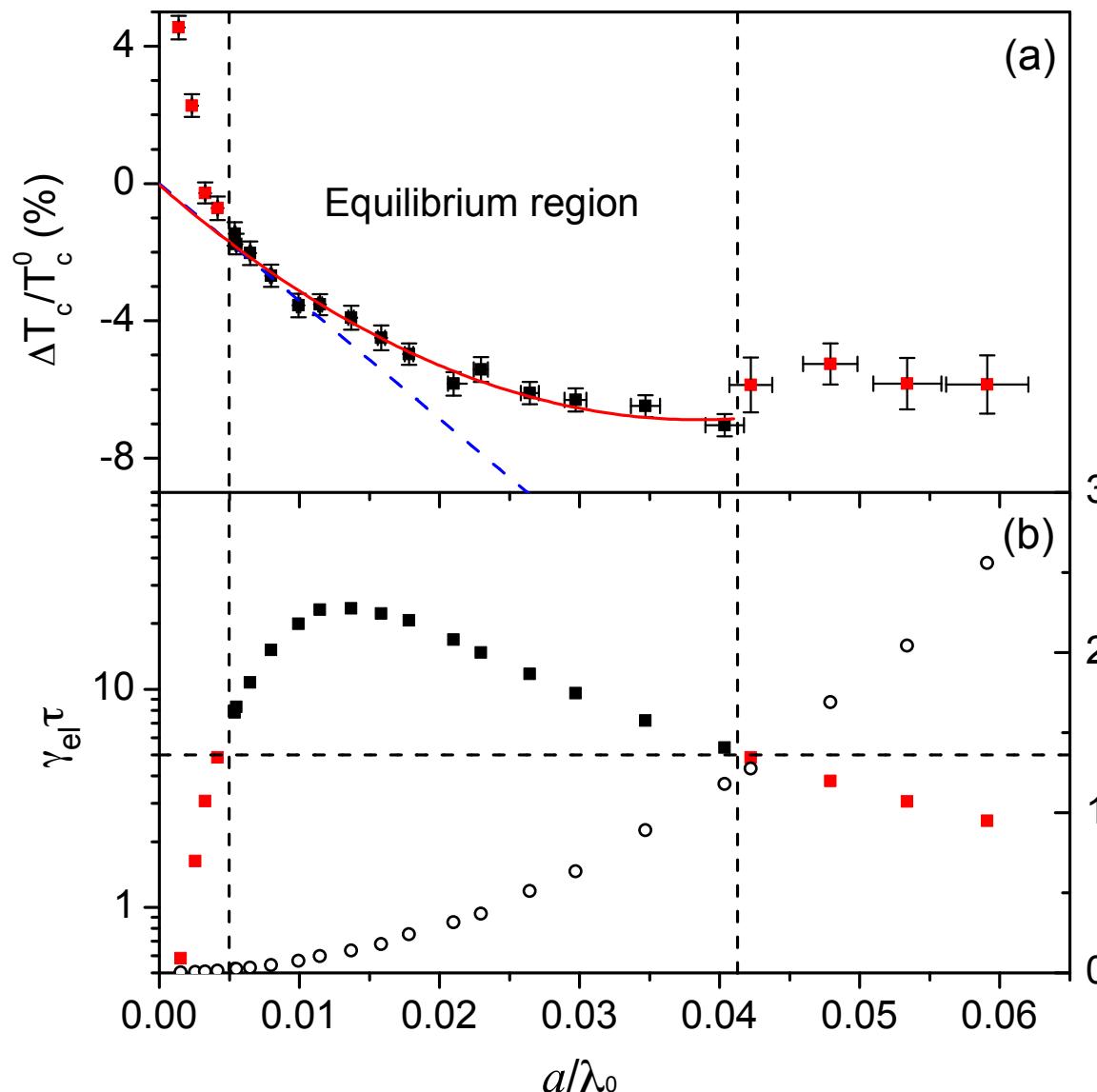
Two kinds of non-equilibrium:

1. “transient” – could reach equilibrium, but  $t_{\text{hold}}$  too short
2. “intrinsic” – thermalization slower than dissipation:

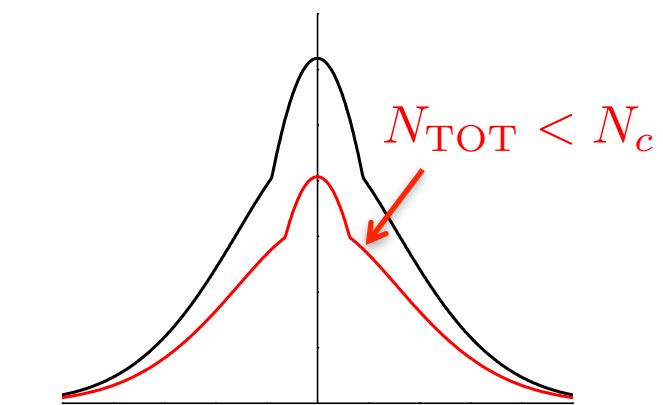
$$t_{\text{hold}} > \tau > 1/\omega \quad \checkmark \quad \gamma_{\text{el}} t_{\text{hold}} > 5 \quad \checkmark$$

but  $\gamma_{\text{el}} \tau < 5 \quad \times$

# Non-equilibrium effects



“super-heated” BECs  
in both extremes



# Uniform vs. harmonic (again)

(subtract the trivial MF part)

new stuff...

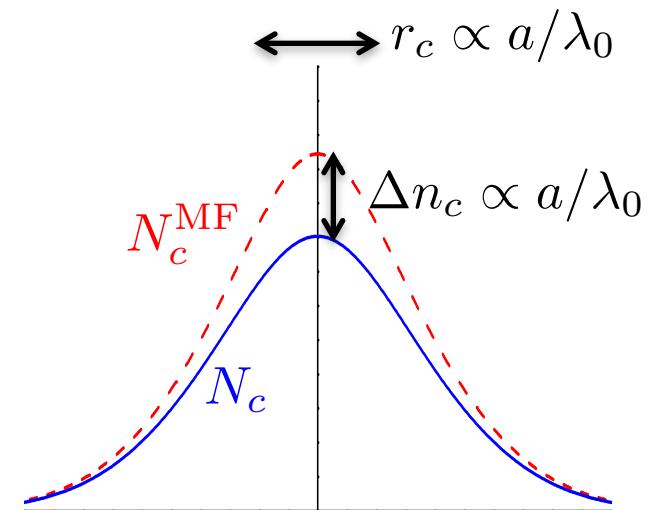
Uniform system:

$$\frac{\Delta T_c}{T_c^0} = c \frac{a}{\lambda_0}$$

$$\Delta n_c \propto \frac{a}{\lambda_0}$$

$$\mu_c^{\text{MF}} - \mu_c \propto \left( \frac{a}{\lambda_0} \right)^2$$

Harmonic trap:



$$N_c^{\text{MF}} - N_c \propto (a/\lambda_0)^2$$

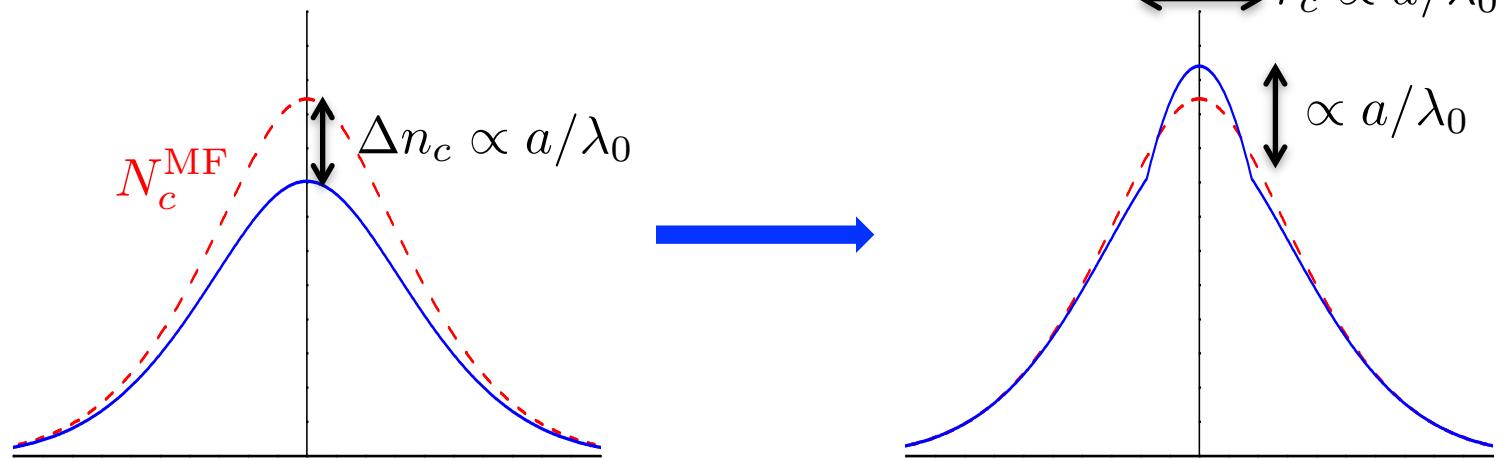
$$\text{not } a/\lambda_0 \text{ or } (a/\lambda_0)^4$$

$T_c$  shift in the harmonic trap dominated by the density shift outside the critical region

# Condensed fraction induced by critical correlations

$$N = N_c < N_c^{\text{MF}}$$

$$N = N_c^{\text{MF}} > N_c$$

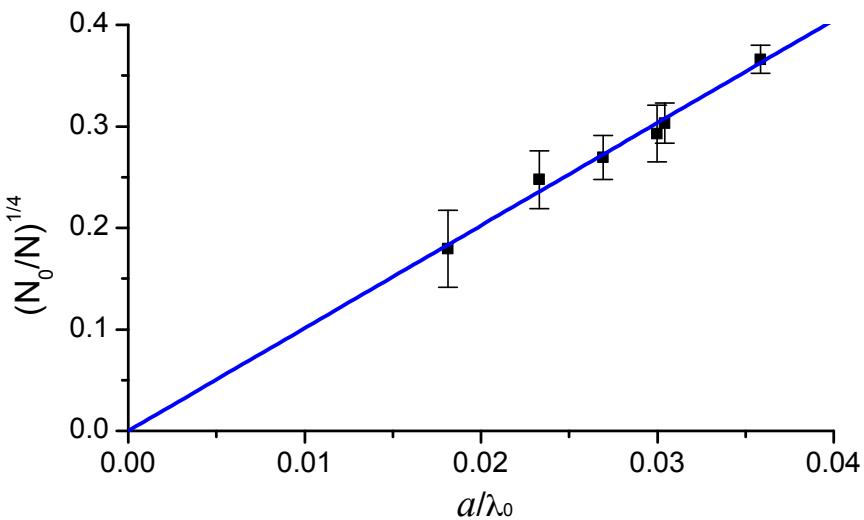
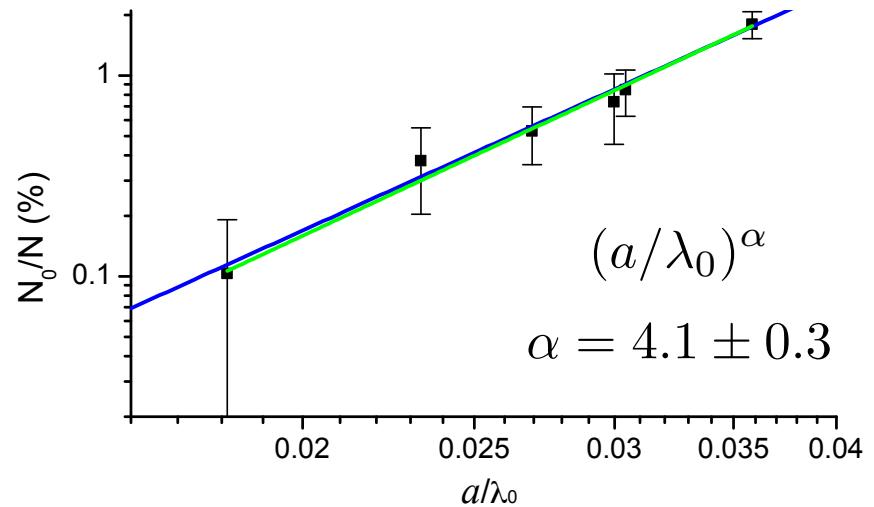
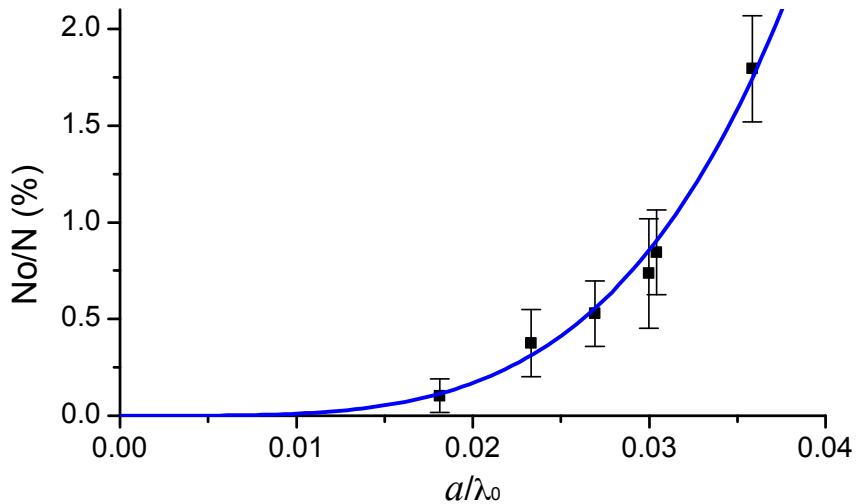


$$N_c^{\text{MF}} - N_c \propto (a/\lambda_0)^2$$

$$\boxed{\begin{aligned} &\text{iff } n_c^{\text{MF}} - n_c \propto a/\lambda_0, \\ &f_0 = N_0/N \propto (a/\lambda_0)^4 \end{aligned}}$$

Baym & Holzmann  
Prokof'ev & Svistunov

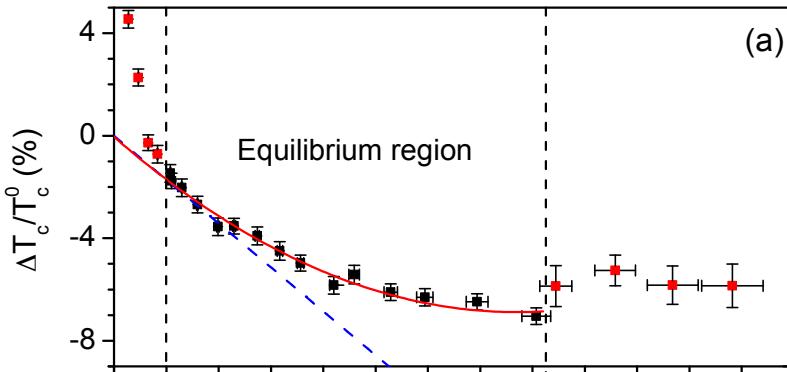
# Preliminary data...



$$(N_0/N)^{1/4} \approx 10 a/\lambda_0$$

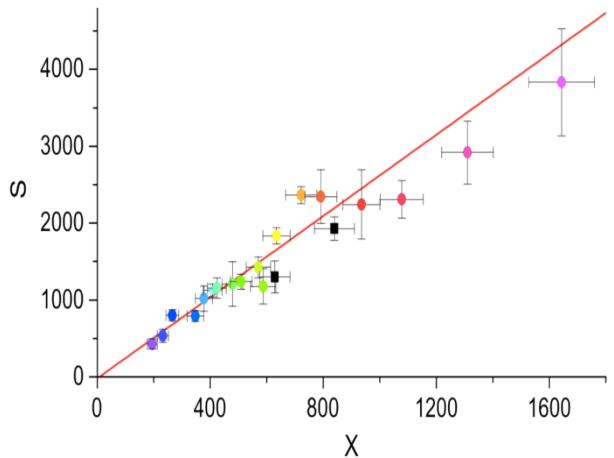
Can be compared with  
uniform system MC + LDA...  
(Prokof'ev & Svistunov)

# Summary



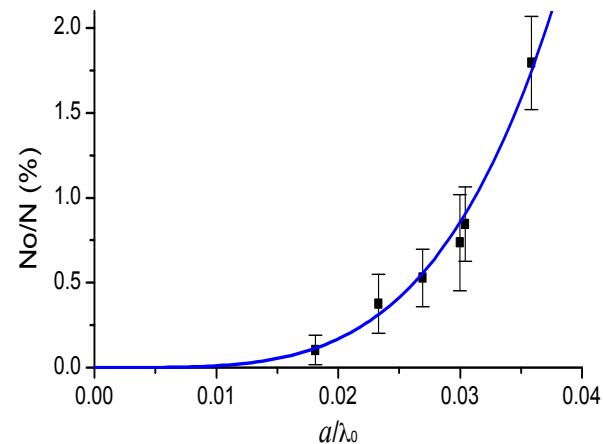
critical point  
beyond-MF  
&  
non-equilibrium

R.P. Smith *et al.*, PRL 2011



thermal  
MF?

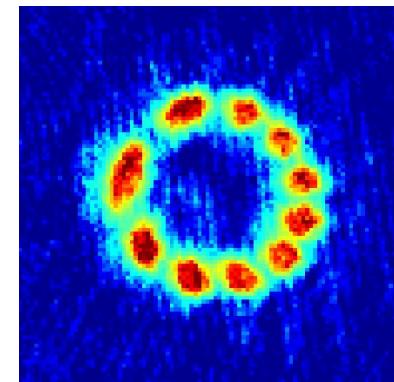
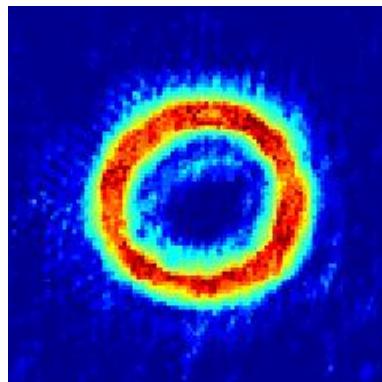
N. Tammuz *et al.*, PRL 2011



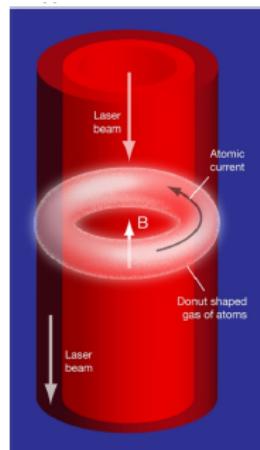
condensed  
beyond-MF

# Sneak preview - Superfluidity in ring geometry...

Laguerre-Gauss  
trapping beam  
w/ various  
 $L$  values  
 $L = 1$  @ NIST



$q=10$  vortex  
interferometrically detected



azimuthal vector potentials,  
superfluid density

N.R. Cooper & ZH, PRL 2010  
S.T. John, ZH & N.R. Cooper, PRA 2011

**THE END**