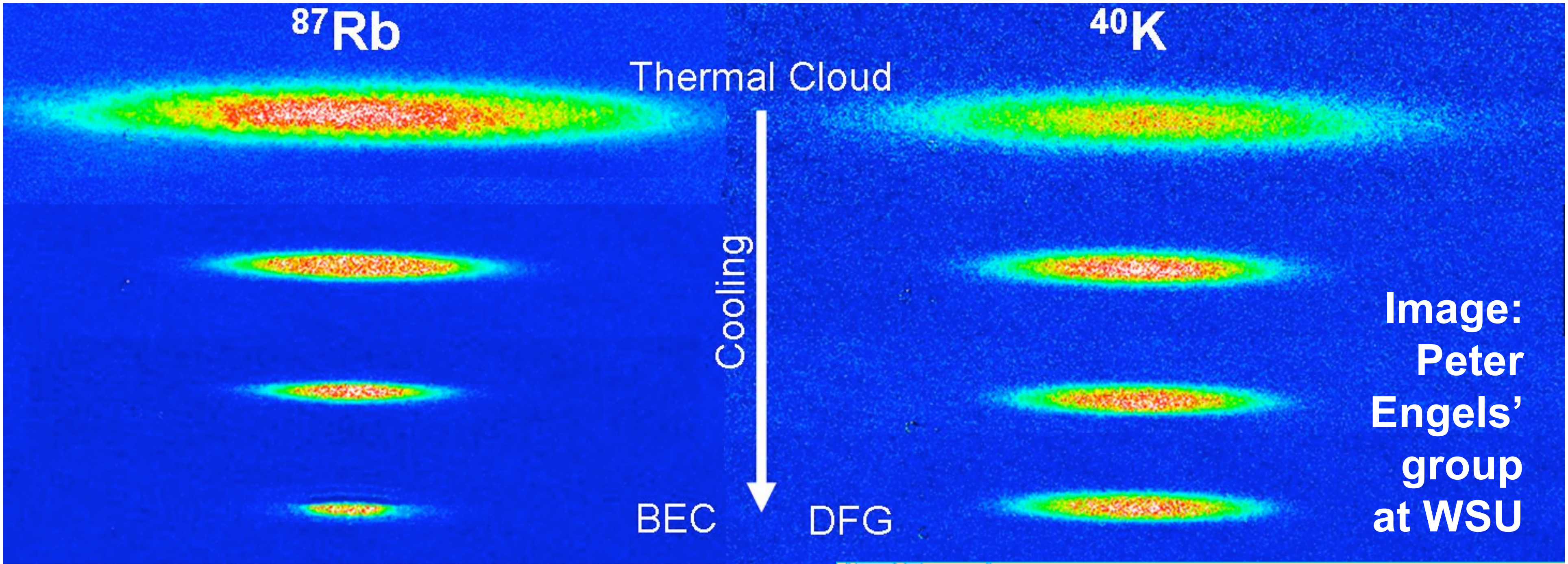


# (Biased) Theory Overview: Few-Body Physics


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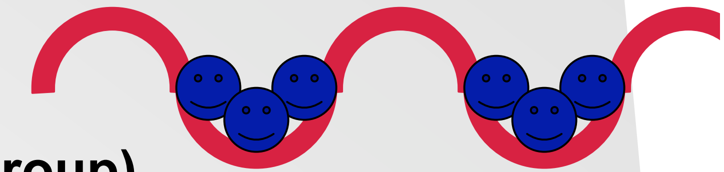
**Supported by NSF and ARO.**

# Few-Body Physics and Cold Atomic Gases

**Three experimental realizations:**

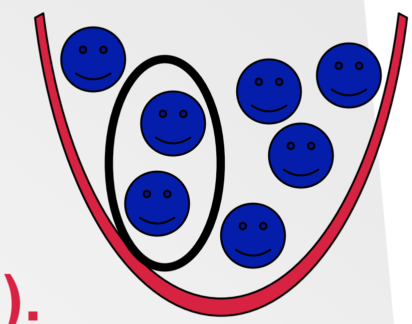
## Optical lattice: Few atoms per site.



- Repulsively bound pairs (Innsbruck).
- Effective many-body interactions (Bloch group).

## Macroscopic sample: Likelihood of finding 2, 3, 4,… particles close together.

- Losses from trap due to two-body, three-body and four-body processes.
- Three-body Efimov effect (next talk by Selim Jochim).
- Four-body Efimov physics.



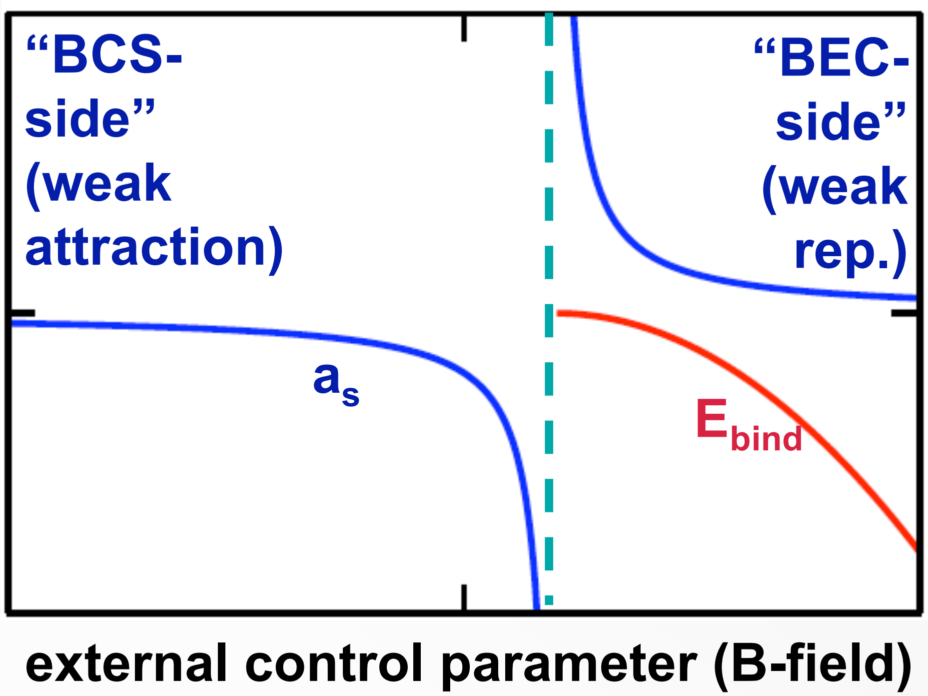
## Microtrap: Controllable number of atoms (2, 3,…).

- Next talk.

# Two-Body System with s-Wave Interactions

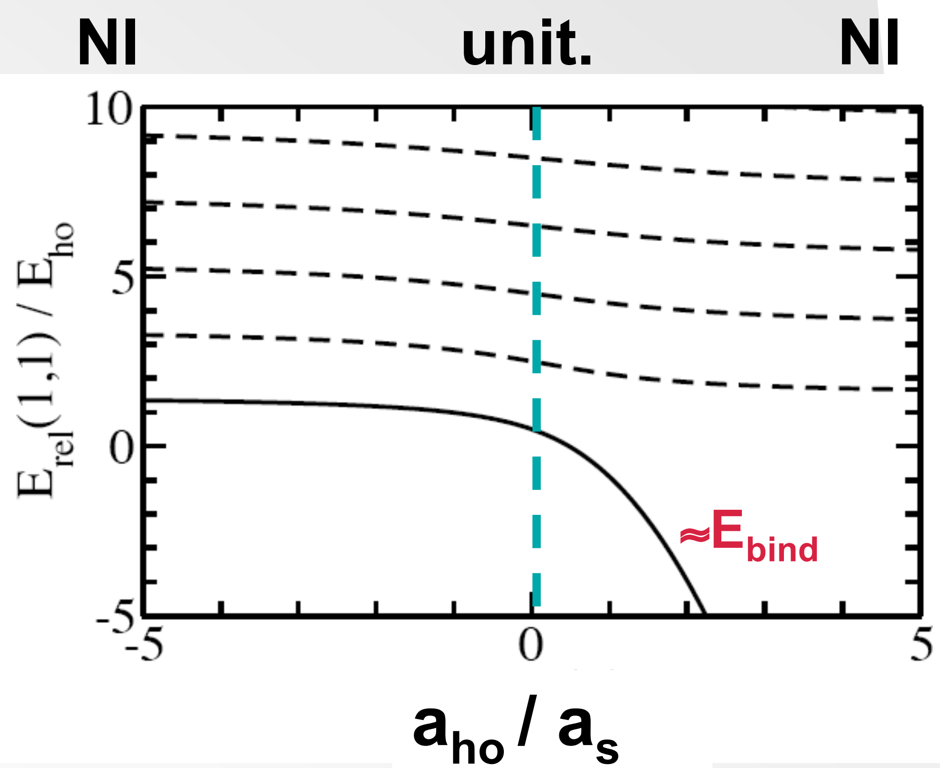
**Free space:**

No low-energy s-wave bound state for negative $a_s$.
One low-energy s-wave bound state for positive $a_s$.




**External spherically symmetric confinement** (Busch et al., Found. of Phys. (1998)):

Quantization of scattering continuum.



# Add a 3rd Particle in Free Space: What Happens Depends on Symmetry…

- FFF' (same isotope but two different internal hyperfine states):
  - No low-energy three-body bound state.

- FFX (X=boson or fermion (heavier or lighter than F)):
  - $a_s>0$ and $L^\Pi=1^-$: Universal low-energy bound state(s) for $8.172<\kappa<13.607$.
  - Large $|a_s|$ and $L^\Pi=1^-$: Three-body resonances can exist for $8.619<\kappa<13.607$.
  - Large $|a_s|$ and $L^\Pi=1^-$: Efimov effect for $\kappa>13.607$.

- FF'F'' (same isotope but three different internal hyperfine states):
  - All three $|a_s|$ large and $L^\Pi=0^+$: Efimov effect (next talk).

- BBB:
  - $|a_s|$ large and $L^\Pi=0^+$: Efimov effect.

References: Braaten and Hammer, Phys. Rep. 428, 259 (2006); Efimov, Yad. Fiz. 12, 1080 (1970); Nucl. Phys. A 210, 157 (1973). D'Incao and Esry, PRA 73, 030702 (2006) and follow-up work. Petrov, PRA 67, 010703(R) (2003). Nishida, Tan, and Son, PRL 100, 090405 (2008). Werner and Castin, PRA 74, 053604 (2006). Kartavtsev and Malykh, JPB 40, 1429 (2007)....

# Three-Body System with Infinitely Large Scattering Length

- Wave function separates (like that of the NI system) into hyperangular and hyperradial coordinates: $\Psi_{rel} = R^{-5/2} F(R)\, \Phi(\Omega)$.

- Eigenvalues of hyperangular Schrödinger equation provide effective potentials for hyperradial coordinate R.

- Two linearly independent hyperradial solutions:
  $f(R) \rightarrow R^{s+1/2}$ as $R \rightarrow 0$.
  $g(R) \rightarrow R^{-s+1/2}$ as $R \rightarrow 0$.

- **s>1:** eliminate g ($a_s$ describes everything).

- **0<s<1:** need f and g (ratio determined by three-body scattering parameter; three-body phase shift of $\pi/2$ corresponds to a divergent three-body "scattering length" (new bound state)).

- **s purely imaginary:** Efimov effect (discrete scale invariance; infinitely many geometrically spaced 3-body bound states).

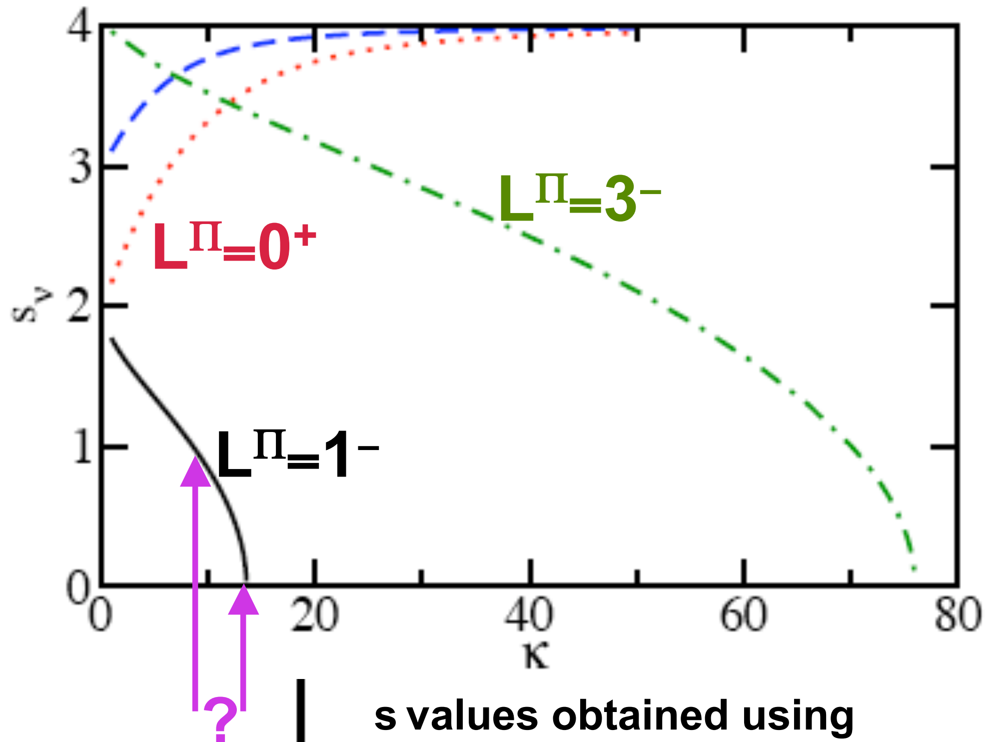
# Application to FFX System with Infinitely Large Scattering Length

**Experiments:**

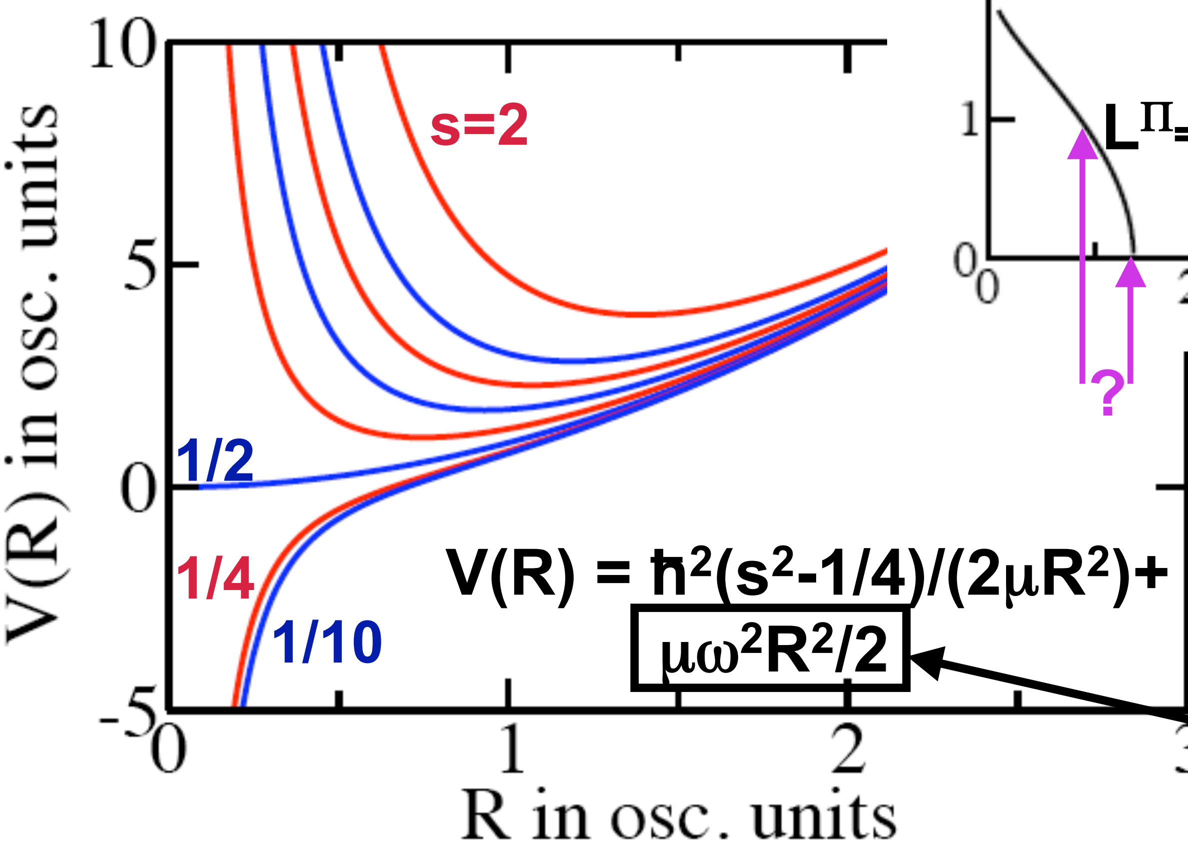
**$^{40}K$-$^{6}Li$: $\kappa \approx 6.7$**

**$^{173}Yb$-$^{6}Li$: $\kappa \approx 28.8$**

**$^{87}Sr$-$^{6}Li$: $\kappa \approx 14.5$**




**s values obtained using formalism developed by Rittenhouse and Greene, PRA 82, 022706 (2010); other approaches exist.**

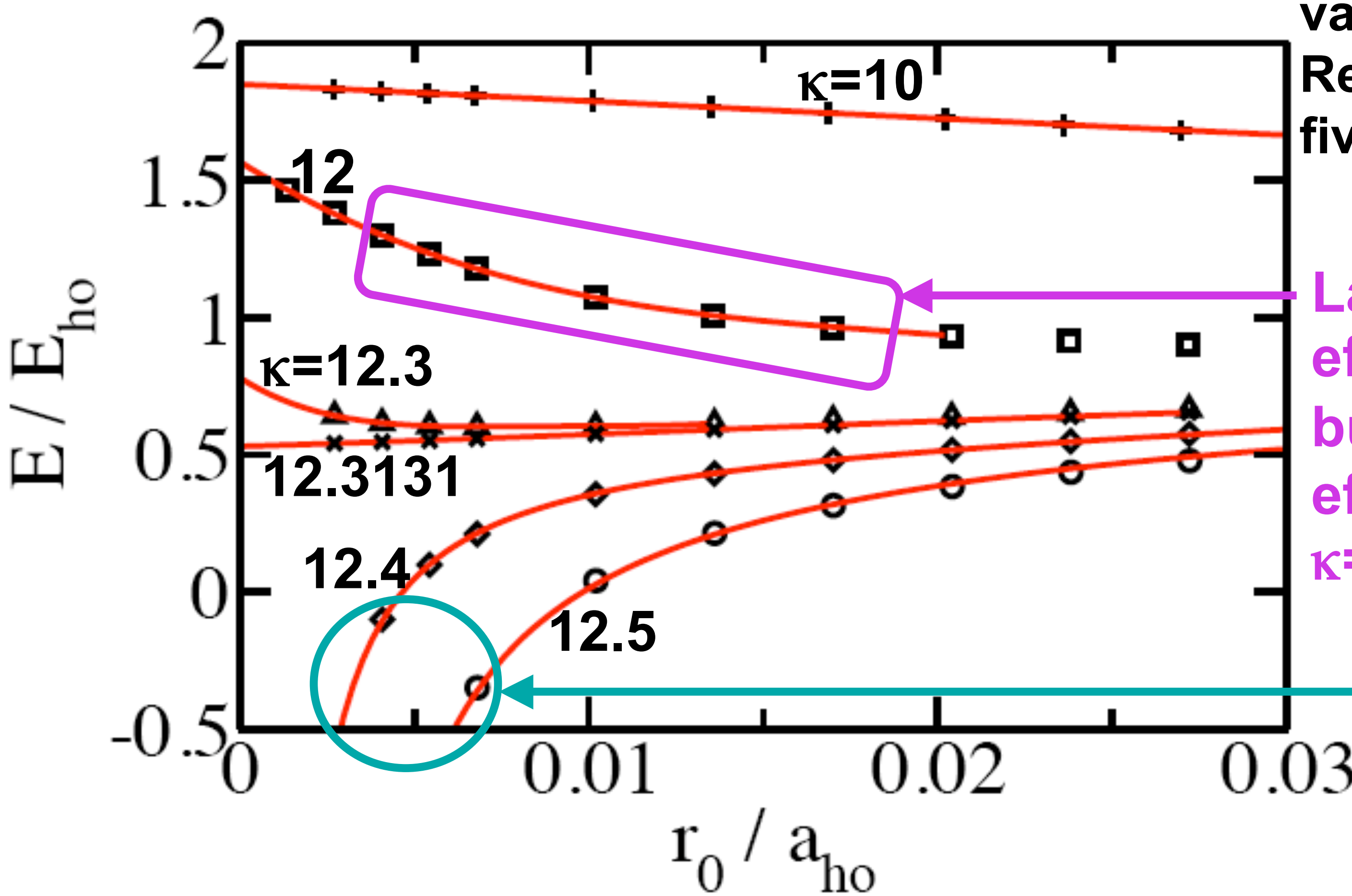



**$V(R) = \hbar^2(s^2-1/4)/(2\mu R^2)+$ $\mu\omega^2R^2/2$**

**Spherically symmetric harmonic trap**

# Trapped FFX System with $L^{\Pi}=1^{-}$ at Unitarity

Symbols: Stochastic variational energies. Red lines: Four to five parameter fits.

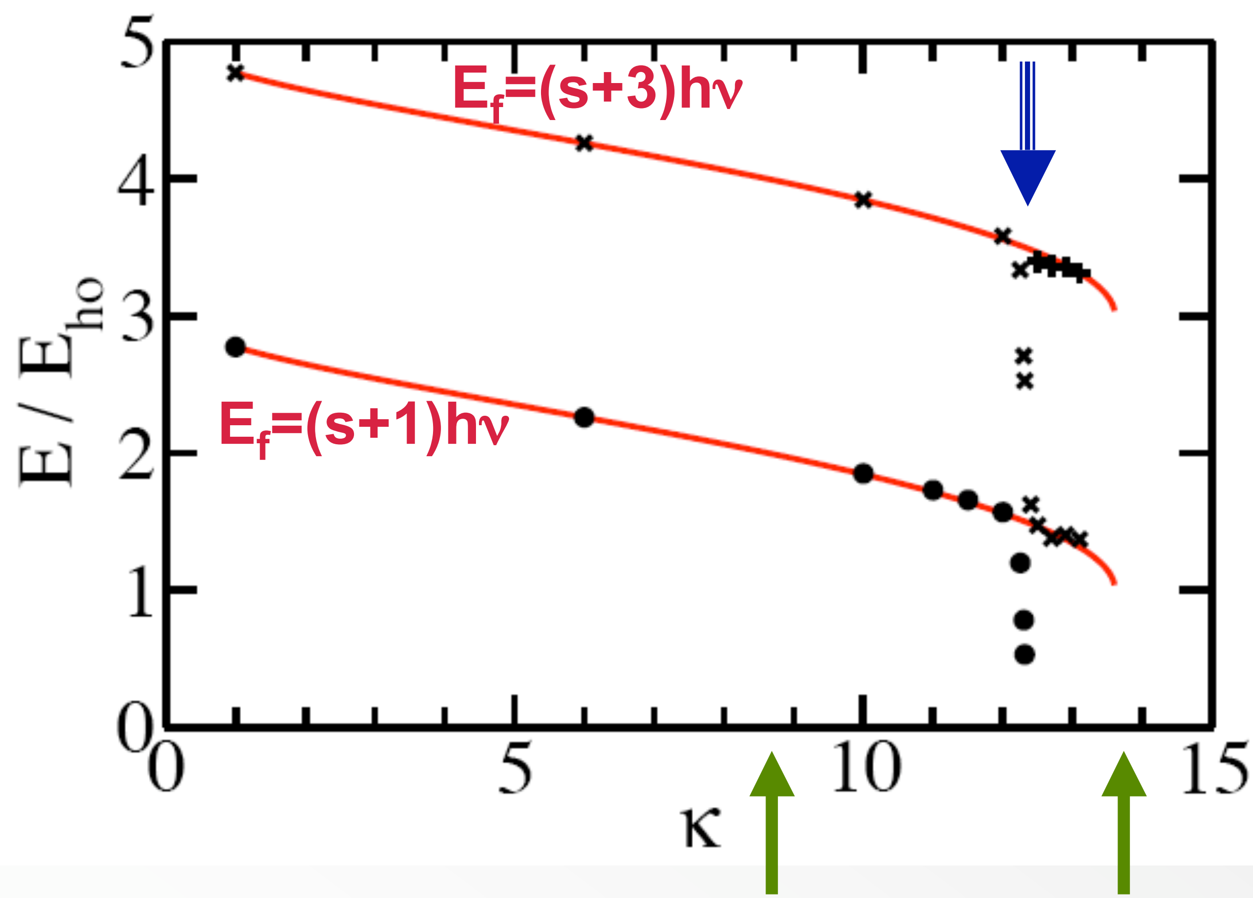



Large FR effects for κ=12; but small FR effects for κ=12.313 (s=1/2)!

Bound states! Diverge as $r_0^{-2}$.

Calculations employ a purely attractive Gaussian potential between FX pairs with range $r_0$. See Blume and Daily, PRL 105, 170403; PRA 82, 063612.

# Energies for Trapped FFX System with $L^{\Pi}=1^{-}$ at Unitarity




κ=8.619 κ=13.607

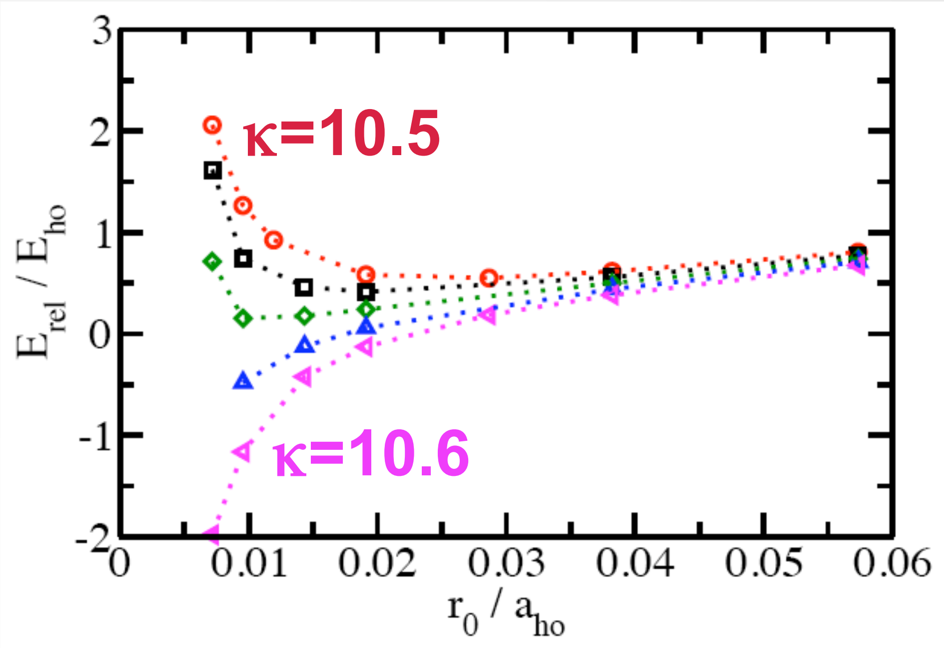
**Black symbols: Stochastic variational energies for finite-range Gaussian extrapolated to $r_0=0$.**

**Away from κ≈12.313 (or s≈1/2), extrapolated FR energies agree well with $E_f$.**

**Effective three-body interaction:**
**$F(R) \sim f(R) - \tan[\delta_{3b}(k)]g(R)$.**
**3-body resonance= phase shift of π/2.**

# Higher-Body Resonances?

**FFFX system with $L^{\Pi}=1^{+}$:**




**Four-body resonance!**

**Calculations are performed for two-body Gaussian model potential.**

**Work by Gandolfi and Carlson (arXiv:1006.5186): 3-, 4-, 5-body resonance but no (N>5)-resonance.**

**Implications?**

- **Collapse…**
- **If trimers or tetramers are stable, we have a system with competing two- and three- or two- and four-body interactions.**
- **Large finite-range effects.**

# Efimov Effect: $1/R^2$ Hyperradial Potential Curve with Imaginary s

- **FFX system with $\kappa>13.607$ (s determined by $\kappa$).**
  - $F(R)\rightarrow R^{1/2}\sin[\mathrm{Im}(s)\log(kR)+\theta]$; $\theta$ three-body phase.
  - $E^{n+1}/E^{n}=\exp[-2\pi/\mathrm{Im}(s)]$
  - Most naïve verification scheme requires observation of two features $\Rightarrow$ larger s is "better".

- **Other systems?**

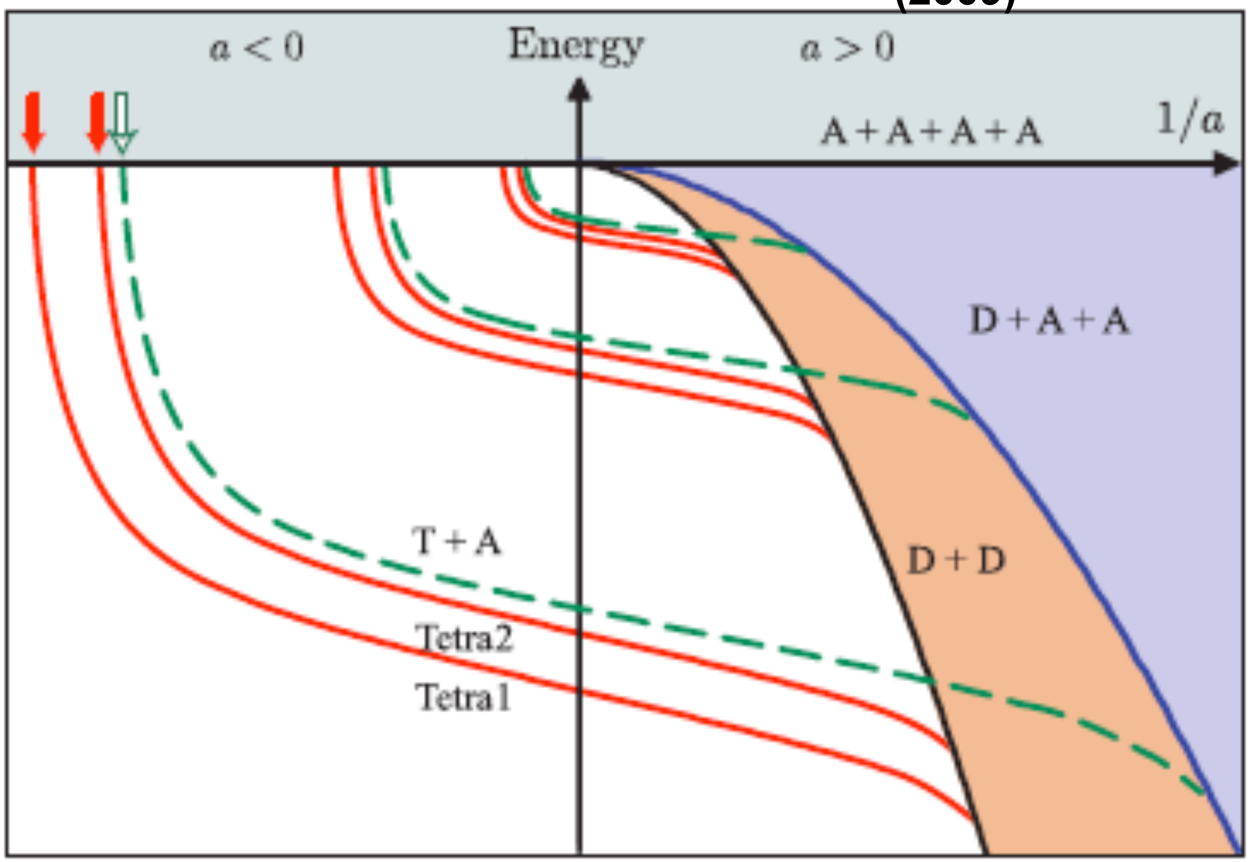
  - **Change symmetry:** FF'F'' and BBB: Im(s)=1.00624.
  - **Change interactions:** BBB system with dipole-dipole interactions (Wang, D'Incao and Greene, arXiv:1103.1406).
  - **Change number of particles:**
    - FFFX: Four-body Efimov effect for $13.384<\kappa<13.607$ (Castin, Mora and Pricoupenko, PRL 105, 223201 (2010)).
    - BBBB: Two four-body states "tagged on" to each Efimov trimer.

# Extended Efimov Scenario for Three- and Four-Boson Systems

**Theoretical prediction:**
**Two Tetra bound states**
**[Platter et al., PRA 70, 052101 (2004); Hammer et al., Eur. Phys. J. A 32, 113 (2007)]**

**“usual” (requires large $a_s$ range)**

**Figure from Ferlaino et al., PRL 102, 140401 (2009)**




**$a_{Tetra1} \approx 0.43 a_{Trimer}$**
**$a_{Tetra2} \approx 0.9 a_{Trimer}$**
**[von Stecher, D’Incao, Greene, Nature Physics 5, 417 (2009)]**

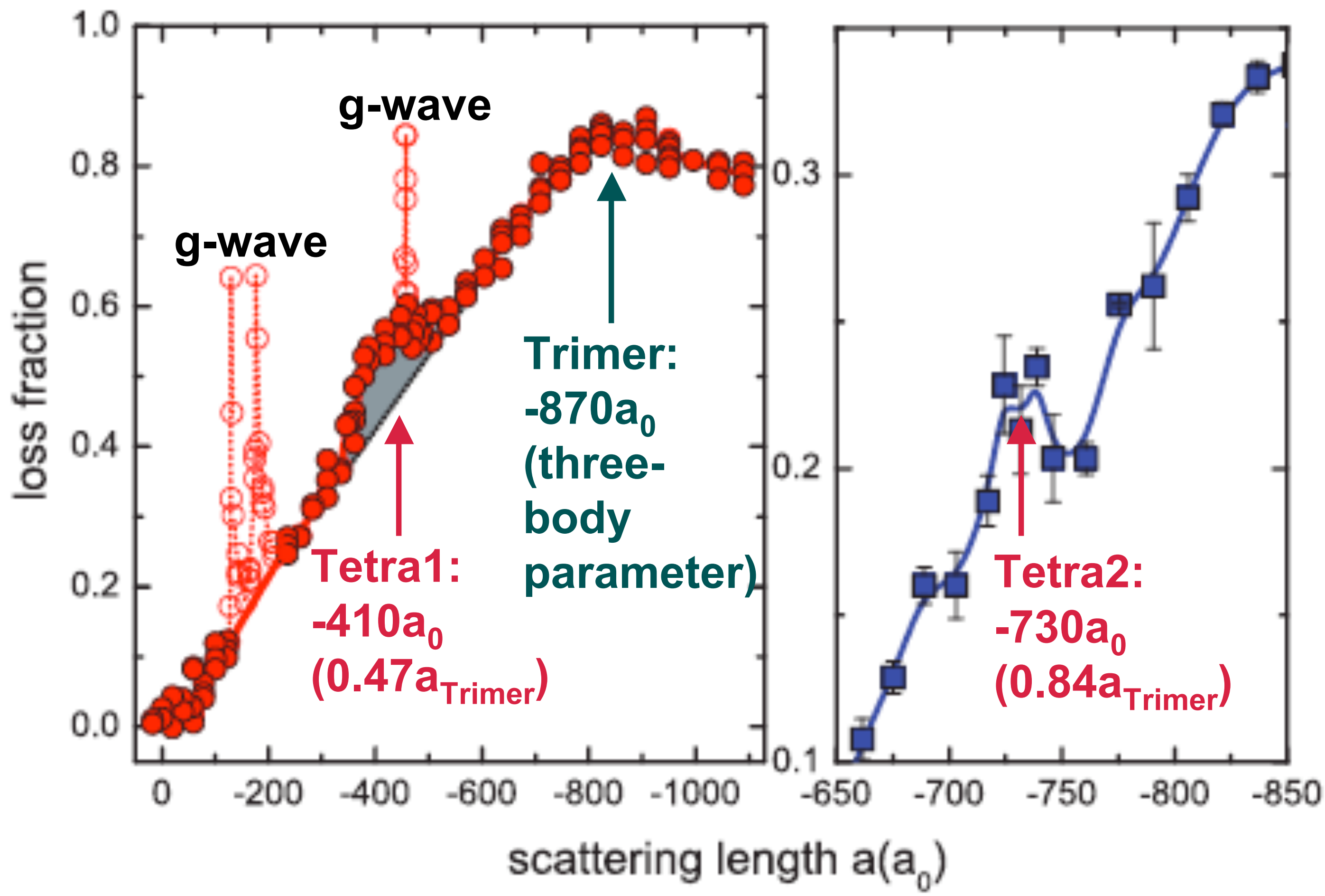
**0.43 in agreement with 0.442 by Hanna, Blume [PRA 74, 063604 (2006)]**

**Experiment:**
**Enhanced losses…**

# Measurement of Loss Rate for Non-Degenerate Bosonic $^{133}$Cs Sample

**First measurement of universal 4-body physics (probe of Efimov physics).**

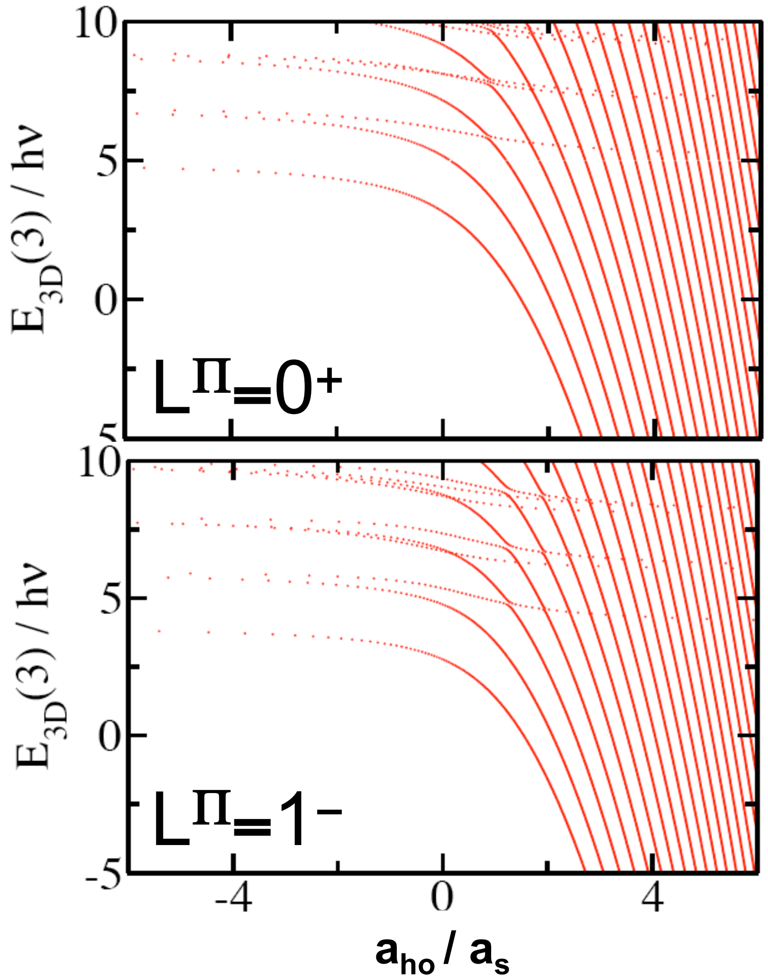
**Figure from Ferlaino et al., PRL 102, 140401 (2009)**




**Other experiments: Pollack et al., Zaccanti et al.,…**

# Return to Equal-Mass Fermions: Harmonically Trapped 3-Body System

**N=3 spectrum for zero-range interactions** [calculated following Kestner and Duan, PRA 76, 033611 (2007)]:




**Few-body spectra determine "high-T" thermodynamics:**

**Start with grand partition function:**

$Z = Tr[-(H-\mu N)/(k_B T)]$

**Perform cluster expansion:**

$Z = 1 + Q_1 z + Q_2 z^2 + \ldots$

where $Q_n = Tr_n[\exp(-H_n/(k_B T))]$;

fugacity $z = \exp[\mu/(k_B T)] << 1$.

**Thermodynamic potential Ω:**

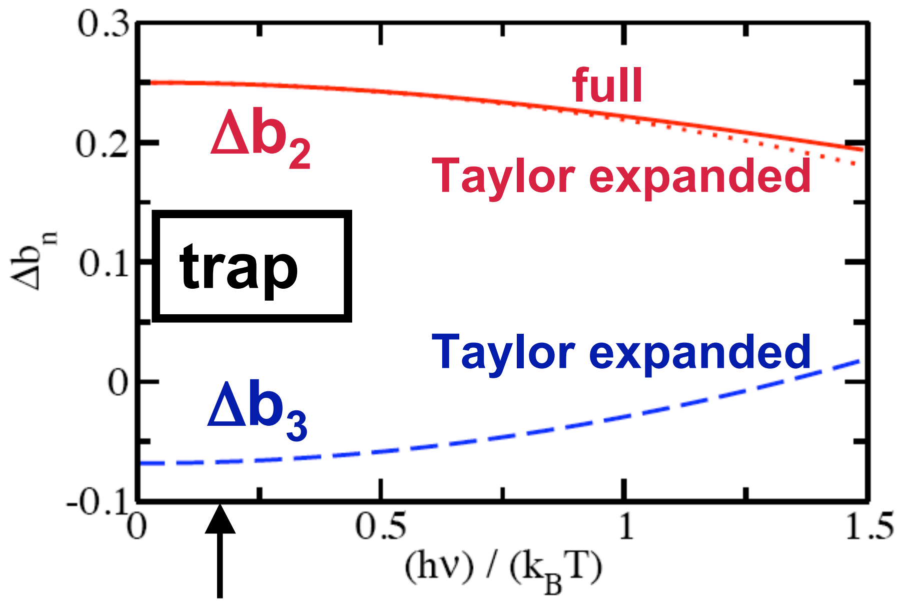
$\Omega = -k_B T \ln(Z)$

$\Omega = -k_B T\, Q_1 (z + b_2 z^2 + b_3 z^3 + \ldots)$

$b_i = b_i(Q_1,\ldots,Q_i)$

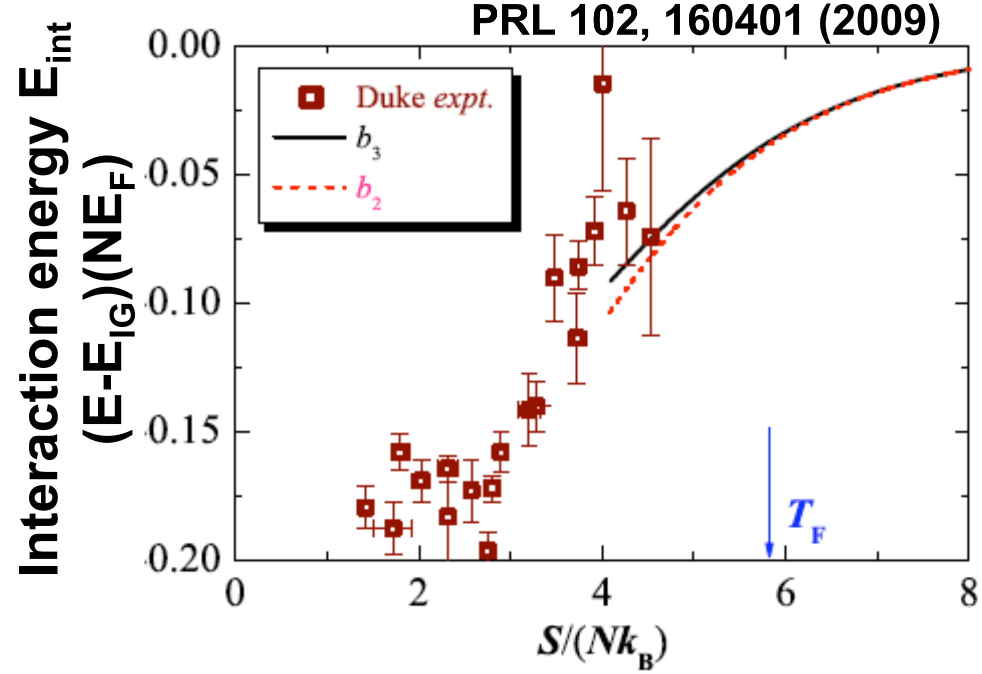
# Virial Expansion for Fermi Gas Based on Two- and Three-Fermion Spectra

**At unitarity ($a_s \to \infty$)**




N=100: $T/T_F$=1

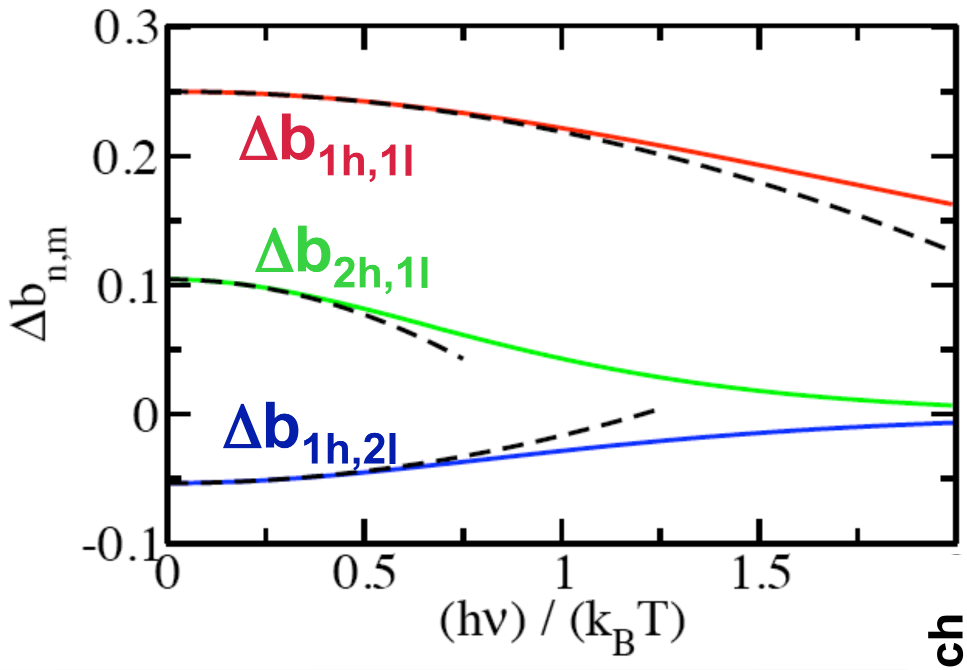
Liu, Hu and Drummond, PRL 102, 160401 (2009)




**Experiment (Nascimbene et al.): $\Delta b_3$(hom)=–0.35(2) [theory –0.355] and $\Delta b_4$(hom)=0.096(15) [no theoretical prediction].**

**Earlier theory work: Ho and Mueller, Rupak,…**
**Other experiments: Horikoshi et al., Zwierlein group,…**

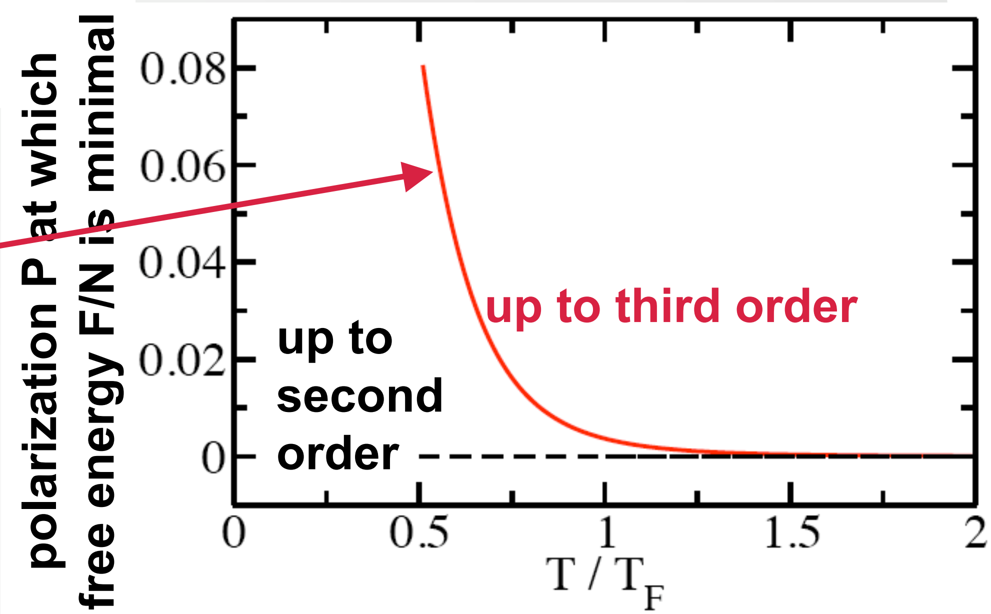
# $^{6}$Li-$^{40}$K Mixture: What Does Virial Expansion Predict?




**Assumptions:**
**Infinite $a_s$.**
**Zero-range interactions.**
**Equal frequencies for Li and K (spherically symmetric trap).**

**Excess of K atoms results in lower free energy per particle.**

**Polarization**
**$P=(N_K-N_{Li})/(N_K+N_{Li})$**



# Few-Body Systems as a Benchmark

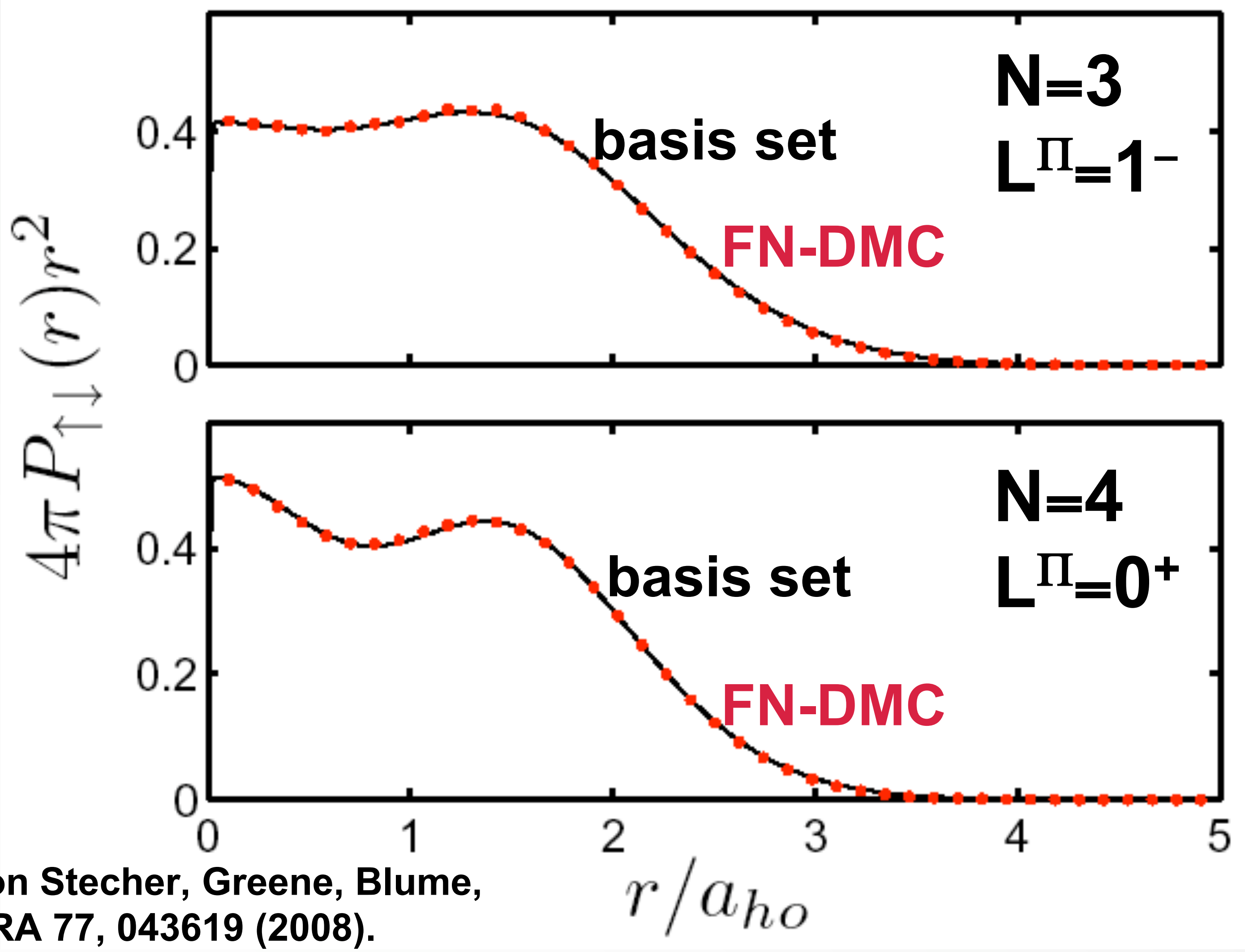
- This is the last week of “Fermions from Cold Atoms to Neutron Stars: Benchmarking the Many-Body Problem” program.

- One approach: Bottom-up…
  - “Exact” results are obtainable for small N.
  - Approximations/techniques can be validated.

- For example:
  - Many of the Tan relations hold for trapped and homogeneous, and small and large systems: Precision benchmark through basis set expansion calculations for equal-mass four-fermion system (Blume and Daily, PRA 80, 053626 (2009)).
  - Equation of state for trapped two-component Fermi gas as T=0…

# FN-DMC and "Exact" Basis Set Calculations: Structural Properties

**Pair distribution function for up-down distance at unitarity:**




Range $r_0=0.01a_{ho}$.

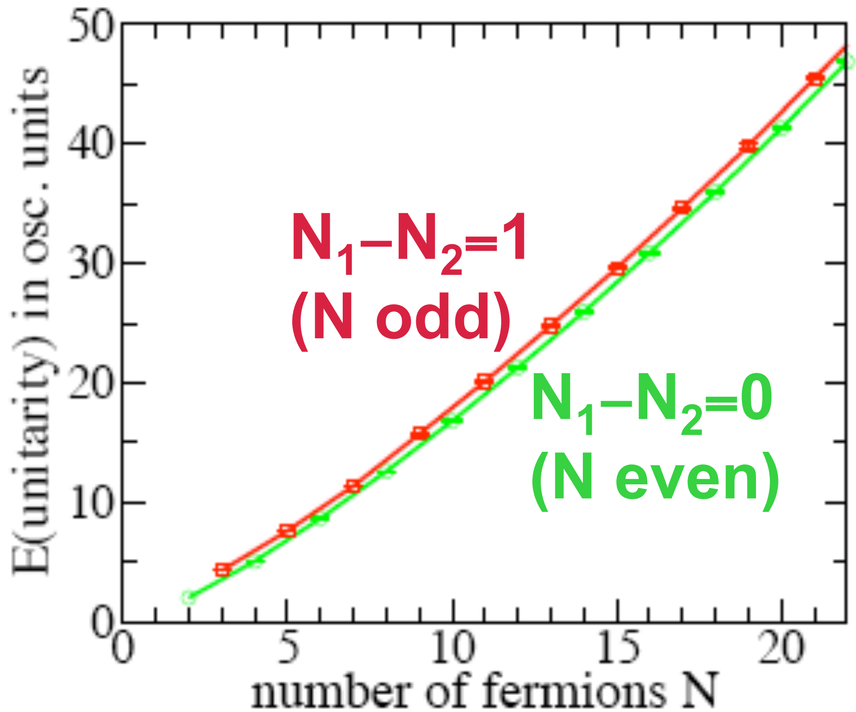
Very good agreement between basis set and fixed-node diffusion Monte Carlo (FN-DMC) results.

von Stecher, Greene, Blume, PRA 77, 043619 (2008).

# $N_1 - N_2 = 0,1$: Energy of Trapped Two-Component Fermi Gas at Unitarity

Blume, von Stecher, Greene, PRL 99, 233201 (2007).

**Fixed-node diffusion Monte Carlo (FN-DMC) energies:**




**Even-odd oscillations.**
**Essentially no shell structure.**

**Local density approximation (even N):**

$$E_{00}(N) = \sqrt{\xi} E_{NI}$$

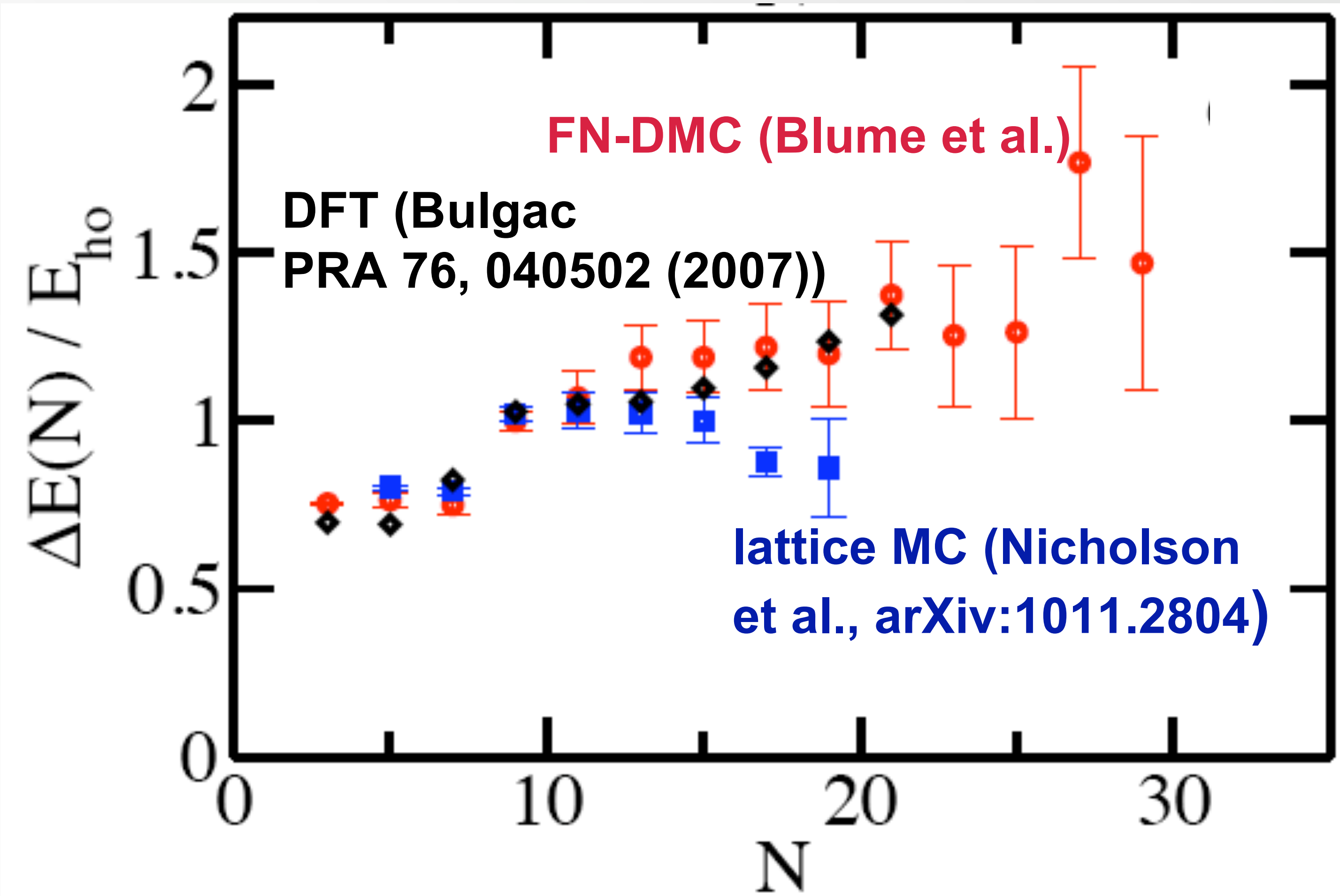
$$E_{fit}(N) = \sqrt{\xi_{tr}} E_{NI,ETF}$$

**We find: $\xi_{tr}$=0.467.**
**For comparison:**
**$\xi_{hom}$=0.383(1)**
**(Forbes et al., arXiv:1011.2197)**

# Excitation Gap: Comparison of Different Approaches

N odd, $N=N_1+N_2$ and $N_1=N_2+1$

$$\Delta(N) = E(N_1,N_2) - [E(N_1-1,N_2) + E(N_1,N_2+1)] / 2$$




Excitation gap of order hν.

Not sufficiently accurate to distinguish between $N^{1/3}$ (LDA) and $N^{1/9}$ (Son, arXiv: 0707.1851) scaling.

# Summary

- A first glance at selected few-body problems:
  - Few-body states whose properties are determined by just one or two parameters.
  - High-T thermodynamics.
  - Benchmark of different analytical and numerical approaches and, in some cases, of the many-body problem.

- Experimentally accessible.

- Future looks bright.