Resonance behavior understood from T-matrix approach and new predictions

Xiaoling Cui

Institute for Advanced Study

Tsinghua University

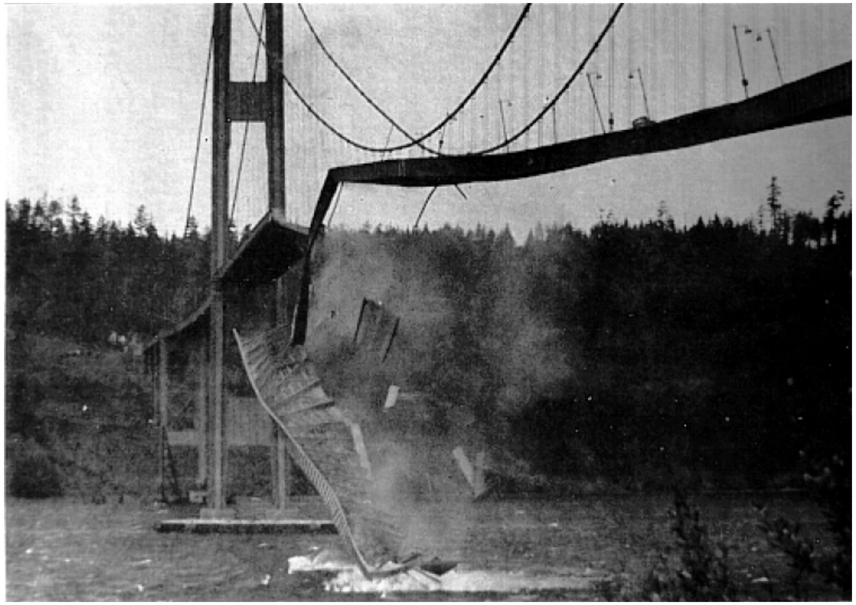
Beijing

INT, 2011.4.19

RESONANCE – widely occur in nature

good!

bad~



Tacoma Narrows Bridge (1940)

In physics: optical cavity, Nuclear magnetic resonance(NMR), Feshbach resonance…

Motivation

- Resonance system in cold atoms:
  i.e., atomic gas with infinite effective coupling strength $g_{eff} \to \infty$

  3D: $a_s \to \infty \quad (a_s \gg 1/k_F \gg \mathrm{r}_0)$

- (existing) strategy to achieve resonance scattering
  - Feshbach resonance, Confinement induced resonance…
  - general ways to understand resonance mechanism

- theoretical methods to solve few-body problem
  - coordinate space (pseudo-potentials, Green function)
  - other approach? better understanding? new prediction?

Outline

- Introduction
  experimental & theoretical studies of resonance scatterings

- A general T-matrix approach to 2~N-body problems
  - **physically transparent**
  - **able to unify various studies in a single framework**
    (bound state, scattering property, reduced interaction…)
  - **easy to identify scattering property**
  - **easy to facilitate new predictions**

- A new system with exotic resonance structure

# of induced resonances: **1 , 2, … , ∞**
√ √ √

--- quasi-1D scattering under separate traps

Basic idea:

To induce resonance in a system

⇕

To find an energy-matched state from a different mode

"closed" channel

Effective coupling (within low-energy space):

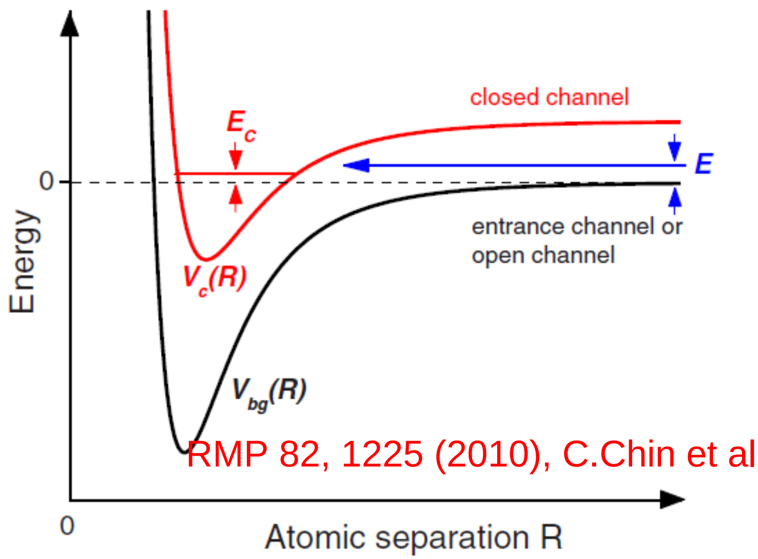
$$g_{eff} = U + \frac{U^2}{E - E_c} + \ldots$$

Resonance: $g_{eff} = \infty$

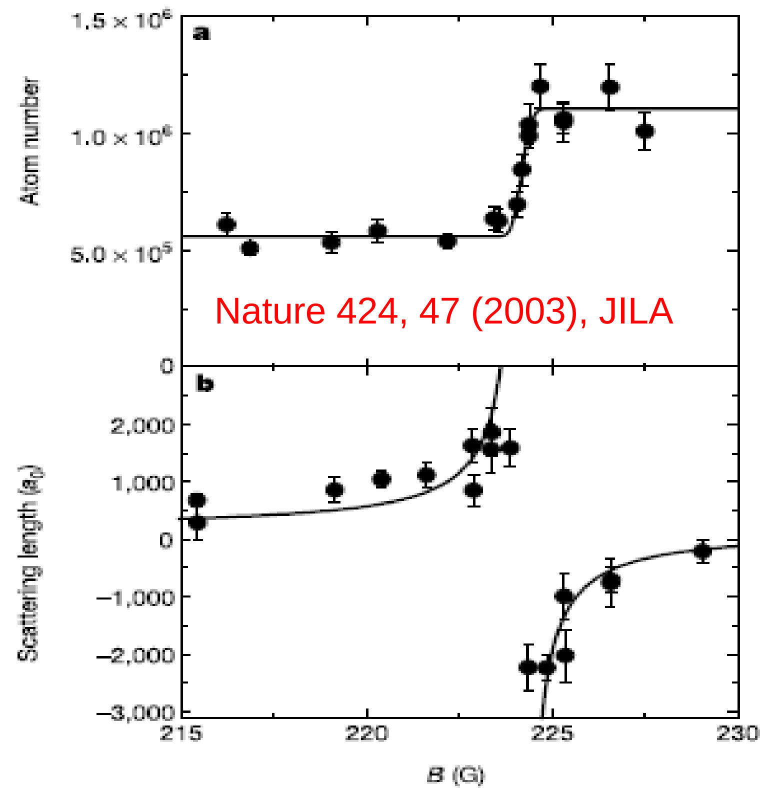
(not necessarily a bound state start to emerge!)

Introduction --- resonance scattering in cold atoms

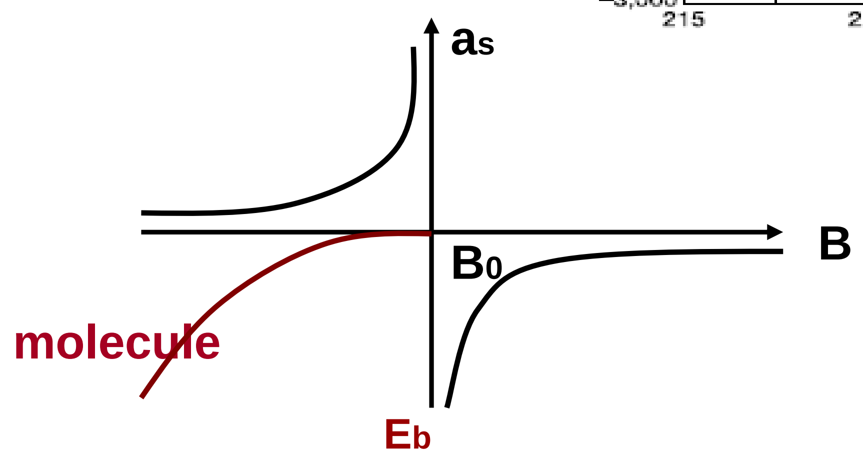
A. Feshbach resonance :



RMP 82, 1225 (2010), C.Chin et al



Nature 424, 47 (2003), JILA



Introduction --- resonance scattering in cold atoms

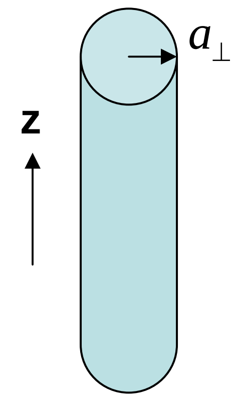
B. Confinement induced resonance :

$$H(r,R) = -\frac{\hbar^2}{2M}\nabla_R^2 - \frac{\hbar^2}{2\mu}\nabla_r^2 + V_{ext}(r,R) + U(r)$$

B1. decoupled (r, R) motions $\longrightarrow$ single resonance

Original CIR in quasi-1D:

$$\left\{\frac{\hat{p}_z^2}{2\mu} + g\delta(\mathbf{r})\frac{\partial}{\partial r}(r\,\cdot) + \hat{H}_\perp(\hat{p}_x,\hat{p}_y,x,y)\right\}\Psi = E\Psi$$



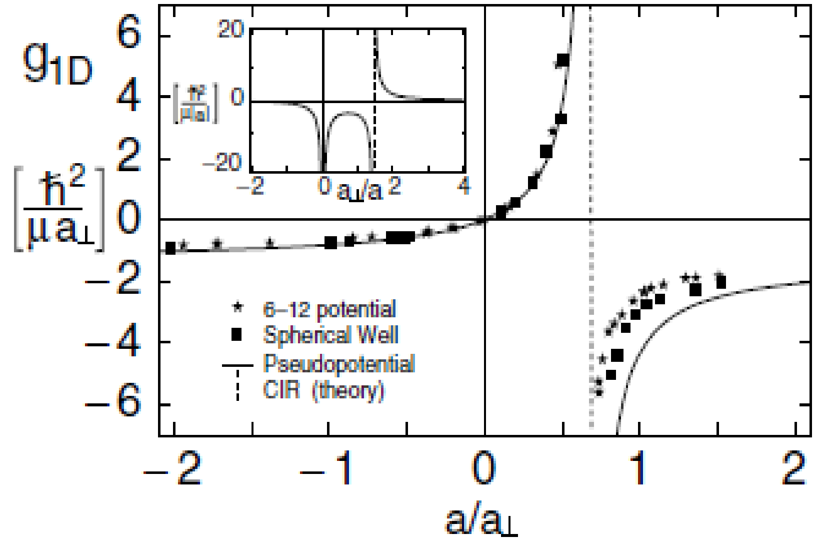
$$\hat{H}_\perp = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2\mu} + \frac{\mu\omega_\perp^2(x^2+y^2)}{2}$$

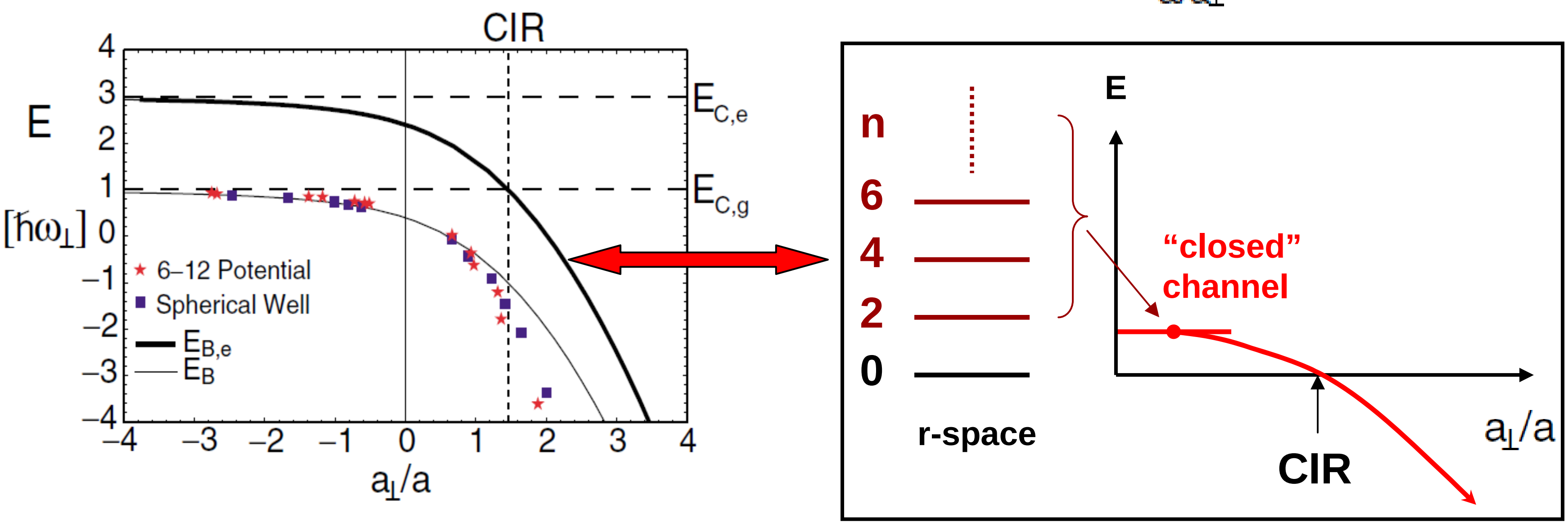
$$\Longrightarrow \quad H_{1D} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + g_{1D}\delta(z),$$

PRL 81,938 (1998); 91,163201(2003), M. Olshanii et al

CIR in quasi-1D:

$$g_{1D} = \frac{2\hbar^2 a}{\mu a_\perp^2} \frac{1}{(1 - Ca/a_\perp)}$$





PRL 81,938 (1998); 91,163201(2003), M. Olshanii et al

expe: PRL 94, 210401 (2005), ETH Zurich (induced molecule)
Science 325, 1224 (2009), Innsbruck (realization of hard-core gases)

Introduction --- resonance scattering in cold atoms

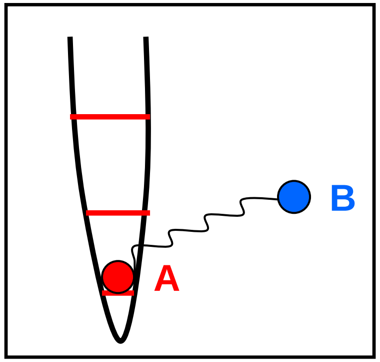
B. Confinement induced resonance :

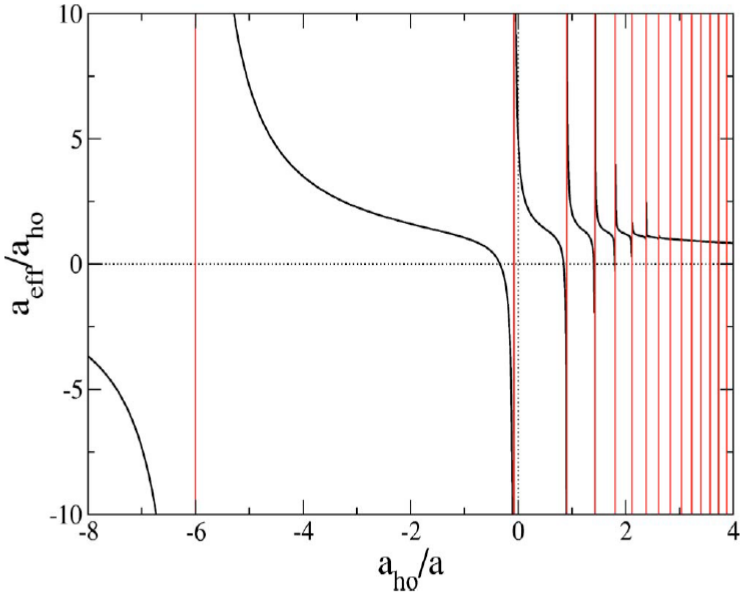
$$H(r,R) = -\frac{\hbar^2}{2M}\nabla_R^2 - \frac{\hbar^2}{2\mu}\nabla_r^2 + V_{ext}(r,R) + U(r)$$

B2. coupled (r, R) motions $\longrightarrow$ infinite # of resonance

Now CIR becomes general (both low-D and 3D)

example 1: mixed dimension

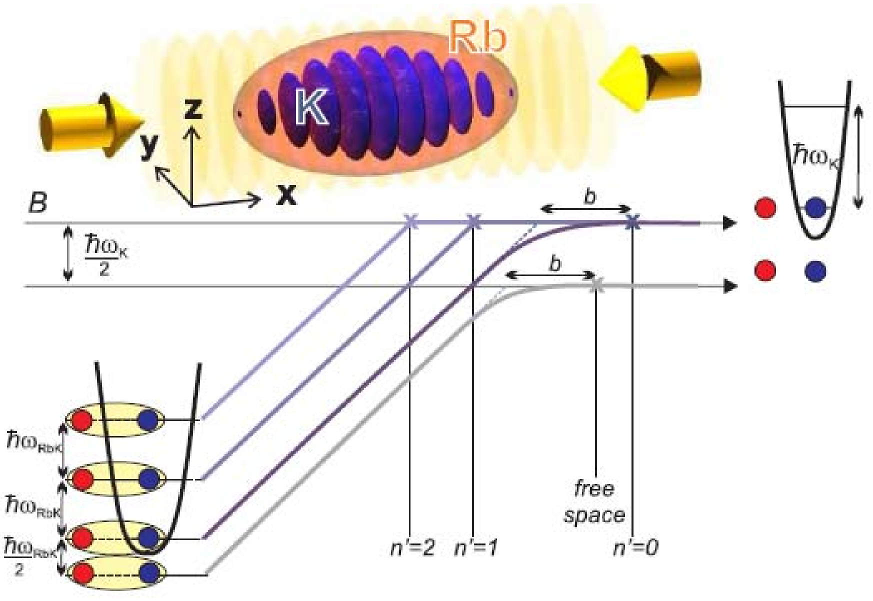


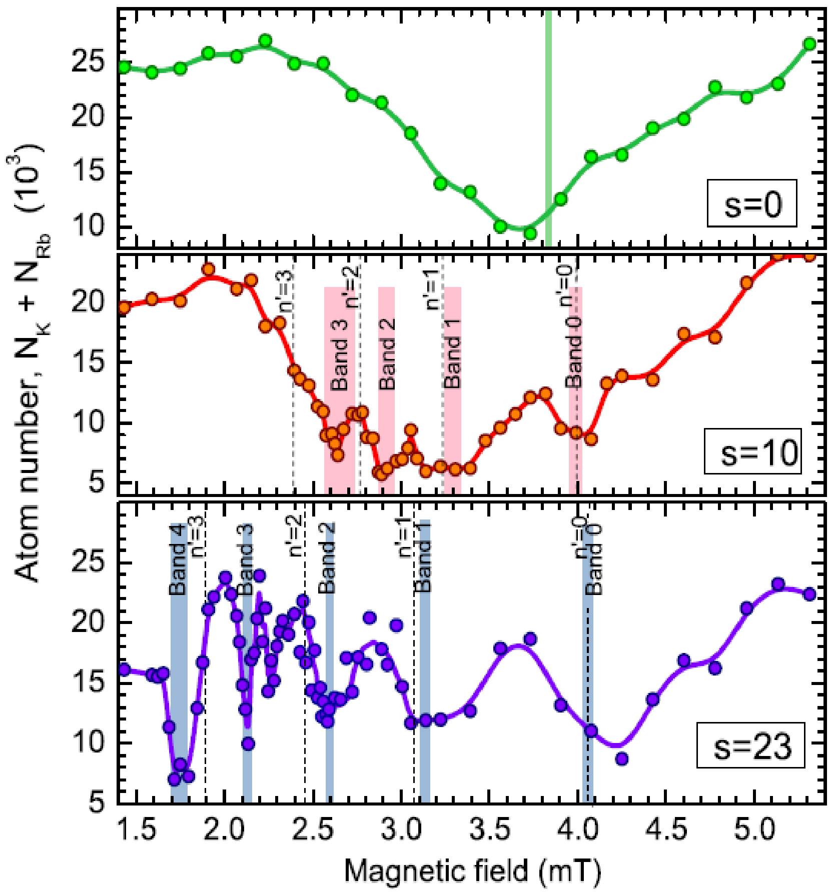


P. Massignan and Y. Castin, Phys. Rev. A 74, 013616 (2006)
Y. Nishida and S. Tan, Phys. Rev. Lett 101, 170401 (2008)

Expe on 2d-3d mixture:

Phys. Rev. Lett 104, 153202 (2008), LENS.





$$\frac{\hbar\omega}{2} = \hbar\omega\sqrt{\frac{m_A}{m_A+m_B}}\left(n'+\frac{1}{2}\right) - E_b$$

no coupling between different COM channels;

good approx only at BEC molecule limit

Introduction --- resonance scattering in cold atoms

B. Confinement induced resonance :

B2. coupled (r, R) motions ⟶ infinite # of resonance:

other examples:

unequal mass or trapping frequency

V. Peano et al, New. J. Phys. 7, 192 (2005).

Bloch-wave scattering in optical lattice :

G. Orso et al, Phys. Rev. Lett. 95, 060402 (2005)

H. P. Buechler, Phys. Rev. Lett. 104, 090402(2010).

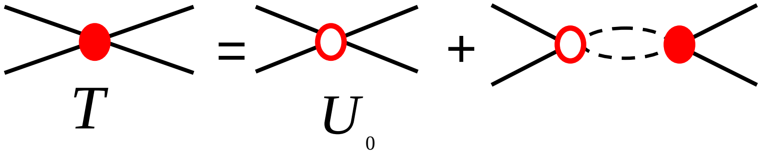
X. Cui, Y. P. Wang and F. Zhou, Phys. Rev. Lett. 104, 153201 (2010).

- **most studies use pseudo-potentials, obtain eqs in coordinate space**
- **an “alternative” way to easily extract resonance(s) ?**

T-matrix approach

I. Basic concept

Free space: Lippmann-Schwinger equation

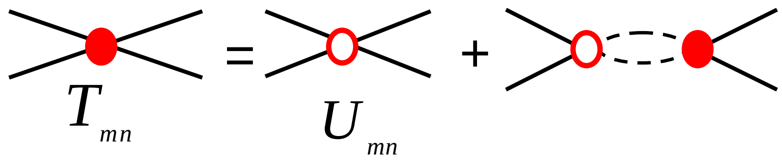


$$T(k,k';E) = U(k,k') + \frac{1}{\Omega}\sum_{k''} U(k,k'')\frac{1}{E - 2\epsilon_{k''} + i\eta} T(k'',k';E)$$

E=0 →
$$\frac{1}{U_0} = \frac{m}{4\pi a_s} - \frac{1}{\Omega}\sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}}$$

implication of renormalization procedure to obtain low-energy physics!

With external potential: $\{\phi_n(r)\}$



$$\frac{\phi_m^*(0)\phi_n(0)}{T_{mn}} = \frac{1}{U_0} - \sum_l \frac{|\phi_l(0)|^2}{E - E_l}$$

$$= \frac{m}{4\pi a_s} - \frac{1}{\Omega}\sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} - \sum_n \frac{|\phi_n(0)|^2}{E - E_n}$$

finite value: universal property of wf at short range

reduced coupling in lower dimension (eg. quasi-1d)

$$\frac{|\phi_0(0)|^2}{g_{1D}} = \frac{m}{4\pi a_s} - \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} - \frac{1}{L_z}\sum_{n>0,k_z} \frac{|\phi_n(0)|^2}{E - E_{n,k_z}}$$

virtual state in high-E space

Q: solution for r&R coupled system? or N(>2)-body system?

T-matrix approach

II. General treatment

$$\hat{H}\left|\psi\right\rangle = E\left|\psi\right\rangle$$

Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{U}$

$$\hat{U} = \sum_{i=1}^{Q}\sum_{m<n}^{N_i} U_i \delta^3(x_m^i - x_n^i) + \sum_{i<j}^{Q}\sum_{m=1}^{N_i}\sum_{n=1}^{N_j} U_{ij}\delta^3(x_m^i - x_n^j)$$

Solve: $|\psi\rangle = |\psi_0\rangle + \hat{G}_0(E)\hat{T}|\psi_0\rangle$

$$\hat{G}_0(E) = \frac{1}{E - \hat{H}_0 + i\delta}$$

$$T = (1 - UG_0)^{-1}U$$

Physical observables: (1) binding energy

(2) scattering amplitude or effective scattering length

(3) reduced interaction in lower dimension

taking advantage of contact U and T, introduce molecular state:

$$\hat{T}|\psi_0\rangle = \sum_{I\lambda} f_\lambda^I |r_I = 0, \lambda\rangle$$

I : pairwise interaction

λ: energy levels

molecular state:

- each is constructed according to fermion/boson statistics;
- each corresponds to one interaction term in U;
- orthogonal to each other, coupled by kinetic term.

$$\sum_{I'\lambda'} f_{\lambda'}^{I'} \left[\frac{\mu_I}{2\pi a_I}\delta_{II'}\delta_{\lambda\lambda'} - C_{\lambda\lambda'}^{II'}\right] = \langle r_I = 0, \lambda|\psi_0\rangle$$

$$C_{\lambda\lambda'}^{II'} = \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}}}\delta_{II'}\delta_{\lambda\lambda'} + \langle r_I = 0, \lambda|\hat{G}_0|r_{I'} = 0, \lambda'\rangle$$

finite value: irrelevance of short-range physics (necessary condition)

Physical meaning :

- **expanded by a set of orthogonal molecular states, describing external center-of-mass motions of pairs of interacting particles;**
- **each matrix element is guaranteed to be finite by a proper renormalization for internal relative motions.**

III. Physical observables:

i. Bound state solution: $\mathrm{Det}(1 - UG_0(E_b)) = 0$

$$|\psi_b\rangle = \sum_{I\lambda} f_\lambda^I \hat{G}_0(E_b)|r_I = 0, \lambda\rangle$$

ii. Scattering amplitude (~a_{eff} for long-distance behavior):

$$\langle\psi_0|\hat{T}|\psi_0\rangle = \sum_{I\lambda} f_\lambda^I \langle\psi_0|r_I = 0, \lambda\rangle$$

$$= \sum_\gamma \frac{\langle\psi_0|\mathcal{F}_\gamma\rangle\langle\mathcal{F}_\gamma|\psi_0\rangle}{\frac{\mu}{2\pi a_s} - c_\gamma}$$

resonance width

resonance position

$$C_{\lambda\lambda'}\mathcal{F}_{\gamma\lambda'} = c_\gamma \mathcal{F}_{\gamma\lambda}$$

iii. Reduced interaction in lower D (long-distance behavior):

$$g_{\mathrm{eff}} = \langle\psi_0|(1 - UG_0^Q)^{-1}U|\psi_0\rangle$$

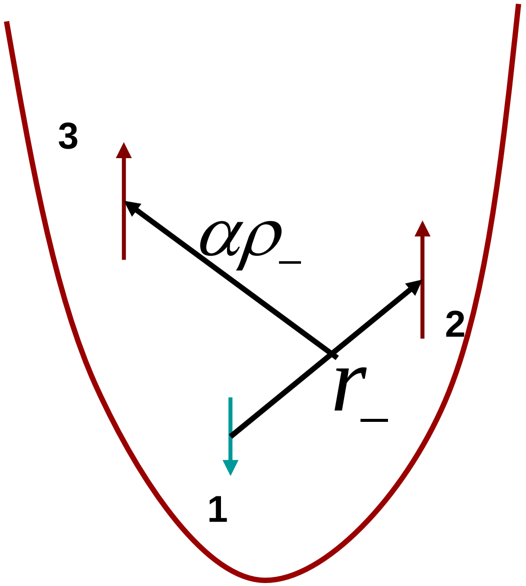
Green functions in "closed" channel: $G_0^Q = \frac{1}{E - H_0^Q + i\delta}$

nothing but molecular state is constructed in a "gapped" closed-channel!

IV. Application to 3-body:

I. Scattering property of 3 trapped fermions

molecular state:



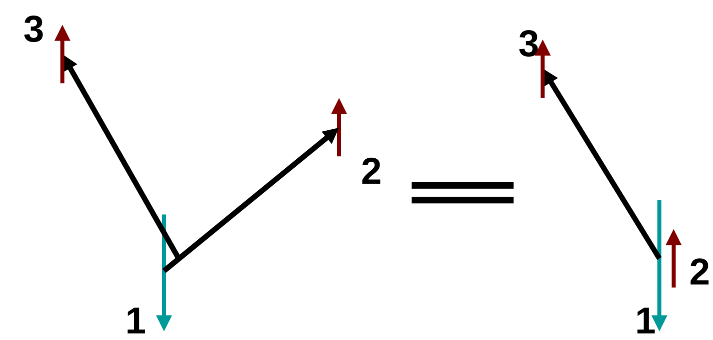
$$|\lambda\rangle = \frac{1}{\sqrt{2}}(|r_- = 0, \lambda\rangle - |r_+ = 0, \lambda\rangle)$$

$$\Rightarrow C_{\lambda\lambda'} = (\frac{1}{V}\sum_{\mathbf{k}}\frac{1}{\epsilon_{\mathbf{k}}} + \sum_{\nu}\frac{|\psi_\nu(0)|^2}{E - E_\lambda - E_\nu + i\delta})\delta_{\lambda\lambda'} + F_{\lambda\lambda'}$$

$$F_{\lambda\lambda'} = \int d\rho\psi^*_\lambda(\rho)\psi_{\lambda'}(-\beta\rho)\sum_\nu \frac{\psi_\nu(0)\psi_\nu(-\alpha\rho)}{E - E_{\lambda'} - E_\nu + i\delta}$$

$$\alpha = \frac{\sqrt{Mm_\downarrow}}{m_\uparrow + m_\downarrow}, \ \beta = \frac{m_\uparrow}{m_\uparrow + m_\downarrow}$$

(i)at two-limits of mass ratio: $u = \dfrac{m_\uparrow}{m_\downarrow}$



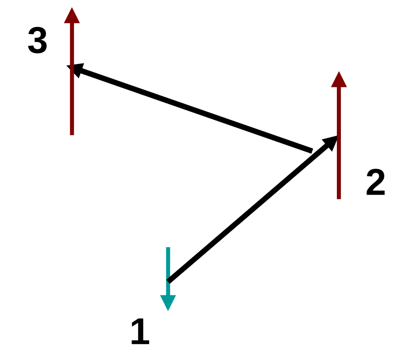
$$u \to 0: \qquad F_{\lambda\lambda'} \sim \delta_{l,0}$$

atom-dimer uncorrelated state for all angular momenta(*l*) except for *l*=0!

$$u \to \infty: \qquad C_{\lambda\lambda'} = \left(\frac{1}{V}\sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}}} + \sum_{\nu} \frac{|\psi_\nu(0)|^2}{E - E_\lambda - E_\nu + i\delta}[1 - (-1)^l]\right)\delta_{\lambda\lambda'}$$

(1) for odd *l*, $C_{\lambda\lambda'}$ not well-renormalized

→ **implication of Efimov physics!**



(2) for even *l*, non-interacting system!

Born-Oppenheimer appro.:

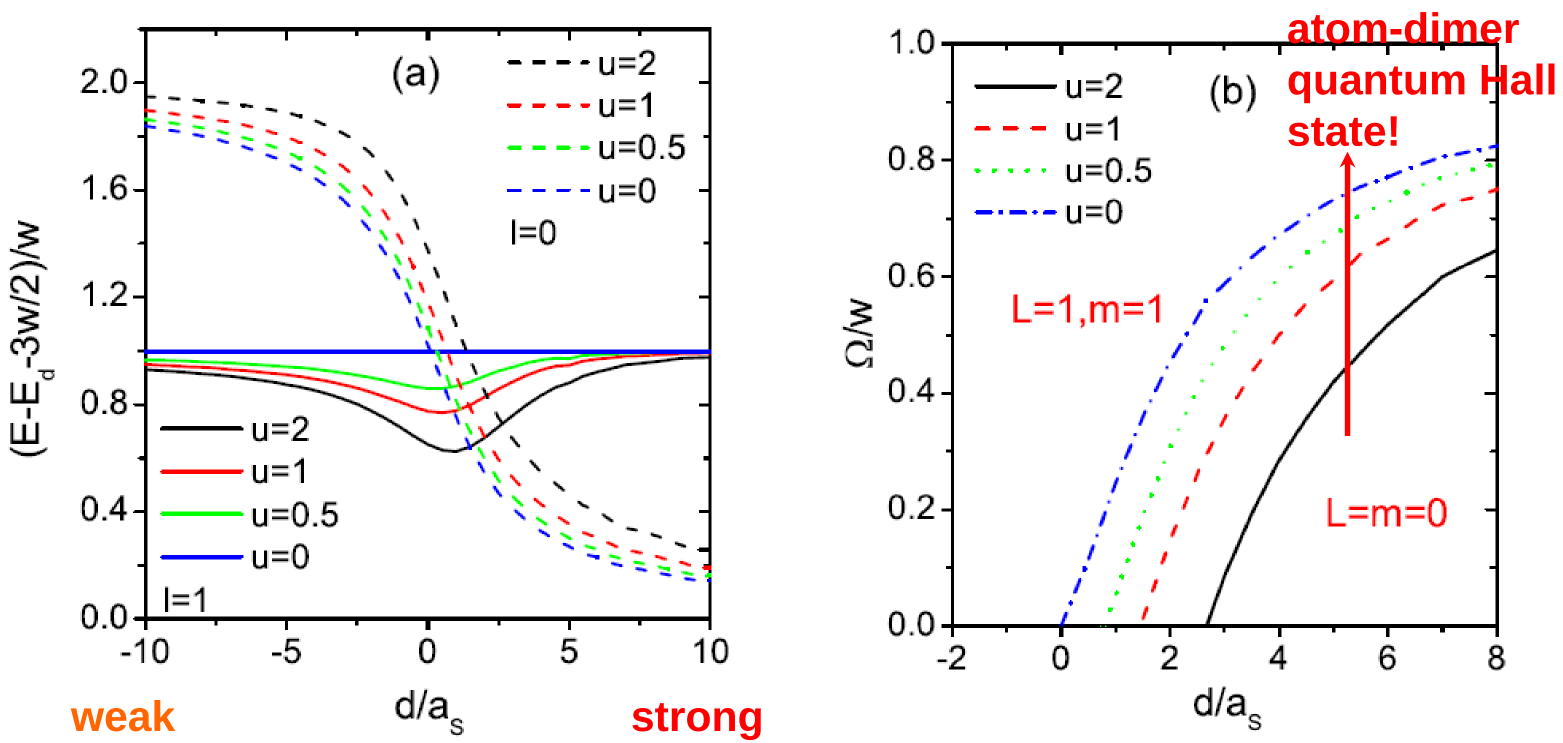
$$\psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = [\varphi(|\mathbf{r}_-|) \pm \varphi(|\mathbf{r}_+|)] f_1(\mathbf{x}_2 - \mathbf{x}_3) f_2(\mathbf{R})$$

+ sign: *l* is odd;
- sign: *l* is even but now $\hat{U}\psi = 0$!

(1,2) are straightforwardly identified by T-matrix!

(ii). Spectra in a rotating harmonic trap and phase transition

$$H_0(\mathbf{r}) = -\frac{\nabla_{\mathbf{r}}^2}{2\mu} + \frac{1}{2}\mu w^2 \mathbf{r}^2 - \Omega L_z(\mathbf{r})$$



weak coupling

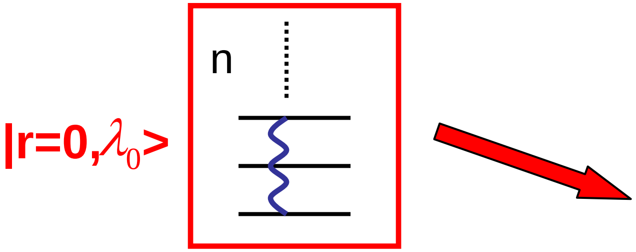
(*l*=1)

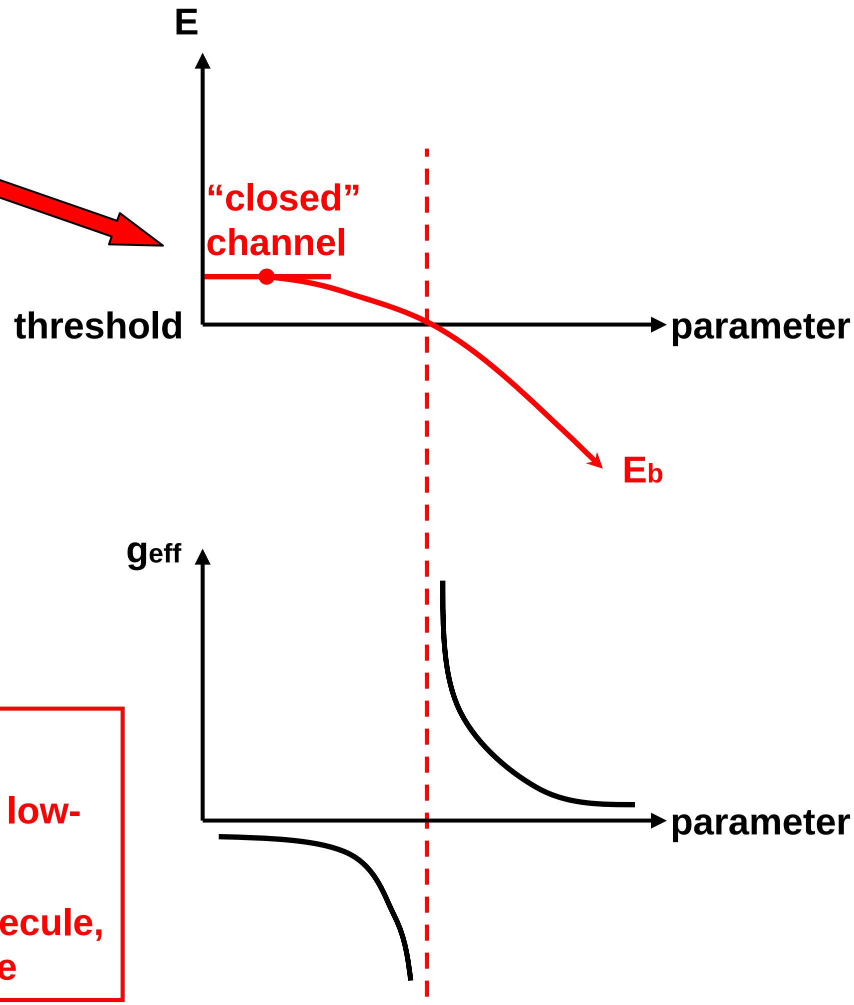
strong coupling

(*l*=0)

General picture of induced resonance(all D) from T-matrix approach :

1) decoupled r&R:





coupled by U

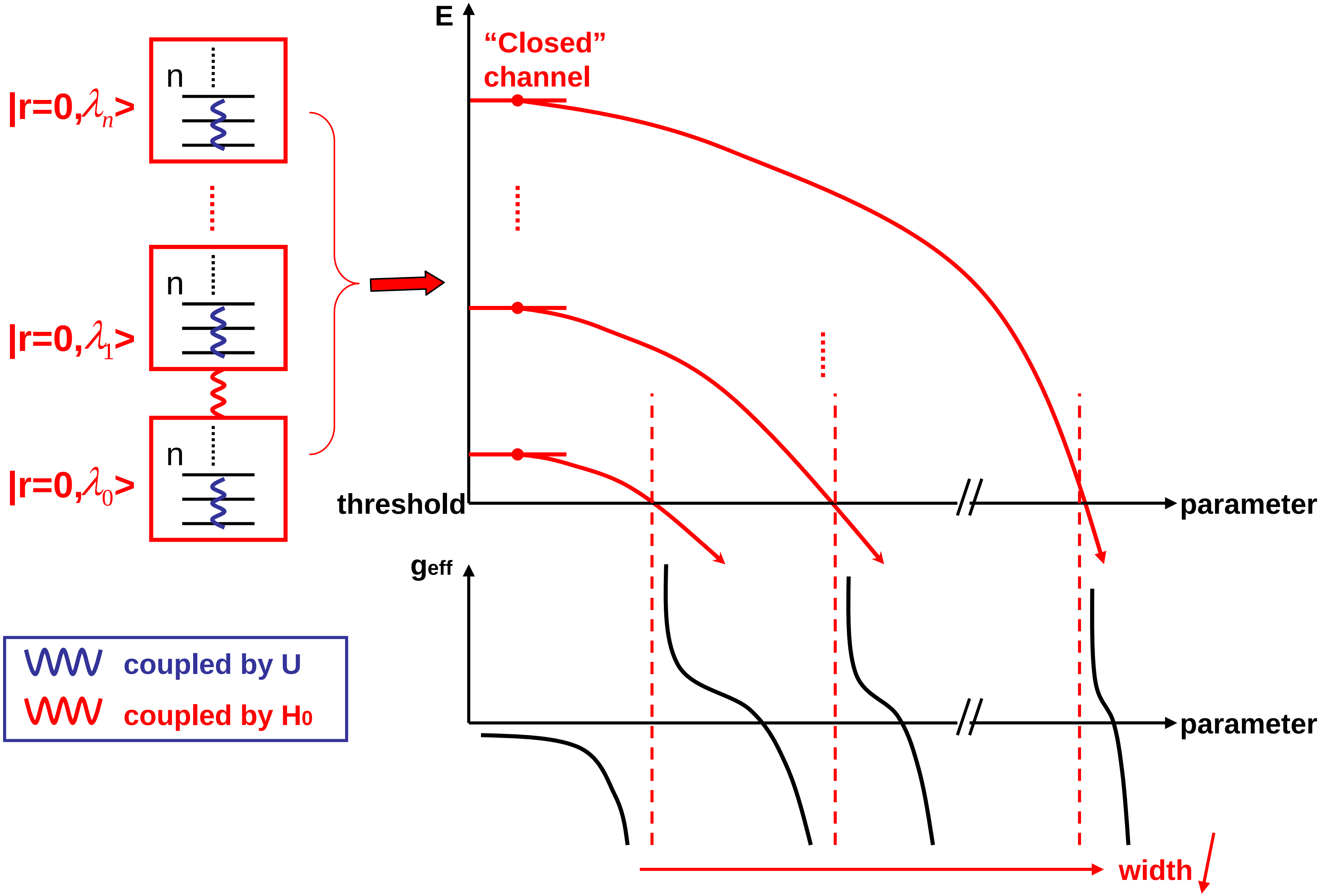
"closed"： high-E space

g_{eff} : effective coupling in low-E space

E_b: "closed" channel molecule, responsible for resonance

General picture of induced resonance(all D) from T-matrix approach :

2) coupled r&R:



COMPARISON

Pseudo-potential	T-matrix
$U(r)=\frac{2\pi a_s}{\mu}\delta^3(r)\frac{\partial}{\partial r}$	$U_0\delta^3(r)$, $\frac{1}{U_0}=\frac{m}{4\pi a_s}-\frac{1}{\Omega}\sum_{\mathbf{k}}\frac{1}{2\epsilon_{\mathbf{k}}}$
universal short-distance behavior ~(1/r-1/a)	absence of ultraviolet divergence for each matrix element
integral equation in real space	matrix equation in energy space (good convergence)
	physically transparent: relative & COM motions effectively separated; **predict single/multiple resonances without any numerical work;** **easier access to unusual scattering properties;**

See X. Cui, arxiv: 1010.0044v2

General conclusion of induced resonances:

under s-wave contact model, by tuning B (or a_s or U_0)

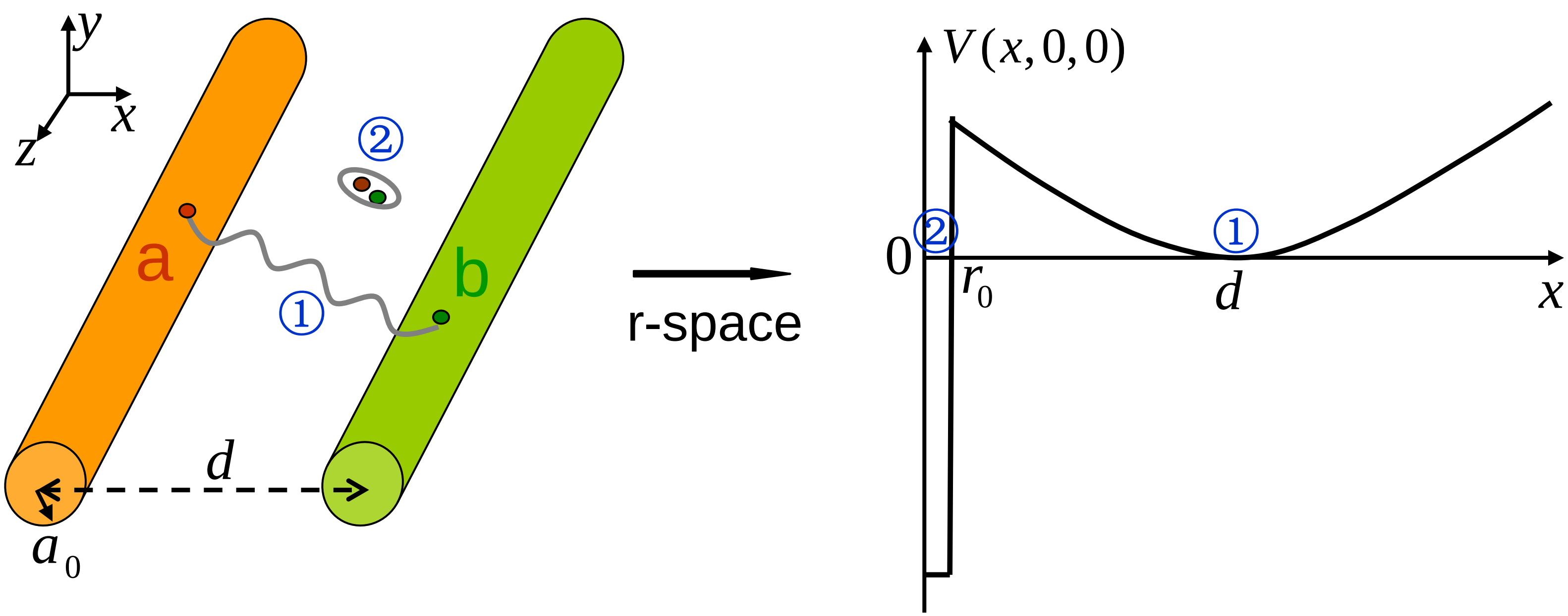
- ✓ **if (r,R) is decoupled, then single renonance**
- ✓ **if (r,R) is coupled, then infinite # of renonance**
- ✓ **a "closed" channel molecule below threshold emerge from each resonance at positive side of g_{eff} (3D, low-D)**
- ✓ **resonance width is wide at large B(weak int.) and narrow at small B(strong int.); all have the same sign**

Q: are there any other induced resonances with different structures? (i.e., #, width …)

A: YES!

---- strong indication from T-matrix approach

Separation induced resonance (quasi-1D)

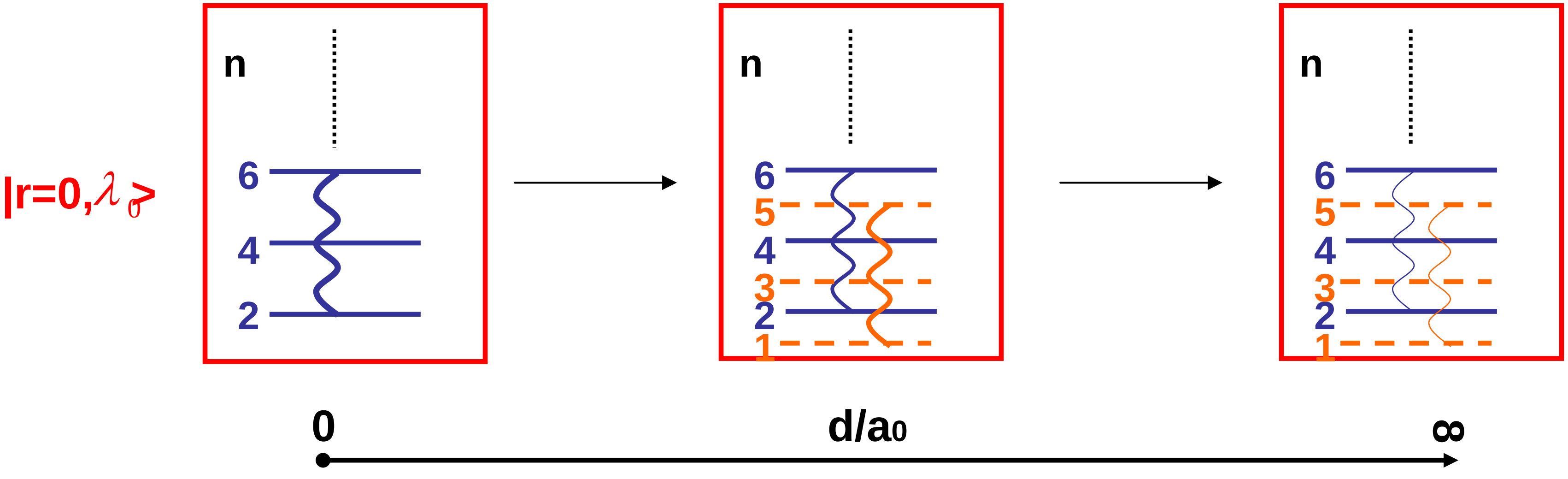


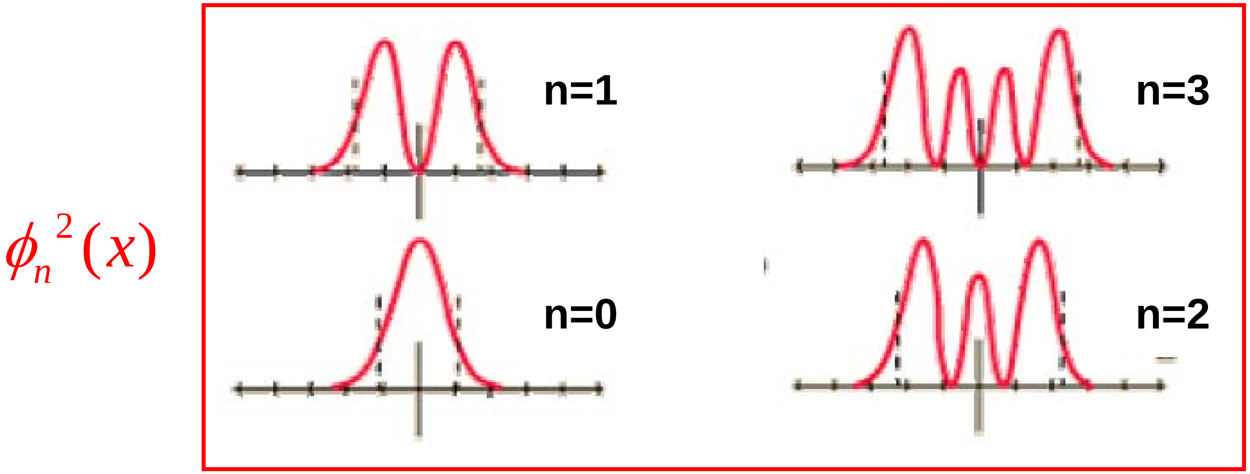
Separation → **?** → **effective 1D scattering**

Separation → **"closed" channel** → **effective 1D scattering**

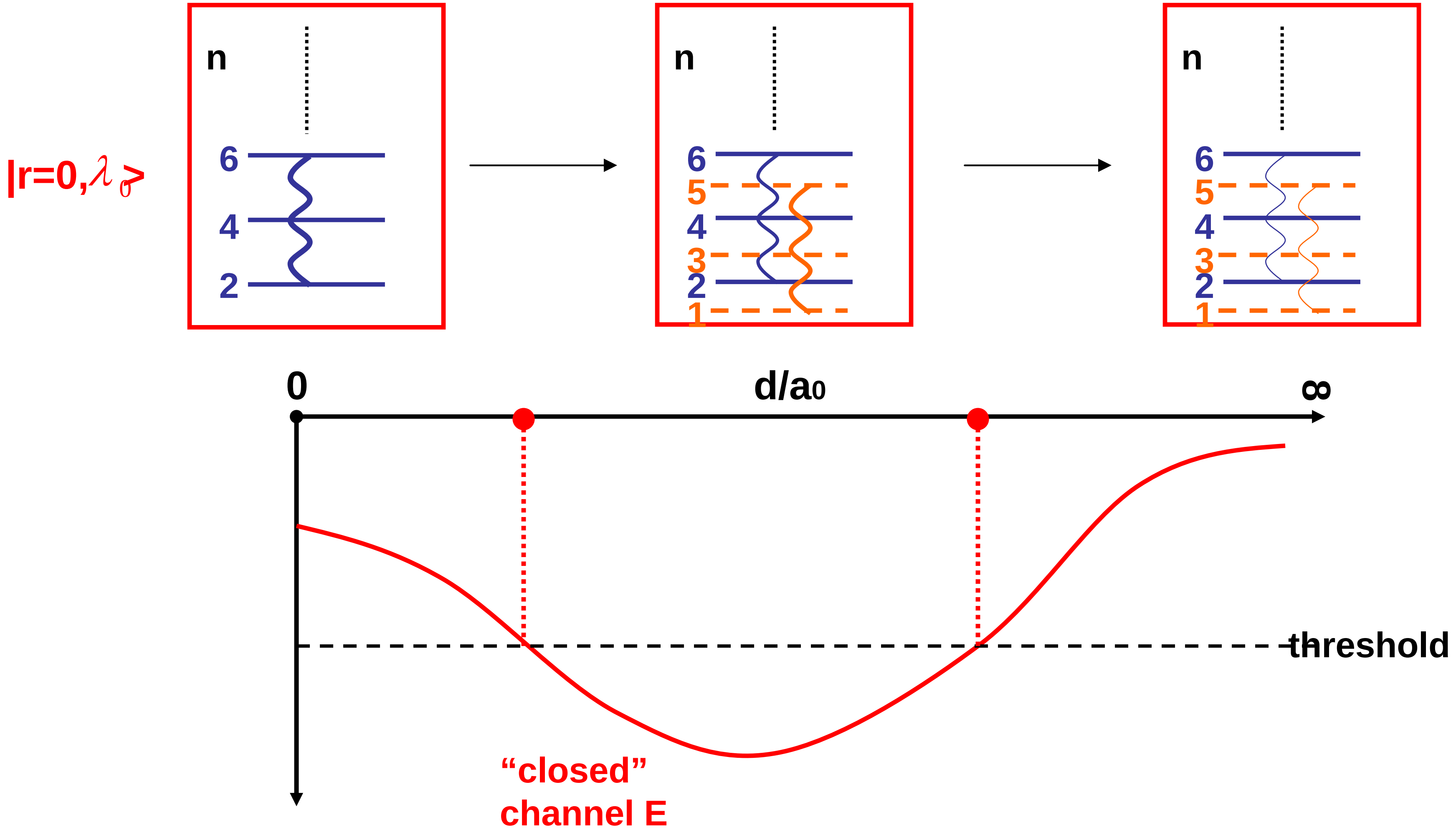
Wenbo Fu (Tsinghua), Zhenhua Yu(OSU) and X. Cui, in preparation

$$0 = \frac{m}{4\pi a_s} - \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} - \frac{1}{L_z}\sum_{\mathbf{n},k_z}' \frac{|\phi_{n_x}(0)\phi_{n_y}(0)|^2}{E_b^c - E_{n,k_z}}$$



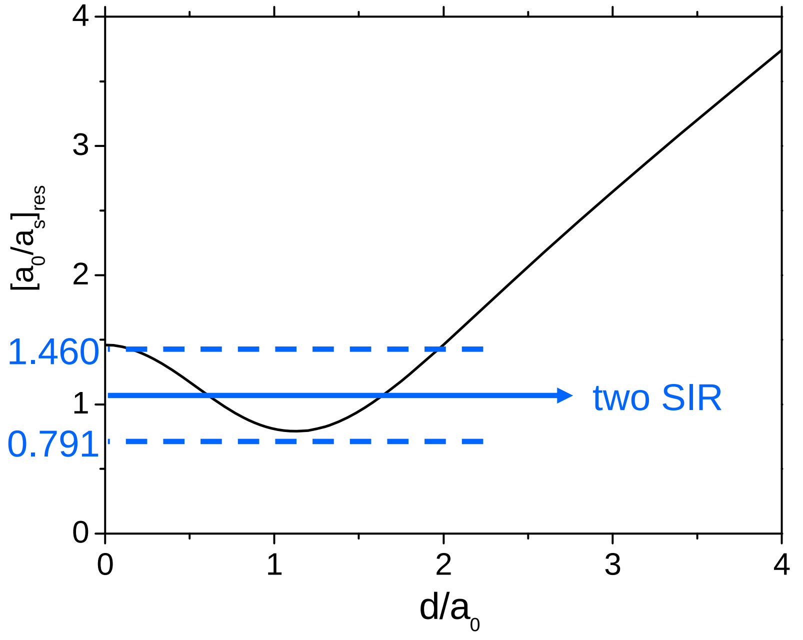


$$0 = \frac{m}{4\pi a_s} - \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} - \frac{1}{L_z}\sum_{\mathbf{n},k_z}' \frac{|\phi_{n_x}(0)\phi_{n_y}(0)|^2}{E_b^c - E_{n,k_z}}$$

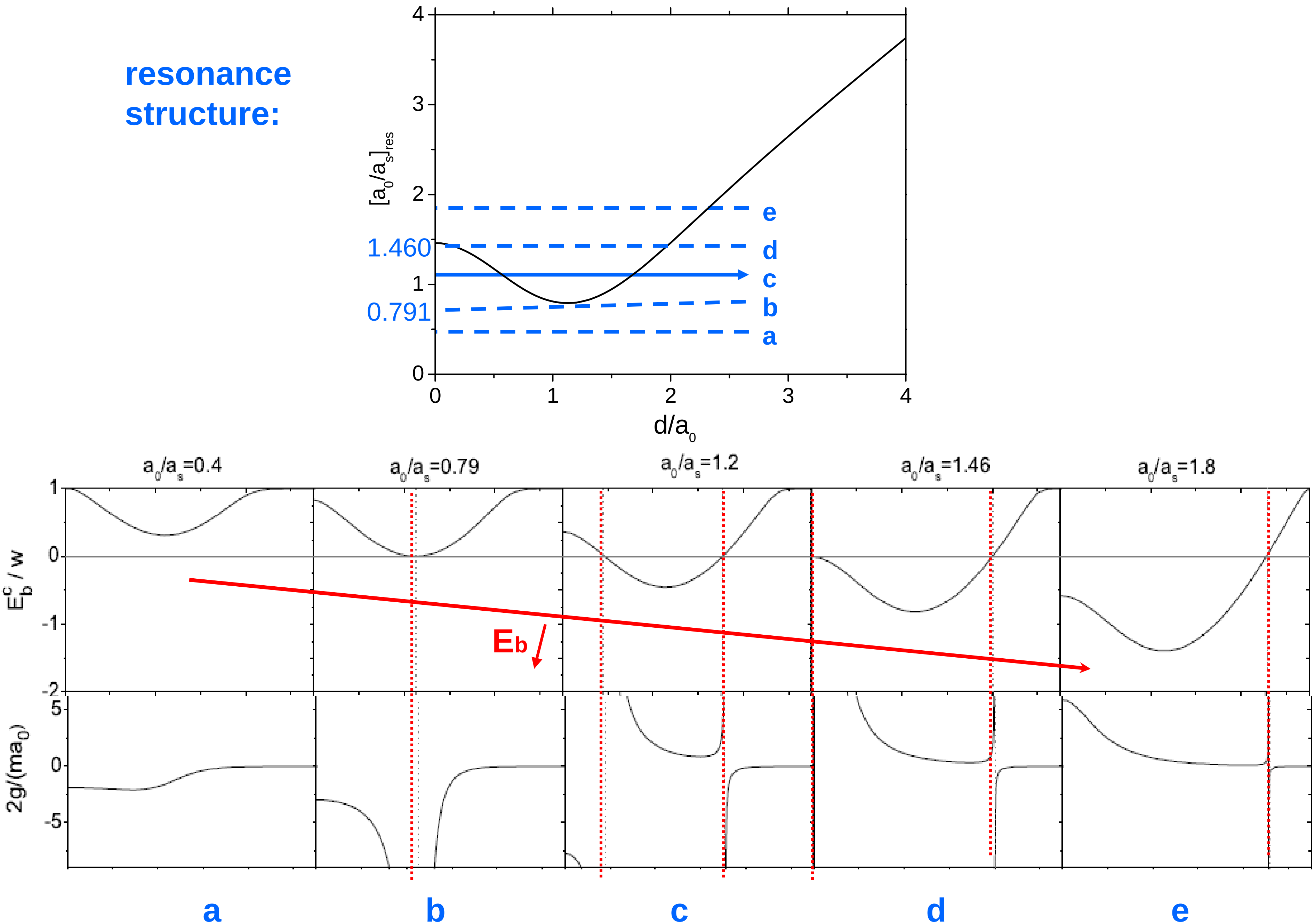


Result of SIR:

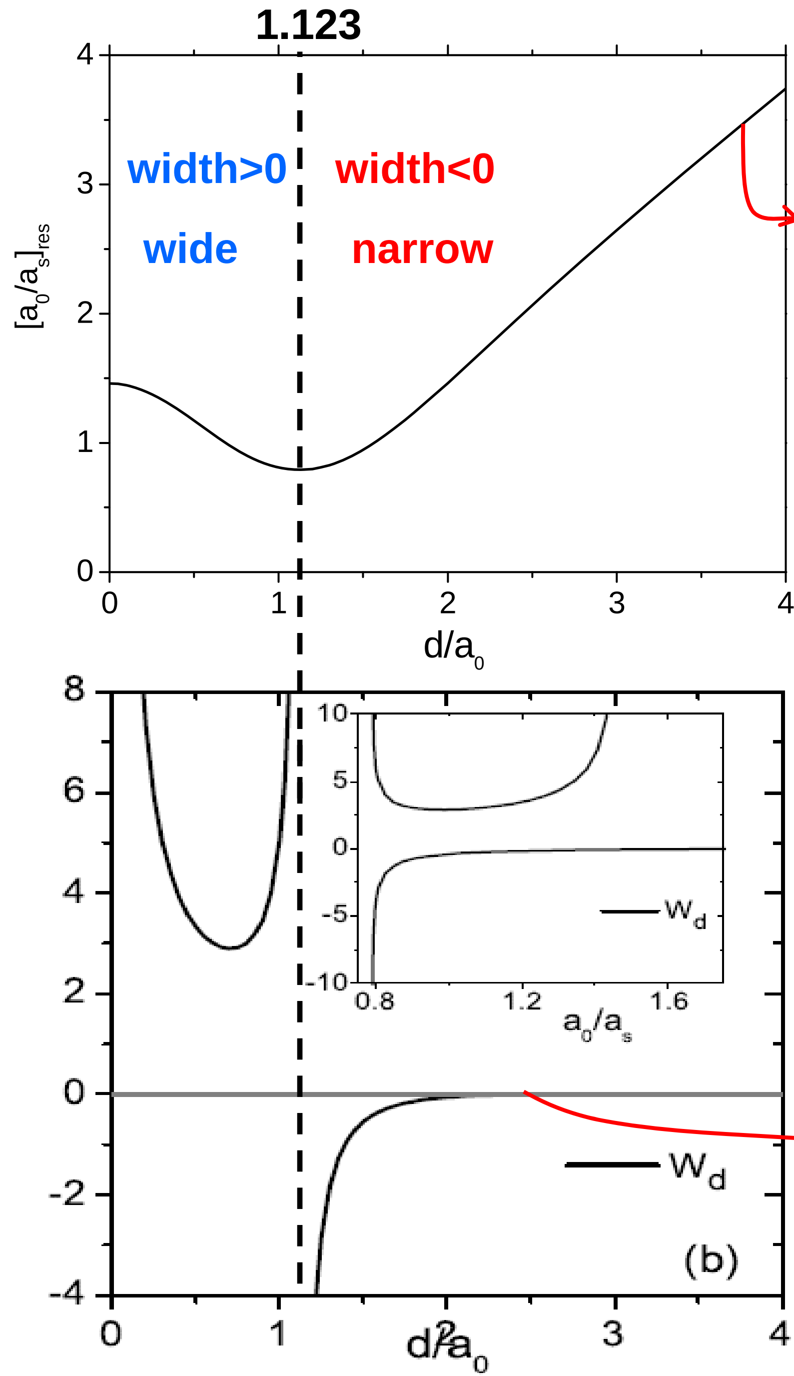
resonance position:



resonance structure:



resonance width:



$$-\frac{1}{ma_s^2}+\frac{m}{4}\omega^2 d^2=\omega$$

"closed" channel

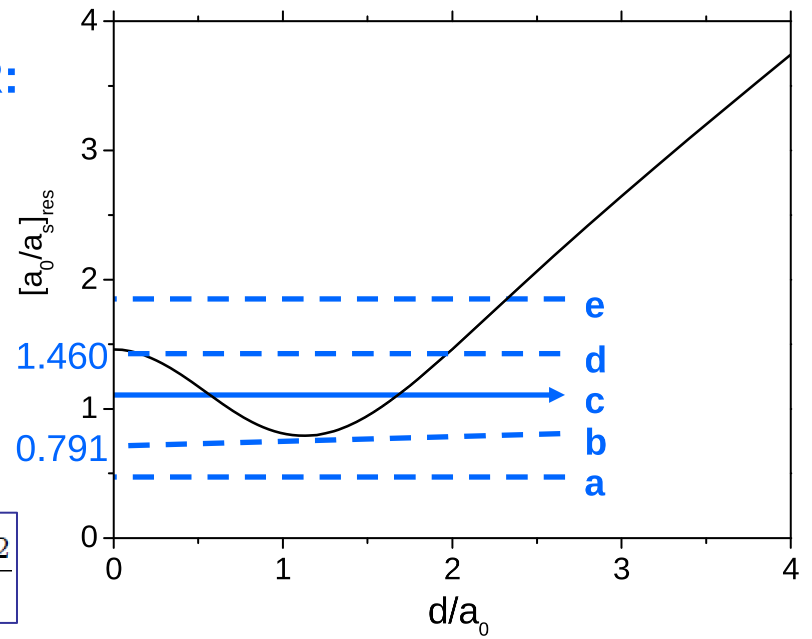
$$\tilde{g}_{1D}=\frac{W_d}{\tilde{d}-\tilde{d}_{res}}$$

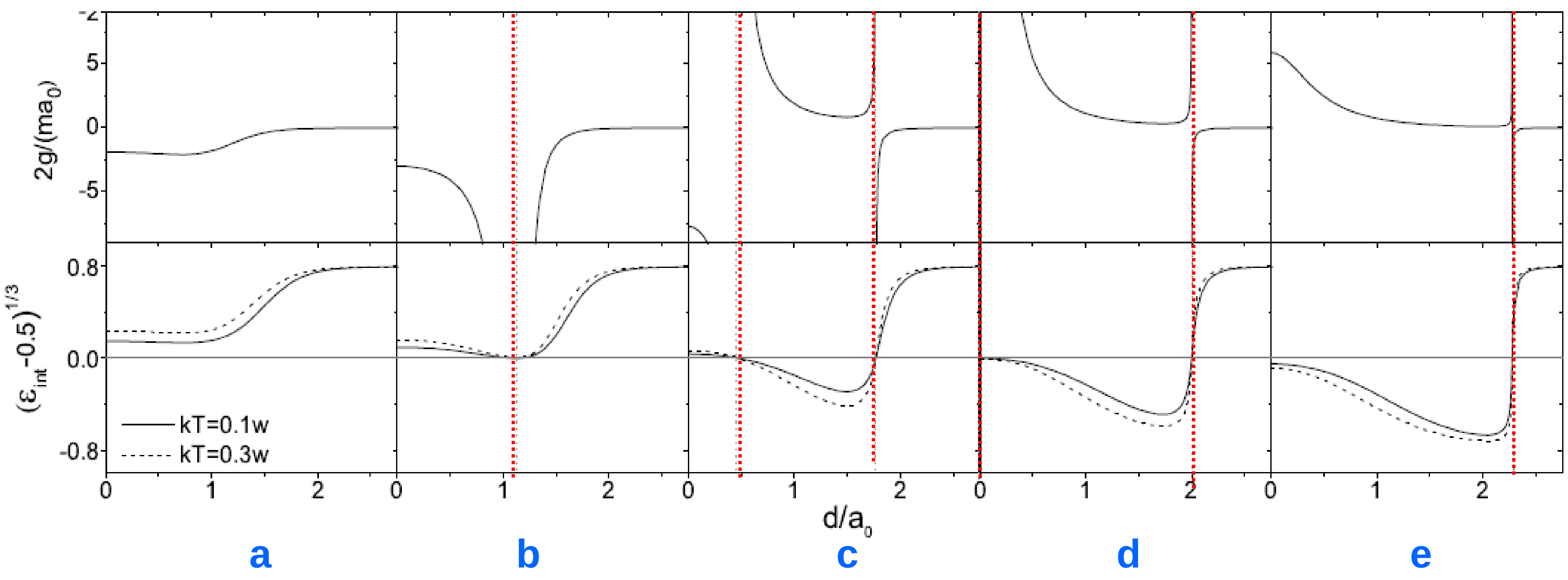
exponentially small

universality at SIR:

(virial expansion)

$$\epsilon_{int} = -\Delta b_2 + 2T\frac{\partial \Delta b_2}{\partial T}$$

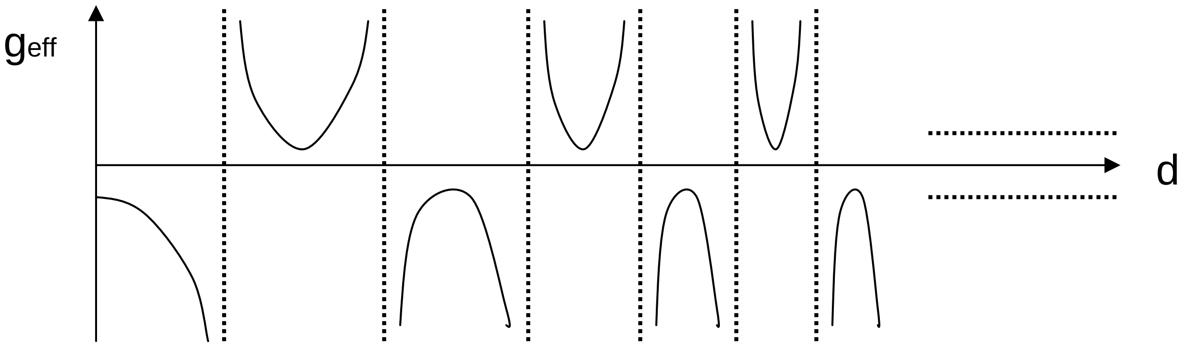




Open questions:

1. r&R coupled system (unequal mass, trapping frequency) in separated trap:

resonance structure:



2. Similar situation in quasi-2d

   --- competing effect, reentrance of strong coupling regime for a single branch

3. many-body property, crossover, universal behavior

4. #=3, 4, …? (more competing effects)

Summary

- General T-matrix approach to 2~N-body problems (bound state, scattering property, reduced coupling…)
  - physically insightful
  - -- easy to make prediction to induced resonance: “closed” channel molecule, resonance #(1 or infty), width…

- Predictions of *two* separation-induced-resonances
  - a new type of induced resonance with competing effects, finite resonance # and novel resonance structure

References:

X. Cui, arxiv 1010.0044v2.

Wenbo Fu, Zhenhua Yu and X. Cui, to appear

Thanks for attention !