Dynamic Mean Field Approximation and the pseudo-gap in unitary Fermi gas

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Outline

1. Dynamic Mean Field Approximation
2. The BCS-BEC crossover
3. The Excitation Spectrum
4. Conclusions

Dynamic Mean Field Approximation
the full story in few lines

- At the limit of large dimensionality $d \longrightarrow \infty$ the self-energy becomes localized.
- One can write self-consistency relations for the self energy $\longrightarrow$ **DMFT**.
- Equivalent to a 1D QFT problem.
- At finite d DMFT becomes an approximation T $\longrightarrow$ A.
- DMFA is a mean field approach, approximating local self-energy.

$$\boxed{\hat{\Sigma}(\boldsymbol{k}, i\omega_n) \longrightarrow \hat{\Sigma}(i\omega_n)}$$

- The DMFA approximation reduces the d-dimensions Hubbard model into a self-consist temporal problem.
- Valid for finite lattice filling (the extrapolation to the continuum is tricky).
- The outcome of this approach is $G(\boldsymbol{k}, i\omega_n)$.

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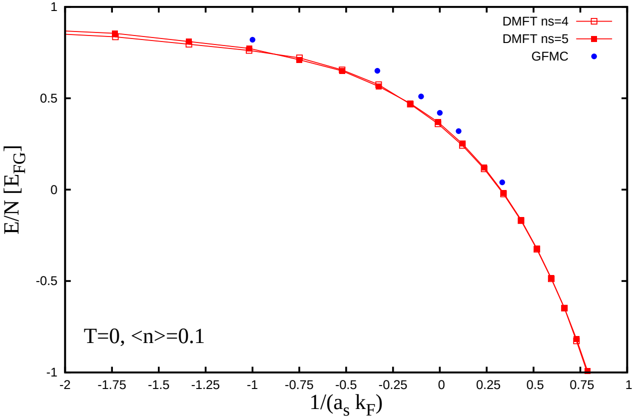
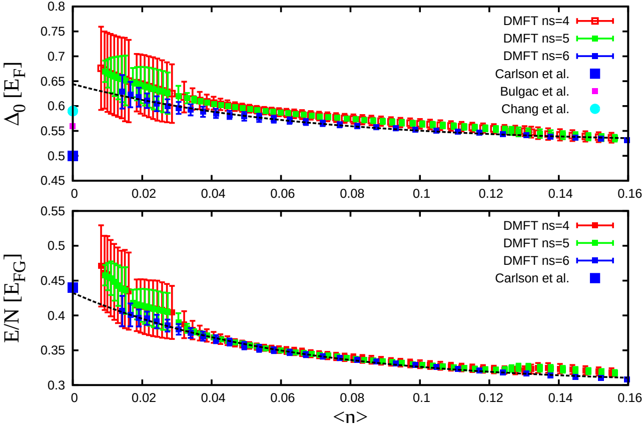
The BCS-BEC crossover

N. Barnea, Phys. Rev. A 78, 053629 (2008).

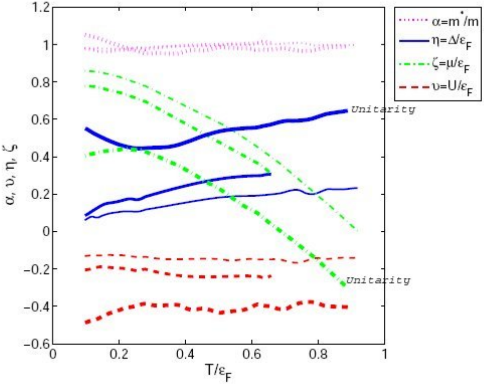


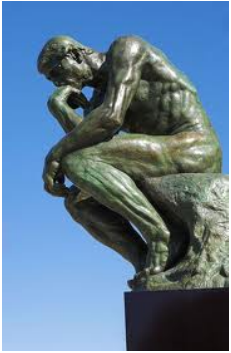
DMFA at lattice filling of $n = 0.1$ with $n_s = 4, 5$ vs the QMC results of Carlson *et al.* PRL **91**, 050401 (2003).



The continuum limit $\langle n \rangle \longrightarrow 0$ for the $T = 0$ energy per particle E/N and Δ_0.

The pseudo-gap - Magiersky *et al.* Arxiv: 0801.150





Graph taken form the first version of the manuscript (Arxiv: 0801.1504). Note that the raising part of Δ_{qp} disappeared in the final version.

- For numerical calculations, analytic continuation to the real axis is a painful procedure.
- To overcome this hardship consider a BCS quasi-particle Green's function

$$G_{qp}(\boldsymbol{k}, i\omega_n) = -\frac{-i\omega_n + \mu - \epsilon_{\boldsymbol{k}} - \Sigma}{(i\omega_n + E_{\boldsymbol{k}})(i\omega_n - E_{\boldsymbol{k}})}$$

- This Green's function contains 3 unknowns $\mu, \Sigma, E_{\boldsymbol{k}}$
and can be used to calculate any physical quantity.
- In particular we can evaluate the susceptibility,

$$\chi(\boldsymbol{k}) = -\int_0^\beta d\tau G(\boldsymbol{k}, \tau) = -\frac{2}{\beta}\sum \frac{1}{i\omega_n} G(\boldsymbol{k}, i\omega_n) .$$

- The occupation probability

$$f(\boldsymbol{k}) = G(\boldsymbol{k}, 0^+) = \frac{1}{\beta}\sum e^{i\omega_n 0^+} G(\boldsymbol{k}, i\omega_n)$$

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$$\zeta(\boldsymbol{k}) = \left.\frac{dG(\boldsymbol{k}, \tau)}{d\tau}\right|_{\tau=0^+} = \frac{1}{\beta}\sum e^{i\omega_n 0^+} i\omega_n G(\boldsymbol{k}, i\omega_n)$$

Energy spectrum

Extracting the physics from the simulation

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Excitation spectrum

Manipulating these quantities, we get

$$E_{\boldsymbol{k}} = \sqrt{-\frac{1}{\chi(\boldsymbol{k})}\left[2\zeta(\boldsymbol{k}) + \frac{2f(\boldsymbol{k})-1}{\chi(\boldsymbol{k})}\right]}$$

few comments

- We use this formula as a definition of $E_{\boldsymbol{k}}$.
- Making it a legitimate physical quantity.
- Interpretation?
- In the DMFA $\chi(\boldsymbol{k}), f(\boldsymbol{k}), \zeta(\boldsymbol{k})$ can be calculated directly.
- $E_{\boldsymbol{k}}$ fits very well to the quasi-particle spectrum

$$E_{\boldsymbol{k}} = \sqrt{(\alpha_{qp}\epsilon_{\boldsymbol{k}} + \Sigma_{qp} - \mu)^2 + \Delta_{qp}^2}$$

where $\alpha_{qp}, \Sigma_{qp}, \Delta_{qp}$ are free parameters.

The quasi-particle spectrum at $T = 0.38T_F \geq 2T_C$

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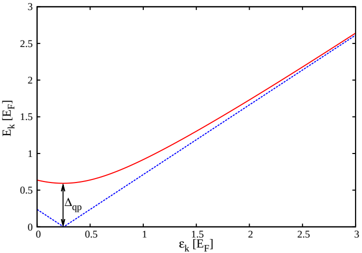

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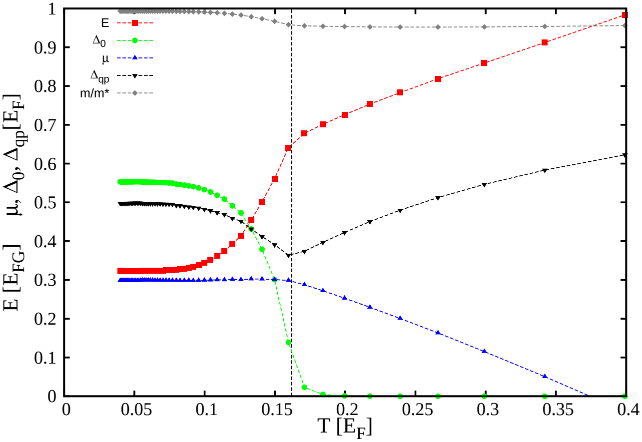
Energy, μ, gaps ...



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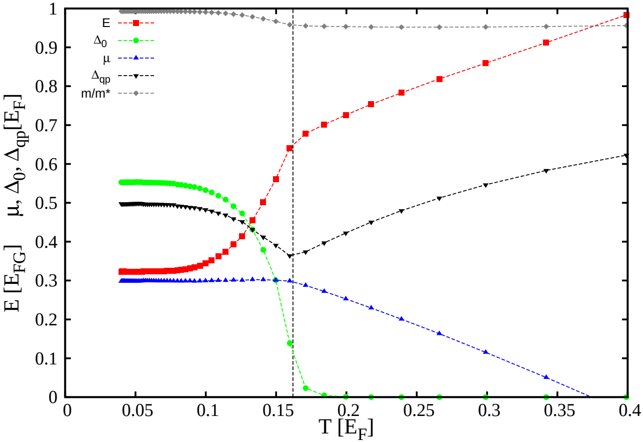
The quasi particle gap, Δ_{qp}, goes a sharp, 2nd order, transition at T_c



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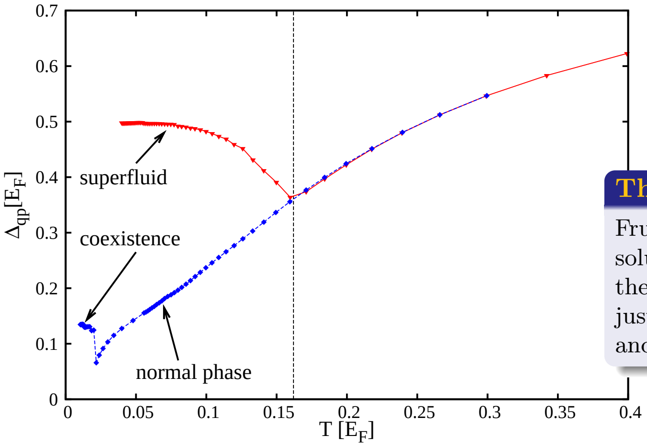

The Gap evolution

Frustrating the superfluid solution it can be seen that the phase transition happens just as the "insulator gap" and the superfluid gap cross.

Conclusions

1. The DMFA reproduce a smooth BCS-BEC transition.
2. The extrapolated continuum values of the energy per particle and the gap function agree very well with QMC results.
3. The pairing phase transition is reproduced. Leading to T_c with overall agreement with the QMC.
4. Pseudo Gap found at $T > T_c$ is associated with the imaginary part of the self-energy.
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