Superfluidity and Crystallization of 2D Polar Fermionic Molecules

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Overview

- MW dressed polar molecules in 2D

- One-component polar fermionic molecules

– $p_x + ip_y$ superfluidity

- Two-component polar fermionic molecules

– ferromagnetism and crystallization

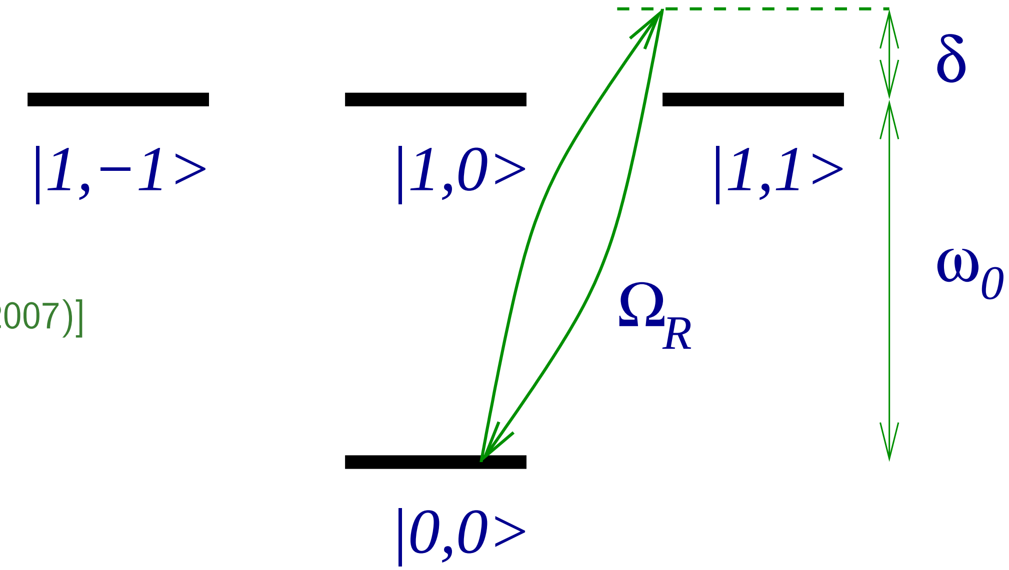
2D Polar Molecules

$^{40}K^{87}Rb$ [Ni *et al.*[JILA], Science (2008)]
$^{7}Li^{133}Cs$ [Deiglmayer *et al.*[Heidelberg], PRL (2008)] $\Rightarrow$ Electric dipole moment

2D centre-of-mass motion, but 3D rotational levels:

$$|J, M_J\rangle \qquad \langle \boldsymbol{d} \rangle = 0$$

- Electric field $E_z \Rightarrow \langle d_z \rangle \neq 0 \Rightarrow 1/r^3$ repulsion [de Miranda *et al.*, arXiv:1010.3731]



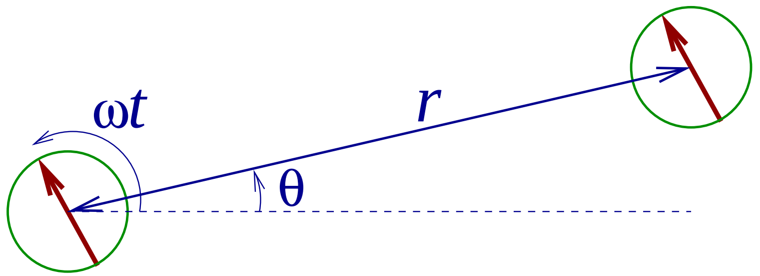
- circularly polarized MW field

[Micheli, Pupillo, Büchler & Zoller, PRA 76, 043604 (2007)]

Rotating wave approximation, $\delta, \Omega_R \ll \omega_0$ $\Rightarrow$field-dressed states

$$|+\rangle = a|0,0\rangle + be^{-i\omega t}|1,1\rangle \qquad |-\rangle = b|0,0\rangle - ae^{-i\omega t}|1,1\rangle$$

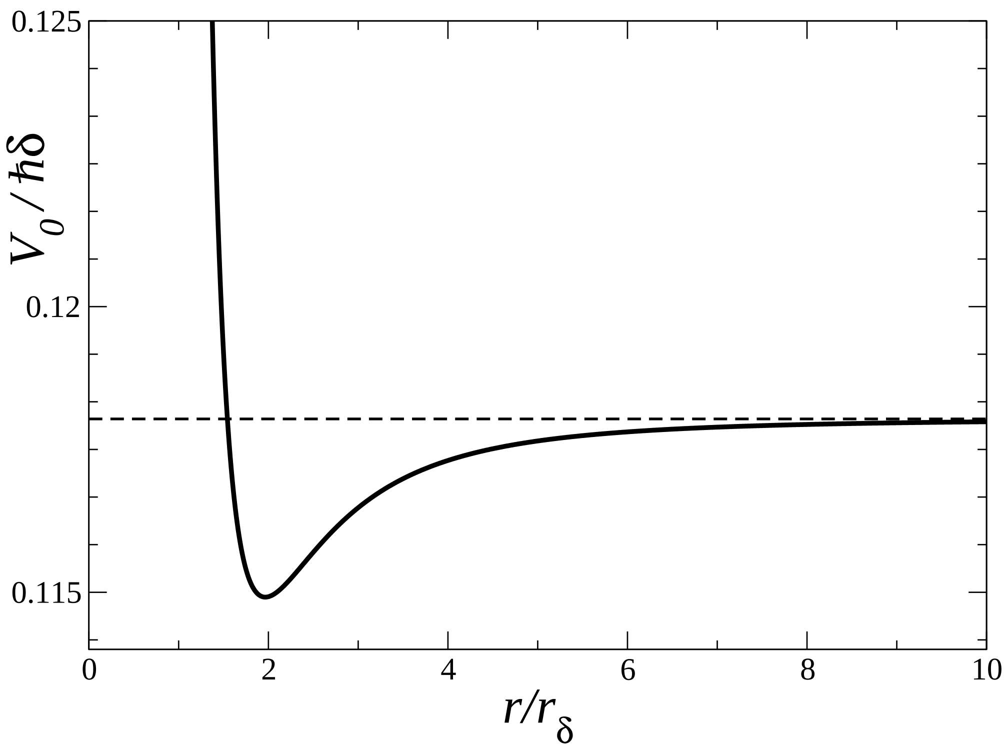
$$\langle +|\boldsymbol{d}|+\rangle = d_{\rm eff}\,(\cos\omega t, \sin\omega t, 0)$$

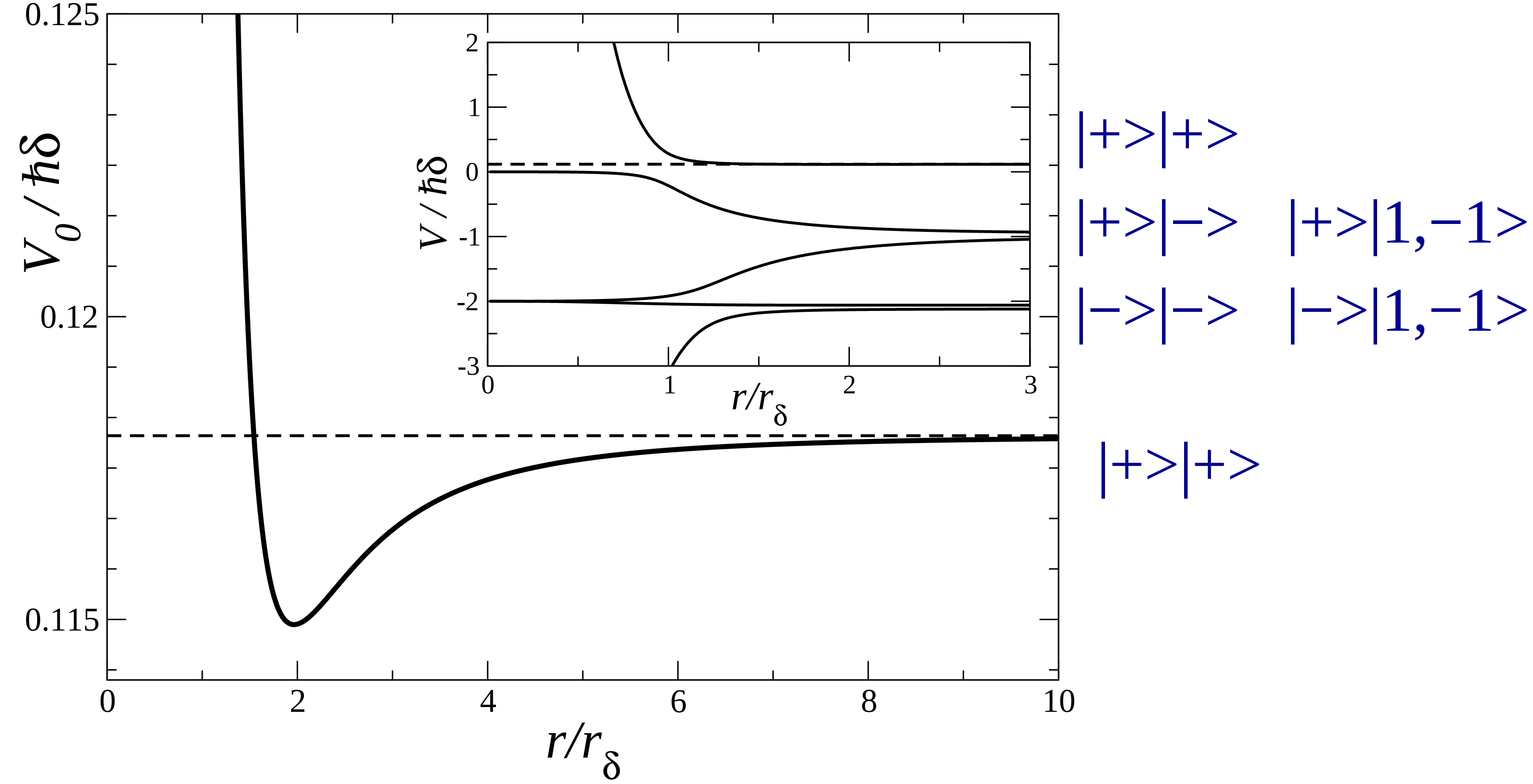


$$V_0(r,\theta;t) = \frac{d_{\rm eff}^2}{r^3}\left[1 - 3\cos^2(\omega t - \theta)\right]$$

$$\overline{V_0}(r) = -\frac{d_{\rm eff}^2}{2r^3}$$

$$r^* \equiv \frac{M d_{\rm eff}^2}{2\hbar^2}$$





"Hard core" repulsion $\qquad \dfrac{d^2}{3r_\delta^3} \equiv \hbar\delta$

Inelastic losses can be made small. Ensure lifetimes $\gtrsim$ 1sec.

[NRC & Shlyapnikov PRL (2009); Levinsen, NRC & Shlyapnikov, arXiv:1103.3859]

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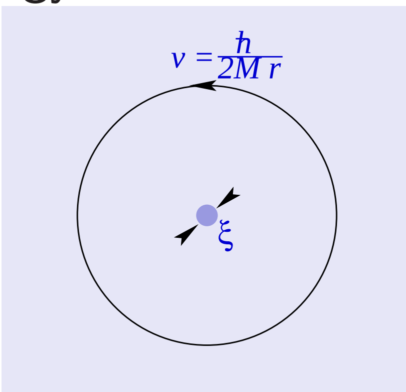
One-Component Polar Fermionic Molecules

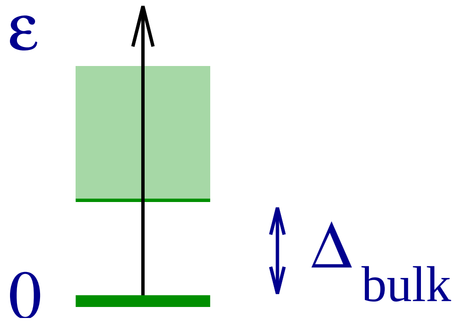
Attractive $1/r^3$ interactions $\Rightarrow$stable $p_x + ip_y$-superfluid phase

[NRC & Gora Shlyapnikov, PRL (2009)]

Topological phase with many interesting physical properties

- Zero-energy fermionic states on quantized vortices. [Volovik; Read & Green]





one "Majorana" fermion per vortex $\Leftrightarrow$ one fermion per pair of vortices

- Adiabatic braiding of vortices $\Rightarrow$"non-abelian" exchange statistics.

"Topologically protected QC" [Nayak, Simon, Stern, Freedman & Das Sarma, RMP (2008)]

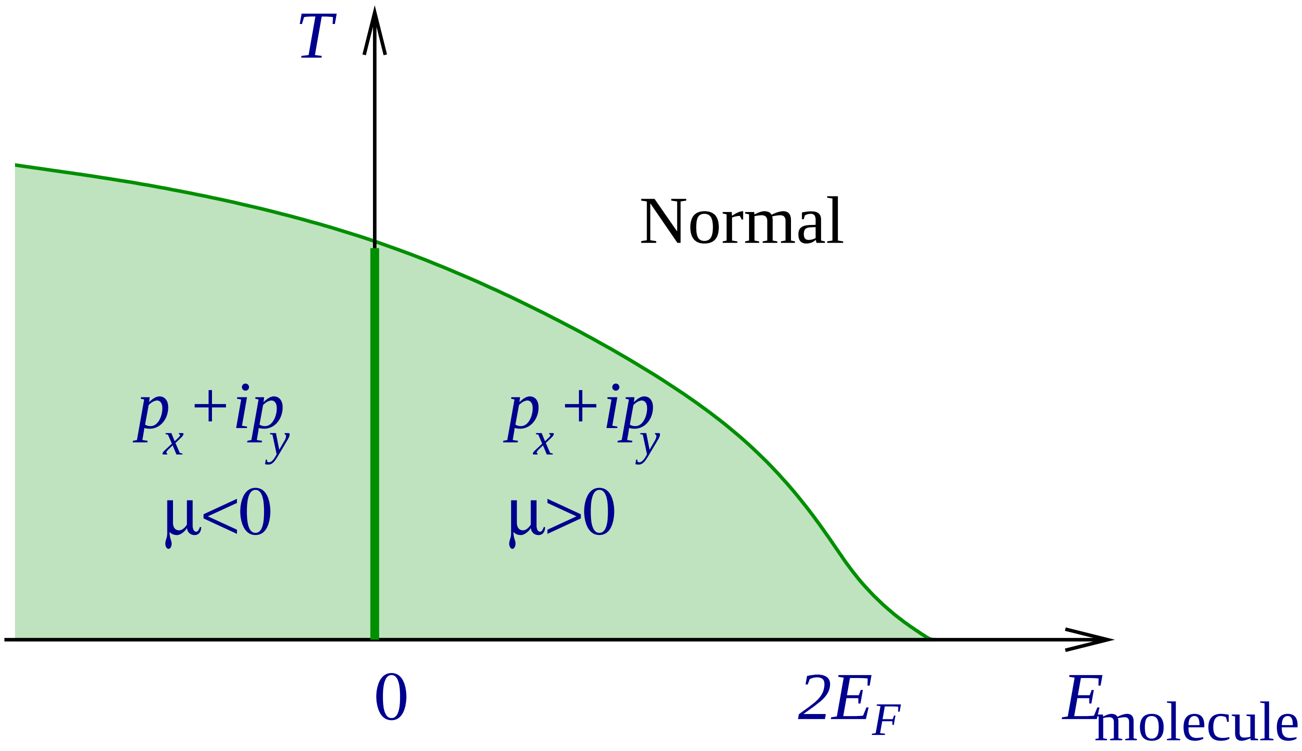
One-Component Atomic Fermi Gas

- Ultracold limit $k_F R_0 \ll 1 \Rightarrow p$-wave interactions very weak

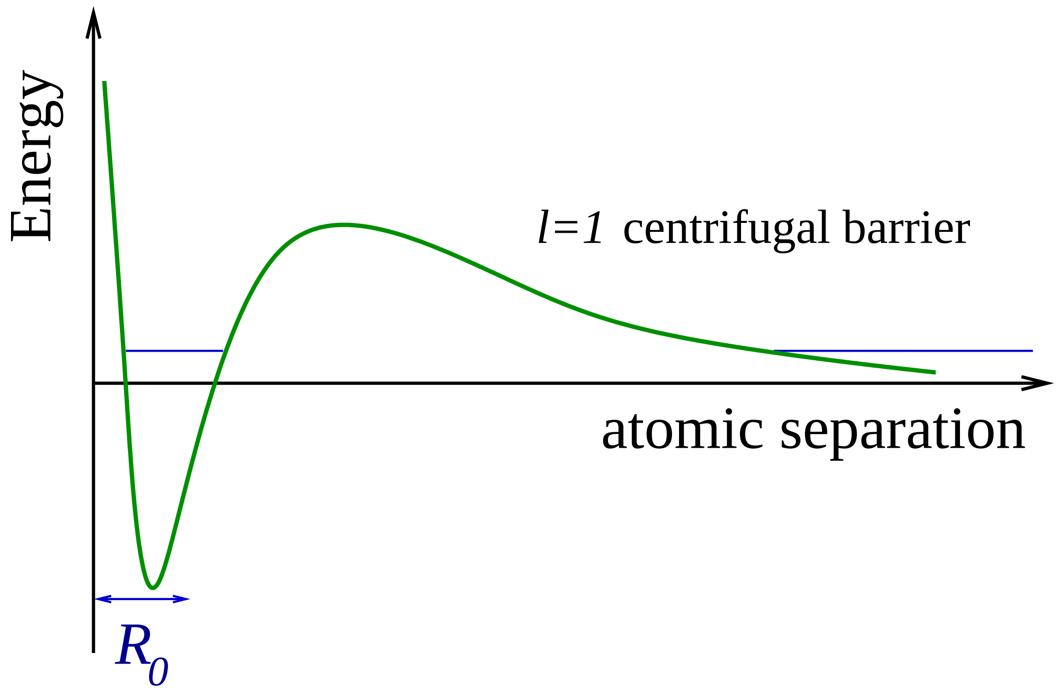
  BCS $T_c \sim E_F e^{-\#/(k_F R_0)^2}$ is vanishingly small.

- Feshbach resonance: formation of p-wave boundstate

  [Gurarie, Radzihovsky & Andreev, PRL (2005); Cheng & Yip, PRL (2005); Ho & Diener, PRL (2005); Botelho & Sa de Melo, J. Low Temp. Phys. (2005)]

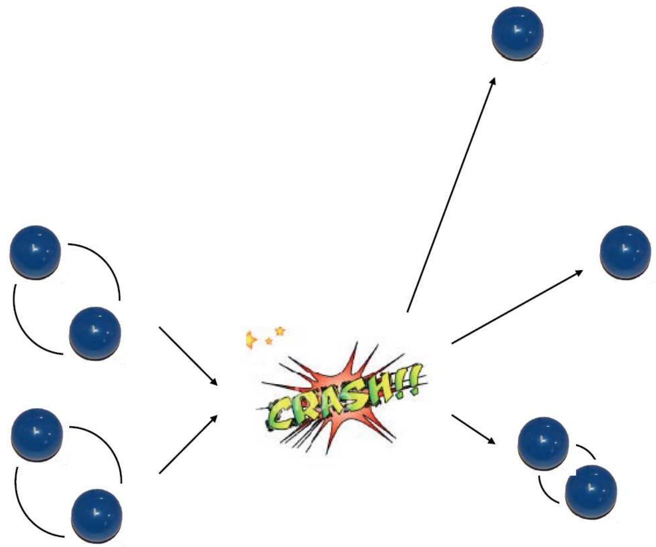


However... close to a p-wave Feshbach resonance



Dimer formation ⇒collisional relaxation
[Levinsen, NRC & Gurarie, PRL **99**, 210402 (2007);

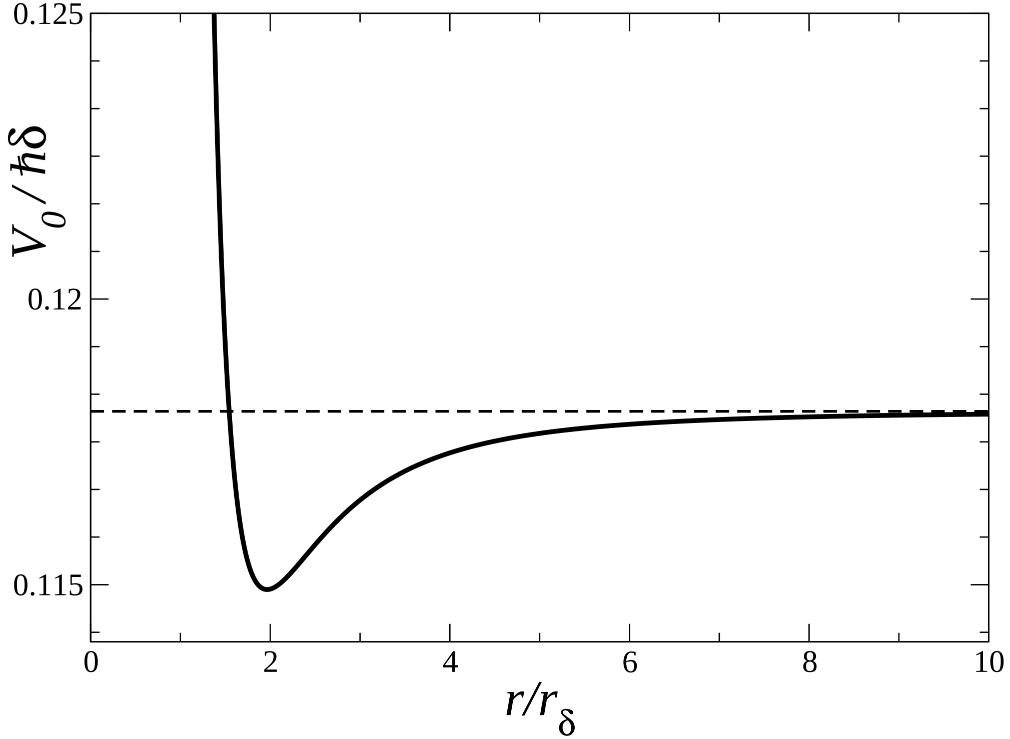
Jona-Lasinio, Pricoupenko & Castin, PRA **77**, 043611 (2008).]



$\sim$ ms lifetimes of ^{40}K and ^{6}Li dimers

[Gaebler *et al.* [JILA], PRL **98**, 200403 (2007); Inada *et al.* [Tokyo], PRL **101**, 100401 (2008)]

Dressed Polar Molecules



$$r^* \equiv \frac{M d_{\text{eff}}^2}{2\hbar^2}$$

Coupling constant $k_F r^*$ can be large without dimer formation

$\Rightarrow p$-wave superfluidity without collisional relaxation

Other ideas

Imbalanced two-component Fermi gas [A. Bulgac, M. M. Forbes, A. Schwenk, PRL (2006)]

Stabilization in a lattice via "quantum Zeno effect"
[Y.-J. Han, Y.-H. Chan, W. Yi, A. J. Daley, S. Diehl, P. Zoller & L.-M. Duan, PRL (2009)]

2D Fermi gas embedded in a 3D Fermi gas [Y. Nishida, Annals Phys (2008)]

2D Fermi-Bose mixture on a lattice [P. Massignan, A. Sanpera & M. Lewenstein, PRA (2010)]

Two-component atomic Fermi gas with "spin-orbit" interaction
[Zhang, Tewari, Lutchyn and Das Sarma, PRL (2008); Sato, Takahashi, Fujimoto, PRL (2009)]

Superfluid Transition

[NRC & Gora Shlyapnikov, PRL (2009)]

Vertex function ($E = 0$)

$$\Gamma(\boldsymbol{k}, \boldsymbol{q}) = V_0(\boldsymbol{k} - \boldsymbol{q}) - \int \frac{d^2\boldsymbol{q}'}{(2\pi)^2} \frac{\Gamma(\boldsymbol{k}, \boldsymbol{q}')V_0(\boldsymbol{q} - \boldsymbol{q}')}{E_{q'}}$$

$$E_q = \hbar^2 q^2 / 2M$$

BCS gap equation

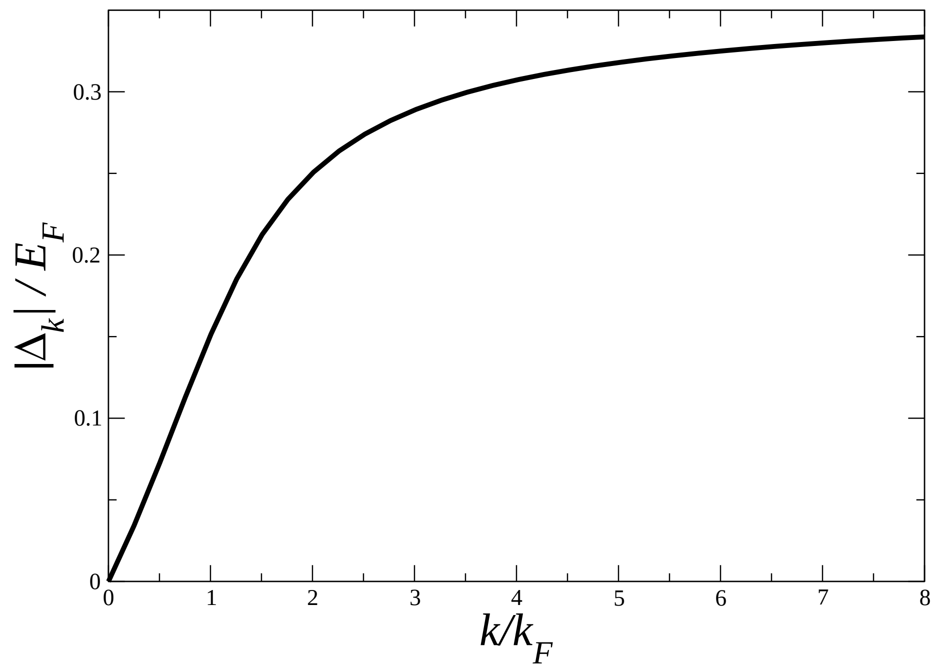
$$\Delta_{\boldsymbol{k}} = -\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \Gamma(\boldsymbol{k}, \boldsymbol{q}) \frac{\Delta_{\boldsymbol{q}}}{2} \left[\frac{\tanh(\epsilon_{\boldsymbol{q}}/2T)}{\epsilon_{\boldsymbol{q}}} - \frac{1}{E_q} \right]$$

$$\epsilon_{\boldsymbol{q}} = \sqrt{(E_q - \mu)^2 + |\Delta_{\boldsymbol{q}}|^2}$$

Born approx. $\Gamma(\boldsymbol{k},\boldsymbol{q}) \simeq V_0(\boldsymbol{k}-\boldsymbol{q})$

- $\underline{T \to 0}$

Most stable phase has $\Delta_{\boldsymbol{k}} \sim (k_x \pm i k_y)$



- BCS limit, $k_F r^* \lesssim 1$

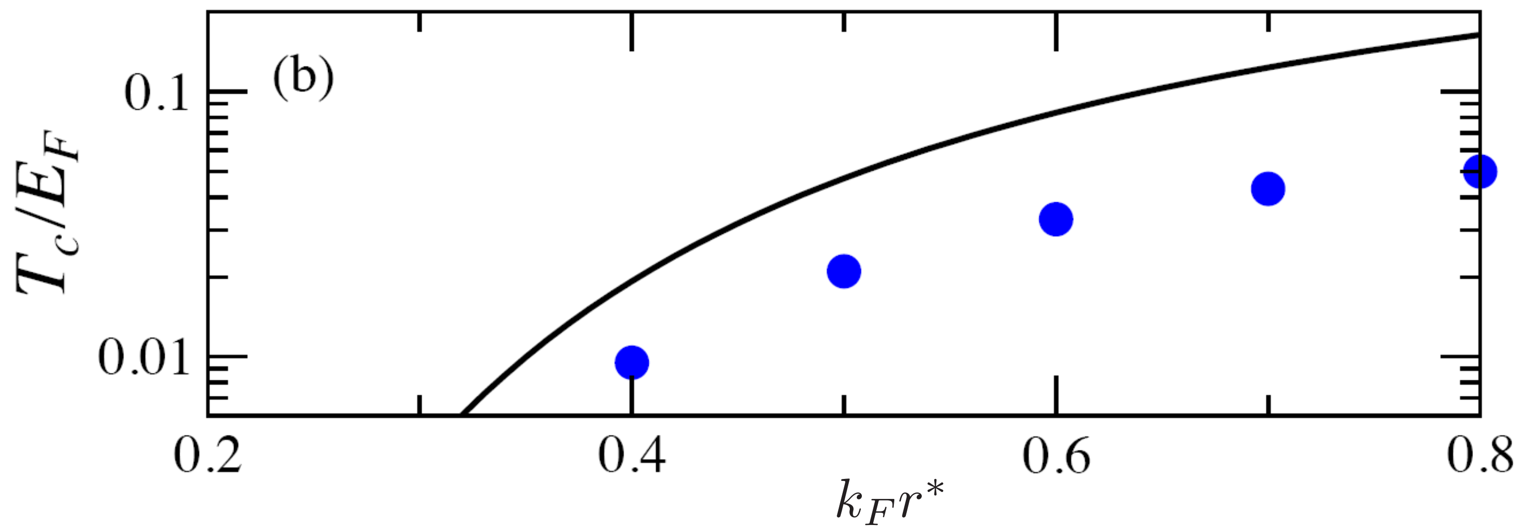
$$T_c \simeq E_F \; \exp\left(-\frac{3\pi}{4k_F r^*}\right)$$

cf. short-range potential, $T_c \simeq E_F \exp\left(-\frac{\#}{k_F^2 R_0^2}\right)$

$1/r^3 \Rightarrow$ "anomalous" scattering from $r \sim 1/k$

Full scattering amplitude + Gorkov Melik-Barkhudarov

[Levinsen, NRC & Shlyapnikov, arXiv:1103.3859]



$T_c \simeq 0.05 E_F$

^{6}Li^{133}Cs dipole moment $d = 5.5$ D

$r^* \simeq 100$ nm, $r_\delta \simeq 15$nm $\quad n = 10^9$cm^{-2}, $k_F r^* = 1$ $\Rightarrow$ $E_F = 200$nK, $T_c \simeq 10$nK

^{40}K^{87}Rb dipole moment $d = 0.566$D [Ni *et al.* [JILA], Science **322**, 231 (2008)]

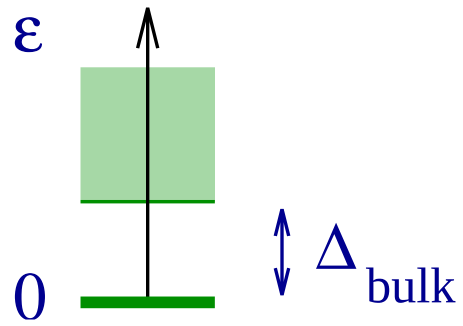
+ shallow optical lattice $m \to 10m^*$

$r^* = 100$nm, $r_\delta \simeq 20$nm$\Rightarrow k_F r^* = 1$ at $E_F \sim 30$nK ($T_c = 1$-2nK)

Experimental Consequences

Field-dressed polar molecules are in the topological $p_x + ip_y$ phase

Vortices carry zero energy
"Majorana" fermions. [Volovik; Read & Green]



- RF absorption [Grosfeld, NRC, Stern & Ilan, PRB **76**, 104516 (2007)]

- Non-abelian statistics [Tewari, Das Sarma, Nayak, Zhang & Zoller, PRL **98**, 010506 (2007)]

Overview

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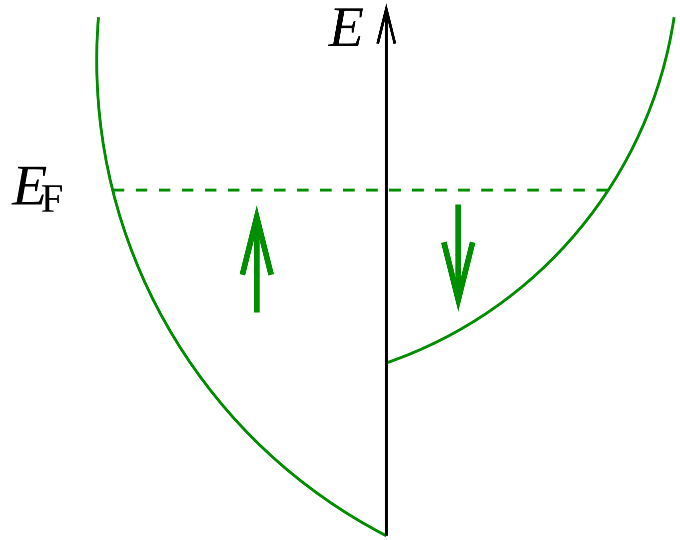
Two-component Polar Fermionic Molecules

Itinerant ferromagnetism?

[G.-B. Jo *et al.*, Science **325**, 1521 (2009)]



a strong coupling phenomenon

s-wave Feshbach resonance: $k_F a_s \gtrsim 1$

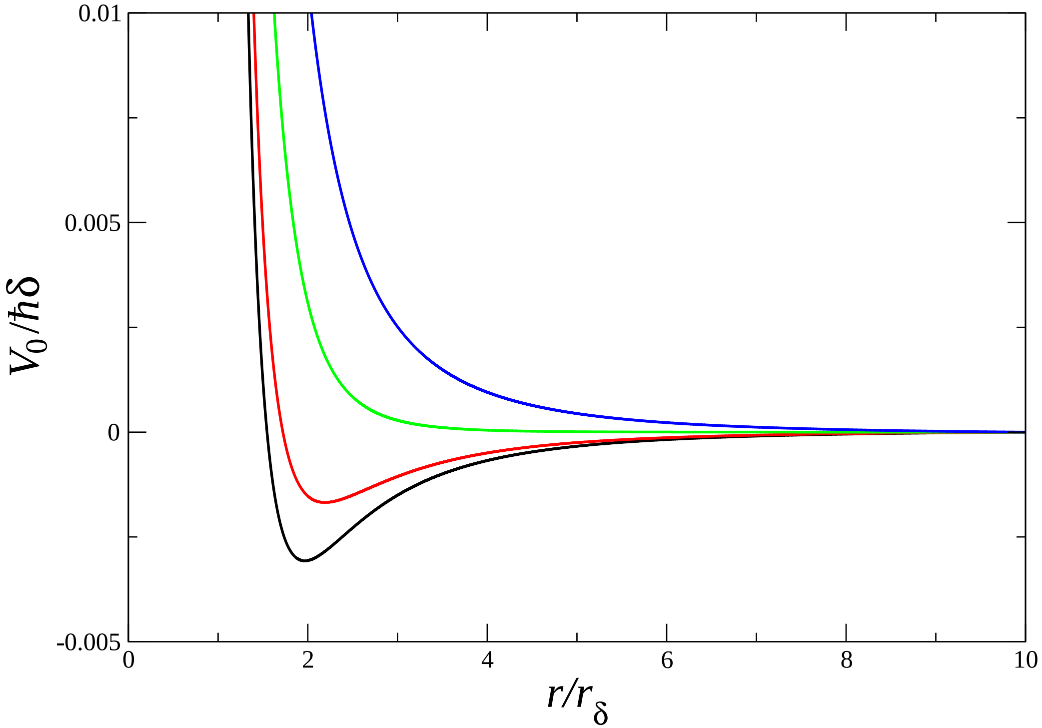
[Chang, Randeria & Trivedi, PNAS (2011); Pilati *et al.*, PRL (2011)]

⇒shallow molecular bound state, $E_{\mathrm{bind}} \sim E_{\mathrm{F}}$

competition of ferromagnetism and molecule formation

[Pekker *et al.*, PRL (2011); Talk of Christian Sanner]

MW dressed 2D polar molecules



Tune long-range potential by E_z

Strong repulsive potential
with no shallow bound states

Hard-core model $V_H(r) = \begin{cases} 0 & \text{if } r > r_0 \\ \infty & \text{otherwise} \end{cases}$

Dimensionless parameter: $D = k_F(2r_0)$

Quantum Monte Carlo Calculations

[N. Drummond, NRC, R. Needs & G. Shlyapnikov, PRB **83**, 195429 (2011)]

– Two-component fermions in 2D, spin-independent hard-core repulsion.

– Diffusion Monte Carlo, with fixed node approximation, $N \leq 50$.

– Slater-Jastrow-backflow trial wavefunctions

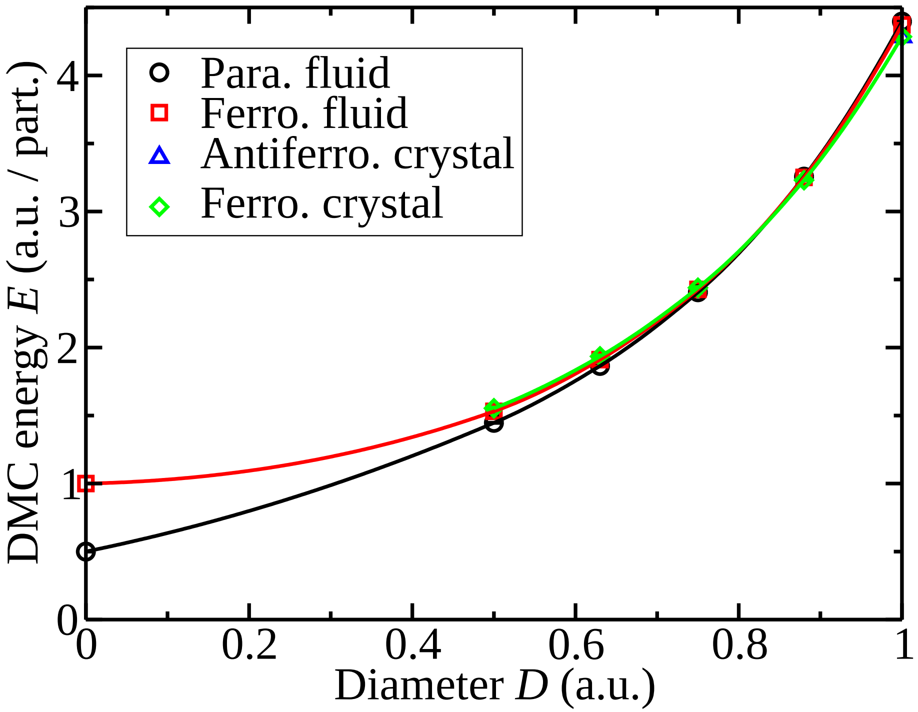
$$\Psi = JS^{\uparrow}S^{\downarrow}$$

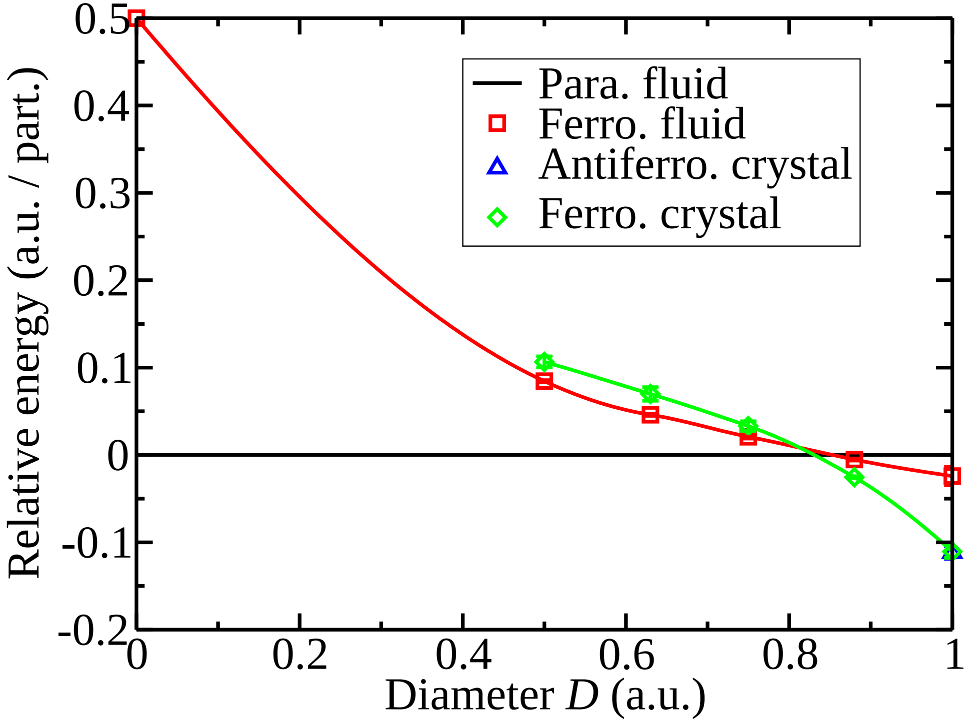
– Jastrow factor imposes hard-core constraint, includes three-body correlations.

– Compare energies of: paramagnetic fluid; ferromagnetic fluid;
anti-ferromagnetic and ferromagnetic crystals (triangular lattice).

QMC Results

[N. Drummond, NRC, R. Needs & G. Shlyapnikov, PRB **83**, 195429 (2011)]





- Exchange energy in crystalline phase negligible.
- Direct transition from paramagnetic fluid to crystal for $D \gtrsim 0.8$.
- Attractive $1/r^3$ interactions favour ferromagnetic fluid over paramagnetic fluid, but require separate study of strong-coupling superfluid phases.

Summary

- Field-dressed polar molecules in 2D have a useful interaction potential with which to investigate interesting many-body phases.

- For a one-component gas of fermionic molecules, attractive $1/r^3$ interactions lead to the topological $p_x + ip_y$ superfluid phase. Energy scales and inelastic lifetimes appear feasible.

- For a two-component gas of hard-core fermionic molecules, the transition to the ferromagnetic state is (just) preceded by crystallization.