The extent of universal physics in three-body collisions

Yujun Wang

Department of Physics
Kansas State University

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Outline

1. Introduction
   - Efimov effect in ultracold atomic gases
   - Theoretical Framework
2. Efimov physics near narrow Feshbach resonances
3. Efimov physics at finite energy
   - $a > 0$
   - $a < 0$
4. Summary & Outlook

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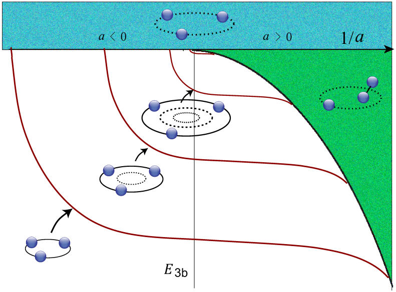
Three-body Efimov effect



Efimov spectrum

An infinite series of three-body bound states with $E_n = E_0 e^{-2n\pi/s_0}$ when two-body scattering length $a \rightarrow \infty$ ($s_0 \approx 1.00624$)

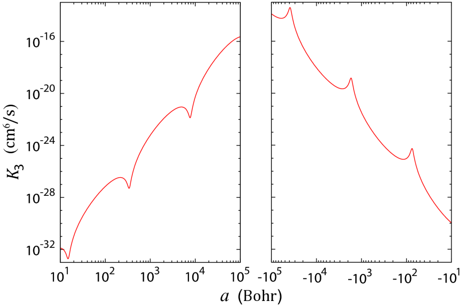
Three-body Efimov effect leads to universal scattering length scaling in the three-body scattering processes which can be observed in ultracold atomic gases.

Efimov effect in ultracold atomic gases

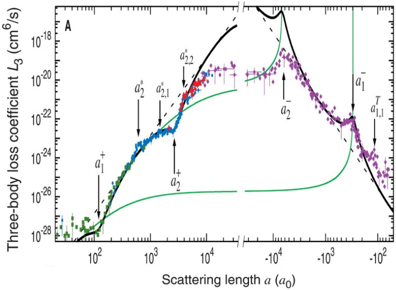
Universal three-body recombination rates:

$$K_3^{(a<0)} = \frac{4590 \sinh 2\eta}{\sin^2[s_0 \ln(|a|/r_0) + \Phi + 1.53] + \sinh^2 \eta} \frac{\hbar}{m} a^4 \quad (B + B + B \rightarrow B_2^* + B)$$

$$K_3^{(a>0)} = 67.1 e^{-2\eta} \left(\sin^2[s_0 \ln \frac{a}{r_0} + \Phi] + \sinh^2 \eta \right) \frac{\hbar}{m} a^4 \quad (B + B + B \rightarrow B_2 + B)$$



Theory



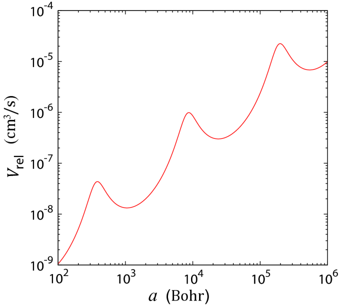
Experiment

Pollack, *et al*, Science (2009)

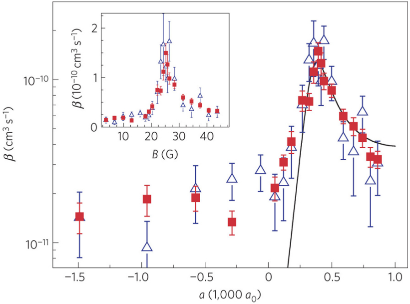
Efimov effect in ultracold quantum gases

Universal three-body vibrational relaxation rates:

$$V_{\rm rel}^{(a>0)} = \frac{20.1 \sinh 2\eta}{\sin^2[s_0 \ln(a/r_0) + \Phi + 1.47] + \sinh^2 \eta} \frac{\hbar}{m} a \quad (B_2^* + B \to B_2 + B)$$



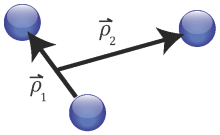
Theory



Experiment

S. Knoop, *et al*, Nature Phys.(2009)

Smith-Whitten hyperspherical coordinates



Jacobi coordinates

- Hyperradius represents the overall size of the three-body system $R = \sqrt{\rho_1{}^2 + \rho_2{}^2}$
- Hyperangle θ characterizes the geometry of three-body triangle
- Hyperangle ϕ characterizes the permutations.

Three-body Schrödinger equation in the hyperspherical coordinates:

$$\left[-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2} + \frac{(\Lambda^2 + \frac{15}{4})\hbar^2}{2\mu R^2} + V_{12} + V_{23} + V_{31}\right]\psi_E = E\psi_E$$

Adiabatic hyperspherical representations

Slow hyperradial motion $\Rightarrow$ Choice of adiabatic representation:

- Intuitive picture and qualitative analysis from the potentials
- Quick convergence for numerical calculations

Generating adiabatic basis $\Phi_\nu(R;\Omega)$ by treating R as a parameter

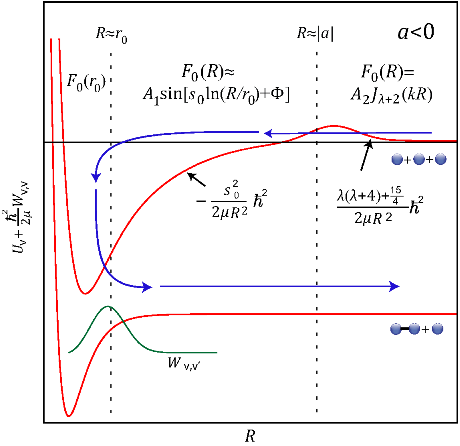
$$\left[\frac{(\Lambda^2+\frac{15}{4})\hbar^2}{2\mu R^2}+V_{12}+V_{23}+V_{31}\right]\Phi_\nu(R;\Omega)=U_\nu(R)\Phi_\nu(R;\Omega)$$

Schrödinger equation reduces to coupled radial equations

$$\left[-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2}+U_\nu(R)\right]F_{\nu E}(R)-\frac{\hbar^2}{2\mu}\sum_{\nu'}W_{\nu\nu'}(R)F_{\nu' E}(R)=EF_{\nu E}(R)$$

Solving three-body problems in the adiabatic hyperspherical representation

Scaling in the hyperradial wavefunction ⇒ Analytic expressions for the inelastic rates



Adiabatic hyperspherical potentials

Matching $F(R)$ at:

- $R \sim r_0$
- $R \sim |a|$

⇒ Transmission coefficient T into deeper channels

$K_3 \propto T$

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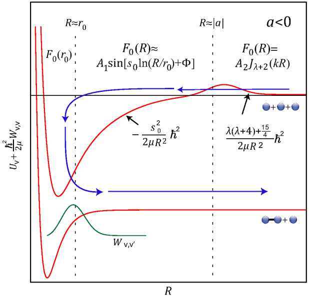
Scaling in the hyperradial wavefunction ⇒ Analytic expressions for the inelastic rates



Adiabatic hyperspherical potentials

Matching $F(R)$ at:

- $R \sim r_0$
- $R \sim |a|$

⇒Transmission coefficient T into deeper channels

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Ab-initio calculations:

- Solve the hyperangular equation in B-spline basis
- Solve the hyperradial equation as a standard multichannel scattering problem

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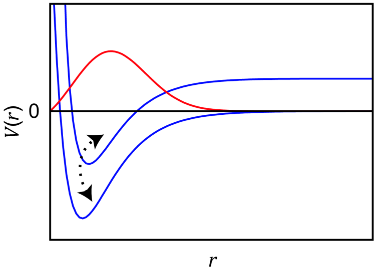
Modeling two-body Feshbach resonance



Two-body potentials

Resonance width Γ connects effective range expansion:
$k\cot(\delta) \simeq -\frac{1}{a} + \frac{1}{2} r_{\mathrm{eff}} k^2$ by $r_{\mathrm{eff}} \sim -\frac{1}{\Gamma}$

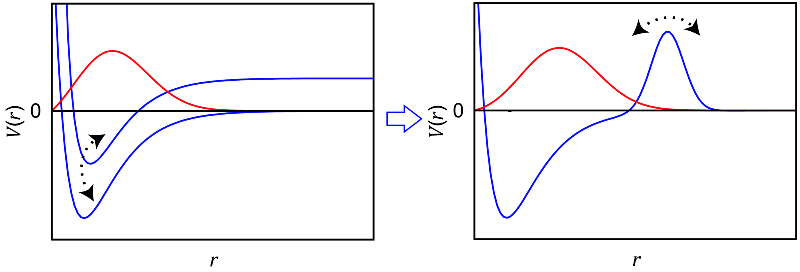
Petrov, PRL (2004); Gogolin, *et al*, PRL (2008).

For small $|r_{\mathrm{eff}}|$ (broad resonance), corrections from r_{eff} have been studied perturbatively:

Platter, Ji, and Phillips, PRA (2009);
Hammer, Länder, and Platter, PRA (2007);
Bratten and Hammer, PRA (2003).

Modeling two-body Feshbach resonance

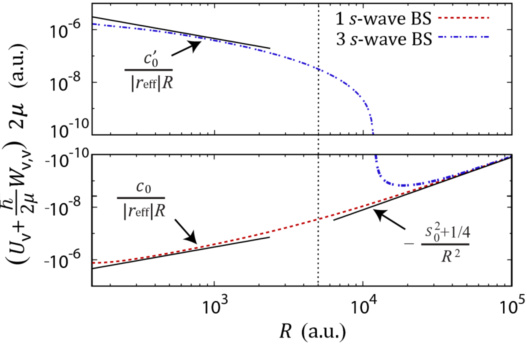
Two-channel Feshbach resonance ⇒ One-channel shape resonance



Two-body potentials

Long-range two-body scattering wavefunctions can be made identical.

Scaling of the adiabatic potentials

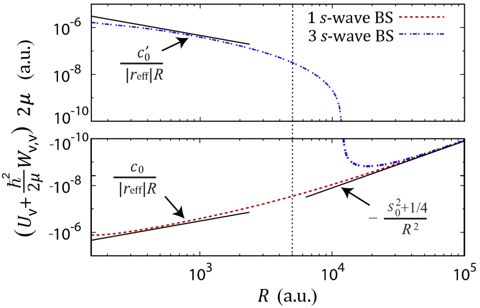


Adiabatic hyperspherical potentials (a=∞, $r_{\rm eff}$=-5×10^3)

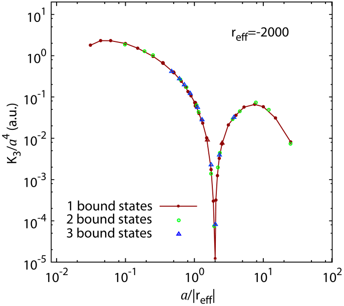
- $\sim \frac{\hbar}{2\mu}\frac{c_0}{|r_{\rm eff}|R}$ Coulomb-like ($\alpha|r_{\rm eff}| \ll R \ll \beta a$)
- Non-universal "effective charge" $c_0/|r_{\rm eff}|$
- Universal rates

Wang, D'Incao, and Esry, arXiv (2009)

Scaling of the adiabatic potentials



Adiabatic hyperspherical potentials
(a=∞, $r_{\rm eff}$=-5×10^3)



Numerical three-body recombination rates

- $\sim \frac{\hbar}{2\mu}\frac{c_0}{|r_{\rm eff}|R}$ Coulomb-like ($\alpha|r_{\rm eff}| \ll R \ll \beta a$)
- Non-universal "effective charge" $c_0/|r_{\rm eff}|$
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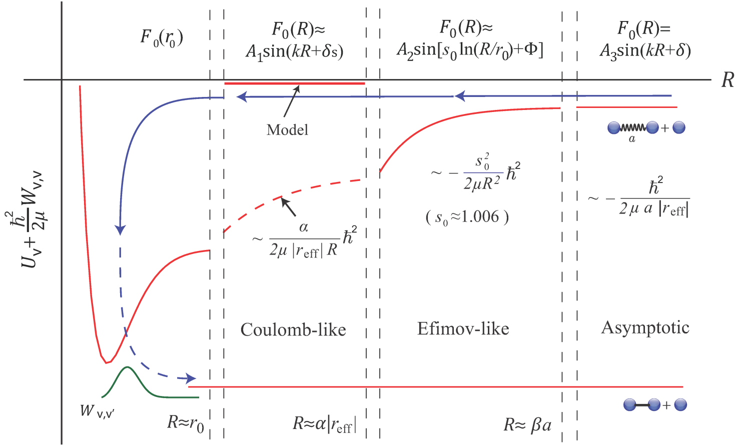
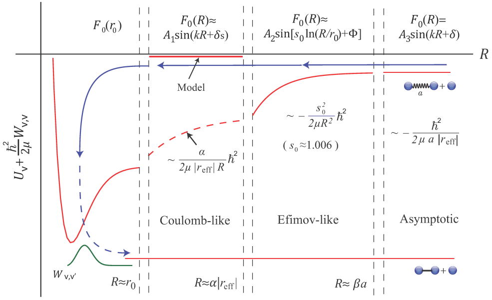
Wang, D'Incao, and Esry, arXiv (2009)

Scaling for the three-body relaxation rates–bosons



Adiabatic hyperspherical potentials ($\beta a \gg \alpha |r_{\rm eff}| \gg r_0$)

Scaling for the three-body relaxation rates–bosons



Adiabatic hyperspherical potentials ($\beta a \gg \alpha |r_{eff}| \gg r_0$)

Matching $F_0(R)$ at:

- $R = r_0$
- $R = \alpha |r_{eff}|$
- $R = \beta a$

$\Rightarrow$ Transmission coefficient T

$$V_{rel} \propto T$$

$$\Rightarrow V_{rel} = \frac{2\sqrt{3}\pi\beta \sin 2\varphi_0 \sinh 2\eta}{\sin^2[s_0 \ln(|a/r_{eff}|)+\Phi+\varphi]+\sinh^2\eta} \frac{\hbar}{m} a$$

where

$$\tan\Phi = 2s_0(\alpha - \mathrm{Re}A/|r_{eff}|)/(\alpha + \mathrm{Re}A/|r_{eff}|),$$
$$\sinh\eta = |\mathrm{Im}A/\alpha r_{eff}| \csc(2\varphi_0)\sin^2(\Phi+\varphi_0),$$
$$\tan\varphi_0 = 2s_0.$$

Non-trivial short-range physics!

Wang, D'Incao, and Esry, arXiv (2009)

Scaling for the three-body relaxation rates

- $1/|r_{\rm eff}|$ suppression for $B_2^* + B \rightarrow B_2 + B$
- $|r_{\rm eff}|^{3.33}$ enhancement for $(FF')^* + F \rightarrow FF' + F$

Implications in ultracold experiments

- Increased collisional stability for dimers of bosonic atoms
- Reduced collisional stability for dimers of fermionic atoms

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Wang, D'Incao, and Esry, arXiv (2009)

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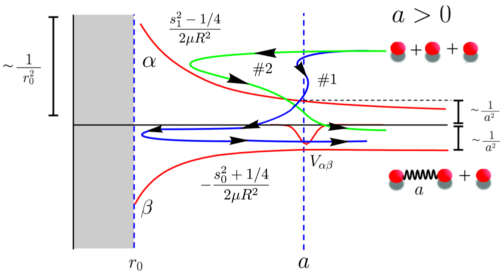
Finite energy recombination for $a > 0$ ($J = 0^+$)



Recombination pathways at finite energy

when $\frac{1}{a} \ll k \ll \frac{1}{r_0}$,

interference between the two pathways gives:

$$K_3 \propto \frac{1}{k^4} \sin^2(-s_0 \ln(kr_0) + \Phi)$$

Wang, D'Incao, Esry, and Naegerl, PRL, to be published (2010)

Connection to zero-energy behavior

At finite energy:

- $K_3 \propto \frac{1}{k^4} \sin^2(-s_0 \ln(kr_0) + \Phi)$

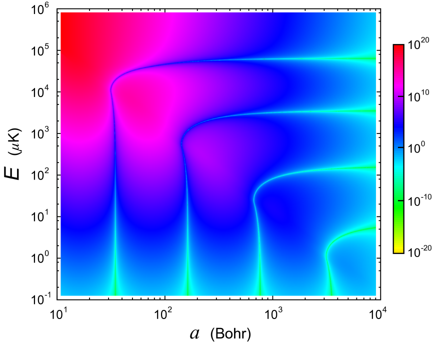
  when $\frac{1}{a} \ll k \ll \frac{1}{r_0}$.

Near zero-energy:

- $K_3 \propto a^4 \sin^2(s_0 \ln(a/r_0) + \Phi'),$

  when $k \lesssim \frac{1}{a}$.

The oscillations are connected.



$K_3(E, a)E^2/a^4$

Wang, D'Incao, Esry, and Naegerl, PRL, to be published (2010)

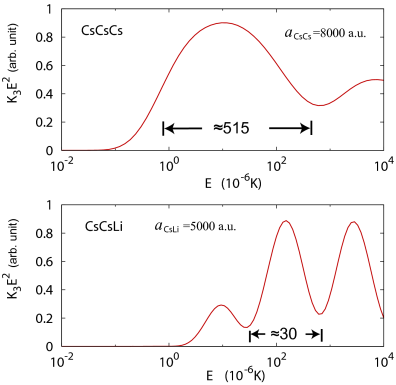
Oscillations in different systems

BBB vs. BBX($m_X/m_B < 1$)

- Smaller oscillatory period for BBX
- Bigger amplitude for BBX

Experimental possible: CsCsLi

$K_3(E)E^2$ for CsCsCs and CsCsLi

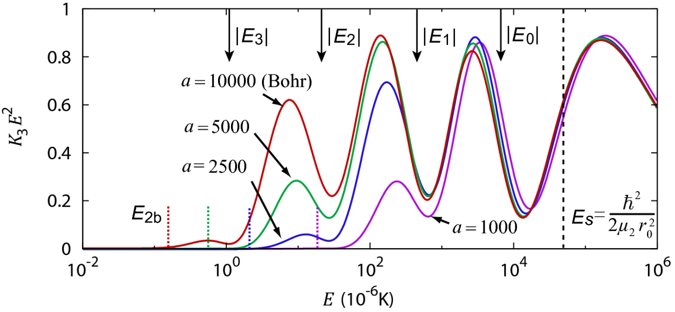


Wang, D'Incao, Esry, and Naegerl, PRL, to be published (2010)

Oscillations and the Efimov states

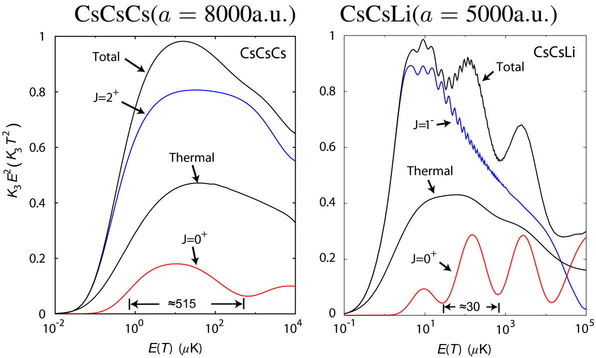
- The energies of the Efimov states E_n are correlated with the oscillations.
- The number of the Efimov states can be counted from the oscillations.

$K_3(E)E^2$ for CsCsLi



Wang, D'Incao, Esry, and Naegerl, PRL, to be published (2010)

$J > 0$ partial waves and thermal effect



- Smooth behavior of $K_3^{J>0}$ does not change the oscillatory structures.
- Thermal average blurs the oscillations dramatically, but the structure can still be seen.

Wang, D'Incao, Esry, and Naegerl, PRL, to be published (2010)

Bypassing thermal effect–BEC collision

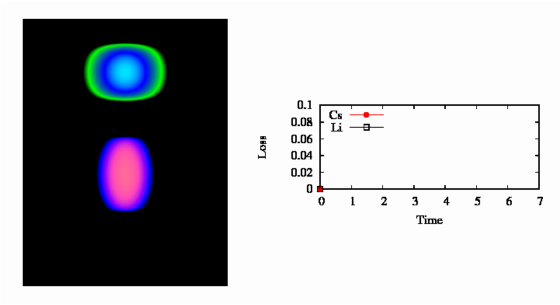
- Advantage: Collisional energy is well-defined and is tunable.
- Question: How does condensate dynamics change the loss?

Solve coupled mean-field equation with loss terms:

$$i\frac{\partial}{\partial t}\phi_{Cs}(\boldsymbol{r},t) = \left(\hat{h}_{Cs} + a_{CsCs}|\phi_{Cs}|^2 + a_{CsLi}|\phi_{Li}|^2\right)\phi_{Cs}$$
$$- i\left(3K_{CsCsCs}|\phi_{Cs}|^4 + 2K_{CsCsLi}|\phi_{Cs}|^2|\phi_{Li}|^2 + K_{CsLiLi}|\phi_{Li}|^4|\right)\phi_{Cs}$$

$$i\frac{\partial}{\partial t}\phi_{Li}(\boldsymbol{r},t) = \left(\hat{h}_{Li} + a_{LiLi}|\phi_{Li}|^2 + a_{CsLi}|\phi_{Cs}|^2\right)\phi_{Li}$$
$$- i\left(3K_{LiLiLi}|\phi_{Li}|^4 + K_{CsCsLi}|\phi_{Cs}|^4 + 2K_{CsLiLi}|\phi_{Cs}|^2|\phi_{Li}|^2\right)\phi_{Li}$$

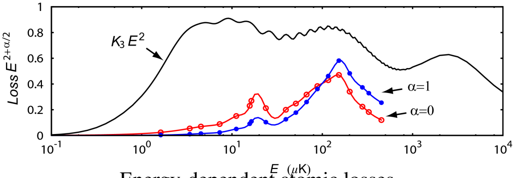
Efimov signatures in the loss



BEC collision at $E = 16\mu\text{K}$ ($a_{\text{CsLi}} = 5 \times 10^3$ Bohr)

The losses for Cs and Li are 2:1

$\Rightarrow$The CsCsLi recombination is the dominant loss process.



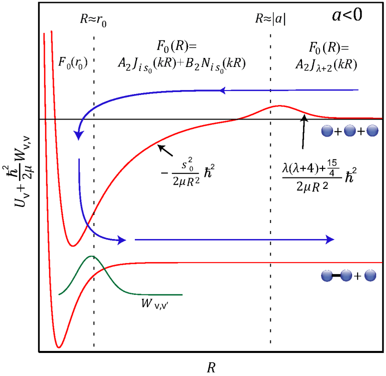
Energy-dependent atomic losses

(Collisional time $\propto E^{\alpha/2}$, where α is in $[0, 1]$.)

The Efimov signature remains in the final losses.

Wang, D'Incao, Esry, and Naegerl, PRL, to be published (2010)

Efimov oscillations in recombination for $a < 0$ at finite energy ($J = 0^+$)



Recombination pathways at finite energy

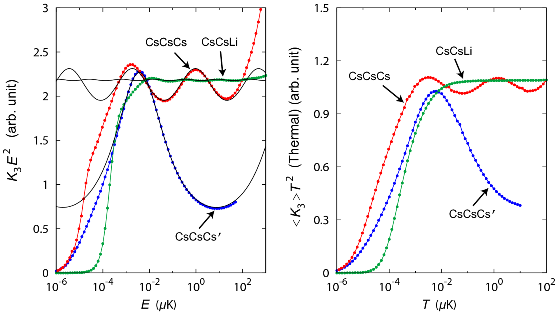
when $\frac{1}{a} \ll k \ll \frac{1}{r_0}$,

$$K_3 = \frac{\frac{384\sqrt{3}\pi^2}{mk^4}\sinh(\pi s_0)\sinh(2\eta)}{\cosh(\pi s_0)+\sin[-2s_0\ln(kr_0)+2\Phi-2\varphi_0]}$$

where

$$\tan\varphi_0 = \frac{\mathrm{Re}[\Gamma(is_0)] - \mathrm{Im}[\Gamma(is_0)]}{\mathrm{Re}[\Gamma(is_0)] + \mathrm{Im}[\Gamma(is_0)]}$$

Oscillations in different systems



Three-body recombination rates for BBB and BBX

- Smaller m_X/m_B, smaller the oscillation period, smaller modulation.
- Thermal averaging has small effect.

Higher partial wave contributions are suppressed!

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Summary

Summary

- We use adiabatic hyperspherical representation to perform analytical and numerical studies on Efimov physics.
- In ultracold atomic systems, the Efimov physics is studied by the scattering processes (recombination, relaxation).
- The Efimov physics extends from broad to narrow Feshbach resonances and from zero-energy regime to finite-energy domain.

Thanks to

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