# THERMODYNAMICS OF ULTRACOLD GASES

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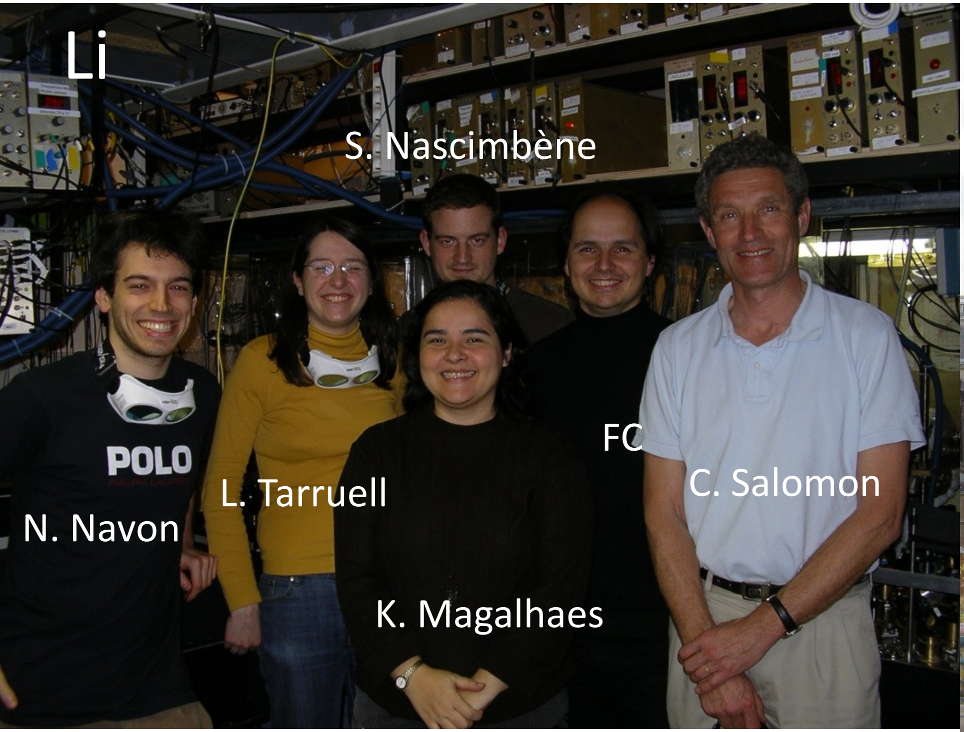
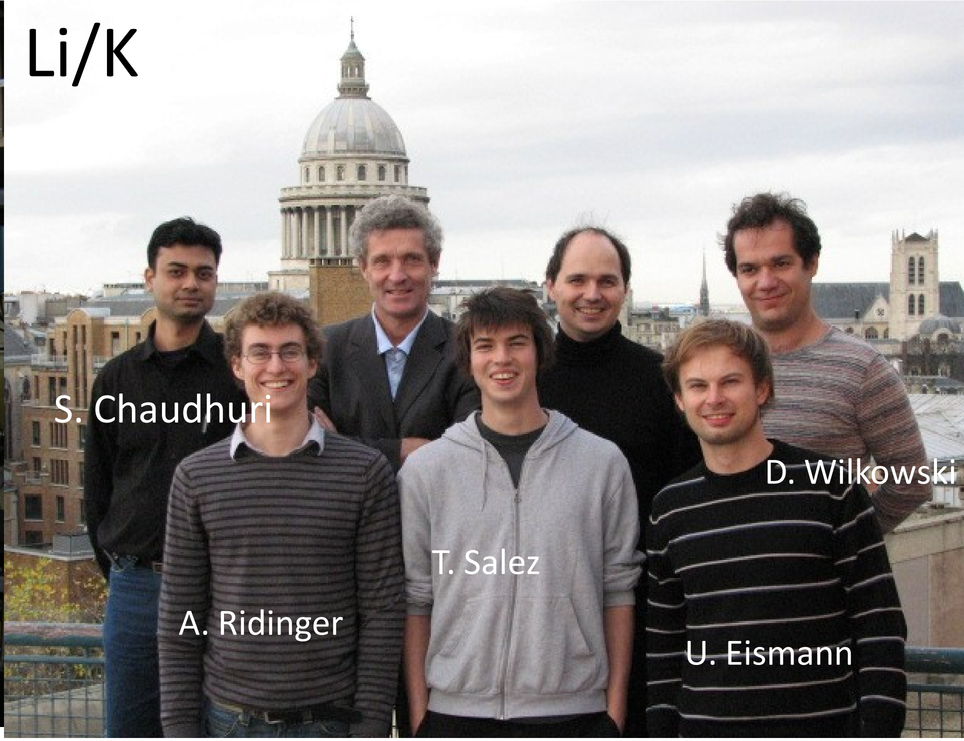
Seattle
May 2011

LETTRES SCIENCES
ECOLE NORMALE SUPERIEURE

Laboratoire Kastler Brossel
Physique quantique et applications

# ENS FERMION GROUPS



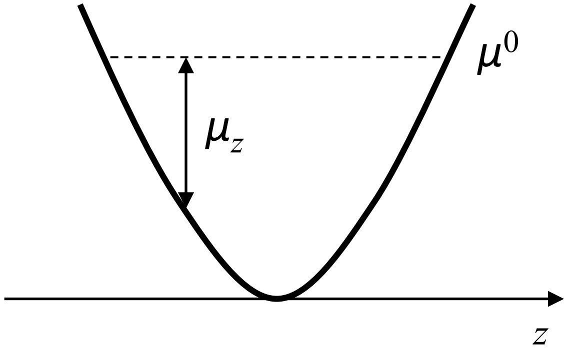


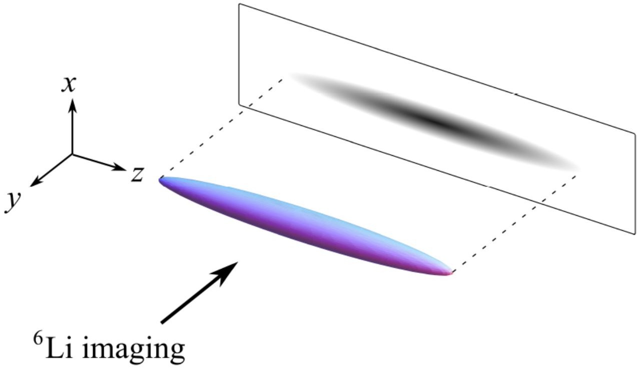
**NEW MEMBERS:** Li: B. Rem, I. Ferrier-Barbut, A. Grier: Li/K: F. Sievers, D. Rio Fernandes, N. Kretzschmar

**COLLABORATIONS:** Y. Castin, F. Werner, C. Mora, R. Combescot, X. Leyronas, W. Krauth, S. Piatecki, A. Georges, S. Giorgini, A. Recati, S. Stringari, C. Lobo, O. Goulko

# MEASUREMENT OF THE EQUATION OF STATE OF A FERMI GAS (T.L. Ho & Q. Zhou, Nature Physics **6**, 131 (2009), suggested by E.A. Mueller)

## LOCAL DENSITY APPROXIMATION



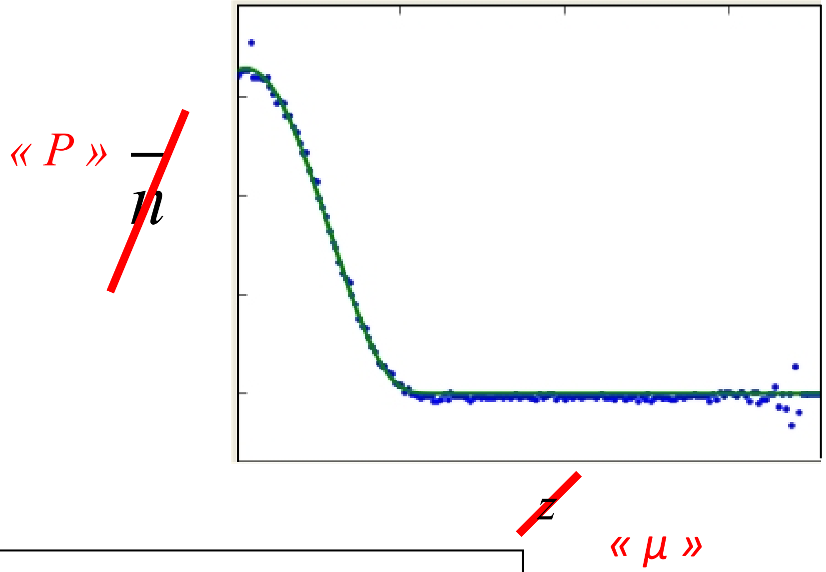


$$P(\mu_{\uparrow z},\mu_{\downarrow z},T)=\frac{m\omega_r^2}{2\pi}\left(\bar{n}_\uparrow(z)+\bar{n}_\downarrow(z)\right)$$

with

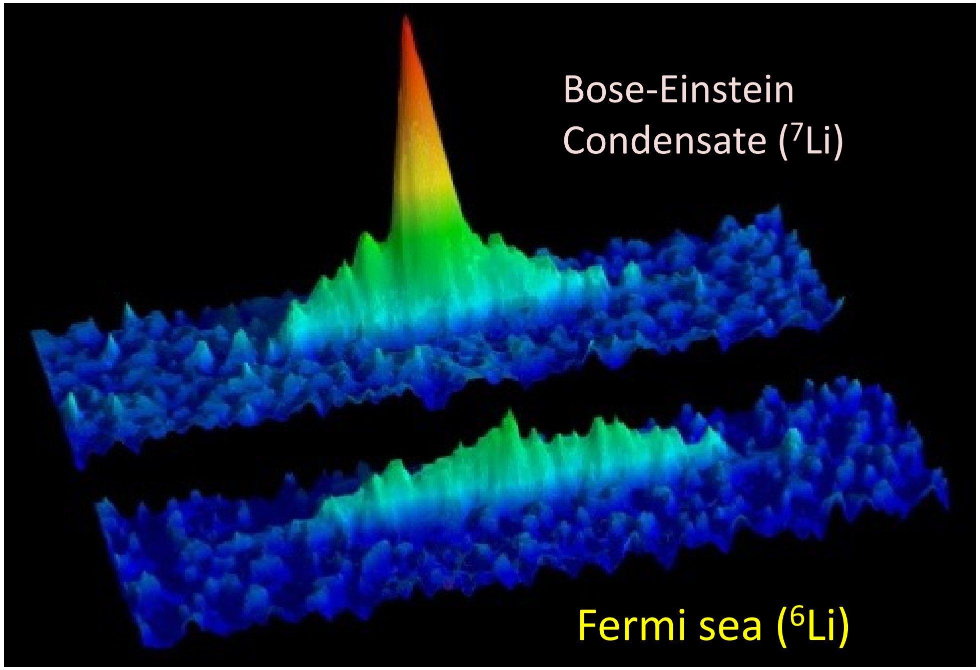
$$\bar{n}_i(z) = \int n_i(x,y,z)\mathrm{d}x\mathrm{d}y$$

$$\mu_{iz}=\mu_i^0-m\omega_z^2z^2/2$$



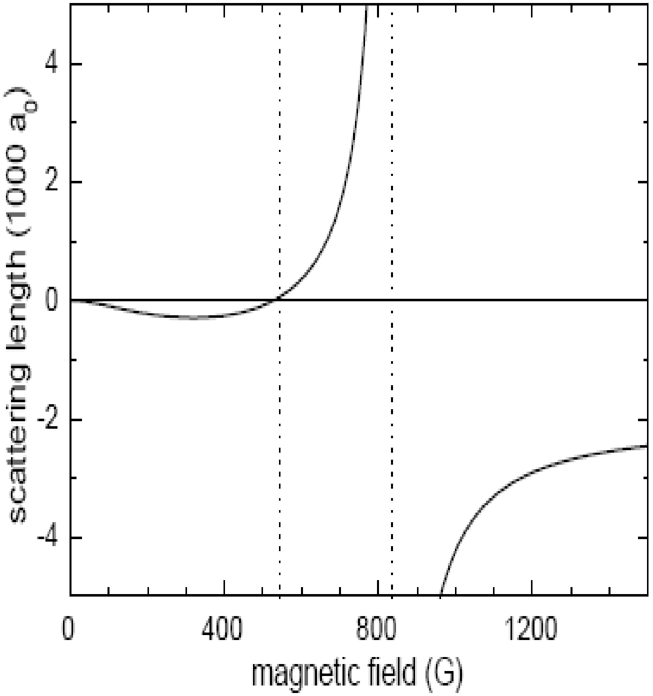
Each picture provides a realization of a *piece* of the equation of state.

# THE ENS LITHIUM EXPERIMENT



- Atom number ~ $10^4$-$10^5$
- Temperature ~100nK ($T/T_F \lesssim 0.05$)

$^6$Li Feshbach resonance



# Fermions

# THE GROUND STATE OF A HOMOGENEOUS FERMI GAS

Our goal : measure the EoS of the *homogeneous* Fermi gas

$$\begin{aligned}\Omega(T,V,\mu_\uparrow,\mu_\downarrow) &= E - TS - \mu N \\ &= -P(\mu_\uparrow,\mu_\downarrow,T)V\end{aligned}$$

Pressure contains all the thermodynamic information

In this talk : Low temperature limit : T = 0 Spin-population balanced gas

Universal function

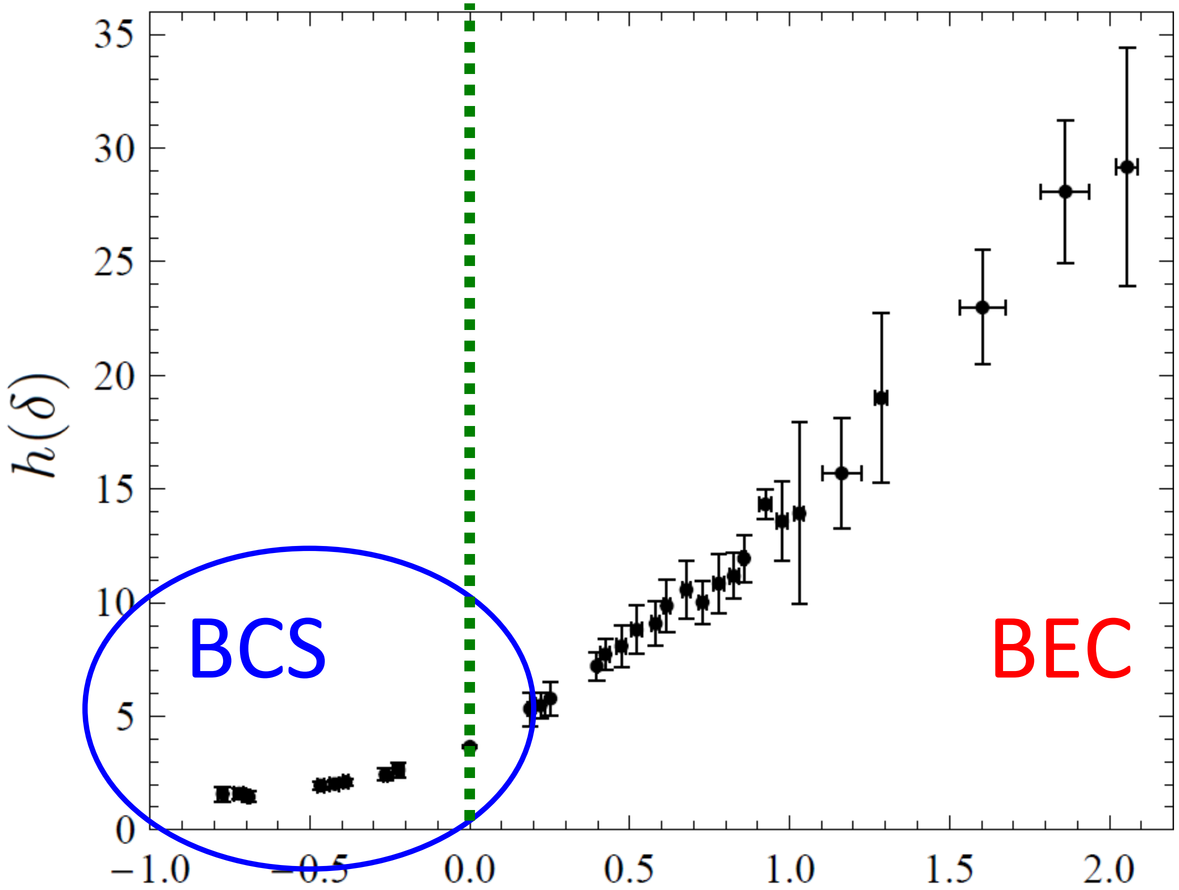
Normalized pressure : $P(\mu) = 2P_0(\mu)h(\delta)$

Fermi pressure of an ideal gas $P_0(\mu, T=0) = \frac{1}{15\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu^{5/2}$

Interaction parameter $\delta = \frac{\hbar}{\sqrt{2m\tilde{\mu}a}}$ with $\tilde{\mu} = \mu - \Theta(a)\frac{E_b}{2}$

grand-canonical analog to $\frac{1}{k_F a}$

# THE EOS OF A FERMIONIC SUPERFLUID WITH TUNABLE INTERACTIONS



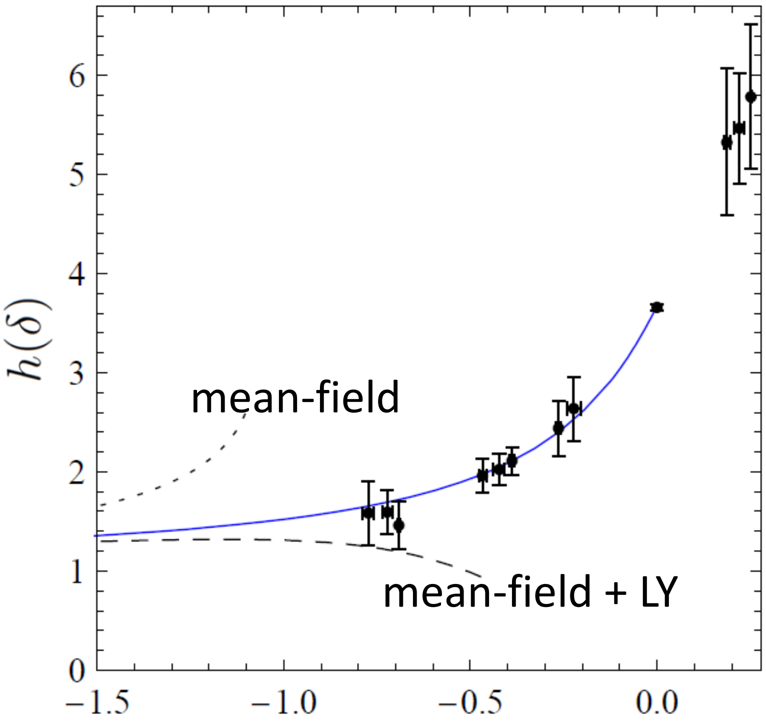
[redacted]

N. Navon, S. Nascimbène , F. Chevy, C. Salomon, *Science* 328, 729 (2010)

# ASYMPTOTICS I: BCS REGIME

$$E = \frac{3}{5} N E_F \left( 1 + \frac{10}{9\pi} k_F a + \frac{4(11 - 2\log 2)}{21\pi^2} (k_F a)^2 \ldots \right)$$

MF Lee-Yang



[redacted]

We find : 0.18(2)

LY calculation : $$\frac{4(11 - 2\log 2)}{21\pi^2} \simeq 0.186$$

T.D. Lee, C.N. Yang, Phys. Rev. 105, 1119 (1957)

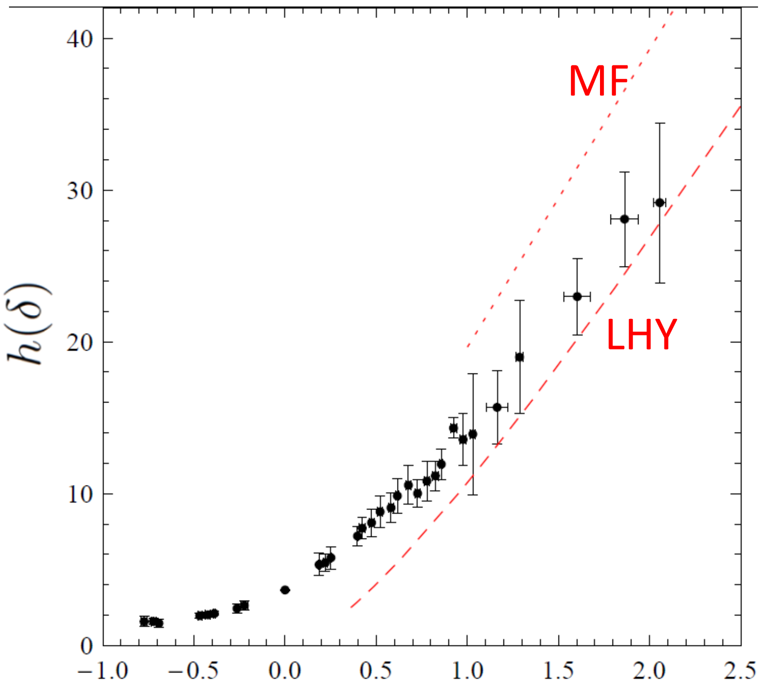
# ASYMPTOTICS II: BEC REGIME

$$E = \frac{N}{2}E_b + N\frac{\pi\hbar^2 a_{dd}}{2m}n\left(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na_{dd}^3} + \dots\right)$$

Binding Energy

MF

Lee-Huang-Yang



[redacted]

We find : 4.4(5)

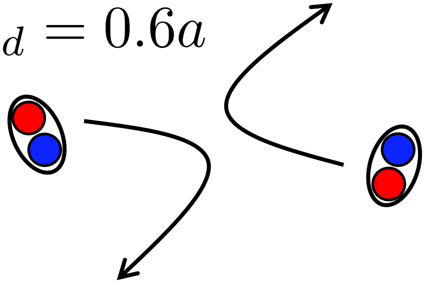
LHY calculation : $\frac{128}{15\sqrt{\pi}} \simeq 4.81$

T.D. Lee, K. Huang, C.N. Yang, Phys. Rev. 106, 1135 (1957)

No effect of composite nature of dimers (except $a_{dd}$)

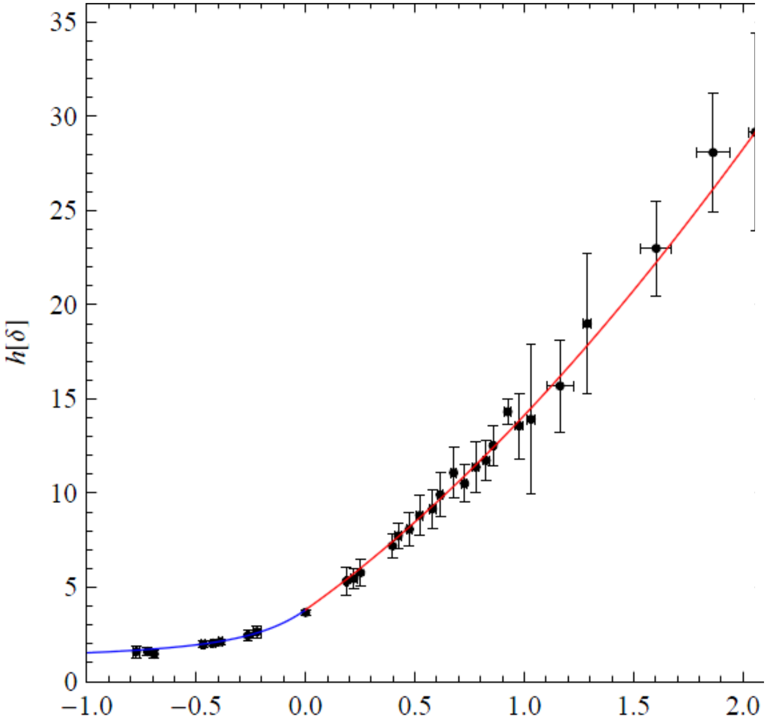
X. Leyronas *et al*, PRL (2007)

$$a_{dd} = 0.6a$$



# ASYMPTOTICS III: UNITARITY

$$E = \frac{3}{5} N E_F \left( \xi_s - \zeta \frac{1}{k_F a} + \ldots \right)$$



[redacted]

$\mu = \xi_s E_F$ Superfluid T=0 compressibility

$$\boxed{\xi_s = 0.41(1)(2)}$$

$C = \frac{2\zeta}{5\pi} k_F^4$ S. Tan's contact constant

[redacted]

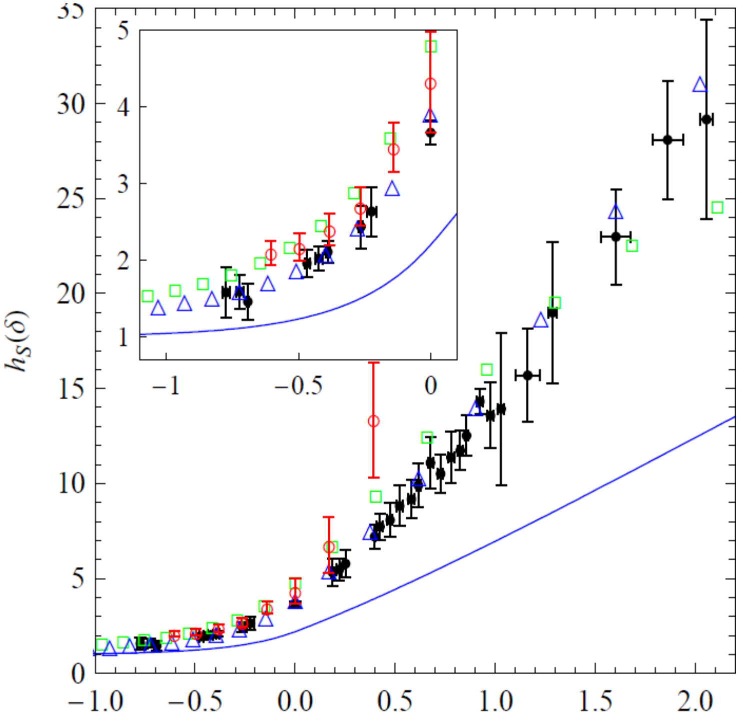
large-k momentum tail

short-range pair correlations

In agreement with Swinburne measurement (Dynamic structure factor) Kunhle *et al.*, PRL **105**, 070402 (2010) and JILA PRL **104**, 235301 (2010)

# DIRECT COMPARISON TO MANY-BODY THEORIES

$$P(\mu, a) = 2P_0(\mu)h(\delta)$$

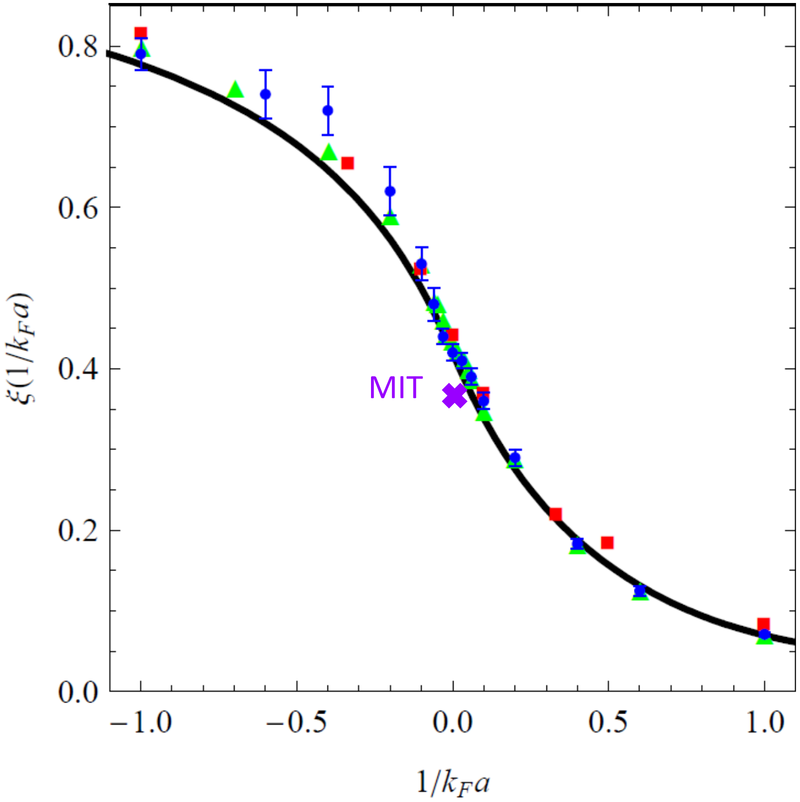


△ Nozières-Schmitt-Rink approximation
Hu *et al*, EPL 74, 574 (2006)

□ Diagrammatic theory
Haussmann *et al*, PRA 75, 23610 (2007)

○ Quantum Monte Carlo
Bulgac *et al*, PRA 78, 23625 (2008)

$$E = \frac{N}{2}E_b + \frac{3}{5}NE_F\xi\left(\frac{1}{k_F a}\right)$$

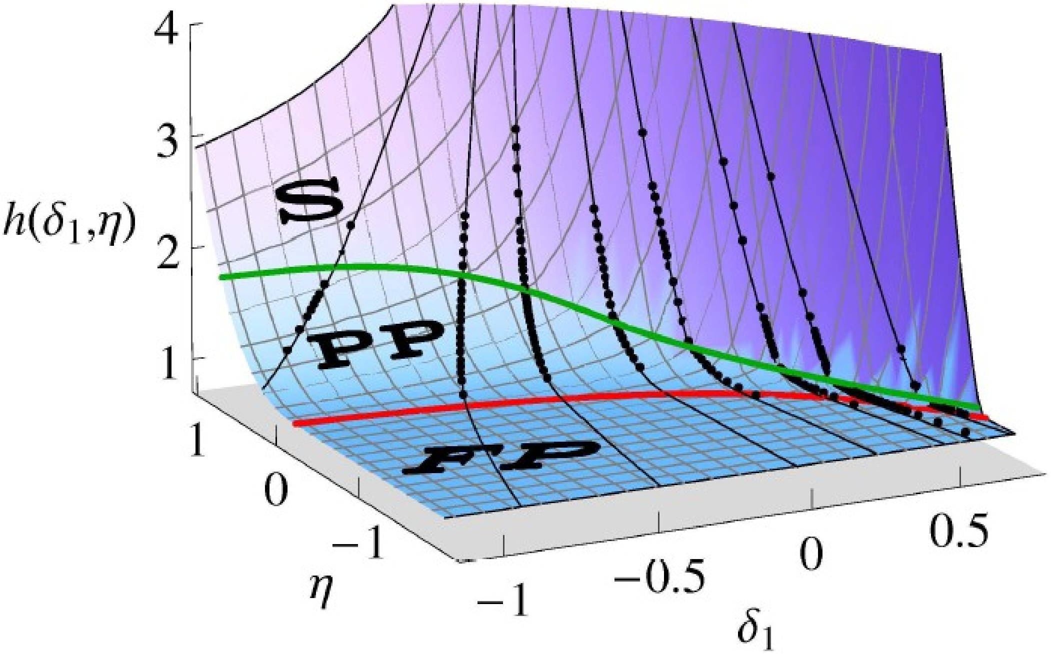


Fixed-Node Monte-Carlo theories

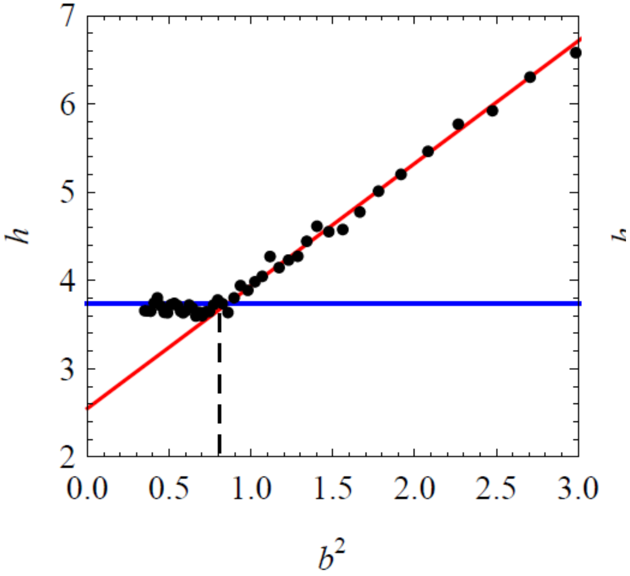
- 🟥 Chang *et al*, PRA 70, 43602 (2004)
- ● Astrakharchik *et al*, PRL 93, 200404 (2004)
- ▲ Pilati *et al*, PRL 100, 030401 (2008)

# EXTENSIONS

- Extension to T > 0  and spin imbalance
- Spin-susceptiblity of an imbalanced Fermi gas



N. Navon et. al., *Science* (2010)



S. Nascimbène et. al. arXiv:1012.4664 (2010)

# Bosons

# THE BOSE GAS WITH SHORT RANGE INTERACTIONS

A well studied quantum many-body system yet few experimental results

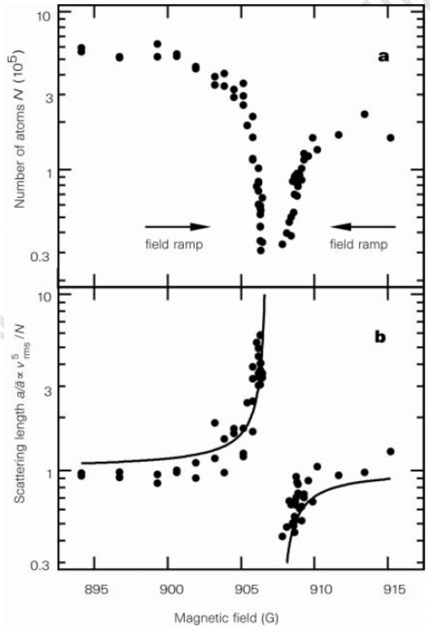
(e.g. Equation of State, Critical Temperature,…)

Notoriously difficult :

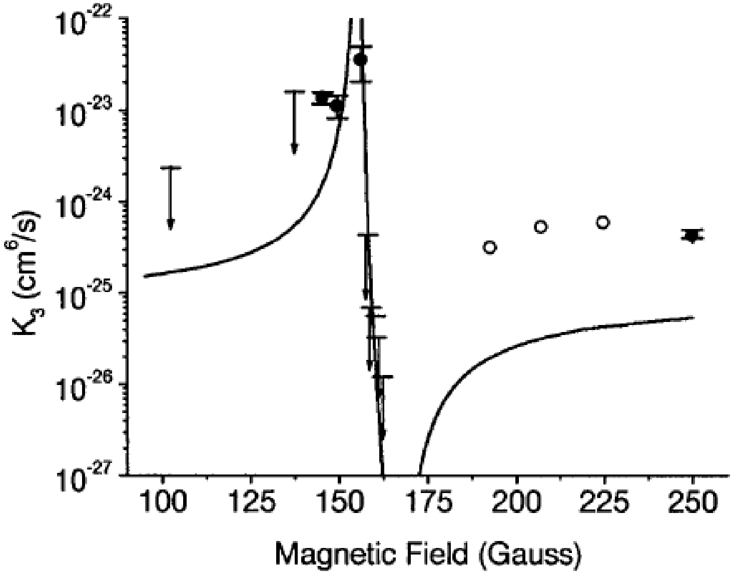
Strong increase of three-body losses with increasing *a*

Typ. $a^4$ *(Fedichev et. al., PRL 1996)*

→ Dramatic decrease of BEC lifetime



*Inouye et. al., Nature (1998)*

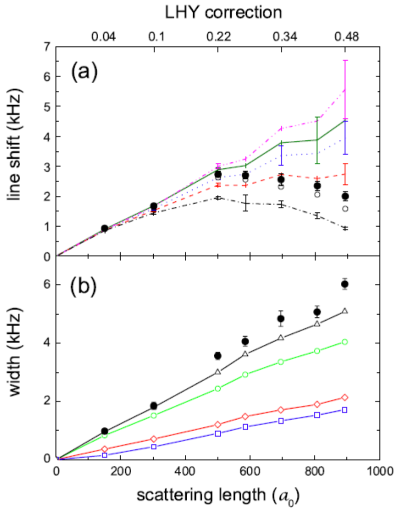


*Roberts et. al., PRL (2000)*

# Beyond Mean Field LHY corrections in atomic Bose gases

- Bragg spectroscopy of a $^{85}$Rb BEC

Complex sequence to reach strongly interacting regime difficult to model → no direct comparison to theory



*Papp et. al., PRL (2009)*

bMF effects sought in molecular BECs (mBEC)

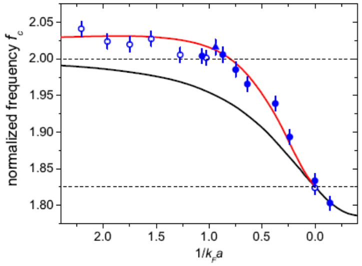
Opposite philosophy :

starting from a (stable) strongly interacting Fermi gas

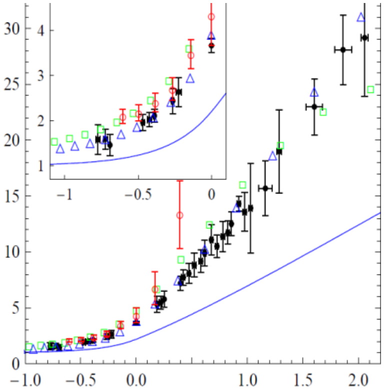
Forming molecules and weakening the interactions

- High precision measurement of collective mode frequencies

*Altmeyer et. al., PRL (2007)*



- First quantitative comparison with LHY using the EoS of a mBEC

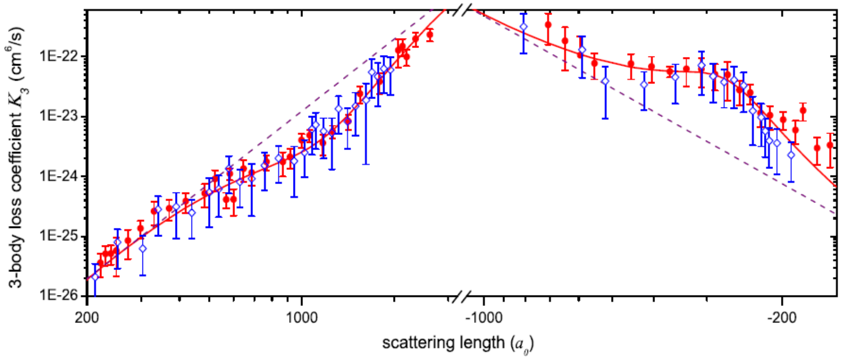


*NN et. al., Science (2010)*

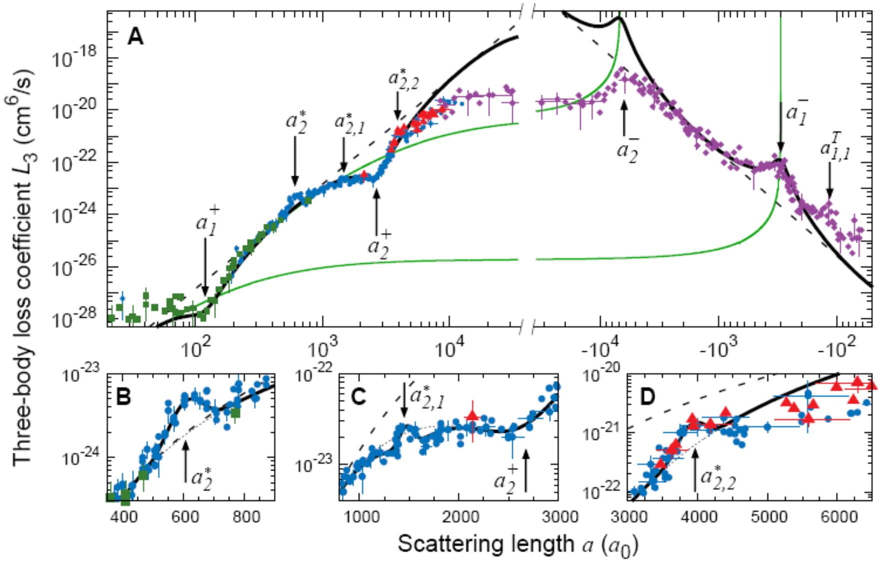
No quantitative measurements of bMF in atomic Bose gases yet !

# $^{7}$Li State of the Art

Use of the |1, $m_F$=0,1> state to probe Efimov physics



*Gross et. al., PRL 2009*



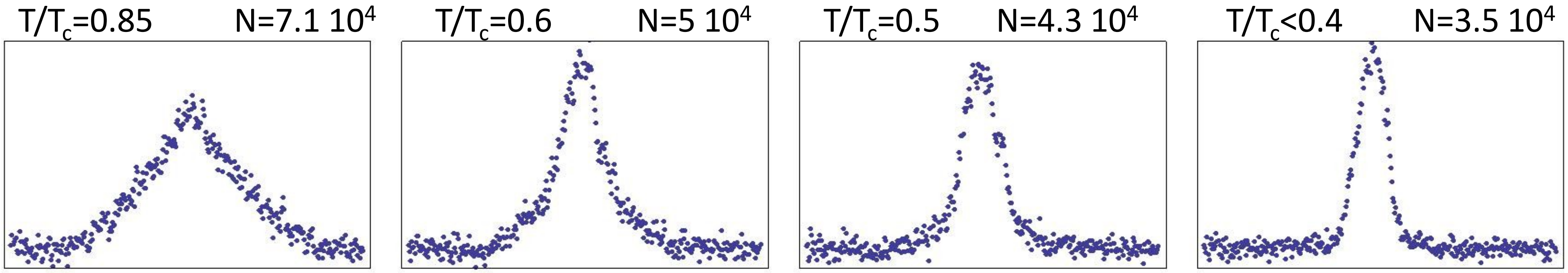
*Pollack et. al., Science 2010*

# BOSE-EINSTEIN CONDENSATION OF $^7$LI

- Possibility to tune the scattering length for evaporative cooling

  Trade-off : not too high (3-body losses), not too low (2-body collisions for rethermalization)

  Optimized value : around 200 $a_0$ in our trap



$T/T_c=0.85$ $N=7.1\ 10^4$

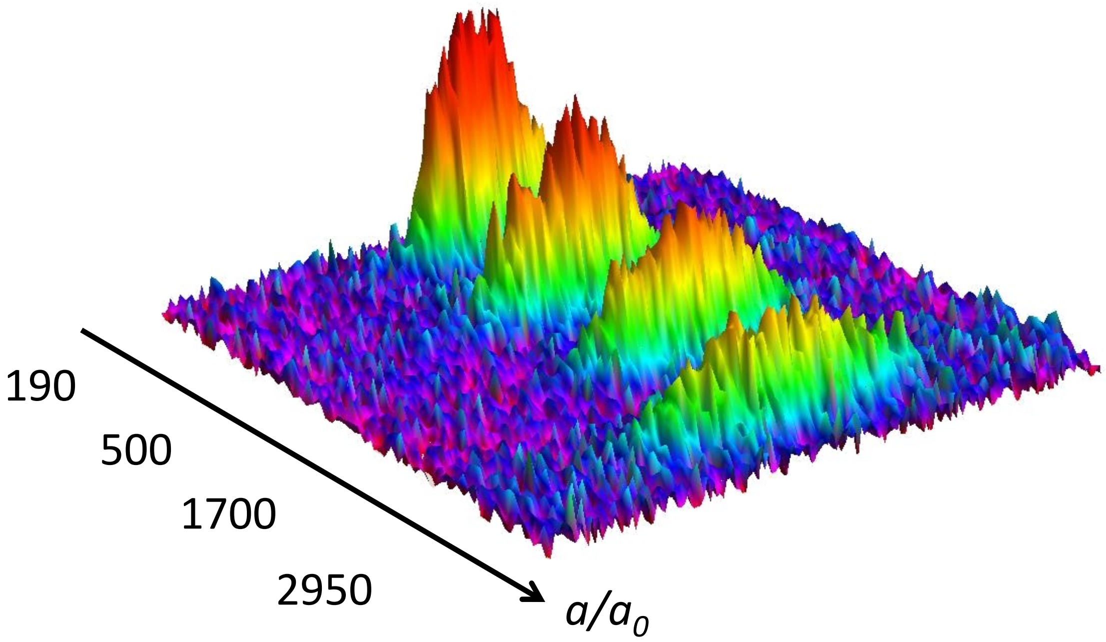
$T/T_c=0.6$ $N=5\ 10^4$

$T/T_c=0.5$ $N=4.3\ 10^4$

$T/T_c<0.4$ $N=3.5\ 10^4$

# ATOMIC BEC WITH TUNABLE INTERACTIONS

- Increasing the magnetic field towards the resonance (t~100 ms, $\omega_z$t~20)



N ~ 4 $10^4$

- For the Bose gas at T=0 :

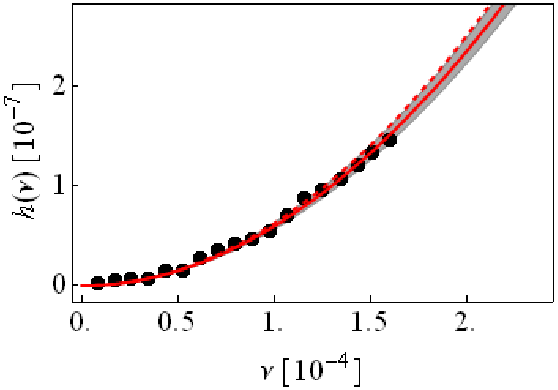
grand-canonical variable $\nu = \dfrac{\mu a^3}{g}$ with $g = \dfrac{4\pi\hbar^2 a}{m}$

Pressure unit $\dfrac{\hbar^2}{m a^5}$

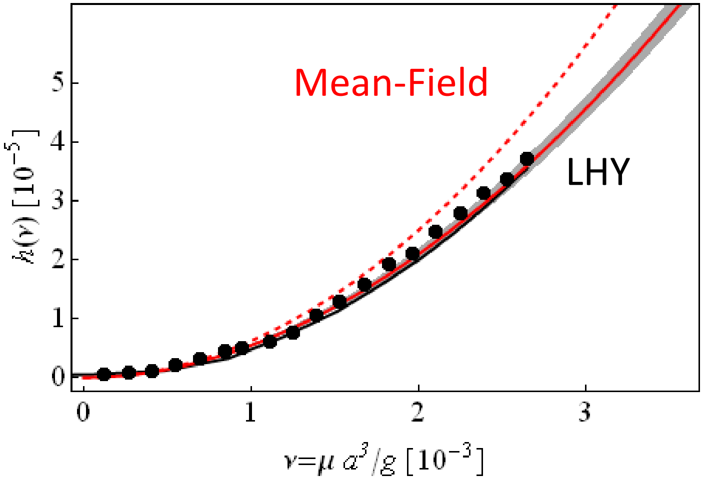
# EQUATION OF STATE OF A HOMOGENEOUS BEC

Calibration of detectivity in the MF regime (700 $a_0$)

Deviation of < 5 % to MF



Averaging images between 1440 and 2150 $a_0$



Navon et. al., arXiv:1103.4449 (2011)

$$E/N = \frac{gn}{2}\left(1 + \alpha\sqrt{na^3}\right)$$

We find : 4.5(7)

$$\frac{128}{15\sqrt{\pi}} \simeq 4.81$$

Together with molecular Bose gas measurement :
Demonstration of the universality of LHY correction

No evidence for *non-universal* effect at our level of precision

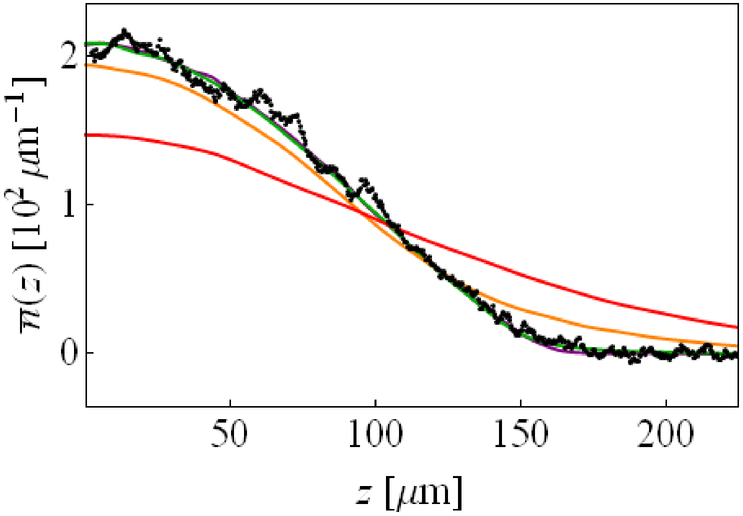
# THERMOMETRY: MONTE-CARLO SIMULATIONS

Question : T=0 assumption ?

Finite-temperature corrections beyond mean-field ? → EoS at T > 0…

Path-Integral Monte-Carlo simulations (S. Piatecki and W. Krauth at ENS)

39000 particles @ 2150 a0



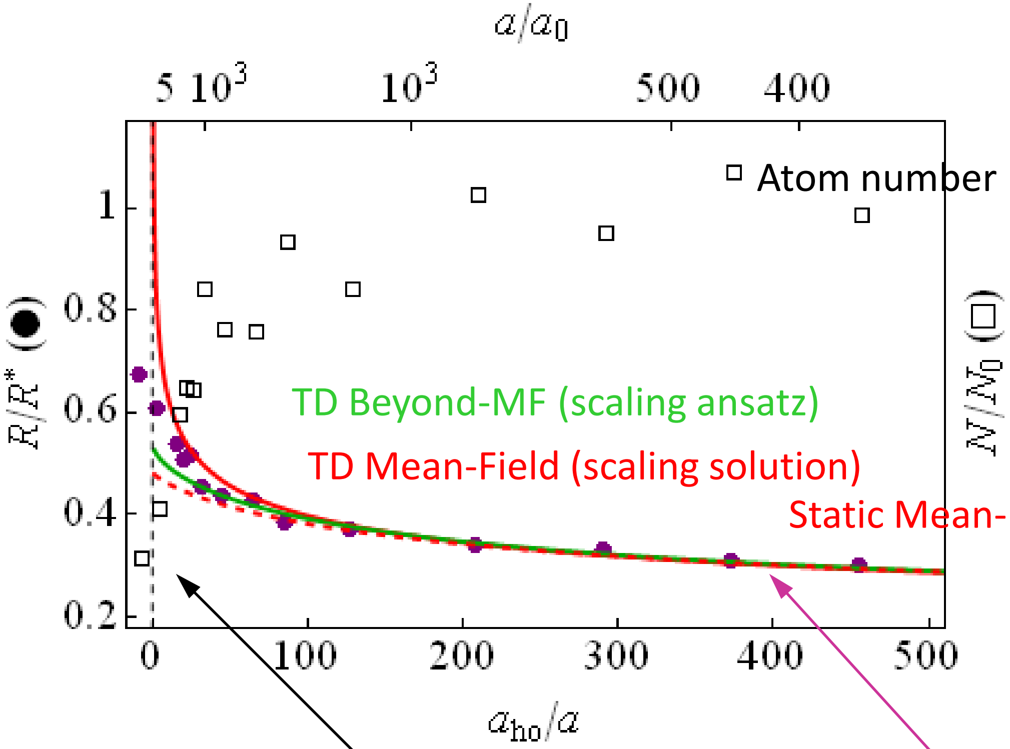
$T/T_c$ = 0.75, 0.5, 0.25, 0.125

We deduce that $T/T_c \leq 0.25$

No sizeable finite-T effects

# DYNAMICAL MEASUREMENT OF THE AXIAL RADIUS

- Pushing to higher values of *a* → faster sweep rates



TD Beyond-MF (scaling ansatz)

TD Mean-Field (scaling solution)

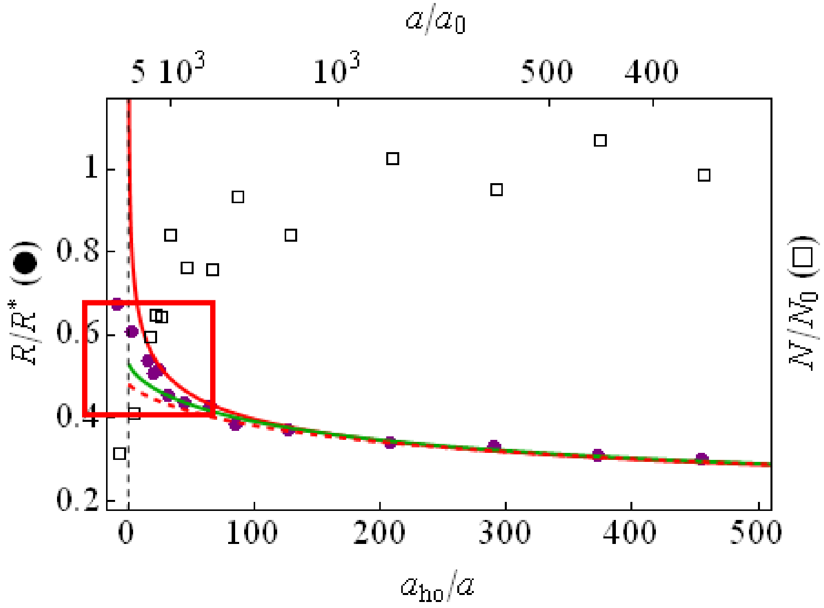
Static Mean-Field ($a^{1/5}$)

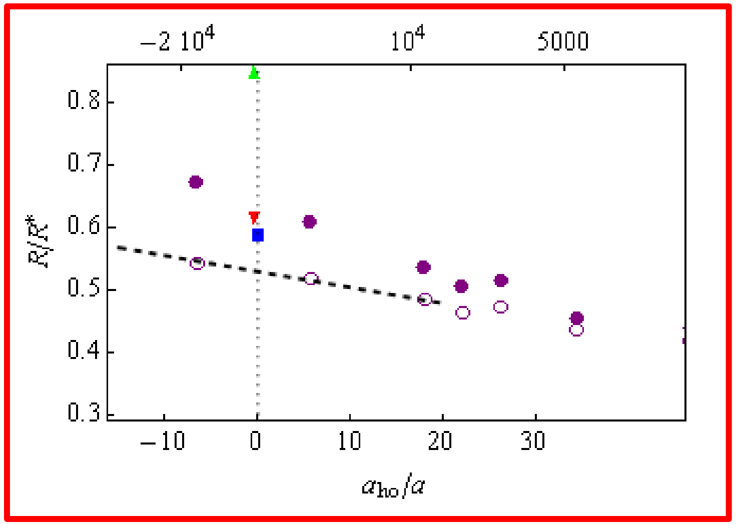
Radius normalized to $N^{1/5}$

Feshbach resonance

- Very good agreement with simple approach to beyond-MF dynamics (hydrodynamics+ scaling ansatz)

# A UNITARY BOSE GAS?





Due to response time of the Bose gas to the changing *a*(t) : $R_{exp}(t) < R_{eq}$

IF a universal state for the Bose gas at unitarity then : $\mu = \xi E_F$ like a Fermi gas !

We deduce from dynamic measurement a lower bound : $\xi > 0.54(8)$

« Variational » calculations : 2.92 (Cowell et al., PRL 2002), 0.80 (Song and Zhou, PRL 2009), and renormalization group method 0.66 (Lee and Lee PRA 2010)