The No Core Shell Model: Its Formulation, Application and Extensions

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MICROSCOPIC NUCLEAR-STRUCTURE **THEORY**

1. Start with the bare interactions among the nucleons

2. Calculate nuclear properties using nuclear many body theory

A. SchwenkWeinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,

- I. Forces among nucleons
- 1. QCD --> EFT --> CPT --> self-consistent nucleon interactions
- 2. Need NN and NNN and perhaps also NNNN interactions

P. Navratil and E. Caurier, Phys. Rev. C 69, 014311 (2004)

H. Kamada, *et al.*, *Phys. Rev. C* 64, 044001 (2001)

PHYSICAL REVIEW C, VOLUME 64, 044001

Benchmark test calculation of a four-nucleon bound state

In the past, several efficient methods have been developed to solve the Schrödinger equation for fournucleon bound states accurately. These are the Faddeev-Yakubovsky, the coupled-rearrangement-channel Gaussian-basis variational, the stochastic variational, the hyperspherical variational, the Green's function Monte Carlo, the no-core shell model, and the effective interaction hyperspherical harmonic methods. In this article we compare the energy eigenvalue results and some wave function properties using the realistic AV8' NN interaction. The results of all schemes agree very well showing the high accuracy of our present ability to calculate the four-nucleon bound state.

No Core Shell Model

"Ab Initio" approach to microscopic nuclear structure calculations, in which all \underline{A} nucleons are treated as being active.

Want to solve the A-body Schrödinger equation

$$
H_A \Psi^A = E_A \Psi^A
$$

R P. Navrátil, J.P. Vary, B.R.B., PRC 62, 054311 (2000) P. Navratil, et al., J. Phys. G: Nucl. Part. Phys. 36, 083101 (2009)

No-Core Shell-Model Approach

• Start with the purely intrinsic Hamiltonian

$$
H_A = T_{rel} + \mathcal{V} = \frac{1}{A} \sum_{i < j = 1}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i < j = 1}^{A} V_{NN} \left(+ \sum_{i < j < k}^{A} V_{ijk}^{3b} \right)
$$

Note: There are **no** phenomenological s.p. energies!

Can use any NN potentials Coordinate space: Momentum space: Argonne V8', AV18 Nijmegen I, II CD Bonn, EFT Idaho

No-Core Shell-Model Approach

• Next, add CM harmonic-oscillator Hamiltonian

$$
H_{CM}^{HO} = \frac{\vec{P}^2}{2Am} + \frac{1}{2}Am\Omega^2 \vec{R}^2; \quad \vec{R} = \frac{1}{A} \sum_{i=1}^{A} \vec{r}_i, \quad \vec{P} = Am\vec{R}
$$

To H_A, yielding

$$
H_A^{\Omega} = \sum_{i=1}^{A} \left[\frac{\vec{p}_i^2}{2m} + \frac{1}{2} m \Omega^2 \vec{r}_i^2 \right] + \sum_{i < j = 1}^{A} \left[V_{NN} (\vec{r}_i - \vec{r}_j) - \frac{m \Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \right]
$$

Defines a basis (*i.e.* HO) for evaluating *V ij*

$H \Psi = E \Psi$

We cannot, in general, solve the full problem in the

complete Hilbert space, so we must truncate to a finite

model space

We must use effective interactions and \implies operators!

Effective Interaction

• Must truncate to a finite model space

- In general, V_{ii}^{eff} is an A-body interaction *ij*
- We want to make an *a*-body cluster approximation

$$
\mathcal{H} = \mathcal{H}^{(I)} + \mathcal{H}^{(A)} \underset{a < A}{\geq} \mathcal{H}^{(I)} + \mathcal{H}^{(a)}
$$

$$
H\Psi_{\alpha} = E_{\alpha}\Psi_{\alpha} \quad \text{where} \quad H = \sum_{i=1}^{A} t_i + \sum_{i \leq j}^{A} v_{ij}.
$$

$$
\mathcal{H}\Phi_{\beta} = E_{\beta}\Phi_{\beta}
$$

$$
\Phi_{\beta} = P\Psi_{\beta}
$$

 P is a projection operator from S into S

$$
\langle \tilde{\Phi}_{\gamma} | \Phi_{\beta} \rangle = \delta_{\gamma \beta}
$$

$$
\mathcal{H} = \sum_{\beta \in \mathcal{S}} |\Phi_{\beta} \rangle E_{\beta} < \tilde{\Phi}_{\beta}
$$

From few-body to many-body

- **NCSM** convergence test
	- Comparison to other methods

P. Navratil, INT Seminar, November 13, 2007, online

P. Navrátil and E. Caurier, Phys. Rev. C **69**, 014311 (2004)

P. Navrátil and W. E. Ormand, Phys. Rev. C **68**, 034305 (2003)

Origin of the anomalous long lifetime of 14 C

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We report the microscopic origins of the anomalously suppressed beta decay of ${}^{14}C$ to ${}^{14}N$ using the ab initio no-core shell model (NCSM) with the Hamiltonian from chiral effective field theory (EFT) including three-nucleon force (3NF) terms. The 3NF induces unexpectedly large cancellations within the p-shell between contributions to beta decay, which reduce the traditionally large contributions from the NN interactions by an order of magnitude, leading to the long lifetime of 14 C.

arXiv: 1101.5124v1 [nucl-th] 26 Jan 2011

P. Navrátil, J. P. Vary and B. R. B., Phys. Rev. C 62, 054311 (2000)

C. FORSSÉN, P. NAVRÁTIL, W. E. ORMAND, AND E. CAURIER

H. Kamada, *et al.*, *Phys. Rev. C* 64, 044011 (2001)

Figure 2. NCSM and GFMC NN pair density in ⁴He.

Stetcu, Barrett, Navratil, Vary, Phys. Rev. C 71, 044325 (2005)

- small model space: expect larger renormalization
- large variation with the model space
- three-body forces: might be important, but not the issue
- $a \rightarrow A$ for fixed model space;
- $P \rightarrow \infty$ for fixed cluster.

Range dependence

Stetcu, Barrett, Navratil, Vary, Phys. Rev. C 71, 044325 (2005)

Beyond the No Core Shell Model

1. The ab initio Shell Model with a Core

- 2. Importance Truncation
- 3. The NCSM in an Effective Field Theory (EFT) Framework

PHYSICAL REVIEW C 78, 044302 (2008)

Ab-initio shell model with a core

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We construct effective two- and three-body Hamiltonians for the p-shell by performing $12\hbar\Omega$ ab initio no-core shell model (NCSM) calculations for $A = 6$ and 7 nuclei and explicitly projecting the many-body Hamiltonians onto the $0\hbar\Omega$ space. We then separate these effective Hamiltonians into inert core, one- and two-body contributions (also three-body for $A = 7$) and analyze the systematic behavior of these different parts as a function of the mass number A and size of the NCSM basis space. The role of effective three- and higher-body interactions for $A > 6$ is investigated and discussed.

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The idea of Importance Truncation

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www.elsevier.com/locate/physletb

No-core shell model in an effective-field-theory framework

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Abstract

We present a new approach to the construction of effective interactions suitable for many-body calculations by means of the no-core shell model (NCSM). We consider an effective field theory (EFT) with only nucleon fields directly in the NCSM model spaces. In leading order, we obtain the strengths of the three contact interactions from the condition that in each model space the experimental ground-state energies of ${}^{2}H$. ${}^{3}H$ and ${}^{4}He$ be exactly reproduced. The first $(0^+; 0)$ excited state of ⁴He and the ground state of ⁶Li are then obtained by means of NCSM calculations in several spaces and frequencies. After we remove the harmonic-oscillator frequency dependence, we predict for ⁴He an energy level for the first $(0^+;0)$ excited state in remarkable agreement with the experimental value. The corresponding ⁶Li binding energy is about 70% of the experimental value, consistent with the expansion parameter of the EFT.

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Effective interactions for light nuclei: an effective (field theory) approach

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Abstract

One of the central open problems in nuclear physics is the construction of effective interactions suitable for many-body calculations. We discuss a recently developed approach to this problem, where one starts with an effective field theory containing only fermion fields and formulated directly in a no-core shell-model space. We present applications to light nuclei and to systems of a few atoms in a harmonic-oscillator trap. Future applications and extensions, as well as challenges, are also considered.

Why $EFT + NCSM$?

EFT:

1. Captures the relevant degrees of freedom/symmetries

- 2. Builds in the correct long-range behavior
- 3. Has a systematic way for including the short-range behavior/order by order
- 4. Many-body and two-body interactions treated in the same framework
- 5. Explains naturally the hierarchy of the (many-body) forces

NCSM:

- 1. Flexible many-body method/easy to implement
- 2. Equivalent SD and Jacobi formulations
- 3. Can handle both NN and NNN interactions
- 4. In principle applies to any nucleus/extensions to heavier nuclei

Pionless EFT for nuclei within the NCSM: Without pions--> Breakdown momentum roughly 100 MeV/c

$$
H = \frac{1}{2m_N A} \sum_{[i < j]} (\vec{p}_i - \vec{p}_j)^2 + C_0^1 \sum_{[i < j]} \delta(\vec{r}_i - \vec{r}_j) + C_0^0 \sum_{[i < j]} \delta(\vec{r}_i - \vec{r}_j) + D_0 \sum_{[i < j < k]} \delta(\vec{r}_i - \vec{r}_j) \delta(\vec{r}_j - \vec{r}_k),
$$
\nt. al., 2007

Stetcu e

-> calculation at Leading order: two N-N contact interactions in the ${}^{3}S_{1}$ ¹S₀ channel and a threebody contact interaction in the 3nucleon $S_{1/2}$ channel

-> coupling constants fitted to the binding energy of the deuteron, triton and ⁴He

Difficulties:

fixing the couplings to few-body states is cumbersome HO: bound states only no immediate connection to the scattering observables

Question : How to construct an EFT within a bound many-body model space beyond Leading-Order?

Answer: by trapping nuclei in a harmonic potential

T. Busch, et al., Found. Phys. 28, 549 (1998)

$$
\frac{\Gamma\left(\frac{3}{4} - \frac{E}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E}{2\hbar\omega}\right)} = -\frac{bk}{2} \cot \delta
$$
\nenergy in the trap (bound state physics) phase shift (scattering physics)\n
\n
$$
k \cot \delta = -\frac{1}{a_2} + \frac{1}{2}r_2k^2 + \dots,
$$
\nEffective Range Expansion

J. Rotureau, ORNL, March 2011

$$
H_{int} = \frac{1}{A} \sum_{i>j=1}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i>j=1}^{A} V_{ij} + \sum_{i>j>k=1}^{A} V_{ijk} + \dots
$$

\n
$$
H = H_{int} + \frac{\vec{P}_{CM}^2}{2mA} + \frac{1}{2}mA\omega^2 \vec{R}_{CM}^2
$$

\n
$$
= \sum_{i=1}^{A} \left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + \sum_{i
\n
$$
h_{12} = \frac{p_1^2}{2m} + \frac{1}{2}m\omega r_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}m\omega r_2^2 + V_{12} - \frac{m\omega^2}{2A}(\vec{r}_1 - \vec{r}_2)^2
$$

\n
$$
h_{12} = h_{rel} + h_{CM}
$$

\nNCSM: unitary transformation h_{rel} Renormalization for trap $\Omega = \omega \sqrt{\frac{A-2}{A}}$
$$

I. Stetcu, TRIUMF, Feb. 2011

FOR TWO PARTICLES IN A T

Original motivation: to understand gross features of nuclear systems from a QCD perspective

At the heart of an effective theory: a truncation of the Hilbert space / all interactions allowed by symmetries are generated / power counting

$$
\frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = \frac{b}{2a_2}
$$

$$
\frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = -\frac{b}{2} \left(-\frac{1}{a_2} + \frac{r_2}{b^2} \varepsilon + \ldots \right)
$$

In finite model spaces:

$$
V_{LO}(\vec{p}, \vec{p}') = C_0
$$

\n
$$
V_{NLO}(\vec{p}, \vec{p}') = C_2(p^2 + p'^2)
$$

\n
$$
V_{N^2LO}(\vec{p}, \vec{p}') = C_4(p^2 + p'^2)^2
$$

 $C_0, C_2, C_4, ...$ Constants to be determined in each model space so that select observables are preserved

I. Stetcu, TRIUMF, Feb. 2011

RENORMALIZA

$$
\Psi(\vec{r}) = \sum_{n=0}^{N_{\text{max}}/2} A_n \varphi_n(\vec{r})
$$

RAPPED NUCLEONS

ν,

Triplet S NN phase shift

3 nucleons at Leading-Order in the trap coupled to $J^{\pi} = \frac{3}{2}^{+}$

for a fixed two-body cutoff (N_2) , the size of the model space (N_a) is increased until convergence

-> convergence of energy as the two-body cutoff $N₂$ increases -> as expected no need for a three body force at Leading Order. J. Rotureau, ORNL, March 2011

SOME REMAINING CHALLENGES

- 1. Understanding the fundamental interactions among the nucleons in terms of QCD, e.g., NN, NNN,
- 2. Determination of the mean field (the monopole effect).
- 3. Microscopic calculations of medium- to heavy-mass nuclei:
	- a.) How to use the advances for light nuclei to develop techniques for heavier nuclei.
	- b.) Building in more correlations among the nucleons in small model spaces, e.g., effective interactions for heavier nuclei.
- 4. Extensions of these microscopic advances for nuclear structure to nuclear reactions.

COLLABORATORS

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S. Quaglioni and P. Navratil, Phys. Rev. Lett. 101, 092501 (2008)

long range (continuum)

P. Navratil

Difficulties:

fixing the couplings to few-body states is cumbersome HO: bound states only no immediate connection to the scattering observables

Question: How to construct an EFT within a bound many-body model space beyond Leading-Order?

Answer: By trapping the nucleons within an HO trap.

$$
h_2 = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2 + V_2(r)
$$

$$
\frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = \frac{b}{2a_2} \qquad \left(b = \frac{1}{\sqrt{\mu\omega}}\right)
$$

T. Busch et. al., Found. Phys. 28 (1998) 549

Strong-Interaction Theory

- 1. Strong Interaction ----> Standard Model
- 2. Standard Model -------> Quarks exchanging gluons

 However, at the energy level of low-energy nuclear physics the quark degrees of freedom are frozen out in favor of nucleon and meson degrees of freedom.

II. Many-Body Techniques for Solving the A-Nucleon Problem

1. Light Nuclei: ab initio approaches: s- and p-shell nuclei Green Function Monte Carlo (GFMC) (R. Wiringa, et al.), No-Core Shell Model (NCSM), Faddeev-Yakubovsky, UCOM, V low-k, SRG, ...

2. sd- and pf-shell nuclei: NCSM, extended NCSM, Standard Shell Model (SSM), Coupled Cluster (CC), Shell Model Monte Carlo (SMMC) (sign problem defeated?), Monte Carlo Shell Model (MCSM) (Otsuka, et al.) ...

3. Heavier Nuclei: Density Functional Theory (DFT) (SciDAC project: UNEDF); CC; Monte Carlo approaches, ...

III. New Methods/Transformative Ideas (???)

- 1. "soft" NN interactions plus weak NNN interactions
- 2. Coupled Cluster calculations with NNN interactions
- 3. Universal Nuclear Energy Density Functional
- 4. Building more correlations into smaller model space: a) Fermionic Molecular Dynamics Approach (T. Neff, et al.) b) Extensions of the NCSM:
	- i) Projected NCSM/SSM
	- ii) Symplectic (3,R) NCSM (J. Draayer, et al.)
	- iii) Importance Truncated NCSM (Navratil and Roth)
	- iv) NCSM + Resonating Group Method (Navratil & Quaglioni)