Benchmarking the Many-body Problem

Precision bounds on the Equation of State

Michael McNeil Forbes

Institute for Nuclear Theory (INT)

and the University of Washington (Seattle)

18 May 2011

Benchmarks

- High precision:
  - QMC etc. usually require small systems
  - Experiments
- How to connect?
  - Density Functional Theory (DFT)

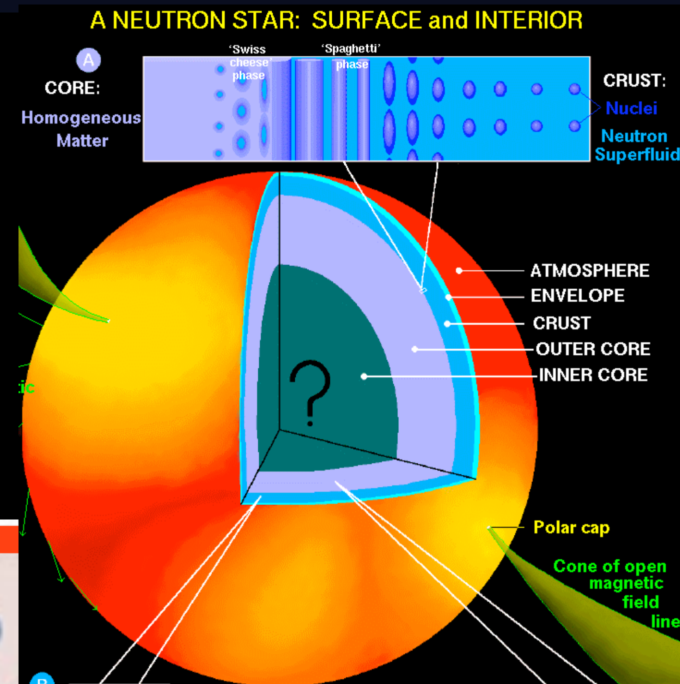
Density Functional Theory (DFT)

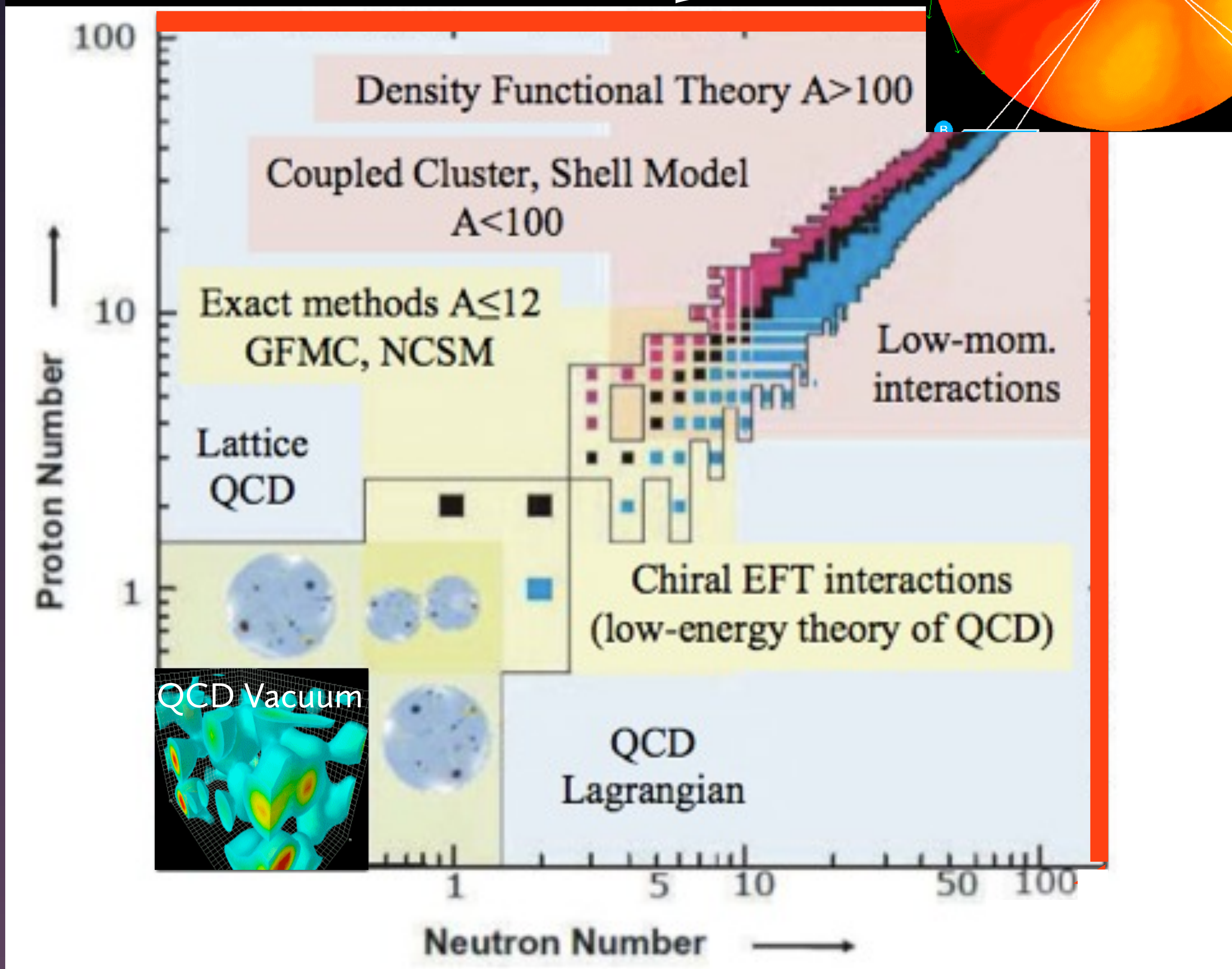
- The ground state density in any external potential $V(x)$ minimizes a functional:

$$\int d^3x \{\mathcal{E}[n(x)] + V(x)n(x)\}$$

- Functional may be complicated (non-local)
- Local Density Approximation (LDA)
- Kohn-Sham introduces kinetic term

The Nuclear Landscape





Nuclear Physics

Complicated interactions

DFT to extrapolate to large systems

QCD Vacuum Animation: Derek B. Leinweber (http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/index.html)
Neutron Star Structure: (Dany Page) Landscape: (modified from a slide of A. Richter)

SLDA: Superfluid Local Density Approximation

$$\mathcal{E}(n,\tau,\nu) = \alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- Three parameters:
  - Effective mass (m/α)
  - Hartree (β)
  - Pairing (g)

BdG (Mean Field)

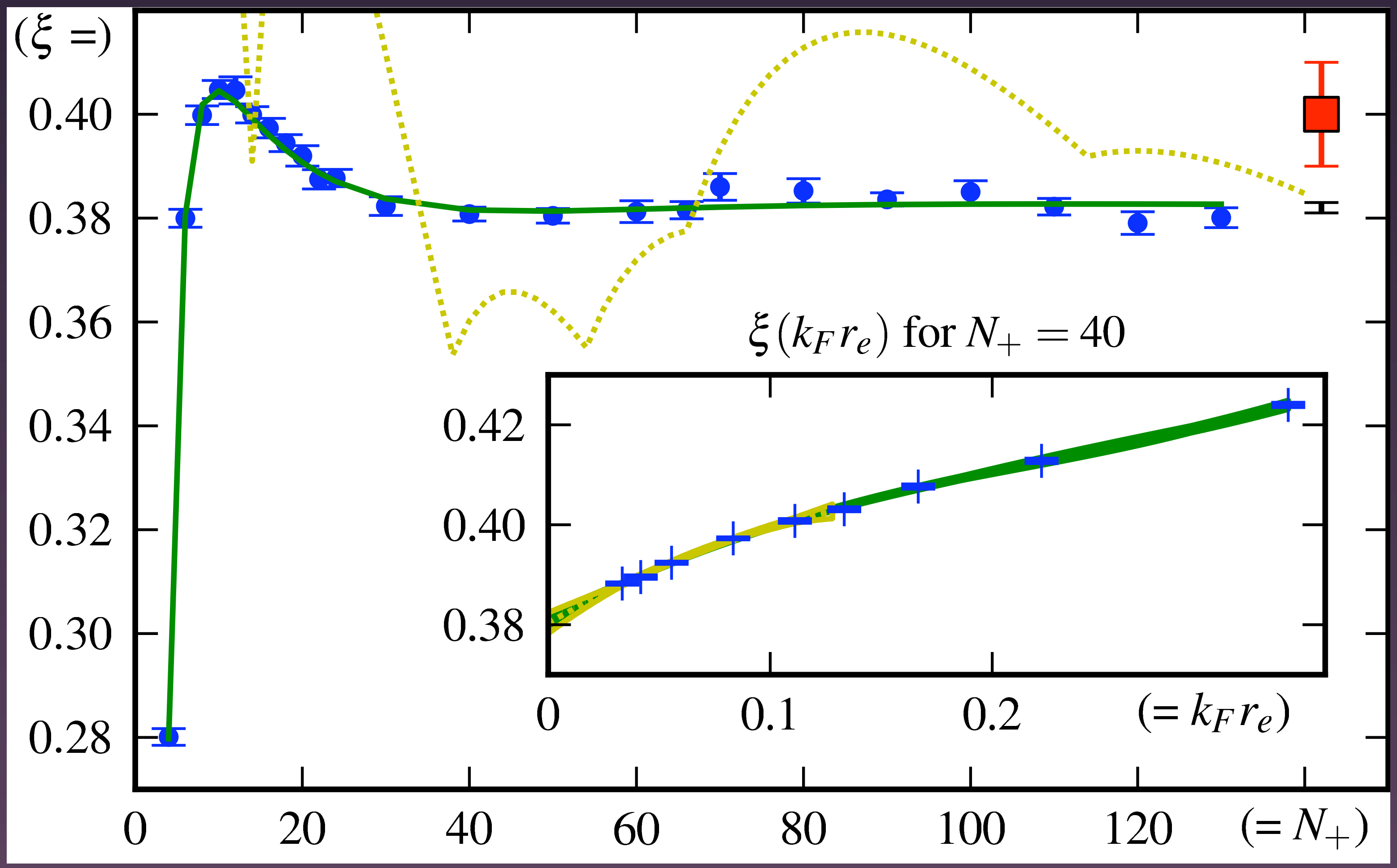
$$\mathcal{E}(n, \tau, \nu) = \alpha \frac{\tau}{m} + \beta \frac{(3\pi^2 n)^{5/3}}{10 m \pi^2} + g_{\text{eff}} \nu^\dagger \nu$$

- The SLDA contains Bogoliubov-de Gennes (BdG)
  - Unit mass
  - No Hartree term
    - (No polaron!)

T=0 Equation of State

- Use Fixed Node Diffusion Monte Carlo (FN-DMC) for small periodic boxes
  - Homogeneous states (no gradient terms)
- Extrapolate to zero range
- Fit SLDA parameters and use to extrapolate thermodynamic value of ξ

Using DFT to extrapolate to N=∞



SLDA matches all finite size effects

No correlation with free "Shell effects"

Essential to extrapolate to zero range

$$\xi \leqslant 0.383(1)$$

$$\chi_{\text{reduced}} = 0.7$$

Forbes, Gandolfi, Gezerlis PRL (2011)

Not Perfect (but close!)

- Gap too large $\Delta = 0.87(2) E_F$

  QMC quasiparticle (q.p.) dispersions give Δ=0.50(5)EF

- Mass too small $\alpha = 1.26(2)$:

  QMC (q.p.) dispersions give and $\alpha = 1.09(2)$

- Local functional can be generalized to fix these issues

Next steps

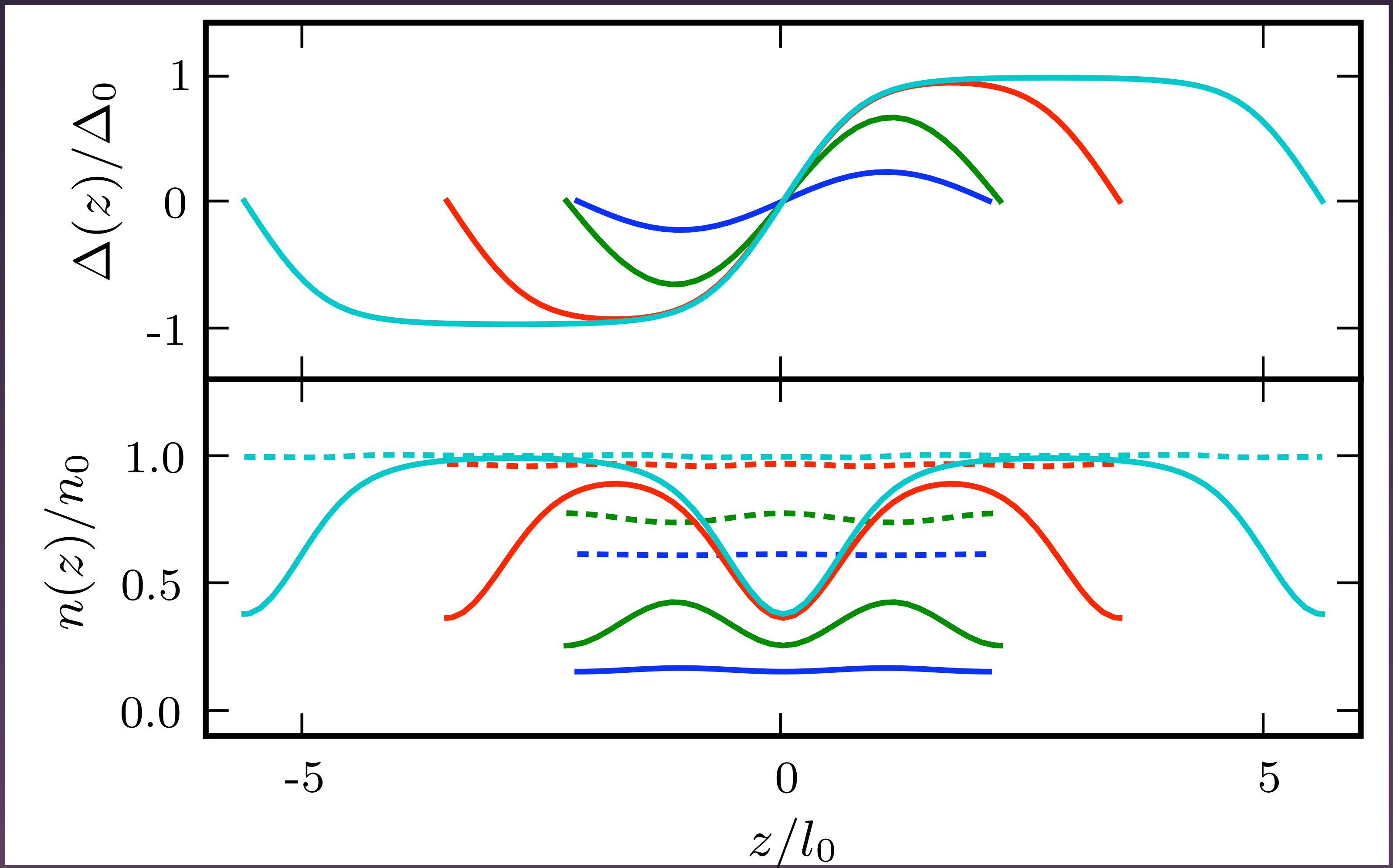
- Evaluate gradient terms (using traps)
- Finite T: Can we build a Pseudo-gap into the DFT?
- Can we exploit Shina Tan's functionals?
- Time dependence (TDDFT) (Thursday)
- Asymmetric (ASLDA)...

ASLDA

$$\mathcal{E} = \alpha_a(x)\frac{\tau_a}{2m} + \alpha_b(x)\frac{\tau_b}{2m} + D(n_a, n_b) + g_{\text{eff}}(x)\nu^\dagger\nu$$

- Introduce dimensionless $x = n_a/n_b$
- Parameters become functions $\alpha(x)$ etc.
- Fit with QMC data (not precision yet)
- Build in superfluid and interacting normal states
- Find that (FF)LO state has lower energy

ASLDA predicts (FF)LO at Unitarity



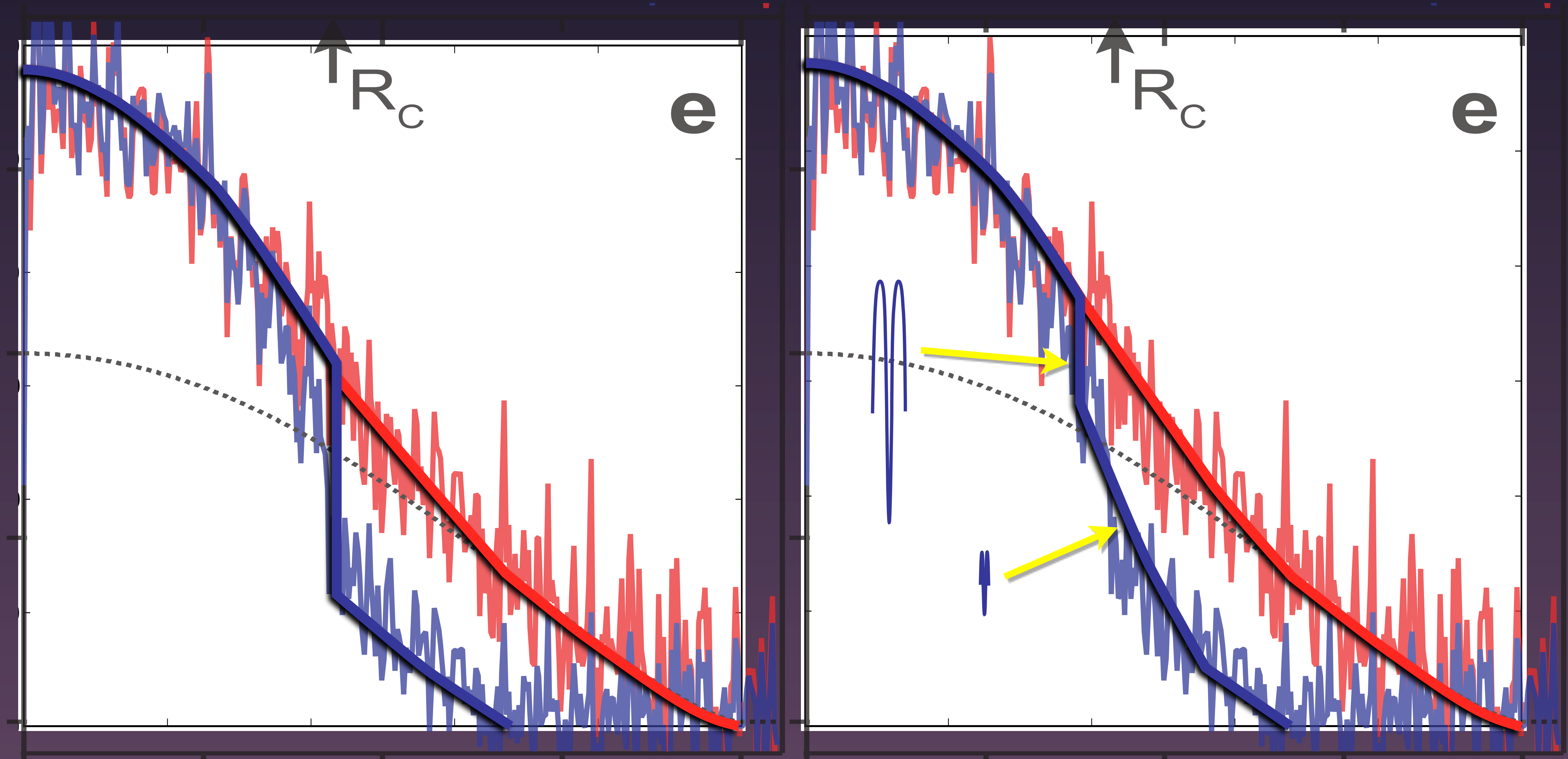
Large density contrast (factor of 2)

Similar to vortex core

Bulgac and Forbes PRL 101 (2008) 215301

Observations

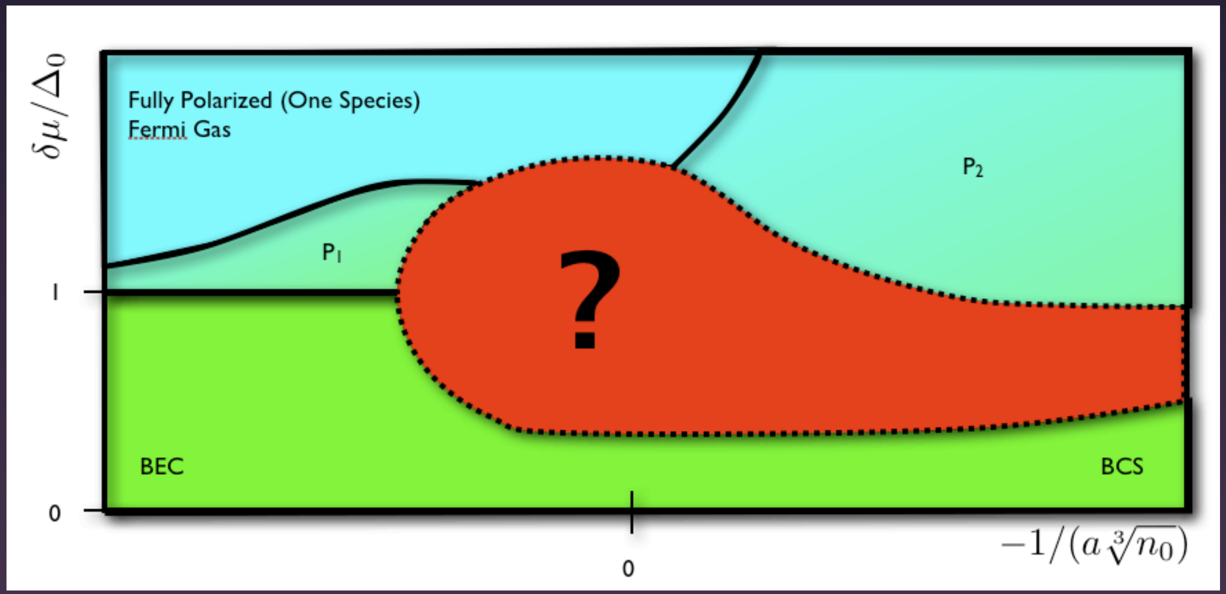
- Need detailed structure or novel signature



MIT Experimental data from Shin et. al (2008)

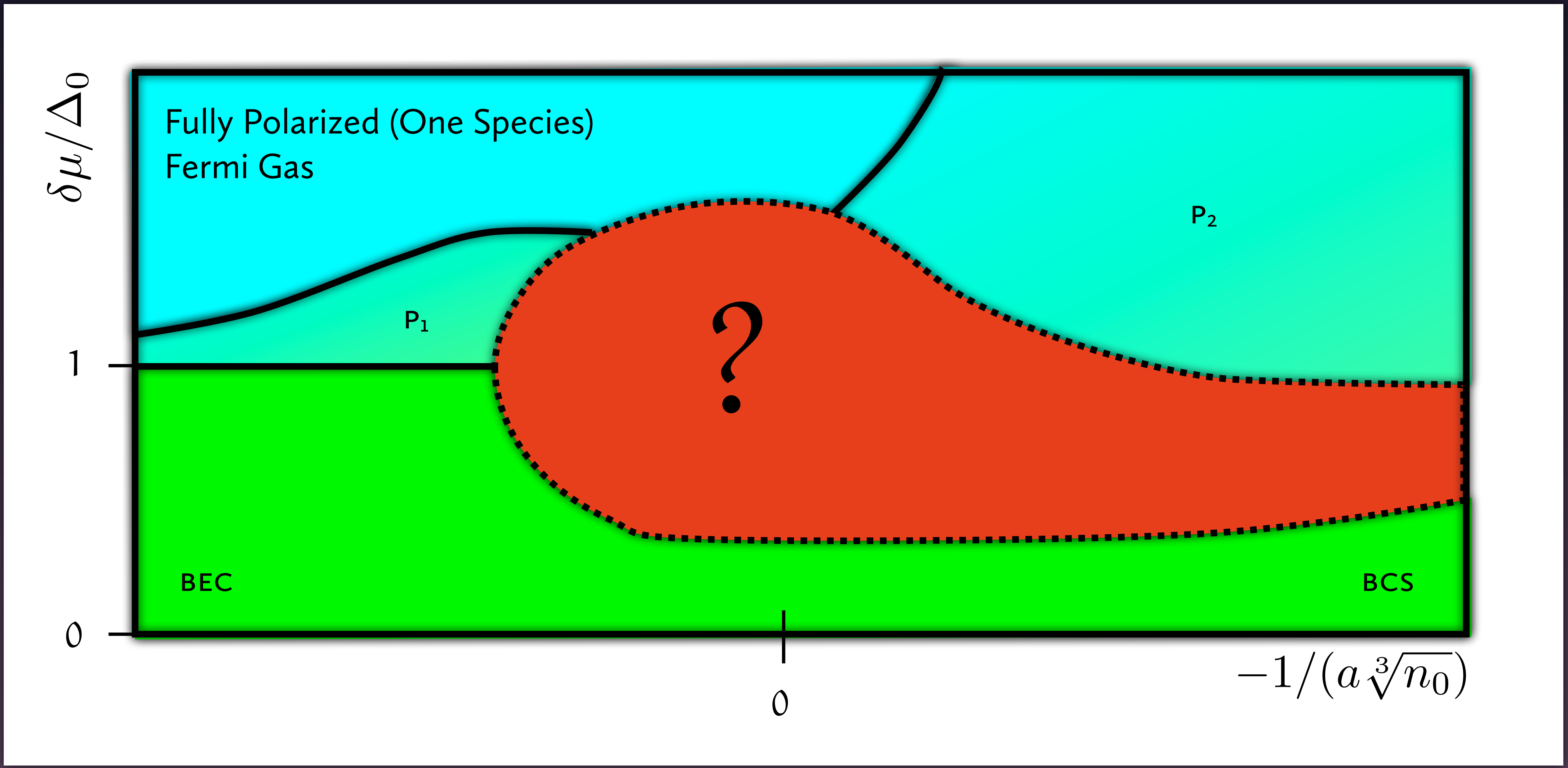
Please Benchmark Asymmetric Phases!

- Challenging for theory
  - Need (unknown) IR structure for nodal structure
  - AFMC has sign problem



- Experiments?
  - Need large physical region (flat traps!)

Rich Phase Structure



Based on D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk PRL 97 020402 (2006)

Theory Wish List (From Program)

- Flat traps! (Slowly oscillating lattices?)
- Tune interactions to model neutron matter ($a_s \sim -7r_e$, $k_F a_s \sim -10$)
- Tensor interactions, Gauge fields (QCD)
- Tuneable masses
- Self bound (dilute) systems?
- Prepare novel states (FFLO from 1D?)
- Vortex pinning (neutron star glitches)

Theory Questions (From Program)

- How to translate from homogeneous systems to traps:
  - When does LDA (Thomas Fermi) work?
  - For what quantities?
  - Other techniques?
- Where can perturbative treatments be used quantitatively? For what quantities? (BCS + Gorkov, Virial expansion etc.)