New developments in few-nucleon scattering

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INT workshop
"Weakly Bound Systems in Atomic and Nuclear Physics"
March 8, 2010

Outline

Collaborators

- C. Romero-Redondo, E. Garrido & R. Alvarez-Rodriguez - *IEM, Madrid (Spain)*
- P. Barletta - *UCL, London (UK)*
- A. Kievsky, L. Girlanda, L.E. Marcucci - *INFN & Pisa University, Pisa (Italy)*

Weakly bound systems in nuclear physics

"Few-nucleon Physics":

- $A \leq 4$:
  - NN/3N forces & Weak/EM currents by EFT
  - Solution of $H\Psi = E\Psi$: quite accurate
  - First part of the talk: "recent results using the HH method"
- Problems:
  - A_y "puzzle" in $N-d$ and $p-{}^3\mathrm{He}$ (see later)
  - Several problems in $N-d$ breakup and $A=4$ scattering
  - Goal: NN/3N/4N observables $\leftrightarrow$ EFT

Weakly bound systems: $A > 4$:

- Extension of the numerical methods very difficult
- Many of these systems are unbound $\leftrightarrow$ need for scattering calculations
- Second part of the talk: "Integral Relations"

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NN & 3N interaction

NN potentials

- ”Old models”: Argonne V18, CD-Bonn, Nijmengen ($\chi^2 \approx 1$)
- Fit of 3N data using non-locality in P-waves (ISuj [Doleschall, 2008])
- Effective field theory
  - J-N3LO [Epelbaum and Coll, 1998-2006]
  - I-N3LO [Entem & Machleidt, 2003]
- Low-q interaction [Bogner and Coll., 2001-2007]

3N potentials

- “Old models”: Tucson-Melbourne [Coon *et al*, 1979, Friar *et al*, 1999]; Brazil [Robilotta & Coelho, 1986]; Urbana [Pudliner *et al*, 1995]
- Effective field theory
  - J-N2LO [Epelbaum *et al*, 2002]
  - N-N2LO [Navratil, 2007]
- Illinois [Pieper *et al*, 2001]
- Under progress: N3LO, Δ, CSB, . . .

Models under study: I-N3LO, AV18, I-N3LO/N-N2LO, AV18/UIX.

Study of the dynamics of 3 and 4 nucleon systems

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \dots$$

Search for accurate solution of $H\Psi = E\Psi$

- Expansion of Ψ on the basis of Hyperspherical Harmonics
- Problems: 1) convergence 2) antisymmetrization of the basis 3) boundary conditions for scattering states, ...
- Accurate, state-of-the-art, calculations of bound and elastic observables
- Treatment of non-local or projecting potential possible
- Hard-core potential $\rightarrow$ inclusion of a correlation factor
- Still to be solved: proper treatment of breakup channels ($N + d \rightarrow N + n + p$)

A high-precision variational approach to three- and four-nucleon bound and zero-energy scattering states, A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, and L. Girlanda J. Phys. G, **35**, 063101 (2008)

The HH method

HH functions

- hyperradius $\rho^2 = \frac{2}{A}\sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{\frac{\xi_1}{\rho}, \ldots, \frac{\xi_{A-1}}{\rho}\}$ (ξ_i Jacobi vectors)
- $T = T\rho + T_\Omega$
- The HH functions $\mathcal{Y}_{[K]}(\Omega)$ are the eigenstates of T_Ω

$$\Phi_n = L_n^{(3A-4)}(\beta\rho)e^{-\beta\rho/2}\,\mathcal{Y}_{[K]}(\Omega)$$

Advantages

Simplified calculation of the matrix elements of

- local/non-local NN & 3N potentials
- coordinate/momentum space interaction

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Scattering calculation

Example: $A - B$ elastic scattering

$$\Omega^F_{LS}(A,B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB} r_{AB})}{q_{AB} r_{AB}}$$

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$$\Omega^\pm_{LS}(A,B) = \Omega^G_{LS}(A,B) \pm i \Omega^F_{LS}(A,B)$$

$$|\Psi_{LS}\rangle = \sum_n a_{LS,n} \Phi_n + |\Omega^F_{LS}(p, {}^3\mathrm{He})\rangle + \sum_{L'S'} T_{LS,L'S'} |\Omega^+_{L'S'}(p, {}^3\mathrm{He})\rangle$$

- $T_{LS,L'S'}$ = T-matrix elements
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Study of the 3N force (1)

Fix the parameters of the 3N force ($\sim$ 5):

1. ^{3}H and ^{4}He binding energies
2. $n-d$ doublet scattering lenght ($J = 1/2$)
3. $n-d$ and $p-d$ elastic scattering (A_y)

Potential	$B(^3H)$ (MeV)	$B(^4He)$ (MeV)	$^2a_{nd}$ (fm)
AV18	7.624	24.22	1.258
I-N3LO	7.854	25.38	1.100
AV18/TM'	8.440	28.31	0.623
AV18/UIX	8.479	28.48	0.578
I-N3LO/N-N2LO	8.474	28.37	0.675
Exp.	8.482	28.30	$0.645 \pm 0.003 \pm 0.007$

n-d zero-energy scattering= first "excited" state of ^{3}H

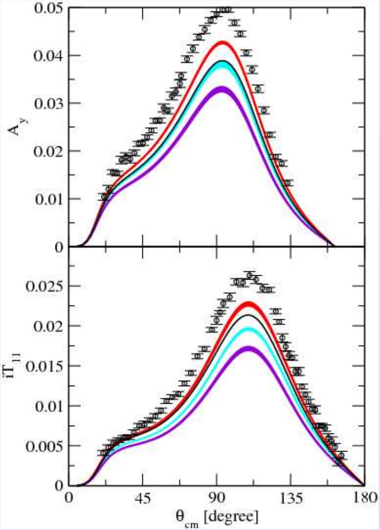
Study of the 3N force (2)

Urbana $\quad W(1,2,3) = aW^{a}_{2\pi}(1,2,3) + cW^{c}_{2\pi}(1,2,3) + U_0 W_R(1,2,3)\ .$

Potential	a	c/a	U_0	$B(^3\text{H})$ (MeV)	$B(^4\text{He})$ (MeV)	$^2a_{nd}$ (fm)
AV18				7.624	24.22	1.258
AV18+URIX	-0.0293	0.25	0.0048	8.479	28.48	0.578
AV18+URIX-1	-0.0200	1.625	0.0176	8.484	28.33	0.644
AV18+URIX-2	-0.0250	1.25	0.0182	8.484	28.34	0.644
AV18+URIX-3	-0.0293	1.00	0.0181	8.484	28.33	0.643
Exp.				8.482	28.30	$0.645 \pm 0.003 \pm 0.007$

- Significative modifications of the 3N force
- We have found 3 families of the 3N force: the N-N2LO, Urbana, and TM- families
- In each case, same operatorial structure (N2LO)
- They differ in the short-range part (regularization)

Study of the 3*N* force (3)



Comparison with p-d: A_y and iT_{11}

$N - d$ elastic scattering
solid curve: AV18+UIX
red band= AV18+N-N2LO-family
cyan band= AV18+Urbana-family
violet band= AV18+TM-family

work in progress

$n - {}^3\mathrm{He}$ scattering lengths

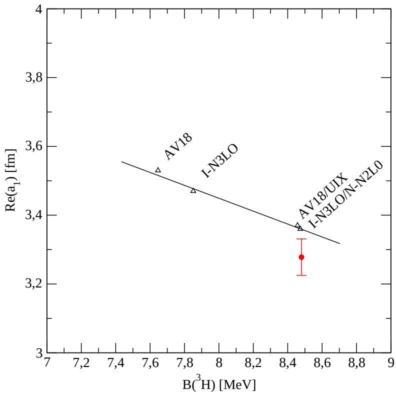
$$|\Psi_{LS}\rangle = \sum_n a_{LS,n}\Phi_n + |\Omega^F_{LS}(n,{}^3\mathrm{He})\rangle + \sum_{L'S'} T^{el}_{LS,L'S'}|\Omega^+_{L'S'}(n,{}^3\mathrm{He})\rangle + \sum_{L'S'} T^{ex}_{LS,L'S'}|\Omega^+_{L'S'}(p,{}^3\mathrm{H})\rangle$$

$$a_S = -\lim T^{el}_{0S,0S}/q_{n^3\mathrm{He}}$$

Int.	Method	a_0 (fm)	a_1 (fm)
AV18	HH	$8.03 - i5.18$	$3.53 - i0.0076$
	RGM	$7.79 - i4.98$	$3.47 - i0.0066$
	FY	$7.71 - i5.25$	$3.43 - i0.0082$
AV18/UIX	HH	$7.89 - i3.34$	$3.37 - i0.0055$
	RGM	$7.63 - i4.05$	$3.31 - i0.0051$
I-N3LO	HH	$7.49 - i5.05$	$3.45 - i0.0058$
	FY		$3.56 - i0.0070$
	AGS	$7.82 - i4.51$	$3.47 - i0.0068$
I-N3LO/N-N2LO	HH		$3.36 - i0.0042$
Exp.		$7.370(58) - i4.448(5)$	$3.278(53) - i0.001(2)$

RGM: Hofmann & Hale (2003); FY: Lazauskas (2009);
AGS: Deltuva (2007)

Triplet $n - {}^3\mathrm{He}$ scattering lenght vs. $B({}^3\mathrm{H})$



Also calculated by Deltuva & Fonseca, (2007)

Integral relations (IR) for the process $A + B \rightarrow A + B$

Example

$A = 2$, central potential, S-wave, no spin

$$(H - E)\Psi(\boldsymbol{r}) = (-\frac{\hbar^2}{2\mu}\nabla^2 + V - E)\Psi(\boldsymbol{r}) = 0 \qquad E = q^2/2\mu$$

$$\begin{aligned} \Omega^F &= \sqrt{\frac{2\mu q}{4\pi}}\frac{\sin(qr)}{qr} \\ \Omega^G &= \sqrt{\frac{2\mu q}{4\pi}}\frac{\cos(qr)}{qr}\Big(1 - \exp(-\gamma r)\Big) \ . \end{aligned}$$

- γ "regularization" parameter
- Normalization chosen so that

$$\langle \Omega^F | H - E | \Omega^G \rangle - \langle \Omega^G | H - E | \Omega^F \rangle = 1$$

$$\psi(r \rightarrow \infty) \longrightarrow A\Omega^F(r) + B\Omega^G(r) \qquad \tan\delta = B/A$$

Exact relations

$$\begin{aligned} \langle\Psi|H-E|\Omega^G\rangle - \langle\Omega^G|H-E|\Psi\rangle &= A \\ \langle\Psi|H-E|\Omega^F\rangle - \langle\Omega^F|H-E|\Psi\rangle &= -B \end{aligned}$$

If Ψ is the exact wave function: $(H-E)\Psi = 0$

$$\frac{B}{A} = -\frac{\langle\Psi|H-E|\Omega^F\rangle}{\langle\Psi|H-E|\Omega^G\rangle}$$

These relations represent an efficient method to extract A & B

1. Solve $H\Psi = E\Psi$ with some method
2. extract A, B (and $\tan\delta$) using the IR

Similar methods

1. Harris PRL **19**, 173 (1967)
2. Nollet *et al.*, PRL **99**, 022502 (2007)
3. Suzuki *et al.*, NP**A823**, 1 (2009)

The variational character of the IR (1)

- exact wave function $\Psi = \Phi + A\Omega^F + B\Omega^G$
  - Φ short range part | A normalization (considered fixed in the following)
- trial wave function $\overline{\Psi} = \overline{\Phi} + A\Omega^F + \overline{B}\Omega^G$

Typical case: Φ_n, $n = 1, \ldots$ complete set of square integrable functions

$$\Phi = \sum_{n=1}^{\infty} a_n \Phi_n \qquad \overline{\Phi} = \sum_{n=1}^{M} \overline{a}_n \Phi_n$$

- $\overline{\Psi} - \Psi \sim \epsilon \quad \epsilon$ "small"

$$IR \qquad \left[\frac{B}{A}\right] = -\frac{\langle\overline{\Psi}|H - E|\Omega^F\rangle}{\langle\overline{\Psi}|H - E|\Omega^G\rangle} \qquad \textit{differs from } B/A \textit{ by } \epsilon^2$$

- $(\overline{a}_n - a_n)_{[n=1,M]} \sim \epsilon \qquad (a_n)_{[n=M+1,\infty]} \sim \epsilon \qquad \overline{B} - B \sim \epsilon$

The variational character of the IR (2)

$$\left[\frac{B}{A}\right] = -\frac{\langle\overline{\Psi}|H-E|\Omega^F\rangle}{\langle\overline{\Psi}|H-E|\Omega^G\rangle} \qquad \overline{\Psi} = \overline{\Phi} + A\Omega^F + \overline{B}\Omega^G \qquad \overline{\Phi} = \sum_{n=1}^{M} \overline{a}_n\Phi_n$$

- Use $\Omega^F = (1/A)(\Psi - \Phi - B\Omega^G)$ (Ψ = exact wave function)

$$\left[\frac{B}{A}\right] = \frac{B}{A} - \frac{1}{A}\frac{\langle\overline{\Psi}|H-E|\Phi\rangle}{\langle\overline{\Psi}|H-E|\Omega^G\rangle}$$

1. $\langle\overline{\Psi}|H-E|\Phi\rangle = \langle\Phi|H-E|\overline{\Psi}\rangle$ (Φ is short-range)
2. Typically $\overline{\Psi}$ is determined by $\langle\Phi_n|H-E|\overline{\Psi}\rangle = 0 \quad n = 1,\ldots,M$

$$\langle\sum_{n=1}^{\infty} a_n\Phi_n|H-E|\overline{\Psi}\rangle = \langle \sum_{n=M+1}^{\infty} a_n\Phi_n|H-E|\overline{\Psi}\rangle = \langle \sum_{n=M+1}^{\infty} a_n\Phi_n|H-E|\epsilon\rangle \sim \epsilon^2$$

3. It can also be derived from the KVP [PRL **103**, 090402 (2009)]

Generalization to $A > 2$

$$\Omega^{F}_{LS}(A,B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{F_L(\eta, q_{AB} r_{AB})}{q_{AB} r_{AB}}$$

$$\Omega^{G}_{LS}(A,B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB} r_{AB})}{q_{AB} r_{AB}} (1 - e^{-\gamma r_{AB}})^{2L+1}$$

- D_{AB} = normalization factors chosen so that

$$\langle \Omega^F(A,B) | H - E | \Omega^G(A,B) \rangle - \langle \Omega^G(A,B) | H - E | \Omega^F(a,B) \rangle = 1 \qquad D_{AB} = \sqrt{2 \mu_{AB} q_{AB}}$$

- $i \equiv LS, AB$: $i = 1, \ldots, \mathcal{N}$ asymptotic channels

$$\overline{\Psi}_i \to A_{ij} \Omega^F_j + \overline{B}_{ij} \Omega^G_j \qquad i = 1, \ldots, \mathcal{N} \qquad K = A^{-1} \overline{B} \qquad K - matrix$$

Generalized IR

$$A_{ij} = < \overline{\Psi}_i | H - E | \Omega^G_j > \qquad \overline{B}_{ij} = - < \overline{\Psi}_i | H - E | \Omega^F_j > \qquad [K] = A^{-1} \overline{B}$$

Applications

Adiabatic HH

- Calculation of $\overline{\Psi}$ using the adiabatic HH expansion
- See next talk by Eduardo
- IR first derived for this case P. Barletta *et al.*, PRL **103**, 090402 (2009)
- Direct application of the boundary condition ($A_{ij} = \delta_{ij}$) only for $\rho \to \infty$

Use of the short range character of the IR

- "interacting region" $\equiv \mathcal{V}_I$ = region where all particles are close together ($\rho \leq 30$ fm)
- $(H - E)\Omega_i^{F,G} = 0$ for r_{AB} outside $\mathcal{V}_I$
- $\overline{\Psi}_i$ need to be known only in $\mathcal{V}_I$

1. Use of "bound-state"–like wave function
2. Phase-shifts of scattering between charged particles using a screened Coulomb interaction

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1. Use of "bound-state"–like wave function
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"Bound-state"–like wave function

- Expansion over a square integrale function basis

$$\overline{\Psi} = \sum_{n=1}^{M} a_n \Phi_n \qquad \Phi_n \to 0 \text{ outside } \mathcal{V}_I$$

- Examples: HO basis, HH basis, etc.

- Eigenvalue problem

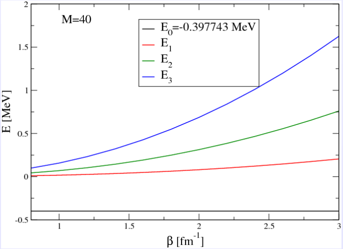
$$\langle \Phi_n | H - E | \Phi_{n'} \rangle a_{n'} = 0$$

- The lowest eigenvalues: $\rightarrow$ bound states
- The others eigenvalues: $\rightarrow$ scattering states (wrong behaviour outside $\mathcal{V}_I$)
- $\rightarrow$ use them in the IR!

Example (1): A=2 case, S-wave

$$V(r) = -V_0 \exp\left(-r^2/r_0^2\right) \quad V_0 = 51.5 \text{ MeV } r_0 = 1.6 \text{ fm}$$

$$\overline{\Psi} = \sum_{n=1}^{M} a_n \Phi_n \qquad \Phi_n = \mathcal{L}_n^{(2)}(\beta r) \exp -(\beta r/2)$$



M	10	20	30	40
E_0	-0.395079	-0.397740	-0.397743	-0.397743
E_1	0.536349	0.116356	0.048091	0.026008
$\tan\delta$	-1.507280	-0.622242	-0.392005	-0.286479
$[B/A]$	-1.522377	-0.621938	-0.392021	-0.286480

Example 2: $A = 3$ case

- Elastic $N - d$ scattering $E_{c.m.} < 2.2$ MeV
- For fixed J and parity
  - $J^\pi = \frac{1}{2}^+$: $LS = 2S_{\frac{1}{2}}, 4D_{\frac{1}{2}}$ (1 bound state)
  - $J^\pi = \frac{1}{2}^-$: $LS = 2P_{\frac{1}{2}}, 4P_{\frac{1}{2}}$ (no bound state)
  - $J^\pi = \frac{3}{2}^+$: $LS = 4S_{\frac{3}{2}}, 2D_{\frac{3}{2}}, 4D_{\frac{3}{2}}$ (no bound state)
  - etc.
- Expansion basis

$$\Phi_n = \mathcal{L}_m^{(5)}(\beta\rho) \exp(-\beta\rho/2) \times \mathcal{Y}_{[K]}$$

- $\rho =$ hyperradius, $\mathcal{Y}_{[K]}$= HH functions
- Typically $m \to 20$

There are 2 non-linear parameters

- β in Φ_n γ in Ω^G

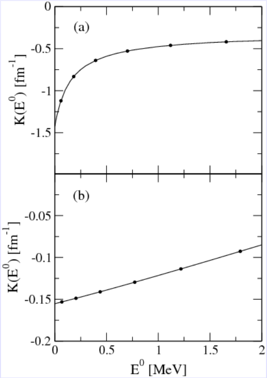
$$\Omega^G_{LS}(A,B) = \sqrt{\frac{1}{N}} D_{AB} \sum_{perm.=1}^{N} \left[Y_L(\hat{\mathbf{r}}_{AB})[\phi_A \phi_B]_S \right]_{JJ_z} \frac{G_L(\eta, q_{AB} r_{AB})}{q_{AB} r_{AB}} (1 - e^{-\gamma r_{AB}})^{2L+1}$$

$n - d$ low energy scattering

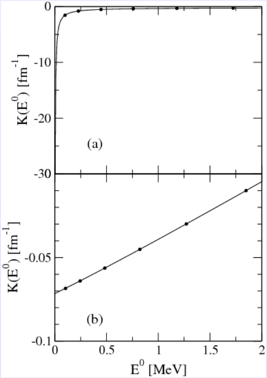
Central potential: Malfliet-Tjon I-III

$$V_{S=0}(r) = \frac{1438.72}{r} e^{-3.11r} - \frac{513.968}{r} e^{-1.55r} \mid V_{S=1}(r) = \frac{1438.72}{r} e^{-3.11r} - \frac{626.885}{r} e^{-1.55r}$$

$$K(E^0) = k \cot \delta$$

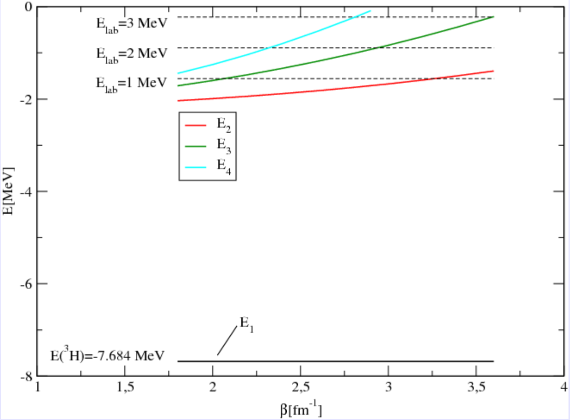


$$K(E^0) = C_0^2(\eta) k \cot \delta + 2k\eta h(\eta)$$

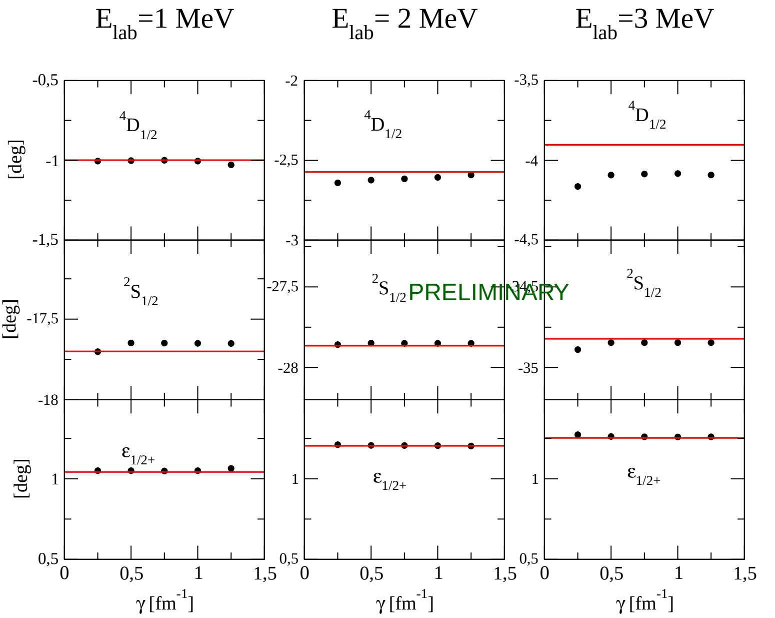


Example 3: $\frac{1}{2}^+$, AV18 potential

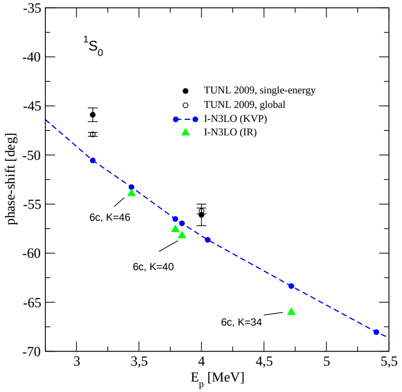
- Problem: how to obtain 2 or more Ψ at the same E?



- (1) Look at the eigenvalues $-2.2 < E < 0$ MeV (2) vary β


E_lab=1 MeV
E_lab= 2 MeV
E_lab=3 MeV
4D1/2
2S1/2
ε1/2+
PRELIMINARY
[deg]
γ [fm-1]
-0,5
-1
-1,5
-17,5
-18
1
0,5
-2
-2,5
-3
-27,5
-28
-3,5
-4
-4,5
-34,5
-35
0
0,5
1
1,5

Example 4: Preliminary results for $A = 4$



PRELIMINARY

$K = 34 \;\; E = -3.59 \;\; E_p = 3.55$
$K = 40 \;\; E = -4.26 \;\; E_p = 2.87$
$K = 46 \;\; E = -4.55 \;\; E_p = 2.58$

Conclusions

Integral Relations

- The IR could allow for a variety of applications for $A > 4$ systems
- They can be used with different methods (GFMC, NCSM, EIHH, AHH, ...)
  - Solution of $H\Psi = E\Psi$ in $\mathcal{V}_I$
  - Calculation of the overlap integrals (via Monte Carlo also for "large" A)
- Tests in $A = 3, 4$ (elastic channels) OK

Future work

- A better treatment of coupled channels
- Extension to breakup channels

In progress

- Extension of our HH method to $A > 4$
- Work in progress with M. Gattobigio, INL, Nice (France)

Non-symmetrical basis

$\Psi = \sum_k a_k \Phi_k$

- It would be easy to use states $\widetilde{\Phi}_k$ constructed without any particular symmetry
- In fact: to construct antisymmetric states very difficult as A increases
  $[\Phi_k = \sum_{k'} b_{k'} \widetilde{\Phi}_k]$
- Idea: solve $H \sum_k a_k \widetilde{\Phi}_k = E \sum_k a_k \widetilde{\Phi}_k$
- since H is a symmetric operators, the eigenstates are also eigenstates of the symmetric group S_A
- Last step: select the antisymmetrical eigenstates
  - multiplicity
  - diagonalizing some operator

"transposition operator" $[(2)] = \sum_{i<j} (i \leftrightarrow j)$

Application for 5 bosons

irreps of S_5	Multiplicity	[(2)]
[5]	1	10
[4, 1]	4	5
[3, 2]	5	2
[3, 1, 1]	6	0
[2, 2, 1]	5	− 2
[2, 1, 1, 1]	4	− 5
[1, 1, 1, 1, 1]	1	−10

Eigenvalues from $H\Psi = E\Psi$

$\Psi = \sum_k a_k \widetilde{\Phi}_k$

Eigenvalue	[(2)]	irrep
− 7.9771E + 00	5.0000E + 00	[4, 1]
−7.9771E + 00	5.0000E + 00	
−7.9771E + 00	5.0000E + 00	
−7.9771E + 00	5.0000E + 00	
−5.0673E + 00	1.0000E + 01	[5]
−1.4406E − 01	5.0000E + 00	[4, 1]
−1.4406E − 01	5.0000E + 00	
−1.4406E − 01	5.0000E + 00	
−1.4406E − 01	5.0000E + 00	
1.9439E + 00	2.0000E + 00	[3, 2]
1.9439E + 00	2.0000E + 00	
1.9439E + 00	2.0000E + 00	
1.9439E + 00	2.0000E + 00	
1.9439E + 00	2.0000E + 00	
6.4904E + 00	−2.0000E + 00	[2, 2, 1]
6.4904E + 00	−2.0000E + 00	
6.4904E + 00	−2.0000E + 00	
6.4904E + 00	−2.0000E + 00	
6.4904E + 00	−2.0000E + 00	

Other publications 2008/2009

1. On a redundancy in the parity-violating 2-nucleon contact Lagrangian, L. Girlanda, Phys. Rev. C **77**, 067001 (2008)
2. Neutron spin rotation in $\vec{n} - d$ scattering, R. Schiavilla, M. Viviani, L. Girlanda L, A. Kievsky, and L. E. Marcucci, Phys. Rev. C **78**, 014002 (2008)
3. Quantum Monte Carlo calculations of magnetic moments and M1 transitions in $A \leq 7$ nuclei including meson-exchange currents, L.E. Marcucci, M. Pervin, S.C. Pieper, R. Schiavilla, R.B. Wiringa, Phys. Rev. C **78**, 065501 (2008)
4. $N - d$ elastic scattering using the hyperspherical harmonics approach with realistic local and nonlocal interactions, L.E. Marcucci, A. Kievsky, L. Girlanda, S. Rosati, and M. Viviani, Phys. Rev. C **80**, 034003 (2009)
5. Electromagnetic currents and magnetic moments in chiral effective field theory (χEFT), S. Pastore, L. Girlanda, R. Schiavilla, M. Viviani, and R. B. Wiringa, Phys. Rev. C **80**, 034004 (2009)
6. Relativity constraint on the two-nucleon contact interaction, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, `arXiv:1001:3676`