INT Workshop on Weakly Bound Systems

THE NO CORE SHELL MODEL APPROACH

- Introduction to the method
- Applications to light nuclei – selected results
- Applications to trapped systems – 2- and 3-body systems

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Description of the method

The nuclear many-body problem

$$H_{int} = \frac{1}{A}\sum_{i>j=1}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i>j=1}^{A} V_{ij} + \sum_{i>j>k=1}^{A} V_{ijk} + \dots$$

- (local and non-local) "high precision" NN interactions
- three-body forces

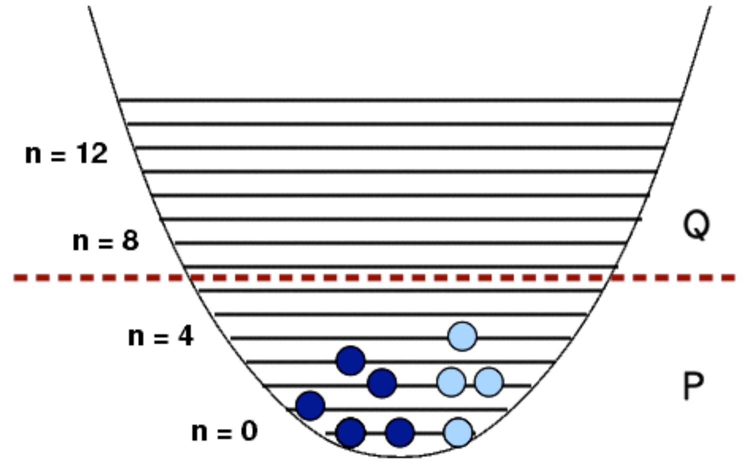
$$H = H_{int} + \frac{\vec{P}_{CM}^2}{2mA} + \frac{1}{2}mA\omega^2\vec{R}_{CM}^2$$

Lipkin 1957

$$= \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + \sum_{i<j=1}^{A}\left(V_{ij} - \frac{m\omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right) + \sum_{i<j<k=1}^{A} V_{ijk} + \dots$$

- used to derive the effective interaction
- mean-field like contribution

NCSM



- direct diagonalization method
- basis states constructed from HO wave functions
- all particles are allowed to interact
- effective interaction constructed via a unitary transformation $\mathcal{H} = UHU^\dagger$
- "cluster approximation"
- short-range effects accounted by the effective interaction
- long-range and many-body effects accounted by increasing the model space
- *quite successful in describing low-energy properties of light nuclei*

$$(P+Q)\mathcal{H}(P+Q)|\Psi\rangle = E_\Psi|\Psi\rangle$$

$$P\mathcal{H}_{eff}P = P\mathcal{H}P + P\mathcal{H}Q\frac{1}{E_\Psi - Q\mathcal{H}Q}Q\mathcal{H}P$$

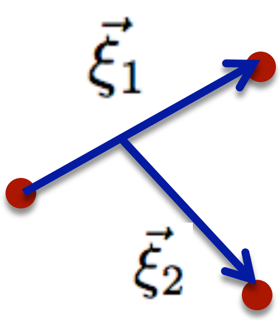
$$P\mathcal{O}_{eff}P = P\mathcal{O}P + P\mathcal{H}Q\frac{1}{E_\Phi - Q\mathcal{H}Q}Q\mathcal{O}P + P\mathcal{O}Q\frac{1}{E_\Psi - Q\mathcal{H}Q}Q\mathcal{H}P$$

$$P\mathcal{H}Q = Q\mathcal{H}P = 0$$

$$+P\mathcal{H}Q\frac{1}{E_\Phi - Q\mathcal{H}Q}Q\mathcal{O}Q\frac{1}{E_\Psi - Q\mathcal{H}Q}Q\mathcal{H}P$$

Basis states in NCSM

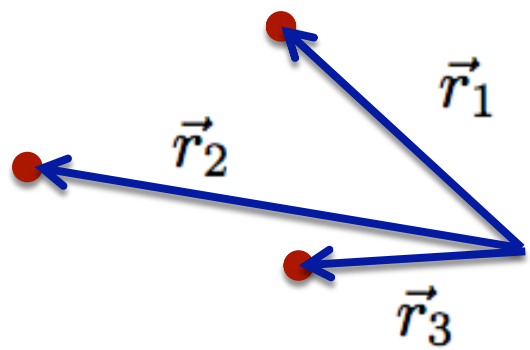
Relative coordinates



$$\Psi(\vec{\xi}_1, \vec{\xi}_2) = \mathcal{A}\left[\phi_{nlj}(\vec{\xi}_1)\phi_{n'l'j'}(\vec{\xi}_2)\right]_{JT}$$

Slater determinant basis

$$\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \psi_{c.m.}\psi_{int}$$

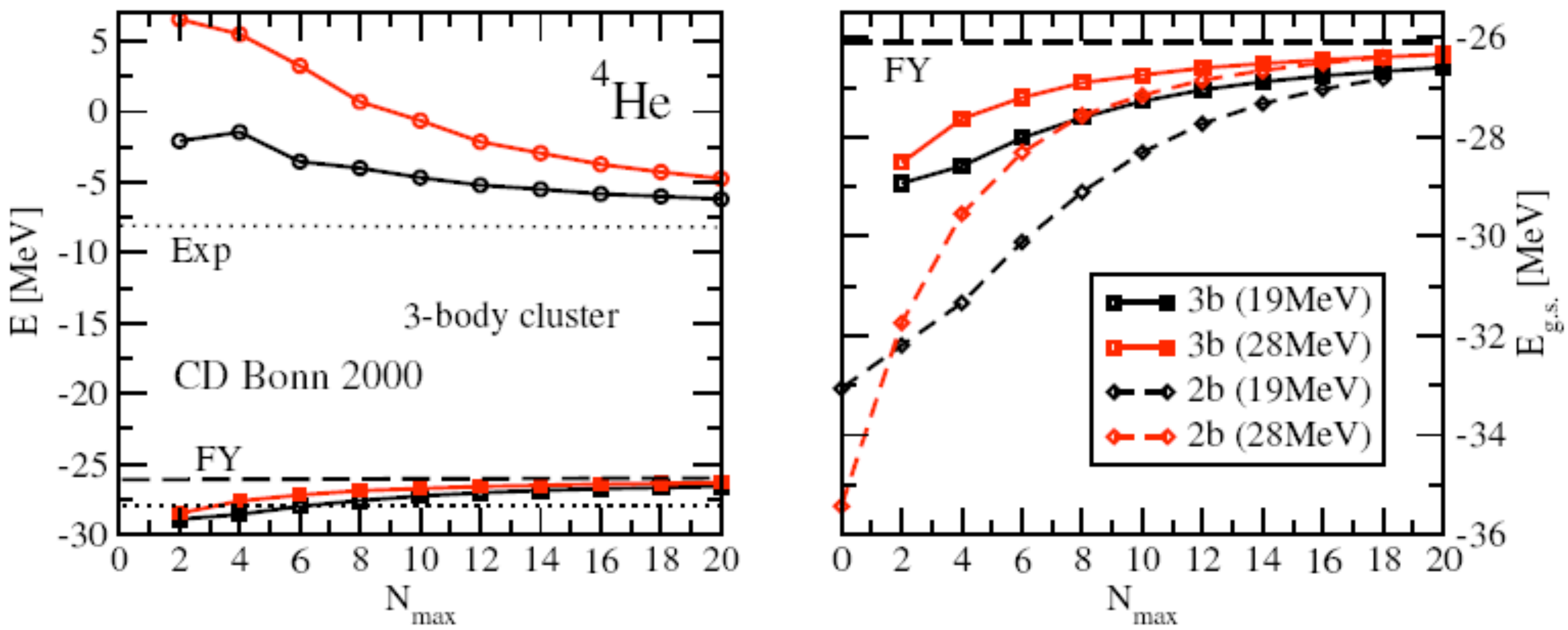


$$\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \begin{vmatrix} \phi_{n_1l_1j_1}(\vec{r}_1) & \phi_{n_2l_2j_2}(\vec{r}_1) & \phi_{n_3l_3j_3}(\vec{r}_1) \\ \phi_{n_1l_1j_1}(\vec{r}_2) & \phi_{n_2l_2j_2}(\vec{r}_2) & \phi_{n_3l_3j_3}(\vec{r}_2) \\ \phi_{n_1l_1j_1}(\vec{r}_3) & \phi_{n_2l_2j_2}(\vec{r}_3) & \phi_{n_3l_3j_3}(\vec{r}_3) \end{vmatrix}$$

$$2n + l + 2n' + l' \leq N_{max} \longleftrightarrow 2n_1 + l_1 + 2n_2 + l_2 + 2n_3 + l_3 \leq N_{max}$$

“Cluster” approximation

- decoupling condition ($P\mathcal{H}Q = Q\mathcal{H}P = 0$) for a-body $(a < A)$ – no expansion
- effective interaction used in solving the A-body problem



Standard approach

$$PH_{eff}P|n\rangle = E_n|n\rangle$$

↓

observables

$$\langle n|\hat{O}|m\rangle \qquad\qquad \sum_{m\neq n} \frac{\langle n|\hat{O}_1|m\rangle\langle m|\hat{O}_2|n\rangle}{E_m - E_n}$$

- all results can depend on the size of the model space
- "bare" operators for other observables are usually employed

Applications to light nuclei

Energy spectrum

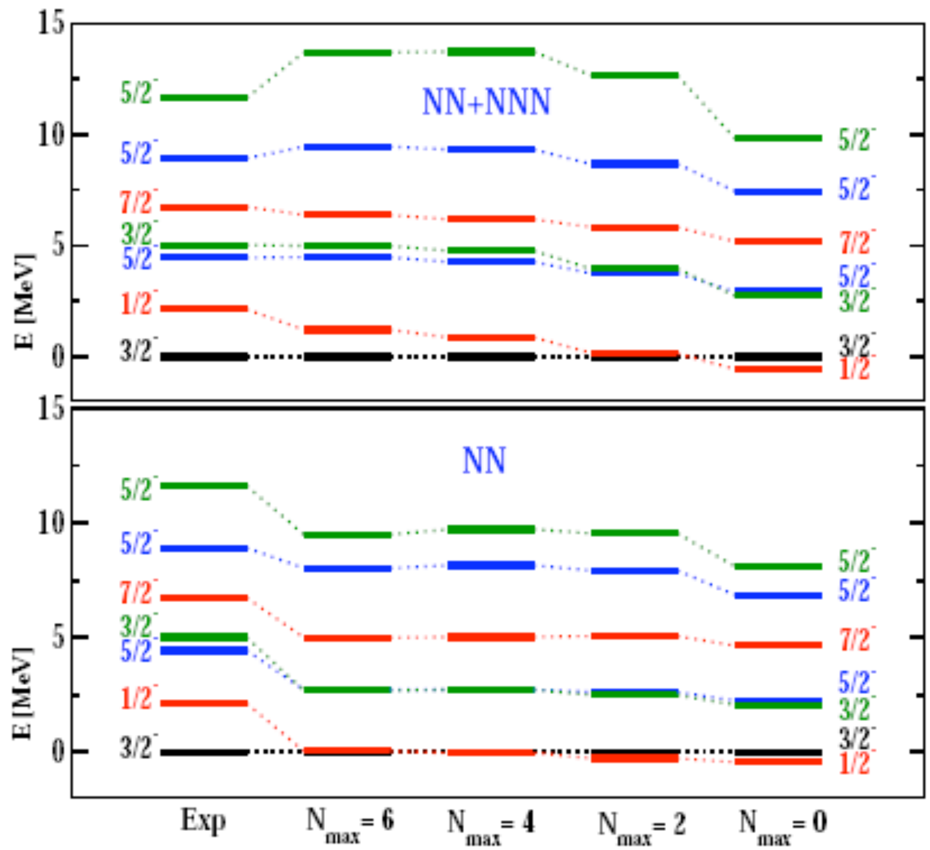


FIG. 3: ^{11}B excitation spectra as function of the basis space size $N_{\max}$ at $\hbar\Omega = 15$ MeV and comparison with experiment. The isospin of the states depicted is T=1/2.

Navratil et. al., 2008

More examples

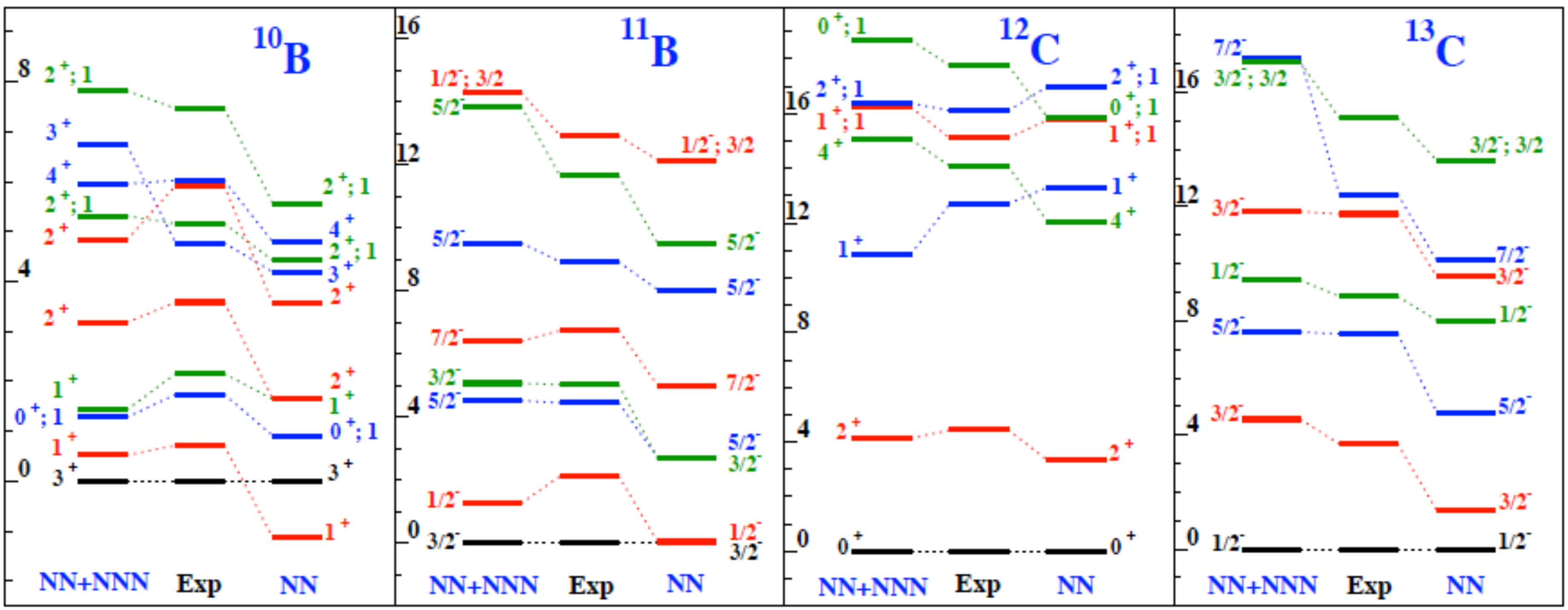


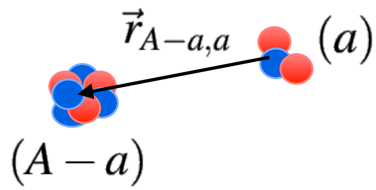
FIG. 4: States dominated by p-shell configurations for ^{10}B, ^{11}B, ^{12}C, and ^{13}C calculated at $N_{\max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin T=0 or 1/2, the isospin label is explicitly shown only for states with T=1 or 3/2. The excitation energy scales are in MeV.

Navratil et. al., 2008

Courtesy S. Quaglioni

Resonating-group method

- Ansatz: $\Psi^{(A)} = \sum_{\nu} \hat{\mathcal{A}} \left[\psi_{1\nu}^{(A-a)} \psi_{2\nu}^{(a)} \varphi_{\nu}(\vec{r}_{A-a,a}) \right] = \sum_{\nu} \int d\vec{r}\, \varphi_{\nu}(\vec{r})\, \hat{\mathcal{A}}\, \Phi_{\nu\vec{r}}^{(A-a,a)}$



$$\Phi_{\nu\vec{r}}^{(A-a,a)} = \psi_{1\nu}^{(A-a)} \psi_{2\nu}^{(a)} \delta(\vec{r} - \vec{r}_{A-a,a})$$

- The many-body Schrodinger equation is mapped onto:

$$H\Psi^{(A)} = E\Psi^{(A)} \longrightarrow \sum_{\nu} \int d\vec{r} \left[\mathcal{H}_{\mu\nu}^{(A-a,a)}(\vec{r}\,',\vec{r}) - E\mathcal{N}_{\mu\nu}^{(A-a,a)}(\vec{r}\,',\vec{r}) \right] \varphi_{\nu}(\vec{r}) = 0$$

Hamiltonian kernel $\langle \Phi_{\mu\vec{r}\,'}^{(A-a,a)} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi_{\nu\vec{r}}^{(A-a,a)} \rangle$

$\langle \Phi_{\mu\vec{r}\,'}^{(A-a,a)} | \hat{\mathcal{A}}^2 | \Phi_{\nu\vec{r}}^{(A-a,a)} \rangle$ **Norm kernel**

- Input: $\psi_{1\nu}^{(A-a)} \psi_{2\nu}^{(a)}$ ← eigenstates of $H_{(A\text{-}a)}$, $H_{(a)}$ in the NCSM basis

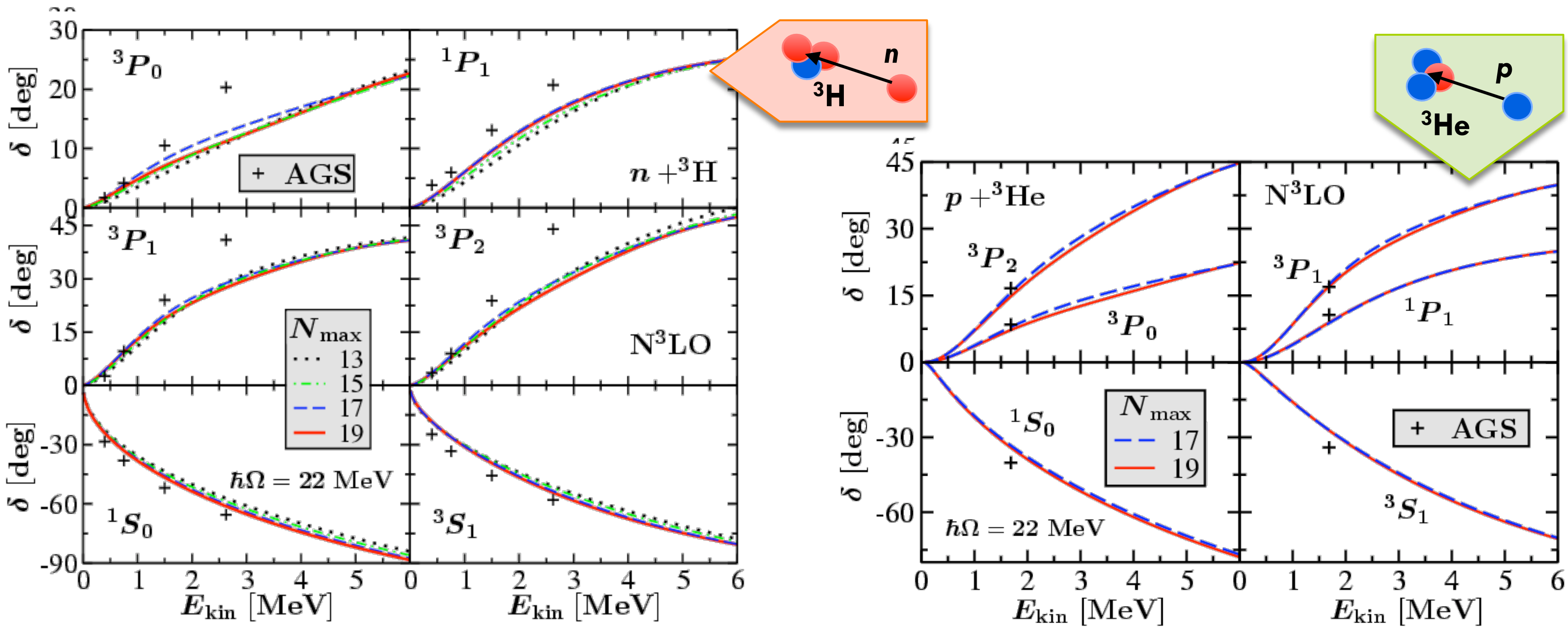
- Output (e.g., *R*-matrix method on Lagrange mesh): $\varphi_{\nu}(\vec{r})$, scattering matrix

NCSM/RGM: NCSM microscopic wave functions for the clusters involved, and realistic (bare or derived NCSM effective) interactions among nucleons.

Courtesy S. Quaglioni

The A=4 system as a test ground for the NCSM/RGM approach within the single-nucleon-projectile basis

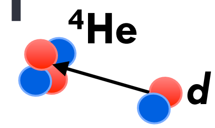
- NCSM/RGM calculation with n + ^{3}H(g.s.) and p + ^{3}He(g.s.), respectively
- χEFT N^{3}LO NN potential: convergence with 2-body effective interaction
- Benchmark: AGS results (+), Deltuva & Fonseca, PRC**75**, 014005 (2007)



The omission of A = 3 partial waves with 1/2 < J ≤ 5/2 leads to effects of comparable magnitude on the AGS results. Need to include target excited (here breakup) states!

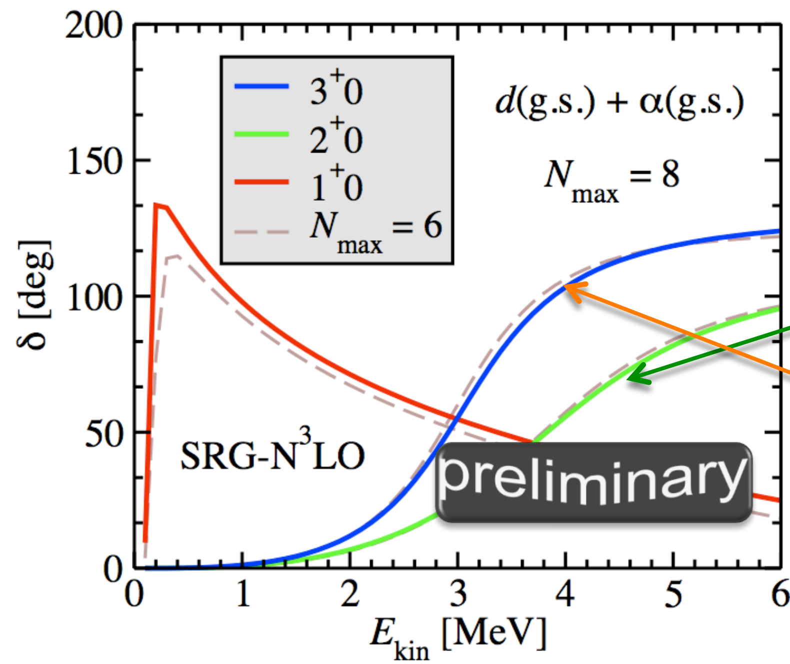
Courtesy S. Quaglioni

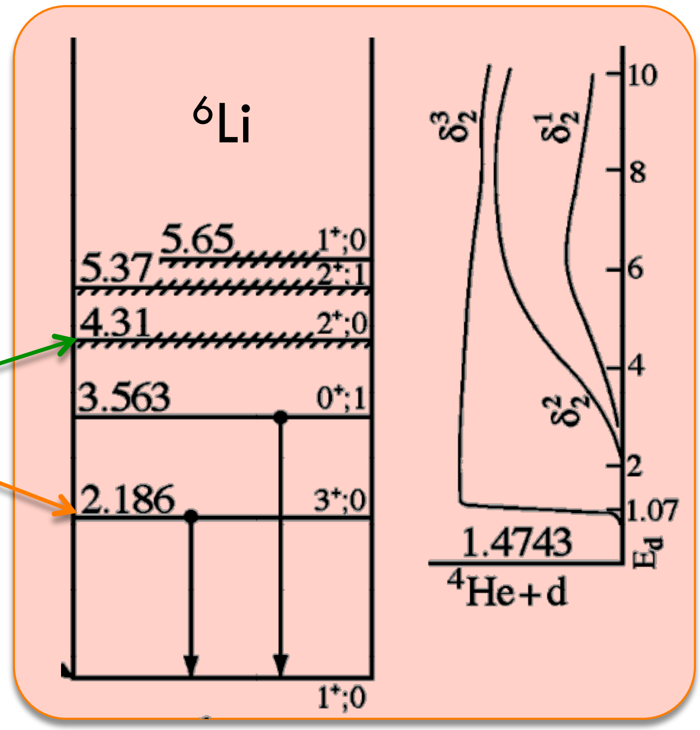
NCSM/RGM *ab initio* calculation of *d*-^{4}He scattering



- N_{max} = 8 NCSM/RGM calculation with *d*(g.s.) + ^{4}He(g.s.)
- SRG-N^{3}LO potential with Λ = 2.02 fm^{-1}

Courtesy S. Quaglioni





- Calculated two resonances: 2^+0, 3^+0
- The 1^+0 g.s. is still unbound: convergence moves towards bound state

NCSM application relevant to physics beyond standard model

One-body contribution:

$$D^{(1)} = \langle 0| \sum_{i=1}^{A} \frac{1}{2} \left[(d_p + d_n) + (d_p - d_n)\, \tau_z(i) \right] \sigma_z(i) |0\rangle$$

$$d_{\substack{p\\n}} \approx \mp \frac{e}{4\,\pi^2\, m_N} (\bar{G}_\pi^0 - \bar{G}_\pi^2) \ln\left(\frac{m_N}{m_\pi}\right)$$

Two-body contributions:

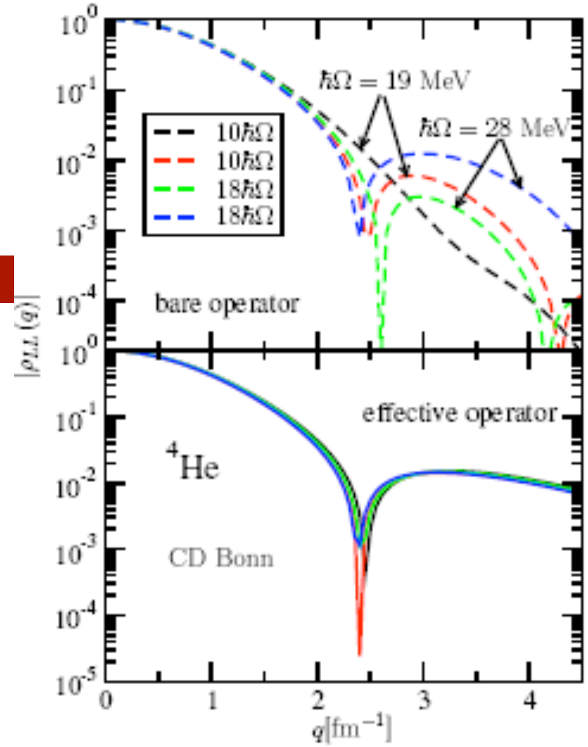
$$D^{(2)} = \langle 0|\, \hat{D}_z |\tilde{0}\rangle + \text{c.c.} \qquad \hat{D}_z = \frac{e}{2} \sum_{i=1}^{A} (1 + \tau_i^z)\, z_i$$

$$|\tilde{0}\rangle = \sum_{n \neq 0} \frac{1}{E_0 - E_n} |n\rangle\langle n| H_{\not P \not T} |0\rangle = G(E_0) H_{\not P \not T} |0\rangle$$

	$\bar{G}_\pi^0$	$\bar{G}_\pi^1$	$\bar{G}_\pi^2$
neutron	0.010	0.000	-0.010
deuteron	0.000	0.015	0.000
^{3}He	0.024	0.023	0.027
^{3}H*	-0.024	0.023	-0.027

Stetcu et. al., 2008

Other observables



$$\omega \to b = \frac{1}{\sqrt{\mu\omega}}$$

Good description only for $r \sim b \ (q \sim 1/b)$

Stetcu et. al, 2005, 2006

Nucleus	Observable	Model Space	Bare operator	Effective operator
^{2}H	Q_0	$4\hbar\Omega$	0.179	0.270
^{6}Li	$B(E2, 1^+0 \to 3^+0)$	$2\hbar\Omega$	2.647	2.784
^{6}Li	$B(E2, 1^+0 \to 3^+0)$	$10\hbar\Omega$	10.221	-
^{6}Li	$B(E2, 2^+0 \to 1^+0)$	$2\hbar\Omega$	2.183	2.269
^{6}Li	$B(E2, 2^+0 \to 1^+0)$	$10\hbar\Omega$	4.502	-
^{10}C	$B(E2, 2_1^+0 \to 0^+0)$	$4\hbar\Omega$	3.05	3.08
^{12}C	$B(E2, 2_1^+0 \to 0^+0)$	$4\hbar\Omega$	4.03	4.05
^{4}He	$\langle g.s.\vert T_{rel}\vert g.s.\rangle$	$8\hbar\Omega$	71.48	154.51

Applications to trapped systems

Many-body problem in a trap

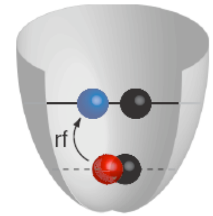
$$H_A = \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + C_0 \sum_{i<j=1}^{A} \delta^{(3)}(\vec{r}_i - \vec{r}_j)$$

$$H = H_{int} + \frac{\vec{P}_{CM}^2}{2mA} + \frac{1}{2}mA\omega^2\vec{R}_{CM}^2$$

$$= \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + \sum_{i<j=1}^{A}\left(V_{ij} - \frac{m\omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right) + \sum_{i<j<k=1}^{A} V_{ijk} + \ldots$$

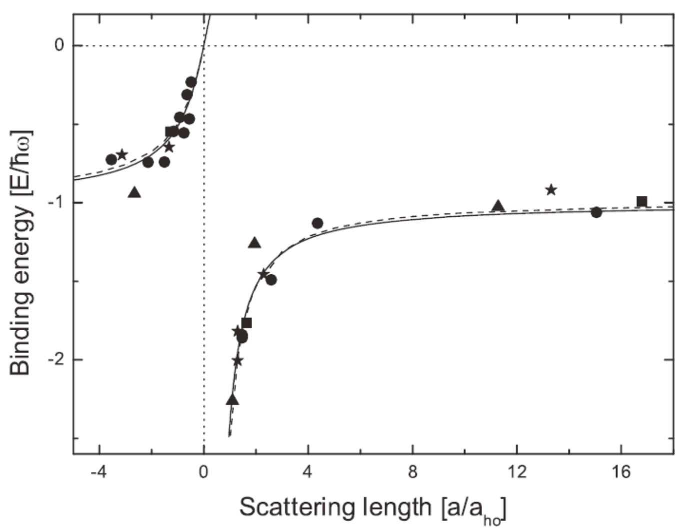
The two many-body systems formally similar

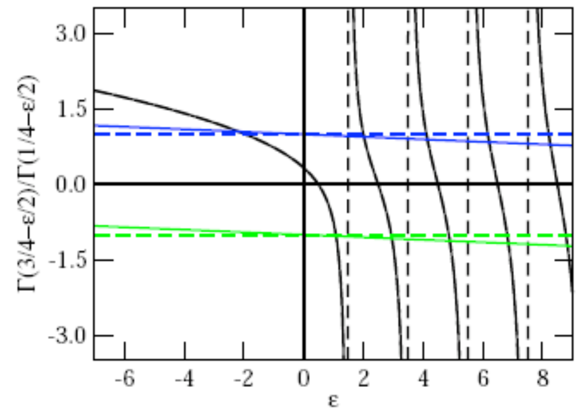
Two particles in a trap



$$h_2 = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2 + V_2(r)$$

T. Stöferle et. al., Phys. Rev. Lett. **96** (2006) 030401





$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = \frac{b}{2a_2} \qquad \left(b = \frac{1}{\sqrt{\mu\omega}}\right)$$

T. Busch et. al., 1998

Continuum limit: $\frac{b}{a_2} \to \infty \qquad \varepsilon \approx -\frac{1}{2\mu a_2}$

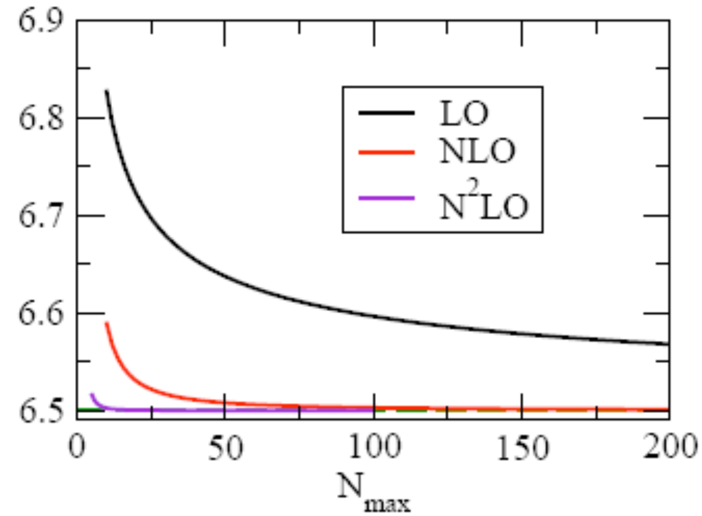
$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = -\frac{b}{2}\left(-\frac{1}{a_2} + \frac{r_0}{b^2}\varepsilon + \ldots\right)$$

Scattering ⟷ Bound states

EFT for two particles in a trap

Assumption: observables given by

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = -\frac{b}{2}\left(-\frac{1}{a_2}+\frac{r_0}{b^2}\varepsilon+\ldots\right)$$



In finite model spaces:

$$V_{LO}(\vec{p},\vec{p}') = C_0$$

$$V_{NLO}(\vec{p},\vec{p}') = C_2(p^2+p'^2)$$

$$V_{N^2LO}(\vec{p},\vec{p}') = C_4(p^2+p'^2)^2$$

$C_0 \quad C_2 \quad C_4 \quad \ldots$

Constants to be determined in each model space so that select observables are preserved

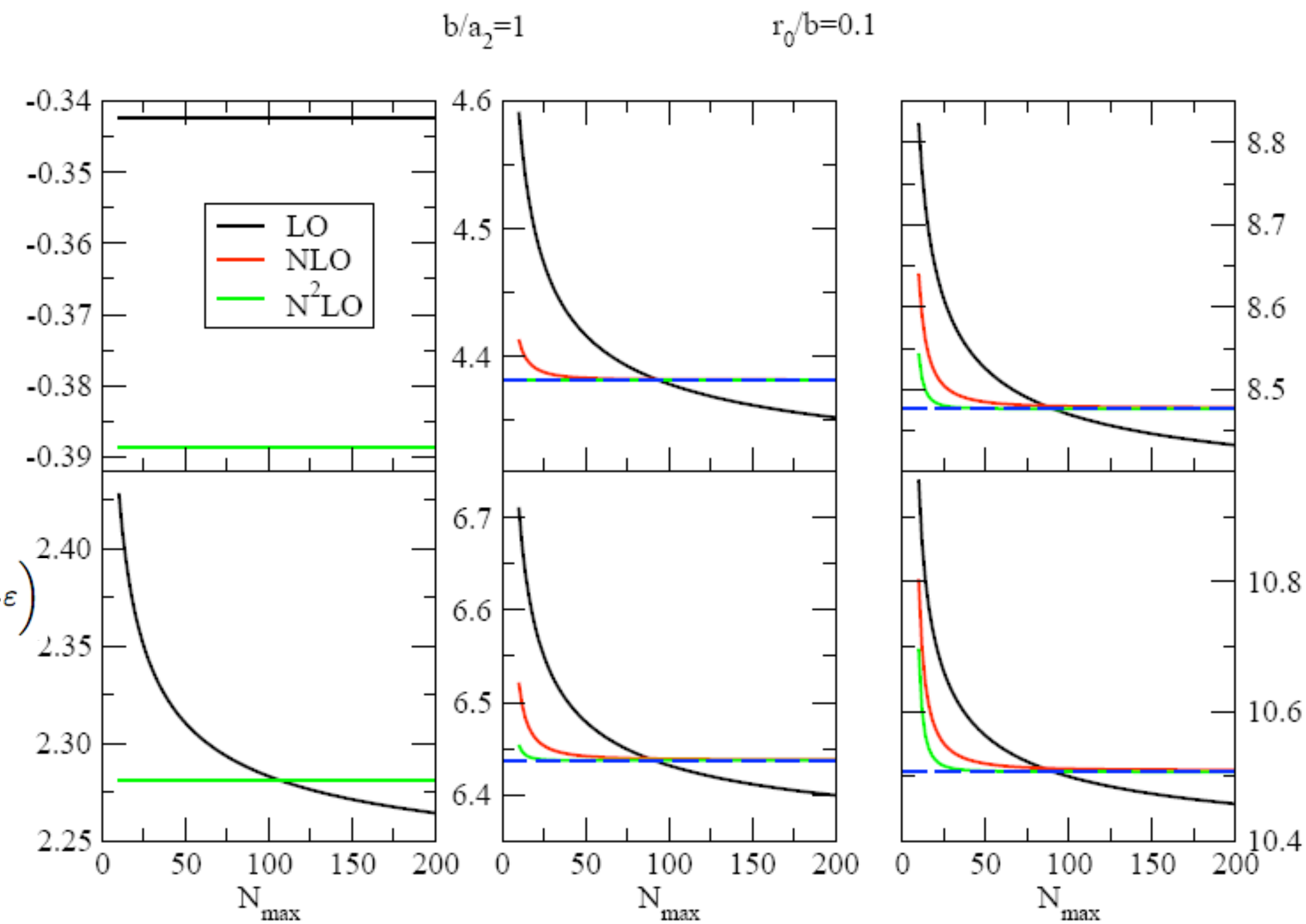
Running of the two-body spectra (finite range)

LO:

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)}=\frac{b}{2a_2}$$

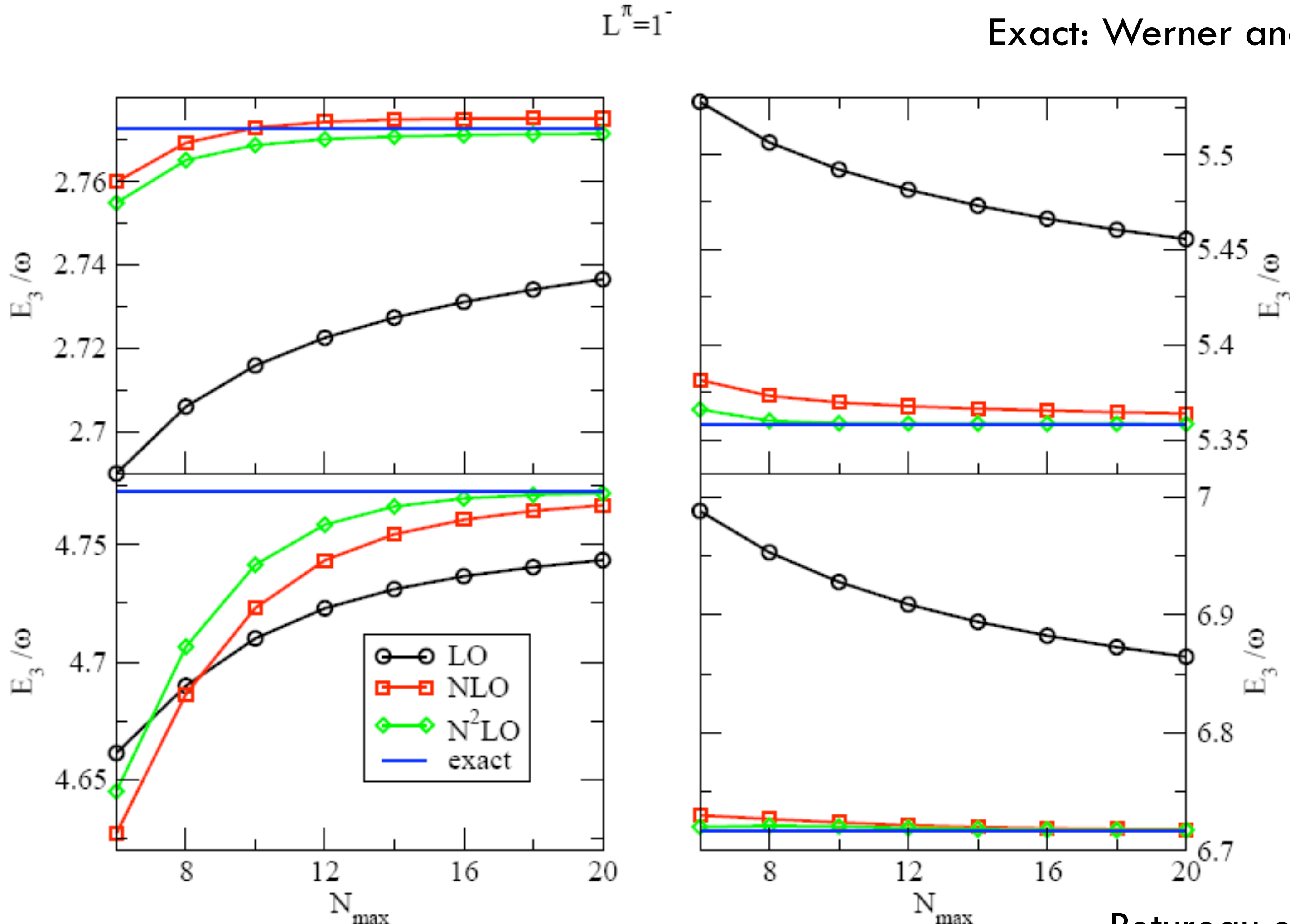
NLO:

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)}=-\frac{b}{2}\left(-\frac{1}{a_2}+\frac{r_0}{b^2}\varepsilon\right)$$



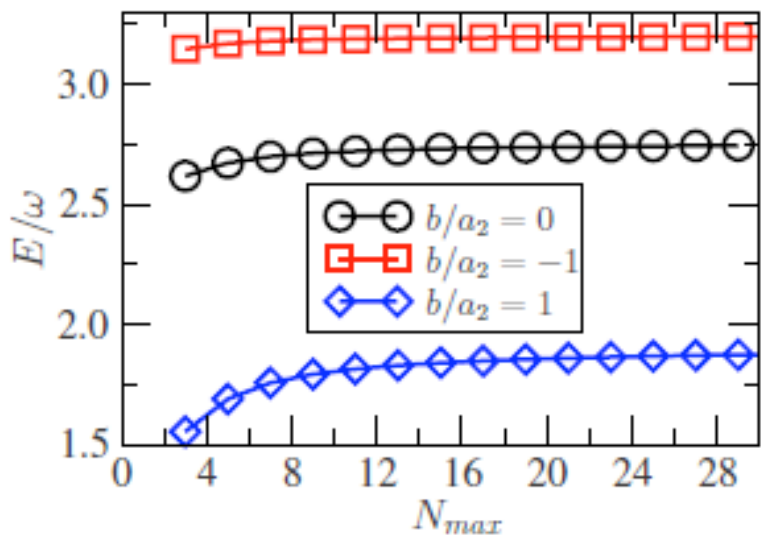
Three-body problem up to N^2LO ($b/a_2=0$)

Exact: Werner and Castin, 2007



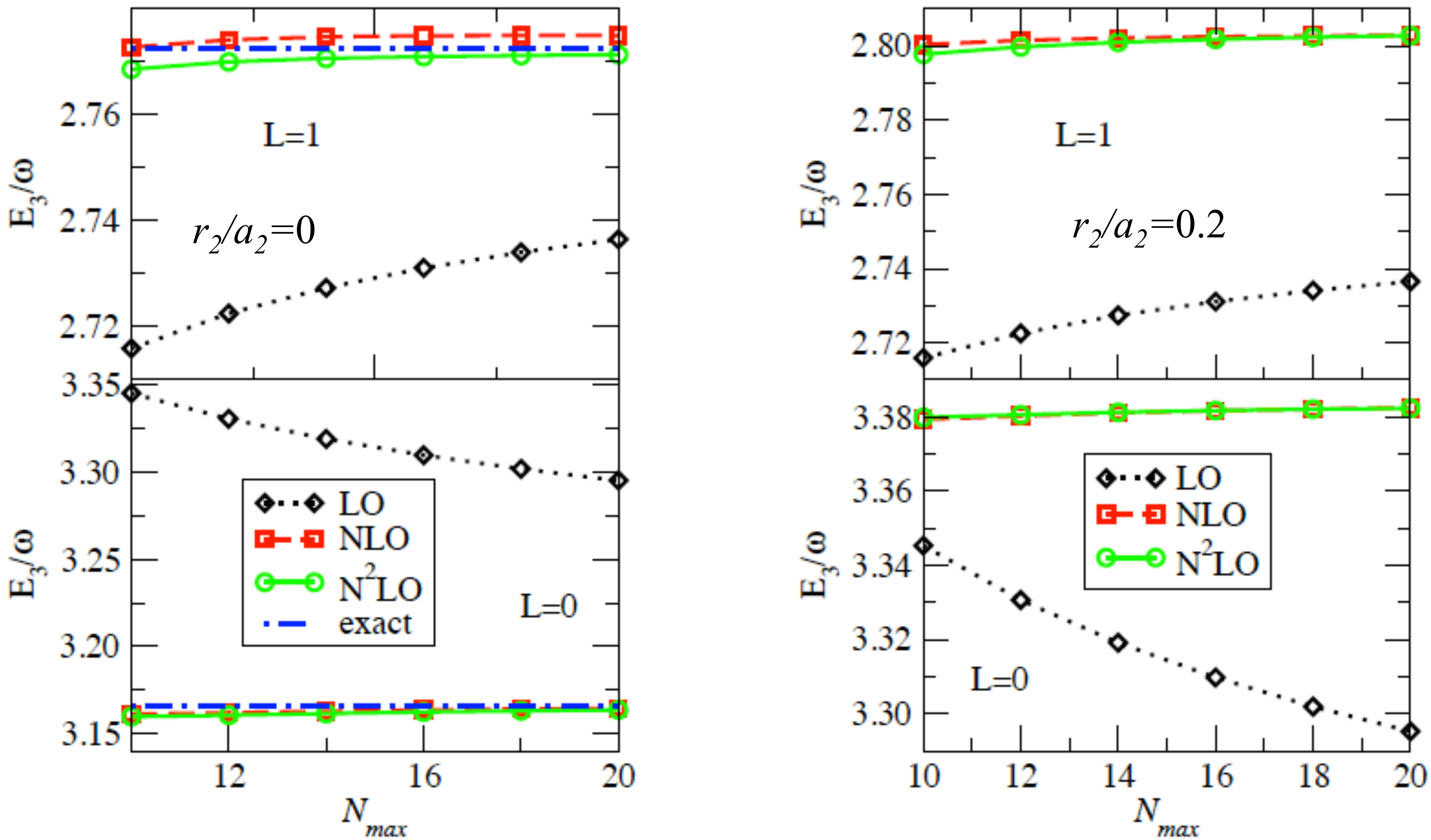
Rotureau et. al., 2010

Away from unitarity (LO only)



Stetcu et. al., 2007

Three-body at unitarity w/ physical range



Summary

NCSM:

- flexible approach to solving few- and many-body systems
- Jacobi / Slater determinant basis equivalent (energy truncation)
- local / non-local interactions
- extension to accommodate clustering effects
- excellent framework for testing new approaches to describing trapped systems

Future:

- lots of exciting developments (core SM, RGM extension, WS, etc.)
- EFT/NCSM application to few-body problems